

## MOVEMENTS IN OFFICIAL INTEREST RATES: PERSISTENCE AND GRADUALISM\*

*Fernando Martins\*\**

## 1. INTRODUCTION

There is a common belief among economists that several central banks have conducted their monetary policy in a gradual manner. This belief is usually supported by the evidence that official interest rate in major central banks, in general, have been adjusted in small movements with infrequent reversals, thus giving rise to a path characterised by a high degree of persistence (Charts 1, 2, 3 and 4).<sup>(1)</sup>

In some literature, this common pattern observed in the behaviour of major interest rates is pointed out as a sign of the preference of central banks for a gradualist monetary policy, also known as “interest-rate smoothing”. According to this perspective, central banks revealed some reluctance in adjusting interest rates more aggressively, rather doing it gradually towards a new optimal level. From an empirical point of view, this behaviour is traditionally incorporated in the models, either by directly introducing a stabiliser of the interest-rate variance in the central bank’s objective function or, alternatively, through a partial adjustment mechanism in which the interest rate is gradually adjusted towards an optimal interest rate defined by a monetary policy rule. These approaches, however, seem to be chiefly motivated by the need to justify the actual persistence of interest rates, rather than by any theoretic

cal considerations justifying their use. Therefore, they do not permit a distinction between the actual persistence of interest rates, as a result of an explicit preference of the authorities for a gradualist monetary policy, and the “natural persistence” due to the actual persistence of the economic variables to which the monetary authority responds. Thus, as explained in the present article, monetary policy can only be characterised as more or less gradualist, when compared with an optimal policy rule.

The results obtained in some literature [see Goodhart (1999) and Sack (1998a)] have shown that actual monetary policy has been characterised by a degree of gradualism that cannot be strictly explained by the dynamic structure of the economy. In other words, the optimal path for the in-

\* The views expressed in this article are those of the author and not necessarily those of the Banco de Portugal.

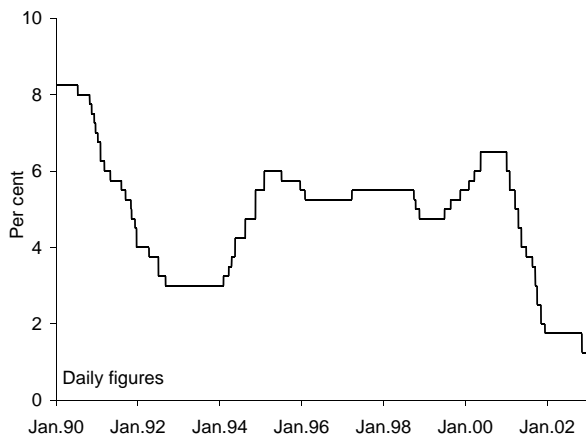
The author is indebted to Carlos Robalo Marques, Isabel Gameiro, Isabel Horta Correia, José Ferreira Machado, Marta Abreu, Maximiano Pinheiro, Nuno Alves e Pedro Duarte Neves for their comments in previous versions of this article.

\*\* Economic Research Department.

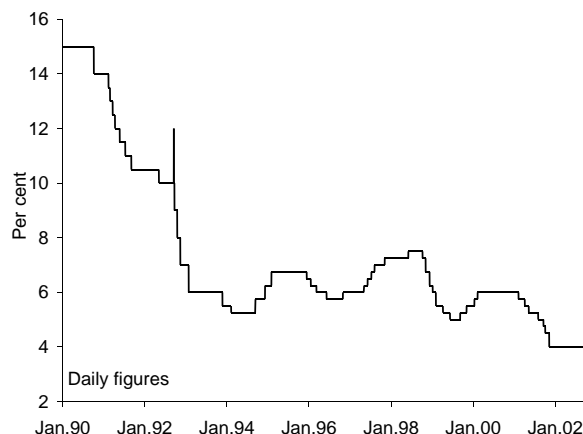
(1) The behaviour of the US Federal Reserve in 2001 stands somewhat in contrast with this approach, since it revealed a historically unprecedented activity level. Indeed, in the course of 2001, the Federal Reserve lowered the target for the Federal funds rate by an accumulated 4.75 percentage points to 1.75 per cent. This decline was achieved through eleven movements. In November 2002, the Federal Reserve lowered it by an additional 50 basis points. By way of example, and assuming that the Federal Reserve will not introduce any additional cuts in the target for the Federal funds rate, it should be mentioned that the previous large interest rate downward cycle in the United States had been significantly longer, spreading from June 1989 to September 1992. Over this period, the Federal Reserve lowered the target for the Federal Funds rates by 6.75 percentage points to 3.0 per cent, through 22 movements, 14 of which of 25 basis points.

(2) The very notion of gradualism depends on the monetary policy instrument considered. For example, an aggressive interest rate rule is consistent with a more gradualist rule for monetary growth and vice-versa, although most works focus the analysis on interest rate rules.

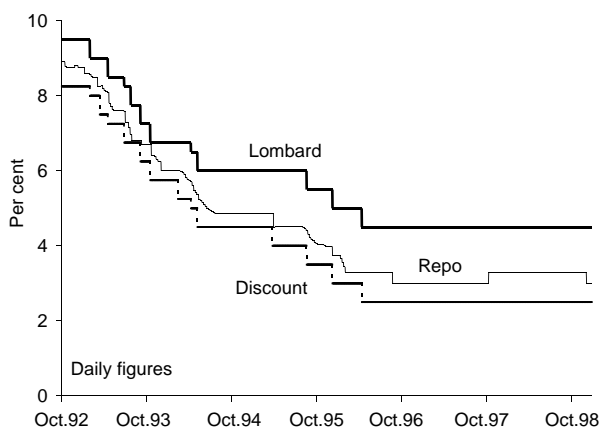
**Chart 1**  
**FEDERAL RESERVE: FED FUNDS TARGETS**  
 January 1990 to December 2002



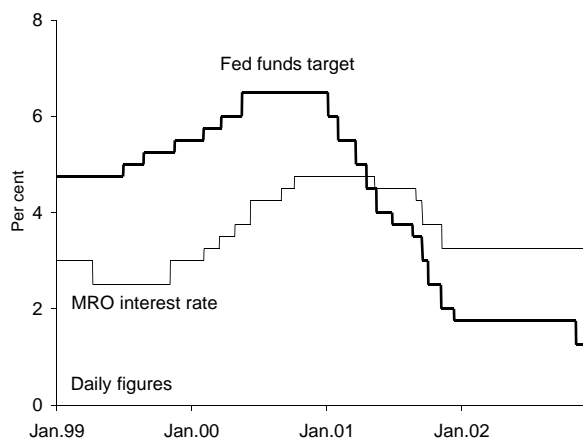
**Chart 2**  
**BANK OF ENGLAND: REPO RATE**  
 January 1990 to December 2002



**Chart 3**  
**BUNDESBANK: REPO, LOMBARD**  
**AND DISCOUNT RATES**  
 October 1992 to December 1998



**Chart 4**  
**FEDERAL RESERVE AND ECB: OFFICIAL**  
**INTEREST RATES**



terest rates emerging from different macroeconomic models is usually less gradual than that actually observed.<sup>(2)</sup>

A possible explanation for the divergence between the indications given by the models and the actual behaviour is that the former may not adequately deal with the uncertainty surrounding monetary policy decisions.<sup>(3)</sup> In fact, monetary authorities face a large number of uncertainties, including the uncertainty about the state of the econ-

omy, uncertainty as to the magnitude of the parameters characterising the transmission mechanism of monetary policy and uncertainty about the model describing the behaviour of the econ-

(3) This article does not discuss to what extent the decision-making process by a collegiate body, such as the EBC Governing Council or the Federal Open Market Committee, and the typically adopted decision rule (simple majority, qualified majority, consensus, etc..) may influence the degree of gradualism of monetary policy.

Table 1

FEDERAL RESERVE, BUNDESBANK, BANK OF ENGLAND AND ECB: MEASURES OF THE DEGREE  
OF PERSISTENCE OBSERVED IN OFFICIAL INTEREST RATES

Based on data available up to 31 December 2002

Official interest rates	Federal Reserve (Fed Funds Target)		Bank of England (Repo)		Bundesbank (Discount)	ECB (MRO interest rate)
	Jan92- Dec98	Jan99- Dec02	Jan92- Dec98	Jan99- Dec02	Jan92- Dec98	Jan99- Dec02
Number of movements . . . . .	17	18	24	15	15	13
Upward movements . . . . .	8	6	10	4	1	7
Downward movements . . . . .	9	12	14	11	14	6
Size of the movements (basis points)						
100 bps . . . . .	0	0	4	0	0	0
75 bps . . . . .	1	0	0	0	1	0
50 bps . . . . .	4	10	7	2	11	6
25 bps . . . . .	12	8	13	13	3	7
Size of the rises						
100 bps . . . . .	0	0	0	0	0	0
75 bps . . . . .	1	0	0	0	1	0
50 bps . . . . .	3	1	3	0	0	2
25 bps . . . . .	4	5	7	4	0	5
Size of the declines						
100 bps . . . . .	0	0	4	0	0	0
75 bps . . . . .	0	0	0	0	0	0
50 bps . . . . .	1	9	4	2	11	4
25 bps . . . . .	8	3	6	9	3	2
Trend reversals . . . . .	4	2	4	2	1	2
Average of days between consecutive movements . . .	148	81	111	71	105	113
Maximum number of days without movements . . . .	553	330	314	364	322	392
Minimum number of days without movements . . . .	15	15	24	16	28	18
Average of days between each reversal and the previous movement . . . . .	411	229	201	227	198	214

omy. In this perspective, the present article analyses the major results obtained in literature as regards the effects of the different types of uncertainty on the degree of gradualism in the conduct of monetary policy.

The article is structured as follows. Section 2 describes a set of characteristics common to the behaviour of the major official interest rates, based on a number of summary statistics normally used to support the notion that monetary policy is conducted in a gradualist fashion. Section 3 explains why the information content of these statistics should be interpreted with caution and presents a definition of gradualist monetary policy and a possible approach to its identification. Section 4

describes the different types of uncertainty faced by monetary authorities, with particular emphasis on the so-called parameter uncertainty, and shows how the “optimal” level of monetary policy gradualism is sensitive to the type of uncertainty specified. Section 5 presents some conclusions.

## 2. COMMON CHARACTERISTICS IN THE BEHAVIOUR OF MAJOR OFFICIAL INTEREST RATES

Notwithstanding the large differences in the pattern of the interest rate changes introduced by the different monetary authorities, there are still important behaviour similarities that ultimately

contribute to the existence of persistence of official interest rates:

1. *Infrequent and minor interest rate changes* — interest rate changes are relatively infrequent, chiefly taking into account the pace of dissemination of relevant information on economic developments. Information that is likely to lead, at least marginally, to changes in inflation and/or growth forecasts is made available on a daily basis. However, most central banks do not change their interest rates more than once a month, and several months may go by without any change. In addition, when there are changes, these, in general, do not exceed 25 basis points. Table 1 shows that, between 1992 and 1998, the average number of days between consecutive movements by the said authorities stood between 105 and 148 days. Note that the magnitude of the movements, in most situations, did not exceed 50 basis points (in the case of the Federal Reserve and the Bank of England, the 25 basis point movements were more frequent, while in the case of the Bundesbank 50 basis point movements were predominant).

2. *Unusual reversals in the interest rate path* — reversals in the interest rate trend are frequently preceded by a sequence of movements in the same direction. In practice, this means that there is a strong persistence in official interest rates. In the case of the Federal Reserve, for instance, between 1992 and 2002, only six of the 35 changes announced in the target for the Federal funds rate corresponded to a trend reversal.

3. *Maintenance of the interest rates for a relatively long period before any reversal in the respective trend* — evidence shows that the time gap between movements in the same direction is rather shorter than the time gap between movements in opposite directions. Furthermore, as the period of time from the last interest change increases, there is a higher probability that the forthcoming interest rate change may reflect a trend reversal.<sup>(4)</sup> Table 1 shows that over the last ten years, for the monetary authorities in question, the average length of the periods preceding interest-rate trend reversals was two to three times longer than the average length of overall movements.

### 3. PERSISTENCE OF THE INTEREST RATES AND MONETARY POLICY GRADUALISM

In some literature, the common pattern of behaviour of major above-mentioned interest rates is considered to be evidence of the adoption of a gradualist policy (“interest-rate smoothing”) that is ultimately responsible for the formation of the so-called “interest-rate cycles”.<sup>(5)</sup> Some economists, such as Goodhart (1997), claim that interest-rate cycles contribute (and do not counter, as would be desirable) to the formation of economic cycles. According to this perspective, if the interest rates would evince a less cyclical behaviour, and if central banks were willing to change their interest rates more frequently and more widely, the length of the economic cycles could be reduced. Expressions like “too little and too late” or “to be behind the curve” are often used to criticise the perceptible reluctance of some monetary authorities to change interest rates more aggressively.

From an empirical point of view, the preference of monetary authorities for a gradualist policy is usually incorporated in the models either by directly introducing in the central bank’s objective function a term that would allow the reduction of the interest rate variance [see, for instance, Söderlind (2001) or Rudebusch and Svensson (1999)] or, alternatively, through a partial adjustment mechanism, in which the central bank gradually changes the official interest rate towards an optimal interest rate defined by a monetary policy rule [see Clarida *et al* (1997) or Batini and Haldane (1999)].<sup>(6)</sup> However, any of the above approaches seems to be chiefly due to the need to justify the actual persistence of interest rates, and not to

(4) A Probit model estimated for the United States between January 1990 and December 2001 — a period that witnessed 49 movements in the Federal funds rates, 9 of which corresponded to trend reversals, showed the following equation for the probability of reversal of the interest rate trend:

$$P(I = 1) = \Phi(-4.546 + 0.024D)$$

where  $I$  assumes the value 1 when the interest rate movement corresponds to a reversal, and 0 in the opposite case.  $\Phi(\cdot)$  represents the distribution function of a normal standard distribution and  $D$  represents the period of time elapsed (in days) between consecutive interest rate changes. The  $t$ -ratio for  $D$  coefficient is 2.31.

(5) See, for instance, Lowe and Ellis (1997).

considerations underpinning their use [see Woodford (1999)].

There are several reasons behind the preference of the authorities for a gradualist monetary policy.<sup>(7)</sup> Nonetheless, the incorporation in the objective function of an interest rate stabiliser does not permit a distinction between actual persistence of interest rates as a result of an explicit preference of the authorities for a gradualist monetary policy and the “natural persistence” due to the inertia observed in the economic variables to which the monetary authority responds. In other words, in the formulation of monetary policy, monetary authorities take into account the dynamics of the so-called fundamental variables of the economy, such as output and inflation. Given that shocks to these variables exhibit strong serial correlation and respond to monetary policy with a substantial lag, some persistence in official rates may be expected even in the absence of an interest-rate smoothing objective. In this perspective, a gradu-

alist monetary policy should be understood as the tendency to limit changes in the official interest rates to a degree greater than the one accounted for the dynamic structure of the economy. Thus, a monetary policy can only be characterised as more or less gradualist when compared with an optimal policy rule.

In order to distinguish between persistence induced by monetary policy and persistence resulting from the dynamic behaviour of non-policy variables, it is necessary, first, to characterise the structural form of the economy, which can be achieved through the estimation of a VAR model formulated as follows:

$$W_t = \sum_{j=0}^q A_j W_{t-j} + \sum_{j=0}^q b_j i_{t-j} + v_t^W$$

$$i_t = \sum_{j=0}^q c_j W_{t-j} + \sum_{j=0}^q d_j i_{t-j} + v_t^i$$

where  $W_t$  is a  $n \times 1$  vector of non-policy variables (i.e. variables not directly controlled by monetary policy) that may include the inflation rate, the GDP growth rate, the unemployment rate or an index of commodity prices,  $i_t$  is the interest rate and  $q$  is the number of VAR lags. The VAR describes both the structure of the non-policy variables in the economy, and the reaction function of the monetary authority. However, in deriving optimal monetary policy, the reaction function of the monetary authority is completely ignored. Thus, after calculating the structural form of the economy estimated from the VAR, it is possible to determine an optimal interest rate that minimises the present discounted value of the sum of the output gap,  $x_t$  and the deviation of the inflation rate ( $\pi_t$ ) from its target ( $\pi^*$ ). A possible objective function to be minimised could be formulated as follows:

$$\frac{1}{2} E_t \left\{ \sum_{i=t}^{\infty} \beta^i \left[ (\pi_{t+i} - \pi^*)^2 + \lambda x_t^2 \right] \right\}$$

where  $\beta$  represents the discount factor ( $0 < \beta < 1$ ) and  $\lambda$  the relative weight assigned to the stabilisation of the output gap. This formulation is relatively conventional in literature [see Woodford (1999)].<sup>(8)</sup> It should be noted, however, that it does not show any term minimising the interest rate variance, given that the purpose is to investigate whether the actual persistence of interest rates can

(6) This latter approach may be represented in formal terms by the following equation:

$$i_t = (1 - \rho) i_t^* + \rho i_{t-1},$$

where  $i_t$  defines an interest rate directly or indirectly controlled by the monetary authority and  $i_t^*$  is an optimal interest rate defined, for instance, on the basis of a Taylor rule. Usually, the empirical results present high values for the parameter  $\rho$  that measures the degree of persistence of the interest rate [see Sack (1998b)].

(7) A number of justifications have been pointed out in literature for the preference of monetary authorities for a gradualist monetary policy. A traditional explanation consists in the willingness of central banks to reduce volatility in the financial markets, a situation that would be facilitated if interest rate changes would be gradual and foreseeable. Some authors sustain that if central banks would frequently reverse their interest rate trend, this might be interpreted as a signal of imperfect knowledge or even of incompetence, which would threaten their credibility. Goodhart (1997) sustains that the major cause behind the preference for a gradualist monetary policy lies in the natural conservatism of monetary authorities, which tend to change official interest rates only when there is solid evidence supporting the decision. Since that evidence emerges only slowly, interest rates are adjusted also slowly. More recently an explanation has been advanced, involving the interaction between a systematic monetary policy and the existence of agents with forward looking expectations (as opposed to expectations based only on the extrapolation of past behaviour). According to this perspective, a gradualist and systematic monetary policy makes possible that small changes in short-term interest rates have a stronger impact on longer-term interest rates and, therefore, on the economy [these arguments can be found, for instance, in Goodfriend (1991), Sack and Wieland (1999) and Amato and Laubach (1999)].



be explained without simply assuming that the authorities prefer to conduct monetary policy in a gradual manner. The parameters of the interest rate resulting from a problem of dynamic programming are  $\pi^*$ ,  $\beta$  and  $\lambda$ , which are exogenously defined and are a function of contemporaneous and lagged values of the model variables that, in addition to the inflation rate, the interest rate and the output gap, could also include other variables:

$$i_t^* = g(x_t, x_{t-1}, \dots, x_{t-k}, \pi_t, \pi_{t-1}, \dots, \pi_{t-k}, i_{t-1}, i_{t-2}, \dots, i_{t-k} | \pi^*, \lambda, \beta)$$

where  $k$  is the result of the lag structure determined by the VAR model. Note that the optimal interest rate  $i_t^*$  depends on the lagged values of the interest rate determined by the VAR model itself. Therefore, even in the absence of any explicit smoothing objective by monetary authorities, the optimal interest rate reveals a natural persistence. Accordingly, since at least one part of the actual persistence of interest rates may result from the dynamic structure of the economy, the mere analysis of the statistics presented in Table 1 does not allow for any conclusion to be drawn in terms of the degree of gradualism of monetary policy. These statistics would be informative if, in the absence of the stabilising term in the objective function, the series of changes in official interest rates had a random-walk type behaviour.

Thus, the evaluation of the degree of gradualism of monetary policy requires a comparison between changes in the actual interest rate ( $\Delta i_t$ ) and changes in the optimal interest rate ( $\Delta i_t^*$ ). As suggested by Sack (1998a) a possible measure could be the comparison of volatility between these two series, by calculating the ratio between their respective variances:

$$\frac{\text{Var}(\Delta i_t^*)}{\text{Var}(\Delta i_t)}$$

The results obtained in some studies [Goodhart (1999) and Sack (1998a)] have shown that actual monetary policy in different countries has been characterised by a degree of gradualism that can-

not be strictly explained by the dynamics of the economic variables to which monetary authorities respond. In other words, the optimal path for interest rates emerging from different macroeconomic models is usually less persistent than the actual path [see, for instance, Goodhart (1999)]. The crucial issue is to ascertain whether this situation implies that monetary authorities have adopted sub-optimal monetary policy strategies or whether, on the contrary, there are factors not included in the models that justify the strategies followed so far.

#### 4. UNCERTAINTY AND DEGREE OF ACTIVISM OF MONETARY POLICY

A possible explanation for the divergence between the indications given by the models and actual practice is that the former may not deal adequately with the uncertainty surrounding monetary policy decisions. The traditional approach for the analysis of monetary policy under uncertainty consists in the specification of an objective function for monetary authorities and of a macroeconomic model, so as to determine how monetary policy may respond to shocks in the economy. The manner in which uncertainty affects monetary policy ultimately depends on the specification of the model and on the type of uncertainty considered.

##### 4.1. Optimal monetary policy under additive uncertainty

Many academic studies assume that the authorities take their monetary policy decisions as if they were under certainty. This results in admitting that uncertainty faced by policy-makers assumes a particular form. Specifically, uncertainty is introduced in the analysis through additive (mean zero) shocks on the objective variables of monetary authorities. This, given the quadratic-linear specification of these models (thus known because the objective function is quadratic and the restriction is linear), does not affect the optimisation problem. This situation corresponds to what in literature is known as “certainty equivalence”, i.e. the optimal rule obtained under uncertainty is the same as if the situation of the economy were perfectly observed (complete information).<sup>(8)</sup>

(8) When compared with other objective functions used in this type of literature [for instance, Woodford (1999)], a salient difference is the absence of future values for inflation and for the output gap. As demonstrated by Woodford (op cit), these future values alone may create an incentive for some interest-rate smoothing.

In this framework, the specified models recognize that uncertainty is independent from the behaviour of monetary authorities. Therefore, the sole uncertainty considered is that arising when the economy variables are lagged from the forecast path — the so-called additive uncertainty. Against this background, authorities should ignore the effects of uncertainty on the economy.

This situation may be illustrated by means of a simple theoretical model for a closed economy, similar to that presented by Svensson (1996). It allows for the consequences of the parameter uncertainty to be discussed in order to determine optimal monetary policy, in a context in which an authority adjusts the interest rate so as to obtain a target for the inflation rate.<sup>(10)</sup> The model, however, due to its simplicity, necessarily has some weaknesses. In particular, it has not microeconomic foundation and, since it presents the equations in their reduced form, it does not permit the identification of the source of shocks hitting the economy. Literature presents alternative models, with microeconomic foundations, but they are less tractable to illustrate the problem under discussion [see, for instance, Khan *et al* (2000)].

The basis of the model is a two-equation system. The first one (a version of the Phillips curve) establishes a relationship between the inflation rate  $\pi_t$  and the output gap  $x_t$ :

$$\pi_{t+1} = a\pi_t + \gamma x_{t+1} \quad (1)$$

The second equation (a version of the IS curve) establishes an inverse relationship between the output gap and real interest rate prevailing in the previous period ( $r_t$ ) defined in terms of a deviation from the neutral or equilibrium level, subject to additive shocks,  $\xi_{t+1}$ , independently distributed with mean zero and variance  $\sigma_\xi^2$ :

$$x_{t+1} = -\delta r_t + \xi_{t+1} \quad (2)$$

(9) Is this case, the so-called “separation principle” is applicable, according to which the problem of selection of the optimal policy (optimisation problem) and the problem of estimation of the current state of the economy (problem of extraction the signal) may be dealt with separately [see, for instance, Svensson and Woodford (2002)].

(10) See Martin (1999) for an application of the same type of model within the framework of an open economy.

The interest rate set at the end of the  $t$  to be in force until  $t+1$  is given by the Fisher equation:

$$r_t = i_t - E_t \pi_{t+1} \quad (3)$$

where  $i_t$  is the nominal interest rate defined in terms of the deviation vis-à-vis the neutral or equilibrium value. By replacing (2) in (1) we obtain the reduced form for the inflation rate:

$$\pi_{t+1} = a\pi_t - br_t + \varepsilon_{t+1} \quad (4)$$

with  $b = \gamma\delta$  and  $\varepsilon_{t+1} = \gamma\xi_{t+1}$ . Monetary authorities set the interest rate with a view to reaching an inflation target  $\pi^*$ . Specifically, it is assumed that the purpose of monetary authorities is to minimise the squared deviation of inflation from the target, the latter being normalised to zero. This is equivalent to minimise both the squared deviation of the expected value of the inflation rate from target (the squared bias) and uncertainty as to future inflation (inflation variance). Formally, the minimisation of the objective function can be written as follows:<sup>(11)</sup>

$$E_t \pi_{t+1}^2 \quad (5)$$

or, alternatively,

$$\left( bias_t \pi_{t+1} \right)^2 + var_t(\pi_{t+1}) \quad (5a)^{(12)}$$

The only source of uncertainty in model (4) results from the shock introduced in the IS equation. Thus, it is assumed that the authorities are definitely fully acquainted with: i) the model parameters; ii) the state of the economy, which implies that, for instance, the inflation rate and the output gap do not reveal measurement errors and that authorities are able to perfectly identify the type of

(11) This problem may be solved through dynamic programming. However, Svensson (1996 and 1997) shows that the solution usually coincides with that obtained with the optimisation for a single period, the objective function being given by equation (5).

(12) The bias of the random variable  $\pi_{t+1}$  is defined as  $E_t \pi_{t+1} - \pi^*$  and measures the difference between expected inflation and the inflation target. Equation (5a) results from the fact that:

$$E_t (\pi_{t+1} - \pi^*)^2 = E_t (E_t \pi_{t+1} - \pi^*)^2 + E_t (\pi_{t+1} - E_t \pi_{t+1})^2,$$

where the second term stands for the variance.

shocks hitting the economy; and, perchance more important, iii) the functional form of the economy (i.e. the manner in which inflation and the output gap are interrelated). Taking into account these hypothesis, by replacing (4) in (5), differentiating in order to  $r_t$  and setting the result equal to zero, the optimal rule is given by:

$$r_t = \frac{a}{b} \pi_t \quad (6)$$

By replacing (6) in (4), we obtain the equilibrium path for the inflation rate:

$$\pi_{t+1} = \varepsilon_{t+1} \quad (7)$$

Based on (6) and (7), the optimal monetary policy rule is obtained in terms of the additive shock:

$$r_t = \frac{a}{b} \varepsilon_t \quad (8)$$

In order to reach this optimal path for the real interest rate, based on (3) and taking into account that inflation expectations are zero, the interest rate should be set according to the following rule:

$$i_t = r_t + E_t \pi_{t+1} = \frac{a}{b} \varepsilon_t \quad (9)$$

This rule fulfils “certainty equivalence principle”: the same optimal rule would be obtained if there were no uncertainty. Once the shock over inflation is observed, the optimal response of the authorities will be to fully cancel its effects, so that the inflation rate may resume the target. This means that, although the authorities are not able to avoid temporary deviations of inflation from target, they may ensure that these deviations are not permanent. Therefore, optimal monetary policy would be characterised by a high degree of aggressiveness. It is obvious that this result depends on the above hypotheses (i) to (iii). These make it possible for the authorities to identify unambiguously the type of shock faced and the manner in which the monetary policy instrument should be adjusted. In practice, however, monetary authorities are not able to identify clearly either the type of shocks faced by the economy or the best response to those shocks.

#### 4.2. Optimal monetary policy under parameter uncertainty

Some more recent studies have attempted to explore the implications for monetary policy of a wider range of uncertainties [see, for example, Sack (1998a)]. One strand of research has tried to gauge the extent to which uncertainty about the parameter magnitude of the transmission mechanism may lead to a less aggressive response of monetary policy to economic shocks. This analysis having its roots in the work of Brainard (1967) is based on the assumption that uncertainty about the relationship between official interest rates and the rest of the economy (a form of parameter uncertainty) creates a trade-off for monetary authorities: the presence of parameter uncertainty may imply that movements in official interest rates could lead to an increase in uncertainty about the future path of the economy. In this case, monetary authorities should be more cautious, even if this would mean a worse outcome on average, in order to reduce the probability of falling a long way short of the target set (Brainard’s principle): “(...) central banks must avoid becoming a source of additional uncertainty themselves when there is only limited knowledge about the economy and the behaviour of economic agents” [Issing (2002)].

The consideration of the so-called parameter uncertainty leads to the elimination of the above hypothesis (i). This type of uncertainty arises when it is assumed that authorities know the structural equations characterising the economy, but ignore the size of the multipliers, thus having to estimate them. For example, if in equation (4), authorities ignore the value of parameter  $b$ , they cannot assess the impact of interest rate changes on the output gap and thus on inflation.

Brainard (1967) assumed that authorities ignore the actual figures of model parameters, but know their distribution. Using the model above, it is assumed that parameters  $a$  and  $b$  follow a normal distribution,<sup>(13)</sup> with means  $\bar{a}$  and  $\bar{b}$ , and variances  $\sigma_a^2$  and  $\sigma_b^2$ , respectively, and covariance  $\sigma_{ab}$ :

$$\begin{pmatrix} a \\ b \end{pmatrix} \sim N \left\{ \begin{pmatrix} \bar{a} \\ \bar{b} \end{pmatrix}; \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix} \right\}$$

(13) The hypothesis of parameter normality is not strictly necessary.



We assume, in a first analysis, that the covariance between these two parameters is zero. In this case, using (4), the objective function (5a) may be written as

$$\left( \bar{a} \pi_t^2 + \bar{b} r_t^2 - 2\bar{a}\bar{b}\pi_t r_t \right) + (\sigma_a^2 \pi_t^2 + \sigma_b^2 r_t^2 + \sigma_\varepsilon^2) \quad (5b)$$

Differentiating in order to  $r_t$  and equalling to zero, we obtain the optimal rule:

$$r_t = \left[ \frac{\bar{a}\bar{b}}{\bar{b} + \sigma_b^2} \right] \pi_t \quad (10)$$

For a better comparison with the situation presented in the previous section, let us represent by  $cv$  the coefficient of variation  $cv = \frac{\sigma_b}{\bar{b}}$  and define the parameter  $h$  as  $h = \frac{1}{1 + cv^2}$ . Thus, equation (10) may be written as follows:

$$r_t = h \frac{\bar{a}}{\bar{b}} \pi_t \quad (10a)$$

Parameter  $h$  defines the gap identified by Brainard, and shows that the response to shock  $\varepsilon_t$ , under parameter uncertainty, is a fraction of the response in the situation in which certainty equivalence occurs (since  $h$  stands between zero and one). This fraction is exclusively determined by the coefficient of variation, i.e. by the relative size of uncertainty (measured by the standard deviation) in relation to the mean of the policy multiplier. When uncertainty is high,  $h$  is small and monetary policy becomes more gradualist. As the relative weight of uncertainty declines,  $h$  tends to one and the optimal response of monetary policy gets closer to the situation described in the previous subsection. In this context, monetary authorities face a trade-off between the desire to bring the inflation rate back to the target (reduction of the inflation bias) and the desire to minimise the risk of increased volatility in inflation with only one instrument available ( $i_t$ ). Deriving from (5b), the variance of inflation depends positively on the real interest rate deviation from its neutral level, and thus monetary policy decisions affect the uncertainty about future inflation. Thus, by contrast with the additive uncertainty model, variance becomes endogenous. Therefore, within this model monetary authorities adjust interest rates to a smaller extent than they would do in a scenario

without uncertainty, (i.e. it is not optimal to completely offset a shock in any period). This situation is what Blinder (1998) calls “Brainard’s conservatism principle”: “Estimate how much you need to tighten or loosen monetary policy to get it right. Then do less”. The response of monetary policy is thus spread over several periods.<sup>(14)</sup>

Replacing the monetary policy rule (10a) in (4) and applying the expectation operator, we obtain:

$$E_t \pi_{t+1} = \bar{a}(1-h)\pi_t \quad (11)$$

The optimal nominal interest rate results from the sum of the optimal rule for the real interest rate (10a) and the expected inflation (11):

$$i_t = \frac{h\bar{a}}{\bar{b}} \pi_t + \bar{a}(1-h)\pi_t \quad (12)$$

Equation (12) shows that the implications in terms of manipulation of monetary policy instrument resulting from a gradualist rule for the real interest rate may be ambiguous. Analysing the right-hand side of equation (12), we verify that: the first term (real interest rate) points to a more gradualist policy, while the second term (inflation expectations) translates the fact that a gradualist rule for the real interest rate leads to an increase in inflation expectations. The combined effect, however, points to a rule for the gradualist nominal interest rate, unless  $b$  is very high, which, according to the estimates presented by Rudebusch and Svensson (1999), seems unlikely.

It is also interesting to consider the situation in which the covariance between parameters ( $\sigma_{ab}$ ) is not zero. In this case, the objective function (5b) will be equal to:

$$\left( \bar{a} \pi_t^2 + \bar{b} r_t^2 - 2\bar{a}\bar{b}\pi_t r_t \right) + (\sigma_a^2 \pi_t^2 + \sigma_b^2 r_t^2 + \sigma_\varepsilon^2 - 2\pi_t r_t \sigma_{ab}) \quad (5c)$$

(14) Some studies have attempted to quantify the importance of “Brainard’s effect”. Sack (1998a) confirms that if the impact of monetary policy on the economy is uncertain, a more aggressive policy may induce excessive volatility on target variables. In that sense, it may be optimal to adjust official interest rates gradually in order to deliberately limit the risks of increased volatility on the economy. The analysis under parameter uncertainty explains an important part of actual persistence in interest rates. However, even considering effects resulting from economy dynamics and parameter uncertainty, an element of inertia which is not explainable by data prevails in interest rates.

and the optimal monetary policy rule is given by

$$r_t = \left[ \frac{\bar{a}\bar{b} + \sigma_{ab}}{\bar{b}^2 + \sigma_b^2} \right] \pi_t \quad (10b)$$

As it derives from equation (10b), a high covariance between the two parameters could lead to a situation in which a more aggressive policy would be optimal (see Annex).<sup>(15)</sup>

Finally, it should be noted that these findings obviously depend on the type of model presented. Starting from a model with an objective function that attaches some weight to output stabilisation, Söderstrom (2000) shows that in some situations parameter uncertainty could induce monetary authorities to choose a more aggressive monetary policy. In particular, when there is uncertainty about the degree of persistence of inflation (parameter  $a$ ), there could be cases in which it would be optimal to change interest rates in a more aggressive fashion, in order to reduce uncertainty about future developments in the economy. However, uncertainty about the impact of monetary policy on the economy (parameter  $b$ ) continues to lead to a less aggressive policy, in line with Brainard's analysis.

#### 4.3. Optimal monetary policy under uncertainty about the state of the economy

The assumed hypothesis that authorities are completely aware of the state of the economy neglects two important sources of uncertainty: the existence of errors in the measurement of variables and the uncertainty about the type of shocks affecting the economy. This could emerge either because certain variables, such as GDP, are only available with some time lag and are subject to frequent revisions, or because there is another type of variables, such as the output gap, whose measurement is not direct and whose results are sensitive to the method of estimation used.

Measurement errors may be considered within models similarly to additive shocks. If this is the case, there is no change in the optimal policy rule.

(15) Although the conclusion pointing to a greater gradualism is better known, Brainard (1967) acknowledges that the existence of high covariances between the parameters of the model may lead to a more aggressive monetary policy. Martin and Salomon (1998) assess the impact of the existence of non-zero covariances for the case of the United Kingdom.

For example, in the previous model, if we assume that the output gap shows measurement errors, equation (2) may be changed to:

$$\hat{x}_{t+1} = -br_t + \varepsilon_{t+1} + \xi_{t+1} \quad (2a)$$

where  $\hat{x}_{t+1}$  represents the output gap measure used by monetary authorities and  $\xi_{t+1}^y$  the measurement error associated. Since authorities cannot distinguish between the contribution to the output gap estimate resulting from the additive shock and that deriving from a measurement error, the optimal policy rule given (1) and (2a) remains (6).

However, within models with different types of shocks requiring different policy responses, the existence of measurement errors could make the problem of identifying shocks particularly complex. For example, a monetary authority, whose purpose is to limit output gap and inflation fluctuations vis-à-vis a target, typically raises the interest rate in the presence of a positive demand shock. However, if the output estimate shows measurement errors it becomes more complicated to know whether an output rise reflects a demand shock, a supply shock or whether it is merely the result of a measurement error. Therefore, raising interest rates on the assumption that the output rise was the result from a demand shock could be a wrong decision.

Smets (1998) and Orphanides (1998) examine the extent to which errors in output gap measurement (Smets) and in output gap and inflation measurement (Orphanides) could affect the optimal response of the Federal Reserve. Both studies assume that monetary policy is defined according to a Taylor rule. Firstly, the optimal rule is derived assuming that there are no measurement errors. Subsequently, this rule is derived admitting the existence of measurement errors. The results suggest that if these errors are significant the optimal response of monetary policy tends to be more gradual:<sup>(16)</sup> "When the noise content of the data is properly taken into account, policy reactions are cautious and less sensitive to the apparent imbalances in the unfiltered data. The resulting policy prescriptions reflect the recognition that excessively activist policy can increase rather than decrease economic instability" [Orphanides (1998)].

#### 4.4. Optimal monetary policy under uncertainty about the economic model

The analysis made in the previous three subsections assumed that authorities know the type of uncertainty they are facing. For example, to estimate the optimal policy rule (10), it is necessary to know the parameter variance and also to identify additive shocks on the economy. Similarly, the work by Orphanides (1998) admits that authorities know the variance of measurement errors of variables. However, the type of uncertainties faced by policy makers is in practice considerably wider. More precisely, monetary authorities ignore the functional form of the “true” model of the economy and which variables should be included in that model.

Some studies have attempted to analyse how monetary policy should be conducted under model uncertainty - also known in the literature as “Knightian uncertainty”. The so-called literature on robust control considers this type of uncertainty and presents some principles about the estimation of the optimal monetary policy rule, taking into account the different models/paradigms at the disposal of a monetary authority. Svensson (2000) identifies three approaches in literature.

The first approach, referred to as Bayesian, starts by calculating for each monetary policy rule  $f$ , the loss in each model ( $m$ ) across all available models and/or paradigms ( $M$ ). Secondly, a subjective probability ( $p_m$ ) is attributed to each actual loss and the so-called expected loss is calculated:

$$E_M L = \sum_{m \in M} p_m L(f, m)$$

where  $L(f, m)$  defines the loss function of the model  $m$ , using the policy rule  $f$ . The optimal policy rule  $f^*$  minimises the expected loss:

$$f^* = \underset{f \in F}{\text{Min}} E_M L$$

Gerdsmeier *et al* (2002) present other weighting choices, as the weighting of the models themselves or of optimal policy rules obtained in each model, concluding that any of these two approaches produces outcomes which are worse than those of the previous method.

More recently, literature on robustness control has adopted an approach that does not require the

advance existence of subjective probabilities to weight the different models under review. The methodology used consists in the so-called *min-max* criterion [see, for example, Onatski and Stock (2000) or Gerdsmeier *et al* (2002)]. For each monetary policy rule  $f$ , the maximum loss across available models is calculated and the optimal policy rule  $f^*$  is the rule that minimises maximum losses. In other words, the optimal policy rule is that showing the best outcome in the range of the different worst-case scenarios. In formal terms:

$$f^* = \underset{f \in F}{\text{Min}} \underset{m \in M}{\text{Max}} L(f, m)$$

Onatski and Stock (2002) use this criterion to identify robust policy rules, starting from the US economy model presented by Rudebusch and Svensson (1999). In this work four different types of uncertainty are considered: parameter uncertainty, uncertainty about data quality, uncertainty about the degree of serial correlation of shocks and uncertainty about the model itself. The key finding was that the different forms of specifying uncertainty produce different implications on the degree of monetary policy activism. Moreover, authors conclude that the main source of uncertainty for policy makers is that associated with the model of the economy itself and aggressiveness found in some estimated policy rules is linked to worst-case scenarios.

Finally, a third approach [see, for example, Levin *et al* (1999)] consists in attempting to identify monetary policy rules performing well across a wide range of models (i.e. robust rules). By definition, this type of rules do not perform as well as the optimal rule derived in each particular model, but is designed to work smoothly within that model and across alternative models.

The impact of this particular source of uncertainty — perhaps the most important one — on the conduct of monetary policy is an evolving matter, and there is neither a consensual manner of identifying robust monetary policy rules nor a clear balance about the degree of monetary policy gradualism. However, an important finding in the most recent literature on robustness control is that in worst-case scenarios interest rate rules point to greater aggressiveness. The underlying view is that some types of uncertainty may lead to a smaller than expected impact of monetary policy

instruments on objective variables. Under these circumstances, in order to prevent worst-case scenarios, it would be optimal to react more aggressively when under uncertainty.

## 5. FINAL REMARKS

The findings in some empirical studies suggest that monetary policy in different countries has been characterised by a degree of interest rate gradualism, which cannot be strictly explained by the dynamic structure of the economy. In other words, the optimal path for interest rates emerging from different macroeconomic models is normally less gradual than that observed in practice. A possible explanation for the divergence between indications given by models and actual practice is that the former may not deal adequately with different sources of uncertainty surrounding monetary policy decisions. Common intuition seems to point to the fact that the introduction of uncertainty should lead to a more cautious position of monetary authorities. This view reflects Brainard's findings (1967). However, from a theoretical point of view, as argued by different authors [see, for example, Onatski and Williams (2002)], greater uncertainty does not necessarily mean more gradual monetary policy.

An important finding in this article relates to the fact that different hypotheses about the form of uncertainty have different implications in terms of the optimal degree of monetary policy gradualism. In that sense, monetary authorities should assess the different sources of uncertainty and combine them comprehensively. A well designed monetary policy strategy should take into account the presence and the implications of those different sources of uncertainty. This suggests that a monetary authority should not rely on a single indicator or model separately. On the contrary, it should assemble several sources of information, cross check their contents and assess their implications.

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ANNEX  
IMPACT ON THE OPTIMAL POLICY RULE OF THE EXISTENCE  
OF NON-ZERO COVARIANCES BETWEEN PARAMETERS

Taking into account that the mean squared error of inflation expectations can be written as (see footnote 12):

$$E_t \pi_{t+1}^2 = E_t^2 \pi_{t+1} + \text{var}_t \pi_{t+1} \quad (\text{I})$$

With the variance being defined as:

$$\text{var}_t \pi_{t+1} = \sigma_a^2 \pi_t^2 + \sigma_b^2 r_t^2 + \sigma_\varepsilon^2 - 2\pi_t r_t \sigma_{ab} \quad (\text{II})$$

When the parameter variances and the respective covariance are zero, (II) does not depend on the real interest rate. This is therefore the certainty equivalence situation.

The marginal change in the inflation variance in period  $t+1$  is given by:

$$\frac{\partial \text{var}_t \pi_{t+1}}{\partial r_t} = 2(\sigma_b^2 r_t - \pi_t \sigma_{ab}) \quad (\text{III})$$

If the covariance between  $a$  and  $b$  is zero, a rise in the real interest rate will then simultaneously lead to an increase in the variance of future inflation. However, if the covariance between the parameters is sufficiently large and positive, the Brainard principle will then cease to occur, because a rise in the real interest rate reduces the inflation variance, and the monetary authorities no longer face a policy dilemma.

This result seems to be relatively intuitive. For example, it is assumed that the multiplier of monetary policy  $b$  and the parameter measuring the persistence of inflation  $a$  are strongly and positively correlated, thereby resulting in a rule that points to a more aggressive monetary policy. Thus, if the persistence of inflation is high, and given the strong correlation between parameters, the actual situation would (desirably) reveal a higher monetary policy efficiency.

The possible existence of a strong positive correlation between parameters is of an empirical nature. For example, Sack (1998a) does not find evidence of a more aggressive monetary policy for the United States, assuming the presence of parameter uncertainty.