

PRINCIPLES OF OPTIMAL STABILIZATION POLICY*

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1. INTRODUCTION

In this article, we derive principles of optimal stabilization policy. We show that optimal policy in response to shocks keeps prices stable, that the nominal interest rate should be low and stable, and that tax instruments play a crucial role. The analysis is based on Correia, Nicolini and Teles (2008).

The model we consider is a stochastic production economy, without capital, with cash and credit goods. Firms are monopolistic competitive and are restricted in setting prices, but are otherwise identical. Government consumption is financed with revenue from labor income and consumption taxes, as well as seigniorage. For simplicity, we assume that there is state-contingent public debt.¹

The model has three sources of distortions. Because firms are monopolistic competitive, there is a mark-up distortion. The price setting restrictions are another source of inefficiency. Finally, the need to raise distortionary taxes to finance public expenditures implies various wedges in marginal decisions. One of those wedges is caused by the nominal interest rate in the marginal decision between money and bonds. The nominal interest on short term, riskless, bonds is the opportunity cost of money. Since the cost of producing money is negligible, a positive nominal interest rate is a distortion. The interplay between the three potential sources of distortions is at the heart of optimal stabilization policy. As shown by Correia, Nicolini and Teles (2008), optimal policy eliminates the distortions associated with sticky prices, as well as the money demand distortion. The markup distortion can be eliminated with an implicit subsidy financed with the lump-sum revenue from profit taxation. The only remaining distortions are the ones arising from the need to raise distortionary taxation in a competitive, flexible price environment.

Once it is clear that optimal policy neutralizes the effects of the nominal rigidity, then we can apply the principles of optimal taxation under flexible prices that are well known after Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991) and many others.

The early approach in the literature (see Rotemberg and Woodford 1997 or Clarida, Gali, and Gertler 1999) is to assume that there are lump-sum taxes. Those taxes finance both government expenditures and a subsidy to production that eliminates the markup distortion. It is also standard to abstract from the money demand distortion by assuming that the economy is the cashless limit of a sequence of monetary economies. By keeping prices flat it is possible to eliminate the only remaining distortion, the nominal rigidity, and achieve the first-best allocation. In this context, price stability is optimal and the nominal interest rate moves one-to-one with the natural real rate of interest, in response to shocks. But lump-sum taxes are needed for these results, and those are obviously not available.

An alternative, more elaborate, approach is to assume that, indeed, lump-sum taxes are not feasible, but to be very selective in the fiscal instruments that are available. Benigno and Woodford (2003),

* The analyses, opinions and findings of this article represent the views of the author, they are not necessarily those of the Banco de Portugal.

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(1) Public debt is normally assumed to be state noncontingent, even if there are ways of using maturities, or consumption taxes, to mimic state contingent debt.

Schmitt-Grohé and Uribe (2004), and Siu (2004) assume that only one tax can be used, either the consumption or the labor income tax. They obtain very different qualitative results, from the first-best ones.² Price stability is not optimal and the nominal rate is not the natural rate.

Correia, Nicolini and Teles (2008) solve the optimal fiscal and monetary policy problem assuming that taxes must be distortionary, but allowing for all the standard taxes. In particular, in this environment, because there is no capital, the taxes that make sense to assume are consumption and labor income taxes, in addition to profit taxes. They recover the first-best principles of price stability and the Friedman rule, i.e., a zero nominal interest rate, even if their analysis is a second-best one, in which distortions must be present.

Why should some distortions be fully eliminated, while others are kept? This is certainly against the general Ramsey principle that distortions should be balanced. There is, however, another well known principle, due to Diamond and Mirrlees (1971), that distortions associated with productive inefficiencies are not optimal even when there are other distortions. Productive inefficiencies bring the economy inside the production possibilities set and it is always better to be on the frontier of that set. It turns out that the distortions caused by sticky prices are productive inefficiencies. Indeed, because price setting decisions may be staggered, otherwise identical firms may set different prices. This is a productive inefficiency.

The reason why the Friedman rule is optimal even when there are other distortions, is also related to the same principle in Diamond and Mirrlees (1971). Money can be modelled as an intermediate good that it is not optimal to tax, precisely to ensure efficiency in production. There is another reason, however, not to tax money. Money is a free good that, regardless of the optimal tax rate, should have a very low price. The nominal interest rate is the price of money, and, therefore, should be close to zero.³

Our analysis proceeds as follows: We start by analyzing an economy with monopolistic competitive firms and flexible prices. We show that every allocation in the economy with flexible prices can be implemented with stable prices. We then show that price stability is optimal when there are sticky price firms. Because, under sticky prices, it is feasible and optimal to replicate the allocations with flexible prices, we solve the optimal taxation problem under flexible prices. We show that the Friedman rule is optimal, for preferences that are separable in leisure and homothetic in the consumption goods. We also show that, for those preferences, the optimal wedges are constant over time and across states, and that the tax rates do not have to vary with contemporaneous shocks.

2. THE MODEL ECONOMY⁴

The model is a standard, Ramsey, optimal taxation model, similar to the one in Lucas and Stokey (1983) or Chari, Christiano and Kehoe (1991), except that firms are monopolistic competitive and that there are restrictions in how they set prices.

The economy is inhabited by identical households, a continuum of firms indexed by i , and a government. In each period t , each firm uses labor n_{it} to produce a good that can be used for private consumption as a cash good c_{1it} , as a credit good c_{2it} , or for public consumption g_{it} . The technology is

$$c_{1it} + c_{2it} + g_{it} = A_t n_{it}$$

(2) Quantitatively, price stability is close to optimal.

(3) For more on the optimal price of money, see Teles (2003).

(4) We describe the model and derive the results with some detail but we advise the more technical reader to revert to the Appendix.

where A_t is the stochastic productivity that is common across goods.

The individual goods aggregate into composite cash goods C_{1t} and credit goods C_{2t} , with the Dixit-Stiglitz aggregator. The households draw utility from these composite goods and disutility from aggregate labor N_t , according to the utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_{1t}, C_{2t}, N_t),$$

Aggregate government purchases G_t are exogenous and stochastic, and must be financed with consumption taxes τ_t^c , taxes on labor income τ_t^n , and taxes on profits τ_t^d and by printing money M_t . For simplicity we allow for debt to be state-contingent.⁵ Again for simplicity, we assume that profits are fully taxed, $\tau_t^d = 1$ and that initial wealth is also fully taxed.

The cash goods C_{1t} must be purchased with money M_t according to the cash-in-advance constraint

$$(1 + \tau_t^c) P_t C_{1t} \leq M_t. \quad (1)$$

The budget constraint of the households can be written as

$$E_0 \sum_{t=0}^{\infty} \frac{\bar{Q}_t}{R_t} [(1 + \tau_t^c) P_t (C_{1t} + C_{2t}) + M_t (R_t - 1) - (1 - \tau_t^n) W_t N_t] \leq 0$$

where \bar{Q}_t is the state-contingent price of one unit of money at time t in a particular state, in units of money at time 0, normalized by the probability of occurrence of that state, and R_t is the gross nominal interest rate from period t to period $t + 1$. W_t is the nominal wage. By arbitrage, because the sum of the prices of the contingent bonds must be equal to the price of the noncontingent bond, it must be that

$$E_t \frac{\bar{Q}_{t+1}}{\bar{Q}_t} = \frac{1}{R_t}. \quad (2)$$

Using the cash-in-advance constraint (1), we can write the budget constraint as

$$E_0 \sum_{t=0}^{\infty} \frac{\bar{Q}_t}{R_t} [(1 + \tau_t^c) P_t (R_t C_{1t} + C_{2t}) - (1 - \tau_t^n) W_t N_t] \leq 0. \quad (3)$$

It is then straightforward to see that the relative price between the cash and the credit goods is the nominal interest rate, R_t , so that the marginal rate of substitution between those two goods must be equal to R_t ,

$$\frac{u_{C_{1t}}}{u_{C_{2t}}} = R_t \geq 1.$$

The cash good is more expensive because it must be bought with money. The nominal interest rate cannot be negative, $R_t \geq 1$, since otherwise households could make arbitrarily large profits by issuing bonds and holding money. The marginal rate of substitution between credit goods and leisure must also be equal to the relative price,

(5) Correia, Nicolini and Teles (2008) obtain the results with noncontingent nominal debt of one-period maturity.

$$\frac{u_{C_{2t}}}{u_{N_t}} = \frac{(1 + \tau_t^c) P_t}{(1 - \tau_t^n) W_t}.$$

The relative price between consumption of the cash good in period t , in a particular state, and in period 0 is $\frac{\bar{Q}_t (1 + \tau_t^c) P_t}{(1 + \tau_0^c) P_0}$, and therefore the marginal rate of substitution must be equal to that price,⁶

$$\frac{\beta^t u_{C_{1t}}}{u_{C_{10}}} = \frac{\bar{Q}_t (1 + \tau_t^c) P_t}{(1 + \tau_0^c) P_0}.$$

Using the arbitrage condition between contingent and noncontingent bonds, (2), we have the Fisher equation,

$$\frac{u_{C_{1t}}}{(1 + \tau_t^c) P_t} = \beta R_t E_t \left[\frac{u_{C_{1t+1}}}{(1 + \tau_{t+1}^c) P_{t+1}} \right],$$

where the relevant prices are gross of the consumption taxes. One unit of money can either be used to

buy $\frac{1}{(1 + \tau_t^c) P_t}$, units of the cash good that give marginal utility $\frac{u_{C_{1t}}}{(1 + \tau_t^c) P_t}$, or used to buy

noncontingent bonds that give a sure gross return R_t that can, then, be used to buy $\frac{1}{(1 + \tau_{t+1}^c) P_{t+1}}$

cash goods, with marginal utility $\beta E_t \left[\frac{u_{C_{1t+1}}}{(1 + \tau_{t+1}^c) P_{t+1}} \right]$. The two marginal benefits must be equal.

For now, we assume that firms set flexible prices. Because all monopolists face the same demand and have the same technology, all set the same price. The common price is equal to a constant markup over marginal cost,

$$P_t = \frac{\theta}{\theta - 1} \frac{W_t}{A_t}.$$

The markup is a function of θ , which is the elasticity of substitution between any of the individual goods. Notice that as the elasticity increases, the markup is reduced, to the point where it is zero, which would correspond to perfect competition.

Since all firms set the same price, they also sell the same quantities, so that the individual quantities are equal to the aggregate. The aggregate resource constraints are, then,

$$C_{1t} + C_{2t} + G_t = A_t N_t.$$

3. EQUILIBRIA WITH FLEXIBLE PRICES

The standard approach in the literature on Ramsey taxation in these kinds of models is to identify the smallest set of conditions restricting the equilibrium allocations of consumption and labor, in order to

(6) Notice that $\bar{Q}_0 = 1$ and that the state-contingent prices \bar{Q}_t are normalized by the probabilities of occurrence of the state.

make it easier to solve the optimal problem. We show formally in the Appendix that the allocations are restricted only by the following implementability conditions,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u_{C_{1t}} C_{1t} + u_{C_{2t}} C_{2t} + u_{N_t} N_t \right) = 0, \quad (4)$$

and

$$u_{C_{1t}} \geq u_{C_{2t}}, \quad (5)$$

and the feasibility conditions

$$C_{1t} + C_{2t} + G_t = A_t N_t. \quad (6)$$

The first condition is obtained replacing the prices and taxes from the marginal conditions of the households in the household budget constraint (3). Because the condition is derived using the conditions of the households only, it does not depend on the price-setting restrictions. The second condition ensures that the nominal interest rate is nonnegative, and also does not depend on whether prices are flexible or sticky.

These conditions are all that is needed to characterize the equilibrium allocations for the consumption of the two goods and labor. They are obviously necessary conditions, since they were obtained using the equilibrium conditions. They are also sufficient, meaning that all the other equilibrium conditions can be satisfied by the choice of policies, prices or quantities other than the consumption of the two goods and labor. It turns out that this can be shown, setting the price level constant over time and equal to some arbitrary number, $P_t = \bar{P}$. This means that each equilibrium allocation can be implemented with a price level that does not depend on the shocks. This result is instrumental for the main point we want to make in this article.

In order to implement an equilibrium allocation with constant prices there is a specific role for the fiscal and monetary policy instruments. To see this, we take a particular allocation for the two consumption goods and labor, satisfying (4), (5) and (6). Then,

$$\frac{u_{C_{1t}}}{u_{C_{2t}}} = R_t \quad (7)$$

pins down the nominal interest rate, $R_t - 1$, which is nonnegative because of the implementability condition (5). For a constant price level, $P_t = \bar{P}$, the intertemporal condition

$$\frac{u_{C_{t-1}}}{(1 + \tau_{t-1}^c) \bar{P}} = \beta R_{t-1} E_{t-1} \left[\frac{u_{C_{1t}}}{(1 + \tau_t^c) \bar{P}} \right] \quad (8)$$

can be satisfied by the choice of consumption taxes τ_t^c . Notice that it is possible to do this for a consumption tax that does not depend on the contemporaneous shocks. Given τ_0^c , we use the condition for $t = 1$, to determine τ_1^c , the conditions for $t = 2$, to determine τ_2^c , and so on.

The money supply is whatever satisfies the cash-in-advance condition (1). The price-setting equations,

$$\bar{P} = \frac{\theta}{\theta - 1} \frac{W_t}{A_t}$$

determine the nominal wage W_t , that must move with the productivity shocks. The household intratemporal conditions,

$$-\frac{u_{C_{2t}}}{u_{Nt}} = \frac{(1 + \tau_t^c) \bar{P}}{(1 - \tau_t^n) W_t},$$

are satisfied by the choice of the labor income tax τ_t^n .

Notice that the household intratemporal conditions together with the price setting conditions can be written as

$$-\frac{u_{C_{2t}} A_t}{u_{Nt}} = \frac{(1 + \tau_t^c)}{(1 - \tau_t^n)} \left(\frac{\theta}{\theta - 1} \right). \tag{9}$$

Suppose, now, that the optimal wedge or distortion between the credit good and labor, $-\frac{u_{C_{2t}} A_t}{u_{Nt}}$, is constant, as is the case for the class of utility functions that we will analyze later. Then the implementation of a constant optimal wedge would mean that the ratio $\frac{1 + \tau_t^c}{1 - \tau_t^n}$ is constant, and if τ_t^c does not move with the contemporaneous shocks, then τ_t^n will also be independent of the shocks.

The equilibrium allocations described by the conditions above are the same here as in Lucas and Stokey (1983) and Chari *et al.* (1991), where firms are assumed to be competitive. Indeed, the implementability conditions are independent of θ , and therefore of the markup. Monopolistic competition creates a distortion but it also creates the lump-sum tax revenue necessary to subsidize production, to eliminate the distortion. The revenue from the full taxation of profits is exactly the revenue needed to finance the implicit subsidy to labor needed to eliminate the monopoly distortion.

We have so far made one very important point, that every allocation under flexible prices, and, in particular, the optimal one, can be implemented with constant prices. The importance of this point is that, if prices can be constant, this means that if firms were restricted in setting prices, those restrictions would not be relevant. It is then possible under sticky prices to achieve the same allocation as under flexible prices. It follows that we cannot do better under flexible prices than under sticky prices.

But maybe we could do better under sticky prices than under flexible prices. This is only surprising if we forget that the economy is distorted, so that adding one more distortion can be better. It turns out that the distortion from sticky prices is of a particular type that should be fully eliminated even if there are other distortions. If prices are sticky, firms that are otherwise very similar may charge different prices. This means that production is inefficient, and productive inefficiencies are not optimal even if there are other distortions.

4. PRICE STABILITY IS OPTIMAL

Suppose now that prices are sticky, and that firms set prices at different times so that there is price dispersion. If otherwise identical firms set different prices p_{it} , then the aggregate resource constraints have to be written as

$$(C_{1t} + C_{2t} + G_t) \int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} di = A_t N_t, \quad (10)$$

where

$$P_t = \left(\int_0^1 p_{it}^{1-\theta} di \right)^{\frac{1}{1-\theta}}.$$

$\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} di$ is the measure of the resource cost due to price dispersion.

The set of implementable allocations under sticky prices must be characterized by the same two implementability conditions under flexible prices (4) and (5), because those were derived with the households conditions only, regardless of how prices were set. In addition, instead of the resource constraints (6), constraints (10) above must be satisfied.

There are certainly other equilibrium restrictions, but they are not relevant for the point we want to make. The point we make now is that allocations under flexible prices dominate the ones under sticky prices. Indeed, because the resource cost is zero, $\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} di = 1$, when prices are the same,

$p_{it} = P_t$, and it is greater than zero, $\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} di > 1$, otherwise, under flexible prices it is possible to

minimize the resource cost due to price dispersion. The resource cost is zero under flexible prices, in which case all the firms set the same price.

The intuition for this result is the following. Firms in this set up are symmetric, so that, if prices are flexible, they must set the same price. Production is then efficient, the economy is on the production possibilities frontier, and the job of the policy maker is to optimally distort along the frontier. Instead, if firms set different prices, there is a productive inefficiency, and the equilibrium will be inside the production possibilities set. This is never optimal, even when the economy is distorted. It is always better to avoid productive inefficiencies and optimally distort along the frontier. This result, that productive inefficiencies are not desirable in a distorted world is due to Diamond and Mirrlees (1971), that applied it to the optimal taxation of intermediate goods. As they show, when there are taxes on the final consumption goods, intermediate goods should not be taxed. Sticky prices act like differential taxation of intermediate goods.

We conclude, then, that price stability is optimal under quite general conditions. Stabilization policy that exploits the nonneutrality of money, to achieve any goal other than price stability is not desirable, unless, of course, taxes cannot be used for stabilization policy.

5. OPTIMAL TAXES – THE FRIEDMAN RULE

Once it is clear that the optimal allocation can be found in the set of allocations under flexible prices, we can solve a Ramsey problem where the optimal allocation is the one that maximizes utility in the set characterized by the implementability conditions under flexible prices, (4), (5), and (6).

Consider now the following utility function, which is separable in leisure and homothetic in the two consumption goods:

$$u(C_{1t}, C_{2t}, N_t) = \frac{C_{1t}^{1-\sigma} - 1}{1-\sigma} + \frac{C_{2t}^{1-\sigma} - 1}{1-\sigma} - \gamma N_t, \text{ with } \sigma, \gamma > 0.$$

The marginal conditions for the maximization of utility, subject to (4) and (6), are

$$\frac{u_{C_{1t}}}{u_{C_{2t}}} = 1$$

and

$$-\frac{u_{C_{2t}} A_t}{u_{N_t}} = \frac{1 + \lambda}{1 + \lambda(1 - \sigma)},$$

where λ is the multiplier of (4), which measures the excess burden of taxation.

It is optimal, then, not to distort between consumption of the cash good and of the credit good, and the optimal wedge between any of the two consumption goods and leisure is constant over time and independent of shocks. The policy that implements these optimal wedges can be seen using the equilibrium conditions for the households, (7), (8), and (9).

The optimal nominal interest rate is zero, $R(s^t) = 1$, so that the Friedman rule is optimal. The opportunity cost of money, which is the nominal interest rate, should be zero, in order not to distort between the two consumption goods. Furthermore, the optimal distortion caused by taxes, $\frac{1 + \tau_t^c}{1 - \tau_t^n}$, should be

constant over time.

We have seen above that the optimal allocation can be implemented with stable prices and with consumption taxes that do not depend on the contemporaneous shocks. How can the nominal interest rate be zero and prices be stable when the real rate is positive and volatile? This does not violate the Fisher equation, (8), because the consumption taxes can move to verify the equation. They move with a lag.

If the optimal wedge between consumption and leisure is constant, then $\frac{1 + \tau_t^c}{1 - \tau_t^n}$ is constant, and there-

fore, because the consumption tax can be predetermined, the labor income tax can also be predetermined. Both taxes have to move, but with a lag, in response to lagged information.

In this economy, in which the utility function is separable in leisure and homothetic in the consumption goods, it is optimal to tax all consumption goods at the same rate, in every date and state. Those conditions on preferences are the conditions for uniform taxation of Atkinson and Stiglitz (1972). This ex-

plains why it is optimal not to distort between cash and credit goods and why the optimal tax distortion is constant over time.

One final point on the money supply. The observation of the cash-in-advance constraint

$$(1 + \tau_t^c) P_t C_{1t} \leq M_t$$

makes it clear that, with stable prices and predetermined consumption taxes, the money supply has to be elastic. It must move in response to shocks to accommodate the movements in transactions.

6. CONCLUDING REMARKS

We have summarized the main principles of stabilization policy assuming that both fiscal and monetary policy can be used to optimally respond to shocks. The optimal policy when prices are sticky is to neutralize the effects of that friction, by making sure that prices are stable. That way, the economy behaves as if prices were flexible. The resulting economy is still a distorted economy because the need to raise distortionary taxes cannot be overcome.

The reason why it is optimal to eliminate one distortion, when there are other remaining distortions, is the same reason why in Diamond and Mirrlees (1971) it is optimal not to tax intermediate goods, even if final goods must be taxed. Sticky prices create productive inefficiencies, just like the taxes on intermediate goods, that are not desirable even when there are other distortions.

The effects of the remaining distortions can be minimized using what we know of optimal taxation under flexible prices. In that context, the Friedman rule is generally optimal, and uniform taxation across different goods is approximately optimal.

In order to follow the Friedman rule and still have stable prices, the consumption taxes have to move. They only have to move with a lag, though. Because uniform taxation is approximately optimal, taxes on labor income also move with lagged information.

In the real world taxes are also sticky, possibly stickier than this model would imply. One conclusion we draw from this analysis, is that those conditions should probably be revised.

The model we analyze is very simple. The world is obviously much more complex; there are certainly many other frictions that we have abstracted from. In a more complex model with other restrictions on decisions, the results we derive in this article will certainly not hold exactly. They may still be approximately correct, though.

APPENDIX

The agents in the model are identical households, a continuum of firms indexed by $i \in [0, 1]$, and a government. The history of events up to period t is denoted by s^t and the initial realization s_0 is given. $\pi(s^t)$ is the probability of the occurrence of state s^t .

Each firm uses labor $n_i(s^t)$ to produce $y_i(s^t)$ that can be used as a cash good $c_{1i}(s^t)$, a credit good $c_{2i}(s^t)$, or public consumption $g_i(s^t)$. The technology is

$$c_{1i}(s^t) + c_{2i}(s^t) + g_i(s^t) = y_i(s^t) = A(s^t)n_i(s^t) \quad (11)$$

where $A(s^t)$ is the productivity that is common across goods.

Households draw utility from composite cash goods $C_1(s^t)$ and credit goods $C_2(s^t)$ and disutility from aggregate labor $N(s^t)$, according to:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(C_1(s^t), C_2(s^t), N(s^t)), \quad (12)$$

with

$$C_1(s^t) = \left[\int_0^1 c_{1i}(s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1, \quad (13)$$

$$C_2(s^t) = \left[\int_0^1 c_{2i}(s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (14)$$

and

$$N(s^t) = \int_0^1 n_i(s^t) di. \quad (15)$$

Aggregate government purchases $G(s^t)$,

$$G(s^t) = \left[\int_0^1 g_i(s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (16)$$

are exogenous and must be financed with consumption taxes $\tau^c(s^t)$, taxes on labor income $\tau^n(s^t)$, and taxes on profits $\tau^d(s^t) = 1$ and by printing money $M(s^t)$.

The households

The households start period t with nominal wealth $W(s^t)$. They decide to buy money balances $M(s^t)$, riskfree nominal bonds $\bar{B}(s^t)$ that pay $R(s^t)\bar{B}(s^t)$ units of money one period later, and $B(s^{t+1})$ units of state-contingent nominal securities. These bonds pay one unit of money at the beginning of period $t+1$ in state s^{t+1} and cost $Q(s^{t+1}|s^t)$ units of money in state s^t . Thus, the purchases of assets by the households must satisfy

$$M(s^t) + \bar{B}(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) B(s^{t+1}) \leq W(s^t). \quad (17)$$

At the end of the period, the households receive labor income $W(s^t)N(s^t)$, where $W(s^t)$ is the nominal wage. The evolution of nominal wealth is governed by

$$\begin{aligned}
W(s^{t+1}) = & R(s^t) \bar{B}(s^t) + B(s^{t+1}) + M(s^t) - [1 + \tau^c(s^t)] \int_0^1 p_i(s^t) c_{1i}(s^t) di \\
& - [1 + \tau^c(s^t)] \int_0^1 p_i(s^t) c_{2i}(s^t) di + [1 - \tau^n(s^t)] W(s^t) N(s^t), t \geq 0.
\end{aligned} \quad (18)$$

Money, $M(s^t)$, is used to purchase consumption of the cash good, $C_1(s^t)$, according to the cash-in-advance constraint

$$[1 + \tau^c(s^t)] P(s^t) C_1(s^t) \leq M(s^t), \quad (19)$$

where $P(s^t)$ is

$$P(s^t) = \left[\int_0^1 [p_i(s^t)]^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \quad (20)$$

which is the money cost to buy one unit of the composite goods.

Households choose the sequence that maximizes utility (12), satisfying (13), (14), (17), (18) together with a no-Ponzi games condition, and (19). The following are necessary marginal conditions, for $t \geq 0$:

$$\frac{c_{1i}(s^t)}{C_1(s^t)} = \left[\frac{p_i(s^t)}{P(s^t)} \right]^{-\theta}, \quad (21)$$

$$\frac{c_{2i}(s^t)}{C_2(s^t)} = \left[\frac{p_i(s^t)}{P(s^t)} \right]^{-\theta}, \quad (22)$$

$$\frac{u_{C_1}(s^t)}{u_{C_2}(s^t)} = R(s^t) \geq 1, \quad (23)$$

$$\frac{u_{C_2}(s^t)}{u_N(s^t)} = \frac{[1 + \tau^c(s^t)] P(s^t)}{[1 - \tau^n(s^t)] W(s^t)}, \quad (24)$$

$$Q(s^{t+1} | s^t) = \beta \pi(s^{t+1} | s^t) \frac{u_{C_1}(s^{t+1}) [1 + \tau^c(s^t)] P(s^t)}{u_{C_1}(s^t) [1 + \tau^c(s^{t+1})] P(s^{t+1})}, \quad (25)$$

and

$$\frac{u_{C_1}(s^t)}{[1 + \tau^c(s^t)] P(s^t)} = \beta R(s^t) E_t \left[\frac{u_{C_1}(s^{t+1})}{[1 + \tau^c(s^{t+1})] P(s^{t+1})} \right]. \quad (26)$$

The last two equations imply the arbitrage condition

$$\frac{1}{R(s^t)} = \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) . \quad (27)$$

Let $Q(s^r | s^t) = Q(s^{t+1} | s^t) \dots Q(s^r | s^{r-1})$ be the price of one unit of money at s^r in units of money at s^t . Imposing the transversality condition, the budget constraint of the households can be written with equality as

$$\begin{aligned} & \sum_{t=0}^{\infty} \sum_{s^t} \frac{Q(s^t | s^0)}{R(s^t)} \{ [1 + \tau^c(s^t)] P(s^t) [C_1(s^t) + C_2(s^t)] \} + \\ & \sum_{t=0}^{\infty} \sum_{s^t} \frac{Q(s^t | s^0)}{R(s^t)} \{ M(s^t) [R(s^t) - 1] - [1 - \tau^n(s^t)] W(s^t) N(s^t) \} = 0 \end{aligned} \quad (28)$$

We can replace in the budget constraint the intertemporal prices $Q(s^t | s^0)$ using (25), and use the intertemporal conditions (26), the intratemporal conditions (23) and (24), and the cash-in-advance constraints (19), to write the budget constraint as the implementability condition

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u_{C_1}(s^t) C_1(s^t) + u_{C_2}(s^t) C_2(s^t) + u_N(s^t) N(s^t) \right] = 0 \quad (29)$$

It is worth noting that the implementability condition (29) does not depend on the price-setting restrictions.

The government

Given the exogenous aggregate government purchases, $G(s^t)$, and the consumer prices, $p_i(s^t)$, the government minimizes the expenditure $\int_0^1 p_i(s^t) g_i(s^t) di$ on $G(s^t)$ given by (16) by choosing

$$\frac{g_i(s^t)}{G(s^t)} = \left[\frac{p_i(s^t)}{P(s^t)} \right]^{-\theta} . \quad (30)$$

Given full profit taxation, $\tau^d(s^t) = 1$ for all s^t , a government policy consists of public consumption of each good, $g_i(s^t)$, money supply, $M(s^t)$, taxes on consumption and labor income, $\tau^c(s^t)$ and $\tau^n(s^t)$, nominal interest rates, $R(s^t)$, and debt supplies, $\bar{B}^g(s^t)$ and $B^g(s^{t+1})$ for all $t \geq 0$ and states $s^t \in S^t$.

If the budget constraint of the households and the market-clearing conditions hold, then the budget constraint of the government is also satisfied.

The firms

Each good $i \in [0, 1]$ is produced by a monopolist firm that faces the constant elasticity demand function

$$y_i(s^t) = \left[\frac{p_i(s^t)}{P(s^t)} \right]^{-\theta} Y(s^t) \quad (31)$$

obtained from the demand functions for the private and public goods, (21), (22) and (30), where $Y(s^t) = C_1(s^t) + C_2(s^t) + G(s^t)$.

We now assume that all firms set flexible prices. The flexible price firms choose prices to maximize profits at each period $t \geq 0$,

$$p_i(s^t) y_i(s^t) - W(s^t) n_i(s^t),$$

given the technology (11) and the demand function (31). All monopolists set the common price

$$p_i(s^t) \equiv P(s^t) = \frac{\theta}{\theta-1} \frac{W(s^t)}{A(s^t)}. \quad (32)$$

Market clearing

Demand must be equal to supply for each good i and for labor according to (11) and (15).

Equilibria

The set of equilibria is characterized by the household marginal conditions (20), (21), (22), (23), (24), (25), (26), and the cash-in-advance constraints (19), together with the nonnegativity constraint on the nominal interest rates, which can be written as

$$u_{C_1}(s^t) \geq u_{C_2}(s^t);$$

the price-setting conditions (32) characterize the optimal behavior of the firms; the government purchases public goods according to (30) and chooses the other policy variables, satisfying the budget constraint, which, given the market-clearing conditions, can be written as the household budget constraint (29); finally, the market-clearing conditions (11) and (15) must hold.

Equilibrium allocations under flexible prices

We can characterize the set of implementable allocations under flexible prices with only a few conditions. In particular, the set of implementable allocations for the consumption goods and labor, $\{C_1(s^t), C_2(s^t), N(s^t)\}$, is characterized by the implementability conditions

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u_{C_1}(s^t) C_1(s^t) + u_{C_2}(s^t) C_2(s^t) + u_N(s^t) N(s^t) \right] = 0, \quad (33)$$

$$u_{C_1}(s^t) \geq u_{C_2}(s^t), \quad (34)$$

and the feasibility conditions

$$C_1(s^t) + C_2(s^t) + G(s^t) = A(s^t)N(s^t). \tag{35}$$

These conditions are necessary and sufficient to characterize the set of equilibrium allocations $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^\infty$. That they are necessary conditions is straightforward. We have shown before that (33) and (34) are equilibrium conditions. Since the prices are the same for all firms, consumption and labor input are also the same for every good $i \in [0, 1]$, so that the resource constraints (11) and (15) imply (35). In order to show that they are sufficient conditions, we need to show that all the other equilibrium conditions are satisfied for the choice of policies, prices or other quantities. We will show this now setting the price level constant over time and equal to some arbitrary number, $P(s^t) = \bar{P}$.

The household marginal conditions on the choice of cash and credit goods, (23), determine uniquely the nominal interest rates, $\{R(s^t)\}_{t=0}^\infty$, which are nonnegative because of (34). Given $\pi^c(s_0)$, and for $P(s^t) = \bar{P}$, (26) for $t \geq 1$, repeated here,

$$\frac{u_{C_1}(s^{t-1})}{[1 + \tau^c(s^{t-1})]\bar{P}} = \beta R(s^{t-1}) E_{t-1} \left[\frac{u_{C_1}(s^t)}{[1 + \tau^c(s^t)]\bar{P}} \right], t \geq 1, \tag{36}$$

restricts the process for $\tau^c(s^t)$. Notice that if the consumption tax was made invariant to the contemporaneous information, given $\tau^c(s_0)$, there would be a single solution for it. If the cash-in-advance constraint, (19), holds with equality, then, given $\tau^c(s_0)$, the money supply is uniquely determined.

The price-setting equations, (32), determine uniquely the nominal wages $\{W(s^t)\}_{t=0}^\infty$. The household intratemporal conditions, (24), given $\{W(s^t)\}_{t=0}^\infty$ and $\{\tau^c(s^t)\}_{t=0}^\infty$, determine, also uniquely, the labor income tax $\{\tau^n(s^t)\}_{t=0}^\infty$. Finally, the prices of the state-contingent debt, $Q(s^{t+1}|s^t)$, are given by (25).

Price stability is optimal.

Suppose now that prices are sticky. The result above that, under flexible prices, it is possible to implement each allocation with a constant price level implies that, if there were sticky price restrictions, for that policy, the restriction would not be binding. This means that it is possible to achieve, under sticky prices, the allocations under flexible prices. It follows, that it is not possible to do worse under sticky prices than under flexible prices, but it might be possible to do better. We now show that it is not the case.

If we add up the market-clearing conditions for each good i , (11), and use the demand functions (21), (22), and (30), as well as the resource constraints (15), we obtain the following aggregate resource constraints:

$$[C_1(s^t) + C_2(s^t) + G(s^t)] \int_0^1 \left[\frac{p_i(s^t)}{P(s^t)} \right]^{-\theta} di = A(s^t) N(s^t). \quad (37)$$

The set of implementable allocations $\{C_1(s^t), C_2(s^t), N(s^t)\}$ under flexible prices is characterized by the implementability conditions (33) and (34) as well as the feasibility conditions (35). The set of implementable allocations under sticky prices must be characterized by the same two implementability conditions, (33) and (34), because those were derived with the households conditions only, regardless of how prices are set. In addition, instead of the resource constraints (35), the constraints (37) must hold. The condition for the price level, (20), must also be satisfied. Because $\int_0^1 \left[\frac{p_i(s^t)}{P(s^t)} \right]^{-\theta} di = 1$ for $p_i(s^t) = P(s^t)$, and $\int_0^1 \left[\frac{p_i(s^t)}{P(s^t)} \right]^{-\theta} di > 1$ for $p_i(s^t) \neq P(s^t)$, under flexible prices it is possible to minimize the resource cost due to price dispersion.

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