

LIQUIDITY AND ORDER FLOW IN THE PORTUGUESE STOCK MARKET

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The major purpose of this paper is the construction of measures characterising the order flow in the Portuguese stock market, and the analysis of the impact of such measures on the probability and time of execution of the orders.

In a market organised as an auction, as is the case of the Portuguese stock market, a bid (ask) order of a certain stock will be executed whenever matched by one or more ask (bid) orders at the required quote or at a more favourable quote. For instance, let us consider that a bid is submitted to the stock exchange at a given moment at quote p for the amount q . If at that moment the total amount of ask orders outstanding at p or a lower quote is greater than q , the bid will then be fully and immediately satisfied. If the number of ask orders outstanding at a satisfactory quote are insufficient to execute in full the bid, the unsatisfied share will be pending until expiration, full execution or cancellation of the order. Therefore, as a result of the volume and quote of the orders for each stock, a given order may (i) be immediately executed; (ii) be pending for some time, until partial or full execution; or (iii) be cancelled.

An important feature of financial markets is the spread in which investors carry out their transactions, and is associated with the notion of market liquidity⁽¹⁾. Thus, an efficient characterisation of the market usually requires the calculation of sev-

eral liquidity measures, such as those presented in this article.

The liquidity notion is relatively wide and is associated with the ability to trade assets into currency, which, by definition, is the most liquid of all assets. Broadly speaking, asset liquidity may be analysed following two approaches: time and cost. According to the first approach, an asset is liquid if it can be traded quickly. Conversely, an asset is illiquid if investors cannot find, for a long period, matching market orders enabling the respective transaction. According to the second approach, an asset is liquid if the price to be paid (received) when buying (selling) the asset is close to its current market price. This leads to consider spreads between bid and ask quotes. The spread between the highest bid quote and the lowest ask quote corresponds to a liquidity measure usually known as the bid-ask spread.

This paper analyses bid and ask orders for all stocks belonging to the PSI-20 index of the Portuguese stock market between January and October 2002. The monitoring of all orders since submission to execution makes it possible to quantify the easiness in the execution of orders. This suggests the construction of specific liquidity indices for every stock or group of stocks. One of these mea-

* The views expressed in this article are those of the authors and not necessarily those of the Banco de Portugal. The authors thank Nuno Caramujo for his comments and suggestions.

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(1) Financial markets are required to offer investors the possibility of executing transactions quickly and with small price changes, maintaining transaction costs at a low level. Against this background, several works show that illiquid assets, i.e., assets facing more difficulties in their transactions, tend to offer higher expected yield (see, inter alia, the works by Amihud and Mendelson (1986) and Brennan and Subrahmanyam (1996)).

asures is the bid-ask spread, for which an average value of 60 basis points was obtained. The distinction between the different stocks of the index shows that the group of highly capitalised corporations has an average spread of 29 basis points, whereas the group of thinly capitalised corporations shows a spread of 105 basis points.

Another interesting measure consists in transactions costs, defined as the percentage change of the transaction price of a stock vis-à-vis its price one minute earlier. This is an additional liquidity measure, and its average value in PSI-20 is 4.11 basis points.

The values of the liquidity indices in the Portuguese stock market are equivalent to those observed in international markets, with bid-ask spreads around 20 and 30 basis points, and transaction costs between 3.79 and 5.03 basis points. It should be noted, however, that market liquidity changes throughout the transaction day, declining from the opening of the session to 12:00 p.m., to increase again approaching close of business. The behaviour in U-shaped pattern is also shared by major international markets.

This study also determines to what extent the orders submitted by investors are affected by market liquidity indicators. The success of an order may be measured either by the probability of execution or by the time to execution. Breaking down the determinants of the execution into specific factors pertaining to overnight, stocks and the market, this paper reveals that an increase in the bid-ask spread and in the stock volatility or a decline in the number of shares supplied at best quote lead to a decrease in the probability and an increase in the time of execution of the orders. It can also be concluded that, when trading high capitalisation stocks, an investor should expect an average increase of 8.5 (9.8) per cent in the probability of execution of the bid (ask) order, compared with trading low capitalisation stocks.

The article is structured as follows. The first section characterises the data and market structure, and calculates liquidity measures. Section 2 presents an analysis of liquidity and determines the behaviour of such measures throughout the day. Section 3 makes use of probit models of non-linear regression and the Cox model of proportional effects to estimate the relationship between liquidity measures and probability and time

to execution. Finally, section 4 presents the major conclusions.

1. CHARACTERISATION OF THE MARKET

1.1. Descriptive data analysis

The database analysed was obtained from Euronext Lisboa and consists of all bid/ask orders relating to the 20 stocks traded in the PSI-20 index during the 195 transaction days between 4 January 2002 and 15 October 2002. This database contains a total of 2,441,490 orders. For each order, the stock exchange reports the ISIN code of the traded stock, the market segment (i.e., if it is a bid or ask order), the type, the expiry date of the order, an identification of the broker launching the order, and the position of the order in the maturity line. This permits to detect whether or not the order was cancelled (by the stock exchange or by the financial operators), changed, executed, or if it has simply expired. In addition, the database includes, for partially or fully executed orders, the number of trades leading to the execution of the transaction. Orders are divided into five types: (i) limit, indicating the maximum (minimum) bid (ask) quote and the quantity; (ii) at best price, which only indicates the quantity to be traded; (iii) opening/closing price, when the orders can only be given during the pre-opening and pre-closing period⁽²⁾; (iv) stop-orders, orders activated only when the market price attains the quote established in the order, giving rise to limit orders; and (v) market, chiefly equivalent to orders at best price, but privileging the swiftness of execution to the detriment of the price of execution, since they accept to "rise" in the order book.

Table 1 presents a summarised characterisation of the orders submitted during the period under analysis, thus making it possible to distinguish be-

(2) Trading of stocks belonging to the PSI-20 is continuous throughout the day. However, at the opening and close of business there are trading periods with different characteristics. These periods are known as pre-opening and pre-closing periods. During these periods, orders are accumulated until when, by matching all bid and ask orders, a single price is determined (opening price and closing price, respectively) satisfying the largest possible quantity of orders on both sides of the market. These transition periods were withdrawn from the analysis, since their characteristics are different from those of continuous transaction periods.

Table 1
DESCRIPTION OF MARKET ORDERS

Per cent

	PSI 20		Thinly capitalised		high capitalisation	
	Number of orders	Amount ^(a)	Number of orders	Amount ^(a)	Number of orders	Amount ^(a)
Buy Orders	45.55	49.28	47.26	48.58	44.36	49.38
Limit Orders	90.62	94.89	92.45	96.45	89.34	94.32
Orders at best price	8.99	5.03	7.00	3.30	10.38	5.38
Buy Limit Orders(% of limit orders)	47.23	49.57	48.28	48.77	46.47	49.74
At best price bid (% of at best price orders)	29.19	44.72	34.90	44.44	26.50	44.95
Validity of the orders (% of at best price orders)						
Validity for the day (% of total)	85.26	93.66	83.62	83.57	86.41	95.80
Order status (% of total)						
Orders executed	50.75	55.83	42.79	44.21	56.31	58.27
Orders expired	26.93	14.72	33.41	26.95	22.40	12.15
Annulled/cancelled/changed orders	22.22	29.47	23.65	28.71	21.22	29.59

Note:

(a) The amount is calculated by multiplying the quantity offered for each order by the closing price on the day the order was submitted.

tween the group of PSI-20 stocks and two sub-groups of corporations: the highly capitalised group, gathering the 5 most highly capitalised corporations listed in the stock exchange, and the low capitalisation group, covering the 15 most low capitalisation corporations of the index⁽³⁾.

The table shows that 54.4 per cent of the orders are ask orders, whereas 45.6 per cent are bid orders, most of the orders with same-day maturity. This result reveals more pressure on the ask side. However, this difference fades when carrying out the value analysis, and does not seem to be significant enough to give rise to imbalances in the order book during the period under analysis.

The limit orders represent 90.6 per cent of total orders submitted, rising to around 95 per cent in value terms. The group of limit orders includes 47.2 percent of bid orders. Orders at best price represent approximately 9 per cent of total orders, nearly 29.2 per cent of which are bid orders. The group of high capitalisation corporations reveals a higher percentage of orders at best price (10.4%)

than the group of low capitalisation corporations (7%). The higher percentage of ask orders in total orders at best price may reveal a different behaviour between bid and ask intentions.

As regards the order position, i.e., the trajectory of the order until it is taken from the order book, it can be concluded that half of the orders submitted are executed, while 27 per cent of the orders expire for exceeding the maturity period. Thinly capitalised corporations have, on average, an execution ratio of 42.8 per cent, which is significantly below the execution ratio of 56.3 per cent, on average, of high capitalisation corporations. This may be interpreted as evidence that high capitalisation corporations have a higher probability of satisfying bid and ask intentions than low capitalisation corporations, denoting the higher liquidity of high capitalisation stocks.

In the preparation of this paper, we only considered the limit orders with maturity on the same day they were submitted. In addition, orders submitted during the pre-opening and pre-closing periods were eliminated. As a result, a total of 1,594,921 observations were used in this study.

(3) The breakdown into the 5 most high capitalisation corporations and the 15 most low capitalisation corporations corresponds to a natural separation in the PSI-20 index. The five largest corporations account for approximately 75% of total capitalisation of the PSI-20 index and are part of the group of 90 corporations with more weight on the Euronext 100 index.

1.2. The order book and liquidity measures

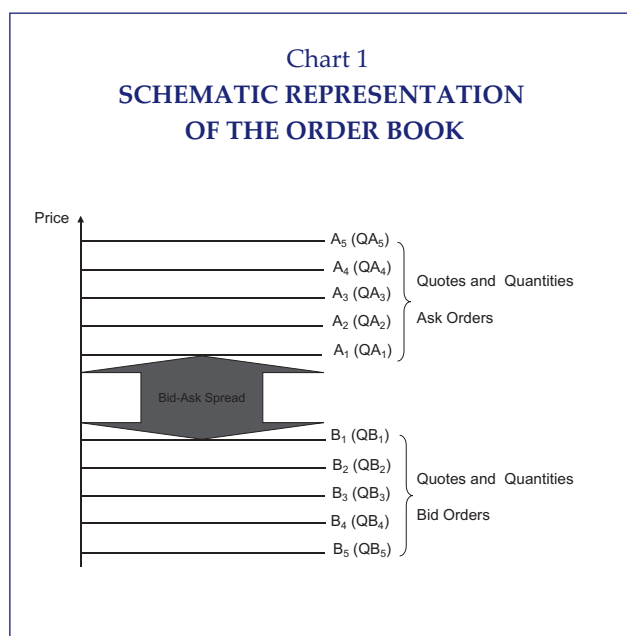
The order book consists of the compilation of bid and ask orders for each stock, at each moment in time. Orders enter in the book during the day and are immediately available to the market. They are removed from the book when executed, cancelled, changed or expired.

For every stock, market side (bid or ask) and transaction time unit (in our case, the second), the order book corresponds to a table comprising all quotes offered for transaction and the respective total quantities of outstanding orders, i.e., the orders that have not yet reached their maturity, have not been cancelled, or have not been fully satisfied. At each moment in time, orders that are cancelled or expired are withdrawn from the order book. Likewise, the quantities offered by outstanding orders that are not immediately satisfied are added to the book. It can be thus concluded that monitoring the order book corresponds to creating a virtual settlement system.

Chart 1 represents the order book of a stock at a given moment in time. Values B_1 to B_5 with $B_1 > B_2 \dots > B_5$, represent the five highest quotes associated with bids and, therefore, have a higher probability of execution. Quantities $QB_1, \dots, QB_5$ with $QB_1 < QB_2 \dots < QB_5$ pertain to the accumulated ask quantities at B_1 to B_5 quotes, respectively. Likewise, A_1 to A_5 , with $A_1 < A_2 \dots < A_5$, represent the five lowest quotes on the ask side and, therefore, show the higher probability of being executed, as well as quantities $QA_1, \dots, QA_5$, with $QA_1 < QA_2 \dots < QA_5$.

The order book is a mechanism for the analysis of bid-ask matching. The bid (B_1 to B_5) and ask values (A_1 to A_5) correspond to the best price quotes offered on the market by agents willing to buy or sell assets, respectively. The spread between the highest bid quote (B_1) and the lowest ask quote (A_1), known as bid-ask spread, is a natural liquidity measure. Stocks with higher spread are characterised by an increased difficulty in obtaining transaction and, as a result, by lower liquidity.

It should be noted that, although each order may be executed in different transactions over time, the construction of the order book assumes that, for fully or partially executed orders, all transactions are carried out at the moment when



the last change is introduced in the order. This simplification introduces some flaws in the book. Therefore, every order is eliminated at moments when the calculation of the order book led to $B_1 = A_1$, which accounts to less than 10 per cent of the orders.

Among the most common criticisms of the bid-ask spread as a measure of liquidity is that it considers only the price component, and not the respective quantities. In order to include the traded volume, an analysis can be made of the changes in the quantities offered in the order book for the different bid/ask quotes (as in the subsequent section), or an additional liquidity measure, called transaction cost, can be considered. The latter represents a percentage change of the transaction price between two consecutive minutes. More liquid stocks will be associated with less price changes, which translates into lower transaction costs.

Table 2 presents a summary of the two previously described liquidity measures. Since higher or thinner liquidity situations may be the result either of spreads between stocks or of different market conditions with periods of larger price changes potentially associated with periods of major disturbances, Table 2 calculates liquidity measures for different daily return intervals. This indicator is intended to measure the general market sentiment on the day an order is submitted. Large price changes should correspond to periods of more difficulties in matching stock bids and asks, which

Table 2
LIQUIDITY MEASURES

	Daily return spreads vis-à-vis the average return for the period					
Bid-ask spread (%)	$\leq \mu - 2\sigma$	$[\mu - 2\sigma, \mu - \sigma[$	$[\mu - \sigma, \mu + \sigma]$	$[\mu + \sigma, \mu + 2\sigma]$	$> \mu + 2\sigma$	Total
Low capitalisation group	1.56	1.27	0.97	1.18	1.26	1.05
High capitalisation group	0.36	0.31	0.27	0.30	0.31	0.29
Total - PSI 20 Index	0.79	0.66	0.56	0.66	0.64	0.60
Transaction costs (in basis points)	$\leq \mu - 2\sigma$	$[\mu - 2\sigma, \mu - \sigma[$	$[\mu - \sigma, \mu + \sigma]$	$[\mu + \sigma, \mu + 2\sigma]$	$> \mu + 2\sigma$	Total
Low capitalisation group	9.02	9.20	5.88	7.23	7.81	6.52
High capitalisation group	3.32	2.84	2.34	2.57	2.77	2.48
Total - PSI 20 Index	5.34	5.15	3.81	4.43	4.54	4.11
Number of orders	$\leq \mu - 2\sigma$	$[\mu - 2\sigma, \mu - \sigma[$	$[\mu - \sigma, \mu + \sigma]$	$[\mu + \sigma, \mu + 2\sigma]$	$> \mu + 2\sigma$	Total
Low capitalisation group	28 051	62 233	476 800	69 045	11 451	647 580
High capitalisation group	50 727	107 342	665 658	102 815	20 799	947 341
Total - PSI 20 Index	78 778	169 575	1142 458	171 860	32 250	1594 921

Note: μ represents the average return of the sample period, and σ the standard deviation. Definition of the variables: the *bid-ask spread* is given by $(A_1 - B_1) / ((A_1 + B_1) / 2) * 100$; the transaction cost is $\sum (B_{1,t} - B_{1,t-1}) * Q_t / \sum P_t * Q_t$, where $B_{1,t}$ is the average value of the best price bid quote in minute t , $P_t = (A_{1,t} + B_{1,t}) / 2$ represents a proxy for the transaction price, Q_t is the quantity executed associated with the order, and the sum is carried out for all orders fully or partially executed entered in minute t . (For ask orders, the definition uses the best price asks quote A_1 .) The number of orders corresponds to the number of bids/asks in any of the distribution intervals of daily return of the PSI-20 index.

may be translated into thinner-liquidity periods. Table 2 groups orders by different categories defined in terms of daily changes in the PSI-20 index. For the period under analysis, the index return was calculated vis-à-vis the previous session, and a daily return distribution was obtained with mean μ and standard deviation σ . For instance, the spread $[\mu - \sigma, \mu + \sigma]$ in the table corresponds to the days when the daily return of the PSI-20 index stood between (i) the daily average return of the index over the period under analysis less one standard deviation of that return ($\mu - \sigma$), and (ii) the average return plus one standard deviation ($\mu + \sigma$). The table also presents the number of orders included in the different daily return spreads of PSI-20.

Table 2 shows that the bid-ask spread of the group of stocks belonging to the index PSI-20 is 60 basis points, with a significant difference between the group of stocks of high and low capitalisation. The bid-ask spread for the low capitalisation group is 1.05 per cent, approximately fivefold that for the high capitalisation group, which is 0.29 per cent. The bid-ask spread varies within the day, with an approximately U-shaped standard. How-

ever, the liquidity distribution shows asymmetry in terms of daily return, with the left tail of distribution of this variable associated with lower liquidity values than those observed in the right tail.

Transaction costs in the case of the group of low capitalisation firms are, on average, at 6.52 basis points, while in the case of the group of high capitalisation companies the figure is 2.48 basis points. Similarly to the bid-ask spread, transaction costs change also with daily return. When daily return is less than two pattern deviations below the sample average, its value stands at 5.34 basis points, decreasing to 3.81 basis points in the interval between minus one and plus one pattern deviation, increasing again to 4.54 when the daily return stands more than two pattern deviations above average return, which also indicates an asymmetric behaviour as regards the distribution of daily return.

In order to better incorporate liquidity measures in the Portuguese market within other stock markets, Table 3 presents the average values of the bid-ask spread and transaction costs for a sample of major international markets.

Table 3

COMPARISON WITH INTERNATIONAL MARKETS

Market – Index	Stock Exchange Capitalisation as at 30/04/2004 (\$M)	Bid-ask spread (%)	Transaction costs (basis points)
USA - Dow Jones	3144935	0.27	3.79
Euronext - Paris	847707	0.14	3.72
Spain- IBEX 35	302030	0.24	5.50
Germany - DAX	607668	0.17	5.03
Italy - MIB 30	429427	0.27	4.40
Greece - ASE 20	3385	0.63	9.43
Austria - ATX	24347	0.94	11.56
Brazil - Bovespa	11931	0.83	13.98

Source: Information published in “Global Equity Markets”, Birinyi Associates, Inc., Deutsche Bank Securities, September 2002. Results reflect the analysis for January 2002.

Although the sample period is not coincident, the comparison of the values in the table with the results obtained for the Portuguese market shows that the liquidity index values fall within the same magnitude range as those observed for international markets. In particular, the sub-group of high capitalisation corporations reveals a bid-ask spread (29 basis points) and an average value of transaction costs (2.48 basis points) at levels that are close to those observed in major international markets.

2. BEHAVIOUR OF THE BID-ASK CURVE AND INTRA-DAILY ANALYSIS

This section analyses in more detail the composition of the bid-ask curves in the order book, with special emphasis on the impact of the quantity on price.

In addition, and similarly to developments in most markets, the intra-daily behaviour of liquidity is analysed, as measured by the average values of the bid-ask spread during the day.

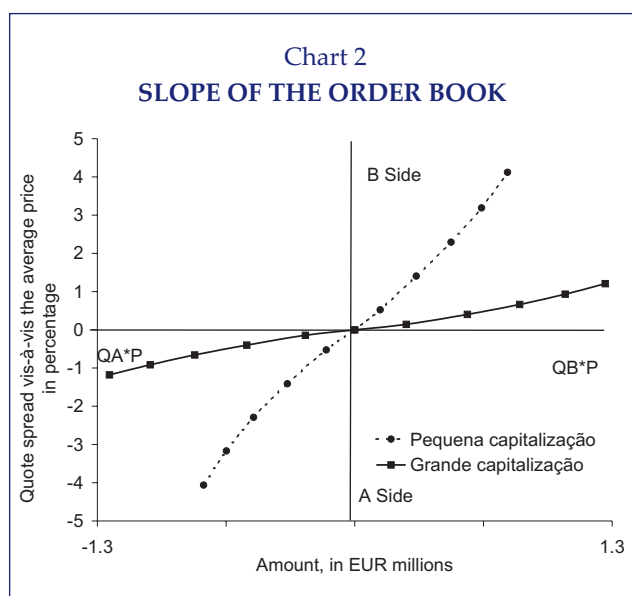
2.1. Behaviour of the bid-ask curve

The full characterisation of the behaviour of stock demand and supply can only be achieved when simultaneously considering the two variables of the order book: quotes (bid-ask spread) and quantities offered at each moment in time. The slope of the book measures the amount that

an investor will have to pay extra (or receive less) to buy (sell) shares of a certain stock. This analysis suggests an additional liquidity measure, which we shall call price robustness (for high-amount orders), and defined as the amount (in euros) that a sell (or buy) order should attain, for its price to change by 0.1 per cent. In a liquid market, large transactions should be absorbed without causing large changes in the transaction price.

The empirical analysis of the order book has been undertaken in other markets. Biais et al. (1995) conclude that the order book in the Paris Bourse is slightly concave, with a bid-ask spread twice as high as the adjacent spreads, on either side. In turn, Al-Suhaibani and Kryzanowski (2000) conclude that the slope of the order book of the Saudi Stock Market does not depart strongly from linearity.

Chart 2 represents the Portuguese market order book, permitting to distinguish between both groups of high and low capitalisation corporations. The vertical axis represents the average bid-ask spread as a percentage of the quote midprice. The horizontal axis presents the amount entered in the order book, evaluated at the quote midpoint. The first quadrant represents the bid segment; the third quadrant the ask segment. The values indicated in the chart are averages of the bid and ask orders of each subgroup of stocks, for the five levels calculated on either side. For instance, the chart section corresponding to high capitalisation corporations, on the right-hand side,



can be interpreted as follows: on average, there are outstanding bid orders amounting to € 1.27 million that may be matched by ask orders standing 1.2 per cent below the regular transaction price.

The group of low capitalisation corporations presents a steeper slope than high capitalisation corporations. This means that the price of low capitalisation corporations is more susceptible to changes in quantities offered. Table 4 quantifies the graphic representation presented in Chart 2, determining the price robustness, i.e., the average amount necessary for an offer to give rise to a 0.1 per cent change in the stock price. (The price is defined herein as the average between the better quotes offered at both sides of the market.) As previously, this measure of impact on prices is calculated for different distribution intervals of daily returns of the PSI-20 index. Table 4 also establishes a distinction between price robustness for bid and ask orders.

The results presented in Table 4 reveal that, on average, a 0.1 per cent change is expected in prices if, at a given moment in time, bid orders to the amount of € 136 thousand or ask orders to the amount of € 140 thousand are settled. As would be expected, the stock price in low capitalisation corporations is more sensitive to changes in quantities offered. In turn, prices in general are more sensitive to changes in quantities offered when the market registers more changes in daily return. This behaviour is consistent with less liquidity of low

capitalisation corporations and/or with situations of sudden changes in market returns.

2.2. Intra-daily behaviour

According to some authors, such as Bias, Hillion and Spatt (1995), the number of orders and transactions changes throughout the day, registering a higher number and average value of transactions towards the end of the session. According to these authors, low average value orders at the beginning of the day may be related to a mechanism of price disclosing. Large orders at the end of the day occur after price disclosure, or also due to the fact that large financial intermediaries, such as investment funds, close their portfolio positions based on the end of the session values instead of intra-daily values.

The change in the volume of transactions seems to be related to a change in market liquidity, wherefore higher liquidity is to be expected when the market registers a higher number of transactions. Chart 3 represents the bid-ask spread behaviour throughout the day between 9:00 a.m. and 4:00 p.m., for one-hour intervals, distinguishing developments in low capitalisation and high capitalisation groups. The values at the opening and the end of the session correspond to a lower number of orders.

As observed in most markets, stock market liquidity in Portugal reveals an intra-day behaviour with approximately a U-shaped standard, decreasing during the morning to increase slightly towards the end of the session.

The bid-ask spread of thinly (highly) capitalised corporations attains its low at approximately 2:30 p.m., accounting for approximately 60 (53) per cent of the value recorded at the opening of the session.

3. DETERMINING FACTORS BEHIND THE RATIO AND TIME OF EXECUTION OF THE ORDERS

The previous sections identified a number of liquidity indicators characterising the market that an investor faces when submitting an order. This section determines the importance of such indicators for the success of the orders, which is defined as the easiness of execution (when the quantity ex-

Table 4

PRICE ROBUSTNESS

0.1% impact on average price (bid) (euro million)	Daily return intervals vis-à-vis average return for the period (μ)					Total
	$<\mu-2\sigma$	$[\mu-2\sigma, \mu-\sigma[$	$[\mu-\sigma, \mu+\sigma]$	$[\mu+\sigma, \mu+2\sigma]$	$>\mu+2\sigma$	
Low capitalisation group.....	0.009	0.028	0.029	0.031	0.016	0.028
High capitalisation group.....	0.130	0.178	0.225	0.197	0.189	0.210
Total - PSI 20 Index.....	0.087	0.123	0.143	0.130	0.128	0.136

0.1% impact on average price (ask) (euro million)	Daily return intervals vis-à-vis average return for the period (μ)					Total
	$<\mu-2\sigma$	$[\mu-2\sigma, \mu-\sigma[$	$[\mu-\sigma, \mu+\sigma]$	$[\mu+\sigma, \mu+2\sigma]$	$>\mu+2\sigma$	
Low capitalisation group.....	0.018	0.025	0.026	0.023	0.016	0.025
High capitalisation group.....	0.128	0.199	0.231	0.201	0.201	0.218
Total - PSI 20 Index.....	0.088	0.135	0.146	0.130	0.136	0.140

ecuted is positive), or the time elapsed from the moment when the order is submitted to the respective execution.

A bid (ask) order will have the more probability of being executed: the more it matches quotes on the opposite side of the market in the order book; the larger is the volume of orders accumulated on the ask (bid) side; or the lower is the bid-ask spread. These factors determine that the rate of execution of the order depends on the characteristics of the order and on its relationship with the order book at the moment it is submitted. For instance, a higher number of orders in the book at the best price quotations will reduce the probability of execution, or increase the average time to execution.

In addition to specific factors inherent to the order, it should be expected that factors related to the stock or the market may also influence the probability of execution. In particular, high capitalisation stocks with lower volatility will probably have more probability of execution. Let us consider that the dummy variable *Exec* corresponds to 1 when the order is fully or partially executed, and to 0 otherwise. The probability of full or partial execution of an order, $\Pr\{Exec=1\}$, may be described as:

$$\Pr\{Exec=1\} = F[\alpha + g(\text{order factors}) + h(\text{stock factors}) + w(\text{market factors}) + \varepsilon]$$

where F is a function associated with the probit model, and it is assumed that g , h , and w are linear functions. In addition:

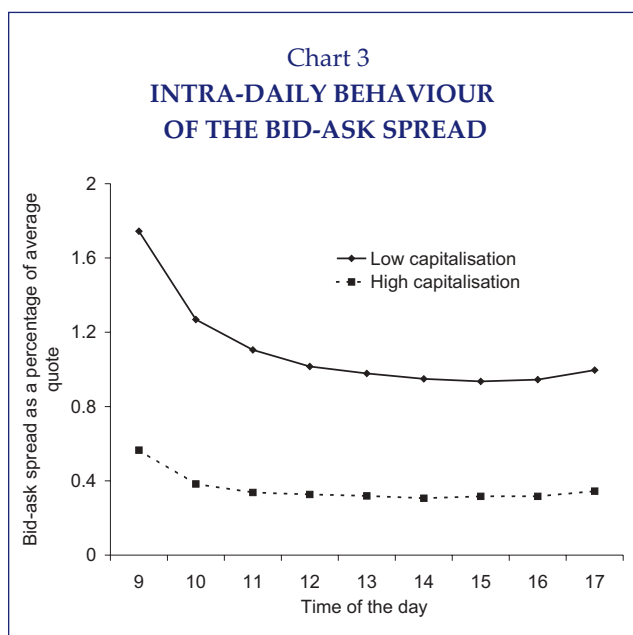
$$(i) \text{ order factors} = (D_{\text{aggressiveness}}, \text{Priority Quantity}, \text{Bid} - \text{ask}, D_9, \dots, D_{16})$$

$$(ii) \text{ stock factors} = (\sigma_{\text{stock}}, D_{\text{high cap.}}, \text{Return}_{\text{stock}});$$

$$(iii) \text{ market factors} = (R_{<\mu-2\sigma}, R_{[\mu-2\sigma, \mu-\sigma[}, R_{[\mu-\sigma, \mu+\sigma]}, R_{[\mu+\sigma, \mu+2\sigma]}, R_{>\mu+2\sigma}).$$

$D_{\text{aggressiveness}}$ is a dummy variable corresponding to 1 in the case when, in an ask order, the order quote, *Quote*, is below the minimum ask quote in the order book of the stock in the minute prior to submitting the order, $V_{1,t-1}$. Otherwise, the variable is zero. In the case of a bid order, the variable corresponds to 1 if $\text{Quote} > C_{1,t-1}$. This is a measure of the aggressiveness of the order. It is to be expected that more aggressiveness will induce a narrowing of the bid-ask spread. Table 5 presents the correlations between the bid-ask spread and the measure of aggressiveness of the order for the minute when the order was submitted and for the three previous minutes. It can be observed that submitting ask (bid) orders at quotes below (above) the lower (higher) ask (bid) quote in the book has a correlation of -8.97 (-8.14) per cent with the bid-ask spread. This correlation will gradually decrease as the lag widens.

The *Priority Quantity* variable for a bid order represents the ratio of the quantity of shares offered at best price bid quote in the total quantity of shares offered at best price bid and ask quotes in the previous minute, i.e., $QB_{1,t-1} / (QB_{1,t-1} + QA_{1,t-1})$, where $QB_{1,t-1}$, for instance, is the quantity outstanding in the order book for the highest bid



quote. For an ask order, the *Priority Quantity* is defined as $QA_{1,t-1} / (QB_{1,t-1} + QA_{1,t-1})$.

The bid-ask variable represents the bid-ask spread of the stock in the minute prior to the order.

As regards the variables associated with the stock, σ_{stock} represents the daily volatility of the stock, and is defined as $(P_{\text{maximum}} - P_{\text{minimum}}) / |P_{\text{close}} - P_{\text{opening}}|$, where P_{maximum} (P_{minimum}) represents the maximum (minimum) transaction price of the stock during the day. (The price of the transaction is proxied by $(A_1 + B_1) / 2$.) The values for P_{close} and P_{opening} are defined in a similar manner. The $\text{Return}_{\text{stock}}$ variable represents the daily return of the stock defined as the basis for closing prices, and is intended to monitor the idiosyncratic movements of stock prices in the PSI-20.

$D_{\text{high cap.}}$ is a dummy variable corresponding to 1 in case the order refers to a high capitalisation stock, which makes it possible to compare the behaviours of both groups of stocks.

Finally, turning to the variables related to the market, and similarly to tables 2 and 4, the dummy variables considered describe the market return intervals vis-à-vis the average for the period. Variable $R_{<\mu-2\sigma}$ is an indicator corresponding to 1, if the daily return is lower than the average return for the period minus two standard deviations, and corresponding to 0 otherwise. Similarly, the dummy variables for the respective interval are defined as: $R_{[\mu-2\sigma, \mu-\sigma]}$, $R_{[\mu-\sigma, \mu+\sigma]}$, $R_{[\mu+\sigma, \mu+2\sigma]}$ and $R_{>\mu+2\sigma}$.

With a view to controlling changes in market behaviour throughout the day, already reported in Chart 3, the regression model also considers dummy variables for the transaction time, $D_9, \dots, D_{16}$, which correspond to 1, if the order was submitted at [9:00,10:00 a.m.], ..., [16:00,16:25 p.m.], respectively. Taking into account that these variables have as their exclusive purpose to monitor the effects estimated for the remaining variables, the value of their estimates is omitted.

Since the dependant variable of the model is a dummy corresponding to 1 if the order is executed and to 0 otherwise, a probit, non-linear regression model is estimated. The second and third columns of Table 6 present the results of the regression model just described. In the table, the estimates of the coefficients represent the marginal effects of the independent variables on the probability that the order is executed. The line below the values estimated for the coefficients presents the respective p-values.

Similarly, it is possible to calculate the impact of the same factors, associated with order, stock and market, on the time of execution, depending on whether or not the order is executed. For that purpose, use was made of a Cox model of proportional effects. The results of the estimates of this model for ask and bid orders are presented in the third and fourth columns of Table 6. This is the ratio (known in the literature as hazard ratio) of the instantaneous rate of execution of the order to the reference rate of execution (known in literature as baseline hazard). Estimates of hazard ratios above 1 represent a decrease in the time of execution vis-à-vis the reference. For instance, the hazard ratio associated with the aggressiveness in ask orders is 2.18, which means that, on average, the aggressive orders (with $D_{\text{aggressiveness}}$ equivalent to 1) have a rate of execution 2.18 times higher than non-aggressive orders (with $D_{\text{aggressiveness}}$ equivalent to 0).⁽⁴⁾

The results presented in Table 6 reveal that the factors related to the order, stock and market are very important in explaining the probability and time of execution of the order. The results in the

(4) In the Cox duration model, the hazard function is defined as $h(t) = h_0(t) * \exp(\beta_1 * x_1 + \dots + \beta_k * x_k)$, where $h_0(t)$ is the baseline hazard, and β_i are the coefficients to be estimated. Table 6 presents estimates of $\exp(\beta_i)$.

Table 5

SAMPLE CORRELATION BETWEEN THE BID-ASK AND AGGRESSIVENESS IN LAGGED MINUTES

	$D_{contemp. \text{ aggress.}}$	$D_{aggressiveness}$	$D_{aggressiveness (-2)}$	$D_{aggressiveness (-3)}$
	Buy orders			
Bid Ask Spread	-0.0897	-0.0869	-0.0904	-0.0856
	Sell orders			
Bid Ask Spread	-0.0814	-0.0778	-0.064	-0.0522

Notes: Definition of the variables: the bid-ask spread is given by $(A_1 - B_1) / ((A_1 + B_1) / 2) * 100$; $D_{contemp. \text{ aggress.}}$ is 1 if $Quote_t < A_{1,t}$ ($Quote_t > B_{1,t}$) for an ask (bid) order, and 0 otherwise. $Quote_t$ is the quote associated with the order and $A_{1,t}$ ($B_{1,t}$) is the average value of the best price ask (bid) quote at minute t . $D_{aggressiveness}$ corresponds to 1 if $Quote_t < A_{1,t-1}$ ($Quote_t > B_{1,t-1}$) for an ask (bid) order, and 0 otherwise. $D_{aggressiveness (-2)}$ corresponds to 1 if $Quote_t < A_{1,t-2}$ ($Quote_t > B_{1,t-2}$) for an ask (bid) order, and 0 otherwise. $D_{aggressiveness (-3)}$ is defined in a similar manner but for $t-3$. All correlations are significant with confidence intervals of 95%. The number of observations is 1,335,266.

second column reveal that, if an ask order is submitted at a quote below its best price ask quote (V_1 in Chart 1) in the previous minute (aggressiveness measure), the probability of execution of the order is raised by 60 per cent (this value stands at 55 per cent in the case of bid orders - third column). As expected, the probability of execution of ask (bid) orders decreases when, in the minute prior to submitting the order, the order book presents a higher number of orders at best price ask (bid) quotes, or when the bid-ask spread is wider. A 1 per cent increase in the quantity of stocks at best price ask quote reduces the probability of execution of the ask orders by 14.9 per cent (15.4 per cent in the case of bid orders). A widening of the bid-ask spread by 1 per cent lowers the probability of execution of ask orders by 5.4 per cent (4.3 per cent in the case of bid orders).

The stock and market factors also influence the probability of execution of bid and ask orders. A 1 per cent rise in the stock volatility is associated with a decline in the probability of execution of bid orders (0.13 per cent) and ask orders (0.07 per cent). In case the stock belongs to the group of high capitalisation corporations, the probability of execution of ask (bid) orders increases by 8.5 (9.8) per cent. This impact is consistent with the results presented in Table 1 (summarised description of the orders). Finally, and as expected, the stock return and the index tend to affect positively (negatively) the probability of execution of the ask (bid) order.

As regards the time of execution of the orders, the results of the duration model are in general consistent with those obtained for the probability of execution. According to the results, the time of execution of ask (bid) orders decreases when the orders are below (above) the best price ask (bid) quote recorded in the previous minute (i.e., when aggressiveness increases), when the number of shares at minimum (maximum) ask (bid) quote decreases vis-à-vis the previous minute, or when there is a narrowing in the bid-ask spread from that recorded in the previous minute. Similarly to the probability of execution, high capitalisation stocks are associated with shorter times of execution than low capitalisation stocks. However, the results reveal that positive changes of the stock returns are associated with an increase in the time of the execution of ask orders or its decrease in the case of bid orders. Stock volatility also shortens the time of execution of both bid and ask orders, albeit not markedly.

4. CONCLUSIONS

This paper analyses liquidity and factors of execution of orders in the Portuguese stock market. The compilation of all orders of stocks belonging to the PSI-20 index from January to October 2002 made it possible to build a market order book and, consequently, to create a virtual system of order transactions. The analysis of the order book permits market liquidity indicators to be identified, such as the spread between the highest bid quote

Table 6
ANALYSIS OF THE EXECUTION RATION AND TIME OF EXECUTION

	Probability of execution		Time of execution	
	Pobit model		Cox duration model	
	Ask orders	Bid orders	Ask orders	Bid orders
	dF/dx	dF/dx	Haz. Ratio	Haz. Ratio
Order factors				
$D_{\text{aggressiveness}}$	0.6060	0.5501	2.1772	2.2695
	0.000	0.000	0.000	0.000
Priority quantity	-0.1489	-0.1539	0.7034	0.6864
	0.000	0.000	0.000	0.000
Bid Ask	-0.0539	-0.0434	0.7648	0.7887
	0.000	0.000	0.000	0.000
Stock factors				
σ_{stock}	-0.0013	-0.0007	1.0060	1.0066
	0.000	0.000	0.000	0.000
D_{stock}	0.0852	0.0983	1.2081	1.2519
	0.000	0.000	0.000	0.000
$\text{Return}_{\text{stock}}$	1.2028	-0.8164	0.4103	2.3180
	0.000	0.000	0.000	0.000
Market factors				
Return $<\mu - 2\sigma$	0.0072	0.0349	1.1381	1.0355
	0.033	0.000	0.000	0.000
Return $[\mu - 2\sigma, \mu - \sigma]$	-0.0204	0.0433	0.9971	1.0003
	0.000	0.000	0.759	0.964
Return $[\mu + \sigma, \mu + 2\sigma]$	0.0380	-0.0092	1.0166	1.0718
	0.000	0.000	0.024	0.000
Return $>\mu + 2\sigma$	0.0904	-0.0722	0.8756	1.3147
	0.000	0.000	0.000	0.000
Observed probability	0.543	0.594		
Expected probability	0.610	0.669		
Pseudo R2	0.292	0.262		
Number of observations	684811	650455	159662	181991

Note: The line below the estimates of the coefficients associated with the coefficients of each variable presents the corresponding p-value of the significance test the coefficient.

(a) The joint null test was rejected at the 99% confidence level.

and the lowest ask quote (bid-ask spread), or indices to be constructed, such as changes in prices in consecutive bid and ask orders (transaction cost) or the impact of the volume of bid and ask orders on prices (price robustness). The analysis shows that the bid-ask spread in the Portuguese market stands at 60 basis points while transaction costs reach, on average, 4.11 basis points. Liquidity indices change from one corporation to the other, and depend the magnitude of changes in market daily returns. The group of low capitalisation corporations presents a bid-ask spread approximately fivefold that of the high capitalisation group, but

this spread may change from 79 basis points to 56 basis points, depending on whether the daily index return lies more than two standard deviations away from the sample average, or within the interval that goes from minus one standard deviation to plus one standard deviation from the average. Liquidity indices calculated for the PSI-20 are within the same order of magnitude than those observed for major international markets. Similarly to most markets, the Portuguese market also reveals a U-shaped standard of intra-day liquidity.

The paper shows that market liquidity factors determine both the probability of execution and

the time of execution. The probability that an ask (bid) order will be executed increases by 60.6 (55.0) per cent submitted within the bid-ask spread. An investor executing an ask (bid) order of a stock belonging to the high capitalisation group will have 8.52 (9.83) per cent more probability of execution of the order.

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