ESTIMATES OF EXPECTED LOSSES IN CREDIT PORTFOLIOS - AN APPLICATION OF SURVIVAL ANALYSIS TO FIRMS WITH DEFAULTED CREDIT*

António Antunes and **Nuno Ribeiro****

1. Introduction

The management and the analysis of risk in loan portfolios have been gradually assuming a more prospective nature, appealing to the notion of expected loss in a given future time horizon. This development results from the technologic progress in the storage, processing and transmission of large volumes of information, in conjunction with advances in financial economics. These factors have been contributing to setting up new paradigms in credit institutions practices, in particular in those with more geographically diversified loan portfolios.

The dissemination of more sophisticated credit risk management techniques catalysed changes in rules concerning the accounting of losses in loan portfolios and in the regulatory approach to banks' provisioning and capital adequacy.

The adoption of the International Accounting Standards and their application in the European Union (Regulation 1606/02) imply adjustments to the value of the loan portfolio, so-called "impairment adjustments". The requirements for considering a loan (or a pool of loans) as impaired involve the observation of objective events deemed as equivalent to default or quasi-default. Even though making appeal to the idea of expected loss, the notion of impairment is not totally coincident with the ex-ante estimate of expected loss of a loan portfolio. The New Basel Accord (or Basel II) introduces the notion of probability of default, which, combined with that of recovery rate, allows for estimates of average losses in a portfolio. Further, it allows for the estimation of extreme values of the distribution of losses, which are underlying the concept of own funds requirements.

In addition, in those countries in which there is a regulatory distinction between specific and general provisions, the former tend to reflect expected losses in loan portfolios and the latter serve as an addition cushion for unexpected events, being eligible, at least partially, as regulatory own funds.

This work approaches the statistical characterisation of the recovery and extinction process of companies with observed episodes of default in their liabilities. Asymptotic probabilities of recovery are analysed, alongside its short run dynamics, both contingent on the companies observed characteristics.

The remaining of this article is organised as follows. In section 2, the specific provisioning regime prevailing in Portugal is summarised, with a focus in its philosophy. In section 3, survival analysis and its specificities when applied to the problem under consideration are described. In section 4, we describe the databases used, as well as the main hypotheses

and procedures taken in the estimates under the methodology described in section 3. In section 5, some of the results are presented in the way most comparable to the specific provisioning concepts, and recovery rates are calculated, the latter with applicability in both impairment assessment and calibration of models of capital requirements. In the final section, the results are summarised, the limitations of this approach are pointed out, and future research avenues are highlighted[1].

2. The specific provisioning regime in Portugal

Specific provisioning levels, as understood in the regime prevailing in Portugal, are the minimum amount necessary to cover the expected losses associated to already observed credit delinquencies (or other specific situations of counterparties with high probability of going into default, for instance different credits concerning the same debtor or companies from an economic sector subject to particularly adverse shocks). The rationale underlying general provisioning is that of a complement to specific provisions, in such a way that the aggregate of both types of provisions should reflect the expected loss in the loan portfolio, both on loans with past due instalments and regular loans.

This work aims at studying companies for which a delinquency event is observed at a given point in time in order to obtain statistics for the expected loss in a loan, given that the debtor (with a set of characteristics) has defaulted.

The specific provisioning rules in Portugal embody severity criteria for defaults, such as the time elapsed since the loan became past due, the ratio between the past due portion of each loan and the gross amount in debt (and cumulatively, the ratio between the past due portion of all loans of the same debtor and the total gross debt of that same debtor), the existence or not of guarantees, and the initial maturity. The information available for undertaking this work uses only the first two of the abovementioned criteria, given the non-availability of information on guarantees and the precise maturity of loans in the *Central de Responsabilidades de Crédito* (*CRC*, the main source of data in this work, which is the compulsory credit register managed by the Banco de Portugal). For the general case of an unsecured loan and with a maturity of no more than 5 years, the minimum provisioning requirement during the first quarter after the identification of delinquency consists of 1 percent of the amount past due, rising to 25 percent in the second quarter, to 50 percent in the third quarter, 75 percent in the fourth quarter and full coverage after one year. In addition, having passed 6 months after the identification of delinquency, or if the ratio between past due instalments and the gross amount in debt in the same loan is at least 25 percent ($r \geq 25\%$), the previous provisioning percentages coverage apply to the full amount of the credit in debt - and not only to the past due portion (see Table 1).

TABLE 1

MINIMUM SPECIFIC PROVISIONING REQUIREMENTS FOR LOANS WITHOUT COLLATERAL AND WITH MATURITIES OF LESS THAN 5 YEARS

	1st quarter	2nd quarter	3rd quarter	4th quarter	5th quarter and afterwards
Share of the overdue credit	1%	25%	50%	75%	100%
Share of the total debt (se $r < 25\%$)	$r \leq 0.25\%$	$r \leq 6.25\%$	50%	75%	100%
Share of the total debt (se $r \geq 25\%$)	1%	25%	50%	75%	100%

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1. See Antunes (2005) for a complete description of the model and its main conclusions.

3. The approach used

Let us define default ratio as the quotient between the total past due amount and the gross credit liability of a firm. In line with Portugal's specific provisioning practices, we shall use this quantity to characterize the severity of default. Let us define a set of "states" for a firm based on its default ratio. Each firm will be in a given state if its default ratio r belongs to the interval associated with that state, with upper and lower bounds defined below. This set of states will be used to operationalise the approach that we shall use (see Chart 1).

CHART 1
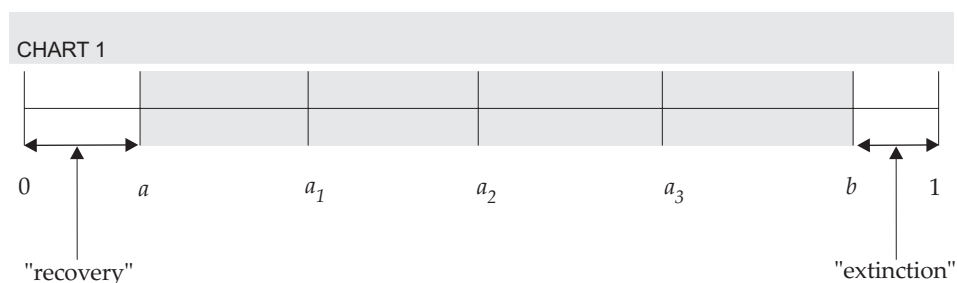
Let us assume that a firm with default ratio smaller than a given threshold a will not be classified as being under actual default. The value for a must be sufficiently low so that such classification would be premature in the case the firm's default ratio has always been negligible, or, in the case the firm has already had a default ratio higher than a, we can consider that the firm has "recovered". Inversely, above b firms are deemed "extinct" and their liabilities will be considered totally unrecoverable. The "recovery" and "extinction" states are absorbing, that is, once a firm moves from an intermediate situation (with ratio between a and b) to each of the absorbing states, any future observation of the same firm in an intermediate situation will be viewed as a new firm. In addition, when the default ratio is between a and b, we can define intermediate default states. These states are defined as intervals of the default ratio, $]a,a_1[$, $[a_1,a_2[$, $[a_2,a_3[$ and $[a_3,b[$, for example. Each of these intervals will correspond to a different range of the default severity: the state defined by interval $[a_3,b[$ will be associated with a higher default severity than the state corresponding to interval $]a,a_1[$.

The intermediate states are, as their designation suggests, non-absorbing, since a firm can move successively from one to the other, and also to the absorbing states. These transitions can be characterized using survival analysis (see below). That will allow us to estimate the probability of transition to each of the absorbing states, both in asymptotic terms and over time. The strategy used for the estimation of recovery and extinction probabilities comprises two different phases: in the first, we econometrically estimate the characteristics of the transitions between states; in the second, we use the estimated models to calculate paths of fictitious firms, so that we can predict their asymptotic behaviour and their evolution over time.

As for the econometric phase, the estimation of transition probabilities between states, conditional on the firm's characteristics and on the time the firm has stayed in the current state, uses survival analysis with competing risks. This approach allows us to estimate the probability per unit of time that a firm transits to a different state, given a set of contemporaneous observable characteristics summarized in vector x, and the time elapsed t since it entered the current state. Introducing some notation, let us denote this conditional probability density by $h_{ik}(t|x)$, where i is the origin state and $k \neq i$ the destination state. In the academic literature, this function is known has the hazard function. Given the state space,

with two absorbing states and, as seen previously, four non-absorbing states, we shall estimate twenty models from the data. Once estimated, these functions will be used to answer to two fundamental questions:

- when will the transition to another state occur?

- once a transition occurs, to which state will the firm go?

We can answer to the first question by calculating the probability density function associated with the moment when the transition occurs. This is obtained as follows. The probability density of a transition from state *i* to any of the other states, conditional on the firm remaining in the current state up until *t*, is the sum of the hazard functions for each of the possible transitions, that is,

$$h_i(t|x) = \sum_{k \neq i} h_{ik}(t|x).$$

The functional form we used for the hazard, known in the literature as the Weibull function, was

$$h_{ik}(t|x) = e^{x\beta} \alpha \lambda^{\alpha} t^{\alpha - 1},$$

where β is a vector of coefficients associated to the firm's characteristics, and α and λ are parameters. All these quantities will have to be estimated for each transition.

Finally, the probability that a firm is still in state *i* after *t* units of time since it entered that state implies the integration of the hazard function in that interval, and is given by:

$$F_i(t|x) = \exp\left\{-\int_0^t h_i(u|x)\,du\right\}.$$

Note that when *t* goes to infinity, if function $h_i(t|x)$ does not converge to 0 too fast, then the probability of remaining in state *i* converges to 0. Note also that if *t* goes to 0, this probability is 1. Finally, the probability density that the transition occurs at time *t*, $f_i(t|x)$, will be given by the product between the probability that the firm remains in state *i* at moment *t*, and the probability density of transition to any other state, given that it has survived up to that moment in that state. Mathematically,

$$f_i(t|x) = h_i(t|x)F_i(t|x).$$

The answer to the second previously stated question is obtained using the hazard functions for the different transitions. The probability that a transition, when it occurs, is to state *k*, is given by expression

$$\pi_{ik}(x) = \int_0^{+\infty} h_{ik}(u|x)F_i(u|x)\,du.$$

The interpretation of this expression is simple: we integrate the probability density that the transition occurs from state *i* to state *k* on all possible durations.

With $f_i(t|x)$ and $\pi_{ik}(x)$ it is possible to simulate the firms dynamics in the state space given their characteristics *x*. These calculations involve the creation of a large number of fictitious firms, which are then observed as time evolves, with the transitions between

states governed by the stochastic processes summarised by $f_i(t|x)$ and $\pi_{ik}(x)$. Since there are two absorbing states - recovery and extinction - the path of a specific firm will end in either of them. We can then calculate the extinction probabilities conditional on the firms' characteristics. Lancaster (1992) presents in a detailed fashion this type of analysis.

This model may be used to characterize the behaviour of firms over the recovery/extinction process. For instance, conditional on the firm's current state (that is, on its default ratio) and its characteristics, we can observe if the recovery, if it occurs, is slow or fast when compared to extinction.

4. Databases and hypotheses

The firms under observation in this exercise are all those firms registered as credit beneficiaries (with the exception of self-employed entrepreneurs) in the *Central de Responsabilidades de Crédito (CRC)* with at least one non-performing loan episode between January 1995 and December 2001. The credit history of the firms prior to the first communication of a bad loan episode is unknown, as is the period of time between two non consecutive communications. When constructing the default ranges referred to in the previous section, we assumed that firms with total liabilities below 100 euros are not classified as having defaulted (as the episode could just be a spurious situation or have resulted from a litigation process initiated by the debtor). Thus, for firms with total liabilities above or equal to 100 euros, four states were defined in terms of the default ratio: the first between 10 and 25 percent (state 1); the second between 25 and 50 percent (state 2); the third between 50 and 75 percent (state 3); and the fourth between 75 and 90 percent (state 4). These limits are motivated by the provisioning rules describe above, and also by robustness and precision tests carried over and presented in Antunes (2005). To the left and right of the lower and upper limit of 10 and 90 percent, respectively, we find the absorbing states "recovery" and "extinction". Thus, we shall consider that a firm recovered if its default ratio falls below 10 percent, and has become extinct if it rises above 90 percent. While arbitrary, these values permit us to achieve three goals: an acceptable precision for the recovery and extinction probabilities; the use of conservative values for the loss given default measure; and the operationalisation of the recovery and extinction concepts. The latter aspect is important in that the legal registration of the bankruptcy of firms is incomplete, has low refresh frequency, and lags the actual bankruptcy event by a sizable lapse of time.

In trying to characterize the firms under study, that is, to quantify vector *x* defined in section 3, we used economic data for firms available for statistical purposes, such as the activity sector, sales, the number of employees and the headquarters' location.

5. Some results compatible with the notion of specific provision in Portugal

5.1 EXAMPLE FOR A TYPICAL FIRM

We can observe in Graph 2 the asymptotic extinction probabilities for firms in different non absorbing states and for different survival times in the current state. The values used in the graph are conditional on the state and survival time in current state, but not on the trajectory after the observation. We used a firm with 20 employees, yearly sales of 1 million euros, located in the Lisbon area, and total credit liabilities between 10 and 100 thousand euros[2].

CHART 2

PROBABILITY OF EXTINCTION WITH PERMANENCE IN THE SAME STATE

Probability of extinction
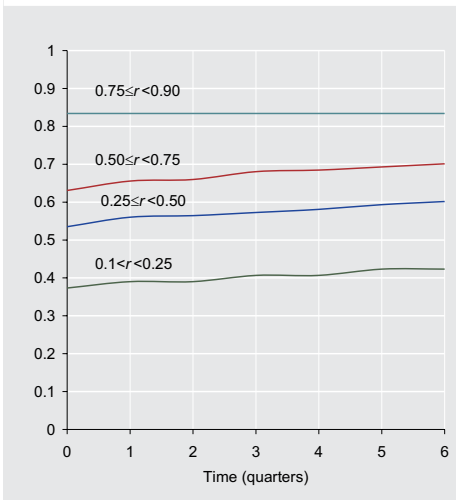
First we can see that the extinction probability varies positively with the default ratio. This means that we can indeed use that measure for gauging the extinction probability. The second relevant observation is that the extinction probability grows slightly as a function of the survival time in the current state. This implies that the entry state largely defines the perspectives of the firm in terms of recovery/extinction. In terms of provisioning rules, this result implies that provisions should be made shortly after the default episode is detected, because the extinction probability will not vary much if the firm stays for a long time in the same state. In Chart 2 we can see that if a typical firm (as described above) without any previous default episode shows up with a default ratio between 10 and 25 percent, the probability that eventually the firm will recover is around 62 percent, and the probability that it will become extinct is 38 percent.

On the other hand, if we know that the firm has had for 5 quarters a default ratio between 10 and 25 percent, the probability of extinction increases to 42 percent, while that of re-covery falls to 58 percent. The increase in extinction probability due to a prolonged stay in a given state exists, but is small.

For the sake of provisioning rules and loss monitoring, it is relevant to know how long it will take until all uncertainty associated with the final outcome of the process dissipates. Table 2 shows the fraction of firms that have recovered as a function of the time elapsed since default, independently of the transitions that might have occurred until recovery. The table also shows the same information for firms that will eventually disappear.

Let us look at an example. Suppose that a typical firm (as described above) shows up at a given moment with a default ratio between 25 and 50 percent (state 2). It is estimated that within one year the probability that it has recovered is 17 percent, the probability that it has become extinct is 12 percent, and the probability that neither of these outcomes has happened is 71 percent[3]. The percentage of dissipated uncertainty will thus be 29 percent.

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2. The figures for the number of employees and total sales are close to their population averages: 18.3 and 1.28 million euro, respectively.
3. Notice that this is the probability of being in any of the non absorbing states.

TABLE 2

| Year | State 1 | | | | State 2 | | | | State 3 | | | | State 4 | | | |
| | $0.1 < r < 0.25$ | | | | $0.25 \leq r < 0.50$ | | | | $0.50 \leq r < 0.75$ | | | | $0.75 \leq r < 0.90$ | | | |
	Recov.	Ext.	Total	LGD	Recov.	Ext.	Total	LGD	Recov.	Ext.	Total	LGD	Recov.	Ext.	Total	LGD
1	0.32	0.06	0.38	0.09	0.17	0.12	0.29	0.14	0.13	0.24	0.36	0.25	0.06	0.63	0.70	0.64
2	0.49	0.17	0.66	0.22	0.33	0.31	0.64	0.34	0.26	0.45	0.71	0.47	0.11	0.73	0.84	0.75
3	0.57	0.28	0.84	0.33	0.40	0.43	0.83	0.47	0.32	0.54	0.86	0.57	0.14	0.78	0.93	0.80
4	0.60	0.33	0.92	0.38	0.44	0.48	0.92	0.52	0.35	0.59	0.93	0.62	0.16	0.81	0.96	0.82
5	0.61	0.35	0.96	0.41	0.45	0.51	0.96	0.55	0.36	0.61	0.97	0.64	0.16	0.82	0.98	0.83

CHART 3A
DEFAULT RATIO
BETWEEN 10% AND 25%

Fraction of firms
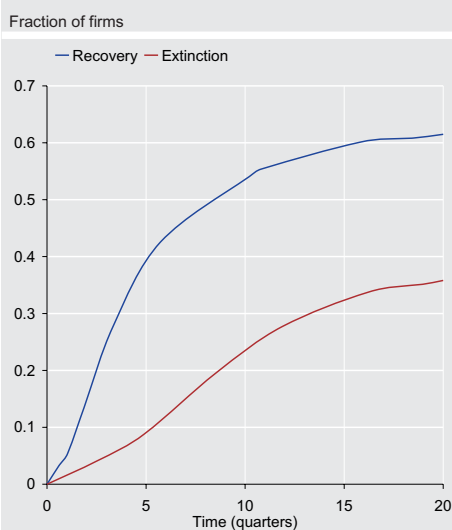
CHART 3B
DEFAULT RATIO
BETWEEN 25% AND 50%

Fraction of firms
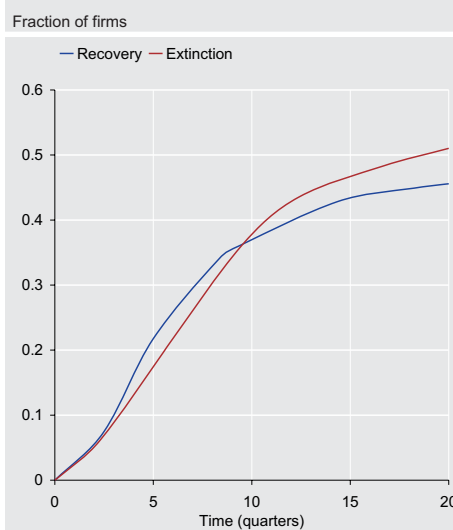
CHART 3C
DEFAULT RATIO
BETWEEN 50% AND 75%
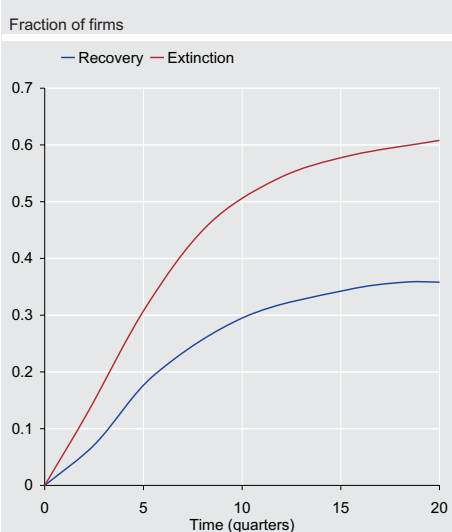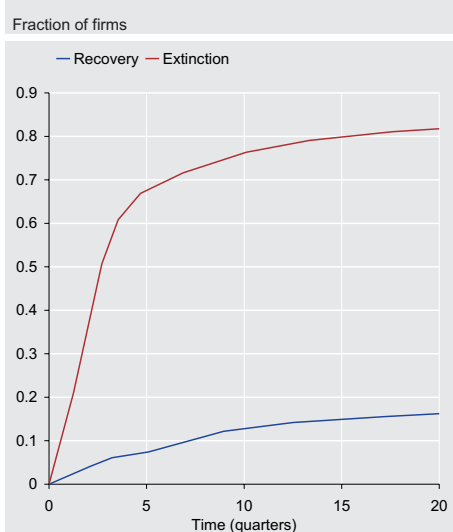
Fraction of firms

CHART 3D
DEFAULT RATIO
BETWEEN 75% AND 90%

Fraction of firms

Let us use the previous example to calculate a measure of loss given default (LGD) within 1 year. This is a measure of the expected value of lost credits in a given horizon once the default episode is identified. We obtain a value not higher than 14 percent. This is calcu-

lated by multiplying the extinction probability (0.12) by the fraction lost (which we shall take as being 1), plus the probability of recovery (0.17) times the lower limit of the least severe state (0.1)[4].

Let us continue with the example of a firm in state 2. The probability that the firm has recovered after 3 years is 40 percent; this figure is 43 percent for extinct firms. This corresponds to 83 percent of resolved uncertainty. For a 3-year horizon, the loss given default measure is estimated in 47 percent, closer to the asymptotic value of 58 percent[5]. These results suggest that the quantification of loss expectations should be performed over a horizon of at least 3 years.

The recovery and extinction pattern is not uniform over time. In Graph 3 we can observe the expected time profile of the recovery and extinction process at the time of default, conditional to the default severity. The information in Graph 3 is similar to the one reported in Table 2; the graph displays the values in continuous form. For moderate default severity (*r* between 0.25 and 0.5), the extinction of the firm will occur slower than recovery. Within one year after default, about 36 percent of firms that will ever recover have already done so[6]; for the firms that will become extinct, that figure is 23 percent. This pattern is reversed for less favourable states. For example, if at the time of default, the default ratio lies between 75 and 90, after one year the firm is extinct in 63 percent of the cases (on a total of 83 percent of firms that will eventually become extinct); this value is approximately 6 percent in the case of recovery (on a total of 17 percent of firms that will recover).

This analysis can be extended to accommodate any firm type and other firm or loan characteristics, namely the existence of collateral - if that information is available. For that purpose, it is enough to calculate the recovery and extinction probabilities for a given set of combinations of characteristics, and then obtain the values for actual firms using interpolation. Given the characteristics of the firms population, this will allow in turn the calculation of loss given default measures directed to each particular situation, and also the aggregation for a given credit portfolio. For example, we estimate in a first approach (using the baseline hazard for each transition and taking into account the distribution of firms in the *CRC* in terms of default severity) an average LGD of 46 percent. This is close to the value indicated in the New Capital Accord for uncollateralized loans, which is 50 percent. However, this value should be used cautiously, as only the average firm characteristics were taken into account, not those of each firm in the database.

5.2 SENSITIVITY OF THE RESULTS TO THE FIRMS' DIMENSION

An interesting question is to know the impact of a firm's size on its recovery and extinction probabilities. Let us suppose a firm identical to the typical firm defined above but 10 times as large. This implies that the firm will have 200 employees, yearly sales of 10 million euros, and total credit between 100 thousand and 1 million euros. We show the evolution of this firm's expected recovery rate in Graph 4, as well as that of the typical firm (denoted by "ref."), for two of the initial states.

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4. The hypotheses that the lost fraction is 1 in the first parcel, and the lower limit of the least severe state in the second parcel, imply that the estimate is an upper bound for LGD.
5. Since there are only two absorbing states, the asymptotic values of the recovery and extinction probabilities should be equal to those with time equal to zero in Chart 2. In this example, the figures are 53 and 47 per cent for extinction and recovery, respectively yielding an asymptotic LGD of 58 per cent.
6. This figure comes from the fact that, in state 2, 17 per cent of firms recover after one year (see Table 2), in a total of 47 per cent of firms that recover (Chart 2).
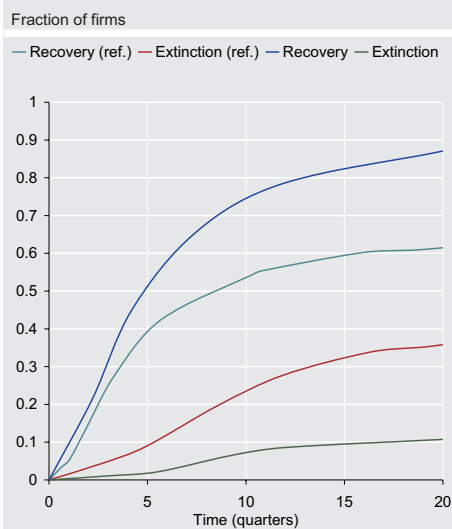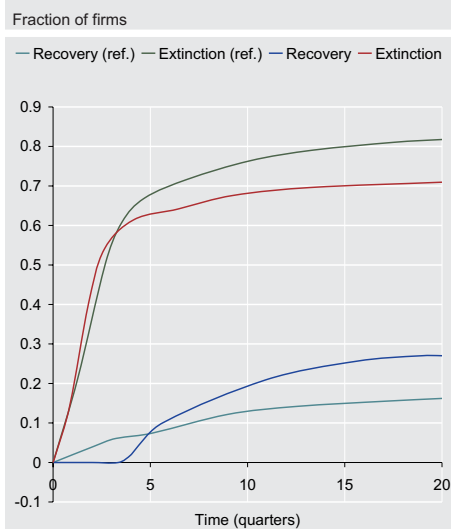
CHART 4A
DEFAULT RATIO
BETWEEN 10% AND 25%

Fraction of firms

— Recovery (ref.) — Extinction (ref.) — Recovery — Extinction

CHART 4B
DEFAULT RATIO
BETWEEN 75% AND 90%

Fraction of firms

— Recovery (ref.) — Extinction (ref.) — Recovery — Extinction

The most evident fact is that the recovery probability grows with the firm's dimension, irrespective of the original state; this finding corroborates existing results in the literature about Portuguese firms (Cabral and Mata, 2003). For instance, the probability of recovery of the larger firm is, for r between 0.1 and 0.25, 88 percent, against 62 percent of the typical firm. If r lies between 0.75 and 0.9, the corresponding figures are 29 and 17 percent, respectively.

In Graph 4 we can also verify that the recovery pattern of firms varies substantially with dimension. For instance, for the larger firm and r between 0.75 and 0.9, we see that only after one year the probability that a recovery is observed becomes positive. This contrasts with the behaviour of the smaller firm: about 36 percent of the firms that are going to recover have done so by the end of the first year.

6. Conclusions

This article presents a methodology for the analysis of the dynamics of non financial firms' recovery/extinction processes, based on survival analysis. The methodology is applied to firms with credit default episodes in Portugal and is explicitly used for obtaining estimates of the recovery rates, both over time and asymptotically.

The results obtained have some interesting applications in terms of the expected values of a creditor's loss once a default episode has been registered. First, the time that a given amount is defaulted seems to be less relevant than the severity of that default, as measured by the ratio of past due credit to total credit liabilities. Second, for moderate default severity, the uncertainty associated to the definitive loss of credit is resolved at a slower pace than the uncertainty associated to recovery. This pattern is reversed in the case of higher default severity. We estimate that the time horizon for calculating the loss given default should be at least 3 years. Finally, we show that the results are sensitive to the firm's size, both in terms of the recovery probabilities and the pattern of recovery over time. The probability of recovery increases with the firm's size irrespective of the original state. Additionally, larger firms in the most severe state only show recovery, in the case it occurs, one year after the default episode has occurred. This is in contrast with the behaviour of smaller firms: after one year, a large fraction of recovering firms will have done so.

The application has limitations in terms of the availability of information of the loans' residual maturity, and the existence of collateral. If such information were available, we would also need to know loan by loan values. Another limitation relates to the fact that the exercise was performed with individual liabilities aggregated on all institutions reporting to *CRC*, as opposed to institution by institution information that could be aggregated later. While not compromising the usefulness of the estimates, this makes the results less comparable to the provisioning rules in Portugal, which apply at the individual institution level.

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