# THE MONETARY TRANSMISSION MECHANISM FOR A SMALL OPEN ECONOMY IN A MONETARY UNION\*1

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#### 1. INTRODUCTION

This paper develops a stylized model of a small open economy integrated in a monetary union. As the small country trades with countries inside and outside of the monetary union, there are three blocks in the model. The small open country, the block represented by all the remaining countries that belong to the monetary union, and the one that includes all countries that do not belong to the monetary union.

As in any monetary dynamic general equilibrium model, in our model too, the behavior of the equilibrium variables is described by a system of difference equations, with as many equations as endogenous variables, and some initial and terminal conditions. In general these additional restrictions to the system are not enough to determine a finite number of solutions.<sup>2</sup> There is, however, a way to obtain a unique locally-bounded equilibrium, that has the property that in its neighbourhood there is no other equilibrium.<sup>3</sup> The procedure is relatively simple, the central bank must follow an interest rate rule that obeys the Taylor principle.<sup>4</sup>

The Taylor principle says that the interest rate rule should be such that the response of the interest rate to a unitary change in the appropriate inflation must be larger than unity. The interest rate relevant for the small country is the one set by the central bank of the monetary union. The central bank of the monetary union follows an interest rate rule that is a function of the union's inflation and output. If the inflation rate in all countries of the union, except the one of the small country, was taken as exogenous the Taylor principle would be violated. The reason is easy to understand. If those union aggregate variables were taken as exogenous, a change in the small country's inflation would imply a negligible change in the interest rate, since the small country contributes little to the union's inflation. Thus, to guarantee local determinacy the variables associated with the countries outside the union can be assumed exogenous, but not the variables associated with the other countries in the union.

To guarantee that the model possesses the local uniqueness property we adopted a straightforward ad-hoc specification of two blocks of equations, each containing three equations, that specify the behavior of some aggregate variables inside and outside the union. One block contains three reduced

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<sup>(1)</sup> There is a working paper version of this article, Adão (2009), that has the technical detailed that this version lacks.

<sup>(2)</sup> J. Cochrane (2007) contains a critical survey of this issue.

<sup>(3)</sup> Besides the local unique equilibrium there are infinitely many explosive equilibria, that cannot be ruled out by transversality conditions.

<sup>(4)</sup> The Taylor principle is described in Taylor (1999). Unlike in less standard models, the Taylor principle is a necessary and sufficient condition to have local determinacy in our model.

form equations, an IS curve, a Phillips curve and an interest rate rule that describe the behavior of output, inflation and the interest rate for the countries outside the union. These three variables are determined entirely inside this block. The other block of equations contains three other similar reduced form equations, regarding the behavior of inflation, output and the interest rate in the union. This block of equations contains three equations and five variables. These variables are: the inflation rate and the output in the small open economy and in the remaining countries of the union, and the interest rate in the monetary union. The arguments in this block of equations associated with the small open economy are scaled down in accordance with the dimension of the small country in the union. These five variables interact with the other variables in the model, and are determined together with them.

Adolfson et al. (2007) developed a model of a small open economy with two blocks only: the small open economy and all the remaining countries. In that context, the Taylor principle is trivially satisfied if foreign inflation, foreign output and the foreign interest rate are taken as exogenously given. This paper can be seen as an extension to their paper, as it considers that the small open economy is integrated in a monetary union. Following their work and the literature, we consider various nominal and real frictions, such as sticky wages, sticky prices, variable capital utilization, capital adjustment costs, habit persistence and volume premium on the foreign interest rate.

Adolfson *et al.* (2007) calibrated and estimated their model using the euro area data. This paper presents a model designed to assess the transmission of monetary policy shocks in a small country integrated in a monetary union. As Portugal can be thought as such an economy, we used the Portuguese data to calibrate several of the parameters of our model. We assume that after the monetary shock, inflation, output and the interest rate outside of the union are unchanged and that inflation, output and the interest rate inside the union change according to the referred three equation block, containing an IS curve, a Phillips curve and an interest rate rule. More specifically, in the quantitative exercise performed we consider parameters for the IS curve, Phillips curve and the interest rate rule that, together with the remaining calibrated equations, generate responses of these variables to the typical euro area monetary shock that mimic the paths of the responses of these variables in the general equilibrium model of Adolfson *et al.* (2007) to the same shock.

Some of the model parameters are calibrated to the steady state values of the Portuguese economy variables, for others we do not have information and they correspond to the estimates obtained (or values assumed) by Adolfson *et al.* (2007) for the euro area. The shape and sign of the impulse responses of the main macro variables to an unanticipated temporary decrease in the nominal interest rate are well in line with the literature. When compared with the euro area, output, employment, investment and real wage in Portugal increase more and inflation adjusts quicker and reacts slightly more on impact. On the other hand, consumption in Portugal has a behavior almost identical to the one of the euro area. In general, economies where inflation adjusts faster in response to a given monetary policy shock, have smaller responses of the real variables to that shock. That is not the case here, since Portugal was assumed to have a higher net foreign debt level and a higher degree of openness than the EMU.

The trade with the two areas responds differently. Due to the change in the exchange rate, the trade with countries outside the euro area changes substantially more than the trade with countries inside the euro area. Both exports and imports to and from the euro area increase, with exports changing less. Imports from outside the euro area decrease initially and exports to outside the euro area increase.

The paper is organized as follows. In Section 2 the model is explained. Section 3 studies the effects of a monetary shock on the Portuguese economy. Section 4 provides some conclusions. The appendix describes how the model is solved and calibrated.

#### 2. MODEL

The model has 3 economic blocks: the small open country, the other countries inside the monetary union and the countries outside the monetary union. We are concerned with the small open economy and assume the developments in the small open economy have small effects over the remaining economic areas. As such, in the model the small open economy is described in detail, but the other economic areas are not.

#### 2.1. Households

There is a representative household in the small open economy, whose preferences over stochastic sequences of consumption  $C_t$  real money  $\frac{M_t}{P_t}$ , and labor  $L_t$  are represented by the utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log \left( C_t - bC_{t-I} \right) + \nu \log \left( \frac{M_t}{P_t} \right) - \xi \frac{L_t^{I+\psi}}{I+\psi} \right] \tag{1}$$

where  $E_0$  is the conditional expectation operator,  $\beta \in (0,1)$  is the discount factor, and b is a parameter that controls the habit persistence. This utility function is similar to the one used by Christiano *et al.* (2005) and Adolfson *et al.* (2007). The aggregate consumption is a bundle given by a CES index of domestically produced and imported foreign goods:

$$C_{t} = \left[ \left( I - \varpi_{o,c} - \varpi_{u,c} \right)^{\frac{I}{\eta_{c}}} + \left( C_{t}^{h} \right)^{\frac{\eta_{c}-I}{\eta_{c}}} + \left( \varpi_{o,c} \right)^{\frac{I}{\eta_{c}}} \left( C_{t}^{o} \right)^{\frac{\eta_{c}-I}{\eta_{c}}} + \left( \varpi_{u,c} \right)^{\frac{I}{\eta_{c}}} \left( C_{t}^{u} \right)^{\frac{\eta_{c}-I}{\eta_{c}}} \right]^{\frac{\eta_{c}}{\eta_{c}}}, \tag{2}$$

where  $C_t^h$  denotes consumption of the home good,  $C_t^o$  denotes consumption of the good produced outside of the union,  $C_t^u$  denotes consumption of the good produced inside of the union,  $\varpi_{o,c}$  is the share of imported consumption from outside of the union in total consumption,  $\varpi_{u,c}$  is the share of imported consumption from inside of the union in total consumption and  $\eta_c$  is the elasticity of substitution between the three consumption goods. Thus, consumers derive utility from the consumption of domestically produced goods as well as from the consumption of goods produced outside and inside of the

union. The price of aggregate consumption (defined as the minimum expenditure required to buy one unit of  $C_t$ ) is given by:

$$P_{t}^{c} = \left[ \left( 1 - \varpi_{o,c} - \varpi_{u,c} \right) (P_{t})^{l - \eta_{c}} + \varpi_{o,c} \left( P_{t}^{o} \right)^{l - \eta_{c}} + \varpi_{u,c} \left( P_{t}^{u} \right)^{l - \eta_{c}} \right]^{\frac{1}{l - \eta_{c}}},$$

where  $P_t$  is the price of the domestically produced good,  $P_t^o$  is the price of the outside of the union imported good and  $P_t^u$  is the price of the inside of the union imported good. All these prices are in units of the domestic currency. Consumers choose quantities of each of these three goods that for a given expenditure maximize aggregate consumption. The individual demands for each good that maximize (2) subject to  $P_tC_t^h + P_t^oC_t^o + P_t^uC_t^u = P_t^cC_t$ , are:

$$C_t^h = \left(I - \varpi_{o,c} - \varpi_{u,c}\right) \left(\frac{P_t}{P_t^c}\right)^{-\eta_c} C_t,$$

$$C_t^o = \varpi_{o,c} \left( \frac{P_t^o}{P_t^c} \right)^{-\eta_c} C_t,$$

and

$$C_t^u = \varpi_{u,c} \left( \frac{P_t^u}{P_t^c} \right)^{-\eta_c} C_t.$$

Also, the aggregate investment is a bundle given by a CES index of domestically produced and imported foreign goods:

$$I_t = \left[ \left( 1 - \varpi_{o,i} - \varpi_{u,i} \right)^{\frac{1}{\eta_i}} \left( I_t^h \right)^{\frac{\eta_{i-1}}{\eta_i}} + \left( \varpi_{o,i} \right)^{\frac{1}{\eta_i}} \left( I_t^o \right)^{\frac{\eta_{i-1}}{\eta_i}} + \left( \varpi_{u,i} \right)^{\frac{1}{\eta_i}} \left( I_t^u \right)^{\frac{\eta_{i-1}}{\eta_i}} \right]^{\frac{\eta_i}{\eta_{i-1}}},$$

where  $I_t^h$  denotes the home good investment,  $I_t^o$  denotes outside of the union investment good,  $I_t^u$  denotes inside of the union investment good,  $\varpi_{o,i}$  is the share of outside of the union investment good in total investment,  $\varpi_{u,i}$  is the share of inside of the union investment good in total investment and  $\eta_i$  is the elasticity of substitution between the three investment goods. The price of aggregate investment is equal to:

$$P_t^i = \left[ \left( 1 - \varpi_{o,i} - \varpi_{u,i} \right) (P_t)^{1 - \eta_i} + \varpi_{o,i} \left( P_t^o \right)^{1 - \eta_i} + \varpi_{u,i} \left( P_t^u \right)^{1 - \eta_i} \right]^{\frac{1}{1 - \eta_i}}.$$

The individual demands for each investment good are

$$I_t^h = \left(1 - \varpi_{o,i} - \varpi_{u,i}\right) \left(\frac{P_t}{P_t^i}\right)^{-\eta_i} I_t,$$

$$I_t^o = \varpi_{o,i} \left( \frac{P_t^o}{P_t^i} \right)^{-\eta_i} I_t,$$

and

$$I_t^u = \varpi_{u,i} \left( \frac{P_t^u}{P_t^i} \right)^{-\eta_i} I_t.$$

Each household is a monopoly supplier of its own labor and can set its wage according to the mechanism described in Calvo (1983), to which we will come back later. In order to guarantee that this friction does not cause households to become heterogeneous we assume complete domestic financial markets against the outcomes of this friction. As a result all households have the same budget constraint:

$$\begin{split} &P_{t}^{c}C_{t} + P_{t}^{i}I_{t} + M_{t} + D_{t} + S_{t}B_{t}^{o} + B_{t}^{u} = P_{t}w_{t}L_{t} + P_{t}(r_{t}u_{t} - a(u_{t}))K_{t-1} + M_{t-1} + D_{t-1}R_{t-1} \\ &+ S_{t}R_{t-1}^{o}\Phi_{o}\bigg(\frac{B_{t-1}}{z_{t-1}}\bigg)B_{t-1}^{o} + R_{t-1}^{u}\Phi_{u}\bigg(\frac{B_{t-1}}{z_{t-1}}\bigg)B_{t-1}^{u} + T_{t} + F_{t}. \end{split} \tag{3}$$

The terms on the left hand side of the equality show how the households use their income and the terms on the right hand side the various sources of that income. Here,  $M_t$  are money holdings,  $D_t$  are deposits that pay nominal gross interest rate  $R_t$ ,  $B_t^o$  are holdings of foreign bonds denominated in foreign currency that pay a nominal gross interest rate  $R_t^o\Phi_o$ ,  $S_t$  is the nominal exchange rate,  $B_t^u$  are holdings of foreign bonds denominated in domestic currency that pay a nominal gross interest rate  $R_t^u\Phi_u$  and  $w_t$  is the real wage. The term  $P_t r_t u_t$  represents the household's earnings from supplying capital services. The function  $a(u_t)K_{t-1}$  denotes the cost of setting the utilization rate of capital to  $u_t$ . We assume  $a(u_t)$  is increasing and convex. These assumptions capture the idea that the more intensely the stock of capital is utilized, the higher are maintenance costs. We assume that  $u_t=1$  in steady state and that a(1)=0, a'>0, and a''>0. The expression  $R_{t-1}^o\Phi_u\left(\frac{B_{t-1}}{z_{t-1}}\right)$  is the level-adjusted gross interest rate on foreign bonds denominated in foreign currency,  $B_t\equiv\frac{S_tB_t^o+B_t^u}{P_t}$ . The term  $z_t$  is a unit root technology shock to be described later. The function  $\Phi_t\left(\frac{B_t}{z_t}\right)$  for t=o,u, is assumed to be strictly decreasing in  $B_t$  and to satisfy  $\Phi_t(\overline{B})=1$ , where  $\overline{B}$  is the steady state value of  $\frac{B_t}{z_t}$ . This function depends on the real holdings of the aggregate foreign assets. This means that domestic households take the functions  $\Phi_t(\cdot)$  as given when deciding on the individual optimal holdings of the foreign bonds.

Functions  $\Phi_i$ try to capture imperfect integration in the international financial markets. If the domestic economy as a whole is borrowing above its steady state, domestic households are charged a premium on the foreign interest rates, if borrowing below its steady state, domestic households pay less. The introduction of this premium is needed in order to ensure a well-defined steady-state in the model (see Schmitt-Grohë and Uribe, 2003, for further details). Without this premium, the stock of bonds and consumption would not be stationary. The remaining variables are  $T_t$  which is a lump-sum transfer, and  $F_t$  that stands for the profits of the firms in the economy.

Investment  $I_t$  induces a law of motion for capital:

$$K_t = (1 - \delta)K_{t-1} + \left(1 - V\left[\frac{I_t}{I_{t-1}}\right]\right)I_t,$$
 (4)

where  $\delta$  is the depreciation rate and V is an adjustment cost function such that  $V\left[\Lambda_i\right]=V'\left[\Lambda_i\right]=0$ , and V''>0, where  $\Lambda_i$  is the growth rate of investment along the balanced growth path. The household chooses  $\left\{C_t, L_t, M_t, D_t, B_t^o, B_t^u, u_t, K_t, I_t\right\}$  to maximize expected lifetime utility (1) subject to constraints (3), (4) and initial values for  $M_0, D_0, B_0^o$ , and  $B_0^u$ .

The labor used by the intermediate good producers is supplied by a representative competitive firm that hires labor to each family *j*. This firm aggregates the differentiated labor of households according to the production function,

$$L_t^d = \left( \int_0^1 L_{jt}^{\frac{\nu^w - 1}{w}} dj \right)^{\frac{\nu^w}{\nu^w - 1}}$$

where  $v^w$  is the elasticity of substitution between the different types of labor and  $L^d_t$  is the aggregate labor demand. This firm maximizes profits taking as given the labor wages  $w_{jt}$  and aggregate labor wage  $w_t$ . Its maximization problem is:

$$\max_{L_{jt}} w_t L_t^d - \int_0^1 w_{jt} L_{jt} dj.$$

The demand for the labor of household j is given by

$$L_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\nu^{w}} L_t^d, \forall j.$$

As referred above households set their wages according to a Calvo's setting, Calvo (1983). In each period, a fraction  $1-\theta^w$  of households can change their wages. All the other households can only partially index their wages to past inflation and past productivity growth. Indexation to past inflation is controlled by the parameter  $\chi^w$  and indexation to past productivity growth by the parameter  $\chi^p$ . Both assume values in the interval [0,1]. Thus, a household that could not change her wage for s periods has real wage  $\times_{\tau=1}^s \frac{\prod^{1-\chi^w} \prod_{t+\tau=1}^{\chi^w} z^{1-\chi^p} z^{1-\chi^p}}{\prod_{t+\tau} w_{jt}} w_{jt}$ , where  $\Pi_t$  is gross inflation of the domestic good in period t,  $\Pi$  is the steady state domestic inflation,  $\bar{z}_t$  is gross productivity growth in period t and  $\bar{z}$  is the steady state productivity growth.

When setting the wage the relevant part of the objective function for the household is,

$$\max_{w_{jt}} E_t \sum_{s=0}^{\infty} \left( \beta \theta^w \right)^s \left[ -\xi \frac{L_{jt+s}^{1+\psi}}{1+\psi} + \lambda_{jt+s} \times_{\tau=1}^{s} \frac{\Pi^{1-\chi^w} \Pi_{t+\tau-1}^{\chi^w} \bar{z}^{-1-\chi^\rho} \bar{z}^{-\chi^\rho}}{\Pi_{t+\tau}} w_{jt} L_{jt+s} \right]$$

subject to

$$L_{jt+s} = \left( \frac{\times_{\tau=1}^{s} \frac{\prod^{1-\chi^{w}} \prod_{t+\tau-1}^{\chi^{w}} z^{1-\chi^{p}} z^{\chi^{p}}}{\prod_{t+\tau}} w_{jt}}{w_{t+s}} \right)^{-\nu^{w}} L_{t+s}^{d}, \forall j.$$

where  $\lambda_{jt+s}$  is the marginal utility of consumption of the domestic good in period t+s of household j.

In each period a fraction  $1-\theta^w$  of the households set  $w_t^*$  as their wage while the remaining fraction index their wage by past inflation. Thus the real wage evolves according to:

$$w_{t}^{1-\nu^{w}} = \theta^{w} \left( \frac{\prod_{t=1}^{1-\chi^{w}} \prod_{t=1}^{\chi^{w}} z^{1-\chi^{\rho}} \overline{z}_{t-1}^{\chi^{\rho}}}{\Pi_{t}} w_{t-1} \right)^{1-\nu^{w}} + \left(1-\theta^{w}\right) \left(w_{t}^{*}\right)^{1-\nu^{w}}.$$

#### 2.2. Final good producer

There is one domestic final good that is produced with intermediate goods:

$$Y_{t} = \left[ \int_{0}^{1} y_{j,t}^{\frac{\nu^{d}-1}{d}} dj \right]^{\frac{\nu^{d}}{\nu^{d}-1}}$$

where  $\nu^d$  is the markup in the domestic goods market. The input demand functions of the final producer are:

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\nu^d} Y_{t,}$$

where  $p_{j,t}$  is the price of intermediate product j and  $P_t$  is the price of the final home good.  $P_t$  is defined as the minimum expenditure in intermediate inputs to produce one unit of final output and is given by

$$P_{t} = \left( \int_{0}^{1} p_{j,t}^{1-\nu^{d}} dj \right)^{\frac{1}{1-\nu^{d}}}.$$

# 2.3. Intermediate producers

There is a continuum of intermediate good producers. Each one has the following technology:

$$\mathbf{y}_{j,t} = \mathbf{A}_t \mathbf{k}_{j,t-1}^{\alpha} \mathbf{1}_{j,t}^{-\alpha} - \rho \mathbf{z}_{t,t}$$

where  $A_t$  is a total productivity technological shock that follows an autoregressive process:

$$A_t = A_{t-1} \exp(\Lambda_A + z_{At})$$
, where  $z_{At} = \sigma_A \epsilon_{At}$ ,  $\epsilon_{At} \sim N(0,1)$ 

and

$$z_t = A_t^{\frac{1}{1-\alpha}}.$$

The parameter  $\rho$  corresponds to the fixed cost of production that guarantees that economic profits are zero in the steady state. We have:

$$z_t = z_{t-1} \exp(\Lambda_z + z_{z,t})$$
, where  $z_{z,t} = \frac{z_{A,t}}{1-\alpha}$  and  $\Lambda_z = \frac{\Lambda_A}{1-\alpha}$ .

Intermediate producers solve two problems. First, given  $w_t$  and  $r_t$ , they rent labor and capital in perfect competitive markets in order to minimize real expenditure.

The second problem intermediate producers must solve is to choose the price that maximizes expected discounted real profits. Firms set prices according to a Calvo set-up too. In each period, a fraction  $1-\theta$  of firms can choose optimally their prices. The price chosen in period t is denoted by  $p_t$ . We suppressed the firm's indexation because all firms that have the opportunity to choose the price set the same price. The remaining  $\theta$  firms cannot choose the price. In period t these firms update their price to  $p_{j,t-1}\Pi^{1-\chi^d}\Pi^{\chi^d}_{t-1}$ , where  $p_{j,t-1}$  is the price firm j was charging in period t-1,  $\Pi_{t-1}$  is past inflation of the domestic good,  $\Pi$  is the steady state domestic inflation and  $\chi^d$  is an indexation parameter. The indexation parameter,  $\chi^d$  assumes values in the interval [0,1]. Each firm uses the stochastic discount factor  $\theta \lambda_t$  to compute the value of its profits.

When a firm can choose its price at date *t* its problem is:

$$\max_{\rho_t} E_t \sum_{s=0}^{\infty} (\beta \theta)^s \lambda_{t+s} \left\{ \left( \frac{\mathbf{x}_{\tau=1}^s \Pi^{1-\chi^d} \Pi_{t+\tau-1}^{\chi^d} \rho_t}{P_{t+s}} - mc_{t+s}^d \right) \mathbf{y}_{j,t+s} \right\},$$

where  $mc_t^d$  denotes the marginal cost, subject to

$$\mathbf{y}_{j,t+s} = \left(\frac{\mathbf{x}_{\tau=1}^{s} \Pi^{1-\chi^{d}} \Pi_{t+\tau-1}^{\chi^{d}} \mathbf{p}_{t}}{P_{t+s}}\right)^{-\nu^{d}} \mathbf{Y}_{t+s}.$$

#### 2.4. Central bank

The central bank sets the nominal interest rate according to the Taylor rule:

$$\frac{R_t}{R} = \left[\frac{R_{t-1}}{R}\right]^{\gamma_R} \left[ \left( \frac{\Pi_t^u}{\Pi^u} \right)^{1-\varsigma} \left( \frac{\Pi_t}{\Pi} \right)^{\varsigma} \right]^{\gamma_\Pi} \left[ \left( \frac{Y_t^u}{Y^u} \right)^{1-\varsigma} \left( \frac{Y_t}{Y} \right)^{\varsigma} \right]^{\gamma_Y} \right]^{(1-\gamma_R)} \exp(m_t)$$

where  $m_t$  is a random shock to the monetary policy that follows  $m_t = \sigma_m \varepsilon_{m,t}$ , where  $\varepsilon_{m,t} \sim N(0,1)$ . Variable  $\Pi^u$  is the target level for the inflation in the union, which is equal to the steady state inflation in the union,  $\Pi^u_t$  is the inflation in period t in the union without including the small open country,  $Y^u$  is the steady-state output in the union,  $Y^u_t$  is the output in period t in the union without including the small open country.

The parameter  $\varsigma$  is the weight of the small open country in the union, while  $\gamma_R$ ,  $\gamma_\Pi$ , and  $\gamma_\Pi$  are the usual parameters of the Taylor rule.

#### 2.5. Government

The budget constraint of the government in the small open economy is:

$$P_tG_t+T_t=M_t-M_{t-1},$$

where  $G_t$  is government consumption, which includes only domestic produced goods and we take as exogenous. The other variables are taxes,  $T_t$  and money supply,  $M_t$ .

#### 2.6. Evolution of net foreign assets

The evolution of net foreign assets at the aggregate level satisfies:

$$S_{t}B_{t}^{o} + B_{t}^{u} = S_{t}R_{t-1}^{o}\Phi_{o}\left(\frac{B_{t-1}}{z_{t-1}}\right)B_{t-1}^{o} + R_{t-1}^{u}\Phi_{u}\left(\frac{B_{t-1}}{z_{t-1}}\right)B_{t-1}^{u} + TB_{t}$$

The trade balance is  $TB_t = P_t X_t^u + P_t X_t^o - P_t^u M_t^u - P_t^o M_t^o$ .

The total imports from the union are  $M^u_t = \varpi_{u,c} \Big( \frac{P^u_t}{P^c_t} \Big)^{-\eta_c} C_t + \varpi_{u,i} \Big( \frac{P^u_t}{P^i_t} \Big)^{\eta_i} I_t$  and the total imports from outside of the union are  $M^o_t = \varpi_{o,c} \Big( \frac{P^o_t}{P^c_t} \Big)^{-\eta_c} C_t + \varpi_{o,i} \Big( \frac{P^o_t}{P^o_t} \Big)^{\eta_i} I_t$ . We assume that the demands from the other countries for the product produced domestically have the same functional form as the demands by the domestic consumers. Thus, the total exports to the union are  $X^u_t = \varpi_{u,u} \Big( \frac{P_t}{P^u_t} \Big)^{-\eta^{u,x}} Y^u_t$  and the total exports to outside of the union are  $X^o_t = \varpi_{o,o} \Big( \frac{P_t}{P^o_t} \Big)^{-\eta^{o,x}} Y^o_t$ , where  $\varpi_{u,u}$  and  $\varpi_{o,o}$  are shares, and  $\eta^{u,x}$  and  $\eta^{o,x}$  are elasticity parameters. The variables  $Y^u_t$  and  $Y^o_t$  denote the output of the other countries in the union and the output of the countries outside of the union, respectively.

# 2.7. Relative prices

When deciding their consumption and investment baskets agents in the small open economy use the following relative prices:  $\xi_t^{c,d} \equiv \frac{P_t^c}{P_t}$  and  $\xi_t^{i,d} \equiv \frac{P_t^i}{P_t}$ . To decide imports consumers use two relative prices: the relative price between imports from the union and the domestically produced good  $\xi_t^{u,d} \equiv \frac{P_t^u}{P_t}$  and the relative price between imports from outside of the union and the domestically produced good  $\xi_t^{o,d} \equiv \frac{P_t^o}{P_t}$ .

From the definitions of prices we have:

$$\left(\xi_t^{c,d}\right)^{\!1-\eta_c} = \left(1-\varpi_{o,c}-\varpi_{u,c}\right) + \varpi_{o,c}\!\left(\xi_t^{o,d}\right)^{\!1-\eta_c} + \varpi_{u,c}\!\left(\xi_t^{u,d}\right)^{\!1-\eta_c},$$

$$\left(\boldsymbol{\xi}_t^{i,d}\right)^{1-\eta_i} = \left(1 - \boldsymbol{\varpi}_{Q,i} - \boldsymbol{\varpi}_{U,i}\right) + \boldsymbol{\varpi}_{Q,i} \left(\boldsymbol{\xi}_t^{Q,d}\right)^{1-\eta_i} + \boldsymbol{\varpi}_{U,i} \left(\boldsymbol{\xi}_t^{U,d}\right)^{1-\eta_i}.$$

#### 2.8. Aggregation

The aggregate demand in the small open economy is:

$$Y_t = C_t^h + I_t^h + a(u_t)K_{t-1} + G_t + X_t,$$

where  $C_t^h$  and  $I_t^h$  denotes consumption and investment of the home good. Thus, the demand for each intermediate good producer is:

$$y_{i,t} = \left(C_t^h + I_t^h + a(u_t)K_{t-1} + G_t + X_t\right) \left(\frac{p_{i,t}}{P_t}\right)^{-\nu^d}, \forall i.$$

Using the production function and market clearing we obtain:

$$A_t(u_t K_{t-1})^{\alpha} L_t^{1-\alpha} - \rho z_t = C_t^h + I_t^h + a(u_t) K_{t-1} + G_t + X_t.$$

#### 2.9. Rest of the world

The rest of the world is composed by two regions: the remaining countries in the union and the countries outside of the union. The output, inflation and the interest rate in the union and outside of the union are given by two blocks one for each region, each containing three equations: an IS equation, a Phillips equation and an interest rate equation. More specifically,

$$Y_t^k = f_Y^k (Y_{t-1}^k, Y_{t+1}^k, \Pi_{t+1}^k),$$

$$\Pi_t^k = f_{\Pi}^k (\Pi_{t-1}^k, \Pi_{t+1}^k, Y_{t+1}^k),$$

and

$$R_t^k = f_R^k \Big( R_{t-1}^k, \Pi_t^k, Y_t^k \Big),$$

for  $Y_t^k = Y_t^o$  or  $Y_t^k = (1 - \varsigma)Y_t^u + \varsigma Y_t$ ,  $\Pi_t^k = \Pi_t^o$  or  $\Pi_t^k = (1 - \varsigma)\Pi_t^u + \varsigma \Pi_t$ , and  $R_t^k = R_t^o$  or  $R_t^k = R_t$ . The parameter  $\varsigma$  is the size of the domestic country in the union.

#### 2.10. Equilibrium

The definition of equilibrium for this economy is standard. It is a vector of prices, policy variables and quantities that satisfies certain conditions. These conditions are the following:

- The conditions that solve the households problem;
- The conditions that solve the firms problem;
- The government's budget constraint;
- The budget constraint with the foreign sector,
- The IS, Phillips and interest rate equations for each region,
- · The markets clearing conditions.

#### 3. IMPULSE RESPONSE FUNCTIONS

After solving and calibrating the model, which we describe in the appendix, we are ready to study the impulse response functions of the various variables to a monetary policy shock. The shock considered is a white noise shock to the nominal interest rate rule.

Except for inflation, the nominal interest rate and the interest rate premium, which are reported as annualized quarterly rates, the graphs associated with the impulse response functions have on the y-axis the percentage deviations of the variables from their steady state values.

There is a feature on the impulse response functions worth noting. As can be seen from Chart 1, with the exception of the variables associated with trade, most of the variables respond in a hump-shaped form, peaking after 3 or 4 quarters and returning to the preshock levels after about three years. The exceptions are the nominal exchange rate, some relative prices and the different imports and exports.

The euro area variables: the nominal interest rate, inflation and output replicate closely the path of the impulse responses of these variables to a monetary shock in the Adolfson *et al.* (2007) model. On impact the nominal interest rate drops by 40 basis points and returns to the steady state four years later. Both output and inflation have hump-shaped responses and achieve their peaks after about one year, around 0.3 percent of the steady state for output and around 10 basis points (annualized) for inflation.

The small open economy responses to the monetary policy shock are roughly similar to the ones obtained in a standard closed economy, even though the model possesses an additional channel of monetary policy transmission. We summarize these responses now. Because prices are sticky, the unexpected decrease in the nominal interest rate implies a decrease in the real interest rate. A lower real interest rate makes bonds less attractive than investment, which leads to an increase in investment. As the stock of capital increases the marginal productivity of labor increases also and firms increase their labor demand.

The temporary lower real interest rate has intertemporal substitution effects on consumption and labor supply. It makes present consumption and present leisure (since nominal wages are sticky) relatively less expensive, which lead households to increase consumption and decrease labor supply. The changes in the labor supply and labor demand lead to an increase in the real wage. Output increases, since consumption and investment increase, and capital utilization increases because there are costs in adjusting capital. As consumption and investment increase the demand for imports increases.

The additional channel for the monetary policy transmission associated with an open economy compels firms to increase production too. In a closed economy households can only smooth out the path of consumption by varying the path of investment. But in an open economy households have another alternative to accomplish that, the possibility of changing the path of net exports. The monetary shock considered leads to an increase in the domestic income. In order to smooth their path of consumption and leisure households increase their net foreign assets. This behavior of foreign assets implies an increase in net exports and a further increase in the output. After the shock, the stock of net foreign assets is above its steady state value for about four years.

There is an additional income effect in Portugal that is absent in the Adolfson *et al.* (2007) calibrated model of the EMU. They assumed that net foreign assets of the EMU in the steady state were zero, instead we assumed that the ratio between the steady state net foreign debt and GDP for Portugal, and the ratio between steady state euro denominated and total net foreign debt were equal to those ratios for Portugal in 2007. Given that most of the net foreign debt of Portugal is denominated in euro, the impact of a drop in the euro interest rate is more favorable for Portugal than for the EMU. For this reason alone, it should be expected that investment and output in Portugal would vary more than investment and output in the EMU.

The degree of openess is relevant too. The more open an economy is, the less it has to rely on investment to partially insulate consumption from economic disturbances, as it has access to more foreign assets, that can be used in addition to investment to smooth out consumption. For that reason, output in a more open economy tends to be more volatile, as the economy can take advantage of the temporary shocks it faces, by increasing production when the shock is positive or decreasing production when the shock is negative, without having to change consumption too much (*i.e.* at a lower cost), than a less open economy. Since the EMU has a smaller degree of openness than Portugal, it should be expected that in reaction to a positive temporary shock, like the monetary policy shock considered, consumption would increase less and output more in Portugal than in the EMU.

The output increases in percentage deviations from the steady state a little more in Portugal than in the euro area. The impulse response function for the output in Portugal is almost all the time above the one for the EMU. For Portugal the maximum response is just over 0.3 and in the EMU it is just below 0.3. The investment, employment and real wage in Portugal and in the EMU, as computed by Adolfson *et al.* (2007), have similar shapes but in Portugal those variables move more. The impulse response functions for investment, employment and real wage in Portugal are almost all the time above the ones for the EMU. For Portugal the maximum response for investment is just about 0.6 for employment is

about 0.25, and for the real wage about 0.1, while for the EMU the maximum response for investment is about 0.5 for employment is about 0.2 and for the real wage is about 0.07.

Inflation in Portugal responds quicker to the shock than inflation in the euro area, on impact it increases by more and returns faster to the steady state. The maximum increase of inflation in Portugal is 16 basis points while that maximum in the EMU is around 10 basis points. This should be associated with the fact that we assumed that in Portugal each quarter 20 percent of the firms change their prices optimally, while Adolfson *et al.* (2007) took that only 10 percent of the firms in the EMU change their prices optimally.

The relative price of the euro area's good is persistently below the steady state in response to the shock due to the referred differentiated behavior of inflation in Portugal and in the euro area. This fact, together with the shock having an impact in the output relatively higher in Portugal than in the euro area, implies that imports from the euro area increase more than exports to the euro area, in response to the shock.

Even though the effects of the shock are stronger in Portugal than in the euro area, consumption in Portugal and in the euro area have almost identical paths. This indicates that households in Portugal are successfully using the available saving instruments to smooth out consumption.

Now we interpret the behavior of the exchange rate. From the first order conditions of the households we obtain the uncovered interest rate parity (UIP) condition, which says that the expected depreciation of the euro is equal to the current interest rate in the monetary union minus the interest rate outside of the union minus the change in the volume premium. The UIP does not restrict the depreciation rate of the euro in the impact period. Apart from the impact period, the nominal exchange rate behavior is the one implied by the UIP. The foreign interest rate is unchanged and the volume premium changes little, as the adjustment costs of the asset stocks were assumed to be relatively small. Thus, just after the impact period, from the second to the fifth quarter the euro appreciates as the decrease in the interest rate is larger in absolute value than the decrease in the volume premium on the foreign debt, and after the sixth quarter it depreciates, little, as the opposite happens.

In the impact period the currency depreciates because, as we observed before, net exports must increase to smooth out consumption. Net exports to the euro area did not increase due to the evolution, described above, of the main aggregate variables in Portugal and in the euro area. Thus, in order for net exports to increase, net exports to outside of the union must increase. That is possible only if the relative price of the good produced outside of the union increases, given that we assumed that the main aggregate economic variables outside of the union were constant. The current relative price of the good produced outside of the union is equal to the price in the previous period plus the current depreciation of the euro plus the differential of inflation between the outside of the euro area and Portugal. For the relative price of the good produced outside of the union to be persistently above its steady state, it is necessary that on impact the euro depreciates sufficiently to compensate its subsequent ap-

preciation, and the persistent domestic inflation. Summing up, in the short run, lower real interest rates in the euro area tend to reduce the foreign exchange value of the euro, which lowers the relative prices of the goods produced in Portugal and in the euro area. This leads to higher outside of the union aggregate spending on goods and services produced in Portugal and in the euro area.

The behavior of the net foreign assets impulse response function, which reflects the evolution of the net exports, is a result of the households choice to smooth out consumption through time. It has an hump-shaped pattern, achieves its peak after four quarters at about 0.18 of the steady state, and returns to the steady state after 4 years.

With respect to the impulse response functions of the exports and imports there is a striking difference between those to and from inside the euro area and those to and from outside of the euro area. Exports and imports to and from the euro area increase but imports increase more, imports achieve a maximum of 0.42 percent above the steady state while exports achieve a maximum of 0.24 percent above the steady state. As referred already, this behavior is explained by two facts. First, because output in Portugal increases more than in the euro area. Secondly, because the relative price of the good produced in Portugal increased due to the fact that inflation in Portugal is slightly above the euro area's inflation. As we noted previously too, the trade with the countries outside of the euro area evolves in a very different way. That is in part explained by the path of the relative price of the good produced outside the euro area, which jumps up on impact and returns with some persistence to its steady state 6 quarters after the shock. Both exports and imports to and from countries outside of the euro area change substantially. Exports on impact are about 0.54 percent above the steady state and imports on impact are about 0.32 percent below the steady state. Imports from outside of the union achieve its maximum when investment achieves its maximum, 5 quarters after the shock.

In our model the effects over the euro exchange rate are smaller than in Adolfson *et al.* (2007). In our model on impact the euro depreciates by about 0.4 percent from the steady state, while in Adolfson *et al.* (2007) it depreciates by about 0.5 percent from the steady state. Most likely, if we were to change some of the parameters in our model in order to get that higher value for the depreciation of the euro, the effects over many of the real variables like output, investment and real wages, which are already bigger in Portugal than in the union, would be further augmented.

#### 4. CONCLUSIONS

In this paper we introduce modifications in the benchmark open economy monetary business cycle model of Adolfson *et al.* (2007) in order to incorporate a small country that trades with countries inside and outside of the monetary union to which it belongs. To guarantee local determinacy the variables associated with the countries outside the union can be assumed exogenous, but not the variables associated with the other countries in the union. As the interest rate rule for the union depends on the inflation and output of the union, if we were to take these variables as exogenous the interest rate would not obey the Taylor principle and there would not be a unique local equilibrium. To surpass this difficulty

we assumed that inflation, output and interest rate inside the union change according to a three equation block, containing an IS curve, a Phillips curve and an interest rate rule. The parameters for these three equations were chosen so that these equations, together with the remaining conditions of the model, could deliver the European Union's inflation, output and interest rate impulse response functions to a monetary shock, obtained by Adolfson *et al.* (2007).

We use the model to study the monetary transmission mechanism in Portugal. There are various findings. The shape and sign of the responses of the variables are similar to the ones obtained in the literature for the euro area. It seems that real variables in Portugal adjust more. When compared with the euro area, the output, investment, real wage and employment in Portugal expand more in response to an unexpected decrease in the interest rate. Inflation in Portugal adjusts quicker and reacts further on impact. The trade with the two areas responds differently to the referred monetary shock. Trade with countries inside the euro area increases, as both exports and imports increase. Imports from countries outside the euro area decrease in the first two quarters, increasing thereafter, while exports to countries outside of the euro area increase substantially on impact.

It could be worthwhile to conduct more empirical work in the context of this model. Different behaviors for the aggregate variables of the countries inside and outside of the euro area can be considered. For instance, to assume that the equations that determine the evolution of these variables are the ones given by an estimated VAR. Another dimension that can be explored is the estimation of some of the parameters of the model using Bayesian methods, as Adolfson *et al.* (2007) do.<sup>6</sup>

The model has many frictions, but is simplistic in various dimensions. As such it could be interestingly extended in various directions. It could incorporate government debt and non-Ricardian households so that fiscal policy could interact with the monetary policy in a less trivial way. It could incorporate a financial sector to study the so called financial accelerator channel of the monetary policy. It could consider the labor market as a more complex market allowing for unemployment. More sectors of production could be considered, in particular the nontradable good sector.

<sup>(6)</sup> According to Canova (2007) in general the estimation of this type of models is problematic for many reasons, but specially because it is prone to identification problems.

#### **REFERENCES**

- Adão, B. (2009), "The Monetary Transmission Mechanism for a Small Open Economy in a Monetary Union", *Working Paper*, Banco de Portugal.
- Adolfson, M., S. Laseen, J. Linde, and M. Villani (2007), "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-through", *Journal of International Economics*, Elsevier, vol. 72(2), pages 481-511, July.
- Altig, D., L. J. Christiano, M. Eichenbaum, and J. Linde (2003), *The Role of Monetary Policy in the Propagation of Technology Shocks*. Manuscript, Northwestern University.
- Backus, D., P. Kehoe, and F. Kydland (1994), "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?", *American Economic Review*, 84, 84-103.
- Calvo, G. (1983), "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics*, vol. 12, p. 383-398.
- Canova, F. (2007), "Methods for Applied Macroeconomic Research", Princeton University Press.
- Cochrane, J. (2007), "Inflation Determination with Taylor Rules: A Critical Review", University of Chicago, *mimeo*.
- Collard, F., and H. Dellas (2002), "Exchange Rate Systems and Macroeconomic Stability", *Journal of Monetary Economics*, 49, p. 571-599.
- Christiano, L., M. Eichenbaum, and C. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy*, vol. 113, no. 1.
- Christoffel, K., G. Coenen, and A. Warne (2008), "The New Area-Wide Model of the Euro Area: A Micro-Founded Open-Economy Model for Forecasting and Policy Analysis", ECB *Working Paper*.
- Deardorff, A., and R. Stern (1990), *Computational Analysis of Global Trading Arrangements*, Ann Arbor, University of Michigan Press.
- Martins, F. (2006), "The Price Setting Behavior of Portuguese Firms Evidence From Survey Data", Working Paper 4-06, Banco de Portugal.
- Smets, F., and R. Wouters (2003), "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area", *Working Paper* 171, ECB.
- Stern, R., J. Francis, and B. Schumacher (1976), *Price Elasticities in International Trade: An Annotated Bibliography*, Toronto, Macmillan.
- Taylor, J. B. (1999), "An Historical Analysis of Monetary Policy Rules", in *Monetary Policy Rules*, ed. by J. B. Taylor, Chicago, University of Chicago Press, pp. 319-341.
- Whalley, J. (1985), Trade Liberalization Among Major World Trading Areas (Cambridge, Mass.: MIT Press).

#### **APPENDIX**

#### 1. Solving the model

The equilibrium conditions can be described by a set of difference equations, with the same number of endogenous variables, see Adão (2009). We want to solve this system of equilibrium equations. However, there are two main difficulties in determining the solution. First, since there is growth in the model, there are variables that are growing while others are stationary. Thus, to solve the model we need to make all the variables stationary. Second, the equilibrium equations are non-linear difference equations and typically their solution is not trivial. The usual procedure involves simplifying each equation of the system. Each equation of the system is approximated by a linear equation. More specifically, each equation is replaced by its first order Taylor approximation, and that approximation is taken around the equilibrium steady state.

To find the steady state it is necessary to give functional forms to a(.), V(.) and  $\Phi(.)$ . We take  $a(u)=\gamma_1\,(u-1)+\frac{\gamma_2}{2}\,(u-1)^2$ . In the steady state we have u=1, then a(1)=0 and  $a'(1)=\gamma_1$ . The investment adjustment cost function is given by  $V\left[\frac{I_t}{I_{t-I}}\right]=\frac{k}{2}\left[\frac{I_t}{I_{t-I}}-\Lambda_I\right]^2$ . Thus, in the steady state,  $V\left[\Lambda_I\right]=V'\left[\Lambda_I\right]=0$ . Finally the volume premium factor is given by  $\Phi_i(\overline{B}_t)=\exp\left(-\phi_i(\overline{B}_t-\overline{B})\right)$  at the steady state  $\Phi_i(\overline{B})=1$  for i=u,o. To obtain a system in stationary variables we redefine the variables, see Adão (2009). The transformation of the original system of equations into a system of equations with stationary variables is involved but trivial.

The next step is to loglinearize the system of equilibrium conditions for the stationarized variables around the deterministic steady state. Once that is done we obtain a system of various difference equations. Let  $state_t$  denote the vector of endogenous state variables,  $nstate_t$  denote the vector of endogenous non-state variables and  $exo_t$  the vector of exogenous variables. The system obtained has the format:

$$AA*state_t + BB*state_{t-1} + CC*nstate_t + DD*exo_t = 0,$$

$$E_t \binom{FF*state_{t+1} + GG*state_t + HH*state_{t-1}}{+JJ*nstate_{t+1} + KK*nstate_t + LL*exo_{t+1} + MM*exo_t} = 0,$$

where the symbols with two equal capital letters denote matrices.

There are a few available algorithms designed to solve this type of difference equations system. We used the one developed by Uhlig (1995). Uhlig's algorithm enables us to write all variables as linear functions of the vectors  $state_{t-1}$ , and  $exo_t$ , which are given at date t. More formally, it gives us matrices P, Q, R and S so that the equilibrium described by the recursive equilibrium law of motion,

$$state_t = P*state_{t-1} + Q*exo_t$$

and

$$nstate_t = R*state_t + S*exo_t$$
,

is stable.

#### 6.2. Calibration

Most of the parameters can be related to the steady state values of the variables in the model and therefore, can be calibrated so as to match the sample mean of these. The remaining were taken from Adolfson *et al.* (2007). Proceeding in this way we minimized the dimensions in which Portugal is different from the euro area. That makes it easier to identify the causes for the differences between the results obtained for Portugal and the ones for the euro area obtained by Adolfson *et al.* (2007).

Many of the parameters were calibrated using the "Quarterly Series for the Portuguese Economy" data set, which refers to the period 77:Q1-07:Q4. This data set is included in the *Economic Bulletin* of the Bank of Portugal, Summer 2008, and is available online. For the period that starts in 1999, the year in which Portugal entered the European Monetary Union, and ends in 2007,  $per\ capita$  Private Consumption grew at 0.29 percent quarterly,  $per\ capita$  Public Consumption grew at 0.32 percent quarterly,  $per\ capita$  Investment grew at -0.11 percent quarterly,  $per\ capita$  GDP grew at 0.25 percent quarterly,  $per\ capita$  Exports grew at 1.05 percent quarterly and  $per\ capita$  Imports grew at 0.79 percent quarterly. We considered the average growth rate of  $z_t$  to be 0.25 percent quarterly, which is the growth rate of real GDP  $per\ capita$  in the period 99:Q1-07:Q4.

The stock of capital was computed using the perpetual inventory method. The quarterly aggregate depreciation rate,  $\delta$ , obtained was 0.011. Adolfson *et al.* (2007) set it equal to 0.013.

For many reasons it is difficult to determine the steady state real interest rate relevant for the representative Portuguese consumer. The average real interest rate measured by the ratio between the 3 month money market interest rate and the realized inflation rate was 0.041 percent quarterly in the period 99:Q1-07:Q4.8 The usefulness of this real interest rate is problematic as it implies a discount factor β, the ratio between the average growth rate and the average real interest rate, larger than one. As such we discarded it and considered alternatives. The other available nominal interest rate series are implicit interest rates. They are obtained either by dividing interest received on bank deposits by bank deposits or by dividing interest received on bank loans by the bank loans. The interest rates on deposits we discard as they also imply a discount factor larger than one. Among the interest rates on credit, the mortgage interest rate is our preferred, as it is the lowest and the most relevant for the representative consumer. The average real interest rate measured by the difference between the implicit mortgage interest rate and the realized inflation rate was 0.46 percent quarterly in the period 99:Q1-07:Q4.

<sup>(7)</sup> The perpetual inventory method is a method of estimating a country's total capital stock, using the level of real investment in each year. Investment is classified by type of capital goods, which we denote by j, such as buildings, machinery, and vehicles. Assuming an appropriate rate of depreciation  $\delta_{i,t}$  for each type of capital, the law of motion of the different types of capital,  $K_{j,t}$ , is  $K_{j,t} = (1-\delta_{j,t})K_{j,t-1} + I_{j,t}$ . Notice that if each type the investment series is sufficiently long, the initial stock of capital is irrelevant, as it is already completely depreciated, to determine the more recent values of the stock of capital. The aggregate stock of capital is obtained by aggregating the various types of capital.

<sup>(8)</sup> This money market interest rate series is the 3-month EURIBOR, and the inflation used was the GDP deflator growth.

This real interest rate together with assumed growth rate of  $z_t$  implies a  $\beta$  equal to 0.998. Adolfson *et al.* (2007) consider a similar value of  $\beta$  for the euro area, 0.999.

Following Adolfson *et al.* (2007), we set the labor supply elasticity,  $\psi$ , to 1 and the habit parameter to 0.65. Christiano *et al.* (2005) consider similar values for these parameters. The constant associated with labor in the utility function,  $\xi$  is chosen so that in the steady state agents work 30 percent of their time. Adolfson *et al.* (2007) assume agents work 30 percent of their time while Chari *et al.* (2002) assume agents work 25 percent of their total time in steady state.

We considered labor income to be the sum of *Remunerações do Trabalho* plus *Contribuições para a Segurança Social* and capital income to be all the remaining domestic income. <sup>9</sup> We took as the value of  $\alpha$ , the sample mean of the ratio between non-labor income and domestic income, which for the period 99:07 is 0.27. The value that Adolfson *et al.* (2007) consider to be the share of capital for EMU is 0.29.

The share of imports in the main components of the domestic expenditure were obtained from the national input-output matrices of INE. That calculation was done for the period 1996-1999. During that period the average share of private consumption that was imported was 27 percent and for the same period the average percentage of investment that was imported was 33 percent. The sample mean, for the period 99:1-07:4, of the share of imports from the euro area was 66 percent and the share of imports from countries outside of the euro area was 34 percent. The percentage of imported consumption from the union was assumed to be proportional to the ratio between total imports from the euro area and aggregate imports. Thus, the share,  $\varpi_{uc}$ , was calibrated to match the sample mean, 0.18. Under this assumption, the other parameters,  $\varpi_{u,i}$ ,  $\varpi_{o,c}$  and  $\varpi_{o,i}$ , were set to 0.22, 0.09 and 0.11. For the period, 99:1-07:4, the ratio between the price of investment and the GDP deflator, and the ratio between the price of consumption and the GDP deflator, which we denoted by  $\xi^{i,d}$  and  $\xi^{c,d}$  respectively. averaged 0.98 and 0.99. The values of the elasticities  $\eta_c$  and  $\eta_i$  could be determined if we had information on the relative prices  $\xi^{o,d}$  and  $\xi^{u,d}$ , which we do not have. Studies seem to indicate that for the United States and Europe the elasticity between home goods and foreign goods is between 1 and 2, and values in this range are generally used in empirical trade models. We set  $\eta_c = 1.5$  and  $\eta_c = 1.6$ , which are the estimates obtained by Adolfson *et al.* (2007) for the euro area, and set both  $\eta^{u,x}$  and  $\eta^{o,x}$ equal to 1.5.

The evidence from survey data, as described in Martins (2006), indicates that the frequency of price changes by Portuguese firms is 1.9 times per year. And of those firms that change prices only about 42 percent use forward looking information to set their price. This implies that in each quarter about 20 percent of the firms change their prices optimally. This value is higher than the one estimated by Smets and Wouters (2003) and Adolfson *et al.* (2007) which is around 10 percent, but lower than the one esti-

<sup>(9)</sup> As the national accounting data of GDP includes net indirect taxes, these need to be subtracted from the GDP to obtain the domestic income.

<sup>(10)</sup> For the US see, for example, the survey by Stern et al. (1976). For Europe see, for example, the discussions of Collard and Dellas (2002), Whalley (1985, Ch. 5) and Deardorff and Stern (1990, Ch. 3). In similar studies, for the United States, Chari et al. (2002) and Backus et al. (1994) set the substitution elasticity between foreign and domestic investment goods equal to 1.5, and for Europe Christoffel et al. (2008) estimate η<sub>c</sub> = 19 and η<sub>i</sub> = 16.

mated by Christiano *et al.* (2005), 40 percent. Following Adolfson *et al.* (2007) we set the mark-up equal to 16 percent. <sup>11</sup> We set the indexation parameter,  $\chi^d$ , equal to 0.22, which is the value estimated by Adolfson *et al.* (2007) for the euro area. Following Adolfson *et al.* (2007) and Christiano *et al.* (2005), the markup power in wage setting is set to 1.05. Following Adolfson *et al.* (2007), the indexation of wages, parameter,  $\chi^w$ , is set to 0.50, and the probability of not being able to change the wage,  $\theta^w$ , is set equal to 0.69.

The steady state of the model is independent of the adjustment cost functions, but the dynamics depend on them. The estimates in the literature concerning the adjustment costs of investment, capital utilization and foreign debt differ substantially. Christiano *et al.* (2005) find estimates of 2.48 and 0.01 for V'' and  $\frac{\gamma_2}{\gamma_1}$ , respectively. Altig *et al.* (2003) find a value of 0.049 for  $\frac{\gamma_2}{\gamma_1}$ . Adolfson *et al.* (2007) estimate a value of a 8.67 for V'' and estimate a value of 0.252 for  $\phi_i$ . We took the values reported in Adolfson *et al.* (2007).

During the period 99:01-07:04 the share of the exports to countries outside of the euro area was 0.37, while as referred before, the share of imports from countries outside the euro area was 0.34. The parameters chosen replicate approximately these ratios as well as the average sample shares in the GDP of private consumption, public consumption, investment, total exports and total imports. It is common in the literature, including Adolfson *et al.* (2007), to assume that in the steady state exports are equal to imports, and the net foreign debt is zero. Instead, we assumed that the level and composition of the steady state net foreign responsabilities were the values for Portugal at the end of 2007. At this date the net foreign responsibilities were a large fraction of GDP, being 83 per cent denominated in euros.

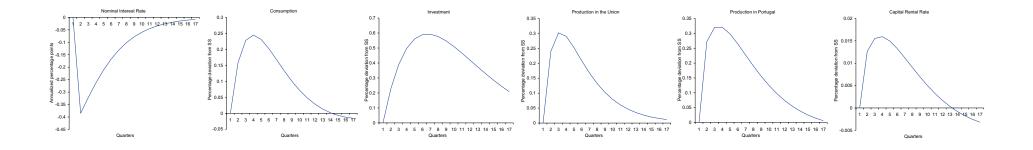
The parameters of the IS curve, Phillips curve and Taylor rule were chosen so that the impulse response functions of the union's inflation, output and interest rate to a monetary shock could mimic the impulse response functions of these variables to a monetary shock in the model estimated by Adolfson *et al.* (2007) for the euro area. The parameters of the interest rate rule are similar to the ones estimated for the euro area by Adolfson *et al.* (2007) and Smets and Wouters (2003). The parameter  $\varsigma$  reflects the size of Portugal and is set to 0.05. The parameters in the equations that determine the nominal interest rate, output and inflation outside the union do not need to be specified as we assumed that these variables are not affected by a monetary policy shock in the union.

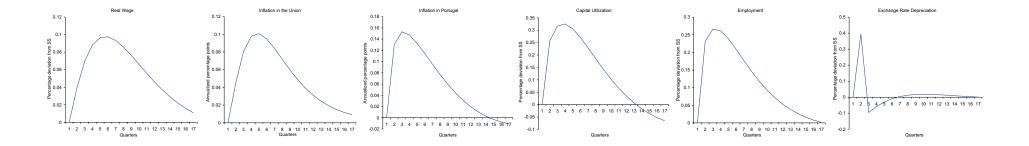
<sup>(11)</sup> Chari et al. (2005) have a mark-up of 11 per cent.

<sup>(12)</sup> These divergent results in the literature are due to the fact that the data sets are divergent and to the different estimation techniques used. Adolfson et al. (2007) use Bayesian estimation techniques while Christiano et al. (2005) match the impulse response functions of the identified shocks.

# Chart 1(to be continued)

#### IMPULSE RESPONSES TO THE MONETARY POLICY SHOCK





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# **Chart 1(continued)**

#### IMPULSE RESPONSES TO THE MONETARY POLICY SHOCK

