UNCERTAINTY IN THE TAYLOR RULE AND MONETARY POLICY ASSESSMENT*

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1. INTRODUCTION

In recent years, one has witnessed a widespread attention on the way monetary policy is conducted and in particular on the role of the so-called monetary policy rules. Several reasons seem to underlie this renewed interest. Perhaps the most important one is that since the second half of the 1980s, a number of studies have concluded that monetary policy significantly influences the short-term performance of the real economy. A part of this strand of literature tries to identify simple monetary policy rules that could reduce the likelihood of inflationary shocks similar to those of the 1970s.

The conventional approach in the literature consists in estimating reaction functions for a monetary authority (the Federal Reserve, in most cases) in which a nominal interest rate, directly or indirectly controlled by that monetary authority, is adjusted in response to deviations of inflation (current or expected) from target and of output from potential. These reaction functions, usually called Taylor rules, following John Taylor's seminal paper published in 1993, seem to match a number of normative principles set forth in the literature for optimal monetary policy[1].

This seems to provide a good reason for the growing prominence of indications given by Taylor rule estimates in debates about current and prospective stance of monetary policy. However, these indications should be interpreted with prudence. Indeed, they are usually presented as point estimates for the interest rate, giving a sense of accuracy, which may be very misleading. Typically, no weight is placed on the discussion of the risks to such estimates and, at least to a certain extent, the reader is encouraged to concentrate on an apparently precise central projection, ignoring the wide degree of uncertainty and operational difficulties surrounding these estimates. As in any forecasting exercise, there is uncertainty regarding both the estimated parameters and the way the explanatory variables evolve during the forecasting horizon [see Martins (2000)].

Our work aims to obtain a methodology to estimate a probability density function for the interest rate resulting from the application of the Taylor rule (the Taylor interest rate) and assuming that not only the explanatory variables but also the parameters of the rule are random variables. Our ap-

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(1) The usefulness of Taylor rules as instruments for monetary policy analysis can be sustained not only on normative grounds, with many studies concluding that simple monetary rules have stabilising properties which are close to those of optimal policy rules, but also on positive grounds, since rules with this kind of formulation seem to depict fairly well the way the major monetary authorities have been conducting monetary policy. Martins (2000) provides a summary of the empirical literature on Taylor rules as well as a discussion on the operational difficulties and limitations associated with the use of this kind of instrument.

proach builds on the work by the Bank of England [see Whitley (1999) and Britton et al (1998)] and the Sveriges Riksbank [see Blix and Sellin (1998)] produced in the context of their inflation forecasting exercises. The method has a Bayesian nature in the sense that involves a subjective component, through a permanent assessment of the state of the economy, based on a central projection and the risks surrounding it. This assessment gives rise to the adoption of asymmetric distributions both for the explanatory variables and the parameters of the Taylor rule. However, unlike the approach followed by the aforementioned central banks, the resulting distribution for the Taylor interest rate is obtained by numerical simulation in line with Esteves, Machado and Martins (2001).

This article is structured as follows. Section 2 presents a brief outline of the Taylor rule and describes the procedure used to compute a probability density function for the Taylor interest rate. This procedure is then applied to the euro area in section 3. Finally, section 4 presents some concluding remarks.

2. TAYLOR RULE: A DISTRIBUTION ASSUMPTION FOR THE EXPLANATORY VARIABLES AND THE SIMULATION OF THE JOINT DISTRIBUTION

The original formulation of the Taylor rule is the following:

$$i_{Tt} = r^* + \pi^* + \beta(\pi_t - \pi^*) + \theta X_t \qquad (1a)$$

where i_T is the interest rate recommended by the Taylor rule (the Taylor interest rate), π_t the average inflation rate over the previous four quarters (measured by the GDP deflator), π^* the inflation rate target, X_t the output gap and r^* the equilibrium (or neutral) real interest rate[2].

Formulation (1a) by taking into account only the contemporaneous inflation rate and output

gap overlooks the forward-looking nature of monetary policy. To overcome this problem, a forward-looking version of the Taylor rule is used in line with Clarida, Gali and Gertler (1997):

$$i_{Tt} = r^* + \pi^* + \beta(\pi^e_{t+2} - \pi^*) + \theta X^e_{t+1} \qquad (1b)$$

The different time horizons considered for the output gap and the inflation forecast (one and two years, respectively), has implicit the stylised fact that, at least in large and relatively closed economies, monetary policy affects economic activity faster than it affects inflation [see Ball (1997)].

As it was referred above, the estimation of Taylor rules involves uncertainty regarding not only the estimated parameters (β and θ) but also the way in which the explanatory variables evolve over the forecasting horizon. As a result, in this article all the arguments of the Taylor rule, excluding the inflation target, are assumed to be random variables. It is also considered that the probabilistic behaviour for each of these variables is characterised by a two-piece normal distribution (TPN)[3]. This distribution, which is also used by the Bank of England and the Sveriges Riksbank in their inflation forecasting exercises, provides a simple way to introduce asymmetrical considerations in the analysis.

A random variable W has a TPN distribution if its probability density function is given by:

$$f(W; \mu_w; \sigma_{w,1}) = C \exp\left[-\frac{1}{2\sigma_{w,1}^2}(W - \mu_w)^2\right], W \leq \mu_w \qquad (2a)$$

$$f(W; \mu_w; \sigma_{w,2}) = C \exp\left[-\frac{1}{2\sigma_{w,2}^2}(W - \mu_w)^2\right], W > \mu_w \qquad (2b)$$

with $C = \sqrt{\frac{2}{\pi}}(\sigma_{w,1} + \sigma_{w,2})^{-1}$.

This distribution can be understood as a merge of two standard normal distributions with the same mode (μ_w) but with different standard deviations $(\sigma_{w,1}; \sigma_{w,2})$, which were adjusted in a way to ensure continuity at μ_w [4]. Chart 1 provides an

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(2) Note that if $\beta > 1$ and $\theta > 0$, the real interest rate adjusts in a way that stabilises both inflation and output; if $\beta < 1$, some inflation is accommodated. In this case, the nominal interest rate change is not sufficient to cause the real interest rate to move in the same direction. This also applies to θ, which has to be non-negative for the rule to be stabilising. In Taylor's seminal paper (1993), the rule arguments were set at $\beta = 1.5, \theta = 0.5, \pi^* = 2$ and $r^* = 2$.

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(3) See Johnson, Kotz and Balakrishnan (1994) for a brief description of this distribution.

(4) The factors of adjustment applied to the normal distribution are $2\sigma_{w,1}/(\sigma_{w,1} + \sigma_{w,2})$ to the left of the mode and $2\sigma_{w,2}/(\sigma_{w,1} + \sigma_{w,2})$ to its right so a to ensure that the probability density function is continuous and the integral adds to 1.

Chart 1

PROBABILITY DENSITY FUNCTION OF A TWO-PIECE NORMAL DISTRIBUTION

$\sigma_{w,1} < \sigma_{w,2}$

$N(\mu_w; \sigma_{w,2})$

$NS(\mu_w; \sigma_{w,1}; \sigma_{w,2})$

$N(\mu_w; \sigma_{w,1})$
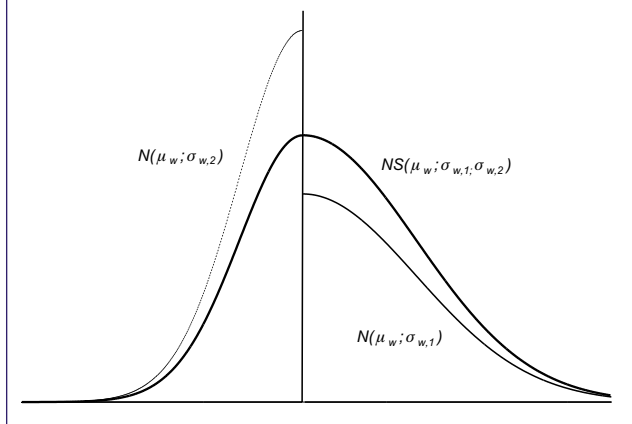
illustration with $\sigma_{w,1} < \sigma_{w,2}$. In this example, the probability mass to the left of the mode is smaller than the probability mass to its right, so that both the mean and the median exceed the mode (positive asymmetry).

The mean and the variance for a random variable with this distribution are given by:

$$E(W) = \mu_w + \sqrt{2/\pi}\left(\sigma_{w,2} - \sigma_{w,1}\right) \qquad (3)$$

$$Var(W) = \left(1 - \frac{2}{\pi}\right)\left(\sigma_{w,2} - \sigma_{w,1}\right)^2 + \sigma_{w,1}\sigma_{w,2} \qquad (4)$$

In our analysis, W denotes each of the arguments of the Taylor rule: inflation forecast, output gap,... In order to obtain the three parameters of the distribution $W\left(\mu_w, \sigma_{w,1}, \sigma_{w,2}\right)$, it is necessary to assign values to:

(i) μ_w, which represents the central projection (i.e. the single most likely outcome);

(ii) $\omega_w = h_w\sigma_w$, which represents the standard deviation calculated using historical data (σ_w) adjusted by a factor of additional uncertainty (h_w);

iii) P_w, represents the subjective probability of W being below the central projection – the downside risk[5]. This parameter plays a key role in the analysis, since the asymmetry builds on the particular value of P_w. Notice that if $P_w = 0.5$, the distribution collapses to the standard normal distribution.

Given the distribution for the arguments of the Taylor rule, the question that arises is how to determine the distribution of the Taylor interest rate itself. Unlike the case when random variables follow a normal distribution, the aggregation of random variables with a TPN distribution does not result in new variable with a TPN or any other known distribution. Contrasting with the Bank of England and the Sveriges Riksbank approaches, in our work the Taylor interest rate distribution is obtained by numerical simulation in line with Esteves, Machado and Martins (2001).

One of the problems to be solved before numerical simulation, is the likely statistical dependence among the Taylor rule arguments. Whereas regarding most of them it seems reasonable to assume independence, that would be little realistic vis-à-vis the inflation forecast and the output gap. To model the dependence between these two variables in a simple manner, it is considered that the inflation forecast π^e can be expressed as a linear combination of two random variables π_1^e and π_2^e, which are independent of the output gap (X^e), and follow TPN distributions with common mode $\left[NS\left(\mu_{\pi^e}; \sigma_{\pi_i^e}\right); i = 1; 2\right]$:

$$\pi^e = \pi_1^e I\left(X^e > \mu_x\right) + \pi_2^e I\left(X^e < \mu_x\right) \qquad (5)$$

where I(.) stands for the indicator function. In addition, it is assumed that $\sigma_{\pi_1^e,2} > \sigma_{\pi_2^e,2}$, which means that, π_1^e, is relatively more skewed to the right than π_2^e. As a result, if the output gap outcome exceeds (falls behind) its central projection, a higher (lower) proportion of agents will be optimistic (pessimistic) about the inflation prospects, that is the mass of probability to the right (left) of the modal inflation forecast will be greater. In other words, "good news" in terms of output/unemployment will lead a higher proportion of people sharing a less favourable outlook for inflation.

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[5] Bearing in mind that $P_w = \int_{-\infty}^{\mu} f(x)dx = \frac{\sigma_1}{\sigma_1 + \sigma_2}$, it is possible to show that the standard deviations of the TPN distribution are given by:

$$\sigma_{1,w}^2(\sigma_\omega, P_w) = h_w^2 \sigma_\omega^2 \left[\left(1 - \frac{2}{\pi}\right)\left(\frac{1 - 2P_w}{P_w}\right)^2 + \left(\frac{1 - P_w}{P_w}\right)\right]^{-1} \text{ and}$$

$$\sigma_{2,w}^2(\sigma_w, P_w) = h_w^2 \sigma_w^2 \left[\left(1 - \frac{2}{\pi}\right)\left(\frac{1 - 2P_w}{1 - P_w}\right)^2 + \left(\frac{P_w}{1 - P_w}\right)\right]^{-1}.$$

Table 1

THE EURO AREA TAYLOR INTEREST RATE: CENTRAL PROJECTIONS, UNCERTAINTY AND BALANCE OF RISKS

	Equilibrium real interest rate (r*)	Inflation target (π^*)	Inflation forecast (π^e)		Output gap (X^e)	Coefficient of the "inflation gap" (β)	Coefficient of the output gap (θ)
Main assumptions:							
Central projections (μ_w)	3.00	1.50	1.90		0.30	1.31	0.25
Adjusted standard deviation (ω_w) . .	0.21	-	0.75		1.21	0.09	0.04
Historical standard deviation (σ_w) . .	0.19	-	0.75		1.10	0.09	0.04
Additional uncertainty (h_w)	1.10	-	1.00		1.10	1.00	1.00
Balance of risks			π^e_1	π^e_2			
Upside $(1-P_w)$	0.60	-	0.60	0.40	0.45	0.60	0.70
Downside (P_w)	0.40	-	0.40	0.60	0.55	0.40	0.30
Memo item:							
Mean	3.07	1.50	1.83[a]		0.11	1.34	0.27

Note:

(a) Computed by numerical simulation.

The distribution for the inflation forecast can be computed by numerical simulation on the basis of equation (5). To do that, it is necessary to postulate some values for the distribution parameters. Regarding the X^e distribution, we have to assign values for the central projection (μ_{X^e}); the historical standard deviation (σ_{X^e}) and the factor of additional uncertainty (h_{X^e}); and the downside risk (P_{X^e}). Concerning the inflation forecast, we also have to assume values for the central projection (μ_{π^e}); the historical standard deviation (σ_{π^e}) and the factor of additional uncertainty (h_{π^e}); and the downside risks $P_{\pi^e_1}$ and $P_{\pi^e_2}$ [6]. The simulation of the inflation forecast distribution assumes that $\sigma_{\pi^e_1,1} = \sigma_{\pi^e_2,1} = \nu$, with ν being a constant to be computed following the aforementioned assumptions for the parameters [see Esteves, Machado e Martins (2001)].

Once the distribution of the inflation forecast is defined, on the basis of equation (1b), it becomes

possible to obtain by numerical simulation a probability density function for the Taylor interest rate that takes into account the statistical dependence between the inflation forecast and the output gap.

3. AN APPLICATION TO THE EURO AREA

Taking into account the above procedure, this section provides an assessment of the euro area monetary policy stance on basis of data available in December 2001. Table 1 presents the central projections for each of the relevant variables of the Taylor rule, as well as the degree of uncertainty and the balance of risks. The underlying assumptions as well as the remaining calculations needed to compute the Taylor interest distribution are listed below.

3.1 Coefficients

Table 2 presents different estimated values for coefficients β and θ. Even though the results are not qualitatively very distinct, the conclusions drawn in each model could be quantitatively different. To define the baseline scenario and the standard deviations, we took the values estimated

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(6) Given equation (5), it is straightforward to demonstrate that the downside risks are related in following manner: $P_{\pi^e} = (1 - P_{X^e}) P_{\pi^e_1} + P_{X^e} P_{\pi^e_2}$. As a result, assuming values for $P_{\pi^e_1}$ and $P_{\pi^e_2}$ is equivalent to assume a value for P_{π^e} and another for $P_{\pi^e_1}$ or $P_{\pi^e_2}$.

for Germany in Clarida, Galí and Gertler (1997), which are similar to those estimated in Peersman and Smets (1998). We did not consider any additional uncertainty. Against the backdrop that most estimates for the United States point to higher coefficients than those of Germany, we considered upside risks greater than 50 percent (60 and 70 percent, respectively for β and θ).

3.2 Equilibrium real interest rate

We assume a value of 3.0 percent for the euro area equilibrium real interest rate. This figure is consistent with estimates derived from a reaction function for the Bundesbank over the last two decades and with the average real interest rates in G7 during the 1990s[7]. Moreover, according to the well-known golden rule of capital accumulation, the marginal product of capital, which in equilibrium equals the real interest rate should not be less than the growth rate of output (otherwise, the economy would be dynamically inefficient). Current estimates for the euro area potential growth rate suggest a lower bound for the real interest rate of around 2.0 to 2.5 percent. For the historical standard deviation, we take the value derived by Smets (1999) from a forward-looking reaction function for the Bundesbank over the period 1979-1997. In this work, the implicit equilibrium real interest rate for an inflation target of 1.5 percent was 3.0 percent as well. As regards the balance of risks, different estimates put forward by the literature for the equilibrium real interest rate for Germany fall overwhelmingly in the range of 2.5 to 3.5 percent, with a slight bias on the upper half. This last evidence seems to indicate that the balance of risks is on the upside, thus justifying the attribution of a 60 percent probability to upside risks. Finally, the possible effects of the so-called "New Economy" induce some uncertainty over the current potential output estimates

(7) The calculation of the equilibrium real interest rate for the euro area on the basis of the average real interest rates prevailing, for example, over the last decade is likely to show an upward bias. Indeed, over this period, the disinflation process in the current euro area countries, may have caused real interest rates to stand above their equilibrium level. Against this background, it seems more appropriate to take past German interest rates as a benchmark for comparison with today's euro area.

Table 2

ESTIMATED COEFFICIENTS β AND θ

		Inflation gap	Output gap
Taylor (1993)...............	US	1.50	0.50
Taylor (1999)...............	US	1.50	1.00
Ball (1997)	US	1.50	1.00
Christiano (1999)	US	3.00	0.50
Clarida, Galí e Gertler (1998) ...	US	1.80	0.12
Clarida, Galí e Gertler (1997) ...	Germany	1.31	0.25
Peersman e Smets (1998).......	Germany	1.30	0.28

and consequently over the equilibrium real interest rate. As a result, we decided to include a factor of additional uncertainty of 10 percent.
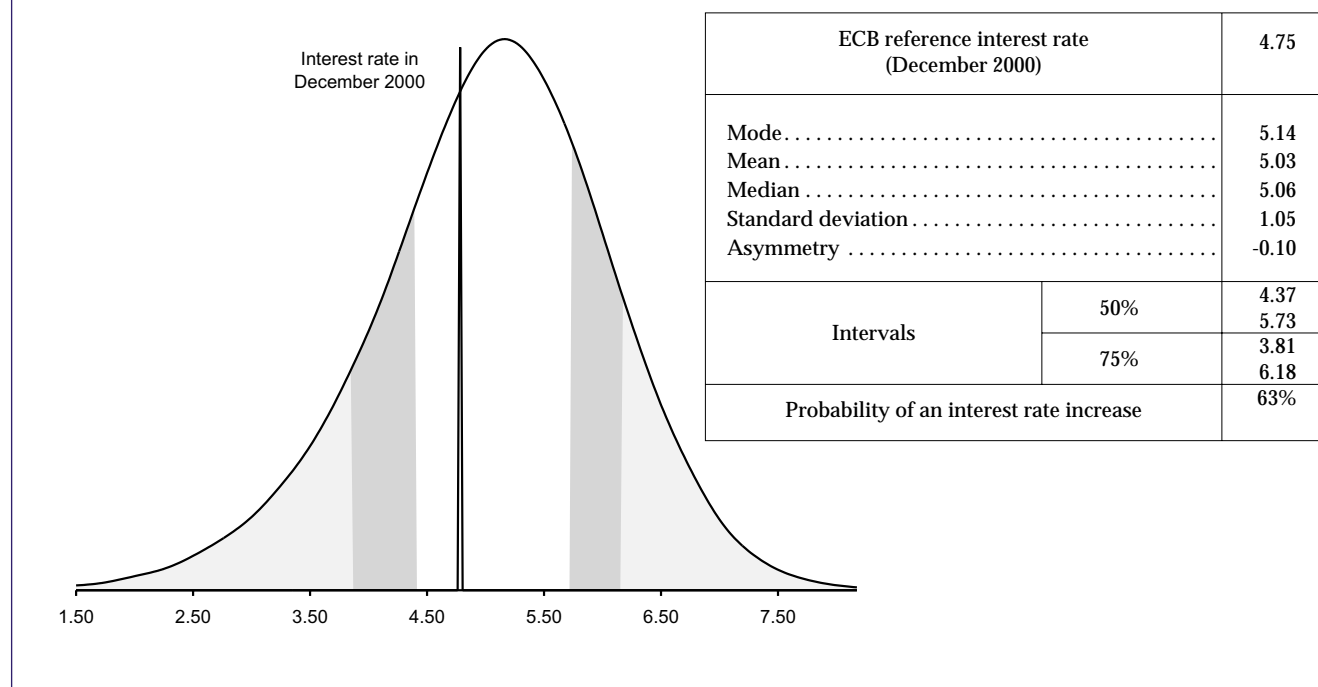
3.3 Inflation target

We assumed an inflation target of 1.5 percent. Recall that within the ECB monetary policy strategy, adopted in October 1998, price stability was defined as an annual increase in the Harmonised Index of Consumer Prices (HICP) of below 2 percent. In addition, the derivation of the reference value for the growth rate of the M3 monetary aggregate had implicit an inflation rate of 1.5 percent. We did not consider any uncertainty regarding this target.

3.4 Inflation forecast

The central projection for the inflation forecast corresponds to the mid-point of the Eurosystem's forecasting interval for the HICP growth in 2002 published in December 2000 – i.e. 1.9 percent. The historical standard deviation was computed taking into account the Eurosystem's forecasting interval (1.3, 2.5) is equal to twice the absolute mean error of the forecasting exercises undertaken over the last years [ECB (2000)]. Considering a normal distribution, this leads to a standard deviation of 0.75. We did not assume any additional uncertainty vis-à-vis the historical standard deviation. Regarding the balance of risks, we took downside risks of 60 percent, if the output gap realisation

Chart 2
PROBABILITY DENSITY FUNCTION OF THE TAYLOR INTEREST RATE

Interest rate in
December 2000

ECB reference interest rate (December 2000)		4.75
Mode.		5.14
Mean.		5.03
Median		5.06
Standard deviation		1.05
Asymmetry		-0.10
Intervals	50%	4.37 5.73
	75%	3.81 6.18
Probability of an interest rate increase		63%

1.50   2.50   3.50   4.50   5.50   6.50   7.50

falls behind the modal forecast, and of 40 percent, if the output gap realisation exceeds the modal forecast.

3.5 Output gap

The central projection for the output gap in 2001 (0.3 percent) was obtained with the Hodrick-Prescott filter, using quarterly data since 1977. This estimate is in line with the European Commission and the OECD projections for 2001, published in October and November 2001, respectively. The historical standard deviation was computed bearing in mind that the Eurosystem's forecasting interval (2.6, 3.6) for the GDP growth rate is equal to twice the absolute mean error of the forecasting exercises undertaken over several years. Taking into account the effects of the so-called "New Economy"[8], we decided to include a factor of additional uncertainty of 1.1. As to the balance of risks, we admitted a downside

risk of 55 percent, given the prospect that the economic slowdown in the United States could be more pronounced than the available projections.

3.6 Results

With the aforementioned assumptions, all the central statistical measures for the Taylor interest rate would be above the ECB reference interest rate in December 2000 (see Chart 2). Nevertheless, given the significant variance implied by the Taylor interest rate distribution, these indications are surrounded by considerable uncertainty, which are confirmed by the width of the confidence intervals. Indeed, the confidence interval for the Taylor interest rate is (4.37;5.37) with a probability of 50 percent, and (3.82;6.19) if the confidence level is increased to 75 percent. The ECB interest rate in December 2000 lied inside both intervals.

It is interesting to assess the impact of each argument of the Taylor rule on the distribution variance. Chart 3 clearly shows that the greatest contribution is given by the inflation forecast variance, which accounts for 96 percent of total variance[9]. It is also shown that the impact of uncertainty related to the Taylor rule specification (equi-

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(8) Estimates for the current output gap are particularly uncertain, both because recent output figures are in most cases preliminar or because many estimation techniques, namely univariate methods such as the HP filter, pose some end-of-sample problems.
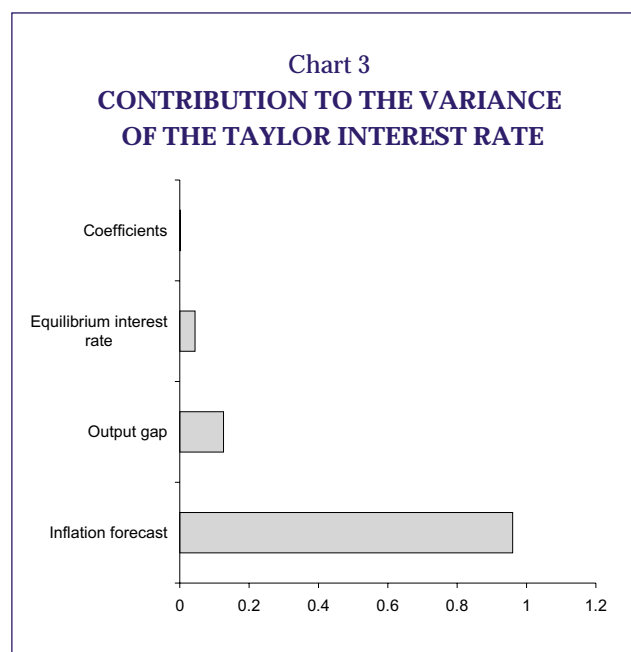
librium real interest rate and coefficients) is negligible.

4. CONCLUDING REMARKS

Empirical evidence suggests that Taylor rules depict fairly well the way major monetary authorities (in particular, the Federal Reserve and the Bundesbank) have conducted monetary policy over the last two decades – a period during which monetary policy is generally considered to be have been rather successful in reducing inflation. In this context, it seems reasonable to sustain that indications given by Taylor rule estimations could be a useful reference when assessing the monetary policy stance.

However, the conventional approach, which consists in presenting these indications as point estimates for the interest rate, seems to be lack prudence, given the high degree of uncertainty and operational difficulties surrounding the derivation of a Taylor interest rate. In particular, the use of Taylor rules in a forward-looking perspective requires the inclusion of macroeconomic forecasts over the period relevant for the monetary policy transmission mechanism. Given the forecasting errors of the past, that requirement provides an important source of uncertainty.

In our work, the informative content of the Taylor rule was presented as a probability density function for the interest rate. This approach makes clear that monetary policy decisions are taken in an uncertain environment, which has to be taken into account explicitly in the context of monetary policy assessment.

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(9) The contribution of each argument is given by the variance of the Taylor interest rate assuming that the variance of other arguments is equal to zero.

Chart 3
CONTRIBUTION TO THE VARIANCE OF THE TAYLOR INTEREST RATE

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