ESTIMATING PROBABILITIES OF DEFAULT UNDER MACROECONOMIC SCENARIOS*

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INTRODUCTION

The assessment of the creditworthiness of current and prospective counterparts in loan operations is pivotal in the banking business, in particular the estimation of the propensity of non-financial corporations to fail their financial obligations in due time. This measure can be assessed in probabilistic terms over some predefined future horizon, conditional on the observable characteristics of the debtor. The implementation of reliable statistical methods to measure and forecast these probabilities implies the consideration of an observation period. In other words, the identification of default events implies monitoring each debtor over time and the identification of a transition from non-default to a default state. The obtained statistics are useful for credit institutions in many ways, spanning from the credit approval process (for instance to implement cut-off points to screen credit applications), to the establishment of risk-sensitive pricing, the decision on whether collateral is required or not, the estimation of provisioning requirements or impairment losses, and the assessment of capital adequacy.

Furthermore, the incorporation of macroeconomic factors in models estimating probabilities of default allows practitioners to stress-test the financial standing of financial institutions and the financial system at large. These stress-tests imply the design of macroeconomic scenarios resulting from large unfavourable shocks, with low probability but still plausible and internally consistent, and have been widely used in the context of the Financial Stability Assessment Program (FSAP) carried out by the International Monetary Fund.

Comprehensive databases with information from companies' financial statements are generally not available, making it difficult to develop complete and reliable models of default. One of the main problems in readily available databases is their bias towards large corporations and the low coverage of small- and medium-sized enterprises.[1] Another relevant issue is the fact that companies in relatively good financial conditions are more likely to transmit their accounting information to institutions collecting data than the others, which may be reluctant to reveal their weak financial standing in order to preserve their ability to get credit and carry on with their regular activity. Both situations have consequences in terms of the accuracy of model estimations, due to the scarcity of relevant observations for an appropriate model adjustment.

This limitation does not apply when information from universal credit registers is used, because all non-financial corporations with debts outstanding *vis-à-vis* the financial system are covered. This is the case of the *Central de Responsabilidades de Crédito (CRC*), the credit register managed by Banco de Portugal, which contains information on any natural or legal person with debts *vis-à-vis* each resi-

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(1) Banco de Portugal is responsible for one of those databases, the *Central de Balanços (CB)*, which includes detailed information on domestic non-financial firms (see Banco de Portugal, 2005).

dent credit institution (named CRC participant).[2] The main goal of the CRC is to provide financial institutions useful information for the loan approval process, such as the debt position and the existence of delinquencies of firms with respect to the set of all participants in the CRC.

Among non-financial corporations filed in the CRC, only a very small fraction is present in other readily available statistical or commercial databases with, for instance, accounting information. For that reason, data from financial statements available in the *Central de Balanços* was not used in this study, even though other information available for statistical purposes on other databases covering the universe of all existing and extinct corporations was used to characterise each company.

The aim of this paper is to present an approach to estimate the probability of default of non-financial corporations using data from the CRC, complemented with data about the debtor's economic sector from another statistical database covering the universe of non-financial corporations. The statistical model also incorporates macroeconomic variables for the period under consideration. In this way, common cyclical factors are taken into account in the probability of default estimation, in a way that allows for the simulation of the impact of adverse shocks on the expected losses of credit institutions.

Two distinct macroeconomic scenarios are used. The first scenario is called "baseline", according to which the Portuguese economy evolves in line with the December 2005 Banco de Portugal's projections undertaken in the context of the Eurosystem's macroeconomic projection exercise. In the second scenario, the "stress scenario", an abrupt correction of global economic imbalances is assumed in early 2006.[3] In both cases probabilities of default are calculated for loans characterised by economic sector and company size in terms of credit. This procedure allows for the computation of aggregate probabilities of default for any structure of portfolio characterised in terms of these two dimensions, even though the figures reported in this paper correspond only to the financial system's aggregate portfolio of loans to non-financial corporations.

Data used and the estimated model allow for the conclusion that smaller enterprises are more prone to default. For instance, non-financial corporations with total debt in CRC ranging from 1 thousand euros to fifty thousand euros have a probability of default around four times larger than those with aggregate debt of over 1 million euros.

Nonetheless, average aggregate probability of default is relatively low, given the relatively high concentration of the system's loan portfolio in large-sized enterprises, which have a low probability of default.

METHODOLOGY

Data

The goal of the model is to estimate the probability of non-repayment in due course of a financial obligation by a particular debtor. In this sense, the unit under observation is the loan (and not the enterprise), implying that at least three different types of variables must be considered. The first set of variables characterises each loan; for instance, a variable denoting the existence of loans in default of the same debtor (other than the loan under observation) is used. The second set characterises the non-financial corporation, which is the counterpart to the financial system in the loan under observation; examples include the total debt of the corporation and its economic sector. The third set of vari-

(2) See Banco de Portugal (2003). Notification is mandatory for exposures above 50 euros.

(3) The macroeconomic scenarios are mere illustrations and do not represent Banco de Portugal's forecast of the evolution of the Portuguese economy during the period under study. For a more detailed description of the macroeconomic unbalances referred to in this study and possible consequences of their correction at a global level, see "Chapter 2 – *Macroeconomic Environment*" of this Report.

ables measures factors that impact all non-financial corporations under study, such as the level of interest rates and a business cycle indicator. This establishes a link between the macroeconomic context and the financial standing of each non-financial corporation.

Two different data sources are used. The first is the CRC, with monthly data about all the non-financial corporations with loans *vis-à-vis* credit institutions participating in the CRC. All loans granted with amount outstanding higher than 50 euros are reported to Banco de Portugal. This information is then centralised and made available on-line to financial institutions. The total amount of loans outstanding is broken down by type of loan[4] and the status of the debtor in the loan (for instance if there is a sole debtor of the loan or more debtors liable for it). The data sample covers the period January 1995 to December 2004 and comprises around 6 million records in 1995 (concerning around 336 thousand enterprises) and more than 11 million of records in 2004 (corresponding to 474 thousand enterprises).

The second data source is a registry of all existing and extinct resident legal persons and includes, along with other information, the economic sector of the enterprise, its legal status and statutory capital. Each company is assigned an economic sector based on the *Classificação das Actividades Económicas Rev.2 (CAE)*, in a total of 15 sectors: agriculture and fishing; wholesale and retail trade; construction; domestic activities; education; real estate development; mining and quarrying; financial services; health services; manufacturing; public services; social services; tourism and transports.

Stratified sampling by economic sector and class of total debt was used to reduce the amount of information involved and keep the dataset manageable in computational terms. The categories used in sampling are depicted in Table 1; companies with total debt under 50 euros were neglected.

All large companies (i.e. companies classified in size class 4), 70 percent of those classified in class 3 and 5 percent of companies in classes 1 and 2 were selected. Company size, proxied by total debt, was used in the econometric procedure in order to ensure that the sampling filter applied would not bias the results.

Table1

CREDIT DIMENSION CLASSES
All figures in euros

Class	1	2	3	4
Definition	$d < 10^3$	$10^3 \leq d < 5 \times 10^4$	$5 \times 10^4 \leq d < 10^6$	$d \geq 10^6$

Table 2

NUMBER OF LOANS AND FIRMS IN THE SAMPLE

	Credit dimension class				
	1	2	3	4	Total
Loans	9021	57922	102330	28031	197304
Firms	5632	18671	13207	1941	39451

(4) Types include, among others, commercial liabilities (type 1), finance at discount (type 2), other short-run liabilities (type 3), and medium- and long-run liabilities (type 4). Impaired credits are registered as defaulted credit (type 7) and credit under litigation (type 8), as well as write-offs (type 9).

As mentioned above, one is interested in analysing the likelihood of a default *vis-à-vis* a credit institution. Taking the information available in CRC, a loan was defined as a bilateral credit relationship between a debtor and a credit institution, meaning that multiple loans of a single firm in a given credit institution count as a single loan. Table 2 presents the number of loans and firms used in the estimation. Data were transformed into quarterly data. The final sample includes around 6 million observations.

One striking feature from the data in Table 2 is the lower number of relationships with different credit institutions exhibited by smaller enterprises. For instance, enterprises with total debt ranging from 1 thousand and 50 thousand euros have, on average 3.1 loans, which compares with 14.4 loans for those enterprises with total debt in excess of 1 million euros.

Statistical Model

The default event was defined as the occurrence of three consecutive months of a positive overdue amount in a loan after a month without any overdue amount. The loan relationship is then removed from the sample and will be considered again only after no overdue amount is booked in the CRC. This means that a default event is counted only the first time it occurs, but the same bilateral loan relationship can display more than one default event in the sample, provided that a period of default is followed by a period in which the full amount outstanding in a loan relationship is in a regular status (i.e., no overdue amounts are booked in the CRC).

Ex-post empirical default rates for a given period can be obtained by contrasting the fraction of new loans in default with the total loans outstanding at the beginning of the period. Figure 1 presents the ex-post annual default rates observed in the period 1995-2004, in each company size class (proxied by total debt booked in CRC). In general, default rates for larger companies are lower than average: the default rate of companies with debt in excess of 1 million euros in 2004, corresponding to class 4, hardly exceeded 1 percent, which compares to 4.3 percent for enterprises with total debt ranging from 1 thousand to 50 thousand euros (class 3). In the time series dimension, in general, default rates decline from 1995 to 2000 and then increase, in line with the stylised facts about the macroeconomic business cycle.[5]

A binary response model with a *probit* specification was used, with default in a credit relationship as the event of interest. A latent variable y was defined as a function of a set of regressors represented by the vector x, as follows:

$$y = x\beta + \varepsilon, \tag{1}$$

where ε is an independent and identically distributed normal error. The observed variable is defined as:

$$y^* = \begin{cases} 1 \text{ if } y \geq 0 \\ 0 \text{ if } y < 0 \end{cases}$$

with 1 the event of interest (default). The conditional probability of default is going to be given by

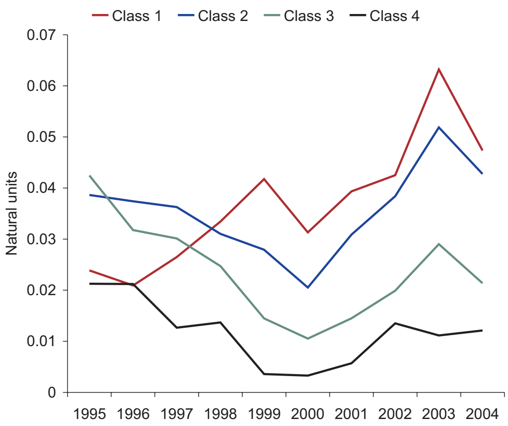
$$\Pr\left(y^* = 1 | x\right) = \Phi\left(x\beta\right),$$

where Φ is the normal cumulative distribution function.

(5) The evolution of default rates for class 1 firms (with total liabilities between 50 and 1000 euros) does not observe this rule, but their weight in the entire Portuguese portfolio is very small (less than 1 percent of total).

Figure 1

YEARLY DEFAULT RATES



The statistical model used in this approach incorporates variables (included in vector *x*) at the loan and debtor level, as well as variables that account for the macroeconomic environment. At the loan level, we used two dichotomous variables. The first is an indicator of default in other loans of the same debtor. The second is a dummy for default in more than 50 percent of the debtor's loans (not counting the loan under observation) during the current quarter. For example, if the debtor has, beyond the loan under observation, two additional loans and defaulted in both of them for two months of the current quarter, then the dummy is 1. If the debtor defaulted in both loans for only one month of the current quarter, then the dummy is 0.

At the firm level we use categorical variables for the activity sector and the credit dimension, according to the classes defined in Table 1. These variables control for the firm's dimension and the specificities of their activity sector. We also use a categorical variable for the number of loans of the firm top coded to 5 loans. It would be useful to include, at the firm level, balance sheet data, but this was not done for the reasons discussed above.

At the macroeconomic level we use the unemployment rate, the short-term interest rate and the GDP deviation from trend. We also included lags of these variables up to 4 quarters. Note that the data cover an entire business cycle, allowing us to take into account periods of economic expansion and contraction.

To accommodate the possible impact of the euro introduction in 1999, we included an indicator equal to 1 prior to 1999, and 0 thereafter. This variable was also interacted with the interest rate, because, in a high inflation context, interest rates were typically high up to 1999.

Also included in the model were interactions between the activity sector and the macroeconomic variables, as well as the credit dimension class and the macroeconomic variables, since different economic sectors and firms of different dimension might respond differently to macroeconomic changes.

Finally, we also included seasonal dummies.

Results

We estimated the model using maximum likelihood, so as to obtain β. The model fit can be ascertained using Figure 2 for two credit dimension classes (class 2, between one thousand an fifty thousand euros; class 4, above 1 million euros). The figure indicates a reasonable fit in both classes.[6] While no exhaustive out-of-sample tests were performed and the model is experimental, measures of the predictive power of the model suggest an adequate performance.[7] Let us evaluate that fit in a more rigorous way.

Although using only data on the credit of firms and the activity sector, the model allows a credit institution with access to the debtor's characteristics – especially the detailed repayment status *vis-à-vis* other credit institutions – to calculate with relative reliability the probability of a default event. For example, if in a given quarter a credit institution knows that a particular debtor defaulted in all its other loans in the previous quarter, the probability of default estimate will be high. If a credit request is at stake, this might entail a refusal; if there is already a credit relationship, the credit institution might provision the loan.

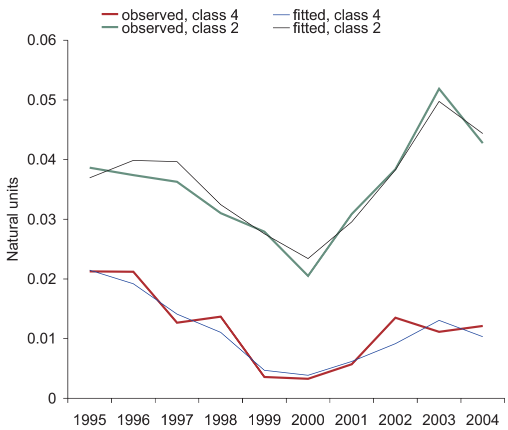
Since the model is a binary response one, we can define a cut-off value a for the expected value of variable y (given by equation (1) with $\varepsilon = 0$), which we shall designate by Ey, in such a way that when Ey is higher than a the loan is "bad" (which might entail the refusal of a credit request or the provisioning of an existing loan); otherwise, the loan is "good". This procedure defines, for each a, a classification for each loan in the sample. Notice that we know if a loan is "good" or "bad" based on what actually happened to each loan of the sample. Naturally, a perfect model would classify correctly all loans. This does not happen in practice.

As a matter of fact, a very negative value for a means that a lot of loans that are good are going to be classified by the model as bad (this is the so-called type I error, and we can think of it as a "false alarm"). As we increase a, more and more good loans are going to be classified as such by the model,

Figure 2

OBSERVED AND FITTED DEFAULT RATES

Annualised mean values



(6) The adjustment quality for other classes is comparable to that observed in this figure.

(7) Partial results suggest that the out-of-sample performance of the model is comparable to the one we have just described. On the other hand, the time horizon is limited to one economic cycle, and this renders it inadequate to simulate the model outside the sample period, that is, estimate the model using data until, say, 2000, and then analyse its performance in the ensuing years.

but some loans that actually are bad are going to be classified as good (this is the type II error, or a “wolf in a sheep’s clothing”). When *a* is really high, all loans are classified as good – and so all bad loans will be classified as good by the model. We call specificity to the fraction of good loans which are classified as good by the model, and sensitivity to the fraction of bad loans that are classified as bad by the model. When *a* is minus infinity, all loans are classified as bad by the loan; therefore, sensitivity is 1 and specificity is 0. When *a* is plus infinity, sensitivity is 0 and specificity is 1. By varying *a* we obtain a set of values for these two measures.

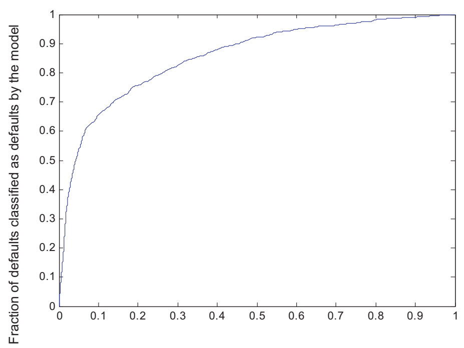
A possible representation of the model’s performance is its ROC curve, which we can see in Figure 3.[8] In the horizontal axis we represent 1 minus specificity, that is, the percentage of good loans classified as bad by the model. In the vertical axis we represent sensitivity, that is, the fraction of bad loans classified as bad by the model. A given point (x, y) in the curve answers the following question: What percentage x of good loans will be rejected by the model in order to classify a percentage y of bad loans as bad? For instance, we can see in the figure that the model will reject 25 percent of the good loans if we want to make sure that about 80 percent of the bad loans are rejected.

As previously said, in a perfect model we would have to reject 0 percent of the good loans in order to reject 100 percent of the bad ones. This means that the perfect model’s ROC curve would be the line segment between points (0,1) and (1,1). On the other hand, a model taking the good/bad decision randomly will have an ROC curve given by the line segment between points (0,0) and (1,1). In other words, the model would reject 25 percent of the good loans to reject 25 percent of the bad, or reject 50 percent of the good loans in order to reject 50 percent of the bad, and so forth. This fact suggests that an adequate measure for the performance of the model is the area below the ROC curve, which we designate by *A*.

This area is 1 for a perfect model and 0.5 for the random choice model. In our case, the value of *A* is 0.86, which suggests a robust performance. Additionally, this value is high even when we consider defaults in only one quarter. For example, in the 4th quarter of 1995 *A* is 0.84, while in the 4th quarter of 2004 its value is 0.86.

Figure 3

ROC CURVE



(8) ROC means Receiver Operating Characteristic. See the Internet site http://www.anaesthetist.com/mnm/stats/roc/ for an intuitive introduction to this topic. Stein (2002) presents a summary of the validation methods of credit default models, including the ROC curve.

In possession of a model with a reasonable ability to discriminate loans in terms of defaults, we can use it to predict the evolution of defaults under given macroeconomic conditions and assuming a particular behaviour for the credit portfolio. This exercise is carried out in the next section.

CREDIT DEFAULT: A STRESS TEST EXAMPLE

In this section we use the model to predict the default probability in each loan under a baseline and a stress macroeconomic scenario. Next, we use that information to calculate the evolution of the default probability for each credit dimension class and activity sector. Finally, we use the Portuguese credit portfolio to non-financial firms to calculate the evolution of the average default rate under each macroeconomic scenario.

Macroeconomic Scenarios

The baseline scenario is in line with Banco de Portugal's projections in the context of the Eurosystem's macroeconomic projection exercise, carried out in December 2005. Table 3 presents a summary of that scenario. We see that GDP growth increases by a small amount and the interest rates remain stable.

The stress scenario builds on the following premises: (i) a sudden decline in the demand for dollar-denominated assets; (ii) a simultaneous and significant appreciation of the euro *vis-à-vis* the dollar; (iii) a marked increase of the long-run dollar interest rates; (iv) global stock markets fall substantially in 2006; (v) the recession then affecting the United States spreads to the world economy. In this scenario, there is in 2006 a strong deceleration of growth in Portugal, and a recession occurs in 2006 and 2007. The euro short-run interest rate decreases, in line with the evolution of economic activity in the euro area.

Default Probabilities

Table 4 presents the model estimates for the default probability by credit dimension class. These estimates are obtained using a credit portfolio to non-financial firms equal to that of end-2004, but with the macroeconomic regressors replaced according to the scenarios. The hypothesis that the credit portfo-

Table 3

MACROECONOMIC SCENARIOS
All values in percentage

	2005	2006	2007
Baseline scenario			
Short-run interest rate	2.2	2.2	2.3
GDP growth rate	0.3	0.8	1.0
Stress scenario			
Short-run interest rate	2.2	1.0	0.8
GDP growth rate	0.3	-1.0	-0.7

Table 4

ESTIMATES OF DEFAULT PROBABILITIES
Yearly values

	Credit dimension class			
Year	1	2	3	4
	Baseline scenario			
2005	6.3%	5.1%	2.6%	1.2%
2006	6.3%	5.1%	2.6%	1.3%
2007	5.8%	4.7%	2.3%	1.1%
	Stress scenario			
2005	6.3%	5.1%	2.6%	1.2%
2006	7.9%	6.4%	3.3%	1.7%
2007	9.0%	7.3%	3.9%	2.0%

Table 5

ESTIMATES OF THE AVERAGE DEFAULT PROBABILITIES OF THE ENTIRE PORTFOLIO

Yearly values

	2005	2006	2007
Baseline scenario	2.20%	2.30%	2.00%
Stress scenario	2.20%	2.90%	3.40%

lio does not change significantly over the simulation horizon is naturally subject to criticism. On the one hand, if defaults increase, bad firms cease to present new defaults because they shut down. Everything else constant, this would improve the average quality of the portfolio. On the other hand, continuing adverse macroeconomic conditions have the opposite sign in the average quality of the portfolio due to the accumulated damage imposed on firms. Finally, the characteristics of entering firms are unknown. In the absence of a model describing these effects, we adopted a static portfolio during the simulation period. In fact, an exploratory analysis of the main characteristics of the credit portfolio suggests that they remain fairly stable, namely in terms of the distribution by credit dimension class and activity sector.

To calculate the average default rate of the entire portfolio of credit to non-financial firms (that is, the probability which multiplied by total exposure is equal to the expected value of defaulted credit), we resort to the estimated default probability of each loan (given by $\Phi(x\beta)$). We perform that calculation for the entire portfolio weighting the default probability of each loan by its amount.

Table 5 presents the average default probability of the credit portfolio for the simulation horizon. For instance, in 2005 the model suggests of value of 2.2 percent, which means that 2.2 percent of total exposure is expected to become overdue. Naturally, the effective loss will be lower since some overdue amounts will be partially or totally repaid, and, even if the debtor shuts down, some credits might be recovered by lenders.

We can observe that, for both scenarios, the weighted values are larger than the default probability of credit class 4 and lower than that of class 3 (see Table 4). This stems from the fact that these two classes account for the largest chunk of the portfolio of credit to non-financial firms (representing around 80 percent of total). The default rate will stay relatively small, since the credit portfolio is concentrated in large firms, which typically present low default rates.

CONCLUSION

This study presents a possible approach to the problem of determining the propensity of a given firm to default a loan. The statistical model incorporates variables at the loan, firm and general macroeconomic level. This work departs from other models by emphasizing the credit information contained in the Banco de Portugal's *Central de Riscos de Crédito* database, as well as its relationship with macroeconomic variables. Some adjustment measures suggest an adequate performance of the model in terms of its capacity to discern "good" from "bad" loans.

As an illustration, we present estimates of a loan's default probability by credit dimension class, and the average default probability of the Portuguese portfolio of credit to non-financial firms, given two different macroeconomic scenarios. Under a scenario featuring relatively moderate economic growth and stable interest rates, the probability of default remains approximately constant. Under a scenario with a strong economic deceleration, the probability of default tends to increase, possibly from 2% to 3.4% at the end of the simulation horizon, that is, after 2 years. The average default rate would still be relatively low given the characteristics of the credit portfolio, which is concentrated in large firms with a typical low default rate.

Naturally, these results should be interpreted cautiously. The econometric model omits important characteristics of firms which may be important in explaining the default event. Their inclusion would better describe the firm and perhaps improve the model's performance. Some hypotheses needed to ensure that our estimates are unbiased might be violated (although some adjustment measures suggest that this might not be serious).[9] In this type of model an over-fitting problem might also occur. This consists of using variables that, by construction or some peculiarity of the way data are generated, induce a good performance but do not account for the real behaviour of firms; the discerning capacity of the model is thus lower than the usual performance measures suggest (Dwyer, 2005). Finally, the results of the simulations under the baseline and the stress macroeconomic scenarios are central values and could be subject to high variability.

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(9) One of the potential problems of this approach is the so-called "neglected heterogeneity", which consists of the non-inclusion in x of independent regressors that are relevant for the default event. However, it can be shown that this problem does not affect, for instance, measure A, since neglected heterogeneity tends to attenuate the magnitude of the coefficients, but not the sorting of the propensity to default (Wooldridge, 2002).