INSTRUMENTS OF MONETARY POLICY*

*Bernardino Adão***
*Isabel Correia***
*Pedro Teles***

1. INTRODUCTION

A classic question in monetary economics is whether the interest rate or the money supply is the better instrument of monetary policy. Until recently practice and theory seemed to be in disagreement. Most will agree that monetary policy decision making has focused on setting a target for the short-term interest rate. However, most theoretical work has considered the monetary policy as being a choice about the trajectory of the money supply. One thing that is frequent in all the literature is that the monetary policy is not specified in sufficient detail. If the interest rate is the chosen instrument it is not described how the associated money supply is determined or vice versa; if the money supply is the instrument it is not explained how the interest rate is determined.

It is confirmed both theoretically and empirically that the demand for real money depends on the nominal interest rate and on the real output level. Thus, unless both the real output level as well as the price level are fixed, setting the nominal interest rate is not equivalent to targeting a monetary aggregate. And vice-versa, fixing money is not equivalent to fixing the nominal interest rate.

There are *ad-hoc* models where there is just one monetary instrument. For instance, the obsolete static IS-LM model with fixed prices has only one instrument. The IS curve is the set of nominal interest rates and output levels for which the good market is in equilibrium when the supply of the good is demand determined. The LM curve is the set of nominal interest rates and output levels for which the money market is in equilibrium. Thus, given the money supply the intersection of the IS and the LM determine the output and the nominal interest rate. And instead, given the nominal interest rate the IS determines the real output, and given the nominal interest rate and the output the LM determines the money supply.

By contrast, this paper considers a standard dynamic macroeconomic model with microeconomic foundations. The main result is that in order to obtain a unique equilibrium, that is, well defined trajectories for variables like inflation and output, the central bank should use both the money supply and the interest rate as instruments. This is a sufficiency result as it is known that in some particular non-robust frameworks(1), uniqueness may be obtained with less instruments.

The rest of the paper is set out as follows: section 2 describes the literature. Section 3 portrays the model. Section 4 shows that the Taylor principle guarantees local determinacy but not unique-

* The views expressed in this article are those of the authors and not necessarily those of the Banco de Portugal. This paper is based on our recent research, the main references being Adão, Correia and Teles, (2003) and (2004).
This paper benefited from comments by Marta Abreu, José Brandão de Brito, José António Machado, Maximiano Pinheiro and Carlos Robalo.
** Economic Research Department.

(1) For instance endowment economies with separable logarithmic utility functions in consumption and real balances, with convertibility of money and no public debt. See Obstfeld and Rogoff (1983).

ness of the equilibrium in the deterministic version of the model. Section 5 reveals which policy variables need to be used as instruments in order to have uniqueness of the equilibrium in the stochastic version of the model. Section 6 concludes. The appendix extends the results of section 4 to the stochastic framework.

2. THE LITERATURE

This section provides a brief description of the main contributions to the literature on the monetary instrument choice problem. The earliest notorious effort was by Friedman (1968), who argues against the use of the interest rate as an instrument. His concern was that if agents have irrational expectations about inflation, the economy would not converge to the rational expectations equilibrium. No matter what nominal interest rate the central bank would choose, if people expected inflation above the rational expectations equilibrium, that would result in lower perceived real interest rate, which would generate a higher demand for current goods, leading to an even higher inflation, which in turn would lead to an even lower real interest rate, stimulating more the economy, and so on without bound.

Unlike Friedman (1968), in the recent literature agents are taken as being rational. The departing point has been that the instrument must be able to generate local determinacy of the equilibrium. Local determinacy means that in the neighbourhood of an equilibrium there are no other equilibria. However, in general besides this locally determined equilibrium there is an infinity of other equilibria that cannot be ruled out. It is very intriguing that all the literature as been satisfied with this local determinacy property. To us the multiplicity of equilibria is a disturbing result. For it implies that the same economic fundamentals are compatible with many values for the macroeconomic variables. Random events completely unrelated to the fundamentals, sunspots, can cause large fluctuations of the output and inflation. From the view point of the central bank this is undesirable, since usually its objective is to promote output and inflation stabilization.

In this literature of local determinacy there have been a few very influential papers. Sargent and Wallace (1975) shows that interest rate rules that depend only on exogenous variables do not guarantee local determinacy and defend instead the use of the money supply as the instrument. Mc Callum (1981) shows that if instead, the central bank chooses interest rate rules that depend on endogenous variables the Sargent and Wallace result does not apply necessarily. The classic Taylor rule, Taylor (1993), is one such example, setting the interest rate as a function of the current estimates of the output gap and inflation. Recently the most forceful defence of the use of the interest rate as the instrument is Woodford's influential book, Woodford (2003).

In this paper we present the concept of equilibrium in a stochastic environment. We show that in general if the monetary authority uses just one instrument, no matter which, there will be a large multiplicity of equilibria. As a corollary, we get that there is an infinite number of equilibria when the monetary authority uses only one instrument, even if it guarantees local uniqueness.

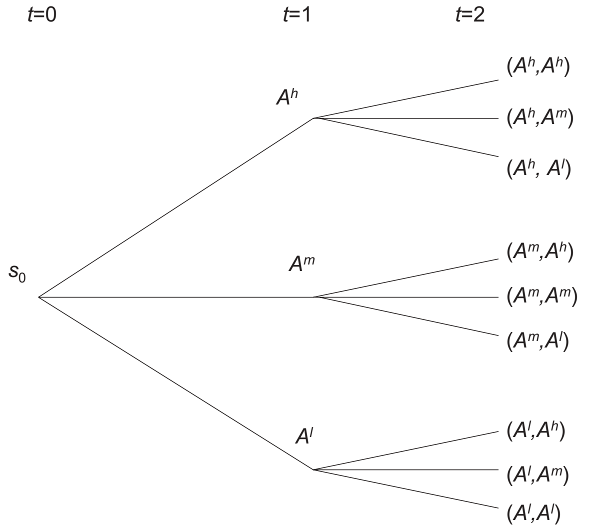
3. MODEL

We consider a cash in advance economy. The economy consists of a representative household, a representative firm behaving competitively, and a government. Production uses labour according to a linear technology. This environment is the simplest to study the instruments of monetary policy. More complex models deliver similar results, as long as agents take decisions for at least two periods.

We consider shocks to technology A_t and government expenditures G_t. The period t vector of shocks is denoted as $s_t=(A_t,G_t)$. The set of all possible shocks in period t is denoted by S_t, the history of these shocks up to period t, which we call state at t, $(s_0,s_1,...,s_t)$, is denoted by s^t, and the set of all possible states in period t is denoted by S^t. The initial realization s_0 is given. To simplify the exposition, we assume that the history of shocks has a discrete distribution. The number of all possible shocks in period t is $\# S_t$ and the number of all possible states in period t is $\# S^t$.

An example may help clarify the terminology. Suppose that G_t is a constant, i.e. $G_t=G$ for all t and A_t for all $t\geq 1$ can assume only 3 values: a high, A_h, a medium, A^m or a low value, A^l. For each period $t\geq 1$, the number of possible shocks is

Chart 1
STATES



3, $S_t = \{(A^h, G), (A^m, G), (A^l, G)\}$. But the number of possible states is different across periods. The number of possible states in the following period is always bigger. In period 0 there is 1 state, the number of possible states in period 1 is 3, the number of possible states in period 2 is 9 and so on. Chart 1 provides a graphical representation of this example.

3.1. Competitive equilibria

Households

The households have preferences over consumption C_t, and leisure L_t. These two variables as all variables in the economy, which we describe in detail below, are a function of s^t, but to shorten the notation instead of writing down $C(s^t)$ we write C_t. The expected utility function is:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right\}, 0 < \beta < 1, \tag{1}$$

where β is a discount factor. The households start period t with nominal wealth $\mathbb{W}_t$. They decide to hold money, M_t, and to buy B_t nominal bonds that pay $R_t B_t$ one period later. The R_t is the gross nominal interest rate at date t. Thus, in the assets market at the beginning of period t they face the constraint

$$M_t + B_t \leq \mathbb{W}_t \tag{2}$$

Consumption must be purchased with money according to the cash in advance constraint

$$P_t C_t \leq M_t \tag{3}$$

At the end of the period, the households receive the labour income $W_t N_t$ where $N_t = 1 - L_t$ is labour and W_t is the nominal wage rate and pay lump sum taxes, T_t. Thus, the nominal wealth households bring to period $t+1$ is

$$\mathbb{W}_{t+1} = M_t + R_t B_t - P_t C_t + W_t N_t - T_t \tag{4}$$

The households' problem is to maximize expected utility (1) subject to the restrictions (2), (3), (4), together with a no-Ponzi games condition on the holdings of assets(2).

The following are first order conditions of the households' problem:

$$\frac{u_L(t)}{u_C(t)} = \frac{W_t}{P_t} \frac{1}{R_t} \tag{5}$$

$$\frac{u_C(t)}{P_t} = R_t E_t \left[\frac{\beta u_C(t+1)}{P_{t+1}} \right] \tag{6}$$

Condition (5) sets the intratemporal marginal rate of substitution between leisure and consumption equal to the real wage adjusted for the opportunity cost of using money, R_t. Condition (6) is an intertemporal marginal condition necessary for the optimal choice of nominal bonds. It says that the utility today of an additional unit of money must be equal to the expected utility tomorrow of R_t additional units of money.

Firms

The firms are competitive and prices are flexible. The production function of the representative firm is linear

$$Y_t \leq A_t N_t.$$

The equilibrium real wage is equal to the marginal productivity of labour,

(2) The implied constraint is that the household must hold a net portfolio at the end of the period that is larger in absolute value than the present value of its future net income.

$$\frac{W_t}{P_t} = A_t. \tag{7}$$

Government

The policy variables are taxes, T_t, interest rates, R_t, money supplies, M_t, and public debts, B_t. The government chooses the policy, which is defined as the behaviour of some, but not all of these policy variables. The government cannot choose the behaviour of all of the policy variables because, as we will see, there are equilibrium conditions that together with the policy determine endogenously the values for the remaining policy variables. A policy is a set of functions, chosen by the government, that map quantities, prices and policy variables into policy variables. One example of a policy is the Taylor rule, that specifies the interest rate as a function of inflation and output. Another example of a policy is a constant growth money supply.

The period by period government budget constraints are

$$M_{t+1} + B_{t+1} = M_t + R_t B_t + P_t G_t - P_t T_t, \; t \geq 0. \tag{8}$$

At each state s^t equation (8) has an intertemporal counterpart that establishes that the present expected value of the future seigniorage flows must be equal to today's government responsibilities plus the present expected value of the future government deficit flows. This stochastic intertemporal condition can be written as a function of only the trajectories for consumption, leisure and policy variables.

Market clearing

Market clearing in the goods and labour market requires

$$C_t + G_t = A_t N_t,$$

$$1 - L_t = N_t.$$

We have already imposed market clearing in the money and debt markets.

Equilibrium

A competitive equilibrium is a sequence of policy variables, quantities and prices such that the private agents, households and firms, solve their problem given the sequences of policy variables and prices, the budget constraint of the government is satisfied and markets clear.

The equilibrium conditions for the 7 variables $\{C_t, L_t, P_t, B_t, R_t, M_t, T_t\}$ are 5. They include the resources constraint

$$C_t + G_t = A_t(1 - L_t), \; t \geq 0 \tag{9}$$

the intratemporal condition that is obtained from substituting the households intratemporal condition (5) into the firms optimal condition (7)

$$\frac{u_C(t)}{u_L(t)} = \frac{R_t}{A_t}, \; t \geq 0 \tag{10}$$

as well as the cash in advance constraint (3), the intertemporal condition (6), and the government intertemporal budget constraint.

These conditions define a set of equilibrium allocations, prices and policy variables. The number of equations at state s^t is equal to 5. The number of equilibrium variables that must be determined at state s^t is equal to 7. If none of the policy variables is chosen exogenously, there is an infinity of allocations, prices and policy variables satisfying the 5 equilibrium conditions. Since there are less equilibrium equations than equilibrium variables there are many equilibria unless the government chooses exogenously some of the policy variables. There can be equilibria with high inflation or low inflation as there can be equilibria with low output or high output. Anything is possible. On the other hand, if all the policy variables, taxes, money supplies, interest rates and debt are chosen exogenously, there is no equilibrium.

There are many ways in which the degrees of freedom can be fulfilled. As we are primarily interested in studying monetary policy we assume that the fiscal policy adjusts to satisfy the intertemporal government budget constraint. In other words, we assume that the fiscal policy is endogenous in the sense that whatever are the choices of the monetary authority, the fiscal instruments, B_t and T_t, adjust to satisfy the intertemporal government budget constraint implied by (8).

Now, the number of relevant variables is 5 and the number of relevant equations 4, being one of them, (6), a stochastic dynamic equation. By counting equations and unknowns, it would seem enough in order to get determinacy that the government would have just one monetary instrument, as that would be equivalent to adding to the remaining equilibrium conditions another condition, which would result in a system with the same number of equations as unknowns. That intuition is wrong because one of the equations, (6), is a stochastic dynamic equation. If the environment was deterministic, (6) would be a first order difference equation and in order to get a unique solution it would be enough to have an initial or terminal condition. Because the environment is stochastic, the number of conditions necessary to get uniqueness is much larger as we will see below.

In section 5 we show that in general by setting only a function for one of the monetary policy variables uniqueness of the equilibrium is not achieved. As we explain in section 4, this implies that by simply following an interest rate rule, even if it guarantees local determinacy, the monetary authority is allowing an infinite number of equilibria, many of which can be associated with very high inflation levels.

4. LOCAL DETERMINACY AND INTEREST RATE RULES

The literature is currently dominated by a rule-based approach to monetary policy. According to the literature local determinacy is among the most desirable properties that a rule must possess. Local determinacy means, as we said before, that in the neighbourhood of an equilibrium there is no other equilibrium. In this section we clarify what is meant by an interest rate feedback rule guaranteeing local determinacy and show that for a standard environment local determinacy is achieved if the Taylor principle is followed. Roughly speaking, the Taylor principle is verified if in response to an increase in inflation the increase in the nominal interest rate is higher.

This section is an exception, as here, to simplify the exposition we consider a deterministic environment, i.e. $A_t = A$ and $G_t = G$ for all t and $u(C_t, L_t) = C_t + v(L_t)$. In the appendix we present the more complex stochastic counterpart. Let $\mathbb{R}$ be the steady state competitive equilibrium for the interest rate and let Π be the steady state competitive equilibrium for the inflation rate. Then, $\mathbb{R} = \frac{\Pi}{\beta}$, where $\frac{1}{\beta}$ is the real interest rate. Assume that the central bank conducts a pure current nonlinear Taylor rule(3):

$$R_t = \mathbb{R}\left(\frac{\pi_t}{\Pi}\right)^{\tau\beta},$$

where $\tau\beta \geq 1$ (the Taylor principle), and $\pi_t = \frac{P_t}{P_{t-1}}$. After substituting the Taylor rule in (6) get

$$z_{t+1} = (z_t)^{\tau\beta}$$

where $z_t = \frac{\pi_t}{\Pi}$. By recursive substitution we get

$$z_{t+k} = (z_t)^{\kappa\tau\beta}, \text{ for all } k \text{ and } t. \tag{11}$$

There is no condition to pin down the initial value for inflation. Since the initial inflation level can be any value there is an infinity of equilibrium trajectories for the inflation rate. Nevertheless, they can be typified in 3 classes. Either inflation is constant, $\pi_t = \Pi$, or there is an hyperinflation, $\pi_t \longrightarrow \infty$, or inflation is approaching zero, $\pi_t \longrightarrow 0$. This is easy to verify. If $\pi_0 = \Pi$ then (11) implies that $\pi_t = \Pi$ for all t. If $\pi_0 > \Pi$ then (11) implies that $\pi_{t+1} > \pi_t$ and $\pi_t \longrightarrow \infty$, since $\tau\beta > 1$. If $\pi_0 < \Pi$, then (11) implies that $\pi_{t+1} < \pi_t$ and $\pi_t \longrightarrow 0$ since $\tau\beta > 1$.

Thus, when the central bank follows a Taylor rule that obeys the Taylor principle it is able to get local determinacy. In a neighbourhood of the steady state inflation Π there is no other equilibrium inflation trajectory. But we have just seen that there is an infinity of other equilibria for inflation which converge to zero or to infinity. These results beg two interrelated questions: Why is local determinacy such an interesting property? Or why has most of the literature assumed that undesirable equilibria do not happen? We do not know the answer to these questions.

(3) Usually the Taylor rule is presented in its linearized form. As can be verified the linearized version is,

$$R_t - \mathbb{R} = \tau(\pi_t - \Pi).$$

There may be institutions that we have ignored in the model, which can be used to eliminate some of these "undesirable" equilibria. For instance, in some models an hyperinflation can be eliminated if the central bank has sufficient resources and can commit to buy back its currency if the price level exceeds a certain level. We are not going to pursue this issue here. Those readers interested in this topic should start by seeing the seminal paper of Obstfeld and Rogoff (1983). In general, there are still an infinity of equilibria that pass these types of tests.

It is easy to verify, using an argument similar to the one above, that if the Taylor rule did not obey the Taylor principle, i.e. $\tau\beta < 1$, there would be just two types of equilibrium. The steady state and an infinity of equilibria converging to the steady state. At first sight it would seem that it would be preferable that a central bank would follow a Taylor rule that did not satisfy the Taylor principle, as "undesirable" equilibria, hyperinflations or hyperdeflations would not be possible. This conclusion is not correct because whenever there is multiplicity of equilibria it may be possible that sunspots can cause large fluctuations in inflation. Inflation can fluctuate randomly just because agents come to believe this will happen. The interested readers should start with Farmer (1993).

5. EXOGENOUS POLICY INSTRUMENTS

We are interested in identifying what are the exogenous instruments of policy that guarantee that there is a unique equilibrium for allocations and prices. This provides a measure of degrees of freedom in conducting policy. This is a question of policy relevance. As mentioned above, it is associated with the instrument problem in monetary economics on whether to use the interest rate or the money supply as the monetary policy instrument.

Under very general conditions the system of equations defining the equilibrium can be summarized by,

$$\frac{u_C\left(C(R_t), L(R_t)\right)}{\frac{M_t}{C(R_t)}} = \beta R_t E_t\left[\frac{u_C\left(C(R_{t+1}), L(R_{t+1})\right)}{\frac{M_{t+1}}{C(R_{t+1})}}\right], t \geq 0 \qquad (12)$$

where $C(R_t)$ and $L(R_t)$ mean that consumption and leisure depend only on the level of the interest rate.

5.1. Conducting policy with constant functions

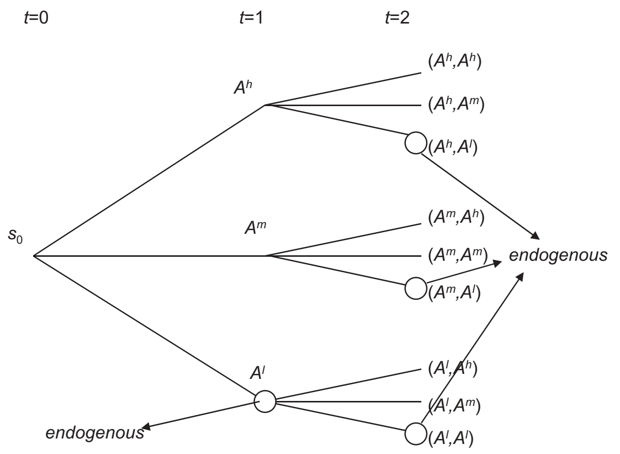
In this subsection, we show that in general when policy is conducted with constant functions for the policy instruments, it is necessary to determine exogenously both interest rates and money supplies.

Suppose the path of money supply is set exogenously in every date and state. In addition, in period zero the interest rate, R_0, is set exogenously and, for each $t \geq 1$, for each state s^{t-1}, the interest rates are set exogenously in $\# S_t - 1$ states that follow. In this case (12) at date $t = 0$ would determine the R_1 in the remaining state, since $\# S_t - 1$ of the R_1s were already given. The usage of (12) for the other dates would determine recursively all the R_ts that were not set exogenously. Thus, there is a single solution for the allocations and prices. Similarly, there is also a unique equilibrium if the nominal interest rate is set exogenously in every date and state, and the money supply is set exogenously in period 0, as well as, for each $t \geq 1$ and state s^{t-1}, in the $\# S_t - 1$ states that follow.

Thus, we have the following result when policy is conducted with constant functions: in general, if money supply is determined exogenously in every date and state, and if interest rates are also determined exogenously in the initial period, as well as in $\# S^t - \# S^{t-1}$ states for each $t \geq 1$, then the allocations and prices can be determined uniquely, similarly, if the exogenous policy instruments are the interest rates in every state, the initial money supply and the money supply, in $\# S^t - \# S^{t-1}$ states, for $t \geq 1$, then there is in general a unique equilibrium.

Chart 2 illustrates this result for the example of section 3. For instance, a unique equilibrium can

Chart 2
INSTRUMENTS



be guaranteed if for the states with a circle one of the instruments, be it the money supply or the interest rate, is determined endogenously by (12) and in the remaining states money supply and interest rate are exogenous(4).

5.2. Conducting policy with feedback rules

It is commonly assumed that policy is conducted with feedback rules, in particular, interest rate feedback rules. In this subsection, we argue that the results of the previous section do not change if instead the monetary policy is conducted with feedback rules for the policy instruments instead of constant functions. The use of interest rate rules that depend on current or past variables (these are the type of rules that guarantee local determinacy) preserves the same degrees of freedom in the determination of the equilibrium. It is still necessary to determine exogenously the levels of money supply in some of the states.

When the policy is conducted with current or backward interest rate feedback rules in order to have a unique equilibrium, it is necessary to determine exogenously the money supply in $\# S_t - 1$ states, for each state s^{t-1}, $t \geq 1$, as well as M_0. We can use the argument used before. At any state s^{t-1}, $t \geq 1$ given M_{t-1} and R_{t-1} there is one equation (12) that relates s^{t-1} with period t, and $\# S_t$ equations for the subsequent R_ts, which are implied by the feedback rule. Thus, to obtain the $\# S_t$ values of the R_ts and the $\# S_t$ values of the M_ts, the monetary authority needs to set $\# S_t - 1$ values for the M_ts.

In general, a similar result holds if the monetary policy is conducted with money feedback rules. When the monetary policy is conducted with a money feedback rule in order to have a unique equilibrium, it is necessary to determine exogenously the interest rate in $\# S_t - 1$ states, for each state s^{t-1}, $t \geq 1$, as well as R_0.

(4) If instead, taxes were exogenous, a single monetary instrument would be enough to get a unique equilibrium. For instance if the central bank set exogenously the interest rate and the fiscal authority set taxes exogenously, the price level would be determined by the government intertemporal budget constraint. This result is known as the fiscal theory of the price level. See Woodford (2003).

6. CONCLUSION

Under the assumption that the fiscal policy was endogenous, a monetary policy that uses just one monetary policy instrument, either the nominal interest rate or the money supply, is not able to eliminate the multiplicity of equilibria. In particular, a Taylor rule that obeys the Taylor principle generates local determinacy. But local determinacy is still consistent with an infinity of equilibria. Any level of inflation can be an equilibrium. Since most central banks have the stabilization of inflation as their main objective it is crucial to know how a unique equilibrium for inflation can be achieved. To obtain uniqueness of the equilibria, it is sufficient for the central bank to use its two instruments simultaneously. The central bank must choose interest rates and money supplies concurrently.

REFERENCES

Adão, Bernardino, Isabel Correia and Pedro Teles, 2003, "Gaps and Triangles", *Review of Economic Studies*, 70, p. 699-713.

Adão, Bernardino, Isabel Correia and Pedro Teles, 2004, "Instruments of Monetary Policy", mimeo, *Federal Reserve Bank of Chicago*.

Farmer, Roger, 1993, "The Macroeconomics of Self-Fulfilling Prophecies", *MIT Press*.

Friedman, Milton, 1968, "The Role of Monetary Policy", *American Economic Review*, 58, 1-17.

McCallum, Bennett, 1981, "Price Level Determinacy with Interest Rate Policy Rule and Rational Expectations", *Journal of Monetary Economics*, 8, 319-329.

Obstfeld, Maurice and Kenneth Rogoff, 1983, "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out", *Journal of Political Economy*, 91, 675-687.

Sargent, T. J. and Neil Wallace, 1975, ''Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule'', *Journal of Political Economy*, 83, p. 241-254.

Taylor, John B., 1993, ''Discretion Versus Policy Rules in Practice'', *Carnegie-Rochester Conference Series on Public Policy* 39, p. 195-214.

Woodford, Michael, 2003, "Interest and Prices", *Princeton University*.

APPENDIX

In the appendix we study local determinacy in the stochastic environment. The introduction of the concept of the time-invariant equilibrium is necessary to study local determinacy. In order to proceed an assumption is made, for each state s^t, the shocks (A_t, G_t) have an identical and independent distribution. The time-invariant equilibrium is a competitive equilibrium with the property that it is just a function of the shock. Formally, the time-invariant equilibrium is a tuple for consumption, leisure, interest rate, money growth and inflation, $\left\{\mathbb{C}(s_t), \mathbb{L}(s_t), \mathbb{R}(s_t), \frac{\mathbb{M}(s_{t+1})}{\mathbb{M}(s_t)}, \Pi\right\}$ that satisfies the relevant competitive equilibrium conditions. These conditions are given by (3), (9), (10) and (12),

$$\Pi = \frac{\mathbb{C}(s_t)}{\mathbb{C}(s_{t+1})}\frac{\mathbb{M}(s_{t+1})}{\mathbb{M}(s_t)},$$

$$\mathbb{C}(s_{t+1}) + G_t = A_t(1 - \mathbb{L}(s_t)),$$

$$\frac{u_C(s_t)}{u_L(s_t)} = \frac{\mathbb{R}(s_t)}{A_t},$$

$$u_C(s_t) = \frac{\beta}{\Pi}\mathbb{R}(s_t)E_t[u_C(s_{t+1})]. \tag{13}$$

For a given $\mathbb{R}(s_t)$ the two middle equations determine $\mathbb{C}(s_t)$ and $\mathbb{L}(s_t)$. Given Π the first equation determines the growth rate of money between a state and any of its subsequent states. Finally (13) determines $\mathbb{R}(s_t)$. To economize on notation we now assume without loss of generality that the utility function is separable and linear in consumption. In this case (13) can be written as

$$\mathbb{R} = \frac{\Pi}{\beta}$$

That is the time-invariant nominal interest rate does not depend on the shocks.

Suppose that the central bank conducts a pure current Taylor rule:

$$R_t = \mathbb{R}\left(\frac{\pi_t}{\Pi}\right)^{\tau\beta} \tag{14}$$

where $\tau\beta \geq 1$ (the Taylor principle), and $\pi_t = \frac{P_t}{P_{t-1}}$.

After substituting (14) in the households' intertemporal condition, (13), we get

$$E_t\left[{z_{t+1}}^{-1}\right] = \left({z_t}^{-1}\right)\tau\beta \tag{15}$$

where $z_t = \frac{\pi^t}{\Pi}$. By recursive substitution we get

$$\left\{E_t\left\{\left\{E_{t+1}\left[\ldots\left(E_{t+k-1}z_{t+k}^{-1}\right)^{\frac{1}{\tau\beta}}\ldots\right]\right\}^{\frac{1}{\tau\beta}}\right\}\right\}^{\frac{1}{\tau\beta}} = {z_t}^{-1}, \text{ for all } k, t. \tag{16}$$

In the following paragraph we supply an heuristic proof that the only equilibria are the time-invariant equilibrium and an infinity of other equilibria which have the characteristic that in some states of nature either inflation is going to infinity or is going to zero.

Since $\tau\beta > 1$ if ${z_t}^{-1} > 1$ then ${z_t}^{-1} \to \infty$ with positive probability. The proof is by contradiction. Assume it was not converging to infinity with positive probability, then it would be bounded with probability one, which means that no matter how arbitrary in the future you take the ${z_{t+s}}^{-1}$ its expected value would be bounded with probability one. But since the exponent is a constant smaller than one by taking s sufficiently large will get the left hand side of (16) smaller than the right hand side. By a similar argument if ${z_t}^{-1} < 1$, have ${z_t}^{-1} \to 0$ with positive probability.

Thus, when the central bank follows a Taylor rule that obeys the Taylor principle it is able to get local determinacy. In a neighbourhood of the time-invariant equilibrium inflation Π there is no other equilibrium. We have just seen that the other equilibria which are infinite in number are either associated with inflation converging with probability bounded from zero to infinity or to zero.