EFFICIENCY OF SECONDARY SCHOOLS IN PORTUGAL: A STOCHASTIC FRONTIER ANALYSIS*

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1. INTRODUCTION

The scores in the national examinations taken by students at the end of their secondary courses have received much public attention in recent years, partly justified by the importance they have among the criteria for admission to higher education. These scores also have been used to evaluate the performance of Portuguese schools offering secondary courses. The aim of this paper is to analyse such performance, employing the stochastic frontier methodology in order to investigate the respective determinants and the degree of efficiency in the utilization of resources. The research presented in this article goes beyond a simple analysis of score-based school rankings that does not take into account the quantity of resources used and the impact of other factors like the environment in which schools operate.

The decision to enrol and invest in schooling is, in the first place, taken by the students and their families considering costs and expected benefits. However, such a decision has important externalities for the society as a whole, since education is believed to be one of the key factors behind a sustained increase in labour productivity. The educational attainment indicators for the Portuguese population lag behind those for other developed countries, and this is often pointed out as the main structural obstacle in Portugal's catching up process to higher income levels. The question is particularly relevant as those indicators coexist with comparatively high levels of public spending on education. In this context, research on the education production function and resource allocation for Portuguese schools is well justified.

The Stochastic Frontier Analysis (SFA) builds on the microeconomic concept of production function which represents the maximum output attainable given a certain quantity of inputs. The transposition of this methodology to the field of education is relatively straightforward in theoretical terms but faces important empirical difficulties. They concern, in the first place, the definition of output and the multiplicity of factors that may influence the learning process. For instance, relevant factors like some teacher characteristics, the innate capacities of students and the interaction with colleagues are difficult to incorporate into an empirical model. Additionally, the relationship between inputs and output in the educational process is rather complex and can only be summarised imperfectly in a production function. Such difficulties have been addressed in detail in the education economics literature and we will touch upon them in the course of this article.

Production frontier estimation in the field of education has mainly used non-parametric techniques like the Data Envelopment Analysis and the Free Disposable Hull (FDH), sometimes complemented with regression analysis (see, for instance, Bessent *et al*. (1982), Ray (1991) and Ruggiero (1996)). SFA has been already used in this context as well, like in Mizala *et al*. (2002). This methodology is more de-

* This article summarises the research presented in Pereira and Moreira (2007). See this reference for more details, in particular concerning the data and the econometric results. The opinions expressed in the article are of the authors and do not necessarily reflect those of the Banco de Portugal.

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manding in terms of assumptions, since it requires the specification of a functional form for the production function, but it is less sensitive to the presence of outliers and allows the possibility of making inference about the contribution of inputs. The last two aspects led us to favour the SFA for this study.

There are very few production frontier-type applications to education based on micro data for Portugal. Most of the evidence available in this domain comes from analyses at a rather aggregated level for groups of countries including Portugal, using non-parametric techniques. Such studies present rankings of countries based on performance and input indicators, like Clements (1999) and Afonso and St. Aubyn (2005). Clements (1999) is a valuable reference because it provides a thorough and critical review of the Portuguese educational system. As far as studies based on disaggregated data are concerned, we are only aware of Oliveira and Santos (2005) that applies the FDH to a sample of public secondary schools, measuring output by approval rates. Carneiro (2006) albeit with a different methodology – OLS regression followed by decomposition of variance – investigates the link between student achievement and a number of school and family background variables, exploiting the database from the 2000 OECD's Programme for International Student Assessment (PISA). The present study differs from those just mentioned as far as methodology is concerned, but also in that it measures output through national examination scores and considers almost the full population of schools offering secondary courses in Portugal, both public and private.

This article is organised as follows. Section 2 presents a brief overview of secondary education in Portugal, while Section 3 describes the stochastic production frontier methodology. Section 4 makes some considerations about the specification of the education production function and data relevant for studies in this area, both in general and for the Portuguese secondary schools. Section 5 presents and analyses the econometric results, focusing on the way the school, teacher and environmental variables determine performance through the education production function. Special attention is devoted to the relative efficiency of schools in public and private sectors. We also address the use of the SFA to rank schools on the basis of the estimates of school-specific technical efficiency. Section 6 concludes.

2. SECONDARY EDUCATION IN PORTUGAL

In the Portuguese educational system basic compulsory education comprises (since 1986) nine years, divided in three cycles: the first cycle covers the first four years, for children from the age of six onwards, the second cycle covers the 5th and the 6th year and the third cycle the 7th through 9th year. Secondary education encompasses three further years of schooling (10th to 12th).[1] Approval in the 12th grade depends partially on the scores in a set of national examinations. Otherwise assessment is made exclusively within each school, even though on the basis of common curricula, which at the secondary level differ according to the field chosen by the student, for example, natural or social sciences, humanities, and arts.

Table 1 presents a number of enrolment and resource indicators for secondary education in Portugal and, in some cases, the corresponding OECD figures. In a context of population ageing, the number of students enrolled in secondary courses has decreased markedly since 1995. This decline occurred along with a stabilization of the enrolment rates (percentage of population at typical school age enrolled), at almost 60 per cent, over the last decade. Most students attend public schools but in the last years the weight of private schools has increased steadily. Due to high dropout and repetition rates, a significant share of the secondary student population consists of overaged students attending special

(1) Corresponding to the third level of ISCED (International Standard Classification of Education), also called upper secondary education. Schools offering secondary courses in Portugal comprise secondary schools proper, secondary schools with the third cycle of the basic education, and basic schools (second and third cycles of basic education) with secondary courses.

Table 1

SECONDARY EDUCATION IN PORTUGAL: SOME ENROLMENT AND RESOURCE INDICATORS	PORTUGAL				OECD
	1989/90	1994/95	1999/00	2003/04	2003
Students enrolled*	309 568	457 194	417 705	382 212	-
by school nature(%)					
public schools	92.3	87.5	84.9	82.4	79.5
private schools	7.7	12.5	15.1	17.6	20.5
by courses (%)					
ensino regular	83.8	82.0	87.1	79.5	-
ensino nocturno / recorrente	16.2	18.0	12.9	20.5	-
Total expenditure as % of GDP**(a)	-	-	1.2	1.2	1.4
Expenditure per student					
in US dollars (PPP adjusted)**(a),(b)	-	-	5, 422	6, 022	7, 582
as % of GDP per capita**(a),(b)	-	-	32	34	28
weight of teachers' compensation (%)**(a),(b),c)	-	-	-	78.3	58.4
Teachers per 100 students**(d),(e)	-	-	12.7	13.7	7.9
public schools	-	-	-	13.2	7.7
private schools	-	-	-	16.4	8.6
Teachers with higher university degree (%)* (f)	68.6	75.9	85.8	-	-
Tenured teachers (%)*(f)	56.6	69.7	75.7	82.3	-

Sources: * Ministério da Educação (2003, 2004) , ** OECD (2002, 2006)
Notes: (a) Portugal's data are for 1999 and 2003. (b) Figures for Portugal include only public schools. (c) Refers to non-tertiary education. (d) Measured in full time equivalents; includes only schools of continental Portugal. (e) Portugal's data are for 2000 and 2004 and OECD's data for 2004. (f) Teachers at the secondary level and third cycle of basic education; includes only schools of continental Portugal.

repeater courses, designated as *ensino recorrente* (former *ensino nocturno*). These are courses designed for students who left school without completing secondary education.

In 2003, the last year with information available, expenditure on secondary education in Portugal amounted to 1.2 per cent of GDP, slightly below the OECD average. The expenditure per student was also lower than the OECD average, although this represented a higher-than-average spending effort when measured against the Portuguese GDP per capita (for which the gap to the OECD average is wider). Considering the composition of outlays, the most salient fact is the very large weight of teachers' compensation in the total. This is mainly due to an abnormally high headcount for teaching staff in Portugal common to all levels of non-tertiary education, as shown by teacher-student ratios much above the OECD average. In addition, teachers' salaries taken in relation to GDP per capita are comparatively high. In contrast there is evidence that both the other current and capital expenses are quite low in relative terms. For instance, the number of computers per student in 2003 was one of the lowest among OECD countries (OECD (2006)). Clements (1999) presents scattered evidence indicating that the wage bill may have squeezed other inputs like teaching materials and infrastructures. An additional aspect worth highlighting is the improvement in the academic qualifications of teachers in public schools since the beginning of the nineties, which took place along with a considerable rise in the proportion of teaching staff with tenure.

Educational attainment in Portugal has improved among recent generations, but remains well below the OECD average. In 2004, less than 25 per cent of the Portuguese adult population aged 35 to 54 had completed the secondary level of education. Considering the age group from 25 to 34 this percentage rose to about 40 per cent, but the corresponding figure for the OECD average was over 75 per cent. The performance of Portuguese students in recent international examinations has shown in general poor competency levels. For example, in the 2003 OECD's PISA for proficiency in mathematics of 15-year-olds, Portugal occupied the 25th position among 29 countries.

Given that overall financial input indicators in Portugal are not much below the OECD average (or even above if taken as a ratio of GDP per capita) while performance indicators are generally poor, it should come as no surprise that studies (like the aforementioned by Clements (1999) and Afonso and St. Aubyn (2005)) find that Portugal achieves little with the resources employed. This is even more the case when the input indicators used relate to physical resources, as the teacher-student ratio.

3. THE STOCHASTIC FRONTIER ANALYSIS METHODOLOGY

3.1. The basic model

The SFA methodology adds to the production frontier an error term with two components: one that allows for technical inefficiency and another that allows for any random events (see, for instance, Kumbhakar and Lovell (2000)). The basic formulation can be represented as

$$y_i = f(x_i, \beta) TEF_i e^{v_i}, \tag{1}$$

where y_i is the output of producer i; x_i is a vector of inputs; β is a vector of K+1 technology parameters to be estimated and $f(x_i, \beta)$ the deterministic production frontier. The variable e^{v_i} represents the random shocks on each producer, and $f(x_i, \beta) e^{v_i}$ the stochastic production frontier. Finally, TEF_i is the output-oriented technical efficiency, computed as the ratio of observed output to the maximum feasible output, given by the stochastic production frontier. Producer i attains the maximum feasible output if, and only if, $TEF_i = 1$, otherwise $o < TEF_i < 1$, and the producer is inefficient.

In order to estimate the stochastic production frontier model in (1), it is necessary to specify $f(.)$ further, which is normally assumed to take a Cobb-Douglas form. In this case and defining $TEF_i = e^{-u_i}$ with $u_i \geq 0$ to ensure that, $TEF_i \leq 1$, the model (in logarithms) is given by

$$\ln y_i = \beta_0 + \sum \beta_k \ln x_{ki} + v_i - u_i, \tag{2}$$

where v_i is symmetric. The error term $\varepsilon_i = v_i - u_i$ is negatively skewed, since it is composed by a two-sided 'noise' term and a nonnegative technical inefficiency term. Model (2) can be estimated by maximum likelihood[2], upon making assumptions about the distributions of v_i and u_i. The original specification put forward in the literature assumed that: (a) v_i has a normal distribution with mean 0 and variance σ_v^2; (b) u_i is a truncation below at 0 of a normal distribution with mean 0 and variance σ_u^2; (c) v_i and u_i are independent of each other and of the regressors. Later other specifications were suggested for the distribution of u_i, in particular, a positive mean μ for the underlying distribution. This assumption, the commonest in the empirical literature, has the advantage of modelling the inefficiency term with a positive mode, fitting better the case of producers with levels of inefficiency farther from zero.

Prior to the maximization of the likelihood function, a reparameterization of the type $\sigma^2 = \sigma_u^2 + \sigma_v^2$ and $\gamma = \sigma_u^2 / (\sigma_u^2 + \sigma_v^2)$ is typically introduced. The parameter γ measures the relative importance of the variances of u_i and v_i. Note that if $\sigma_v^2 \to +\infty$ and/or $\sigma_u^2 \to 0$, then $\gamma \to 0$, case in which the production frontier would comprise the deterministic frontier plus a two-sided error. In the context of this methodology, testing the significance of γ assumes particular importance, since if the null hypothesis $\gamma = 0$ was accepted, no stochastic frontier methodology would be necessary and all parameters in β could be consistently estimated by OLS.

After the estimation, it is possible to obtain the 'composed error' term for each producer ($\hat{\varepsilon}_i$), which is then used to obtain the estimates for the producer-specific efficiency scores (TEF_i). These efficiency

(2) The estimation of equation (2) by OLS is consistent for all parameters in β, except for β_0, since $E(\varepsilon_i) \leq 0$.

predictors are always based on the distribution of u_i conditional on the value assumed by ε_i. We will use throughout this article the predictor proposed by Battese and Coelli (1988) which takes the expression for $E(e^{-u_i} | \varepsilon_i)$ (remember that $TEF_i = e^{-u_i}$).

3.2. Incorporation of 'exogenous' influences on efficiency

The SFA methodology may have two components. The first one concerns the estimation of the stochastic production function, used as a reference to estimate the degree of technical efficiency, as explained in the preceding section. The second one is related to the incorporation of 'exogenous' variables that are not at the discretion of the producer but nevertheless influence the outcome of the production process (in the literature this is sometimes referred to as producer heterogeneity), as it can be the case of variables that characterize the environment where production takes place. Such variables are not supposed to influence the shape and/or location of the production frontier, but determine how far away the producer is from it.

The incorporation of inefficiency determinants into the SFA has initially been done in a kind of second step, after estimating the frontier in the first one, by regressing the TEF_i on a vector of producer-specific variables. While this approach may give an informal indication of possible explanatory variables for efficiency, it is econometrically flawed (see Kumbhakar and Lovell (2000)). Several approaches have been suggested in the literature to incorporate appropriately the inefficiency effects into the SFA. The one we follow in this paper was introduced by Battese and Coelli (1993), and assumes that u_i is a truncation below at zero of a normal distribution with mean $\mu_i = \delta_0 + \sum \delta_m z_{mi}$ (and variance σ_u^2), where z_{mi} are M producer specific variables that determine inefficiency. In case the δ's, except δ_0, are statistically equal to zero, then $\mu_i = \delta_0$ for all producers and the specification reduces to the model presented in the preceding section. The model with this modified assumption about the distribution of u_i can likewise be estimated by maximum likelihood.

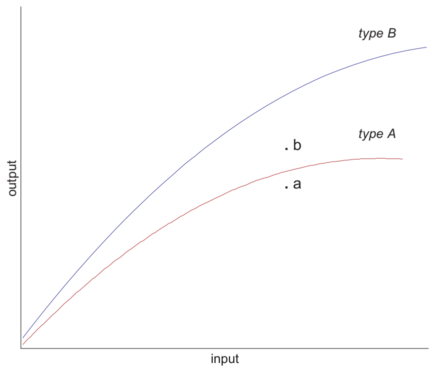
The trickiest issue in modelling 'exogenous' variables is that, very often, the location of a variable outside the discretion of the producer in the inefficiency term, as opposed to the production function, is a matter of judgement. For instance, variables related to environment may be nevertheless a determinant of technology. The econometric results do not always provide guidance. In fact, if a relevant variable is omitted from the production function, producers that 'use' it more intensively are likely to appear more efficient. That is, as the efficiency scores were estimated without controlling for such variable (see the Section 3.3), the latter may appear to have explanatory power for efficiency. Therefore, in doubtful cases there is merit in testing alternative specifications.

3.3. Model specification and measured technical efficiency

The SFA yields predictors that, by definition, measure technical efficiency after controlling for (i.e. net of) all variables in the production frontier. However, there may be variables that determine the production possibilities, but that one would not like to control for when measuring efficiency. At this point, it is useful to make a (simplifying) distinction between two categories of variables entering the production frontier. The first category refers to inputs proper whose variation implies a change in the utilization of costly resources from the point of view of the producer. One will always want to control for those by definition of efficiency as a relationship between outcomes and costly inputs. The second category covers variables that influence production, but whose variation does not come at a (visible) cost to the producer. These variables may, for instance, relate to 'organisation' of production or may be environmen-

Chart 1

PRODUCTION FRONTIER FOR TYPE-A AND TYPE-B UNITS



tal variables. When measuring efficiency, one will typically want to control for such variables only if the producers take them as given, for the sake of comparability among units.

Consider the case in which there are two types of producers (A and B) with different management practices, so that type B-producers always attain more output, for each given combination of other inputs. Such an effect should be incorporated into the production function, and a way of doing this is by means of a dummy variable that differentiates both types of producers. This amounts to estimating two separate production frontiers, and the efficiency scores from this model will be measured against each of two, depending on the type of producer. Chart 1 illustrates this point, for the case of one input (plus the dummy variable) and a deterministic frontier. Unit 'b' of type B vis-a-vis the outer frontier is less efficient than unit 'a' vis-a-vis the inner frontier; however, vis-a-vis a common (outer) frontier, unit 'b' is more efficient than unit 'a'.

When one wants to measure efficiency without controlling for one or more variables in the production function, it is not correct to exclude them because that would cause a problem of omitted variables. Nor it is appropriate in many cases to model them as 'exogenous' in the inefficiency term (as presented in the previous section), for instance, when such variables are at the discretion of the producer. Therefore it is necessary to introduce a modified efficiency predictor (TEF_i^*), which can be obtained by replacing the original estimate for the 'composed error', $\hat{\varepsilon}_i = \ln y_i - \ln \hat{y}_i$, by $\hat{\varepsilon}_i^* = \ln y_i - \ln \hat{y}_i^*$, where $\hat{y}_i^*$ is calculated taking, instead of the value for the variable at issue for producer i (x_i), the value x^* that maximizes the contribution of that input to output over the whole sample (see Coelli *et al.* (1999)).

4. THE PRODUCTION FUNCTION OF PORTUGUESE SECONDARY SCHOOLS

4.1. Conceptual issues

The first step in order to assess school performance using the SFA is to specify a production function. Summarizing the learning processes in a function is quite problematic due to its complexity.[3] The first issue arising is how to measure the outcomes of schooling. Most studies use standardized achievement test scores, however, other indicators have been used such as school approval rates (Oliveira and Santos (2005)) or dropout rates (Kanep (2004)). Theoretically, the main purpose of education is to develop the skills and knowledge of students in order to make them more productive in the labour market. The fact that empirical investigations tend to detect correlation between the level of schooling and post-school achievement offers some support for concentrating on examination results. In a broader sense, schools have the role of promoting values and contributing to integration in society, aspects hardly measurable by an indicator.

The second difficult issue concerns the factors determining the educational output. Ideally, the analysis should include not only school inputs, but also family background and influence of peers, as well as innate endowments and learning capacities. Many factors affecting the educational production process are not observed and/or quantifiable, and ultimately are difficult to incorporate mechanically into a production function. Inputs relating to teachers are typically included using proxies of their objective characteristics, like qualifications and experience, but ignoring other non-quantifiable features that can be important such as communication skills, teaching methods or classroom management. Furthermore the information on some school organisational aspects like curricula, textbooks or school day is limited or otherwise difficult to incorporate into the models. Another shortcoming concerns accuracy of measurement, since for some inputs (in particular those related to the school) one should possibly use a 'value added' specification by employing measures of cumulative influence over the years. This approach is very demanding in terms of data, which may explain the low number of studies that followed it. It is fair to say that some of these difficulties are more relevant in the context of disaggregated studies that attempt to model individual student performance, than in the context of modelling performance across schools, as we do. This is, in particular, true for student inputs like innate endowments and learning capacities which should average out at the same level across schools. Studies taking the school as a reference are less informative to the extent they do not consider intra-school heterogeneity, but are less demanding in terms of data.

4.2. Data

4.2.1. Output indicator

The output measure selected in this study was the average score in the 12th grade national examinations by school, for the academic year 2004/05 (see Table 2 for some descriptive statistics of the variables).[4] In Portugal, national exams have an important role as a selection mechanism for further schooling, thus relating directly to 'real' outputs. Students, parents and policy makers use them to as-

(3) Hanushek (1979) and Hanushek (1986) provide a comprehensive discussion of this topic.

(4) In Pereira and Moreira (2007) additional details about the variables and their sources are presented.

sess secondary schools' performance and implicitly the quality of education that they provide. Moreover other indicators such as school completion rates have the drawback that they are not comparable across schools due to different approval or success criteria.

National examinations evaluate student's knowledge on specific subjects. There is no set of exams obligatory for all pupils; the requirements depend on the area in which the students are and the post-secondary courses they wish to attend. In this study, we use examination scores for all subjects, which might pose a comparability problem, since the weight of the various disciplines is not uniform among schools. However, the alternative of focusing on one discipline seemed also quite arbitrary, as our input variables relate to the school as a whole (or to the environment).

4.2.2. School and teacher variables

The school data comprise the number of students (split between *ensino regular* and *recorrente*), teachers and classes, all variables relating only to secondary courses, and whether the school has private or public management. Arguably we are lacking a measure of capital, for instance regarding school facilities. The school data were supplied by the *Gabinete de Informação e Avaliação do Sistema Educativo* and cover the academic year 2004/05. Concerning teacher data, we dispose of information about seniority, age, academic background, tenure (only for public schools) and average wage, for the whole school. For these variables the source was the *2º Recenseamento Geral da Administração Pública* (2nd General Government Census) for the public schools and the *Quadros de Pessoal* survey for the private.[5] Since the last General Government Census dates from December 1999, for the sake of comparability, we used the *Quadros de Pessoal* survey of 1999 (October).[6]

Concerning the set of regressors included in the model, we chose the average number of teachers per 100 students as an input quantity measure. However, this variable was not available in 'full time equivalents', thus being an imperfect measure of the 'teaching effort' put into the educational production process. Therefore we included another variable in the regression, the teacher-class ratio, as an indicator for the degree of 'intensity' in the utilization of the teaching staff. In fact, tenured teachers in public schools may be relieved of teaching duties for several reasons.[7] On the other hand, due to the decline of the student population over the last decade, it may also happen that there is an excess of teaching staff allocated to some public schools for their needs. While it is reasonable to assume that other tasks carried out by teachers also contribute positively to school performance, their contribution is likely to fall short of that of teaching.

There is a general perception that students are not equally involved in their educational project. Students in *ensino recorrente* perform on average worse than their counterparts in *ensino normal*, and thus it is important to control for the relative weight of both groups in schools. Further we included in the regression an indicator for the (secondary education) production scale - a ranking based on both the number of students and teachers - given that our input quantity variable does not have a scale dimension. We also considered a dummy variable to differentiate between public and private institutions.

Concerning teacher data, in the place of seniority in the job that was not defined using the same criteria in the two sources used, it was taken on board in the regression the average teacher age (see Pereira

(5) *Quadros de Pessoal* is an annual mandatory employment survey, carried out by the *Ministério do Trabalho e da Solidariedade Social*, and covers private sector employees.

(6) Thus teacher data refer to the academic year 1999/2000, a different academic year from that of the school data (2004/05). We take them on board under the assumption that the relative position of schools regarding the characteristics of the teaching staff has not changed substantially in the intervening period.

(7) Apart from particular cases like disability and performing administrative tasks at school, teachers are entitled to a progressive reduction of weekly teaching hours when they are 40 or older and have 10 years of seniority or more. This reduction can add up to a maximum of 8 hours (out of a normal workload of 20 hours at the secondary level).

Table 2

DESCRIPTIVE STATISTICS

	All schools				Public		Private	
	mean	std.dev.	p25	p75	mean	std.dev.	mean	std.dev.
Output								
Average scores in exams	108	12	101	115	107	11	113	17
Inputs								
Schools								
Teachers per 100 students	12.6	4.1	10.2	14.7	12.7	3.9	12.5	5.4
Teachers per class	2.8	0.8	2.2	3.3	2.9	0.8	2.3	0.8
Students per class	22.4	4.0	20.3	24.8	22.8	3.6	19.6	5.3
Size (ranking)	-	-	-	-	320	148	144	106
Share of students in ens. recorrente	0.17	0.17	0.00	0.27	0.17	0.16	0.13	0.24
Teachers								
Average age (years)	38.9	4.0	35.7	41.7	39.0	3.9	38.2	4.4
Average wage (euros)	1399	256	1235	1571	1456	213	1051	216
Share with university education	0.97	0.04	0.96	1.00	0.98	0.03	0.93	0.07
Environmental								
Health status (index)	223	19.0	212	238	-	-	-	-
Household electricity consumption (Kw/h)	2.0	0.6	1.6	2.4	-	-	-	-
Illiteracy rate (perc.)	12.7	5.2	8.7	16.4	-	-	-	-

Notes: Statistics based on 490 schools, 419 public and 71 private, except for teachers' average wage and share with university education which refer to 489 schools, 419 public and 70 private. Statistics for environmental variables are based on data for 241 municipalities.

and Moreira (2007)). Average teachers' wage was not considered, given the high colinearity with seniority (and correlated variables), in particular for public schools. We tested the significance of other candidates to enter the regression, namely, the proportion of teachers with a university degree but results pointed to its non-inclusion. In fact such proportion is likely to have increased significantly over the last years (in line with the evidence in Table 1) and shows currently reduced variability (Table 2).

4.2.3. Environmental variables

One can expect that the region where schools are located influences attainment. In the public debate about the examination scores it is often stated that schools in predominantly rural areas have worse outcome than their counterparts in more developed urban centres. In order to study the impact of school location on output, the educational production frontier must include environmental variables. We considered three environmental indicators at the municipality (*concelho*) level, related to the living standard, education level and health conditions. Those were, respectively, the average household electricity consumption, the illiteracy rate and a Health Status Index elaborated by Santana *et al.* (2004).[8]

4.3. Model specification

We first considered a stochastic production frontier without environment given by

$$\ln y_i = \beta_0 + \beta_1 \left(\frac{T}{S}\right)_i + \beta_2 \left(\frac{T}{S}\right)_i^2 + \beta_3 \left(\frac{T}{C}\right)_i + \beta_4 \ln A_i + \beta_5 \ln S_i + \beta_6 P_i + \beta_7 R_i + v_i - u_i \qquad (3)$$

where *i* refers to the ith school, *y* is the average score in the national examinations; *(T/S)* and $(T/S)^2$ are the number of teachers per 100 students and its square; *T/C* is the number of teachers per class; *A*

(8) For each indicator it was considered the last year available (see Pereira and Moreira (2007)). Note that the illiteracy rate is an 'inverse' indicator of the educational level.

is average age of teachers; *S* is a measure of school size; *P* is a dummy variable which takes on value 1 if the school is private and 0 if the school is public; *R* is the share of the student population in *ensino recorrente*. Variables *v* and *u* are defined as described in the Section 3.1. We followed a log-linear specification for the teacher-student ratio (approximated by a quadratic function), average age and school size, in order to allow for a decreasing marginal contribution to output (see Pereira and Moreira (2007)), while the coefficients of the remaining variables are semi-elasticities.

As to the environmental regressors, the fact that schools do not have control over the environment would speak for modelling them in the inefficiency component. This is nevertheless a debatable assumption since in the traditional education production modelling, socioeconomic characteristics enter the production function.[9] This question is related to the discussion presented in Section 3.2 about whether the impact of 'exogenous' variables should be considered in the technology or in the efficiency. Both alternatives were estimated, the first one consisting of an extended version of equation (3) encompassing the three socioeconomic variables - living standard (*LivSt*), educational level (*Educ*) and health conditions (*Health*). The second one including those variables in the mean of the distribution underlying u_i that becomes $\mu_i = \delta_0 + \delta_1 \, LivSt_i + \delta_2 \, Educ_i + \delta_3 \, Health_i$.

5. RESULTS

5.1. Estimated stochastic production function

The estimation results are shown in Table 3. All models were estimated by maximum likelihood using FRONTIER 4.1 (Coelli (1996)). The null hypothesis of absence of random technical inefficiency ($\gamma = 0$) is rejected in the different specifications and thus the SFA seems quite appropriate for the data. In the models with the environment modelled in the one-sided error component, the estimate of γ goes down, in line with a lower value of σ_u, since some heterogeneity previously captured by this parameter now goes into the producer specific mean. The parameter μ, or the $\delta' s$ in the model with inefficiency effects, are statistically not significant at a conventional level of significance, pointing to a null mode for the distribution of u_i. The average measured efficiency level is near 90 per cent (see also Section 5.3, where we propose a different efficiency predictor), but this result is sensitive to the academic year on which the estimation is based.[10]

The school and teacher variables are significant in all specifications, also in the ones including environmental variables, although in some cases the respective impacts change when the latter are taken on board in the regressions (see below). As to the environmental regressors, the health conditions indicator is not significant in any of the models, a result that may be explained by the homogeneous situation in the different municipalities in this regard, mirrored by the reduced variance of the indicator. The other environmental variables are significant only when incorporated into the production function, which suggests this modelling alternative as more adequate.[11] Note that the sign of such variables in the production frontier is the opposite of that in the inefficiency term, and rightly so, because they determine the maximum output level in the former, and the deviation from it in the latter.

(9) See Coelli *et al.* (1999) for a similar discussion in another context.

(10) When we include data for the academic year 2003/04, in a panel-type formulation, the measured efficiency is near 10 per cent lower (and the null hypothesis $\mu = 0$ is rejected) (see Pereira and Moreira (2007)).

(11) This conclusion is not so clear-cut in the panel formulation, in which an inclusion in the inefficiency term is also not rejected by the data. However, as regards the adequacy of the model and the impacts of the different variables, the results are largely coincident. The main exception concerns the teacher-student ratio. In fact, its significance is below the conventional level in the models corresponding to the ones presented in the first two columns of Table 3.

Table 3

ESTIMATED STOCHASTIC PRODUCTION FRONTIER

		Without environment	Environment in production	Environment in inefficiency
Production Function	Teacher/student	0.014	0.014	0.013
		(2.8)	(2.8)	(2.5)
	(Teacher/student)$^{2(a)}$	-0.040	-0.040	-0.040
		(-2.4)	(-2.4)	(-2.4)
	Teachers/class	-0.024	-0.023	-0.021
		(-2.3)	(-2.3)	(-2.1)
	ln(Age)	0.369	0.217	0.235
		(7.6)	(4.0)	(3.9)
	ln(Size)	0.037	0.025	0.026
		(4.6)	(3.2)	(3.1)
	Share *ens.recorrente*	-0.071	-0.056	-0.074
		(-3.0)	(-2.3)	(-2.9)
	Private school	0.096	0.064	0.069
		(6.9)	(4.4)	(4.7)
	Living std.	-	0.026	-
			(2.8)	
	Educacional level$^{(a)}$	-	-0.396	-
			(-2.9)	
	Health Conditions$^{(a)}$	-	0.033	-
			(1.2)	
	Constant	3.142	3.663	3.708
		(19.2)	(17.8)	(17.5)
Distributions of u and v	Living std.	-	-	-0.074
				(-1.4)
	Educacional level	-	-	0.006
				(1.8)
	Health Conditions	-	-	-0.001
				(-0.9)
	μ or δ_0	-0.351	-0.316	0.233
		(-1.7)	(-1.3)	(1.6)
	σ^2	0.037	0.031	0.013
		(2.9)	(2.4)	(2.9)
	γ	0.835	0.806	0.560
		(13.6)	(9.8)	(3.0)
	σ_u	0.176	0.158	0.087
	σ_v	0.078	0.078	0.077
Efficiency Predictors	Average TEF_i	0.94	0.94	0.93

Notes: Results based on data for 490 schools, for the academic year 2004/05, except for the average age (academic year 1999/00). Environmental variables are for the last year available (see Pereira and Moreira (2007)). t-ratios in brackets. **(a)** Coefficient multiplied by 100.

The impact of the number of teachers per 100 students on output is positive and marginally decreasing, since the quadratic term is negative. In the calculation of this impact it is assumed that the average relationship between teachers and classes is kept, that is, in the case of an increase in the number of teachers, that 'new' teachers are engaged in teaching to the same degree as the 'older'. Thus the number of classes increases proportionally, or the number of students per class (class size) goes down. It is also possible to estimate the impact on performance of a positive variation in the number of teachers not accompanied by a change in the class size, which also reflects the offsetting effect of the rise in the teacher-class ratio. In this case, the marginal impact is much lower, because of the negative sign of the coefficient associated with this last variable.

As referred in Section 2, in Portugal over the last decade there has been a fall in the number of students in secondary courses, implying a strong increase in the teacher-student ratio in some schools. Given the low flexibility to move teachers with tenure across schools, the positive impact on output that should have ensued might have been restricted to the schools affected by the phenomenon and with relatively high figures for the class size (along with a reduction in that variable). Schools already featuring small class sizes had a reduced margin to obtain output gains from further reductions. For those, as there are government regulations that fix a minimum value for that size[12], the decrease in the number of students was most likely accompanied by a reduction in the number of classes. In our specification this would be captured by a rise in the teacher-class ratio, offsetting the impact of the change in the teacher-student ratio.

Hanushek (1986) provides a survey of much econometric work in this area, most of which finds no significant impact of the teacher-student ratio on output. Hanushek points out, as a possible reason for this, the fact that the relationship between the number of students and teachers, or students and classes, is often subject to regulations that reduce much the sample variability. As said, there are such regulations in Portugal, applying to the class size, but the variable shows sample variability (Table 2).

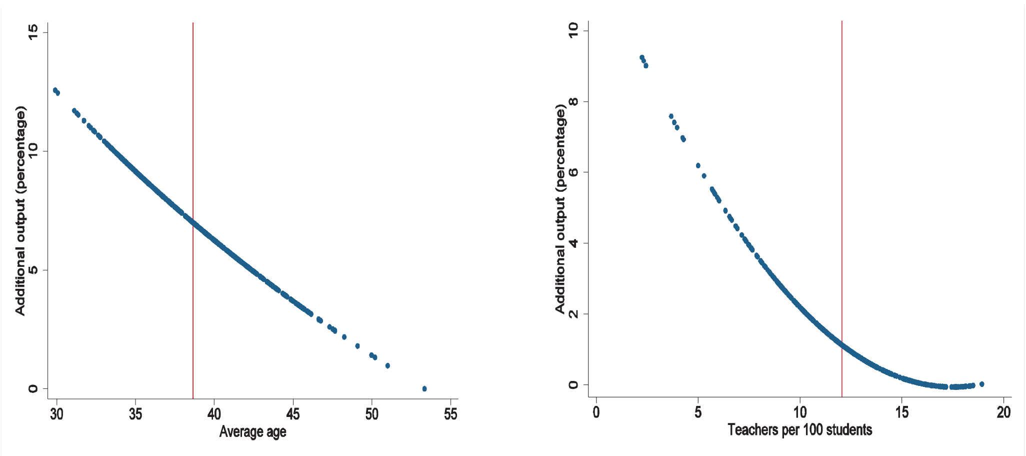
Teacher seniority, proxied by age, appears important for educational output in all specifications considered. However, controlling for the influence of environmental variables on output, the estimated elasticity goes down from 0.369 to 0.217 or 0.235, when those are included in the production function or in the inefficiency term, respectively. This may be explained by the fact that this regressor in the model without environment, beyond the pure effect of teacher experience, is most likely capturing the impact of school location. Clements (1999) states that there is 'a systematic movement of teachers from less desirable areas to developed urban centres' as they become more experienced. This smaller impact of experience, correcting for the fact that more developed regions tend to attract more senior teachers, is likely to be more accurate.

It is interesting to compare the potential output gains stemming from an increase in the number of teachers (per 100 students) and more experience of the teaching staff. Chart 2 presents the output gains at the estimated frontier (in percentage) of an increase in each of these factors from the current level where the school is, to the sample level yielding the maximum output. The vertical lines indicate the respective sample medians. Contrary to seniority, for the teacher-student ratio a great proportion of schools operate at levels where output gains are very low. Such evidence is likely to reflect the abnormally high value of the teacher-student ratio in Portuguese schools vis-a-vis international standards.

The share of *ensino recorrente* influences negatively the outcome, as expected. School size (comprising only secondary education) appears as a positive determinant of output. This speaks for concentrat-

Chart 2

ADDITIONAL OUTPUT FROM INCREASES IN THE UTILIZATION OF RESOURCES



(12) As a general rule, 24 pupils (and a maximum of 28) (see *Despacho 13 765/2004 do Ministro da Educação*).

ing resources in fewer schools, whenever possible. Other studies like Rainey and Murova (2003) and Mizala *et al*. (2002) also find scale economies in education production, albeit in different contexts. The impact of the private school dummy on educational output is positive and precisely estimated across the specifications, meaning that it is possible to estimate separate frontiers for the two groups. When location is taken into account (which corrects for the prevalence of private schools in relatively more developed regions), private institutions feature output gains between 6 and 7 per cent. This result is, as always, conditional on the variables included in the regression and this point deserves a more detailed discussion, so we come back to it in a separate section below.

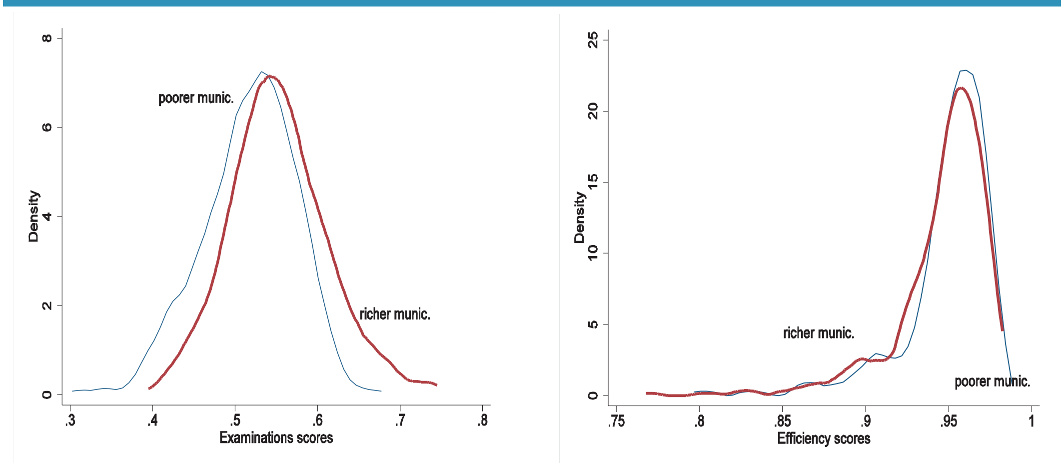
As already mentioned, socioeconomic variables are globally significant and therefore influence school output, in line with the conclusions reached by Oliveira and Santos (2005), although they use a different methodology and other variables. In order to highlight the impact on efficiency scores of controlling for school location, Chart 3 (right side) presents the respective density functions for schools in poorer and richer municipalities.[13] The predictor densities, net of the effect of the environmental variables, almost overlap. By contrast, on the left side, the density function of the examinations scores (divided for 200, the maximum) for richer municipalities is clearly shifted to the right in comparison to the corresponding density for poorer municipalities.

5.2. Efficiency of public and private schools

Clements (1999) stated in his conclusions that evidence suggested that private schools were more efficient than their public sector counterparts, as they achieved higher success rates employing less resources, in particular, as far as the relationship between teachers and students was concerned. Regarding the performance in national examinations, the general perception is that private schools outdo those in the general government. Nevertheless, a careful analysis of the distribution of examination scores (see Pereira and Moreira (2007)) shows that private schools have better results only at the upper percentiles, while at the intermediate and lower percentiles the results in both groups are similar. On the other hand, figures in Table 2 indicate that schools in public and private sectors employ a similar

Chart 3

EXAMINATION SCORES AND EFFICIENCY SCORES IN POORER AND RICHER MUNICIPALITIES



(13) The criterion used to differentiate between poorer and richer municipalities was the average value for the *Indicador per capita do Poder de Compra Concelhio 2004* (see Pereira and Moreira(2007)).

level of resources, as far as one can measure by the teacher-student ratio, although private institutions employ teachers 'more intensively', allowing them to have lower class sizes. The average teacher age is almost identical in both groups.

We have seen in the preceding section that, after controlling for the use of resources and other school and environment variables, the results clearly indicate that private schools are more efficient. It is worth noting that if we had run a regression with financial inputs (like average expenditure per student), instead of physical one, the efficiency gap between private and public schools would have been most likely larger, as average teachers' salary in general government seems to be considerably higher than in the private sector (see Table 2).

The results obtained are likely to be biased by the fact that students attending private schools typically come from households with higher social status. Carneiro (2006) presents evidence pointing to a strong importance of family background variables as determinants of educational outcomes in Portugal, and this is very much in line with the empirical findings for other countries. Unfortunately information on the socioeconomic background of students who took the examinations (or, more generally, of the students attending a given school) was not available. Actually, for this type of insights, it would be important to focus on a lower aggregation level - that of the student. By considering school averages one already looses information on intra-school variance, very important in this context.

We conjecture that controlling for the family background of students would reduce the magnitude of the private sector dummy, but it would not obliterate its significance. In the first place, not all private schools are financed by students' families. About 1/4 of pupils in private schools attend institutions privately run but financed by government, and for those the household background argument does not apply. Secondly, such an argument is normally put forward in connection with private schools featuring outstanding results, at the very top of the distribution of examination scores. However, observing that distribution, one sees considerable dispersion for private schools, with a number of institutions ranking very low in terms of performance. In the light of abovementioned findings in Carneiro (2006), for those schools there seems to be no reason to assume *a priori* that their students come from advantaged households.

5.3. A proposal for an efficiency ranking of secondary schools

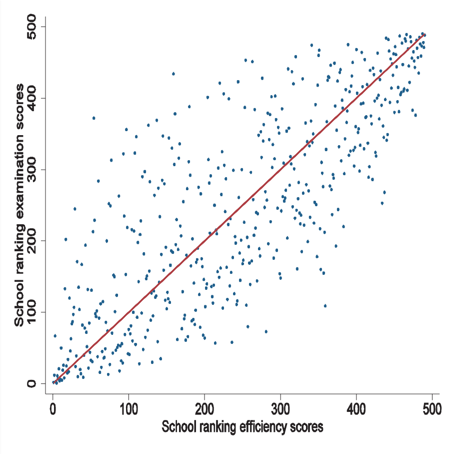
In Section 3.3 it was explained that it is possible to measure efficiency not controlling for some of the variables in the production frontier. This idea is here applied to the construction of a ranking of Portuguese secondary schools. This ranking is based on a modified predictor that controls for all regressors considered above, except for the teacher-class ratio and the private school dummy (see Pereira and Moreira (2007)). Indeed, these two variables capture aspects relative to the organisation and management of schools that are under its discretion (in case of public schools, under the discretion of the Ministry of Education) and whose change would affect output without an evident impact on the production costs.

This approach leads to a measured efficiency level of about 90 per cent.[14] Chart 4 presents the loci of individual schools in the ranking resulting from the original examination scores and the ranking on the basis of the efficiency scores computed with the proposed methodology. The latter ranking entails considerable changes in comparison with the original one (the 45º line indicates the schools whose position is unchanged).

(14) In the panel formulation, the corresponding figure is slightly under 80 per cent.

Chart 4

ORIGINAL AND PROPOSED RANKINGS OF SCHOOLS



6. CONCLUSIONS

In this article we studied the determinants of the educational output of Portuguese secondary schools – measured by the average scores in the 12th grade national examinations – and provided estimates of the respective technical efficiency level. SFA models with school, teacher and environmental variables were estimated. Further a proposal for an efficiency ranking of secondary schools was put forward.

Our analysis points to the existence of technical inefficiency: examination scores could be on average about 10 per cent higher for the current level of resources. This value is sensitive to the academic year on which the estimations are based. On the other hand, the production frontier underlying the measurement of efficiency takes as a reference the most efficient Portuguese schools that implicitly establish a benchmark. It would be interesting to apply the same techniques to a panel of schools from several countries. The cross-country analysis of global performance and input indicators suggests that the Portuguese benchmark might be inefficient when compared to other countries. In this case, the measured inefficiency level of the Portuguese schools could be higher.

Results indicate that the 'quality' of teachers has more effect on output than the 'quantity'. That is, the variation in the number of teachers per student appears to have less influence on output than differences in their characteristics proxied by seniority. On the other hand, there seems to be a high proportion of schools operating at teacher-student levels for which changes in this variable have little effect on output. This should reflect, in particular, the fact that many schools have lost students over the recent years and did not adjust the number of teachers. Therefore, enhancing the flexibility in the allocation of teachers could free resources without a noticeable effect on scores. Part of the reduction in outlays obtained could be applied on non-personnel spending items, in which Portugal ranks very low in international comparisons. Such added flexibility would also be the way to achieve a greater uniformity in the average class size across schools.

As far as the school network is concerned, we found evidence of scale economies in secondary education production, indicating potential gains from the concentration of resources.

The study demonstrates that there is a sizeable influence of geographical location of schools on outcome. Schools located in municipalities featuring higher living standards and education levels achieve a comparatively better performance.

The comparison between schools in the general government and in the private sector shows that the latter are more efficient, after resources and other outcome determinants have been taken into account. It was not possible to control for the influence of the socio-economic background of students in the regressions which introduces a bias in the results. Nonetheless evidence points to the attainment of efficiency gains with the introduction in the general government schools of some management practices of the best schools in the private sector.

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