

# WELFARE COSTS OF INFLATION WITH DISTORTIONARY TAXATION\*

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## ABSTRACT

We show that the welfare cost of inflation decreases when distortionary taxation is taken into account. The estimates of the welfare cost of inflation usually consider that governments are able to use lump sum taxation to finance their budget. However, governments can only use distortionary taxation, such as labor income taxes. When only distortionary taxation is available, the government can decrease the size of distortionary taxes by compensating the decrease in revenues with the revenues generated by inflation. We compare the case in which the government has access to lump sum taxes with the case in which only distortionary taxes are available. We keep the level of government expenditures as a percentage of output constant. We find that the welfare cost of an increase in inflation from 0 to 10% per year decreases from 1.3% in terms of income to 0.8%.

## 1. Introduction

The popular belief is that inflation is harmful, but in general its effects are not well understood. That is due to the fact that the effects of inflation are very diversified and many times subtle.

Inflation may have important distributional effects. Surprises in the inflation rate lead to redistributions of income and welfare between various groups of the population. Unexpected inflation increases redistribute wealth from lenders to borrowers, and unexpected reductions of inflation redistribute wealth in the opposite direction. This principle applies to other financial contracts besides the loan contracts. In general those that hold financial assets that are not fully indexed to inflation lose with unexpected increases in inflation and win with unexpected decreases. For instance an inflation higher than expected redistributes wealth to the younger generations, since the older generations have a higher portion of nominal assets. It also redistributes income from those that have fixed nominal income contracts to those that have variable incomes that follow inflation. Two examples: an inflation above what was expected implies for the pensioners a deterioration in their real pension and for workers a deterioration in their real wage. A redistribution of income can occur between countries. When the exchange rate is fixed, a higher inflation rate in one country is going to make that country exports more expensive and affect that country trade account.

Moreover, a variable inflation rate makes it difficult to distinguish changes in the relative prices from changes in the aggregate price, which implies an efficiency loss in the allocation of the resources in the economy. For instance, assume that a firm expects low inflation and inflation turns up to be high. When

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the firm observes the price of the good it produces increasing more rapidly than expected, it might believe that there was an increase in the demand for its product. As it confuses the increase in inflation with an increase in the demand for its product it increases production. If this behavior is repeated by many firms there will be an increase in aggregate supply that leads to a distorted level of output in the economy.

Menu costs are another effect of inflation. This costs are associated with the resources spend by sellers to adjust to the inflation the prices of the goods and services they sell. The concept is associated with the image of restaurants incurring in costs of printing new menus with higher prices for the dishes as the price of the ingredients used increase.

Inflation has effects over the tax system. Inflation increases the effective marginal tax rates. If the marginal tax brackets are set in nominal terms, or are not fully indexed to inflation, the tax payers are pushed to higher marginal tax rates by the effect of inflation. Also, the effect of inflation over the depreciation allowed by the firms' tax code discourages productive investment. The value of the depreciation that firms can take depends on the historical value of its physical capital, and with inflation the real value of the depreciation falls. A similar situation occurs with capital gains on assets. The tax on capital gains is taken on the difference between the sale price and the purchase price of the asset. If the purchase price considered is its historical value, then investors will be taxed on capital gains even when the real price of the asset is unchanged.

Inflation is a regressive tax. As the income elasticity of the demand for money is less than one, the richer taxpayers pay a smaller portion of their income as inflation tax than poorer taxpayers.

Inflation is a tax and as all taxes introduces distortions in the economy, implies smaller disposable income for the private agents and revenue for the government. Part of that income can be recovered by the private agents through more public services or less of the other taxes. However, as the agents in the economy are going to reduce their demand for money they are going to have more difficulty in carrying their transactions. Unlike the other costs referred above, this cost does not vanish when the economic agents are homogeneous or prices are fully flexible.

In this paper we quantify only this effect of inflation. Thus, the figure for the cost we compute is the lower bound of the total inflation cost. We consider that inflation is completely anticipated. We determine the real effects of inflation, when the agents' inflation expectations coincide with the realized inflation and increases in inflation have a zero impact on the government revenue. In this context the social welfare variation caused by an increase in inflation is known in the literature as the welfare cost of inflation.

The experiment that we have in mind is the one in which the government increases the amount of money 10 percent every period and gives back to the economic agents the revenue in excess of the necessary to finance government consumption. Additionally, all contracts can be adjusted to the rate of inflation and everyone knows the value of inflation. In this case, everyone incorporates the higher rate of inflation into their plans. House and apartment rentals, negotiated labor contracts, loan contracts, income tax brackets in the tax code, etc. will be adjusted upward by 10 percent every period. All decisions incorporate the price changes.

The literature concluded that this experiment imposes costs. The most important cost is the efficiency loss caused by the inflation tax. Inflation increases the opportunity cost of money, that is, the interest rate. As a result, people substitute away from activities that require cash, such as consumption, for activities that do not require cash, such as leisure. This result is associated with the assumption that revenues obtained from inflation (known as seigniorage) are redistributed back to the public in lump sum form. As lump sum taxes do not affect the opportunity cost of money, it is not possible to counteract the distortionary effect of inflation. It follows that anticipated inflation decreases welfare.

The computation of the welfare cost of inflation has been done for many countries and the results are analogous across them. The U.S. experience of the post-World War II is the one mostly studied. The first

approach to measure the welfare cost of inflation, introduced by Bailey (1956), was to compute the area under the money demand curve. In the first papers, Fischer (1981) and Lucas (1981), found the cost of inflation to be relatively low. Fischer (1981) computes the deadweight loss generated by an increase in anticipated inflation from zero to ten percent as 0.3 percent of GDP using the monetary base as the definition of money. Lucas (1981), conducting the same experiment, places the cost of a ten percent inflation at 0.45 percent of GDP using M1 as the measure of money.

Subsequently, general equilibrium models have been used as an alternative to econometric estimates of the triangle under an estimated money demand curve. Cooley and Hansen (1989) calibrated a cash-in-advance version of a business cycle model. They found that the welfare cost of ten per cent inflation was just below 0.4 per cent of GNP. Thus, the costs of inflation were along the same order of magnitude as suggested in previous studies. More recently, models with variable money velocity have been used. Lucas (1994) and Pakko (1998) discussed the welfare costs of inflation in the context of a shopping time model of money demand and estimated the costs of a ten percent inflation to be about 1.3 percent of the output. Burstein and Hellwig (2008) considered a model with money in the utility function and obtained values similar to the ones found in the shopping time models of money. Silva (2012) adopts a more fundamental approach. When the timing for the portfolio decisions is taken as exogenous, in which case money velocity is constant, the welfare cost of ten percent inflation instead of zero inflation is 0.4 percent of the output, as in Cooley and Hansen. On the other hand, with endogenous timing for the portfolio decisions, money velocity is variable and the welfare cost of ten percent inflation instead of zero inflation increases to 1.3 percent of the output.

However, there are two features of real economies that these models ignore: government consumption and distortionary taxation. These features could be important in assessing the welfare benefits of bringing down inflation because government consumption is a large component of aggregate spending and because lump sum taxation is usually not part of the available fiscal instruments. The welfare cost estimates above ignore the interaction of the inflation tax with other distortionary taxes. We analyse this issue here.

The further apart are the marginal rate of substitution between consumption and leisure and the marginal rate of transformation, the greater the degree of inefficiency in the economy. Inflation introduces a wedge between two fundamental marginal rates: the marginal rate of substitution and the marginal rate of transformation. With lump sum taxation, the percentage change in the wedge is equal to the percentage change in the inflation rate. With distortionary taxation, the wedge depends on the inflation rate and on the tax rate over consumption and labor. Moreover, the wedge increases if the inflation rate or the tax rate increase.

If the fiscal instrument available is a distortionary consumption tax, instead of lump sum taxation, then an increase in inflation allows for a decrease in the distortionary consumption tax. Therefore, in comparison with the case in which lump sum taxation is available, the impact over the wedge is smaller, since the distortionary tax rate and the inflation rate move in opposite directions. This paper confirms this intuition. The welfare costs of inflation are smaller when lump sum taxation is not available.

We consider an endogenous general equilibrium Baumol-Tobin model to quantify the welfare benefits of a reduction in anticipated inflation. The model is similar to the one described in Silva (2012). There is a cash in advance constraint for consumption expenditures, but the timing of the financial transactions is endogenous. Typically, models to tackle this question with variable money velocity have been ad hoc models, with assumptions on shopping time or money in the utility function. Generally, it is assumed that lump sum taxes are available. Instead, we consider the more realistic case, that the only fiscal instruments are distortionary taxes.

The results confirm the intuition. The welfare cost of ten percent instead of zero inflation decreases from 1.3 percent of income with lump sum taxation to 0.8 percent with distortionary taxation. Eighty basis

points of the USA income is 80 billion dollars in 2000 dollars, which is a substantial figure.

The paper is organized as follows. Section 2 presents an example to focus on the intuition of the result. Section 3 specifies the model. Section 4 explains how the steady state equilibrium is determined. Section 5 has the main result: the effect of an increase in inflation. Section 6 concludes.

## 2. Example

The economy has a representative household with preferences over consumption  $c$  and labor  $h$ ,

$$u(c, h) = \log c + \alpha \log(1 - h),$$

where  $\alpha > 0$  is a parameter. As discussed in King et al. (1988), these preferences are compatible with a balanced growth path. There is a cash in advance constraint that requires that consumption expenditures to be done with money

$$c \leq m.$$

Production is linear in labor,

$$y = Ah,$$

where  $A > 0$  is a parameter. Firms pay a wage  $w$  equal to the marginal productivity of labor,

$$w = A.$$

The government satisfies its budget constraint

$$rm + \tau wh + T = g,$$

where  $r$  is the nominal interest rate,  $m$  is real money holdings,  $\tau$  is the tax rate on labor income,  $T$  is the lump sum tax and  $g$  is government consumption. Market clearing implies that the production of the good is equal to its demand,

$$y = g + c.$$

Utility maximization by the household implies equality of the marginal rate of substitution between leisure and consumption, and the real wage, taking into account taxes and the interest foregone of money,

$$\frac{\alpha c}{1 - h} = \frac{w(1 - \tau)}{1 + r}.$$

The Ramsey problem for this economy is to maximize the representative household's utility subject to the government financing condition, the production function constraint and the condition that the ratio between the marginal rate of substitution and the marginal rate of transformation is equal to the distortion caused by the policy instruments. This problem can be formalized as

$$\max \left\{ \log c + \alpha \log(1 - h) \right\}$$

subject to

$$rc + \tau Ah + T = g,$$

$$Ah = g + c,$$

$$\frac{\alpha c}{1 - h} = \frac{1 - \tau}{1 + r}.$$

We consider two cases with different policy instruments available. In the first case, the available instruments are the labor income tax and the interest rate. In the second case, the available instruments are a lump sum tax and the interest rate.

In the first case, with  $T = 0$ , using the first two restrictions of the Ramsey problem we get  $\frac{c}{Ah} = \frac{1-\tau}{1+r}$ . Using this equality and the third restriction of the Ramsey problem we obtain  $h = \frac{1}{1+\alpha}$ . And using the production function we get  $c = \frac{A}{1+\alpha} - g$ . Without loss of generality, we set  $\alpha = 2$ ,  $A = 3/2$ , and  $g = 0.2$ . The solution for the allocation vector  $(c, h)$  is  $(0.3, 1/3)$ . Replacing in the third restriction of the Ramsey problem we get the value for the distortion,  $\frac{1-\tau}{1+r} = 0.6$ . There are many combinations of  $r$  and  $\tau$  that imply  $g = 0.2$  and  $\frac{1-\tau}{1+r} = 0.6$ , and so the same welfare. Three examples: (i)  $r = 0$  and  $\tau = 0.4$ ; or (ii)  $r = 2/3$  and  $\tau = 0$ ; or (iii)  $r = 0.1$  and  $\tau = 0.34$ .

In the second case, with  $\tau = 0$  and lump sum taxes, the solution to the Ramsey problem is not to have any distortion. In other words, the optimal allocation is achieved by setting  $r = 0$  and  $T = 0.2$ . The Friedman rule applies (Friedman 1969). Any other pair  $(\tau, r)$  that satisfies the restrictions in the Ramsey problem is associated with a lower utility level.

Two conclusions can be reached from the analysis above. First, when lump-sum taxes are not part of the policy instruments available, changing the nominal interest rate does not have any welfare effects since the labor income tax rate can be adjusted accordingly. Second, when the labor income tax rate is not available, increasing the nominal interest rate implies a decrease in the lump sum tax and a decrease in welfare. In this case with 0% interest rate there is no distortion between the marginal rate of substitution and the marginal rate of transformation, but with a 10% interest rate, the distortion between the two margins is equal to  $\frac{1}{1.1}$ .

### 3. The Model

We use the general equilibrium Baumol-Tobin model of Silva (2012). Money must be used to purchase goods, only bonds receive interest payments, and there is a cost to transfer the money from bond sales to the goods market. As a result, households accumulate bonds for a certain time and exchange bonds for money infrequently. The infrequent sales of bonds for money occur as in the models of Grossman and Weiss (1983), Rotemberg (1984) and, more recently, Alvarez, Atkeson, and Edmond (2009). The difference is that the timing of the financial transfers is endogenous. We allow for distortionary taxes and inflation tax to finance government consumption.

Time is continuous and denoted by  $t \in [0, \infty)$ . At any moment, there are markets for assets, consumption goods, and labor. There are two assets: money and nominal bonds. The markets for assets and the market for goods are physically separated.

There is an unit mass of infinitely-lived households with preferences over consumption and leisure. Households have two financial accounts, a brokerage account, in which they hold bonds, and a bank account, in which they hold money. We assume that readjustments in the brokerage account have a fixed cost. As only money can be used to buy goods, households need to maintain an inventory of money in their bank account large enough to pay for consumption expenditures until the next transfer of funds.

Firms are perfect competitors and hire labor and capital to produce the consumption good. There is a government, which must finance its expenditures with labor income taxes or seigniorage.

#### 3.1. Firms

At date  $t$ , the firms combine labor  $H_t$  and capital  $K_t$  to produce goods of date  $t$ . The production function is Cobb-Douglas,

$$y_t = AK_t^\theta H_t^{1-\theta}, \quad (1)$$

where  $y_t$  is output,  $A$  is a technological parameter, and  $\theta$  is a parameter,  $0 < \theta < 1$ .

Firms maximize profits, which are given by

$$P_t A K_t^\theta H_t^{1-\theta} - W_t H_t - r_t^k P_t K_t,$$

where  $P_t$  is the price of the good,  $W_t$  is the nominal wage received by the worker, and  $r_t^k$  is the real rental price of capital. As firms are perfect competitors, profit maximization implies demand for labor

$$\frac{W_t}{P_t} H_t = (1 - \theta) y_t, \quad (2)$$

and demand for capital

$$r_t^k K_t = \theta y_t. \quad (3)$$

### 3.2. Government

The government finances consumption expenditures  $g_t$  with taxes on labor income at rate  $\tau$  and seigniorage  $r_t^k m_t$ , where  $r_t$  is the nominal interest rate and  $m_t$  real money holdings.<sup>1</sup> The budget constraint of the government is

$$r_t^k m_t + \tau (1 - \theta) y_t = g_t. \quad (4)$$

### 3.3. Households

As referred above each household has a brokerage account and a bank account. The funds deposited into the brokerage account cannot be used to buy goods but receive nominal interest  $r_t^k$ . Only the money in the bank account can be used to buy goods. The transfer of funds between accounts, as said before, has a real fixed cost  $\gamma$ .

Household  $i$  sells hours of labor  $h_t(i)$  to the firms and rents capital  $k_t(i)$  to the firms. The labor income is then  $W_t (1 - \tau) h_t(i)$  and the rental income is  $r_t^k P_t k_t(i)$ . Labor income and capital income are deposited into the brokerage account. The instantaneous utility function of household  $i$  is

$$u(c_t(i), h_t(i)) = \frac{\left[ c_t(i) (1 - h_t(i))^\alpha \right]^{1-1/\eta}}{1 - 1/\eta},$$

where  $1/\eta$  is the relative risk aversion, and  $\alpha$  the relative preference parameter for leisure  $l_t \equiv (1 - h_t)$ . These preferences are compatible with a balanced growth path (King et al. 1988). Household  $i$  decides consumption  $c_t(i)$ , labor supply  $h_t(i)$ , capital  $k_t(i)$ , the dates when transfers between accounts are made  $T_j(i)$ ,  $j = 1, 2, \dots$ , money holdings in the bank account  $M_t(i)$ , and bond holdings in the brokerage account  $B_t(i)$  so that these allocations solve the problem

<sup>1</sup> We do not consider capital income taxes because, in this economy, it is optimal not to tax capital.

$$\max \sum_{j=0}^{\infty} \int_{T_j}^{T_{j+1}} e^{-\rho t} \frac{\left[ c_t(i) (1 - h_t(i))^\alpha \right]^{1-1/\eta}}{1 - 1/\eta} dt$$

subject to the intertemporal budget constraint

$$\sum_{j=1}^{\infty} Q_{T_j(i)} \left[ M_{T_j(i)}^+ (i) + P_{T_j(i)} \gamma \right] \leq B_0(i) + P_0 k_0(i) + \int_0^{\infty} (1 - \tau) W_t h_t(i) dt,$$

and to the cash in advance constraint

$$M_{T_j(i)}^+ (i) = \int_{T_j(i)}^{T_{j+1}(i)} P_t c_t(i) dt,$$

where  $Q_{T_j(i)}$  is the price at  $t = 0$  of a bond that pays  $T_{j+1}(i)$ , and  $M_{T_j(i)}^+(i)$  denotes money holdings just after  $t = T_j(i)$ . Formally,  $M_{T_j(i)}^+(i) \equiv \lim_{t \rightarrow T_j(i), t > T_j(i)} M_t(i)$ .

The first order conditions of this problem as well as the description of the steady state equations are described in the Appendix.

## 4. Welfare Cost of Inflation

### 4.1. Costs

We define the welfare cost of anticipated inflation by the amount of compensation needed to make households as well off with 10% inflation as they are with zero inflation.

Let  $r$  be the lower nominal interest rate and  $\bar{r}$  the higher nominal interest rate that prevails under a higher rate of inflation. Let  $U(r)$  denote the steady state aggregate intertemporal utility from all households, each with equal weight, when the steady state nominal interest rate is  $r$ . We have<sup>2</sup>

$$U(r) = \frac{1}{N} \frac{1}{\rho} \int_0^N \frac{\left[ c_0 e^{g_c t} \left( (1 - h_0) e^{g_t t} \right)^\alpha \right]^{1-1/\eta}}{1 - 1/\eta} dt.$$

Let  $U(\bar{r}, \Delta)$  denote the steady state intertemporal utility for all households when each household receives a compensation  $\Delta$  and all remaining equilibrium variables are set at their steady state values under the nominal interest rate  $\bar{r}$ . The compensation that makes the households indifferent between  $\bar{r}$  and  $r$  is  $\Delta_r$ , and is defined as

$$U(\bar{r}, \Delta_r) = U(r, 0).$$

<sup>2</sup> It can be shown that  $U(r) = \frac{1}{\rho} \frac{c_0^{1-1/\eta} (1-h_0)^{\alpha(1-1/\eta)}}{1-1/\eta} \frac{e^{(g_c+\alpha g_t)N(1-1/\eta)} - 1}{(g_c+\alpha g_t)N(1-1/\eta)}$ .

## 4.2. Calibration and Results

We set standard values for the parameters. Usually, the estimates for  $\eta$ , the elasticity of intertemporal substitution, are above 0.1 and below 10. We set it equal to 1 the same value used in Silva (2012), Cooley and Hansen (1989) and Cooley and Hansen (1991). The intertemporal discount  $\rho$  is calibrated so that  $r = 3\%$  implies zero inflation. The transfer cost  $\gamma$  is calibrated so that  $m(r)$  matches the U.S. 1900-1997 annual average when  $r$  is the historical average nominal interest rate, 3.64%. That is,  $m(3.64) = 0.26$ . The preference parameter  $\alpha$  is set so that labor hours is 30% of total time. The share of capital income in total income  $\theta$  is set to one third. The depreciation rate  $\delta$  is set to 5%, so that the share of investment in total expenditure is 19%. Government consumption  $g$  is set so that it corresponds to 18% of output when  $r = 3.64$ .

The value of  $\Delta_r$  associated with an increase in inflation from 0% to 10% is equal to 0.8% of the output produced in the economy.

## 5. Conclusions

The demand for money decreases when inflation increases. Therefore, inflation imposes welfare costs because households divert resources to financial services to decrease their demand for money when inflation increases. The households change their demand for money by increasing the frequency of bond trades. In contrast, standard cash-in-advance models assume that the frequency of trades is fixed. Letting the frequency of trades vary implies a more elastic demand for money, a better fit to the data, and a higher estimate of the welfare cost of inflation.

In general, changes in inflation imply reactions on other fiscal instruments such as labor income taxes. That will be the case if the government wants to maintain an unchanged budget deficit. In this case, the other taxes usually decrease. These changes in the fiscal policy instruments have been ignored in the literature because lump sum taxes are assumed to be available.

We make two changes here. First, we consider that lump sum taxation is not available. Second, we take into account that households react to fiscal policy by changing their demand for money. As an increase in inflation that is revenue neutral implies smaller distortionary taxation, the demand for money decreases less and the increase in financial services is smaller than in the case with lump sum taxes. As a result, the welfare cost of inflation is smaller when only distortionary taxes are available.



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### Appendix

Among the first order conditions of the households' problem, we have the intratemporal rate of substitution between leisure and consumption

$$\frac{\alpha c_t}{l_t} = (1 - \tau)w(t)e^{-r(t-T_j)}, \text{ para } t \in [T_j(i), T_{j+1}(i)]$$

where  $w_t \equiv \frac{W_t}{P_t}$ . The growth rates of consumption and leisure for each holding period

$[T_j(i), T_{j+1}(i)]$ ,  $j = 1, 2, \dots$ , are

$$g_c \equiv \frac{\dot{c}}{c} = \frac{\alpha(\eta - 1) - \eta}{1 - \alpha(\eta - 1)}r,$$

e

$$g_l \equiv \frac{\dot{l}}{l} = \frac{1 - \eta}{1 - \alpha(\eta - 1)}r.$$

If  $\eta = 1$ , in particular, then  $g_c = -r$  and  $g_l = 0$  on each holding period, that is, consumption decreases at the nominal interest rate and leisure is constant. Let  $c_0$  and  $h_0$  be the levels of consumption and labor at the beginning of a holding period. We have

$$c_t = c_0 e^{g_c(t-T_j)} \text{ e } 1 - h_t = (1 - h_0) e^{g_l(t-T_j)} \text{ para } t \in [T_j(i), T_{j+1}(i)], j = 1, 2, \dots$$

The first order condition with respect to  $T_j(i)$  implies

$$c_0 \frac{1 - e^{(r+g_c)N_j(i)}}{\eta - 1} - rc_0 \frac{e^{(\pi+g_c)N_j(i)} - 1}{(\pi + g_c)} - (1 - \theta)(1 - \tau)Y \frac{(1 - h_0) \left(1 - e^{g_l N_j(i)}\right)}{1 - (1 - h_0)(1 - \tau) \frac{e^{g_l N_j(i)} - 1}{g_l N_j(i)}} = \gamma(r - \pi),$$

where  $N_j(i) = T_{j+1}(i) - T_j(i)$ .

The first order conditions with respect to bonds and capital imply the standard non arbitrage condition

$$(r_t - \pi_t) = (r_t^k - \delta),$$

which says that the rate of return on bonds, on the left hand side, must be equal to the real return on physical capital, on the right hand side. The households must be indifferent between investing in bonds or capital.

The demand for money at time  $t$  of an household that made  $j + 1$  transfers is  $M_t(i) = \int_t^{T_{j+2}(i)} P_s c_s(i) ds$ , while the demand for money of an agent at time  $t$  that made  $j$  transfers is  $M_t(i) = \int_t^{T_{j+1}(i)} P_s c_s(i) ds$ , for  $j = 1, 2, \dots$ . The aggregate real money demand at date  $t$  is  $m_t \equiv \int_0^1 M_t(i) di / P_t$ .

We want to study the steady state equilibrium. The equilibrium steady state has the properties that holding periods, across households and across time, have the same duration,  $N$ , and that all households behave similarly during their holding periods. Thus, all households readjust their portfolio in the same way, being equal the fraction of households that readjust their portfolio at any moment in this interval. Household  $i \in [0, 1]$ , which initially adjusts the portfolio at date  $n(i) \in [0, N)$ , also readjusts the portfolio at dates  $n(i) + jN$  for  $j = 1, 2, \dots$ .

As we are concerned with the steady state equilibria, we drop the subscript  $t$  from the notation. There are nine independent equilibrium static equations that can be used to determine nine steady state equilibrium variables,  $c_0$ ,  $N$ ,  $\tau$ ,  $m$ ,  $h_0$ ,  $w$ ,  $Y$ ,  $K$ , and  $H$ .

The steady state equilibrium equations are: the production function

$$y = AK^\theta H^{1-\theta},$$

the demand for capital

$$(\rho + \pi)K = \theta y,$$

the demand for labor

$$wH = (1 - \theta)Y,$$

the aggregate supply of hours by households

$$1 - H = (1 - h_0) \frac{e^{g_t N} - 1}{g_t N},$$

the intratemporal condition of households

$$(1 - h_0) = \frac{\alpha c_0}{w(1 - \tau)},$$

the government budget constraint

$$rm + \tau(1 - \theta)y = g,$$

the market clearing condition for goods

$$c_0 \left( \frac{e^{g_c N} - 1}{g_c N} \right) = \left( y - \frac{\gamma}{N} - g - \delta \frac{\theta y}{\rho + \delta} \right),$$

the condition on the choice of the duration of the holding period by households

$$\begin{aligned} & \frac{rN}{\alpha} \left[ \left( \frac{1}{\eta - 1} \right) \left( \frac{e^{rN} - 1}{rN} \right) + \frac{e^{\pi N} - 1}{\pi N} \right] + \frac{\gamma}{w} (r - \pi) \\ & = \left[ r \frac{e^{(\pi + g_h)N} - 1}{(\pi + g_h)N} \left( 1 + \frac{1}{\alpha} \right) + \left( \frac{1}{\eta - 1} - 1 \right) (r + g_h) \left( \frac{e^{(r + g_h)N} - 1}{(r + g_h)N} \right) \right] N h_0, \end{aligned}$$

and the money demand

$$m = \frac{c_0}{g_c + \pi} \left( e^{g_c N} \frac{e^{\pi N} - 1}{\pi N} - \frac{e^{g_c N} - 1}{g_c N} \right).$$