

AN EVALUATION OF GOVERNMENT EXPENDITURES' EXTERNALITIES*

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ABSTRACT

We estimate, using US data, a general equilibrium model with two salient features. First, we allow government consumption to directly affect the welfare of agents. Second, we allow public capital to shift the productivity of private factors. On the one hand, private and government consumption are estimated to be substitute goods, that is, all else equal an increase in government consumption leads agents to partially substitute private consumption with the newly available government consumption. Because of this, labor supply reacts little to a government consumption shock and, hence, the estimated output multiplier is much lower than in models with separabilities, peaking - on impact - at 0.33. On the other hand, non-defense public investment enhances mildly or negligibly, depending on the specification, the productivity of private factors. In the specifications where non-defense public investment is found to be productive, a non-defense investment shock generates the following estimated responses (after several quarters): a positive reaction for private consumption, the value of firms, private investment and real wages. Unlike in models with unproductive government investment, the estimated output multiplier builds up over time, reaching roughly 1.5 after six years.

1. Introduction

Assessing the mechanisms through which government spending affects the private sector has occupied a large portion of the macroeconomic literature. The current debate on the design of fiscal stimuli and consolidations across industrialized countries has renewed the interest on such mechanisms. This article contributes to the debate by conducting a positive analysis on the effects of “externalities” produced by government spending. By externality we mean that government consumption may influence the welfare of agents or, more precisely, it can affect households’ marginal utility of consumption and, therefore, the level of consumption itself. This occurs if some items of government consumption act as substitutes or complements for private consumption. For example, public health care can reduce the need for private health services, or, public education services can reduce the need for private tutors and schools but, on the other hand, increase the demand for textbooks or personal computers. These potential relations between private consumption and different items of public spending make government consumption, on the aggregate, to be either substitute or complement for private consumption. Thus, omitting a priori the channel of substitutability/complementarity can produce biased estimates of the response of private consumption to a government consumption shock. Importantly, even the reaction of labor supply, and

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hence of output, to such a shock depends on the relation in preferences between private and public consumption. Also, government investment can create externalities for the private sector. More precisely, public capital can act as a shifter of the productivity of private factors, such that a shock to public investment has the potential to affect the dynamics of several variables such as private investment, consumption as well as output. For example, an efficient system of public highways built in place of an old route can enhance the productivity of private firms operating in that area (e.g. by fostering within-country trade). Again, omitting this role for government investment can bias estimates of its effects.

Although the potential effects of government consumption on private consumption through preferences have long been considered (e.g. Barro, 1981), the standard hypothesis of the bulk of the macroeconomic literature (e.g. Baxter and King, 1993 or Smets and Wouters, 2007) is that private and government consumption are separable in preferences, *i.e.*, the marginal utility of consumption is independent of the level of public consumption. Moreover, public capital is often omitted since it is assumed to be “unproductive” (exceptions are Baxter and King, 1993 or Pappa, 2009, or Leeper *et al.*, 2010). Within these models, the so called *negative wealth effect* is the main driver of spending shocks. If government spending increases, then the present discounted value of taxes to be paid by households also increases and so permanent income is lower. The well known consequence of this effect is the negative correlation between public spending and private consumption conditional on government spending shocks.¹ The negative wealth effect impacts positively on labor supply which, in turn, generates an increase in output and a decrease in real wages.² Finally, private investment usually falls in response to a (temporary) government spending shock.

Against this background, it is clear that the externalities we explore have the potential to flip the usual sign of the reactions to government spending shocks or, even if the sign is correct, to assess the likely bias of these responses. Importantly, if one is interested on output effects of fiscal stimulus, it's obvious that the responses obtained in Uhlig (2010) - focusing on distortionary taxes - or in Christiano *et al.* (2011) - focusing on the zero lower bound - or in Monacelli, Perotti and Trigari (2010) - focusing on the labor market - can be affected by the government spending externalities.

Our objective is thus answering three main questions: is it reasonable to assume independence in preferences between private and government consumption (*i.e.*, separability)? Is there evidence that public capital shifts the productivity of private factors? What are the effects produced by these externalities? To answer these questions we add the two “externalities channels” into an otherwise standard general equilibrium model with flexible prices, *i.e.*, a Real Business Cycle (RBC) model.

On the one hand, estimation of various versions of the model indicates that government and private consumption are substitute goods. This means that an increase in government consumption makes private consumption less enjoyable and, all else equal, leads agents to partially substitute private consumption with the newly available government consumption. As a consequence, agents want to work less relative to a world with separable government consumption. Hence, the estimated output multiplier is much lower (approximately one third) compared to the one obtained in models with separabilities, peaking - on impact - at 0.33. On the other hand, non-defense public investment enhances mildly or negligibly, depending on the specification, the productivity of private factors while investment in defense appears not to have any such impact. In our benchmark specification, a non-defense investment shock generates

¹ Notable exceptions can be found in the literature. For example, Galí, López-Salido and Vallés (2007) introduce a market imperfection in a new Keynesian model, namely, a share of the population cannot borrow or lend. Because of this, aggregate consumption can increase after a government spending shock. Following a different route, Linnemann (2006) builds a neoclassical model in which leisure and consumption are not separable in preferences. This type of non-separability can allow consumption to react positively to government spending shocks.

² Real wages surely react negatively within a flexible prices model since the labor demand schedule remains unchanged. Instead, in versions of sticky prices models, real wages can happen to increase in response to a government spending shock (see Linnemann and Schabert, 2003).

the following estimated responses after several quarters: a positive reaction for private consumption, the value of firms, private investment and real wages. The estimated output multiplier builds up over time, reaching roughly 1.5 after six years. Note that the output multiplier measured from the model with unproductive government investment, behaves in the opposite way. It starts higher on impact, *i.e.* around one, and then falls gradually over time.

The remainder of the article is organized as follows: section 2 gives an overview of the empirical evidence and relates it to our work. Section 3 outlines the model while section 4 describes the estimation exercise and results. In section 5, concluding remarks are presented.

2. Review of The Empirical Evidence

Virtually all the estimates of the degree of substitutability/complementarity between private and government consumption are obtained within partial equilibrium models or based on a few equilibrium conditions (mainly Euler equations). The empirical evidence obtained by estimating Euler equations is not conclusive. Aschauer (1985) finds a significant degree of substitutability between the two variables in the case of the U.S. whereas Amano and Wirjanto (1998) find weak complementarity. Focusing on the UK, Ahmed (1986) finds substitutability while Karras (1994), examining the relationship between private and public consumption across thirty countries, finds that the two types of goods are best described as complementary (but often unrelated). Fiorito and Kollitznas (2004) split government consumption in two groups named “public goods” and “merit goods”. The first includes spending in defense, security forces and judicial system; the second contains health, education and other services that can be provided privately. They use dynamic panel methods motivated by Euler equations and show that, for twelve European countries, public goods slightly substitute while merit goods complement private consumption. Using general equilibrium models provides at least two contributions. First, estimates of the elasticity of substitution within partial equilibrium models are prone to suffering from omitted variables bias (e.g. the negative wealth effect can bias the estimates of such elasticity). Second, a general equilibrium framework allows us to study the effects of government spending on several variables simultaneously. Along this line of research, Bouakez and Rebei (2007) estimate private and government consumption to be complement goods within a general equilibrium framework, using Maximum Likelihood estimation and U.S. data. Our analysis differs from theirs in various aspects, other than in the results.³

The other class of papers related to our work focuses on the importance of public capital in boosting output growth. Aschauer (1989) estimates an aggregate production for the U.S. economy, with inputs being labor, private capital but also public capital, finding that the output elasticity of government capital is 0.39. Following a similar approach, Finn (1993) estimates much lower output elasticities of various items of government capital (the largest is 0.16 for highways) and surrounded by great uncertainty. The implication of these two papers is that public capital is an important explanatory factor for changes in the productivity of the economy. Other authors, like Tatom (1991), find, instead, that the best estimate for the mentioned elasticity is zero. Belo and Yu (2011) report movements in stock returns compatible with a specification, very similar to ours, where public investment is directly productive. Unlike all these papers, we estimate the productivity shift caused by public capital within a general equilibrium model. As a consequence, we can study the effect of a government investment shock, while controlling for general equilibrium effects. There are virtually no estimates of the effects of a government investment

³ Crucially, unlike us, they do not use public spending data throughout the estimation; we believe that using this data is essential to correctly identify and measure the elasticity of substitution between private and public consumption, especially within closed economy models. Further, unlike us, they use HP filter detrended data, which implies a loose mapping between the model's variables and the data. Finally, they fix some relevant parameters, e.g. the parameter measuring the weight of private consumption in the effective consumption aggregator. On the contrary, we estimate it since this parameter is fundamental to establish whether or not government consumption affects the welfare of agents.

shock within an estimated general equilibrium model. However, it is worth noting the work of Leeper *et al.* (2010) who analyze, within an estimated general equilibrium model, scenarios with different values for the output elasticity of public capital. Conditional on choosing the value of 0.1 for this elasticity, they find an output multiplier ranging from 0.90 to 1.14 within the first three years. Baxter and King (1993), within a fully calibrated framework, find a long-run output multiplier equal to 4.12, conditional on choosing 0.1 for the output elasticity of public capital. Straub and Tchakarov (2007) conduct a calibration exercise for the Euro area within a general equilibrium framework, finding that under reasonable parameter values both permanent and temporary public investment shocks generate a much larger multiplier than the one obtained upon exogenous increases in government consumption. Finally, turning to the analysis of vector autoregressions (VARs), Perotti (2004) uses a structural VAR identified with institutional information and finds that a government investment shock creates an output multiplier which peaks on impact at 1.68, for the U.S.. Interestingly, Pappa (2009), using a sign-restrictions identified VAR and U.S. state data, finds that government investment increases both employment and real wages.

3. The Model

We now briefly describe our model economy, making clear the problems solved by households and firms. We also describe the behavior of the government, or fiscal authority. In a nutshell, we will be looking at an otherwise standard RBC model augmented with two ingredients aimed at assessing the role of government consumption and investment on private decisions. More precisely, government consumption is allowed to affect the marginal utility of consumption and public investment is allowed to enhance the productivity of private factors by entering the final goods' production function. Further, we borrow from the literature ingredients that have proven useful to fit the data: external habit formation in consumption, monopolistic competition in factor markets, investment adjustment costs, costs of adjusting capacity utilization and distortionary taxation. Uncertainty arises from six orthogonal shocks: a preference shock, total factor productivity, investment adjustment, wage markup (wedge) as well as public consumption and public investment shocks. Most often we break public investment into defense and non-defense items which results in the addition of another shock.

3.1. Households

The economy is populated by a continuum of households. We assume that the representative household derives utility from effective consumption, $\tilde{C}_t$, and disutility from working, L_t , in each quarter t . Effective consumption is assumed to be an aggregator of private consumption, C_t , and government consumption, G_t :

$$\tilde{C}_t = \left[\varphi (C_t)^{\frac{\nu-1}{\nu}} + (1-\varphi) G_t^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \quad (1)$$

where φ is the weight of private consumption in the effective consumption aggregator, and $\nu \in (0; \infty)$ is the elasticity of substitution between C_t and G_t .⁴ Note that if $\varphi = 1$ then $\tilde{C}_t = C_t$ and the standard hypothesis of separability emerges. The lifetime expected utility is given by:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t e^{\varepsilon_t^b} \left[\ln(\tilde{C}_t - h\tilde{C}_{t-1}^A) - \chi \frac{1}{1+\sigma_L} (L_t)^{1+\sigma_L} \right] \right\} \quad (2)$$

⁴ When $\nu = 0$, we have a Leontief" aggregator, i.e. C_t and G_t become perfect complements. When $\nu = 1$, we have a Cobb-Douglas" aggregator of the form $\tilde{C}_t = C_t^\varphi G_t^{(1-\varphi)}$. As $\nu \rightarrow \infty$, we have a linear aggregator of the form $\tilde{C}_t = \varphi C_t + (1-\varphi) G_t$, the two goods are perfect substitutes.

$\tilde{C}_{t-1}^A$ is the aggregate level of effective consumption at time $t-1$ which creates external habit formation in consumption. The parameter $h \in (0,1)$ measures the degree of habit formation in effective consumption whereas $\beta \in (0,1)$ is the subjective discount factor, σ_L is the inverse of the Frisch elasticity of labor supply and χ is a positive number fixing the steady-state level of labour. ε_t^b represents a preference shock, assumed to follow a first-order autoregressive process with an i.i.d.-normal error term. To analyze the substitutability/complementarity mechanism channel, we look at the derivative of the instant marginal utility of consumption with respect to government consumption. Given a steady-state level of consumption, this is given, in log-linearized form, by:

$$U_{cg} = (1 - \varphi) \left(G / \tilde{C} \right)^{\frac{v-1}{v}} \left(\frac{1}{v} - \frac{1}{(1-h)} \right), \quad (3)$$

where G and $\tilde{C}$ are the steady-state levels of government consumption and effective consumption, respectively. The parameters which are important in delivering the sign to U_{cg} are the elasticity of substitution between private and government consumption, v and the level of habit persistence, h . When U_{cg} is greater than 0, private and government consumption are defined to be complements; when U_{cg} is less than 0, private and government consumption are defined to be substitutes; when U_{cg} is equal to 0, private and government consumption are not related in preferences. Obviously, if we set φ equal to 1 government consumption does not enter the utility function and U_{cg} collapses to zero. For values of φ less than one, U_{cg} can be either positive or negative depending on the other parameters in U_{cg} . In particular, U_{cg} is strictly positive if $v < 1 - h$ and negative otherwise. Since $0 \leq h < 1$, $v > 1$ guarantees that U_{cg} is negative.

Households maximize their lifetime expected utility by choosing consumption, C_t , labor supply, L_t , next period's physical capital stock, K_{t+1} , the level of investment, I_t , and the intensity with which the installed capital stock is utilized, u_t . We present the version of the model with distortionary taxation on labor, consumption and capital, with marginal rates given, respectively, by τ^w, τ^c and τ^k . The agents thus face the following budget constraint (expressed in real terms):

$$(1 + \tau^c)C_t + I_t = (1 - \tau^w)W_t L_t + (1 - \tau^k) \left[r_t^k u_t - a(u_t) \right] K_t + D_t - T_t, \quad (4)$$

where r_t^k is the net return on capital, $W_t L_t$ is labor income, $a(u_t)$ represents the cost of using capital at intensity u_t (see, e.g., Schmitt-Grohé and Uribe, 2006), D_t are the dividends paid by household-owned firms while T_t are lump-sum taxes/transfers to/from the government.⁵

The capital stock evolves according to the following equation:

$$K_{t+1} = (1 - \delta_k) K_t + I_t \left[1 - S \left(e^{\varepsilon_t^I} \frac{I_t}{I_{t-1}} \right) \right], \quad (5)$$

where δ_k is the depreciation rate and the function $S(\cdot)$ introduces investment adjustment costs à la Christiano, Eichenbaum, and Evans (2005). Specifically, $S(\cdot) = \frac{\kappa}{2} \left(e^{\varepsilon_t^I} \frac{I_t}{I_{t-1}} - e^{\bar{\gamma}} \right)^2$, where ε_t^I is a shock to the investment cost function assumed to follow a first-order autoregressive process with an i.i.d.-normal error term, and $\bar{\gamma}$ is the steady-state growth rate of productivity (see the next section for details).

⁵ In order to justify the existence of a representative agent we complete the markets by making agents able to trade a full set of state-contingent claims. In equilibrium these are in zero net supply.

3.2. Firms and Labour Market

Here we describe very briefly the labour and product markets. In order to simplify the exposition we postulate directly the price/wage setting process. The general and widely used setup rationalizing the resulting equilibrium conditions can be found in Ercolani and Valle e Azevedo (2012). We assume there is a continuum of monopolistically competitive firms each of which produces a single variety of final goods, $Y_{j,t}$. All firms are identical and all varieties have the same degree of substitutability (elasticity of substitution) vis-a-vis any other variety. In equilibrium, households split their consumption uniformly over varieties and we are able to abstract from index j for firms. We focus throughout on the representative firm. This firm produces the final good Y_t , using as inputs capital services, K_t and labor services, L_t from competitive suppliers. Moreover, we augment the standard production function with K_t^G , representing the “productivity” of public capital. The production function is given by:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} (K_t^G)^{\theta_g} - \Phi, \quad (6)$$

where A_t is a productivity shock.⁶ The process for $\ln(A_t)$ evolves according to:

$$\ln(A_t) = \bar{\gamma} + \ln(A_{t-1}) + \varepsilon_t^a, \quad (7)$$

where $\bar{\gamma}$ is the steady-state growth rate of productivity (and hence of the economy) and the productivity shock evolves according to $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$, where η_t^a is an i.i.d.-normal sequence. The parameter Φ represents a fixed cost of production while $\theta_g \in (0; \infty)$ is the output elasticity with respect to public capital productivity. Conditional on $\theta_g > 0$, K_t^G has a direct effect on firm's output and, as a consequence of this, a positive influence on the productivity of private factors. The higher θ_g is, the more effective is K_t^G in boosting firm's output and private productivities. Notice that if $\theta_g = 0$, the standard production function pops up and K_t^G doesn't produce any externality effects. The productivity of public capital is assumed to evolve according to:

$$K_{t+1}^G = (1 - \delta_{Kg}) K_t^G + \xi_t^{ig}, \quad (8)$$

where δ_{Kg} is the depreciation rate and ξ_t^{ig} is the public investment rate (in our case, public investment, I_t^g , over total output, i.e. $\xi_t^{ig} = I_t^g / Y_t$). We will later specify how ξ_t^{ig} evolves. For now, we refer that ξ_t^{ig} follows a stationary process (which seems consistent with the data), implying that K_t^G is stationary. This is convenient for technical reasons (see Belo and Yu, 2011 and references therein for a similar specification and reasoning) and avoids keeping track of - poorly measured - public capital.

The solution of the profit maximization problem for the firms amounts to setting the price P_t as a markup, equal to $\lambda_{p,ss}$, over marginal cost.

In this article we only consider versions of the model with public investment split into defense and non-defense items. The production function for final goods varieties producers becomes:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} (K_t^G)^{\theta_g} (K_t^{G,def})^{\theta_{g,def}} - \Phi,$$

where the productivity of defense capital, $K_t^{G,def}$, is assumed to evolve according to $K_{t+1}^{G,def} = (1 - \delta_{Kg,def}) K_t^{G,def} + \xi_t^{ig,def}$ and $\xi_t^{ig,def}$ is defense investment over output, i.e. $\xi_t^{ig,def} = I_t^{g,def} / Y_t$.

Public investment, I_t^g , is then assumed to exclude defense items.

Regarding the labor market, we assume there is a continuum of monopolistically competitive households

⁶ The chosen specification implies substitutability between private capital and the productivity of public capital as typical in the literature, see e.g. Aschauer (1989), Finn (1993), Baxter and King (1993) or Leeper *et al.* (2010).

who are able to set their wage as a markup over the marginal rate of substitution between consumption and leisure. The markup is stochastic and follows a first order autoregressive process.

3.3. Government

First, we specify the evolution of public consumption, G_t , and public investment, I_t^g (we use exactly the same formulation for $I_t^{g,def}$) These can always be expressed as a varying fraction of output:

$$G_t = \xi_t^g Y_t ; \quad I_t^g = \xi_t^{ig} Y_t.$$

We further specify ξ_t^g and ξ_t^{ig} as follows:

$$\xi_t^g = \exp(\varepsilon_t^g + ss^g) / (1 + \exp(\varepsilon_t^g + ss^g)) ; \quad \xi_t^{ig} = \exp(\varepsilon_t^{ig} + ss^{ig}) / (1 + \exp(\varepsilon_t^{ig} + ss^{ig})).$$

This formulation (basically a reparametrization) ensures ξ_t^g and ξ_t^{ig} are always between 0 and 1. The exogenous shocks ε_t^g and ε_t^{ig} to government consumption and investment, respectively, follow autoregressive processes:

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g \quad (9)$$

$$\varepsilon_t^{ig} = \rho_{ig} \varepsilon_{t-1}^{ig} + \eta_t^{ig}, \quad (10)$$

where η_t^g and η_t^{ig} are normal i.i.d. and mutually independent with mean zero. Note that ss^g and ss^{ig} fix the average (or steady-state) levels of ξ_t^g and ξ_t^{ig} (denoted, respectively, by $\xi^{g,ss}$ and $\xi^{ig,ss}$). Since Ricardian equivalence holds in the model, we abstract from government debt and assume that the government balances its budget.

3.4. Solution

We start by deriving the first order conditions associated with the households' and firms' problems, combining them with market clearing conditions and exogenous processes while recognizing that all firms and households are ex-ante identical. The equilibrium concept is standard. The labor market is in equilibrium when labour demanded by firms equals the differentiated labour services supplied by households at the aggregate wage rate W_t . The market for capital is in equilibrium when the demand for capital services by the firms equals the capital supplied by households at the market rental rate r_t^k . Finally, the final goods market is in equilibrium when the supply by firms equals the demand by households and government:

$$Y_t = C_t + I_t + a(u_t)K_t + G_t + I_t^g + I_t^{g,def}. \quad (11)$$

All the equilibrium conditions can be found in Ercolani and Valle e Azevedo (2012). Before proceeding to the estimation, we log-linearize the model equations around the deterministic steady-state. The exception to log-linearization occurs with the variables ξ_t^g , ξ_t^{ig} and $\xi_t^{ig,def}$, which are fractions of total output.

4. Estimation

We estimate the model using standard Bayesian techniques, using U.S. data from 1969Q1-2008Q3, see the Appendix for a detailed description of the dataset. We take as observables log differences of quarterly real output (GDP), real consumption, real investment and real wages as well as a particular transformation of public consumption and public investment (the latter split into defense and non-defense items), consistent with the formulation of the processes for $G_t = \xi_t^g Y_t$ and $I_t^g = \xi_t^{ig} Y_t$ described earlier. We believe that using data until 2012 could open (further) issues of misspecification in our empirical model (e.g. driven by the absence of financial frictions and the lack of consideration of the zero lower bound on

nominal interest rates).⁷ Importantly, the mapping of data to variables in the model is exact, that is, we do not filter data before proceeding to estimation (for details see Ercolani and Valle e Azevedo, 2012).

Bayesian estimation entails specifying prior distributions (beliefs of the analyst) for the parameters that are not fixed. Let $P(\theta | m)$ be the prior distribution of the parameter vector $\theta \in \Theta$ for some model $m \in M$ and $L(X_T | \theta, m)$ be the likelihood function for the observed data $X_T = \{x_t\}_{t=1}^T$, conditional on the parameter vector θ and model m . The posterior distribution of the parameter vector θ for model m , $P(\theta | X_T, m)$, is then obtained combining the likelihood function for X_T (i.e., information from the sample) with the prior distribution of θ :

$$P(\theta | X_T, m) \propto L(X_T | \theta, m)P(\theta | m), \quad (12)$$

$P(\theta | X_T, m)$ can be thought of as the probability of θ taking a particular value given the prior information and information from the data. This function can be numerically maximized to obtain the mode (the most likely value of θ given prior information combined with sample information), which is often seen as a point estimate of the parameter vector θ . Simulation algorithms allow us to obtain numerically the distribution $P(\theta | X_T, m)$ as well as distributions of functions of the parameter vector θ (e.g., impulse response functions), see An and Schorfheide (2007). As discussed in Geweke (1999), Bayesian inference also provide tools to compare the fit of various models. For a given model m , the marginal likelihood is:

$$L(X_T | m) = \int_{\theta \in \Theta} L(X_T | \theta, m)P(\theta | m)d\theta, \quad (13)$$

which gives an indication of the overall likelihood of a model conditional on observed data. Below we discuss the priors employed in our estimation and an analysis of the posterior distribution for the parameters of greatest interest to us.

4.1. Calibration and Prior Distributions

We will focus exclusively on the parameters related to public spending externalities. The interested reader can find all the details in Ercolani and Valle e Azevedo (2012); in particular, the values of the parameters that are fixed (calibrated) and the prior distributions employed. Concerning the choice of the priors, we refer that they are independent and we keep them mostly uninformative, i.e., we don't favor disproportionately any particular value of the parameters while centering them around values common in the literature. This is specially true in what regards the parameters related to government spending externalities. The utility parameter φ follows a uniform distribution (equal probability) with support in $[0,1]$. Concerning the parameter $\nu = \exp(\nu_b)$ we decide to reparametrize it such that $\nu = \exp(\nu_b)$, where now $\nu_b \in (-\infty; \infty)$. Then, in assigning the prior to ν_b we want to be as agnostic as possible, so we decide again for a uniform distribution with support in $[-4,20]$ (meaning that ν is in the range $[0,018$, almost perfect substitutes], say), which covers a wide range of possibilities in the complementarity/separability space.⁸ Regarding the choice for the prior mean of θ_g (or $\theta_{g,def}$), we also hold to uniform distributions with support in $[0,4]$.

4.2. Estimation Results

This section presents the estimation results. We analyze various versions of the model, focusing on the following variations:

⁷ Interestingly, Fernandez-Villaverde (2010) analyzes the effects of fiscal policy within a DSGE model with financial frictions.

⁸ The aim is to avoid providing prior information in favor either of substitutability or of complementarity.

- Full-sample, *i.e.* from 1969Q1 through to 2008Q3 or only the so-called “Great Moderation”, *i.e.* from 1984Q1 through to 2008Q3

- Restricted models: without public spending externality channels ($\varphi = 1$, $\theta_g = 0$, $\theta_{g,def} = 0$), with the utility function channel only ($\theta_g = 0$, $\theta_{g,def} = 0$), with the production function channels only ($\varphi = 1$) and, within this one, consideration of no output effects of defense investment ($\varphi = 1$, $\theta_{g,def} = 0$).

Table I reports some selected estimated specifications for the full sample, the associated values for the marginal data density, and a summary of the posterior (mean and mode) of the externalities’ parameters. The model with the highest marginal data density is the one with the productivity of public capital’s channel closed. This specification reveals that government consumption affects the marginal utility of consumption, since φ is estimated to be less than 1, and that government and private consumption are substitute goods. In fact, the high estimated value for ν_b implies a very high elasticity of substitution between private and government consumption. Given these results, U_{cg} in equation (Ucg) is unambiguously negative. Focusing now on those specifications where θ_g and $\theta_{g,def}$ are estimated, we underline the following facts: first, the estimates for φ and ν_b are very close to the ones obtained in the version where θ_g and $\theta_{g,def}$ are restricted to 0. Second, whenever estimated, both the mean and the mode of $\theta_{g,def}$ are 0. Third, in some specifications the *posterior mean* of θ_g is above 0, ranging from 0.09 to 0.28. Finally, models with $\theta_{g,def}$ restricted to 0 are very clearly preferred to models with $\theta_{g,def}$ left unrestricted.⁹

All in all, the results suggest clear evidence of strong substitutability between public consumption and private consumption and mixed evidence on the positive effects of non-defense public investment on private sector productivity, and hold also in the post ‘84 sample. Indeed, the 90% posterior intervals associated to φ and ν_b are tight around the estimated mean, while the one for θ_g is much wider (see Ercolani and Valle e Azevedo, 2012 for more details). Next, we turn to the analysis of the dynamics of the estimated model with the highest marginal data density, comparing several impulse responses to what obtains in the case of separable government consumption.

Table 1

POSTERIOR MEAN AND MODE OF EXTERNALITIES PARAMETERS											
Parameter	POSTERIOR post 1969										No Channels
	$\theta_{g,def}=0$						$\theta_{g,def}$ Unrestr.				
	$\varphi=1$		$\theta_g=0$		unrestr.		$\varphi=1$		unrestr.		
	Mode	Mean	Mode	Mean	Mode	Mean	Mode	Mean	Mode	Mean	
A. Utility function											
$\nu_b = \log(\nu)$	-	-	7.9	10.2	7.6	10.7	-	-	8.1	10.5	
φ	-	-	0.65	0.63	0.65	0.63	-	-	0.65	0.61	
B. Production function											
θ_g	0.0	0.0	-	-	0.0	0.09	0.0	0.28	0.0	0.0	
$\theta_{g,def}$	-	-	-	-	-	-	0.0	0.0	0.0	0.0	
Laplace Log Data Dens.	2522.3		2544.8		2539.6		2518.9		2518.5		2541.4
Log Data Dens.	2534.9		2555.1		2551.6		2534.7		2533.3		2548.3

Source: Authors’ calculations.

⁹ We should refer that we have also estimated versions of the model with public investment not split into defense and non-defense items. In this case, estimates of the posterior mode and mean of the parameter measuring the productivity of public investment, θ_g , were almost always exactly 0. This indicates perhaps the increased difficulty of identifying public investment as a shifter of productivity once defense and non-defense items are considered jointly.

4.3. Impulse-Response Analysis

In this section we study the dynamics of the economy where the only externality channel is represented by non separable government consumption. Figure 1 describes the estimated model variables' reactions to a government consumption shock in the first 100 quarters after the shock. The size of the shock is set to the estimated *posterior mean* of the standard deviation and the impulse response functions are expressed as percentage points of steady-state deviations. Each plot presents two lines, a black and a dark grey one. The black one is the *posterior mean* of the estimated responses and is named *Posterior Mean*. The dark grey line is the reaction obtained by fixing all the parameters at the respective *posterior mean*, with the difference that the "externality" parameter, ϕ , is set to 1. The latter summarizes the reactions when the externality channel is shut down, and is labeled *No Channels*.¹⁰ The shaded area within each plot draws the 80% Bayesian posterior credibility interval of the estimated impulse response functions.

The behavior of the variables in Figure 1 can be explained as follows: because of substitutability, the increase in government consumption lowers the marginal utility of consumption, leading households to substitute part of their private consumption with the newly available government consumption. As a consequence, private consumption (black line) decreases more than private consumption in the *No Channels* specification (dark grey line). That is, in the estimated model, both the negative wealth effect and the substitutability effect sum up. Additionally, because of substitutability households work less since - for given negative wealth effect - the marginal utility of consumption is lower than in the *No Channels* model. As a result, wages decrease less. Importantly, the impact on output is smaller in the estimated model because of the lower increase in labor supply. To accommodate the new path for private consumption, the real interest rate increases by less on impact, so that investment is crowded-out less. Notice that Ercolani and Valle e Azevedo (2012) includes also the analysis of the dynamic effects generated by a shock to government investment.

The next section completes the quantitative analysis, resorting to the analysis of dynamic multipliers induced by non separable government consumption.

4.4. Dynamic Multipliers

We analyze here public spending multipliers associated with the estimated effects of a government consumption shock on output, consumption and investment. We use the notion of present value multipliers formulated in Mountford and Uhlig (2009); the present value multiplier of output, say, t quarters after an increase in government consumption is:

$$\phi_t = \frac{\sum_{k=1}^t (1 + r_{ss})^{-k} \Delta y_k}{\sum_{k=1}^t (1 + r_{ss})^{-k} \Delta g_k}$$

where Δy_k represents the actual deviation of stationarized output from its steady-state at time k , Δg_k represents the actual deviation of stationarized public consumption from its steady-state at time k and r_{ss} is the steady-state real interest rate on the risk free asset. The expression generalizes for the case of consumption and investment.

Table II shows the present value multipliers for Y , C and I and for various periods in response to a government consumption shock. We look at the *posterior mean* of the multipliers and also at an 80% Bayesian credibility interval. Further, the corresponding *No Channels* multipliers are reported.

¹⁰ Hence, this restricted model is not estimated in order to guarantee that the only parameters changing are those related to public spending externalities. However, we should note that the estimated *posterior mean* of the impulse response functions obtained with the imposed restrictions are very similar to the reported ones.

Chart 1 (continue)

IMPULSE RESPONSE FUNCTIONS: EFFECTS ON OUTPUT (Y), CONSUMPTION (C), INVESTMENT (I), WAGES (W), HOURS (L) AND RETURN ON CAPITAL (R^k) OF A GOVERNMENT CONSUMPTION SHOCK

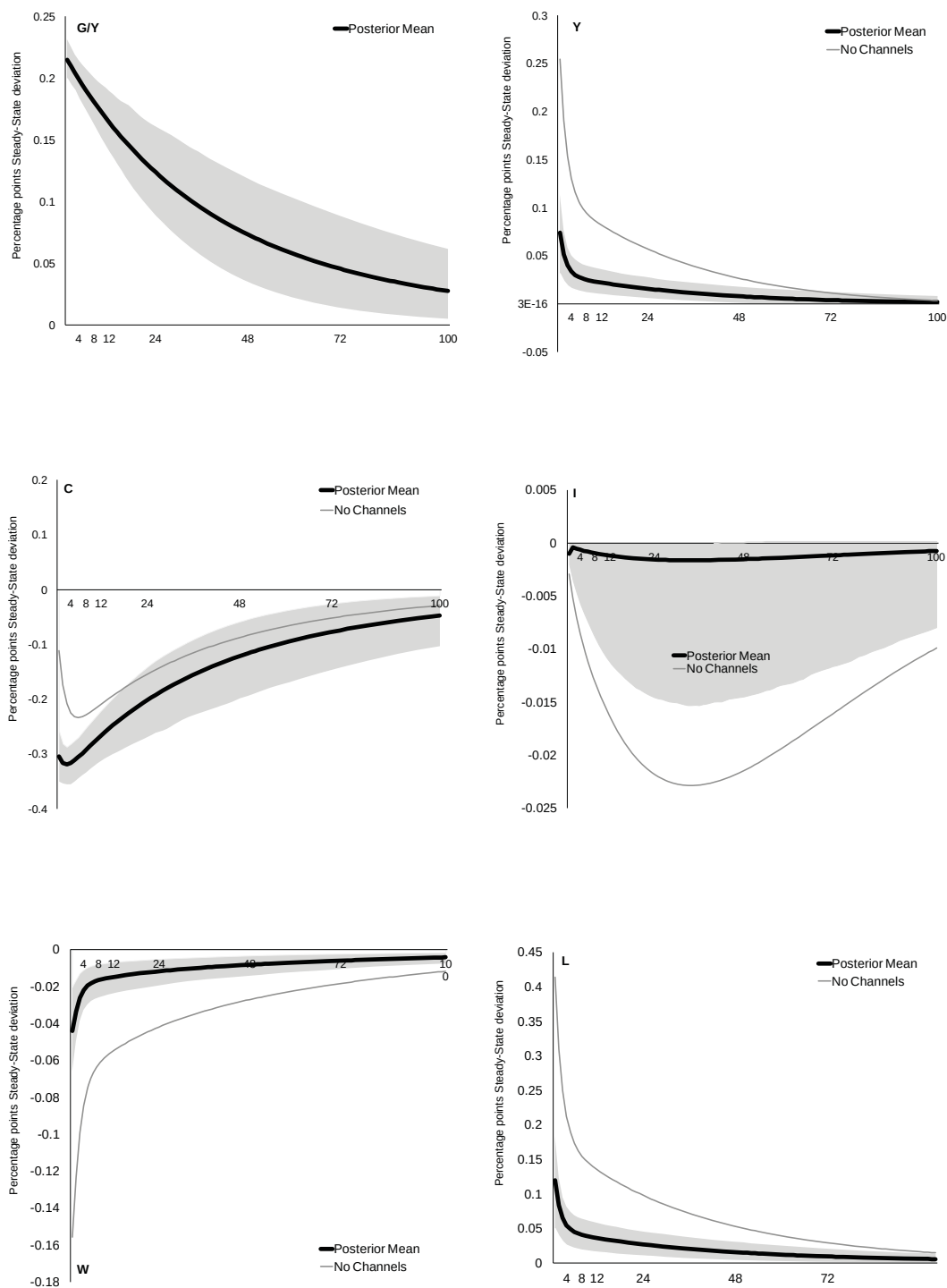
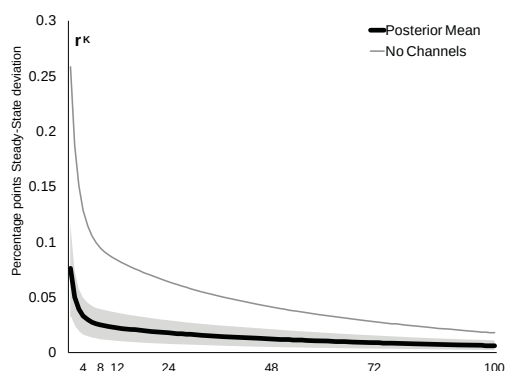


Chart 1 (continuation)

IMPULSE RESPONSE FUNCTIONS: EFFECTS ON OUTPUT (Y), CONSUMPTION (C), INVESTMENT (I), WAGES (W), HOURS (L) AND RETURN ON CAPITAL (R^k) OF A GOVERNMENT CONSUMPTION SHOCK



Source: Authors' calculations.

Notes: The government consumption shock is equal to the estimated standard deviation. The black line is the posterior mean of the IRF and denoted by *Posterior Mean*. The grey area is the associated 80% Bayesian credibility interval. The dark grey line is the IRF obtained with the "externality" channel closed (i.e. $\phi = 1$) and the other parameters set to the posterior mean obtained from the estimated model (denoted by *No Channels*).

The estimated output multiplier reaches its maximum at 0.33 on impact, and then slowly decreases. The output multiplier calculated within the *No Channels* specification turns out to be 0.99 on impact, i.e. three times bigger than the one generated by non separable government consumption. Our estimated multiplier is close to the impact output multiplier found by Mountford and Ugh (2009), which equals 0.31. This is obtained within a VAR identified through sign restrictions, where taxes are forced to adjust so as to fully finance the increase in government spending during the first four quarters after the shock. Clearly, our 80% posterior interval does not even contain the impact values found by Blanchard and Perotti (2002), which are 0.90 (under a deterministic detrending of the data) and 0.84 (under a stochastic one).¹¹

As expected, the multipliers for consumption and investment are negative, though the 80% posterior interval for investment contains zero. In the case of consumption the multipliers are clearly below those obtained in the *No Channels* model, especially at short horizons, whereas for investment they are above, but still negative. Notice that Ercolani and Valle e Azevedo (2012) includes the analysis of the multipliers generated by a shock to government investment.

¹¹ It is worth noting that many of the analyzes of the effects of government spending focus on military spending, instead of government consumption, as it is unlikely that this type of expenditures is endogenous. Among others, Barro and Redlick (2010) estimate an output multiplier ranging from 0.6 to 0.7 at the median unemployment rate (reaching 1.0 when the unemployment rate is around 12%); also, they find a crowding out effect for investment and net exports. Hall (2009)'s range for the output multiplier is 0.7-1.0. Finally, Ramey (2011), using news shocks obtained with a narrative approach, finds output multipliers in the range 0.6-1.2 (at peak GDP) and slightly negative consumption multipliers.

Table 2

DYNAMIC MULTIPLIERS, ESTIMATED VS NO CHANNELS MODEL, GOVERNMENT CONSUMPTION SHOCK									
Quarters	1	2	4	8	12	24	48	72	100
Y Estimated	0.33	0.28	0.25	0.19	0.17	0.16	0.14	0.14	0.13
	(0.14,0.50)	(0.12,0.43)	(0.11,0.35)	(0.09,0.29)	(0.08,0.27)	(0.07,0.25)	(0.06,0.24)	(0.05,0.24)	(0.04,0.24)
Y No Channels	0.99	0.89	0.77	0.65	0.60	0.55	0.51	0.49	0.48
C Estimated	-0.75	-0.79	-0.82	-0.85	-0.86	-0.87	-0.89	-0.89	-0.90
	(-0.87,-0.64)	(-0.89,-0.68)	(-0.92,-0.73)	(-0.96,-0.77)	(-0.98,-0.75)	(-1.04,0.73)	(-1.13,-0.67)	(-1.19,-0.63)	(-1.24,-0.61)
C No Channels	-0.24	-0.32	-0.43	-0.52	-0.56	-0.61	-0.63	-0.64	-0.65
I Estimated	-0.001	-0.001	-0.001	-0.001	-0.001	-0.002	-0.002	-0.003	-0.003
	(-0.002,0.000)	(-0.003,0.000)	(-0.003,0.000)	(-0.003,0.000)	(-0.003,0.000)	(-0.003,0.000)	(-0.003,0.000)	(-0.003,0.000)	(-0.003,0.000)
I No Channels	-0.003	-0.004	-0.005	-0.009	-0.012	-0.019	-0.029	-0.035	-0.039

Source: Authors' calculations.

Notes: 80% Bayesian credibility interval in parenthesis.

5. Conclusions

This article has posed attention on the potential externalities produced by public expenditures, focusing on how these externalities affect the response of the economy to two government spending shocks: a government consumption shock and a government investment shock. To this effect, we have built an otherwise standard flexible prices model extended with two important features. First, we have allowed government consumption to affect the welfare of agents, by entering directly households' utility function. Second, we have allowed public capital to shift the productivity of private factors, by entering in the firms' production function.

On the one hand, our results question the standard hypothesis of separability between private and government consumption, as the two goods are robustly estimated to be substitutes. Because of substitutability labor supply reacts little to a government consumption shock, so the estimated output multiplier is lower (approximately one third) than the one measured in models with separable government consumption. On the other hand, we find that non-defense public investment enhances mildly, in some specifications, the productivity of private factors while investment in defense appears not to have any such impact. When non-defense public investment is found to shift the production frontier, shocks to it generate non-standard responses that manifest themselves only after several quarters (a positive reaction of private consumption, Tobin's q , private investment and real wages). Further, the estimated output multiplier builds up over time, in contrast to what obtains in the corresponding model with unproductive government investment. These results show that incorporating the channels we study into general equilibrium models can be important to understand and measure more thoroughly the expected impacts of fiscal stimuli and consolidations, as well as to conduct welfare analysis of fiscal policy.

Finally, it will be worth investigating how our measures interact with several important features of fiscal and monetary policy such as debt smoothing details, implementation delays, or the zero lower bound on nominal interest rates.

References

- Ahmed, S. (1986), "Temporary and Permanent Government Spending in an Open Economy", *Journal of Monetary Economics*, 17 (3), 197-224.
- Amano, A.R. and T.S. Wirjanto (1998), "Government Expenditures and the Permanent-Income Model", *Review of Economic Dynamics*, 1 (3), 719-30.
- An, S. and F. Schorfheide (2007), "Bayesian Analysis of DSGE Models", *Econometric Reviews*, 26 (2-4), 113-172.
- Aschauer, D. A. (1985), "Fiscal policy and aggregate demand", *American Economic Review*, 75 (1), 117-127.
- Aschauer, D. A. (1989), "Is public expenditure productive?", *Journal of Monetary Economics*, 23 (2), 177-200.
- Baxter, M. and R. G. King (1993), "Fiscal Policy in General Equilibrium", *American Economic Review*, 83 (3), 315-334.
- Barro, R. (1981), "Output Effects of Government Purchases", *Journal of Political Economy*, 89 (6), 1086-1121.
- Barro, R. and C. Redlick (2010), "Macroeconomic Effects from Government Purchases and Taxes", Harvard University, mimeo.
- Belo, F. and Yu, J. (2011), "Government Investment and the Stock Market", University of Minnesota, mimeo.
- Blanchard, O. J. and R. Perotti (2002), "An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output", *Quarterly Journal of Economics*, 117 (4), 1329-1368.
- Bouakez, H. and R. Rebei (2007), "Why Does Private Consumption Rise after a Government Spending Shock?", *Canadian Journal of Economics*, 40 (3), 954-979.
- Christiano, L., M. Eichenbaum and C. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy*, 113 (1), 1-45.
- Christiano, L., Eichenbaum, M., Rebei, S. (2011), "When is the Government Spending Multiplier Large?", *Journal of Political Economy*, 119 (1), 78-121.
- Ercolani, V. and J. Valle e Azevedo (2012), "The Effects of Public Spending Externalities", *WP 12/2012*, Banco de Portugal.
- Fernandez-Villaverde, J. (2010), "Fiscal Policy in a Model With Financial Frictions", *American Economic Review P&P*, 100 (2), 35-40.
- Finn, M. (1993), "Is All Government Capital Productive?", mimeo Federal Reserve Bank of Richmond.
- Fiorito, R. and T. Kollitznas (2004), "Public goods, merit goods, and the relation between private and government consumption", *European Economic Review*, 48 (6), 1367-1398.
- Galí, J., J. D. López-Salido and J. Vallés (2007), "Understanding the Effects of Government Spending on Consumption", *Journal of the European Economic Association*, 5 (1), 227-270.
- Geweke, J. F. (1999), "Using Simulation Methods for Bayesian Econometric Models: Inference, Developments and Communication", *Econometric Reviews*, 18, 1-126.
- Hall, R. (2009), "By How Much Does GDP Rise if the Government Buys More Output?", *Brookings Papers on Economic Activity*.

- Karras, G. (1994), "Government spending and private consumption: Some international evidence", *Journal of Money, Credit and Banking* 26 (1), 9--22.
- Leeper, E., Walker, T. B. and Yang, Shu-Chun S. (2010), "Government investment and fiscal stimulus", *Journal of Monetary Economics*, 57(8), 1000-1012.
- Linnemann, L. (2006), "The Effects of Government Spending on Private Consumption: A Puzzle?", *Journal of Money, Credit, and Banking* 38 (1), 1715-1736.
- Linnemann, L. and A. Schabert (2003), "Fiscal Policy in the New Neoclassical Synthesis, *Journal of Money, Credit and Banking*, 35 (6), 911-929.
- Monacelli T., R. Perotti, and A. Trigari (2010), "Unemployment fiscal multipliers", *Journal of Monetary Economics*, 57 (5), 531-553.
- Mountford A. and, H. Uhlig (2009), "What are the Effects of Fiscal Policy Shocks?", *Journal of Applied Econometrics*, 24 (6), 960-992.
- Pappa E. (2009), "The Effects of Fiscal shocks on Employment and The Real Wage", *International Economic Review*, 50 (1), 217-244.
- Perotti, R. (2004), "Public Investment: Another (Different) Look", *Universita Bocconi*, mimeo.
- Ramey, V. A. (2011), "Identifying Government Spending Shocks: It's all in the Timing", *The Quarterly Journal of Economics*, 126 (1), 1-50.
- Schmitt-Grohé, S. and Uribe, M. (2006), "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model", *NBER Macroeconomics Annual 2005*, 20, 383-462.
- Smets, F. and R. Wouters (2007), "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach", *American Economic Review*, 97 (3), 586-606.
- Straub, R. and I. Tchakarov (2007), "Assesing the Impact of a Change in the Composition of Public Spending: a Dsge Approach", *ECB Working Paper 479*.
- Tatom, J. A. (1991), "Public capital and private sector performance", *Federal Reserve Bank of St. Louis Review*, 3-15.
- Uhlig, H. (2010), "Some fiscal calculus", *American Economic Review*, 100 (2), 30--34.

Appendix: Data

We follow closely Smets and Wouters (2007) in treating the data. We use the September 30, 2010 vintage of data. The tables below clarify sources and transformations.

Table 1

DATA SOURCES			
Variable	Designation	Source	CODE
Gross Domestic Product (Nominal)	GDP	U.S. Dep. of Commerce - BEA	A191RC1
Personal Cons. Expenditures (Nominal)	C	U.S. Dep. of Commerce - BEA	DPCERC1
Personal Cons. Expenditures - Durables (Nominal)	Durables	U.S. Dep. of Commerce - BEA	DDURRC1
Private Fixed Domestic Investment (Nominal)	PFI	U.S. Dep. of Commerce - BEA	A007RC1
Federal Cons. Expenditures (Nominal)	G_Federal	U.S. Dep. of Commerce - BEA	A957RC1
State & Local Cons. Expenditures (Nominal)	G_StateLocal	U.S. Dep. of Commerce - BEA	A991RC1
Federal Gross Investment, Non-Defense (Nominal)	IG_Federal	U.S. Dep. of Commerce - BEA	A798RC1
Federal Gross Investment, Defense (Nominal)	IG_Defense	U.S. Dep. of Commerce - BEA	A788RC1
State & Local Gross Investment (Nominal)	IG_StateLocal	U.S. Dep. of Commerce - BEA	A799RC1
Gross Domestic Product Deflator	GDPDEF	U.S. Dep. of Commerce - BEA	GDPDEF
Hourly Compensation, Non Farm Sector (Nominal)	Wages	Bureau of Labor Statistics	PRS85006103
Civilian noninstitutional population, 16 years and over	POPULATION	Bureau of Labor Statistics	LNU00000000Q

Note: All series are seasonally adjusted at annual rates.

Table 2

D2 - OBSERVABLES FOR MEASUREMENT EQUATIONS

$$Y_t^{obs} = (GDP/GDPDEF)/POPULATION$$

$$C_t^{obs} = ((C - Durables)/GDPDEF)/POPULATION$$

$$I_t^{obs} = ((PFI + Durables)/GDPDEF)/POPULATION$$

$$W_t^{obs} = (Wages/GDPDEF)/POPULATION$$

$$\xi_t^{g, obs} = (G_Federal + G_StateLocal)/GDP$$

$$\xi_t^{ig, obs} = (IG_Federal + IG_StateLocal)/GDP$$

$$\xi_t^{ig, def, obs} = IG_Defense/GDP$$