

UNCERTAINTY AND RISK ANALYSIS: AN APPLICATION TO THE PROJECTIONS FOR THE PORTUGUESE ECONOMY IN 2004*

*Paulo Soares Esteves***

*Álvaro A. Novo***

1. INTRODUCTION

Economic forecasts are generally presented as point projections. However, these central projections are subject to risks that can be translated into a probability distribution function whose estimation can supply significantly important indications. The existence of uncertainty as well as asymmetric risks is reflected in forecast intervals, which can result in distinct probabilities of the variable standing above/below that point projection.

In practice, while the uncertainty analysis is often quantified in the form of forecast intervals, the implication of the asymmetric risk, given their higher degree of technical complexity, are generally not quantified. Instead, only qualitative assessments of the direction of projection risks are produced⁽¹⁾.

With a pioneer approach, since 1996 the Bank of England has carried out quantified analyses of the risks underlying its forecasts. The communication to the public is made using fan charts, which quantify the (usually asymmetric) probability distribution underlying a point projection. These charts superimpose, with different shades of colour⁽²⁾, asymmetric projection intervals for different degrees of confidence⁽³⁾.

Novo and Pinheiro (2003) have recently proposed an alternative procedure to construct such probability distribution functions, trying to overcome some limitations inherent to the Bank of England procedure. This note summarises the approach by Novo and Pinheiro (2003), illustrating its application with an example for the Portuguese economy projections for 2004 published in this issue of the *Economic bulletin*.⁽⁴⁾

The second section introduces the problem. The method of definition of uncertainty factors and of risks balance is presented in the third section. The fourth section illustrates the aggregation of these uncertainty and risk factors, and presents the resulting probability distribution underlying the projected values. Finally, the fifth section applies the procedure to the projections for the Portuguese economy in 2004.

2. HOW CAN ASYMMETRIC AND CORRELATED DISTRIBUTIONS BE AGGREGATED?

In order to illustrate the problem a simple example will be considered, in which the variable to be forecast, y , can be expressed as a local linear approximation of k conditioning variables, x_i :

* The views expressed in this article are those of the authors and not necessarily those of the Banco de Portugal.

** Economic Research Department.

(1) For example, this has been the practice followed by the Banco de Portugal in the articles that publish the projections for the Portuguese economy.

(2) Also known in literature as «rivers of blood» due to their red colour.

(3) This approach was subsequently adopted by other central banks, in particular, the Bank of Sweden.

(4) The procedure is applicable to forecasts at more than a period ahead, although for illustrative purposes only 2004 is considered.

$$y_t = \sum_{i=1}^k \alpha_i x_{i,t} \tag{1}$$

The point projection of variable y , for moment t , \hat{y}_t , can be written as a linear combination of the contemporaneous values projected for variables x_i , $\hat{x}_{i,t}$:

$$\hat{y}_t = \sum_{i=1}^k \alpha_i \hat{x}_{i,t} \tag{2}$$

Similarly, the point projection errors for variable y , $\varepsilon_{y_t} = y_t - \hat{y}_t$, can be written as a linear combination of the projection errors for variables x_i , $\varepsilon_{x_{i,t}} = x_{i,t} - \hat{x}_{i,t}$:⁽⁵⁾

$$\varepsilon_{y_t} = \sum_{i=1}^k \alpha_i \varepsilon_{x_{i,t}} \tag{3}$$

The point projections assumed for variables x are subject to uncertainty, which naturally passes through to the projections made for variable y . Thus, the problem lies firstly in establishing distributions for the projection errors of variables x_i and secondly in aggregating these distributions (which may be correlated), to obtain the probability distribution underlying the projection error of variable y ⁽⁶⁾. In general terms, two cases can be considered.

A simpler case would be to consider that the forecast errors of variables x are symmetrically distributed⁽⁷⁾, assuming therefore balanced risks.

(5) This simpler example is intended to be illustrative, considering only one variable to be forecast – not depending on a random term – and that there is no uncertainty with respect to the aggregation coefficients. More generally, this will not happen, because the variable to be forecasted will also depend on a specific random disturbance, which will represent all the factors influencing the variable, but that are not expressed in the linear approximation considered (e.g. specification errors associated with the functional form and the variables included, as well as measurement errors of the variables).

In a more general analysis, let Y stand for a variable vector to be forecasted and X represent the conditioning variables vector. It can be admitted that there is a local linear approximation to the forecast generating process. In matricial terms:

$$A_y Y = A_x X_t + u_t \Leftrightarrow Y = A_y^{-1} A_x X_t + A_y^{-1} u_t \tag{1a}$$

$$\hat{Y}_t = \hat{A}_y^{-1} \hat{A}_x \hat{X}_t \tag{2a}$$

$$\hat{A}_y \varepsilon_{y_t} = \hat{A}_x \varepsilon_{x_t} + v_t \Leftrightarrow \varepsilon_{y_t} = \hat{A}_y^{-1} \hat{A}_x \varepsilon_{x_t} + \hat{A}_y^{-1} v_t \tag{3a}$$

The total error of each variable of vector Y is accounted for by the error of the conditioning variables ε_{x_t} and by the pure errors v_t that cannot be accounted for by errors made in the conditioning variables. It should be noted that (3a) expresses ε_{y_t} as a linear combination of ε_{x_t} and v_t .

In this case, the expected value and the mode of the projection errors of variables x coincide in zero. Consequently, the distribution for the forecast error of y is also characterised by a symmetrical distribution centred in zero. From the technical point of view, in this case the problem boils down to the estimation of a dispersion measure of the projection error of y .

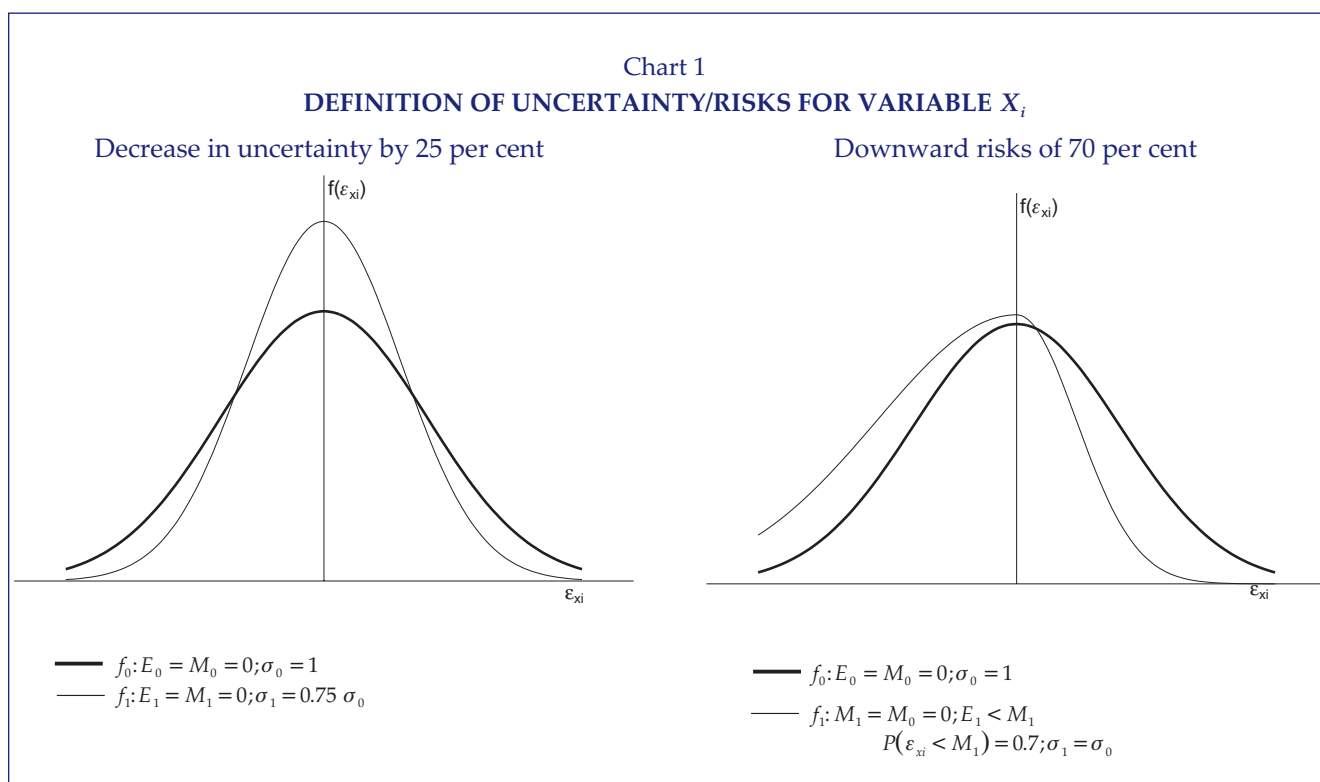
In a more realistic alternative, some projection errors for variables x_i may have asymmetric distributions. In this case, the problem is more complex and, in addition to the dispersion, it is also necessary to estimate the degree of asymmetry underlying the error in the forecast of y . Moreover, the distribution function of variable y is not normally the same as that of variables x . In such cases, different solutions to the problem can be considered:

- (i) numerically simulating uncertainty and risk factors (variables x_i) and aggregating them. The disadvantage of this procedure is related to the difficulty in considering the existence of correlations between the distributions of projection errors for variables x_i ⁽⁸⁾.
- (ii) assuming that the asymmetry of the projection is the linear combination of the asymmetries defined for the risk factors. This is the procedure used by the Bank of England to construct fan-charts⁽⁹⁾. In this procedure, the dispersion of the projection error of y is, in turn, obtained through the linear combination of the variances of projection errors for variables x_i , thereby assuming the inexistence of correlations between these errors.

(6) An hypothesis to overcome this problem would be the direct use of projection errors subsequently observed in the projection of variable y . This procedure is used to define the projection intervals currently published by the European Central Bank [see ECB (2000)]. However, this analysis («to make as many errors as in the past») does not consider the introduction of elements of uncertainty and risks distinct from the past, nor the analysis of their impact on projections.

(7) Conveniently, the most common parametric hypothesis is the normal distribution of errors.

(8) Martins et al (2003) present an application of a numerical procedure which only considers a binomial correlation between two input distributions. The use of the method shown in Mardia (1970) only considers correlations between two uncertainty factors – applied to inflation and GDP in Sweden by Blix and Sellin (2000).



(iii) Novo and Pinheiro (2003) suggest a more global procedure. This approach, under certain conditions, overcomes the restrictive hypotheses of the linear aggregation used by the Bank of England.

3. DISTRIBUTION OF UNCERTAINTY AND RISK FACTORS

Chart 1 illustrates the procedure used to define the level of uncertainty and risk for any conditioning variable. This procedure — which includes the judgment of the forecaster in the definition of the probability distribution around the values underlying the central scenario — is the same used by the Bank of England, and is also followed in Novo and Pinheiro (2003).

In Chart 1, the probability density function f_0 represents the initial distribution of the forecast error of variable x_i . It is a distribution centred around zero, reflecting the most probable value for

projection errors. However, as there is uncertainty, these errors are characterised by a probability distribution, which is approximated by a normal distribution due to the fact that the baseline was admittedly constructed without considering asymmetric risks. Thus, in this distribution the most probable value (M_0) coincides with the mean (E_0). In turn, the variance can be estimated taking into account the historical projection errors made for variable x_i (the example considers $\sigma_0 = 1$). However, the forecaster should evaluate whether it is reasonable to project this historical variability for the future, changing it if he deems appropriate. For example, for a euro area country, the variability of the effective exchange rate will tend to be lower than the volatility seen in the past, when there were significant changes in bilateral rates across the countries currently comprising the euro area.

In order to illustrate the type of intervention which can be carried out by the forecaster, let's consider a first case where uncertainty on developments in variable x_i admittedly decreases by 25 per cent. This type of intervention is justified when it is considered that the uncertainty associated with the projection of this variable will be different from that historically observed⁽¹⁰⁾. Thus, the definition of these additional uncertainty factors only

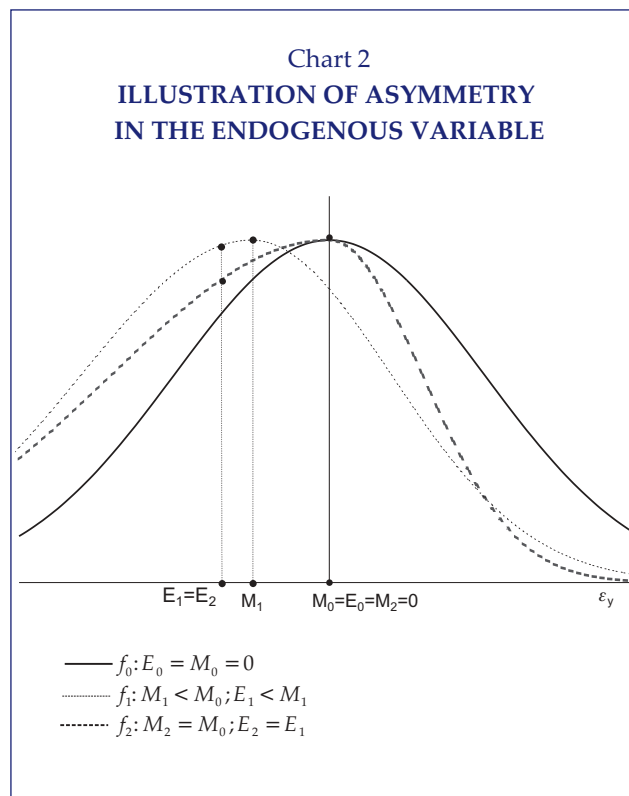
(9) This procedure considers that the difference between the expected value and the mode is the linear combination of the differences between the mean and the mode of risk factors. The problem in this hypothesis, as actually acknowledged by the Bank of England, results from the fact that the mode of a linear combination cannot be expressed as the linear combination of the respective modes (contrary to the expected value).

changes the dispersion of distribution, and does not affect location measures — both the most probable value (M_1) and the mean (E_1) for the forecast error of this variable continue to be equal to zero.

The second panel of Chart 1 considers the definition of an asymmetric risk factor with regards to developments in variable x_t . Although the most probable value of this error is still considered to be zero — i.e. the value of x_t implied in the baseline continues to be the most probable — it is assumed a different probability that this error may stand below or above zero (70 and 30%, respectively, in the example shown). With the definition of this asymmetry, the expected value (E_1) deviates from the mode (M_1), shifting to the tail of the distribution with higher probability mass. In this case, the parametric hypothesis of the normal distribution no longer serves the purposes of the forecaster. Thus, it is necessary to select distributions that accommodate both situations — symmetry and asymmetry, as is the case of the distributions used by the Bank of England and in Novo and Pinheiro (2003). In the former case the distribution chosen was the two-piece normal (*tpn*) and in the latter case the authors chose the skewed generalized normal (*sgn*)⁽¹¹⁾.

4. DISTRIBUTION OF THE PROJECTION

Upon defining the risk and uncertainty factors on developments in the conditioning variables, aggregating these distributions makes it possible to obtain the distribution underlying the projections of endogenous variables. Chart 2 illustrates the output resulting from this aggregation, showing



the impact of the adoption of risks (asymmetry). Its analysis can be broken down into three parts:

- (i) The initial distribution, f_0 , corresponds to the distribution underlying the initial point projection, defined in a context of inexistent asymmetric risks. In this case, the projection errors for variable y follow a normal distribution, with a mean zero and a specific standard deviation.
- (ii) The definition of asymmetric risks changes the distribution of forecast errors. The distribution f_1 illustrates the result of the procedure by Novo and Pinheiro (2003). The shifting of the mean (from E_0 to E_1) reflects the fact that the risk analysis has also changed the expected value of the variables affecting the projection of y . However, the introduction of asymmetric risks also changes the projection mode (from M_0 to M_1). This last change results from the fact that the procedure aggregates, in a statistically consistent way, the distributions of x_t , i.e. without simplifications in the aggregation of the mode. Thus, the effect of the introduction of asymmetric risks must consider the shifting of the distribution, which can be broken down into two parts: (i) shifting of the most probable

(10) In this context, a pure statistical fact can justify reducing the uncertainty associated with the variable in question. The historical variance of the projection errors of this variable may have been significantly affected by an irregular observation, in particular when small samples are used.

(11) The adoption of the *sgn* distribution (linear combination of a normal distribution with a gamma distribution) is justified by its better properties of linear aggregation when compared with the *tpn*. In terms of the definition of the elementary factors of uncertainty and risk, the results are quite similar to those of the Bank of England, given the resemblance of the two distributions. In the case of symmetry, the results are the same, because both distributions have as a particular case the normal distribution.

value; (ii) differentiation between the expected value and the mode due to the asymmetry of the new distribution. This difference between the mode and the mean will be smaller than the linear combination of the differences between the mean and mode of the variables on which risks are defined, in particular when there is an increase in the number of variables. This new distribution will be the basis for the definition of forecast intervals and for the calculations made regarding the probability that the variable in question may stand above/below the value projected in the central scenario.

- (iii) For illustrative purposes, the distribution resulting from applying the procedure used by the Bank of England (f_2) is also plotted. In this case, while the shifting of the expected value is equal to the method of Novo and Pinheiro (2003)⁽¹²⁾, the mode is artificially maintained in the initial projection.

Thus, with the approach by Novo and Pinheiro (2003), two location measures can be considered in order to measure the effects of the risk analysis in the central scenario: (i) the shifting of the mode, $M_1 - M_0$; (ii) the shifting of the mean, $E_1 - E_0$. It should be noted that the signal of the differences indicates the direction of risks. Thus, negative (positive) differences are associated with downward (upward) risks in the forecast.

The use of the mode has the advantage of measuring the change in the most probable value resulting from the analysis of risks. In fact, given the analysis of uncertainties and risks considered, the value M_1 can be interpreted as the most likely realisation for the variable in question. The use of the expected value is less consistent with the interpretation that the baseline corresponds to a modal forecast on which uncertainties and risks are defined. However, the use of this measure has a com-

munication advantage. Its shifting can be easily accounted for by the risks introduced in variables x_i , i.e. the effect on the mean of dependent variable can be broken down in an additive manner by the effects on the means of conditioning variables.

5. APPLICATION TO CURRENT PROJECTIONS FOR THE PORTUGUESE ECONOMY IN 2004

The use of the procedure by Novo and (2003) requires the adoption of a set of working hypotheses. Thus, for the 2004 Portuguese economic projections, which were prepared within the scope of the Eurosystem autumn exercise and disclosed in this issue of the Economic bulletin, the following options were used:

(i) Variables considered

The current Eurosystem projection exercise considers a set of variables on which risks and uncertainty factors can be defined. These variables can be broken down into three major groups.

- Conditioning variables on which technical hypotheses are made (includes the exchange rates of the euro, the oil price and long and short-term interest rates).
- Other conditioning variables on which common hypotheses are assumed for the Eurosystem countries (developments in the economic activity of countries outside the euro area, which, together with projections for the remaining Eurosystem countries, determine foreign demand relevant for the Portuguese economy), or whose nature as a policy variable naturally renders them conditioning variables of the exercise (public consumption).
- Endogenous variables on which uncertainty and risk factors can be defined through the behaviour of the residual of the respective equations (private consumption, investment, exports, imports, wages, employment, inflation)⁽¹³⁾. The effect on the behaviour of an endogenous variable will depend not only on the factors directly defined for this variable, but also on the impacts associated with the factors defined for both the conditioning

(12) In approximate terms, given the differences in parametric hypotheses, t_{pn} and sgn , used for defining risk factors. The expected values of these two distributions are different, even if they share the same mode, mode percentile and standard deviation. Thus, the shifting of the expected value of distribution resulting from the definition of the same specific risk factor (see figure 1) is not exactly the same, thereby giving rise to slight differences in the shifting of the expected value of endogenous variables.

variables and the remaining endogenous variables.

(ii) Historical errors

The collection of historical errors for both the conditioning variables and the projections made for endogenous variables is instrumental to estimate the standard deviations and corresponding correlation matrix among projection errors of the several variables considered. As already mentioned, this information is viewed as a working basis, which the forecaster can judge and assess whether it is reasonable to project this historical information into the projection horizon.

This historical information was constructed taking into consideration the autumn projection exercises prepared by the Banco de Portugal from 1995 to 2003, within the framework of the European Monetary Institute up to 1998 and of the Euro-system from that date onwards. In order to deal with the usual series revisions, the actual value for each variable in a given year was considered to correspond to the value projected in the autumn exercise of the following year.

(iii) Linear proxy

When defining the linear approximation around the central scenario that characterises relationships among the variables considered, the shock multipliers of both the conditioning variables and the endogenous variables, were obtained by means of innovations in the respective behavioural equations. In addition to permit the aggregation of the several variables, these multipliers are instrumental to filter the projection errors of each endogenous variable of the part that can be explained by errors made in the projection of the other variables considered.

These multipliers were obtained on the basis of the model normally used in the Banco de Portugal in the projection exercises for the Portuguese economy. With regard to conditioning variable shocks (with the exception of public consumption), it should be noted that the model multipliers were amplified by considering equivalent simulations

(13) GDP projection errors are determined by aggregating the errors of its expenditure components.

for the remaining euro area countries. Thus, these multipliers also take into account the effects on the Portuguese economy associated with the fact that these variables affect the remaining euro area countries with which Portugal maintains trade flows. For example, a change in the euro interest rates, in addition to having direct effects on the Portuguese economy, also generates indirect effects through foreign demand relevant for the Portuguese economy, given that euro area countries are also affected by this change in interest rates.

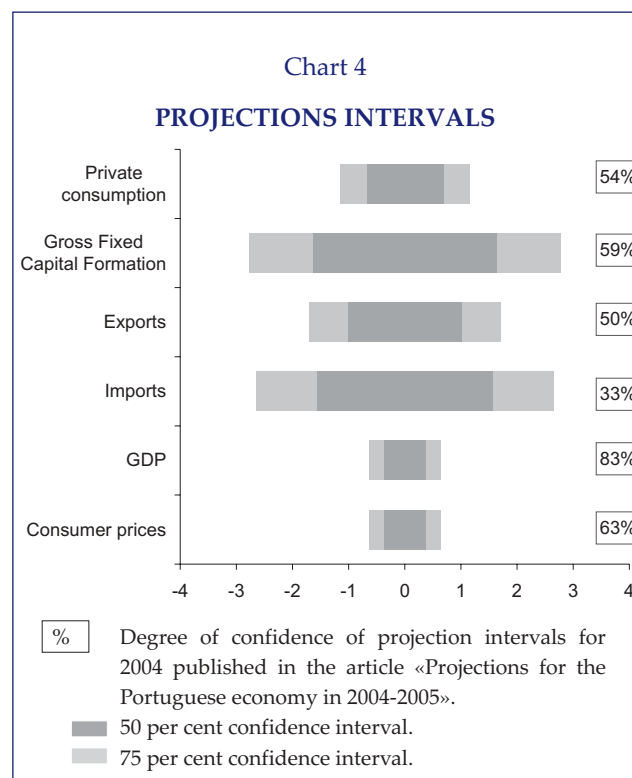
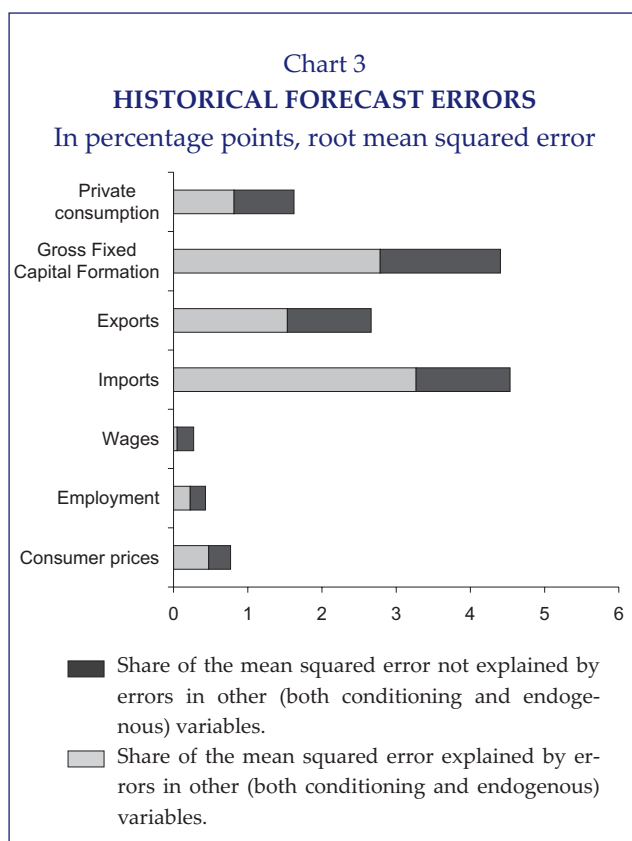
5.1. Filtering of errors

As a consequence of applying the procedure, it is possible to analyse the projection errors made in the past in similar projection exercises. In addition, the filtering of such errors makes it possible to recognise their origin. Chart 3 illustrates this analysis for projection errors one year away, by showing a numerical measure of these errors for each endogenous variable — Root Mean Squared Error () — as well as the percentage of this indicator which is not attributable to «pure» projection errors of this variable⁽¹⁴⁾.

Not surprisingly, projection errors are larger for investment (strong sensitivity to the business cycle), exports (higher dependence on the hypotheses assumed for developments in foreign demand) and imports (strongly conditioned by the projection errors made when projecting the various global demand components). In these cases, a significant percentage of the error will tend to be accounted for by the errors made in the remaining variables considered.

Projection errors have been less significant for the remaining variables — wages, employment and inflation. This, however, cannot be directly attributed to a greater skill in the projection of these variables, since this result will reflect the fact that these series have recorded less variability in the period under consideration⁽¹⁵⁾. Among the variables considered, the projection errors made in the autumn exercises for wage developments in the

(14) Notice that the variables used in the linear proxy model around the baseline do not necessarily coincide with those used by the Banco de Portugal in its forecast models. Thus, the percentages reported in this article must only be interpreted in the light of the model used for risk analysis.



following year are the smallest. Simultaneously, however, they tend to be the least explained by projection errors made in the remaining variables considered — showing the low flexibility of wages with regard to the events of the year.

5.2. Uncertainty analysis

Regardless of the set of risks that may be considered, applying the procedure by Novo and Pinheiro (2003) always makes it possible to construct confidence intervals for economic projections. If the sole purpose is to obtain a measure of the degree of imprecision of projections, there is no need to define asymmetric risks. The estimated dispersion of projection errors is based on the historical variances and covariances and on the set of model multipliers.

Chart 4 summarises the results for 2004⁽¹⁶⁾. Considering confidence degrees of 50 and 75 per cent, confidence intervals are shown for the projection

(15) When obtaining projection skill indicators, the comparison of projection errors with the variability of the series itself gains importance. Errors made in the projection of more unpredictable series are more tolerated [see for example, Diebold and Killian (2001)].

errors of variables for which projection intervals are usually disclosed. In addition, the Chart shows the degree of confidence that this procedure assigns to the projection intervals for 2004 considered in the article of this issue of the Economic bulletin that presents projections for the Portuguese economy.

According to the results obtained, considering all uncertainty associated with both the adoption of external hypotheses resulting from the Eurosystem's exercise and specific behaviours of the endogenous variables considered, GDP growth in

(16) Several hypotheses have been considered in the construction of these confidence intervals. First, translating a usual hypothesis, it was assumed that projection errors of conditioning variables were not correlated with the residuals of the behavioural equations («pure» errors) of endogenous variables. Second, the historical standard deviations of projection errors were calculated excluding the more «abnormal» observation, since the presence of outliers is particularly prone to influence the results when small samples are taken into consideration. Finally, the adoption of non-truncated distributions gives rise to a different-from-zero probability of the projected variable shifting to very negative or very positive values, despite its low economic reasonability. In the case of the Bank of England's fan charts, the original forecast distribution is truncated at the limits defined by the scale of the chart itself. In this case, a 98 per cent truncation was considered, i.e. a distribution defined only for the central «most likely» 98 per cent values.

2004 is likely to stand, with a 50% degree of confidence, within a range of 0.8 percentage points defined around the baseline (or 1.2 in the case of a 75% confidence interval). The limits of these intervals for the inflation rate are also 0.8 and 1.2 percentage points, respectively.

Turning to developments in the various expenditure components, there is a significantly higher uncertainty than that underlying the GDP projection. There are two possible explanations for this result. First, in the absence of correlations, the variability of a more aggregated indicator (which can be expressed as an average of other indicators) can be attenuated (being lower than the average indicator variability), since positive innovations in a variable can be offset by negative innovations in another variable. In addition, given the strong correlation between imports and global demand, part of the innovations in the various demand components is reflected in import developments, not affecting output growth.

5.3. Risk scenarios

Applying this procedure also makes it possible to quantify the impacts of a given balance of risks in the projections made. As an example, let's consider of a balance that translates a possible quantification of the risk factors referred to in the article that presents the projections for the Portuguese economy: (i) appreciation of the exchange rate of the euro; (ii) higher oil price; (iii) lower foreign demand growth; (iv) higher public consumption growth; (v) higher labour market adjustment — translated into a lower growth pace of employment and wages; (vi) higher consumer price growth due to the possibility of a greater increase in administered prices. In all cases an intermediate level of risks is considered (0.4 or 0.6 mode percentile). In the case of the exchange rate, given the significant appreciation of the euro since the Eurosystem's exercise, a sharper risk balance is assumed (70% subjective probability that the exchange rate of the euro in 2004 may appreciate vis-à-vis the technical hypothesis considered in the central scenario).

Table 1 presents the balance of risks for the projections for the Portuguese economy in 2004, showing the probability that the GDP growth rate and the inflation rate may stand below the central

projection, as well as the respective minimum magnitude confidence intervals⁽¹⁷⁾.

The results confirm that the risks regarding economic activity in 2004 are mainly on the downside — a higher than 55 per cent probability that the GDP growth rate may stand below the initial point projection. Inflation risks seem to be more balanced. The probability that the inflation rate may stand above the below projection stands only slightly above 50 per cent.

One important aspect is the fact that confidence intervals cease to be centred around the initial projection, with the limit of the interval widening towards the branch with higher concentration of risks. Considering this balance of risks defined around a point projection of 0.75 per cent (central value of the interval disclosed in the article that presents the projections for the Portuguese economy), GDP growth rate intervals in 2004 would be [0.28; 1.03] with 50 per cent confidence and [0.02; 1.29] with 75 per cent confidence. In the case of the inflation rate, these intervals defined around the central scenario of 2.5 per cent would be [2.19; 2.86] and [1.89; 3.13].

Considering the effects of the risk balance on the expected projection value (mean effect), it is possible to analyse which risk factors can contribute the most to a deviation of the GDP growth rate and the inflation rate vis-à-vis the central scenario in 2004 (Chart 5).

Most risk factors considered, in particular those related to developments in foreign demand relevant for the Portuguese economy, can negatively influence the GDP growth rate in 2004. These factors will tend to override the risk that public consumption will not record such negative developments as those assumed in the central scenario of the projection.

(17) In the presence of asymmetric distributions the choices are (i) confidence intervals in which the tails of the distribution not considered by the interval have the same probability mass or (ii) intervals whose limits are defined for the points whose density function reaches the same value so as to minimise the magnitude of the interval — option underlying the fan chart of the Bank of England [see Wallis (1999)]. No technical absolute arguments seem to support the adoption of one of the solutions. Thus, the criterion of showing minimum magnitude intervals may prevail. However, it should be mentioned that the probability that this variable may stand above or below this interval is not the same.

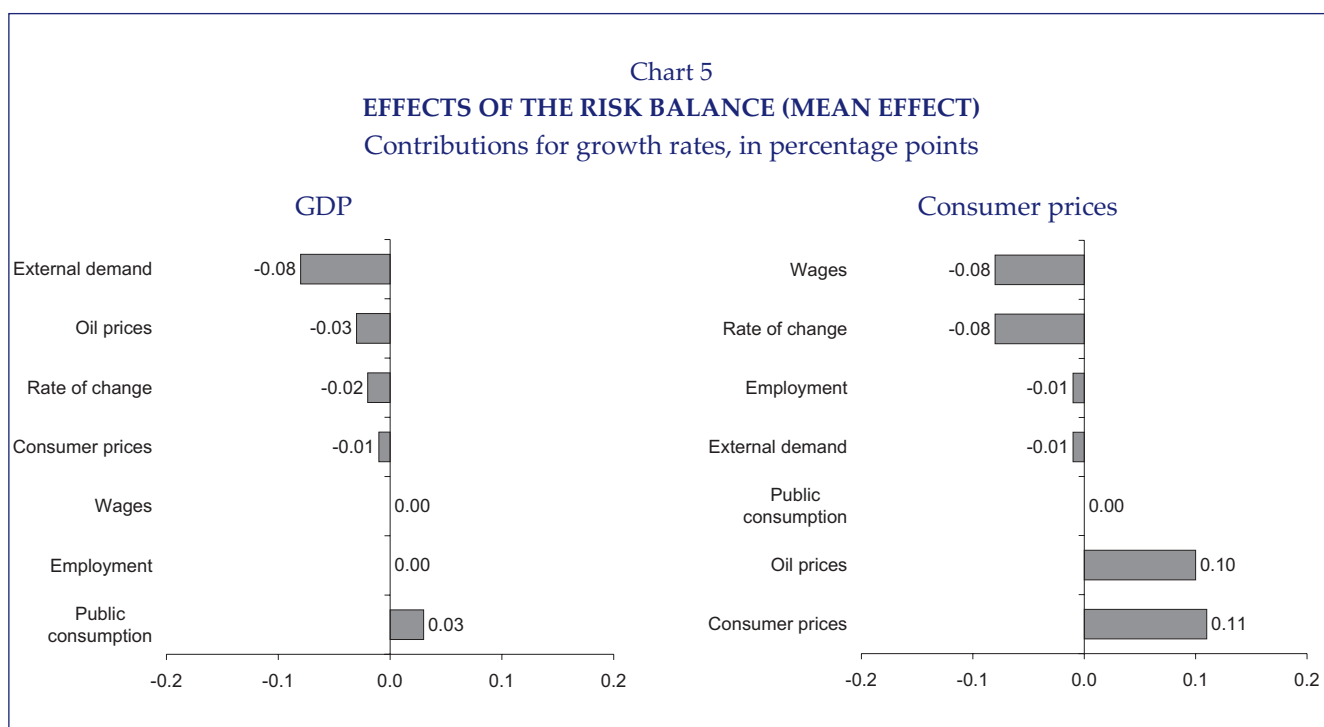
Table1
RISK ANALYSIS

<i>Input</i>		Subjective probability (mode percentile)
<i>Conditioning variables</i>		
Exchange rate ^(a)		0.7
Oil price		0.4
Foreign demand		0.6
Public consumption		0.4
<i>“Pure errors”</i>		
Employment		0.6
Wages		0.6
Inflation		0.4
<i>Output</i>		
GDP	Central scenario percentile	0.57
	Confidence interval (50%) ^(b)	[0.28; 1.03]
	Confidence interval (75%) ^(b)	[0.02; 1.29]
Inflation	Central scenario percentile	0.49
	Confidence interval (50%) ^(b)	[2.19; 2.86]
	Confidence interval (75%) ^(b)	[1.89; 3.13]

Notes

(a) An increase (decrease) corresponds to a depreciation (appreciation).

(b) Defined on truncated distributions at 98 per cent.



With regards to inflation, the risks associated with the possibility of an oil price level above that assumed in the point projection or of a higher increase in administered prices, will tend to be offset by the risks of an appreciation of the euro and lower wage growth.

REFERENCES

- Britton, E., P. Fisher and J. Whitley (1998), The inflation report projections: understanding the fan chart, Bank of England, *Quarterly Bulletin*, February.
- Blix, M. and P. Sellin (2000), A bivariate distribution for inflation and output forecasts, Riksbanc, *Working Paper*.
- Diebold, F. and Lutz Killian (2001): Measuring predictability: theory and macroeconomic applications, *Journal of Applied Econometrics* 16, 657-669.
- ECB (2000), European Central Bank, *Monthly Bulletin*, December.
- Mardia, K. (1970), Families of bivariate Distributions, Griffin, London.
- Martins, F., J. Machado and P. Esteves (2003), Modelling Taylor rule uncertainty: an application to the euro area, *Economic Modelling*, forthcoming.
- Novo, A. and M. Pinheiro (2003), Uncertainty and risk analysis of conditional macroeconomic forecasts: Fan charts revisited, Banco de Portugal, *Working Paper*, December.
- Wallis, K. (1999), Asymmetric forecasts of inflation and the Bank of England's fan-chart, *National Institute of Economic Review*, January.