ENSURING PRICE STABILITY WITH AN INTEREST RATE RULE*

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1. INTRODUCTION

The primary concern of monetary policy is to ensure price stability. Such is the mandate of the ESCB established by the Maastricht Treaty, but the objective, spelled out in various ways, is common to every other central bank. In a somewhat old fashioned language, the objective of price stability requires that monetary policy provide a nominal anchor, that it anchor expectations. For once, the objective seems easier to attain in practice than in theory.

Central banks in developed countries have been very successful in the last twenty five years in targeting low inflation. The success has been attributed to a somewhat mechanical interest rate rule where the short term nominal interest rate is set in response to deviations from trend of inflation and economic activity, a Taylor rule, named after John Taylor who first estimated it (Taylor, 1993). It turns out that no policy rule of this type is able to achieve in a monetary model what it appears to achieve in reality. The same models that give very reasonable answers to other questions, generate multiple equilibria when monetary policy is conducted with an interest rate feedback rule whether it may respond to future, current or past inflation.

There is an extensive literature on this issue of central importance to monetary policy making, dating back to Sargent and Wallace (1975) who showed that a policy that targets the interest rate gives rise to multiple equilibria. Most of the later literature has focused on conditions for local determinacy, meaning that, while the multiplicity of equilibria remains, there might be only one equilibrium in a particular neighborhood of interest. McCallum (1981) is the main responsible for triggering this literature, showing that there are indeed interest rate feedback rules that guarantee local uniqueness. Technically, this has been very useful because it has allowed economists to abstract from a problem that was not easy to solve, and concentrate on other issues focusing on the unique local equilibria. Unfortunately, as pointed out by Benhabib, Schmitt-Grohe and Uribe (2001, 2002), the same policy rules that ensure locally determinacy typically generate global indeterminacy, so that the alternative equilibria can converge to other steady states or cycle around the original one.

In this note, and based on Adao, Correia and Teles (2006), we discuss how interest rate rules can be used to implement a unique equilibrium with stable prices. We first consider an economy with a finite horizon and show that nominal interest rates are not a sufficient policy instrument. In a finite horizon economy a finite number of equilibrium variables is restricted by a finite number of equilibrium conditions. If policy specifies restrictions for the nominal interest rates only, then there are more unknowns than equations and there are multiple equilibria. This result does not depend on whether there is an exogenous target for the interest rate or whether it responds to endogenous variables. The number of equations is the same. Similarly, whether prices are flexible or sticky also does not matter.

We proceed to considering an economy that lasts forever. We first discuss in a simplified model how economists usually approach the multiplicity problem, by concentrating on conditions for local determinacy. Finally, we show, as in Adao, Correia and Teles (2006), that there is a policy rule that implements a unique global equilibrium. This rule would not work in the finite horizon economy.

The interest rate feedback rule that implements a unique equilibrium globally in a price targeting the rule where the nominal interest rate responds to the forecast of the future price level as well as the forecast of future economic activity. Unfortunately the rule is not as robust as one would hope. Whether the time horizon is infinite or finite, even if arbitrarily large, makes a fundamental difference and that is not an issue that economists can take a stand on. But, furthermore, in order to be effective the rule would require a knowledge of the economic structure that is not realistic.

Robert Lucas (1996) wrote in his Noble lecture that "Central bankers and even some monetary economists talk knowledgeably of using high interest rates to control inflation, but I know of no evidence from even one economy linking these variables in a useful way (...)." We will have to add that some of us, monetary economists, are still not reasonably confident of a theory linking the two variables in a useful way.

2. THE MODEL

The economy consists of large number of identical households, a representative firm behaving competitively, and a government. The economy has a finite horizon T. There are shocks to technology and to government expenditures.

The representative household has preferences over consumption C_t, and leisure L_t, described by the expected utility function

$$U = E_0 \left\{ \sum_{t=0}^{T} \beta^t u\left(C_t, L_t\right) \right\}$$

(2.1)

where β is a discount factor. The technology uses labor only and is linear

$$Y_t \leq A_t N_t$$

for $0 \leq t \leq T$, where C_t is aggregate consumption, $N_t = 1 - L_t$ is labor and A_t is the technology parameter.

We assume that households must purchase consumption with money according to the cash-in-advance constraint

$$P_t C_t \leq M_t$$

(2.2)

for $0 \leq t \leq T$, where P_t is the price of the consumption good in units of money and M_t are money balances used for transactions. Each period is divided into two subperiods, with the assets market operating in the first subperiod and the goods market in the second.

The households start period t with nominal wealth $\mathbb{W}_t$. They decide to hold money, M_t, and to buy B_t riskless nominal bonds that pay $R_t B_t$ one period later. R_t is the gross nominal interest rate at date t. Thus, in the assets market at the beginning of period t they face the constraint

$$M_t + B_t \leq \mathbb{W}_t$$

(2.3)

for $0 \leq t \leq T$.

At the end of the period, the households receive the labor income $W_t N_t$ where W_t is the nominal wage rate, and pay lump sum taxes, T_t. Thus, the nominal wealth households bring to period $t+1$ is

$$\mathbb{W}_{t+1} = M_t + R_t B_t - P_t C_t + W_t N_t - T_t, \tag{2.4}$$

for $0 \leq t \leq T$. After period T, there is a subperiod for the clearing of debts, where money can be used to pay debts. Wealth in the terminal period cannot be negative,

$$\mathbb{W}_{t+1} \geq 0. \tag{2.5}$$

The households' problem is to maximize expected utility (2.1) subject to the restrictions (2.3), (2.2), (2.4), together with the no-Ponzi games condition (2.5).

The first order conditions of the households problem include

$$\frac{u_L(t)}{u_C(t)} = \frac{W}{P_t} \frac{1}{R_t}, \tag{2.6}$$

for $0 \leq t \leq T$, and

$$\frac{u_C(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right], \tag{2.7}$$

for $0 \leq t \leq T-1$. Condition (2.6) sets the intratemporal marginal rate of substitution between leisure and consumption equal to the real wage adjusted for the cost of using money, R_t. Condition (2.7) is an intertemporal marginal condition necessary for the optimal choice of risk-free nominal bonds. The other conditions are the constraints with equality and the terminal condition

$$\mathbb{W}_{T+1} = 0. \tag{2.8}$$

The firms are competitive and prices are flexible. The firms maximize profits so that the equilibrium real wage is

$$\frac{W_t}{P_t} = A_t, \quad 0 \leq t \leq T. \tag{2.9}$$

The policy variables are lump sum taxes, T_t, interest rates, R_t, money supplies, M_t, state noncontingent public debt, B_t. The period by period government budget constraints are

$$M_0 + B_0 = \mathbb{W}_0$$

$$\begin{aligned} M_t + B_t \\ = M_{t-1} + R_{t-1} B_{t-1} + P_{t-1} G_{t-1} - P_{t-1} T_{t-1}, \\ 1 \leq t \leq T \end{aligned}$$

$$\mathbb{W}_{T+1} = M_T + R_T B_T + P_T + G_T - P_T T_T = 0 \tag{2.10}$$

Market clearing in the goods and labor market requires

$$C_t + G_t = A_t N_t,$$

and

$$N_t = 1 - L_t,$$

for $0 \leq t \leq T$.

Equilibrium

An equilibrium is a sequence of policy variables, quantities and prices such that the private agents solve their problems given the sequences of policy variables and prices, the budget constraint of the government is satisfied, markets clear, and the policy sequence is in the set defined by the policy.

The equilibrium conditions for the variables $\{C_t, L_t, R_t, M_t, B_t, T_t\}$ are the resource constraints

$$C_t + G_t = A_t\left(1 - L_t\right), 0 \leq t \leq T, \tag{2.11}$$

the intratemporal conditions

$$\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t}, 0 \leq t \leq T, \tag{2.12}$$

those are obtained from the households intratemporal conditions (2.6) and the firms optimal conditions (2.9), the cash in advance constraints (2.2), the intertemporal conditions (2.7) and the budget constraints (2.10), as well as the government policy rules, to be specified below.

3. INTEREST RATE POLICY DOES NOT ENSURE PRICE STABILITY

An equilibrium in the economy described above is characterized by a finite number of equations and unknowns. A necessary condition for there to be a unique equilibrium is that the number of equations equals the number of the unknowns. Interest rate rules, whether these are sequences of numbers or feedback rules, functions of future, current or past variables, are not sufficient restrictions. They are never able to pin down unique equilibria.

To see this, notice taht from the resource constraints, (2.11), the intratemporal conditions (2.12), and the cash in advance constraints holding with equality, (2.2), we obtain the functions $C_t = C(R_t)$ and $L_t = L(R_t)$ and $P_t = \dfrac{M_t}{C(R_t)}$, $0 \leq t \leq T$. We can substitute these variables in the intertemporal conditions (2.7), so that the system of equilibrium conditions restricting the policy variables $\{R_t, M_t, P_t\}$ can be summarized by the following dynamic equations

$$\frac{u_C\left(C(R_t), L(R_t)\right)}{\dfrac{M_t}{C(R_t)}} = \beta R_t E_t \left[\frac{u_C\left(C(R_{t+1})L(R_{t+1})\right)}{\dfrac{M_{t+1}}{C(R_{t+1})}} \right], t = 0, \ldots, T-1 \tag{3.1}$$

together with

$$P_t = \frac{M_t}{C(R_t)}, t = 0, \ldots, T.$$

The budget constraints restrict, not uniquely, the levels of state noncontingent debts and taxes. Assuming these policy variables are not set exogenously we can ignore those restrictions.

Suppose the interest rates are determined exogenously. In that case there are still more variables than unknowns. If the money supplies were also set exogenously in every state in the terminal period, then

the conditions above would determine the money stock in every state in the previous periods. That way the price levels would also be pinned down in every period and state. The degrees of multiplicity are therefore the number of states in the terminal period. The nominal interest rates restrict the conditional average growth rate of money supply, they do not restrict how money supply is distributed across states. In this economy with uncertainty there is still the need for a nominal anchor for every history. In a deterministic economy only one money supply would be missing, one nominal anchor.

In this economy, if instead of targeting the interest rate, there was a feedback rule for the interest rate where it would be responding to endogenous variables, nothing would change. There would still be the same number of equations and unknowns and the degree of multiplicity would be the same. Similarly this result does not depend on preferences or technology, and it also does not depend on whether prices are flexible or sticky.

In the following section, the economy lasts forever. It is the same economy, but with an infinite horizon. We illustrate how this problem of multiplicity is usually handled, by imposing conditions such that there is a single equilibrium in a neighborhood of interest, so that a particular equilibrium may be locally determinate. We also show that in the infinite horizon economy there are rules that implement unique global equilibria.

4. AN INFINITE HORIZON ECONOMY

4.1. Local determinacy

In monetary models with multiple equilibria it is possible to conduct policy so that there is a single equilibrium in the neighborhood of a steady state. In this case we say that there is a determinate equilibrium. Most of the analysis in monetary models focus on that particular equilibrium.

We will now consider the analogous infinite horizon model but will simplify the structure assuming that the utility is linear in consumption. For linear consumption the log-linearized intertemporal conditions approximated around a deterministic steady state with constant nominal interest rate and constant inflation π^*, which is also the target, are

$$\hat{R}_t - E_t \left(\hat{P}_{t+1} - \hat{P}_t \right) = 0.$$

Suppose now that the interest rate rule is the forward rule

$$\hat{R}_t = E_t \left(\tau \hat{\pi}_{\tau+1} \right)$$

so that the interest rate is raised above the steady state when the inflation forecast is above the target π^*. Then we have

$$(\tau - 1) E_t \left(\hat{\pi}_{t+1} \right) = 0,$$

so that for $\tau \neq 1$, expected inflation is pinned down, but the price level in each date and state is not.

Suppose now that the interest rate rule is

$$\hat{R}_t = \tau \hat{\pi}_t$$

where $\tau > 1$. Then, from

$$\hat{R}_t = E_t \left(\hat{\pi}_{t+1} \right),$$

we have

$$\tau \hat{\pi}_t - E_t \left(\hat{\pi}_{t+1} \right) = 0$$

With $\tau > 1$, there is a bounded solution and a continuum of unbounded solutions. If $\hat{\pi}_0 = 0$, then $\hat{\pi}_t = 0$, $t \geq 0$ but if $\hat{\pi}_0 = \varepsilon > 0$, the inflation path is explosive.[1]

The bounded solution is the determinate equilibrium

$$\hat{\pi}_t = 0.$$

The equilibrium inflation is equal to the target and given an historical price level P_{-1}, the path for the price level is pinned down. The unbounded solutions cannot be analyzed with the linearized model, which is only valid for small deviations around the steady state. In general there are other equilibria in the nonlinear model, which may cycle or converge to other steady states (see Benhabib, Schmitt-Grohe and Uribe, 2001, 2002)

In the following section we show that there are interest rate rules that do not react to inflation but rather to a forecast of the price level that implement unique global equilibria. These are the rules analyzed in Adao, Correia and Teles (2006).

In the log-linearized model the rule would be

$$\hat{R}_t = E_t \hat{P}_{t+1} .$$

Then from the intertemporal condition

$$\hat{R}_t = E_t \hat{P}_{t+1} - \hat{P}_t ,$$

we have

$$\hat{P}_t = 0$$

so that there is indeed a unique global solution.

4.2. Rules that implement unique global equilibria

Here we consider the original nonlinear model for general preferences but with an infinite horizon. Suppose monetary policy was conducted with the following interest rate rule

$$R_t = \frac{\xi_t}{E_t \dfrac{\beta u_c (t+1)}{P_{t+1}}}, \qquad (4,1)$$

where ξ_t is an exogenous variable. Then there is a unique global equilibrium. To see this, notice that the intertemporal condition (2.7) can be written as

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$$\frac{u_c(t)}{P_t} = \xi_t, t \geq 0, \tag{4.2}$$

so that

$$R_t = \frac{\xi_t}{\beta E_t \xi_{t+1}}. \tag{4.3}$$

Given the functions $C_t = C(R_t)$ and $C_t = C(R_t)$ obtained using the resource constraints, (2.11), and the intratemporal conditions, (2.12), we can use (4.2) above to determine the sequence of price levels P_t. The money supply is determined endogenously using the cash-in-advance condition.

Depending on the process for ξ_t, a particularly desirable outcome can be implemented. In this model the first best equilibrium can be achieved with the Friedman rule of a zero nominal interest rate. This can be implemented with the policy rule (4.1), above, where $\xi_t = \frac{1}{\beta^t}$.

We saw in the previous section that in the finite horizon economy there were no rules that implemented unique equilibria. Indeed in the finite horizon economy this rule cannot be used in the last period, since there are no forecasts ahead. The rule would have to be

$$R_t = \frac{\xi_t}{E_t \frac{\beta_{uc}(t+1)}{P_{t+1}}}, \ 0 \leq t \leq T-1 \tag{4.4}$$

For period T, the rule cannot be used because there is nothing to forecast at T, and if the nominal interest rates are set exogenously, the price level in the different states is not pinned down. If the economy lasted forever, there would be no last period and the rule would always work.

The forward looking interest rate feedback rules that implement unique global equilibria resemble to some extent the rules that appear to be followed by central banks. The nominal interest rate reacts positively to the forecast of future consumption[2]. It also reacts positively to the forecast of the future price level. This last feature of the rules is less conventional (see Woodford (2003) for Wicksellian price targeting rules).

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(2) It does not react to labor if the utility function is separable in leisure.

5. CONCLUDING REMARKS

In this note we discuss, based on Adao, Correia and Teles (2006), how monetary policy can be used to implement a unique stable price equilibrium. We show that in a monetary model with a finite horizon, where the degrees of freedom in conducting policy can be counted exactly, interest rate rules will not implement unique equilibria. Instead in an infinite horizon there are feedback rules that can achieve that.

The rules that implement unique equilibria are price targeting rules where the interest rate is raised when the forecast of the future price level goes up. It also reacts positively to the forecast of future economic activity. Unfortunately in order for the rule to be effective there is the need for a knowledge of the economic structure that is not realistic. Also, the fact that the rule would not work in a finite horizon model even if arbitrarily large is also an obvious fragility.

In the end we have to conclude that more work has to be done in the development of models or alternative ways of conducting policy so that we can be reasonably confident that policy is able to implement a unique stable price equilibrium.

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