THE FORWARD *PREMIUM* OF EURO INTEREST RATES*

*Sónia Costa***

*Ana Beatriz Galvão****

1. INTRODUCTION

The expectations hypothesis is based on the idea that expectations about future interest rates affect the current level of long rates. In these conditions, interest rates expectations may be extracted from the yield curve, which represents interest rates at a given point in time for different maturities. A popular way of extracting market expectations about future interest rates is the computation of the forward rates (i.e., the interest rates contracted today to start in the future) implicit in the yield curve. Such estimates may not be accurate as the forward rate may differ from market expectations by a forward *premium*. In a context of uncertainty, market participants might demand a different return from the expected value to protect themselves from possible surprises.

This paper shows that indeed there is a forward *premium* in the euro area forward rates. We use an extension of the model for forecasting the yield curve of Diebold and Li (2006) to estimate this forward *premium*. The first step of the procedure applies the Nelson and Siegel (1987) regression for computing yield factors (level, slope and curvature). In a second step, these factors are modelled jointly with macroeconomic variables in a vector autoregression (VAR). The estimates of the VAR are used to compute forecasts of the yields factors, which are used in the Nelson and Siegel (1987) regression to obtain yield forecasts. At date t, the estimate of the τ-period forward *premium* for horizon h is the difference between the implicit forward rate of maturity τ contracted in t to start in $t+h$ and the forecasts at t for the yield of maturity τ in $t+h$.

We also develop a method for calculating confidence intervals for the forward *premia*. The evaluation of the significance of the forward *premia* at each point in time is an important feature of our approach because it helps to decide whether a change in the forward rate is due to a change in the risk *premium* or instead to a change in expected interest rate.

Our sample starts in 1999 and the interest rates consist of euro money market rates and swap rates.[1] The estimation of the risk *premia* using only euro data is not frequently found in the literature.

Our estimates of forward *premia* are positively correlated with the key ECB interest rate. This relation might be explained by the fact that agents change their probability distributions about future interest rates when policy rates are increasing (decreasing) such that they increase (decrease) the probability of future interest rates being larger (smaller) than their actual expected values. Because interest rates higher than expected are bad news, a move of the distribution to the right implies a higher risk *premium*.

* The views expressed are of the authors and do not necessarily reflect those of the Bank of Portugal. We would like to thank Rita Lourenço for the computation of the option-implied skewnesses and the comments and suggestions made by Isabel Gameiro and João Sousa.

** Economic Research Department.

*** Department of Economics, Queen Mary, University of London.

(1) In Costa and Galvão (2007) we use also a longer set of data which consists of German interest rates in the period before the introduction of the euro. When using that set of data we identify a break in the dynamic correlation between yield factors in the period 2000-2001. Because this break affects the estimates of the forward *premium*, we advice to use only information after 1999 to compute the euro forward premium.

The second section of the paper reviews the definitions of risk *premium* and the expectations hypothesis. The section that follows describes our method to compute the forward *premia*. Section 4 presents empirical results on the biases of the forward rates when forecasting 3-month interest rates. It also shows that this is in part caused by a forward *premium*. Section 5 presents our estimates of the forward *premia*. Section 6 includes some concluding remarks.

2. THE EXPECTATIONS HYPOTHESIS AND THE RISK *PREMIUM*

This section presents notation and concepts that are important to understand the different definitions of risk *premia*. The risk *premium* on interest rates depends on the definition of the expectations hypothesis of the term structure of interest rates employed to compute the risk neutral component of the interest rates.

Cochrane (2001) defines the *yield* of a bond as "the fictional, constant, known, annual, interest rate that justifies the quoted price of a bond, assuming that the bond does not default" (p. 348). From this definition, the gross yield of a zero-coupon bond with maturity n and price $P_t^{(n)}$ is $Y_t^{(n)}$ that satisfies $P_t^{(n)} = \frac{1}{\left[Y_t^{(n)}\right]^n}$. Assuming that the price of a bond at the maturity date is 1 $\left(P_t^{(0)} = 1\right)$, the return of holding a n-period bond until maturity (return to maturity) corresponds to $R_t^{(n)} = \frac{1}{P_t^{(n)}}$. Using these variables in logs, we can see that the log-yield of a zero-coupon bond $\left(y_t^{(n)} = \frac{p_t^{(n)}}{n}\right)$ corresponds to the log-return per period $\left(y_t^{(n)} = \frac{r_t^{(n)}}{n}\right)$.[2] In addition to the yield and the return to maturity, the holding period return and the forward rate are also obtained from the log-price of a zero-coupon bond. The *holding period return* is the return from holding a bond with maturity n over the next period (i.e, from t to $t+1$): $hpr_{t+1}^{(n)} = p_{t+1}^{(n-1)} - p_t^{(n)}$. The *one-period forward rate* is the interest rate contracted today to start in n periods from now and with maturity in $n+1$ periods from now, that is, $f_t^{(n,n+1)} = p_t^{(n)} - p_t^{(n+1)}$. The one-period forward rate can be also written using the yields as $f_t^{(n,n+1)} = y_t^{(n+1)} + n\left(y_t^{(n+1)} - y_t^{(n)}\right)$.

"The yield curve is a plot of yields of zero-coupon bonds as a function of their maturity" (Cochrane, 2001, p. 352). Most of the times, the yield curve is upward sloping. Based on the definition of the forward rate using the yields, one can show that the one-period-forward rate is above the yield for the same n if the yield curve is upward slopping.

The expectations hypothesis describes the relationship between returns on zero-coupon bonds of different maturities. It is based on the idea that expectations about future interest rates affect the current level of long rates. As Cox, Ingersoll and Ross (1981) point out there are four different formulations of the expectations hypothesis:

(i) The yield of a zero-coupon bond that matures in n periods is equal to the average of the expected one-period yields (yield to maturity hypothesis):

$$y_t^{(n)} = \tfrac{1}{n} E_t\left(y_t^{(1)} + y_{t+1}^{(1)} + \ldots + y_{t+n-1}^{(1)}\right)$$

(2) In Cochrane (2001) notation, the yield (and the return) correspond to one plus the interest rate, that is, it corresponds to the gross yield (return). In logs, this distinction is not relevant since $\ln(1+i) \approx i$. It is also important to notice that $y_t^{(n)}$ corresponds to the continuously compounded interest rate. This is so because the relation between the continuously compounded interest rate $\left(i^c\right)$ and the price of a bond with maturity n is $e^{ni^c} = \frac{1}{P_t^{(n)}}$, and the relation between i^c and an annually compounded interest rate $\left(i^a\right)$ is $i^c = \ln\left(1+i^a\right)$.

(ii) The return to maturity of a *n*-period bond is equal to the expected return of rolling over a series of single-period bonds (return to maturity hypothesis):

$$r_t^{(n)} = E_t\left(r_t^{(1)} + r_{t+1}^{(1)} + \ldots + r_{t+n-1}^{(1)}\right)$$

(iii) The one-period forward rate *n*-periods ahead is equal to the corresponding expected one-period spot rate (unbiased expectations hypothesis):

$$f_t^{(n.n+1)} = E_t\left(y_{t+n}^{(1)}\right)$$

(iv) The expected holding period return of a bond with maturity *n* is equal to the current one-period interest rate (local expectations hypothesis):

$$E_t\left(hpr_{t+1}^{(n)}\right) = y_t^{(1)}$$

In practice the above relations do not necessarily hold. The difference between the left-hand side and the right-hand side of the above equations is then called the risk *premium*. If the expectations hypothesis holds in its pure form, the risk *premium* will be zero. However, it is generally considered that the expectations hypothesis holds if the risk *premium* is constant over time. For the empirical testing of the expectations hypothesis, the definition (i) is the most popular, because it can be tested using restrictions in a vector autoregression of yields of different maturities and/or using cointegration. The results depend in general on how large *n* is. For large *n*, the hypothesis of a constant risk *premium* is in general not valid.

The four different formulations of the expectations hypothesis imply different measurements of the (time-varying) risk *premium*. The premium derived from definition (i) is frequently called as yield *premium*, term *premium* or rollover term *premium*. The *premia* arising from definitions (iii) and (iv) are called (one-period) forward *premium* and (one-period) holding *premium*. Finally, the *premium* corresponding to definition (ii) is not usually calculated. The nomenclature of the different risk *premia* is sometimes confusing. In fact, many authors (as, for example, Singleton, 2006) use the name term *premium* to refer generically to risk *premium* because it arises from the existence of different maturities in the yield curve. In this paper we will refer to the *premia* derived from definitions (i) up to (iv) respectively as yield *premium*, return *premium*, one-period forward *premium* and one-period holding *premium*.

In Costa and Galvão (2007) we show that in this particular case, where the variables are in logs and time is discrete, these four different ways of defining the expectations hypothesis are mathematically equivalent. As a consequence the yield *premium* of a bond with maturity *n* is equal to its return *premium* divided by *n* to the average of the one-period forward *premia*, and to the one-period holding *premia*.

3. PROCEDURES FOR THE ESTIMATION OF THE FORWARD *PREMIA*

In the previous section, we described four different ways of writing the expectations hypothesis and their implications for the definition of the risk *premia*. In this section, we describe the method we will use for computing the risk *premium*.

The method used here is an extension of Diebold and Li (2006), which is based on the Nelson and Siegel (1987) parametric yield curve fitting. This method will be applied specifically for the forward *premium*, but it could also be useful to compute the other definitions of risk *premia*. The emphasis on the forward *premium* arises from the fact that we want to use the information in the yield curve to obtain market forecasts of future interest rates.

In general one might be interested in forecasting interest rates of maturity higher than one period. The τ-period forward rate, i.e., the interest rate contracted today to start in n periods and with maturity in $n+\tau$ periods, corresponds to:

$$f_t^{(n,n+\tau)} = \frac{1}{\tau}\left[\tau y_t^{(n+\tau)} + n\left(y_t^{(n+\tau)} - y_t^{(n)}\right)\right] = \frac{1}{\tau}\left[(n+\tau)y_t^{(n+\tau)} - ny_t^{(n)}\right]. \tag{1}$$

The τ -period forward *premium* is:

$$frp_t^{(n,n+\tau)} = f_t^{(n,n+\tau)} - E_t\left(y_{t+n}^{(\tau)}\right). \tag{2}$$

The forward rates $f_t^{(n,n+\tau)}$ can be computed using the yield (spot) rates $y_t^{(n+\tau)}$ and $y_t^{(n)}$. However, the maturities of observable yields may not match the ones required to compute the forward rates that we are interested. Thus, it is necessary to fit a curve for the observable yields, so that one can use the fitted yields to compute forwards for any desired maturity and horizon. We use the Nelson and Siegel (1987) parametric approach for fitting the yield curve. The Nelson and Siegel equation for the spot rate (yield on zero-coupon bonds) with maturity τ at a given point in time t is:

$$y_t^{(\tau)} = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\theta_t\tau}}{\theta_t\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\theta_t\tau}}{\theta_t\tau} - e^{-\theta_t\tau}\right), \tag{3}$$

where β_{1t}, $\beta_{1t}+\beta_{2t}$ and θ_t must all be positive. The parameters β_{1t}, β_{2t} and β_{3t} are called yield factors and are interpreted as the level (L_t), the symmetric of the slope $(-S_t)$ and the curvature (C_t) of the yield curve. θ_τ is the parameter that measures the rate of the exponential decay of the loading of the second and the third factors. Smaller θ_τ implies slower decay. This parameter also defines the maturity at which β_{3t} has larger weight. Following Diebold and Li (2006) we fix θ_τ in the value that the maturity of almost 3 years has the highest loading for β_{3t}.[3] An advantage of keeping θ_τ fixed is that the factors can be estimated by the usual least squares formula. Diebold and Li (2006) also argue that the estimates of the factors are more stable over time when θ_τ is fixed, which is an advantage when one is interested in predicting the yield factors.

For the estimation of the τ-period forward *premium*, one also needs $E_t\left(y_{t+n}^{(\tau)}\right)$, which can be estimated as a n-period ahead forecast for the yield with maturity τ $\left(\hat{y}_{t+h|t}^{(\tau)}, \text{where } h=n\right)$. The Nelson and Siegel (1987) approach for fitting the yield curve can be used to forecast yields of different maturities. Using the name of factors and a fixed θ_t, the Nelson and Siegel regression for forecasting a yield of maturity τ at h-steps ahead conditional on information at t is:

$$\hat{y}_{t+h|t}^{(\tau)} = \hat{L}_{t+h|h} - \hat{S}_{t+h|t}\left(\frac{1-e^{-\bar{\theta}\tau}}{\bar{\theta}\tau}\right) + \hat{C}_{t+h|t}\left(\frac{1-e^{-\bar{\theta}\tau}}{\bar{\theta}\tau} - e^{-\bar{\theta}\tau}\right). \tag{4}$$

Diebold and Li (2006) suggest the estimation of an autoregressive model of order one (AR(1)) for each factor to be able to compute $\hat{L}_{t+h|t}$, $\hat{S}_{t+h|t}$ and $\hat{C}_{t+h|t}$. However, there is some important dynamic correlation between the slope, the level and the curvature. Thus, we consider a VAR(1) to be more adequate.

(3) In Costa and Galvão (2007) we also use other methods for fitting the yield curve: Nelson and Siegel (1987) without fixed θ_t and Svensson (1994). We conclude that the improvement in the fit of the yield curve of using these alternative methods is small. In particular, it is not large enough to reduce significantly the error of using the forward rate for forecasting the interest rates.

Using the estimated factors at each time $(t = 1,\ldots,T)$, we define a VAR(1) for modelling the vector $x_t = \left(\hat{L}_t \ \ -\hat{S}_t \ \ \hat{C}_t\right)'$ as:

$$x_t = c + \Phi_1 x_{t-1} + \varepsilon_t. \tag{5}$$

Conditional on the estimated parameters, we use this VAR to generate *h*-step-ahead forecasts as:

$$\hat{x}_{t+h|t} = \left(1 + \hat{\Phi}_1 + \ldots + \left(\hat{\Phi}_1\right)^{h-1}\right)\hat{c} + \left(\hat{\Phi}_1\right)^h x_t. \tag{6}$$

There are several recent papers modelling the relation between factors of the yield curve and some important macroeconomic variables (for example, Ang and Piazzesi, 2003; Diebold *et al*, 2006; Rudebush and Wu, 2004; Hordahl *et al*, 2006). One simple way of adding information of macroeconomic variables to predict the factors of the yield curve is to augment the VAR of equation (5) with a small group of variables. We estimate two specifications of the VAR: one with only the yield factors and another including also inflation and real activity growth. These macroeconomic variables were chosen because there is evidence in the literature of a strong dynamic relation between them and yield factors.[4]

An alternative to our approach of estimating the factors in a first step and the dynamic relation of the factors in the second step is the one proposed by Diebold *et al* (2006). Diebold *et al* (2006) show how to jointly estimate the yield factors (eq. 3) and the coefficients of a VAR of yield factors and macroeconomic variables (eq. 5), using a state-space representation by Kalman filtering and maximum likelihood estimation. A disadvantage of the joint estimation of the parameters and the three unobserved factors, which are non-linearly related with observable yields, is the challenge in the numerical optimization procedure. In addition, because the yields have a high persistence over time, the time-dependence of the factors may be captured even when the regression is computed independently each time. Another issue is that the three factors explain most of the variation of the yields, implying that the inclusion of the macro variables does not affect the estimation of the factors. However, when using the VAR for forecasting yield factors, it is important to consider the dynamic relation between the macro variables and the yield factors because there is a strong relation between them. With the support of previous statements, our two-step estimation may generate yield forecasts similar to the ones implied by the model of Diebold *et al*. (2006). In addition, the use of a method which is less demanding in computation reduces the problem of using a sample as short as the one available after the euro introduction. Similar two-step approaches have been employed by Carriero *et al* (2006) and Favero and Kaminska (2006).

Based on forecasts of the yields $y^{(\tau)}_{t+h|t}$ and on the forward rates, the forward *premia* can be computed with equation (2). However, the computation of the forward *premia* does not give information on whether they are statistically different from zero at each point in time. Even if the expectations hypothesis is rejected for the whole sample, it may be the case that at a specific point in time, it may not be possible to exclude that the forward *premia* may not be statistically significant. We propose a bootstrap procedure for computing the confidence intervals of the estimates of the forward *premia* at each point in time. The procedure, described in detail in Costa e Galvão (2007), is based on the empirical predictive density of $y^{(\tau)}_{t+h|t}$, which is the main source of uncertainty on the estimation of the *premia*.

(4) For empirical evidence in the case of euro area or some euro area countries see, for instance, Estrella *et al* (2003), Moneta (2003), Duarte *et al* (2005), Hordähl *et al* (2006) and Cappiello *et al* (2006).

4. IS THERE A FORWARD *PREMIUM*?

In this section, we start by evaluating the fit of our method in the estimation of the yield curve. Then we calculate the implied 3-month forward rates and evaluate the bias in forecasting the observed 3-month interest rates. Finally, we compute forecasts for the 3-month interest rates with the method described in the previous section and compare the mean forecast errors with the ones obtained with the forwards.

Our sample starts in January 1999 and ends in June 2006. The interest rate data are EURIBOR rates for maturities 1, 3, 6, 9 and 12 months and euro swap rates for each year between 2 and 10 years. The euro swap rates are from Thomson Financial DataStream. The EURIBOR rates are transformed into continuously compounded rates to be compatible with the Nelson and Siegel (1987) parameterization. The interest rate data correspond to end-of-month values. Inflation is measured by the annual growth rate of the euro area Harmonized Index of Consumer Prices. Real activity growth is measured by the annual growth rate of the euro area industrial production excluding construction. Both series were taken from the Eurostat.

Table 1 presents the square root of the mean of the squared residuals for each maturity of estimating a yield curve with Nelson and Siegel (1987) method, when θ_t is kept fixed and equal to 0.0542, implying that the curvature has the largest loading at a 3-year maturity.[5] The worst fit is detected for the one-month maturity and some intermediate maturities (1 and 2 years) but the the maximum error is only of 6 basis-points.

Based on the estimated yields and using expression (1), the 3-month forward rates $(\tau = 3)$ are computed quarterly for horizons up to three years (n corresponds to $h = 3, 6, \ldots, 36$). Table 2 includes the average forecast errors of using the forwards to predict the 3-month spot interest rates. Both the forward rates and the observed 3-month interest rates $\left(y_{t+h}^{(3)}\right)$ are transformed to be annually

Table 1

FIT OF THE YIELD CURVE WITH NELSON AND SIEGEL (1987) WITH FIXED θ

Square root of the mean of squared residuals of each maturity.

τ (maturity measured in months)													
1	**3**	**6**	**9**	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**
0.061	0.039	0.033	0.046	0.052	0.053	0.032	0.030	0.036	0.031	0.017	0.009	0.022	0.036

Table 2

MEAN FORECAST ERRORS OF THE 3-MONTH FORWARD RATES [(a)]

t+h (*h* corresponds to the forecasting horizon measured in months)											
***t*+3**	***t*+6**	***t*+9**	***t*+12**	***t*+15**	***t*+18**	***t*+21**	***t*+24**	***t*+27**	***t*+30**	***t*+33**	***t*+36**
0.131	0.278	0.457	0.673	**0.900**	**1.164**	**1.450**	**1.741**	**2.004**	**2.240**	**2.426**	**2.548**
(0.061)	(0.149)	(0.262)	(0.365)	(0.441)	(0.464)	(0.450)	(0.414)	(0.374)	(0.327)	(0.297)	(0.294)

Note: (a) For example, for a horizon of 12 months it corresponds to the difference between the 3-month forward rate contracted in t to start in *t*+12 and the 3-month interest rate observed in *t*+12. The values in brackets are standard deviations of the forecast errors computed using the Newey-West estimator with lag truncation *h*-1.

(5) The estimates of the curve are obtained with Gauss CML package (with BFGS).

compounded in order to be compatible with the usual form of presentation of these series. The emboldened values of the 3-month horizon and of horizons higher than 1 year mean that the *t*-statistics, calculated using the estimates and standard errors, are larger than 2, implying a rejection of the null that the forecast error is equal to zero.

The bias identified when forward rates are directly used as forecasts of future spot rates may be the result of either a forward *premium* or of poor forecasts of the future short-term interest rates, due to unexpected shocks. As a consequence, we evaluate whether the approach for forecasting the yields described in the previous section can generate forecasts of short-term interest that are unbiased. If there is still a failure in predicting at long horizons, we would attribute it to errors in forecasting interest rates. In fact, unexpected changes in interest rates take longer to be incorporated when forecasting long horizons because of the longer time required to observing them.

For computing the forecasts, we estimate the VAR (eq. 5) using only the information of yield factors ($x_t = \left(\hat{L}_t \ \ -\hat{S}_t \ \ \hat{C}_t\right)'$) and also adding information of real activity growth $\left(g_t\right)$ and inflation $\left(\pi_t\right)$ ($x_t = \left(\hat{L}_t \ -\hat{S}_t \ \ \hat{C}_t \ \ g_t \ \ \pi_t\right)'$). We then use the estimated VAR to generate *h*-step-ahead forecasts of the 3-month interest rate (eq. 6 and eq.4).

Table 3 presents the mean of the forecast errors in predicting the 3-month interest rates at horizons between 3 and 36 months. The forecasts use full sample information on the estimation of the VAR parameters, but they use information on yields up to *t*. The emboldened entries indicate again the rejection of the null that the average forecast error is equal to zero. There is some evidence of bias at long horizons, but the bias is on average three to five times smaller than using the forward. There is also some weak evidence that the inclusion of macroeconomic variables improves the forecasting performance.

Summarizing the results indicate that our approach generates unbiased forecasts at horizons shorter than two and half years and that the forward rates generate biased forecasts for horizons larger than 1 year. This suggests that the bias obtained when using forward rates as forecasts is in part caused by a forward *premium*. For long horizons, part of the bias arises from unexpected changes in interest rates. The estimates of the biases at the 3-year horizon presented in Table 3 suggest that around 1/4 of the bias incurred by the use of forward rates as forecasts is caused by unexpected shocks, while around 3/4 of the bias could be explained by forward *premium*.

Table 3

MEAN FORECAST ERRORS OF THE EXTENDED DIEBOLD AND LI (2006) APPROACH [(a)]

_t+h_ (_h_ corresponds to the forecasting horizon measured in months)

t+3	*t*+6	*t*+9	*t*+12	*t*+15	*t*+18	*t*+21	*t*+24	*t*+27	*t*+30	*t*+33	*t*+36
-0.008	-0.045	-0.061	-0.040	0.003	0.094	0.223	0.372	0.508	**0.629**	**0.713**	**0.743**
(0.062)	(0.137)	(0.237)	(0.328)	(0.396)	(0.419)	(0.410)	(0.380)	(0.341)	(0.284)	(0.225)	(0.180)
Results with macroeconomic variables											
-0.011	-0.046	-0.064	-0.056	-0.040	0.016	0.105	0.213	0.313	0.403	**0.463**	**0.476**
(0.056)	(0.105)	(0.171)	(0.231)	(0.279)	(0.303)	(0.310)	(0.302)	(0.286)	(0.251)	(0.208)	(0.174)

Note: (a) For example, for a horizon of 12 months it corresponds to the difference between the extended Diebold and Li (2006) 12-month ahead forecast of the 3-month interest rate and the 3-month interest rate observed in *t*+12. In the results with macroeconomic variables, the VAR employed in the forecast includes in addition to the yield factors inflation and economic growth. The values in brackets are standard deviations of the forecast errors computed using the Newey-West estimator with lag truncation *h*-1.

5. THE FORWARD *PREMIUM* OF EURO INTEREST RATES

The results presented in the last section support the existence of forward *premia* on the euro area 3-month interest rates. In the first part of the present section, we present our estimates of the average forward *premia* for different horizons. In the second part, we evaluate the behaviour of the estimates of the forward *premium* along the sample period. We first compare the estimates with the estimated confidence intervals. Finally, we analyse our estimates in comparison with the key ECB interest rate and the skewness of the option-implied probability distribution of interest rates futures.

5.1. Forward *premium* for different horizons

The 3-month forward *premia* for each horizon are calculated as the difference between the implied 3-month forward rates and the forecasts with the extended Diebold and Li (2006) approach of the 3-month interest rate:

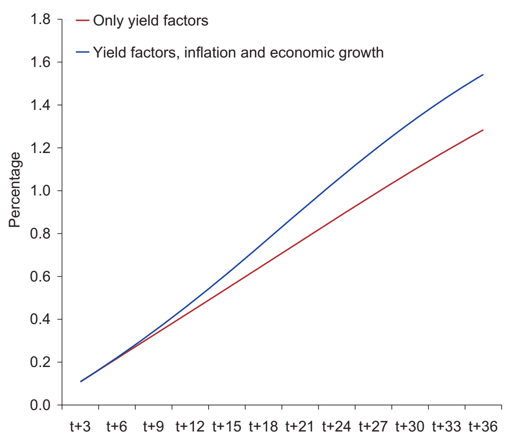
$$\hat{frp}_t^{(h,h+3)} = \hat{f}_t^{(h,h+3)} - \hat{y}_{t+h|t}^{(3)}.$$

Chart 1 presents the mean in our sample period of the estimated 3-month forward *premia* for horizons from 3 to 36 months with and without the inclusion of macroeconomic variables. One can see that the forward *premium* monotonically increases with the horizon. The fact that the risk *premia* increase with maturity is a standard result also obtained for German and euro area data in Hordähl *et al* (2006) and Capiello *et al* (2006).

The inclusion of macroeconomic variables increases the average of the estimated forward *premia*. In general, as referred in the evaluation of Table 3, the reductions in the positive bias are marginal. The inclusion of these additional factors leads also to marginal reductions of the variance of the shocks employed in the computation of the confidence intervals for the forward *premia* over time. Therefore, we will present estimates of the forward *premia* only with the inclusion of macroeconomic variables in the remaining of the paper. Our results do not change qualitatively if these variables are removed from the VAR.

Chart 1

MEAN OF THE 3-MONTH FORWARD *PREMIA* FOR EACH HORIZON



5.2. Forward *premia* and confidence intervals over time

Chart 2 presents the estimated 3-month forward *premia* for forecasting horizons 3, 12, 24 and 36 and their 90 per cent confidence intervals. The forward *premia* are not significantly different from zero for the 3-month horizon, even though the point estimates are positive. In the remaining forecasting horizons, the forward *premia* are significantly positive in some periods between 1999:6 up and 2002:10, with the duration of these periods being higher for longer horizons. The variability of the forward *premia* increases with the horizon because the variability of the forecast yields with the extended Diebold and Li (2006) approach decreases with the horizon. The finding that the variability of the *premia* increases with the horizon is also obtained by Hordähl *et al* (2006) with an affine term-structure model augmented with a dynamic structural macro model.

The time-varying behaviour of the estimated forward *premia* has some resemblance with the yield *premia* computed by Werner (2006) with an affine term-structure model. For the longest horizons, the *premia* increase in 1999, start to decrease in 2000, and only reverse the downward trend in mid-2005. Our results are also similar to the one-year yield *premium* estimated with data after 1999 by Capiello *et al* (2006). The fact that our measure of forward *premium* does not differ significantly from the ones presented in the literature based on affine term-structure models gives support to the use of our method, which is less demanding in computation.

Chart 2

ESTIMATES OF THE 3-MONTH FORWARD *PREMIA* AND 90% CONFIDENCE INTERVALS

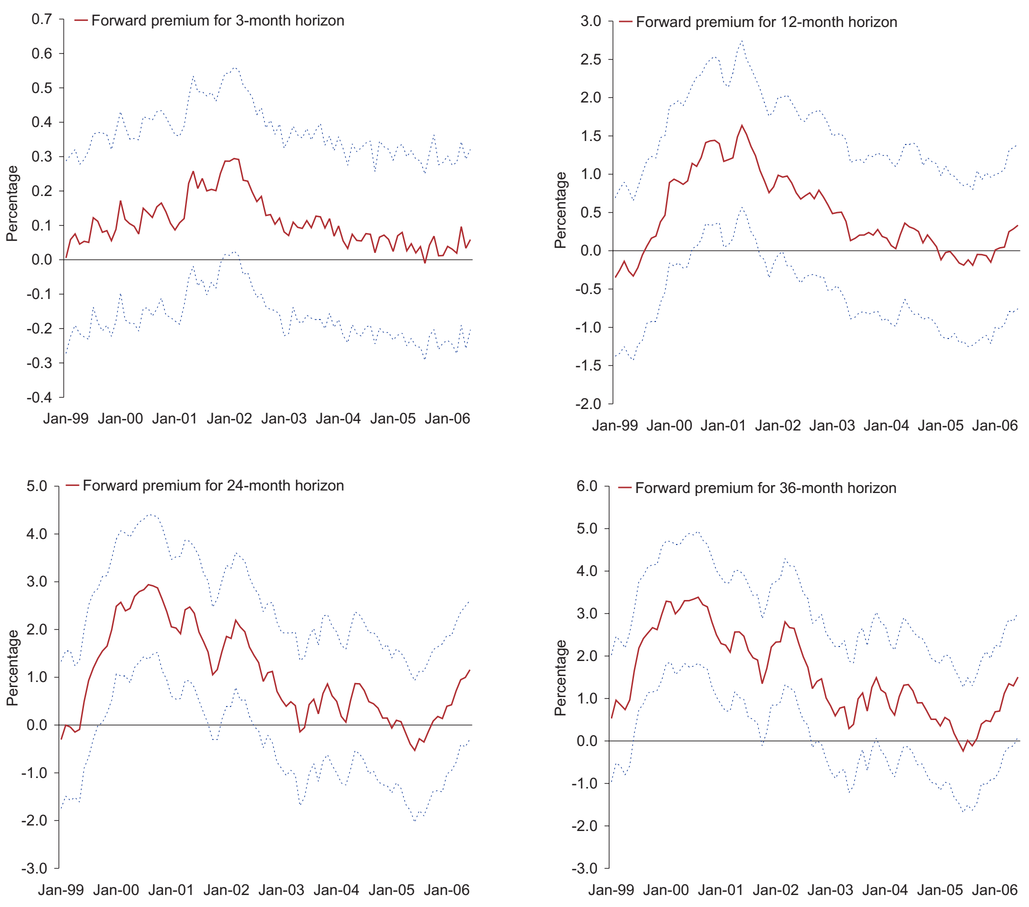


Chart 3 shows that our estimated forward *premia* are positively correlated with the key ECB interest rate. Capiello *et al* (2006) also indicate a positive correlation between the one-year yield *premium* and the level of the short-term interest rate after 1999. An economic interpretation of the relation between the risk *premium* and short-term interest rates may be the existence of a relation between movements in official interest rates and the probability attached by market participants to increases in interest rates in the future in comparison to their actual mean values (Vähämaa, 2004). In particular it is natural to expect that investors will demand a higher protection for potential capital looses if there is an increase in the probability that future interest rates will turn out to be higher than their expected values.

One way to assess this is by looking at the skewness of the distribution of expected future short-term interest rates. This can be done by using options on EURIBOR futures contracts. We calculate the skewness of option-implied probability distribution of the one-year-ahead 3-month EURIBOR futures contracts in our sample period. The option-implied probability distribution is calculated using the one-year-ahead 3-month EURIBOR nearest contract (for example, in January 1999 we use the contract for December 2000, and in February, March and April 1999 the contracts for March 2000). The skewness is measured by the Fisher coefficient, that is, the ratio of the third central moment to the cubed standard deviation. It has a positive (negative) value when the distribution is positively (negatively) skewed, that is, when there is a higher probability that the values stay below (above) the average of the distribution than above (below). Chart 4 shows that the skewness is negatively correlated with the one-year ahead 3-month forward *premium*. When the skewness is decreasing, the probability distribution is moving to the right, implying that the probability associated with future rises in interest rates as compared to their expected values is increasing. It is important to notice that the futures contracts used in the estimation of the distribution might also incorporate a risk *premium*. Therefore, increases in the probability attached to future rises in the interest rates may result either from a review of the risk neutral assessment of the likelihood of future rises or from an increase in the risk aversion.

Chart 3

3-MONTH FORWARD *PREMIA* AND KEY ECB INTEREST RATE

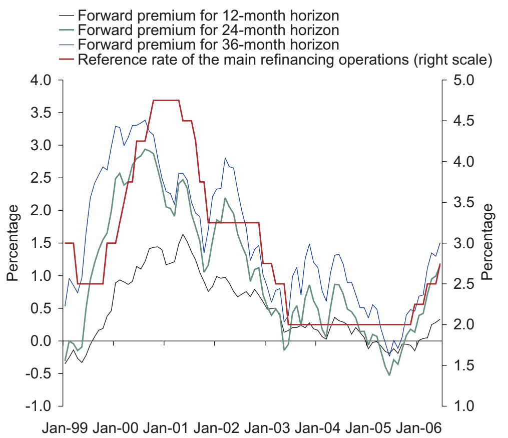
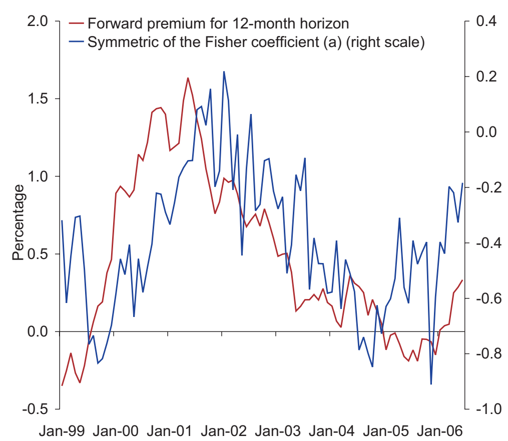


Chart 4

3-MONTH FORWARD *PREMIA* AND SKEWNESS OF THE OPTION-IMPLIED PROBABILITY DISTRIBUTION OF THE 3-MONTH EURIBOR FUTURES CONTRACTS



Note: (a) Fisher coefficient of the option-implied probability distribution of the 3-month EURIBOR futures contracts for *t*+12.

6. CONCLUDING REMARKS

We present evidence that forward rates are biased forecasts of euro interest rates and that this is, at least in part, associated with the existence of a forward *premium*. Our estimates show that the level of the *premium* and its variability increase with the forecast horizon.

Even though the forward *premia* of horizons from 3 to 36 months are on average positive, the observation of the evolution of the forward *premia* over time together with their confidence intervals indicate that they are significantly equal to zero in some periods of time. The forward *premia* are positively correlated with the policy rates. One reason for this positive relation might be the fact that when monetary policy is tightening, the market perception changes towards attributing a higher probability to the "bad news" scenario that interest rates could be higher than expected.

In addition to these empirical results, this paper contributes with a competitive method to compute the forward *premium* and its confidence interval. These are required to evaluate whether a change in the forward rate is due either to modifications in the compensation for risk or in the market expectations. Our approach is easy to estimate and it is flexible to include information outside the yield curve. It does not impose no-arbitrage restrictions but it is able to capture the time-varying behaviour of the forward *premium* such as the dynamic term structure models by Capiello *et al* (2006) and Werner (2006). Another advantage is that the *premium* and its confidence interval can be computed daily.

REFERENCES

Ang A. and Piazzesi M. (2003). "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables". *Journal of Monetary Economics*, 50: 745-787.

Cappiello L., Hördahl P., Kadareja A. and Maganelli S. (2006). "The impact of the euro on financial markets". European Central Bank. *Working Paper* n. 598.

Carriero A., Favero C. A. and Kaminska I. (2006). "Financial factors, macroeconomic information and the expectations theory of term structure interest rates". *Journal of Econometrics*, 131: 339-358.

Cochrane J. H. (2001). "Asset Pricing". Revised Edition. Princeton: Princeton University Press.

Costa S. and Galvão A. B. (2007). "The forward *premium* of euro interest rates". *Working Paper* n. 2, Banco de Portugal.

Cox J. C., Ingersoll J. E. and Ross S. A. (1981). "A re-examination of traditional hypotheses about term structure of interest rates". *The Journal of Finance*, 36: 769-799.

Diebold F. X. and Li, C. (2006). "Forecasting the term structure of government bond yields". *Journal of Econometrics*, 130: 337-364.

Diebold F.X., Rudebusch G.D. and Aruoba S. B. (2006). "The macroeconomy and the yield curve: a dynamic latent factor approach". *Journal of Econometrics*, 131: 309-338.

Duarte A., Venetis I. A. and Paya I. (2005). "Predicting real growth and the probability of recession in the euro Area using the yield spread". *International Journal of Forecasting*, 21: 261-277.

Estrella A., Rodrigues A. P. and Schich S. (2003). "How stable is predictive power of the yield curve? Evidence from Germany and the United States". *The Review of Economics and Statistics*, 85 (3): 629-644.

Favero C. and Kaminska I. (2006). “Measuring term *premium*. Evaluating alternative dynamic term structure models. Bocconi University”, *mimeo*.

Hordähl P., Tristani O. and Vestin D. (2006). “A joint econometric model of macroeconomic and term-structure dynamics”. *Journal of Econometrics*, 131: 405-444.

Moneta F. (2003). “Does the yield spread predict recessions in the euro area?” European Central Bank. *Working Paper* n. 294.

Nelson C. R. and Siegel, A. F. (1987). “Parsimonious modelling of yield curves”. *Journal of Business*, 60: 473-489.

Rudebush G. D. and Wu T. (2004). “A macro-finance model of the term structure, monetary policy and the economy”. Federal Reserve Bank of San Francisco. *Working Paper* n. 2003-17.

Singleton K. J. (2006). “Empirical dynamic asset pricing: model specification and econometric assessment”. Princeton: Princeton University Press.

Svensson L. E. O. (1994). “Estimating and interpreting forward interest rates: Sweden 1992-1994". National Bureau of Economic Research. *Working Paper* n. 4871.

Vähämaa, S. (2004) “Option-implied asymmetries in bond market expectations around monetary policy actions of the ECB”. European Central Bank. *Working Paper* n. 315.

Werner T. (2006). “Term *premia* developments in the euro area: an affine term structure model estimated with survey data”. European Central Bank. *mimeo*.