

AN APPLICATION OF CONTINGENT CLAIM ANALYSIS TO THE PORTUGUESE BANKING SYSTEM*

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1. INTRODUCTION

Following the current financial crisis, economic agents as well as their regulators, have been alerted for the need of models able to quantify the risks taken both at individual level and as a whole. In this context, more than ever, analyzing credit risk has become critical. This article presents contingent claim analysis as one of the models that has been most prominent in assessing this type of risk. This approach is based on an extensive literature that began in the 70s (Merton, 1974, Black and Scholes, 1973, Vasicek, 1977) and has been expanded since then. Recently, the ideas of these seminal papers have been applied on assessing the risk of firms and economic sectors. Gray and Malone (2008) have a reasonably complete picture of the latest applications of this approach.

Following the presentation of the model, the methodology is applied to a set of three Portuguese banks listed in Lisbon's stock exchange. This exercise shows that contingent claim analysis is able of synthesizing the perception of the markets about the solvency state of financial institutions by providing measures for time series and cross section analysis. Although affected by the global financial crisis, our results show that the three banks under analysis have been able to overcome the instability in the markets. In February 2009, the probability of default of the whole financial system did not exceed 2 percent. The ex-ante expected loss also proved to be very low, not exceeding 20 million Euros. Nevertheless, the distance to distress fell near 80 percent as compared to the value reached in May 2007. At the balance-sheet level, our results show that the strong fall in banks' equity market value was compensated by an increase in nominal debt leading to a relative stabilization on their risk adjusted asset value between June 2007 and May 2009 at around 155 billion Euros. As a consequence, the ratio between their debt and their risk adjusted assets rose from 81 percent in May 2007 to around 96 per cent in February 2009. The second half of 2009 has seen some improvements on these indicators in face of banks' stock market valuation. However, these improvements were partially reversed after the recent climate of distrust on the ability of some countries to solve their budget problems.

This study finishes with some suggestions for future research.

This study has five sections. Section 2 presents Merton's contingent claim analysis model. Section 3 presents the results concerning the application of Merton's model to the Portuguese banking system. Section 4 discusses the limitations of the model applied and proposes lines for future research. Section 5 concludes. The reader less interested in technicalities can skip sections 2.2, 2.3 and 2.4.

* The analyses opinions and findings of this article represent the views of the authors, which are not necessarily those of Banco de Portugal or the Euro-system. All errors and omissions are the sole responsibility of the author.

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2. A BRIEF DESCRIPTION OF THE CONTINGENT CLAIM ANALYSIS MODEL

2.1. A Simple idea

The idea behind contingent claim analysis is to use Merton's (1974) model to assess the creditworthiness of a debt issuer, which we will call the "firm", but which could be a bank or an economic sector. Conceptually, this is a very simple model. Consider a firm that issues debt at a given time with a certain maturity. The question that arises is whether the firm has enough assets to honour its obligations at maturity.

In simple terms, the firm will honour its commitments if the value of its assets exceed, at maturity, its debt. If not, the firm declares bankruptcy and all assets are liquidated to creditors. The negative difference between its assets and total liabilities will then be debt holder's loss.

Deciding on whether or not to pay back debt at maturity is very similar to the process of exercising a call option. Recall that a call option gives the holder the right, but not the obligation, to buy some security (underlying asset) at a predetermined price (strike price or exercise price) at the option's maturity.¹ In this context, the option holder will buy the underlying asset if its market price at maturity exceeds the strike price; otherwise the call option is not exercised. To understand the similarities between these two decisions, note that in this simple model without frictions, a firm's equity market value equals the difference between its assets and its debt current market value. At maturity, the firm will pay its liabilities if the value of its assets exceeds its nominal debt, and will declare bankruptcy otherwise. Each firm's equity market value can therefore be seen as equal to that of a call option with its assets as the underlying and exercise price equivalent to its nominal debt.

In this discussion we have omitted a relevant fact. Since debt payments are contingent on assets value, there is no certainty on their payment. Therefore, applying a discount factor based on the risk free interest rate on the nominal debt is not enough to calculate the current market value of debt. One needs to take uncertainty into account. Once again we turn to option pricing theory, but this time to put options. Put options give the holder the right but not the obligation to sell the underlying asset at a predetermined price at the option maturity. In this case, the current value of riskless debt should be equal to the current value of that debt (that is, assuming you have the risk of not being repaid) plus a put option on the underlying asset with exercise price equal to nominal debt at maturity. This is equivalent to say that an investor should be indifferent between taking an amount of riskless debt, or take the same amount at risk but ensuring that in case of non-repayment, it can recover the difference between what it has received (the value of the assets of the firm) and what he should have received (debt repayment). This is achieved through the put option, which will be executed if the exercise price (the value of debt) is higher than the underlying asset (total assets of the firm).

In practice, knowing a firm's equity market value, the volatility of its equity returns, its nominal debt

(1) We are considering only "European" options, in which the option can be exercised only at maturity, as opposed to "American" options, in which the option can be exercised at any time. In our application, this means that a firm can go bankrupt only at debt's maturity. Although there are results showing that in the absence of dividends, an American call option should only be exercised at maturity, this is not always the case, leading to differences in prices. The relevance of this simplifying assumption is open to discussion. Nevertheless, this question tends to be of less importance when the time to maturity is sufficiently small.

and the current risk free interest rate one can use contingent claim analysis to calculate a series of risk measures, namely the distance to distress, the probability of default and the ex-ante expected loss.

2.2. Formalization of the problem

The reader less interested in formal aspects of the model can go straight to Section 3. Define A as assets market value. Similarly, define B and E as a firm's risky debt and its equity market value, respectively. As mentioned in 2.1, in the absence of financial frictions and assuming liquidity at maturity of all assets of the company,

$$A = E + B \quad (1)$$

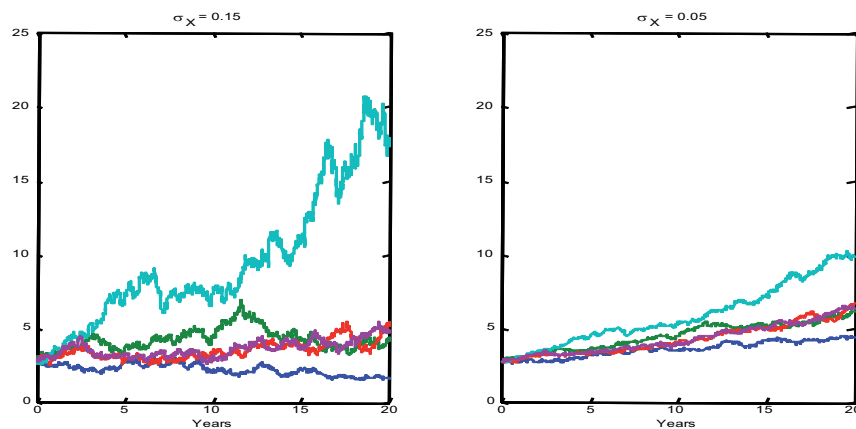
i.e. the market value of equity should equal the difference between assets and the market value of the risky debt.

Suppose that A follows a stochastic diffusion process with a deterministic trend determined by the risk free interest rate. Chart 1 shows some examples of such diffusion processes, usually known as geometric Brownian motions. Now consider that at $t = 0$, the firm issues zero coupon bonds, B_T , amounting to all its liabilities. As shown in Chart 2, a firm is bankrupted if its assets, A , are less than its nominal debt, B_T , at maturity.

It follows that, in accordance with option pricing theory, a firm's equity market value, E , equals an European call option on the underlying assets, A , with maturity $t = T$ and strike price equal to its nominal debt, B_T .

Chart 1

EXAMPLES OF DIFFERENT PATHS FOR SEVERAL STOCHASTIC PROCESSES SIMILAR TO A WITH TREND $\mu_x = 0.05$ AND DIFFERENT VALUES FOR ANNUAL VOLATILITY σ_x . ASSETS STARTING VALUE IS $EXP(1)$ FOR BOTH CASES



Source: Authors' calculations.

Applying Itô's Lemma,² imposing no arbitrage and frontier conditions equivalent to a call option, and defining $\tau = T - t$, one can obtain the following equation for E ,

$$E = A\Phi(d_1) - B_T e^{-r\tau} \Phi(d_2) \tag{2}$$

where

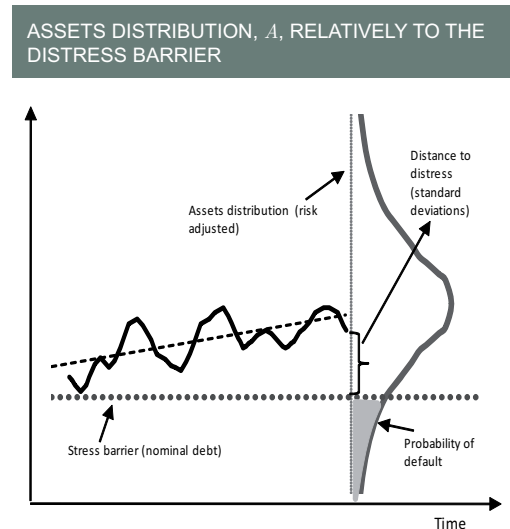
$$d_1 = \frac{\ln \frac{A}{B_T} + (r + \frac{1}{2})\tau}{\sigma_A \sqrt{\tau}} \tag{3}$$

$$d_2 = \frac{\ln \frac{A}{B_T} + (r - \frac{1}{2})\tau}{\sigma_A \sqrt{\tau}} \tag{4}$$

In the equations above, σ_A stands for the volatility of assets returns, r is the risk free interest rate, which we considered to be constant, τ is the time interval up to maturity and Φ is the cumulative normal distribution. Equation (2) has a simple interpretation. The first term evaluates assets weighted by a coefficient related to the probability of the call option being exercised; the second term weights the discounted nominal debt by a coefficient slightly smaller (there is a negative signal on Φ argument) given that losses are limited.

The formula above corresponds to a situation with no transaction costs or other financial frictions,

Chart 2



Source: Authors' calculations.

(2) See Hull (2009).

total divisibility of assets, no dividend distribution, no arbitrage opportunities, continuous transaction, constant risk free interest rate and where bankruptcy can only occur at maturity.

As explained in Merton (1974), this expression can also be obtained through Black and Scholes (1973) option pricing theory by means of the so-called put-call parity, which uses a no arbitrage argument to show that any investor should be indifferent between holding A until T or, alternatively, holding a portfolio with (i) a risk free asset with value equal to K in T ; (ii) a call option on A with maturity T and strike price K ; (iii) and a short position on a put option on A with maturity T and strike price K .³

Translating this parity for our case, where K is BT, it comes that

$$A = e^{-rT} B_T - P + E \quad (5)$$

Rearranging the above equation one can then obtain the value of the put option, P . The value of P is a very interesting measure in risk analysis, normally interpreted as a risk premium in sight of the debt's contingent character. Note that, considering that the risky debt (B) equals the discounted risk free debt ($e^{-rT} B_T$) less the risk premium (i.e. the put option value), then equation (5) is equivalent to equation (1).

2.3. Model Estimation

Equation (2) has two unknowns, A and σ_A . In order to obtain their value, one needs to impose a second condition. One possibility is to say that equity, E , also follows a geometric Brownian motion but with different parameters than A .

Applying Itô's Lemma and equating the volatility terms, we obtain

$$E\sigma_E = A\sigma_A\Phi(d_1) \quad (6)$$

where σ_E is the volatility of equity returns.

Solving the system composed by equations (2) and (6) for each moment it is possible to obtain a time series for A and σ_A . Substituting A on E in equation (1), one can then obtain B and calculate the risk measures listed ahead.

This approach allows for changes on the volatility of assets returns, σ_A , signalling that asset risk is not constant. However, if someone wants to be loyal to the theory underneath this model, assets return volatility, σ_A , must be constant. The best way to do this is to solve the following problem:

(3) For $A_T > K$, this investor receives $A_T - K$ from exercising the call option. Since the put option is not exercised, it ends with $K + A_T - K = A_T$. For $A_T < K$ the call option is not exercised but its counterparty exercises the put option. Thus, this investor has to pay K and receive A_T ending with $K - K + A_T = A_T$.

$$\min_{\{A, \sigma_A\}} \sum_t \left[A\Phi(d_1) - B_T e^{-rT} \Phi(d_2) - E \right]^2 + \sum_t \left[A\sigma_A \Phi(d_1) - E\sigma_E \right]^2 \quad (7)$$

Note that this minimization is done for several values of A but for a single value of σ_A . In the somatorium, t stands for each observation for equity, the stress barrier and the risk free interest rate. In this study, we will follow the latter approach. The first option would be preferable if there were equity options on these banks available in the market, which is not the case.

2.4. Risk Measures

The model described leads to a series of measures used in risk analysis. First of all, d_2 represents the number of standard deviations of $\ln(A_T)$ separating the firm assets from its distress barrier. For this reason this measure is commonly called distance to distress.

Another risk measure frequently used in the context of Merton's model is the probability of default, defined as

$$pd = 1 - \Phi(d_2) \quad (8)$$

Though useful, this measure has the caveat of being very sensitive to the curvature of the normal distribution.

Finally, one may also be interested in the ex-ante expected loss. In our model this value is given by the implicit put option. The rationale behind this interpretation is rather simple. Note that the value of a put option at maturity equals $\text{Max}\{0, B_T - A_T\}$. The first value corresponds to the case where the firm is not bankrupted (i.e. $A_T \geq B_T$) while the second value corresponds to the case where the company filed for bankruptcy. In this context, the current value of the put option is usually interpreted as the ex-ante expected loss or, similarly, as the risk premium expected by investors for holding risky debt. Solving (5) for P and substituting E by equation (2), it follows that

$$P = B_T e^{-r\tau} (1 - \Phi(d_2)) - A(1 - \Phi(d_1)) \quad (9)$$

This is the expression for the *ex-ante* expected loss.

3. AN APPLICATION TO THE PORTUGUESE BANKING SYSTEM

In order to assess the risk perception of market participants on the Portuguese financial system, this study uses the Merton model to estimate the risk measures listed in section 2.4 for three Portuguese banks: BCP, BES and BPI. All results are presented in aggregate terms, meaning that all values for the distress barrier and market value of equity were summed up and the model was then estimated as if they were a single bank.

These banks were chosen based on two criteria. On the one hand, they are the only significantly large Portuguese banks listed in Lisbon's stock exchange.⁴ These banks are part of the PSI-20 index, which ensures ease of access to data and reduces low turnover issues. On the other hand, these banks together account for near 44 percent of the Portuguese credit market. The dataset under analysis is composed by monthly observations ranging from January 2002 to March 2010.

Before estimating the model, some calibration procedures were needed. Therefore, for σ_E we calculated the standard deviation of the annual return on equity.⁵ Then, σ_E was obtained as a three period moving average of monthly values. The stress barrier was defined as the sum of short term debt and 50 per cent of long term debt, which is the standard in the literature. Regarding the time horizon of analysis, we have also used the standard in the literature, i.e a maturity of 1 year. Finally, the 3-month Euribor rate was used as a proxy for the short term interest rate.

After applying the model to our dataset, we obtained values for total assets, the volatility of assets returns, risky debt, and all risk measures mentioned in section 2.4. Chart 3 shows the market value of equity, E , and risky debt, B , as estimated by the model for the aggregate of the three banks. The sum of these two components correspond to total assets, A . Notice that, as a consequence of the high leverage ratios common in the banking sector, the value of B is much higher than E . Despite this structural feature of the sector, in the last two years the gap between these two aggregates increased. This movement is visible, either graphically through the narrowed area corresponding to the value of E , or through the market value of the popular debt-to-equity ratio, that by the end of 2007 went above its usual interval between 5 and 10, reaching more than 20 in February 2009. Similarly, the ratio between B and A increased significantly from around 81 per cent in May 2007 to 96 per cent in February 2009. The second half of 2009 saw these indicators falling slightly to 14 and 93 per cent, respectively. During the first quarter of 2010, these indicators have remained relatively stable.

Chart 4 decomposes the nominal debt (approximated by the distress barrier) for the aggregate of the three banks. B_T equals the sum of the risky debt (B), the time discount ($(1 - e^{-rt})B_T$) and the expected loss (estimated through the put option). The expected loss is almost negligible and difficult to see on the chart.

Figure 5 shows the distance to distress (measured in standard deviations of the volatility of assets returns) for the aggregate of the three banks. The vertical bars indicate how results for each bank are dispersed. Graphical inspection suggests the existence of a common trend driving the three banks under analysis. This trend can be divided in three distinct cycles. The period between 2002 and early 2003 is marked by some international accounting scandals (eg Enron) which led to a price collapse in the stock market and increased volatility. Next is a phase of recovery in prices, between 2003 and 2007. These years were characterized by a very favourable framework of expectations regarding the overall economic activity. However, following a sharp deterioration in the housing market in the

(4) Although Banif and Finbanco are listed in Lisbon's stock exchange, they were excluded from this study in face of their low trading volume (they are only listed in PSI-Geral). It adds that these banks represent a small part of the total credit stock in Portugal. Despite being listed in Lisbon's stock exchange, Santander and Banco Popular were not included in the analysis since their share price is determined by their parent companies.

(5) The daily equity return on special dates (eg. ex-dividend days, issuance of new shares) were eliminated in order to avoid spurious increases in volatility.

Chart 3

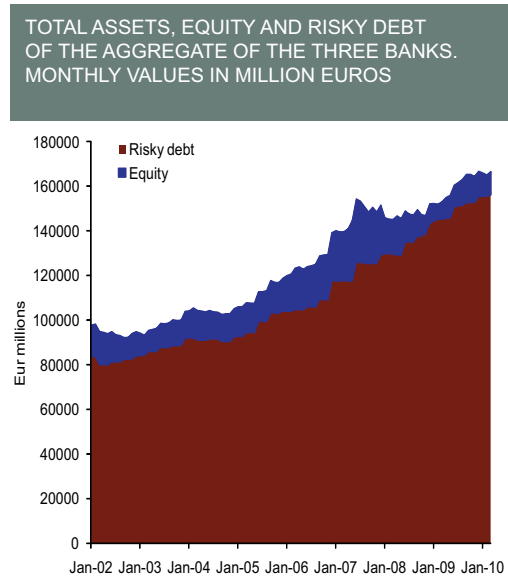
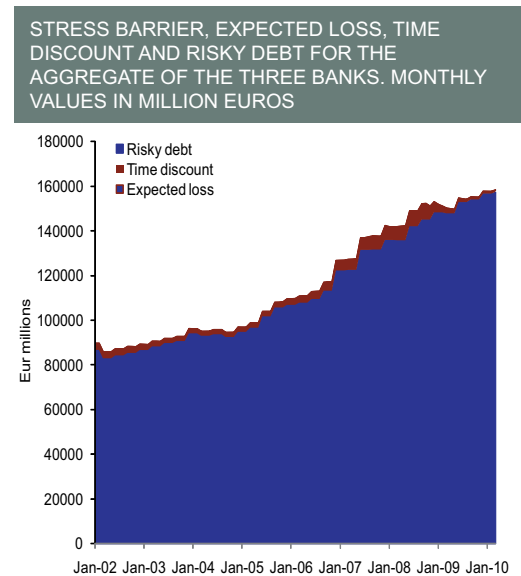


Chart 4



U.S., there was a reversal in the economic cycle from the second quarter of 2007 on. Given the high exposure of several international institutions to this market, the losses tended to be quickly transmitted across the world, generating an atmosphere of uncertainty that led to strong declines in assets value. The downward trend was reversed only in the second quarter of 2009, when stock exchanges started to climb. Though there has been some recovery, the last months of 2009 were characterized by the uprising of some doubts regarding the sustainability of public finances and the credibility of the process of fiscal consolidation in many countries, including Portugal. As a consequence, stocks devaluated and volatility increased, leading to a further decrease on the distance to distress. Nevertheless, it remained above the levels reached in February 2009.

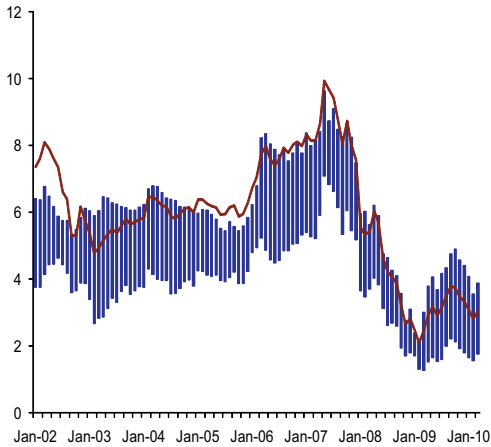
In spite of following a common trend, the wide vertical bars on Chart 5 (corresponding to the maximum and minimum of the distance to distress of the three banks) suggest the existence of different resilience levels. The period between September 2008 and April 2009 is the only exception.

As defined in 2.4, the probability of default of the aggregate of the banks (Chart 6) is exclusively determined by the distance to distress and by the cumulative normal function. Given the shape of the normal distribution, the probability of default stays near zero for most of the period under analysis. The only exception is February 2009, when the probability of default reached almost 2 per cent.

The expected value of losses showed a behavior very similar to the probability of default. This was already expected given that these measures are closely related. The highest value achieved for the period under analysis is about 20 million Euros in February 2009. However, this value is very close to zero under most of the sample period. In the current context, where governments have been willing to give their guarantees on banks' debt issues, this value gives some idea on the market perception regarding the possible cost of insuring against the bankruptcy of the aggregate of these three banks.

Chart 5

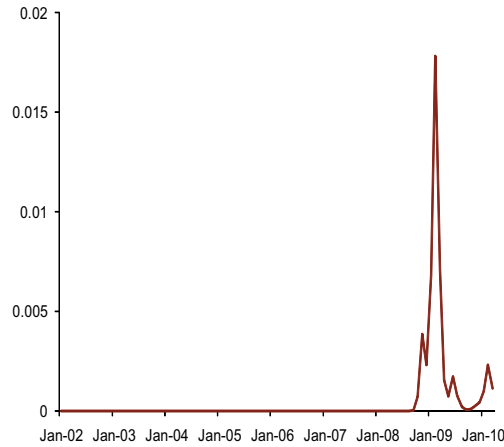
DISTANCE TO DISTRESS FOR THE AGGREGATE OF THE THREE BANKS. MONTHLY VALUES



Source: Authors' calculations.

Chart 6

PROBABILITY OF DEFAULT FOR THE AGGREGATE OF THE THREE BANKS. MONTHLY VALUES

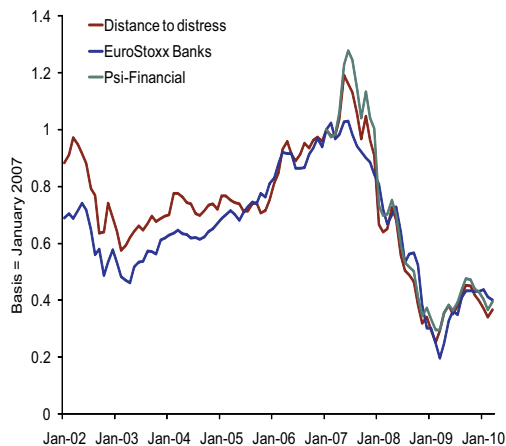


Source: Authors' calculations.

Finally, we compared the behaviour of the distance to distress of the aggregate of the three banks with the evolution of the major stock indices for the banking sector, including the DJ EuroStoxx Banks and the PSI-Financials. Chart 7 shows that in terms of growth rate, the distance to distress tends to mirror the behaviour of the PSI-Financials, which is largely determined by these three banks. Both seem to be highly correlated with the DJ Euro Stoxx Banks Index, which reflects the international nature of the crisis. This common behaviour suggests that these indicators, in spite of having a forward looking nature, particularly when compared with measures based on the history of entities under analysis (e.g. percentage of non-performing loans or financial ratios based on accounting statements), suffer from the same advantages and disadvantages of all other indicators that are based on capital markets. In other words, they do not provide significant additional information.

Chart 7

DISTANCE TO DISTRESS, PSI-FINANCIALS AND EUROSTOXX BANKS. MONTHLY VALUES



Source: Euronext and Authors' calculations.

4. FUTURE RESEARCH

As any methodology, contingent claim analysis has some advantages and disadvantages. The latter can be divided in two groups. First, this approach suffers from the limitations inherent in most pricing methods. Regarding the Black-Scholes pricing model, it ignores liquidity risk, transaction costs, the non-continuous nature of trading, the existence of dividends, the possibility of asset returns not following a normal distribution; or, finally, the fact that put options may be exercised before maturity. Furthermore, these methodologies, by relying exclusively on market prices, are vulnerable when the latter are not conscious of certain elements of systemic risk. Similarly, the dependence on short-term changes in “market sentiment” may lead these indicators to send false alarms on the overall financial situation of many institutions. Secondly, there are a number of model assumptions that are either violated or ignored in practice. For instance, the empirical volatility of equity returns σ_E can be calculated in different ways. Similarly, the stress barrier and the time to maturity are determined in an ad-hoc fashion.

Despite these limitations, our analysis suggests that contingent claim analysis can be a valuable tool in risk analysis. Thus, this study points to three lines for future research. First, it would be interesting to extend the analysis to more banks. This study focuses only in three banks, which represent roughly 44 percent of the Portuguese credit market. In order to have a better picture of the Portuguese banking system, it would be therefore interesting to include some other banks that are not listed in Lisbon’s stock exchange. In these cases, however, some proxies would then be needed. One hypothesis would be to use the spreads on credit default swaps to estimate the spread implied in Merton’s model. The high correlation between these two measures suggests that good results may be achieved in this way. The model can then be estimated reversely, i.e. we first obtain the value for the risky debt. Assets are then calculated using equation (1).

A second line for research consists in focusing the analysis on the entire economic system and not on the banks individually. Indeed, the model applied so far does not take into account transmission effects within the financial system, between the financial system and all other sectors, and between the national economy and the rest of the world. In this context, it would be important to study how the implicit guarantees usually assumed to be given to the banking system have evolved during the financial crisis.

Finally, it would be appropriate to incorporate Merton’s methodology in a VAR model, as in Gray and Walsh (2008). However, these authors estimate equations only for the leading Chilean banks. Thus, the impulse response functions derived do not account for feedback effects between the banks and are not suitable for the study of systemic crises. One alternative would be to consider a VAR model for the whole economy.

5. CONCLUSION

Generally speaking, this study aims, firstly, to evaluate the potential of contingent claim analysis as a tool to quantify risks in the economy and, secondly, to apply the model to the Portuguese economy. Regarding its potentials and limitations, this study found that contingent claim analysis is easy to implement and able of producing sound results on the perception of markets participants about the financial situation of the institutions under review. Although it has not been explored in its broader perspective, contingent claim analysis has shown to be especially useful while analyzing changes in corporate debt value, risk transmission mechanisms and the dynamics created by the existence of implied guarantees among sectors. The latter are particularly difficult to incorporate in most models since they are highly non-linear.

Among all risk indicators, the distance to distress has shown to be the most interesting since it allows comparability among institutions and countries and is less dependent on considerations regarding neutrality towards risk. Nevertheless, the ability of contingent claim analysis to produce early warning indicators should be assessed carefully to the extent that its forward looking nature is limited by the perception of market participants. Thus, these indicators are unable to detect a “bubble” that has not busted; similarly, they can signal negative perspectives in the market that are not based on economic fundamentals. Regarding other risk measures, the probability of default proved to be very dependent on the curvature of the normal distribution. In turn, the expected loss is highly dependent on the probability of default.

As regards the application of contingent claim analysis to the Portuguese economy, our results indicated that, in aggregate, the three banks under analysis ended the first quarter of 2010 with total risk adjusted assets slightly above 165 billion Euros, i.e. 70 percent above January 2002 figures. Concerning leveraging, risky debt stood at 94 percent of total assets, i.e. 156 billion. As a consequence, the ratio between the risky debt and the market value of equity stood at near 15. Nevertheless, these values are below those obtained in February 2009, when they were 96 and 22 percent, respectively. Regarding other risk measures, the distance to distress showed a great degree of variability, with a maximum slightly above 10 standard deviations in May 2007 and a minimum of 2.1 standard deviations in February 2009. Except for February 2009, when it reached 1.8 percent, the probability of default has remained very low for most of the sample. In late March 2010, the distance to distress was at 2.1 standard deviations, corresponding to a probability of distress of 0.11 percent. The expected loss showed a trend in line with the probability of default not exceeding 20 million Euros in February 2009.

Finally, we have indicated some lines for future research, such as increasing the number of banks under analysis, focusing the analysis on the entire economic system instead of banks individually and incorporating contingent claim analysing in a VAR for the whole economy.

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