Investment Hangover and the Great Recession*

Matthew Rognlie[†] Andrei Shleifer[‡] Alp Simsek[§]

May 2015

Abstract

We present a model of investment hangover motivated by the Great Recession. In our model, overbuilding of residential capital requires a reallocation of productive resources to nonresidential sectors, which is facilitated by a reduction in the real interest rate. If the fall in the interest rate is limited by the zero lower bound and nominal rigidities, then the economy enters a liquidity trap with limited reallocation and low output. Nonresidential investment initially declines due to low demand, but later booms due to low interest rates during the liquidity trap. The boom induces a partial and asymmetric recovery in which the residential sector is left behind, consistent with the broad trends of the Great Recession. In view of aggregate demand externalities, welfare can be improved by ex-post policies that slow down the decumulation of residential capital, as well as ex-ante policies that restrict the accumulation of capital.

JEL Classification: E32, E22, E4

Keywords: The Great Recession, housing, residential investment, overbuilding, reallocation, liquidity trap, zero lower bound, investment, capital, accelerator, aggregate demand externality.

*We thank Monika Piazzesi, Giacomo Ponzetto, John Shea, and Michael Woodford for very helpful comments. We also thank the seminar participants at Columbia University, Cornell University, Georgetown University, Institute for International Economic Studies, London Business School, MIT, Northwestern University, Norwegian Business School, Princeton University, Sveriges Riksbank, Stanford University, University of Chicago; conference participants at Koc University for their comments.

[†]MIT, mrognlie@mit.edu

[‡]Harvard University and NBER, shleifer@fas.harvard.edu

[§]MIT and NBER, asimsek@mit.edu

1 Introduction

Since 2008, the US economy has been going through the worst macroeconomic slump since the Great Depression. Real GDP per capita declined from more than $49,000 in 2007 (in 2009 dollars) to less than $47,000 in 2009, and surpassed its pre-recession level only in 2013. The civilian employment ratio, which stood at about 63% in January 2008, fell below 58% by January 2010, and remained below 59% in June 2014.

Recent macroeconomic research emphasizes the bust of the housing bubble—and its effects on financial institutions, firms, and households—as the main culprit for these developments. The collapse of home prices arguably affected the economy through at least two principal channels. First, it triggered the *financial crisis*, which led financial institutions that suffered losses related to the housing market to cut back their lending to firms and households (Brunnermeier (2009), Gertler and Kiyotaki (2010)). Second, the reduction in home prices also generated a *household deleveraging crisis*, in which homeowners that suffered leveraged losses from their housing equity cut back their consumption so as to reduce their outstanding leverage (Guerrieri and Lorenzoni (2011), Eggertsson and Krugman (2012), Mian and Sufi (2014)). Both crises reduced aggregate demand, plunging the economy into a Keynesian recession. The recession was exacerbated by the zero lower bound on the nominal interest rate, also known as *the liquidity trap*, which restricted the ability of monetary policy to counter these demand shocks (Hall (2011), Christiano, Eichenbaum, Trabandt (2014)).

A growing body of empirical evidence shows that these views are at least partially correct: the financial and the household crises both appear to have played a part in the Great Recession.[1] But these views also face a challenge in explaining the nature of the recovery after the Great Recession. As Figure 1 illustrates, the recovery has been quite asymmetric across components of aggregate private spending. Nonresidential investment and consumption—measured as a fraction of output—reached or exceeded their pre-recession levels by 2013, while residential investment remained depressed. One explanation for this pattern is that households are unable to buy houses due to

[1]Several recent papers, such as Campello, Graham, and Harvey (2010) and Chodorow-Reich (2014), provide some evidence that financial crisis affected firms' investment before 2010. Mian, Rao, Sufi (2013) and Mian and Sufi (2014, 2015) provide evidence that household deleveraging reduced household consumption and employment between 2007 and 2009.

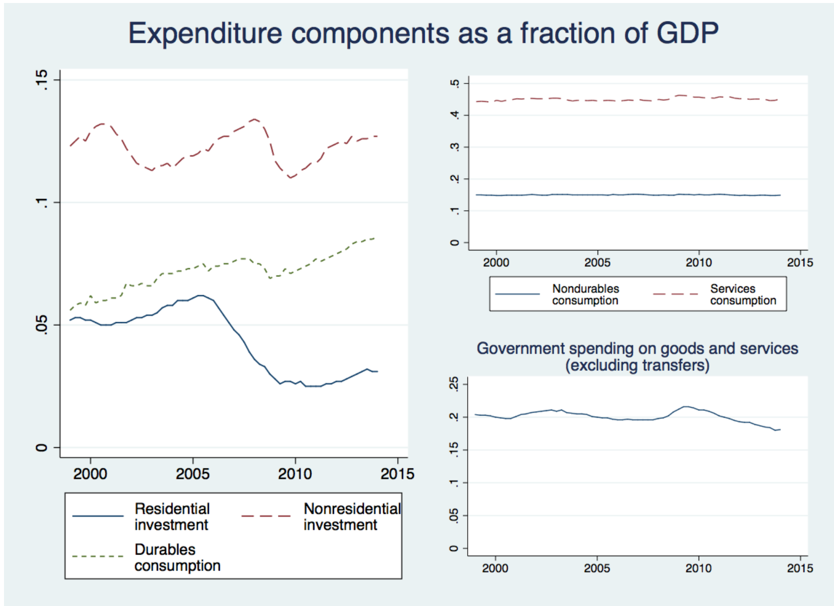

Figure 1: The plots illustrate different components of aggregate expenditure in the US as a fraction of GDP. The data is quarterly and reported as the seasonally adjusted annual rate. Source: St. Louis Fed.

ongoing deleveraging. But the left panel of Figure 1 casts doubt on this explanation: sales of durable consumption goods such as cars—which should also be affected by household deleveraging—rebounded strongly in recent years while residential investment has lagged behind. Another potential explanation is that there is a feature of the housing market in the US that makes residential investment generally lag behind in recoveries. This explanation is also incorrect: Leamer (2007) shows that residential investment has typically *led* the post-war recoveries in the US.

In this paper, we supplement the two accounts of the Great Recession with a third channel, which we refer to as the *investment hangover*, which could help to explain the asymmetric recovery. Our key observation is that the housing bubble was an *investment bubble* as much as an asset price bubble. Overbuilding during the bubble years created excess supply of housing capital by 2007, especially certain types of capital such as owner occupied housing. The top panel of Figure 2 illustrates that, between 1996 and 2005, the share of US households living in their own homes rose from about 65% to about 69%. The homeownership rate fell back below 65% in 2014, suggesting that the housing capital might have been in excess for many years after 2005. Our model's first prediction is that the excess housing capital lowers residential

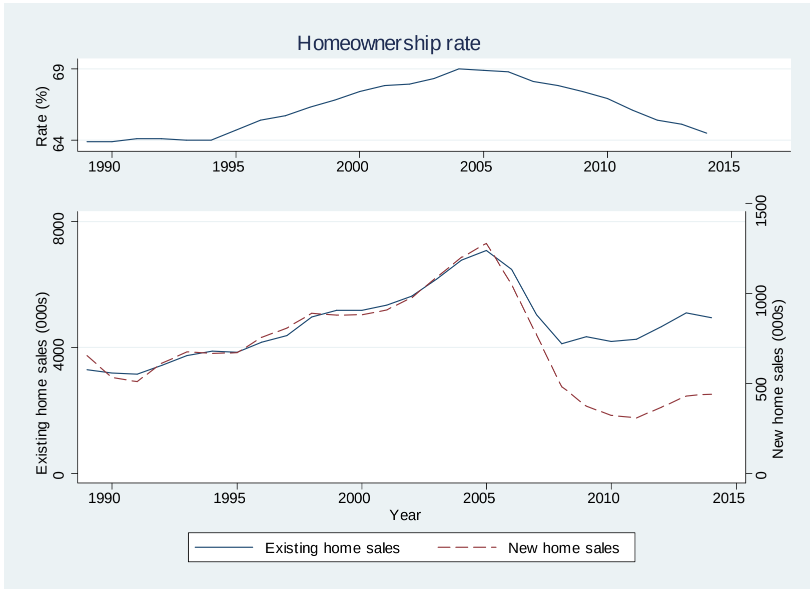

Figure 2: The top panel plots the homeownership rate in the US (source: US Bureau of the Census). The bottom panel plots the total sales of existing and new homes (source: National Association of Realtors). The data is annual.

investment because the existing capital provides a substitute for new investment. The bottom panel of Figure 2 provides evidence consistent with this prediction. The sales of newly constructed homes, which have historically changed in tandem with the sales of existing homes, sharply fell starting in 2005.

Our argument so far is similar to the Austrian theory of the business cycle, in which recessions are times at which excess capital built during boom years is liquidated (Hayek (1931)). The Hayekian view, however, faces a challenge in explaining how low investment in the liquidating sector reduces aggregate output and employment. As noted by Krugman (1998), the economy has a natural adjustment mechanism that facilitates the reallocation of labor (and other productive resources) from the liquidating sector to other sectors. As the interest rate falls during the liquidation phase due to low aggregate demand, other sectors expand and keep employment from falling. This reallocation process can be associated with some increase in frictional unemployment. But it is unclear in the Austrian theory how employment can fall in *both* the liquidating and the nonliquidating sectors, which seems to be the case for major recessions such as the Great Recession. To fit that evidence, an additional—Keynesian—aggregate demand mechanism is needed.

Accordingly, we depart from the Hayekian view by emphasizing that, during the Great Recession, the aggregate reallocation mechanism was undermined by the zero lower bound constraint on monetary policy. If the initial overbuilding is sufficiently large, then the interest rate hits a lower bound and the economy enters a liquidity trap. As this happens, low investment in the residential sector cannot be countered by the expansion of other sectors. Instead, low investment reduces aggregate demand and output, contributing to the Keynesian slump.

The overbuilding of residential capital can initially *reduce* nonresidential investment. The Keynesian slump reduces the return to nonresidential capital, such as equipment and machines, in view of the low demand. We show that this can generate an initial reduction in nonresidential investment, despite the low interest rate and the low cost of capital. This mechanism is reminiscent of the acceleration principle of investment that was emphasized in an older literature (see Clark (1917) or Samuelson (1939)), but there are also important differences that we clarify in Section 4.1.

As the economy liquidates the excess residential capital, nonresidential investment gradually recovers in anticipation of a recovery in output. In fact, the initial bust in nonresidential investment is followed by an even greater boom due to low interest rates. From the lens of our model, then, the recession can be roughly divided into two phases. In the first phase, both types of investment decline, generating a severe slump. In the second phase, nonresidential investment rises, generating a partial recovery, but residential investment remains low. The residential sector is left behind in the recovery, as in Figure 1.

We finally investigate the implications of our analysis for policies directed towards controlling investment. A naive intuition would suggest that the planner should not interfere with residential disinvestment, since the problems originate in this sector. We find that this intuition is incorrect: if the recession is sufficiently severe, then the planner optimally stimulates residential investment. Intuitively, the planner recognizes that raising residential investment in a liquidity trap stimulates aggregate demand and output. In view of these *aggregate demand externalities*, the planner perceives a lower cost of building during the liquidity trap compared to the private sector. We also find that, before the economy enters the liquidity trap, the planner (that respects agents' beliefs as well as preferences) optimally restricts investment in capital—including nonresidential capital. Intuitively, the planner reduces the accumulation of capital in earlier periods, so as to stimulate investment and aggregate demand during the liquidity trap. Taken together, our welfare results support policies that intertemporally substitute investment towards periods that feature deficient demand.

The rest of the paper is organized as follows. The next subsection discusses the related literature. Section 2 describes the basic environment, defines the equilibrium, and establishes the properties of equilibrium that facilitate subsequent analysis. The remaining sections characterize the dynamic equilibrium starting with excess residential capital. Section 3 presents our main result that excessive overbuilding induces a recession, and establishes conditions under which this outcome is more likely. Section 4 investigates the nonresidential investment response, and discusses the relationship of our model with the acceleration principle of investment. Section 5 addresses the welfare implications of our analysis and Section 6 concludes.

1.1 Related literature

Our paper makes contributions to several strands of the literature. First, we identify the ex-ante overbuilding of housing as an important source of deficient aggregate demand during the Great Recession. A large literature emphasizes other types of demand shocks such as those driven by financial frictions (Christiano, Eichenbaum, Trabandt (2014)) or household deleveraging (Eggertsson and Krugman (2012)).[2] Other papers emphasize long-run factors that might have lowered demand more persistently (Summers (2013), Eggertsson and Mehrotra (2014), Caballero and Farhi (2014)). We do not claim that other demand-reducing channels were unimportant during the Great Recession—in fact, we illustrate that these channels complement our mechanism. However, our analysis reveals that overbuilding of housing can reduce nonresidential investment earlier in the recession, even without any financial frictions or deleveraging. These confounding effects should be taken into account by quantitative analyses of the Great Recession.

Several recent papers investigate the role of housing during the Great Recession

[2] In addition to the papers mentioned earlier, see Gertler and Karadi (2011), Midrigan and Philippon (2011), Jermann and Quadrini (2012), He and Krishnamurthy (2014) for quantitative dynamic macroeconomic models that emphasize either banks', firms', or households' financial frictions during the Great Recession. There is also a vast theoretical literature that analyzes the amplification mechanisms that could have exacerbated the financial crisis (see Brunnermeier et al. (2013) for a survey). Another strand of the literature emphasizes uncertainty shocks as a contributing factor to the Great Recession (see, for instance, Bloom et al. (2012)).

using quantitative macroeconomic models, but without necessarily focusing on overbuilding. Among others, Iacoviello and Pavan (2013) and Favilukus et al. (2015) emphasize the collateral channel by which housing might have tightened household borrowing constraints, Kaplan and Violante (2015) emphasize that the high-returns associated with (illiquid) housing wealth might have increased the number of constrained households in equilibrium.[3] In recent work, Boldrin et al. (2013) also focus on overbuilding and the resulting slowdown in construction. Their paper complement ours in the sense that we illustrate how overbuilding might have reduced aggregate demand, whereas they investigate the supply-side input-output linkages by which the slowdown in construction might have spilled over to other sectors.

Second, and more broadly, we illustrate how having too much of a durable good can trigger a recession. As DeLong (1990) discusses, Hayekian (or liquidationist) views along these lines were quite popular before and during the Great Depression, but were relegated to the sidelines with the Keynesian revolution in macroeconomics. Our paper illustrates how Hayekian and Keynesian mechanisms can come together to generate a recession. The Hayekian mechanism finds another modern formulation in the recent literature on news-driven business cycles. A strand of this literature argues that positive news about future productivity can generate investment booms, occasionally followed by liquidations if the news is not realized (see Beaudry and Portier (2013) for a review). This literature typically generates business cycles from supply side considerations (see, for instance, Beaudry and Portier (2004), Jaimovich and Rebelo (2009)), whereas we emphasize a demand side channel.

In recent work, Beaudry, Galizia, Portier (BGP, 2014) also investigate channels by which overbuilding can induce a recession driven by deficient demand. Their paper is complementary in the sense that they use different ingredients and emphasize a different mechanism. In BGP, aggregate demand affects employment due to a matching friction in the labor market, whereas we obtain demand effects through nominal rigidities. In addition, BGP show how overbuilding increases the (uninsurable) unemployment risk, which exacerbates the recession due to households' precautionary savings motive. In contrast, we describe how overbuilding exacerbates the recession

[3] There is also a large literature that develops quantitative macroeconomic models with housing, but without focusing on the Great Recession or overbuilding, e.g., Greenwood and Hercowitz (1991), Gervais (2002), Iacoviello (2005), Campbell and Hercowitz (2005), Davis and Heathcote (2005), Fisher (2007), Piazzesi, Schneider, and Tuzel (2007), Kiyotaki, Michaelides, and Nikolov (2011).

due to the endogenous nonresidential investment response. We also apply our model to explain the asymmetric recovery from the Great Recession.

Third, our analysis illustrates how the liquidity trap (or more generally, constrained monetary policy) restricts the efficient reallocation of resources between sectors. A large macroeconomics literature investigates the role of reallocation shocks relative to aggregate activity shocks in generating unemployment (see, for instance, Lilien (1982), Abraham and Katz (1986), Blanchard and Diamond (1989), Davis and Haltiwanger (1990)). Our paper shows that the liquidity trap blurs the line between reallocation and aggregate activity shocks. In our setting, reallocation away from residential investment triggers a Keynesian recession. Moreover, nonresidential investment also declines earlier in the recession, generating sectoral comovement that resembles an aggregate activity shock. Caballero and Hammour (1996) describe a supply-side channel by which reallocation is restricted because the expanding sectors are constrained due to a hold-up problem.

Fourth, we obtain several positive and normative results for investment when the economy features a temporary liquidity trap. These results apply regardless of whether the episode is driven by overbuilding or some other (temporary) demand shock. A growing applied theoretical literature investigates various aspects of the liquidity trap, but often abstracts away from investment for simplicity (see, for instance, Krugman (1998), Eggertsson and Woodford (2003), Auerbach and Obstfeld (2005), Adam and Billi (2006), Jeanne and Svennson (2007),Werning (2012)).

On the positive side, we show that nonresidential investment can decline earlier in the liquidity trap, even if the real interest rate remains low and there are no financial frictions, because low aggregate demand also lowers the rate of return to investment. In recent work, Schmitt-Grohe and Uribe (2012) also show that the liquidity trap can generate an investment slump driven by low return (due to low aggregate demand).[4]

On the normative side, we show that the private investment decisions during or before a liquidity trap are typically inefficient, and characterize the optimal constrained policy interventions.[5] These results complement a recent literature that describes

[4]This mechanism is also present in many other New Keynesian models with capital, especially when the monetary policy is constrained, but it is not always emphasized. For instance, Christiano et al. (2011) illustrate that the investment slump during the liquidity trap exacerbates the government spending multiplier. However, they argue that the investment decline is mainly driven by an increase in the real interest rate (due to deflation) as opposed to the decline in the rate of return.

[5]Coreira et al. (2013) also analyze the optimal policy in a New Keynesian model with capital

the inefficiencies driven by aggregate demand externalities when the monetary policy is constrained. Korinek and Simsek (2014) and Farhi and Werning (2013) illustrate the inefficiencies associated with ex-ante financial market allocations, such as leverage and insurance, whereas we establish the inefficiencies associated with physical investment.[6]

2 Basic environment and equilibrium

The economy is set in infinite discrete time $t \in \{0, 1, ...\}$ with a single consumption good, and three factors of production: residential capital, h_t, nonresidential capital, k_t, and labor, l_t. For brevity, we also refer to nonresidential capital as "capital." Each unit of residential capital produces one unit of housing services. Capital and labor are combined to produce the consumption good according to a neoclassical technology that we describe below.

As we will show, absent shocks, the economy converges to a level of residential capital denoted by h^*, which we refer to as *the target level*. We analyze situations in which the economy starts with an initial residential capital that exceeds the target, $h_0 > h^*$ (see Eq. (17) below), so that an adjustment is necessary.

The assumption, $h_0 > h^*$, can be interpreted in several ways. Our favorite interpretation is that it captures an unmodeled overbuilding episode that took place before the start of our model. In particular, suppose the (expected) housing demand increased in the recent past relative to its "historical" level. The economy has built residential capital to accommodate this high level of demand, captured by h_0. At date 0, the economy receives news that that the high demand conditions are not sustainable. The residential capital stock needs to adjust to its historical average, captured by h^*. Section 5.2 introduces an ex-ante period and formalizes this interpretation.

An alternative and mathematically equivalent interpretation is that h_0 corresponds to the historical housing demand, whereas h^* represents "the new normal"

and the liquidity trap, and show that the first best allocation can be replicated with a sufficiently rich set of tax instruments. Our constrained planning exercise applies as long as the planner cannot completely avoid the liquidity trap with alternative policy tools, which seems to be consistent with the recent experience in various developed economies.

[6] A separate literature describes the inefficiencies in physical investment driven by pecuniary externalities (see, for instance, Lorenzoni (2008), Hart and Zingales (2011), Stein (2011), He and Kondor (2014), Davila (2015)).

with permanently low housing demand (e.g., due to financial frictions in the mortgage market). One could also imagine intermediate interpretations in which both overbuilding in the past (i.e., the supply overhang) and low current demand (i.e., the demand shortage) plays some role in driving the adjustment. We would like to understand how the economy decumulates the excess residential capital.

In our baseline setting, we also abstract away from adjustment costs. In particular, one unit of the consumption good can be converted into one unit of residential or nonresidential capital or vice versa (see Section 4.1 for a version with adjustment costs for residential capital). Thus, the two types of capital evolve according to,

$$h_{t+1} = h_t \left(1 - \delta^h\right) + i_t^h \text{ and } k_{t+1} = k_t \left(1 - \delta^k\right) + i_t^k. \tag{1}$$

Here, i_t^h (resp. i_t^k) denote residential (resp. nonresidential) investment, and δ^h (resp. δ^k) denotes the depreciation rate for residential (resp. nonresidential) capital.

Households The economy features a representative household with preferences over consumption, labor, and housing services. We assume households' per period utility function takes the following form,

$$U\left(\hat{c}_t, l_t, h_t\right) = u\left(\hat{c}_t - v\left(l_t\right)\right) + u^h \mathbf{1}\left[h_t \geq h^*\right]. \tag{2}$$

Here, the functions $u(\cdot), v(\cdot)$ satisfy the standard regularity conditions. The expression $\mathbf{1}\left[h_t \geq h^*\right]$ is equal to 1 if $h_t \geq h^*$ and zero otherwise, and u^h is a large constant.

This specification of preferences relies on two simplifying assumptions. First, households always demand the target level of residential capital h^* in the sense that they receive a large marginal utility from units up to this level but zero marginal utility from additional units. Given this specification and the lack of adjustment costs, the household optimally chooses,[7]

$$h_{t+1} = h^*, \text{ which implies } i_t^h = h^* - h_t\left(1 - \delta^h\right). \tag{3}$$

[7] Residential capital not only provides housing services but it also represents an investment technology. Hence, Eq. (3) also requires the gross interest rate, $1 + r_{t+1}$, to be greater than the gross return (on empty houses), $1 - \delta^h$, which will be the case in equilibrium.

In particular, starting with some $h_0 > h^*$, the household decumulates its excess residential capital in a single period. Hence, a period in the baseline setting should be thought of as long as necessary (arguably several years) to adjust the housing capital to its steady-state level. This specification is admittedly extreme and mechanical, but it considerably simplifies the residential investment part of the model, and enables us to focus on the effect of overbuilding on the rest of the equilibrium allocations. Residential investment would qualitatively follow a similar pattern in models with more elastic housing demand.

Second, the functional form $u(\hat{c}_t - v(l_t))$, implies that the household's labor supply decision does not depend on its consumption (see Greenwood, Hercowitz and Huffman (1988)). Specifically, households' optimal labor solves the static optimization problem,

$$e_t = \max_{l_t} w_t l_t - v(l_t). \tag{4}$$

Here, e_t denotes households' net labor income, that is, labor income net of labor costs. We also define $c_t = \hat{c}_t - v(l_t)$ as *net consumption*. Households' consumption and saving problem can then be written in terms of net variables as:

$$\max_{\{c_t, a_{t+1}\}_t} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{5}$$

$$\text{s.t.} \qquad c_t + a_{t+1} + i_t^h = e_t + a_t(1 + r_t) + \Pi_t.$$

Here, a_t denotes her financial assets, w_t denotes the wage level, and Π_t denotes profits received from firms described below, and i_t^h denotes the optimal level of residential investment characterized by Eq. (3). The optimal household behavior is summarized by Eq. (3) and problems (4) and (5).

Investment, the interest rate, and the liquidity trap The capital stock of the economy is managed by a competitive investment sector. This sector issues financial assets to households, invests in capital, and rents the capital to the production firms that will be described below. The optimality conditions for the sector imply that the interest rate (equivalently, the cost of capital) satisfies,

$$r_{t+1} = R_{t+1} - \delta^k, \tag{6}$$

where R_{t+1} denotes the rental rate of capital. The capital market clearing condition is $a_t = k_t$.

Our key ingredient is that the nominal interest rate is bounded from below, that is, $r^n_{t+1} \geq 0$. This constraint emerges because cash in circulation provides households with transaction services.[8] If the nominal interest rate fell below zero, then individuals would switch to hoarding cash instead of holding financial assets. Therefore, monetary policy cannot lower the nominal interest rate below zero. The situation in which the nominal interest rate is at its lower bound is known as *the liquidity trap*.

The constraint on the nominal interest rate might not affect the real allocations by itself. However, we also assume that nominal prices are completely sticky (as we formalize below) which ensures that the nominal and the real interest rates are the same, and thus the real interest rate is also bounded,

$$r^n_{t+1} = r_{t+1} \geq 0 \text{ for each } t. \tag{7}$$

Production firms and output We introduce nominal price rigidities with the standard New Keynesian model. Specifically, there are two types of production firms. A competitive final good sector uses intermediate varieties $\nu \in [0,1]$ to produce the final output according to the Dixit-Stiglitz technology,

$$\hat{y}_t = \left(\int_0^1 \hat{y}_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\varepsilon/(\varepsilon-1)} \text{ where } \varepsilon > 1. \tag{8}$$

In turn, a unit mass of monopolistic firms labeled by $\nu \in [0,1]$ each produces the variety according to,

$$\hat{y}_t(\nu) = F(k_t(\nu), l_t(\nu)), \tag{9}$$

where $F(\cdot)$ is a neoclassical production function that satisfies the standard regularity conditions.

[8] To simplify the notation and the exposition, however, we do not explicitly model cash or its transaction services. We could incorporate these features into the model without changing anything essential. For instance, suppose households' state utility function takes the separable form, $u(c_t) + \varphi(m_t)$, where $m_t = M_t/P_t$ denotes households' real money balances and $\varphi(\cdot)$ is an increase and concave function with $\varphi'(\overline{m}) = 0$ for some $\overline{m}$. With this specification, the non-monetary equilibrium allocations remain unchanged. The equilibrium money balances are obtained by solving $\varphi'(m_t) = \frac{r^n_{t+1}}{1+r^n_{t+1}} u'(c_t)$ for each t, given the equilibrium levels of the nominal interest rate, r^n_{t+1}, and net consumption, c_t.

We make the extreme assumption that each monopolist has a preset and constant nominal price, $P_t(\nu) = P$ for each ν. This also implies that monopolists are symmetric: they face the same real price (equal to one) and they choose the same level of inputs and outputs subject to an aggregate demand constraint. In particular, the representative monopolist's problem can be written as:

$$\Pi_t = \max_{k_t, l_t} F(k_t, l_t) - w_t l_t - R_t k_t \text{ s.t. } F(k_t, l_t) \leq \hat{y}_t. \tag{10}$$

In the equilibria we will analyze, the monopolist's marginal cost will be below its price so that it will find it optimal to meet all of its demand. Thus, the output satisfies $\hat{y}_t = F(k_t, l_t)$. In view of GHH preferences, we find it more convenient to work with *the net output*, that is, output net of labor costs,

$$y_t = \hat{y}_t - v(l_t) = F(k_t, l_t) - v(l_t).$$

Efficient benchmark and the monetary policy It follows that the outcomes in this model are ultimately determined by the (net) aggregate demand for the final good, $y_t = c_t + i_t^k + i_t^k$. Since the price level is fixed, we assume that the monetary policy focuses on replicating the efficient allocations.[9] We next define the efficient allocations, which we then use to formalize the monetary policy.

Given date t with state variables $k_t, h_t \geq h^*$, *the efficient benchmark* is the continuation allocation that maximizes households' welfare subject to the feasibility constraints. The appendix shows that the efficient benchmark is characterized as the solution to the problem,

$$\begin{aligned} V(k_t, h_t) &= \max_{\{c_{\tilde{t}}, k_{\tilde{t}+1}\}_{\tilde{t}=t}^{\infty}} \sum_{\tilde{t}=t}^{\infty} \beta^{\tilde{t}} u(c_{\tilde{t}}), \\ \text{s.t.} \quad & c_{\tilde{t}} + k_{\tilde{t}+1} - \left(1 - \delta^k\right) k_{\tilde{t}} + i_{\tilde{t}}^h = s(k_{\tilde{t}}). \end{aligned} \tag{11}$$

[9] In particular, the monetary policy does not replicate the "frictionless" equilibrium that would obtain if monopolists could reset their prices at every period. Instead, the monetary policy also corrects for the distortions that would stem from the monopoly pricing. Ideally, these distortions should be corrected by other policies, e.g., monopoly subsidies, and the monetary policy should focus on replicating the frictionless benchmark. We ignore this distinction so as to simplify the notation.

Here, $i_{\tilde{t}}^h$ is given by Eq. (3), and the function $s(\cdot)$ is defined as,

$$s(k_t) = F(k_t, l_t^*) - v(l_t^*), \text{ where } l_t^* \in \arg\max_{\tilde{l}} F\left(k_t, \tilde{l}\right) - v\left(\tilde{l}\right). \tag{12}$$

In particular, the efficient benchmark maximizes the net output in every period. We refer to $y_t^* = s(k_t)$ as *the efficient (or supply determined) level of net output,* and l_t^* as the efficient level of employment. The residential investment path is the same as in the competitive equilibrium. The rest of the efficient dynamic allocations are found by solving a standard neoclassical planning problem.[10]

Given the efficient benchmark, we assume the monetary policy sets the interest rate at date t according to,

$$r_{t+1}^n = r_{t+1} = \max\left(0, r_{t+1}^*\right). \tag{13}$$

Here, r_{t+1}^* is recursively defined as *the natural interest rate* that obtains when employment and net output at date t are at their efficient levels, and the monetary policy follows the rule in (13) at all future dates. In particular, the monetary policy replicates the statically efficient allocations subject to the lower bound constraint on the interest rate. This policy is constrained efficient in our environment as long as the monetary policy does not have commitment power.

Definition 1. *The equilibrium is a path of allocations,* $\left\{h_t, k_t, l_t, \hat{c}_t, c_t, i_t^h, i_t^k, \hat{y}_t, y_t\right\}_t$*, and real prices and profits,* $\{w_t, R_t, r_{t+1}, \Pi_t\}_t$*, such that the household allocations satisfy* (3) *and solve problems* (4) *and* (5)*, a competitive final good sector produces according to* (8)*, the intermediate good monopolists solve* (10) *for given fixed goods prices, the interest rate is set according to* (13)*, and all markets clear.*

3 Investment hangover and the recession

We next turn to the characterization of equilibrium. The following lemma establishes the basic properties of the equilibrium within a period.

Lemma 1. *(i) If* $r_{t+1} > 0$*, then* $y_t = s(k_t), l_t = l_t^*$*, and* $R_t = s'(k_t)$*.*

[10] To ensure an interior solution, we assume the parameters are such that the economy is able to afford the required residential investment at all dates. A sufficient condition is $\min(s(k_0), s(k^*)) > \delta^k k^* + \delta h^*$.

(ii) If $r_{t+1} = 0$, then the net output is below the efficient level, $y_t \leq s(k_t)$, and is determined by net aggregate demand, $y_t = c_t + i_t^k + i_t^h$. The labor supply is below its efficient level, $l_t \leq l_t^*$, and is determined as the unique solution to,

$$y_t = F(k_t, l_t) - v(l_t) \text{ over the range } l_t \in [0, l_t^*]. \tag{14}$$

The rental rate of capital is given by $R_t = R(k_t, y_t) \leq s'(k_t)$, where the function $R(k_t, y_t)$ is strictly decreasing in k_t and strictly increasing in y_t.

Part (i) describes the case in which the interest rate is positive and the monetary policy replicates the efficient outcomes. Part (ii) describes *the liquidity trap* scenario in which the interest rate is at its lower bound. In this case, the economy experiences a recession with low net output and employment.

Lemma 1 also characterizes the rental rate of capital in each case, which determines the return to investment. To understand these results, consider monopolists' factor demands, captured by the optimality conditions for problem (10),

$$(1 - \tau_t) F_k(k_t, l_t) = R_t \text{ and } (1 - \tau_t) F_l(k_t, l_t) = w_t. \tag{15}$$

Here, $\tau_t \geq 0$ denotes the Lagrange multiplier on the demand constraint in (10), which is also *the labor wedge* in this model. If the interest rate is positive, then employment is at its efficient level and the labor wedge is zero, $\tau_t = 0$. In this case, the demand constraint effectively does not bind and the factors earn their marginal products. If instead the interest rate is zero, then the employment is below its efficient level and the labor wedge is positive, $\tau_t > 0$. In this case, the demand shortage lowers capital's (as well as labor's) rental rate relative to the efficient benchmark. The second part of the lemma shows further that the return to capital in this case can be written as a function of the capital stock and net output. Greater k_t reduces the rental rate due to diminishing returns, whereas greater y_t increases it due to greater demand.

Combining Eq. (6), Lemma 1, and the lower bound in (7) further implies that the capital stock is bounded from above, that is,

$$k_{t+1} \leq \overline{k} \text{ for each } t, \text{ where } s'\left(\overline{k}\right) - \delta^k = 0. \tag{16}$$

Here, the upper bound $\overline{k}$ is the level of capital that delivers a net return of zero absent a demand shortage. Investing beyond this level would never be profitable given the lower bound to the cost of capital in (7) (as well as the possibility of a demand shortage). This bound will play a central role in the subsequent analysis.

3.1 Investment hangover

We next characterize the dynamic equilibrium under the assumption that the economy starts with too much residential capital,

$$h_0 = (1 + b_0)\, h^*, \text{ where } b_0 > 0. \tag{17}$$

Here, b_0 parameterizes the degree of past overbuilding as a fraction of the target level of residential capital, h^*.

By Eq. (3), the residential investment level at date 0 is given by,

$$i_0^h = h^* - \left(1 - \delta^h\right) h_0 = \left(\delta^h - b_0 \left(1 - \delta^h\right)\right) h^*. \tag{18}$$

Note that residential investment is below the level required to maintain the target, $i_0^h < \delta^h h^*$. Overbuilding represents a negative shock to the residential investment demand. The equilibrium depends on how the remaining components of aggregate demand—nonresidential investment and consumption—respond to this shock.

To characterize this response, we solve the equilibrium backwards. Suppose the economy reaches date 1 with $h_1 = h^*$ and some capital level $k_1 \leq \overline{k}$. Since the residential capital has already adjusted, the continuation equilibrium does not feature a liquidity trap, that is, $r_{t+1} > 0$ for each $t \geq 1$. Consequently, monetary policy replicates the efficient benchmark starting date 1. The equilibrium is the solution to problem (11) given $h_1 = \overline{h}$ and $k_1 \leq \overline{k}$. The appendix shows that the solution converges to a steady-state (c^*, k^*), characterized by

$$s'(k^*) - \delta^k = 1/\beta - 1 \text{ and } c^* = s(k^*) - \delta^k k^* - \delta^h h^*. \tag{19}$$

The initial consumption can be written as $c_1 = C(k_1)$, where $C(\cdot)$ is an increasing function.

Next consider the equilibrium at date 0. The key observation is that both nonresidential investment and consumption are bounded from above due to the lower bound on the interest rate. In particular, the bound on capital in Eq. (16) also implies the following bound on nonresidential investment,

$$i_1^k \leq \overline{k} - \left(1 - \delta^k\right) k_0.$$

Intuitively, only so many investment projects can be undertaken without violating the lower bound on the safe (or more broadly, risk adjusted) cost of capital. Likewise, we have a bound on consumption determined by the Euler equation at the zero interest rate,

$$c_0 \leq \overline{c}_0, \text{ where } u'\left(\overline{c}_0\right) = \beta u'\left(C\left(\overline{k}\right)\right). \tag{20}$$

Intuitively, the household can only be induced to consume so much without violating the lower bound on the interest rate.

Combining the bounds in (16) and (20) with the demand shock in (18), the aggregate demand (and output) at date 0 is also bounded from above, that is,

$$y_0 \leq \overline{y}_0 \equiv \overline{k} - \left(1 - \delta^k\right) k_0 + \overline{c}_0 + \left(\delta^h - b_0\left(1 - \delta^h\right)\right) h^*. \tag{21}$$

The equilibrium depends on the comparison between the maximum demand and the efficient level, i.e., whether $\overline{y}_0 < s\left(k_0\right)$. This in turn depends on whether the amount of overbuilding b_0 exceeds a threshold level,

$$\overline{b}_0 \equiv \frac{\overline{k} - \left(1 - \delta^k\right) k_0 + \overline{c}_0 + \delta^h h^* - s\left(k_0\right)}{\left(1 - \delta^h\right) h^*}. \tag{22}$$

Proposition 1 (Overbuilding and the Liquidity Trap)**.** *Consider the model with* $b_0 > 0$ *(and thus* $h_0 > \overline{h}$*).*

(i) Suppose $b_0 \leq \overline{b}_0$*. Then, the date* 0 *equilibrium features*

$$r_1 \geq 0, y_0 = s\left(k_0\right) \text{ and } l_0 = l_0^*.$$

(ii) Suppose $b_0 > \overline{b}_0$*. Then, the date* 0 *equilibrium features a liquidity trap with*

$$r_1 = 0, k_1 = \overline{k}, y_0 = \overline{y}_0 < s\left(k_0\right) \text{ and } l_0 < l_0^*.$$

Moreover, the net output y_0 *and the labor supply* l_0 *are decreasing in the amount of overbuilding* b_0*.*

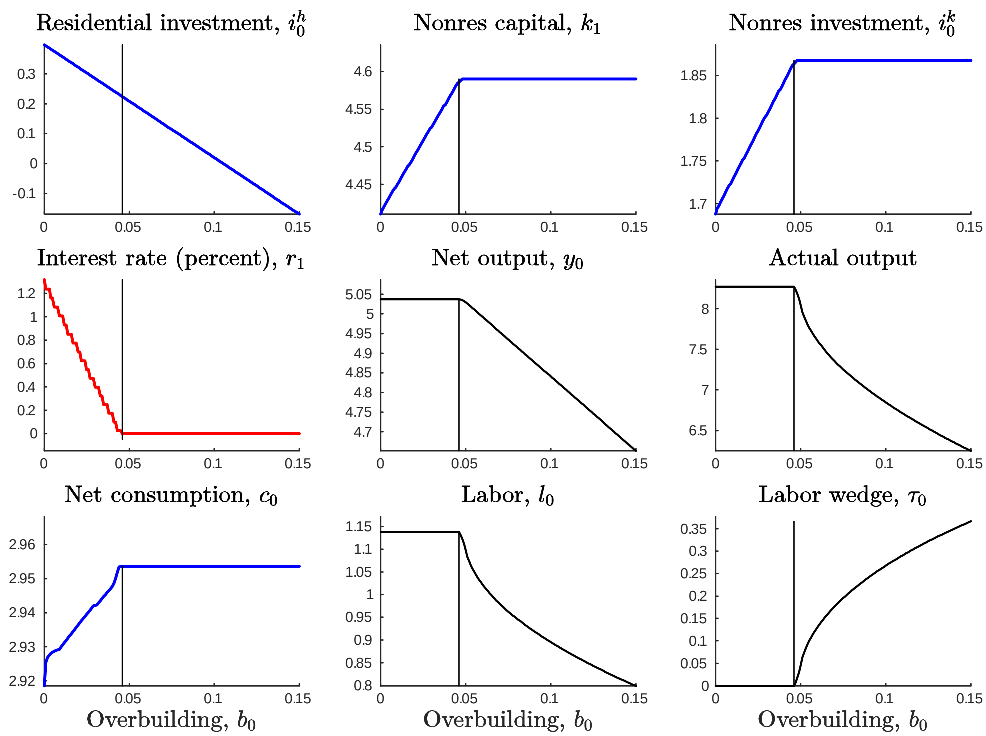

Figure 3: Date 0 equilibrium variables as a function of the initial overbuilding b_0 (measured as a fraction of the target residential capital stock, h^*).

In either case, starting at date 1*, the economy converges to the steady state* (k^*, c^*) *characterized by* (19)*.*

Part (i) describes the equilibrium for the case in which the initial overbuilding is not too large. In this case, the economy does not fall into a liquidity trap. Residential disinvestment is offset by a reduction in the interest rate and an increase in nonresidential investment and consumption, leaving the output and employment determined by productivity. The left part of the panels in Figure 3 (the range corresponding to $b_0 \leq \bar{b}_0$) illustrate this outcome. This is the Austrian case.

Part (ii) of Proposition 1, our main result, characterizes the case in which the initial overbuilding is sufficiently large. In this case, the demand shock associated with residential disinvestment is large enough to plunge the economy into a liquidity trap. The lower bound on the interest rate prevents the nonresidential investment and consumption sectors from expanding sufficiently to pick up the slack aggregate demand. As a consequence, the initial shock translates into a Keynesian recession with low output and employment. Figure 3 illustrates this result. A greater initial residential shock—driven by greater overbuilding—triggers a deeper recession. This is the Keynesian case of our model.

Figure 3 also illustrates the welfare implications of overbuilding. In the liquidity trap region, $b_0 \geq \bar{b}_0$, increasing the initial stock of housing, b_0, leaves the initial net consumption, c_0, unchanged. It can also be checked that it leaves the welfare of the representative household unchanged.[11] Perhaps surprisingly, starting the economy with more residential capital (or conversely, destroying some residential capital) neither raises nor lowers welfare. This effect is driven by *aggregate demand externalities* associated with residential investment, which we explain further in Section 5.

3.2 Comparative statics of the liquidity trap

We next investigate the conditions under which a given amount of overbuilding b_0 triggers a liquidity trap. As illustrated by Eq. (22), factors that reduce aggregate demand at date 0, such as a higher discount factor β (that lowers $\overline{c}_0$), increase the incidence of the liquidity trap in our setting. More generally, other frictions that reduce aggregate demand during the decumulation phase, such as household deleveraging or the financial crisis, are also complementary to our mechanism. Intuitively, this is because the liquidity trap represents a nonlinear constraint on the interest rate. In particular, a demand shock that lowers the interest rate leaves a smaller slack for monetary policy, increasing the potency of other demand shocks such as overbuilding.

Eq. (22) illustrates that a higher initial level of nonresidential capital stock k_0 also increases the incidence of a liquidity trap. A higher k_0 affects the equilibrium at date 0 through two main channels. First, a higher k_0 reduces nonresidential investment at date 0 and lowers aggregate demand. Second, a higher k_0 also increases the efficient output, $s(k_0)$, which makes it more likely that the economy will have a demand shortage. Overbuilding of the two types of capital is complementary in terms of triggering a liquidity trap.

A distinguishing feature of residential capital is its high durability relative to other types of capital. We next investigate whether high durability is conducive to triggering a liquidity trap in our setting. To isolate the effect of durability, consider a slight

[11] In this model, the initial net consumption, c_0, is a sufficient statistic for welfare, both because the net consumption takes into account labor costs and because it changes in tandem with future net consumption.

variant of the model in which there are two types of residential capital that mainly differ in terms of durability. Specifically, the two types depreciate at different rates given by δ^{h^d} and δ^{h^n}, with $\delta^{h^d} < \delta^{h^n}$. Thus, type d (durable) residential capital has a lower depreciation rate than type n (nondurable) residential capital. Suppose the preferences in (2) are modified so that each type has a target level $h^*/2$. Suppose also that $\left(\delta^{h^d} + \delta^{h^n}\right)/2 = \delta^h$ so that the average depreciation rate is the same as before. Let $h_0^d = \left(1 + b_0^d\right)(h^*/2)$ and $h_0^n = (1 + b_0^n)(h^*/2)$, so that b_0^d and b_0^n capture the overbuilding in respectively durable and nondurable capital. The case with symmetric overbuilding, $b_0^d = b_0^n = b_0$, results in the same equilibrium as in the earlier model. Our next result investigates the effect of overbuilding one type of capital more than the other.

Proposition 2 (Role of Durability)**.** *Consider the model with two types of residential capital with different depreciation rates. Given the average overbuilding $b_0 = \left(b_0^d + b_0^n\right)/2$, the incidence of a liquidity trap $1\left[l_0 < l_0^*\right]$ is increasing in overbuilding of the more durable residential capital b_0^d.*

To obtain an intuition, consider the maximum aggregate demand at date 0, which can be written as [cf. Eq. (21)],

$$\overline{y}_0 = \overline{k} - \left(1 - \delta^k\right) k_0 + \overline{c}_0 + \delta^h h^* - b_0^d \left(1 - \delta^{h^d}\right) \frac{h^*}{2} - b_0^n \left(1 - \delta^{h^n}\right) \frac{h^*}{2}. \qquad (23)$$

Note that $1 - \delta^{h^d} > 1 - \delta^{h^n}$, and thus, overbuilding of the durable residential capital (relative to the nondurable capital) induces a greater reduction in aggregate demand at date 0. Intuitively, depreciation helps to "erase" the overbuilt capital naturally, thereby inducing a smaller reduction in investment. When capital is more durable, there is less natural erasing. This in turn leads to lower investment and aggregate demand, and makes a liquidity trap more likely. This result suggests that overbuilding is more of a concern when it hits durable capital such as residential investment, structures, or infrastructure (e.g., railroads), as opposed to less durable capital such as equipment or machinery.[12]

[12] This observation should be taken with caution since different types of capital often differ in many dimensions in addition to durability, and those other dimensions might also interfere with the overbuilding mechanism. One important difference between nonresidential and residential capital is that the former affects the market production function whereas the latter does not. In particular, overbuilding nonresidential capital not only reduces demand, but it also increases the production

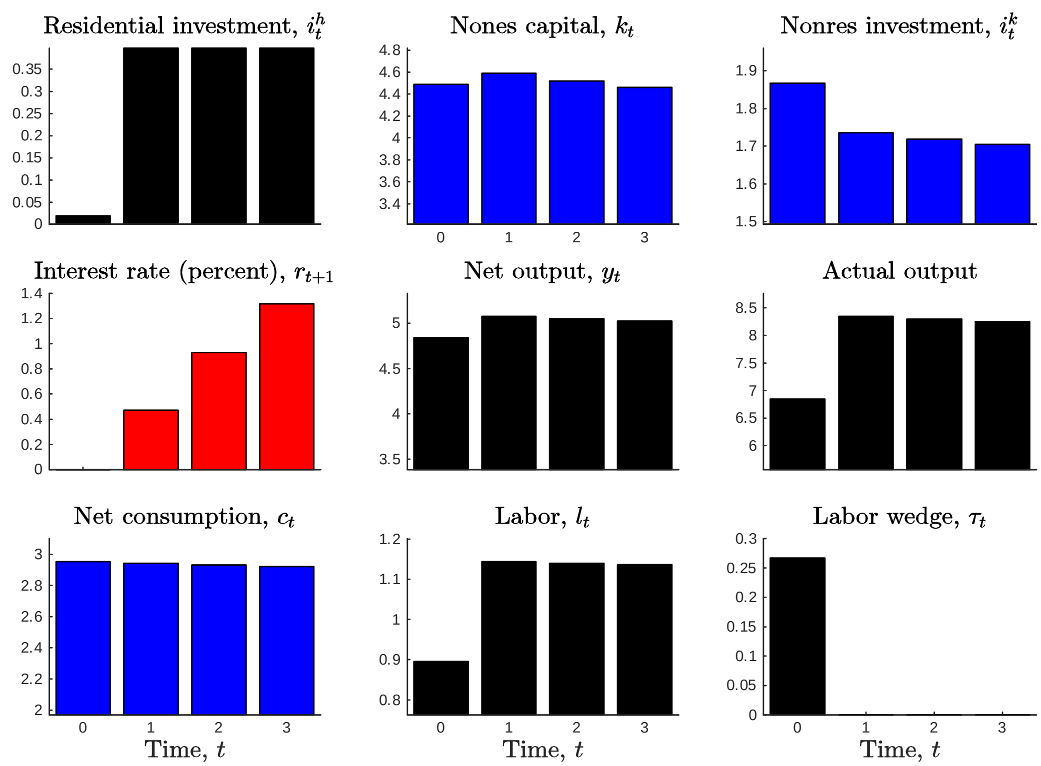

Figure 4: The evolution of equilibrium variables over time, starting with $b_0 > \bar{b}_0$.

3.3 Aftermath of the recession

We next investigate the equilibrium behavior in the aftermath of the liquidity trap. Figure 4 plots the full dynamic equilibrium in the original model with single residential capital (and in the liquidity trap scenario). The initial shock generates a temporary recession, followed by a neoclassical adjustment after the recession.

The interest rate gradually increases during the aftermath of the recession, and might remain below its steady-state level for several periods. This is because the economy accumulates capital during the liquidity trap thanks to low interest rates. The economy decumulates this capital only gradually over time, which leaves the rate of return low after the recession. These low rates are reminiscent of the secular stagnation hypothesis, recently revived by Summers (2013). According to this hypothesis, the economy could permanently remain depressed with low interest rates due to a chronic demand shortage (see Eggertsson and Mehrotra (2014) for a formalization). In our model, the economy eventually recovers. But the low rates in the aftermath suggest that the economy remains fragile to another demand shock. Intuitively, the economy has used much of its investment capacity to fight the reduction of demand at date 0, which leaves little capacity to fight another demand shock going forward.

Figure 4 illustrates further that, while there is a recession at date 0, several components of aggregate demand—especially nonresidential investment—actually expand. The recession is confined to the residential investment sector in which the shock originates. This prediction is inconsistent with facts in major recessions, such as the Great Recession, in which all components of aggregate demand decline simultaneously. To address this puzzle, we next analyze the investment response in more detail.

4 Investment response and the accelerator

This section investigates a variant of the model in which the liquidity trap persists over multiple periods. We show how the overbuilding of residential capital can induce an initial bust in nonresidential investment followed by a boom. We also discuss the relationship of our model to the acceleration principle of investment. We finally

capacity—captured by the efficient supply, $s(k_0)$, in our model. This channel suggests that, ceteris paribus (in particular, controlling for durability), overbuilding nonresidential capital might be more likely to trigger a liquidity trap than overbuilding residential capital.

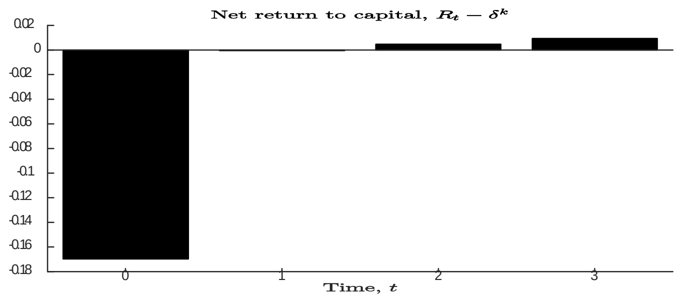

Figure 5: The evolution of net return to capital over time, starting with $b_0 > \bar{b}_0$.

briefly discuss how our model can be further extended to induce an initial reduction in consumption.

The analysis is motivated by Figure 5, which illustrates the evolution of the net return to capital $R_t - \delta^k$ corresponding to the equilibrium plotted in Figure 4. The near-zero return during the recovery phase reflects the high level of accumulated capital. The figure illustrates that the net return at date 0, given the predetermined capital stock k_0, is even lower. Intuitively, the recession at date 0 lowers not only the output but also factor returns, including the return on capital (see Lemma 1). This suggests that, if nonresidential investment could respond to the shock during period 0, it could also fall.

To investigate this possibility, we modify the model so that the residential disinvestment is spread over many periods. One way to ensure this is to assume that there is a lower bound on housing investment at every period.

Assumption 1. $i_t^h \geq \underline{i}^h$ for each t, for some $\underline{i}^h < \delta^h h^*$.

For instance, the special case $\underline{i}^h = 0$ captures the irreversibility of housing investment. More generally, the lower bound provides a tractable model of adjustment costs. To simplify the exposition, we also assume that the initial housing capital, $h_0 = h^* (1 + b_0)$, is such that the economy adjusts to the target level in exactly $T \geq 1$ periods.

Assumption 2. $\delta^h h^* = \left(\delta^h h_0\right) \left(1 - \delta^h\right)^T + \underline{i}^h \left(1 - \left(1 - \delta^h\right)^T\right)$ for an integer $T \geq 1$.

With these assumptions, the residential investment path is given by

$$i_t^h = \begin{cases} \underline{i}^h < \delta^h h^* & \text{if } t \in \{0, ..., T-1\} \\ \delta^h h^* & \text{if } t \geq T \end{cases}, \tag{24}$$

For future reference, note that the parameter i^h also provides an (inverse) measure of the severity of the residential investment shock.

As before, we characterize the equilibrium backwards. The economy reaches date T with residential capital $h_T = h^*$ and some $k_T \leq \overline{k}$. The continuation equilibrium is characterized by solving problem (11) as before. In particular, consumption is given by $c_T = C(k_T)$, where recall that $C(\cdot)$ is an increasing function.

Next consider the equilibrium during the decumulation phase, $t \in \{0, .., T-1\}$. We conjecture that—under appropriate assumptions—there is an equilibrium that features a liquidity trap at all of these dates, that is, $r_{t+1} = 0$ for each $t \in \{0, .., T-1\}$. In this equilibrium, the economy reaches date T with the maximum level of capital, $k_T = \overline{k}$ (since $r_T = 0$). Consumption is also equal to its maximum level, that is, $c_t = \overline{c}_t$ for each t, where

$$u'(\overline{c}_t) = \beta u'(\overline{c}_{t+1}) \text{ for each } t \in \{0, 1, .., T-1\}.$$

It remains to characterize the path of the capital stock $\{k_t\}_{t=1}^{T-1}$ during the decumulation phase.

To this end, consider the investment decision at some date $t-1$, which determines the capital stock at date t. The net return from this investment is given by $R(k_t, y_t) - \delta^k$ (cf. Lemma 1). The net cost of investment is given by $r_t = 0$. The economy invests at date $t-1$ up to the point at which the benefits and costs are equated,

$$R(k_t, y_t) - \delta^k = 0 \text{ for each } t \in \{1, .., T-1\}. \tag{25}$$

Recall that the return function $R(\cdot)$ is decreasing in the capital stock k_t and increasing in net output y_t. Hence, Eq. (25) says that, if the (expected) output at date t is large, then the economy invests more at date $t-1$ and obtains a greater capital stock at date t.

The level of output is in turn determined by the aggregate demand at date t,

$$y_t = \overline{c}_t + k_{t+1} - \left(1 - \delta^k\right) k_t + i^h \text{ for each } t \in \{0, ..., T-1\}. \tag{26}$$

Eqs. (25) and (26) represent a difference equation that can be solved backwards starting with $k_T = \overline{k}$. The resulting path corresponds to an equilibrium as long as $s(k_0) > y_0$, so that there is a liquidity trap in the first period as we have conjectured.

The next result establishes that this is the case if the shock is sufficiently severe, as captured by low i^h, and characterizes the behavior of nonresidential capital in equilibrium.[13]

Proposition 3 (Nonresidential Investment Response)**.** *Consider the model with the adjustment length $T \geq 2$. Suppose Assumptions 1-2 and Assumption 3 in Appendix B hold.*[14]

(i) There exists $i^{h,1}$ such that if $i^h < i^{h,1}$, then there is a unique equilibrium path $\{k_t, y_{t-1}\}_{t=1}^T$, which solves Eqs. (25) − (26) along with $k_T = \overline{k}$. The equilibrium features a liquidity trap at each date $t \in \{0, .., T-1\}$ with $r_{t+1} = 0$ and $y_t < s(k_t)$.

(ii) There exists $i^{h,2} \leq i^{h,1}$ such that, if $i^h < i^{h,2}$, then the nonresidential capital declines at date 1, and then increases before date T:

$$k_0 > k_1 \text{ and } k_1 < k_T = \overline{k}.$$

The main result of this section is the second part, which establishes conditions under which the nonresidential capital (and investment) follow a non-monotone path during the recession: falling initially, but eventually increasing.

To understand the drop in investment, note that a negative shock to residential investment reduces aggregate demand and output. This in turn lowers nonresidential investment as captured by the break-even condition (25). When the shock is sufficiently severe, the aggregate demand at date 1 is sufficiently low that capital declines. Intuitively, the economy is optimally responding to the low return to capital depicted in Figure 5.

In later periods, aggregate demand and output gradually increase in anticipation of the eventual recovery. As this happens, the low cost of capital becomes the dominant factor for nonresidential investment. Consequently, the economy starts reaccumulating capital, and in fact—exits the liquidity trap with the maximum level of capital $\overline{k}$ as in the earlier model.

Figure 6 illustrates the dynamic evolution of the equilibrium variables for the case

[13] If the condition $i^h < i_1^h$ is violated, then there is an alternative equilibrium in which there is a partial liquidity trap at dates $t \in \{T_b - 1, .., T-1\}$ for some $T_b \geq 2$. We omit the characterization of these equilibria for brevity.

[14] Assumption 3 is a regularity condition on shocks and parameters that ensures an interior liquidity trap equilibrium at date 0 with positive output. This assumption is satisfied for all of our numerical simulations and is relegated to the appendix for brevity.

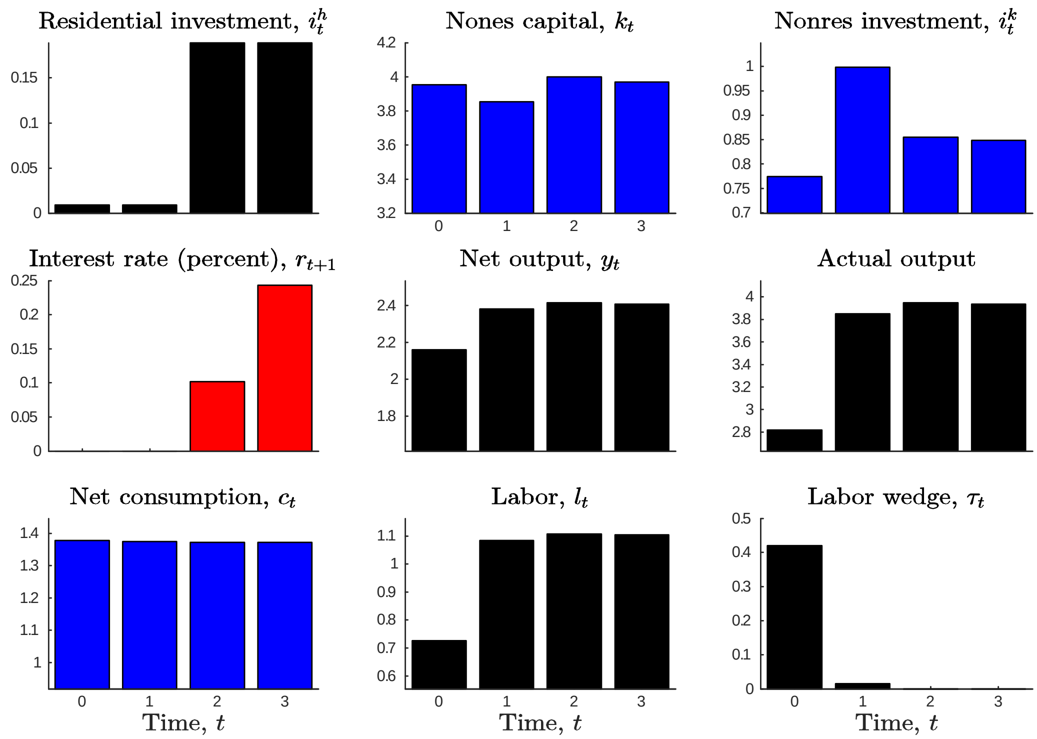

Figure 6: The evolution of equilibrium variables over time, given the length of decumulation $T = 2$.

$T = 2$. The parameters are chosen so that the figure can be compared to Figure 4 after replacing a single period with two periods. The lower panels on the left illustrate the non-monotonic response of capital and investment identified in Proposition 3. The figure illustrates that the recession can be roughly divided into two phases. In the first phase, captured by date 0, both types of investment fall. This induces a particularly severe recession with low output and employment. In the second phase, captured by date 1 in the figure (and dates $t \in \{1, .., T-1\}$ more generally), residential investment remains low whereas the nonresidential investment gradually recovers and eventually booms. The nonresidential investment response also raises aggregate demand. Hence, the second phase of the recession in our model represents a partial and asymmetric recovery in which the residential sector is left behind, similar to the aftermath of the Great Recession (see Figure 1).

4.1 Comparison with the acceleration principle

Our analysis of nonresidential investment bears a certain resemblance with the accelerator theory of investment (see Clark (1917)). To illustrate the similarities, let us linearize Eq. (25) around $(k, y) \simeq \left(\overline{k}, s\left(\overline{k}\right)\right)$, to obtain the approximation

$$k_t \simeq \alpha + \beta E_{t-1}\left[y_t\right] \text{ for each } t \in \{1, .., T-1\},$$

where $\beta = -R_y/R_k > 0, \alpha = \overline{k} - \beta s\left(\overline{k}\right)$, and $E_{t-1}\left[y_t\right] = y_t$. We introduce the (redundant) expectations operator to compare our rational expectations approach with the previous literature. Taking the first differences of this expression, and assuming that the depreciation rate is small, $\delta^k \simeq 0$, we further obtain

$$i_t^k \simeq k_{t+1} - k_t \simeq \beta\left(y_{t+1} - y_t\right) \text{ for each } t \in \{1, .., T-2\}. \tag{27}$$

Starting at date 1, our model implies a version of the acceleration principle, which posits that investment is proportional to changes in output (see Eckaus (1953) for a review).

Our model, however, has several important differences. First, the accelerator theory posits a relationship between the investment flow and the changes in output (or consumption) flows, without explicitly keeping track of the capital stock. In contrast, the capital stock plays an important role in our analysis. In fact, our main point is that an excessive level of the initial capital stock reduces aggregate demand. To see this, consider the analog of Eq. (27) for date 0,

$$i_0^k \simeq k_1 - k_0 \simeq \alpha + \beta y_1 - k_0.$$

Similar to Eq. (27), a reduction in expected output, y_1, or an increase in the initial capital stock, k_0, reduces investment. However, unlike Eq. (27), the initial stock, k_0, is a given of the model and is not necessarily related to the initial output, y_0. To the contrary, Eq. (21) shows that these two variables are actually *inversely* related. This is because an increase in the initial stock of capital reduces aggregate demand and output. Our analysis, thus, suggests that the acceleration principle should be qualified for the early stages of episodes, such as the one we consider, in which the capital stock might adjust to a new level.

A second difference is that the relationship in (27) is mechanically assumed in the accelerator literature, whereas we obtain Eq. (25) by combining the optimal investment behavior with the liquidity trap. In particular, our analysis suggests that the liquidity trap (or constrained monetary policy) is important for obtaining strong accelerator effects. Otherwise, the interest rate response would also affect investment. Moreover, the interest rate would typically respond in a way to dampen the accelerator effects (e.g., if the monetary policy focuses on output stabilization).[15]

A third difference is that the agents in our economy hold rational expectations, whereas the macroeconomic applications of the accelerator theory often use versions of Eq. (27) with backward looking expectations (for instance, $E_{t-1}[y_t] = y_{t-2}$). In particular, our model does not feature the periodic oscillations of output emphasized in Samuelson (1939) or Metzler (1941), which are driven by adaptive expectations.

4.2 Consumption response

While our model can account for the decline in investment in the earlier part of the recession, it cannot generate a similar behavior for consumption. As Figure 6 illus-

[15] In his review of the accelerator theory, Caballero (1999) notes: "the absence of prices (the cost of capital, in particular) from the right-hand side of the flexible accelerator equation has earned it disrespect despite its empirical success." The liquidity trap provides a theoretical rationale for excluding the cost of capital from the investment equation.

trates, (net) consumption expands during the recession due to the Euler equation.[16] However, the Euler equation—and the permanent income hypothesis that it implies—cannot fully capture the behavior of consumption in response to income changes in the data. After reviewing the vast empirical literature on this topic, Jappelli and Pistaferri (2010) note "there is by now considerable evidence that consumption appears to respond to anticipated income increases, over and above by what is implied by standard models of consumption smoothing."

To make consumption more responsive to income, Appendix A.1 extends the model by introducing additional households that have high marginal propensities to consume (MPC) out of income. The main result shows that, if there are sufficiently many high-MPC households, then aggregate consumption initially declines. Intuitively, the low output earlier in the recession lowers all households' incomes, which in turn reduces aggregate consumption due to the high-MPC households. As output increases later in the recession, so does consumption. Hence, consumption also responds non-monotonically to overbuilding.

The appendix also shows that the model with high-MPC households features a Keynesian income multiplier with two implications. First, the recession is more severe than in the baseline model, because the decline of consumption exacerbates the reduction in aggregate demand and output. Second, the accelerator effects are more pronounced in the sense that investment decreases more early in the recession, while also increasing more later in the recession. In this sense, the multiplier and the accelerator effects reinforce one another.

5 Policy implications

We next investigate the welfare implications of our analysis. We first discuss ex-post policies by which the government can improve welfare once the overbuilding is realized and the economy is in a recession. We then discuss ex-ante policies that the government can implement (prior to date 0) so as to mitigate the damage during the overbuilding episode. Throughout, we focus on constrained policy exercises in which the planner has access to a limited set of policy tools. Since investment plays a central

[16] Actual consumption, $c_t = \hat{c}_t + v(l_t)$, might fall in view of the reduction in employment, l_t. We do not emphaisze this result since it is mainly driven by the GHH functional form for the preferences, which we adopted for expositional simplicity.

role in our analysis, we only focus on policy interventions directed towards controlling investment. We also abstract away from adjustment costs so that the competitive equilibrium decumulates the excess capital in a single period as in Section 3.

All of the results in this section hold in our baseline analysis with GHH preferences. However, the GHH preferences make the aggregate demand externalities very powerful, as illustrated by the result in Section 3.1 that destroying houses does not affect welfare. To provide a more transparent cost-benefit analysis for policy interventions, in this section we work with a slight modification of the model. Suppose at date 0 (and only at date 0) households' preferences over consumption and labor are given by the separable form, $u(c_0) - v_0(l_0)$, as opposed to the GHH form, $u(\hat{c}_0 - v(l_0))$. With a slight abuse of terminology, we use c_0 to denote consumption at date 0 as opposed to net consumption, and $y_0 = F(k_0, l_0)$ to denote output at date 0 as opposed to net output. The labor wedge in this version of the model is given by,

$$\tau_0 = 1 - \frac{v_0'(l_0)}{u'(c_0) F_l(k_0, l_0)}. \tag{28}$$

As before, the labor wedge captures the severity of the demand shortage in this economy. In equilibrium, it is always nonnegative, $\tau_0 \geq 0$, and it is strictly positive when the firms' demand constraint in problem (10) binds.

Our next result establishes the analog of Proposition 3 in this setting. To state the result, let $\bar{c}_0, \bar{k}$ and $\bar{y}_0$ respectively denote the maximum level of consumption, investment, and output characterized in Section 3. Let $l_0^{*,sep}$ denote the efficient level of output at date 0 (when there is a liquidity trap) characterized by setting the labor wedge to zero when $c_0 = \bar{c}_0$. Finally, let $\bar{b}_0^{sep}(k_0)$ denote the threshold level of overbuilding as a function of the initial capital stock k_0, characterized by Eq. $(B.5)$ in the appendix.

Lemma 2. *Consider the modified model with separable preferences at date* 0*. The competitive equilibrium decumulates the excess residential capital in a single period,* $h_1 = h^*$*. If the overbuilding is sufficiently large,* $b_0 > \bar{b}_0^{sep}(k_0)$*, then the date* 0 *equilibrium features a liquidity trap with,*

$$r_1 = 0, \quad \tau_0 > 0, \quad y_0 = \bar{y}_0 < F\left(k_0, l_0^{*,sep}\right), \text{ and } l_0 < l_0^{*,sep}.$$

In addition, the rental rate of capital at date 0 *is given by* $R_0 = (1 - \tau_0) F_k(k_0, l_0)$*,* *and it is strictly below the efficient rate,* $F_k\left(k_0, l_0^{*,sep}\right)$.

5.1 Ex-post policies: Slowing down disinvestment

Consider the optimal policy in this environment given $b_0 > \bar{b}_0^{sep}(k_0)$. Since our model features a liquidity trap, several policies that have been discussed in the literature are also relevant in this context. In particular, welfare can be improved with unconventional monetary policies as in Eggertsson and Woodford (2003), or unconventional tax policies as in Correira et al. (2013). Once we modify the model appropriately to include government spending, welfare can also be improved by increasing government spending during the recession as in Werning (2012) and Christiano et al. (2011). We skip a detailed analysis of these policies for brevity.

A natural question in our setting concerns the optimal government policy regarding residential investment. On the one hand, since overbuilding is associated with residential capital, a naive intuition might suggest that the planner should not interfere with the decumulation of this type of capital. On the other hand, policies that support the housing market have been widely used during and after the Great Recession. We next formally analyze the desirability of these types of policies.

Before stating the planner's problem, it is useful to revisit the representative household's equilibrium trade-off for residential investment. Consider the analog of the household's problem (5) in this context, stated formally in Appendix B.3. Let $W_0(h_1)$ denote the household's perceived value function when she chooses $h_1 \geq h^*$ and makes the remaining decisions optimally.[17] The appendix shows that,

$$\frac{d_+ W_0(h_1)}{dh_1}|_{h_1=h^*} = u'(\bar{c}_0)\left(\frac{1-\delta^h}{1+r_1} - 1\right) < 0, \tag{29}$$

where $\frac{d_+ W_0(h_1)}{dh_1}$ denotes the right derivative, and the inequality follows since $r_1 = 0$. Thus, the household's marginal value of increasing the residential capital beyond h^* is strictly negative. Intuitively, given the preferences (2), the household does not receive any housing services from the additional housing units. Moreover, the present discounted value of the nondepreciated part of the housing units is strictly below their cost. Consequently, the household optimally chooses $h_1 = h^*$.

[17] Choosing $h_1 < h^*$ is sub-optimal in view of the preferences (2).

Next consider a constrained planner who can fully determine residential investment at date 0 (e.g., by using various policies in the housing market), but who cannot interfere with the remaining market allocations either at date 0 or in the future. Let $W_{0,pl}(h_1)$ denote this planner's value function from setting $h_1 \geq h^*$. When h_1 is in a neighborhood of h^*, the value function can be written as,

$$W_{0,pl}(h_1) = \max_{c_0,k_1,y_0,l_0} u(c_0) - v_0(l_0) + \beta V(k_1, h_1), \tag{30}$$
$$\text{s.t. } k_1 = \overline{k} \text{ and } u'(c_0) = \beta u'(C(h_1)),$$
$$\text{and } y_0 = F(k_0, l_0) = k_1 - \left(1-\delta^k\right) k_0 + c_0 + h_1 - \left(1-\delta^h\right)(1+b_0) h^*. \tag{31}$$

Here, $V(k_1, h_1)$ denotes the efficient value function characterized as the solution to problem (11), and $C(h_1)$ denotes the efficient level of consumption. The second line captures the liquidity trap constraint that consumption and nonresidential investment are determined by the zero interest rate. The third line captures that output and employment are determined by the aggregate demand at date 0. Importantly, the output is increasing in h_1 because a greater level of residential investment increases aggregate demand.

Using problem (30), Appendix B.3 calculates the planner's marginal value from increasing residential capital as,

$$\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*} = u'(\overline{c}_0)\left(\left(1-\delta^h\right) - (1-\tau_0) + \frac{dc_0}{dh_1}\tau_0\right). \tag{32}$$

Comparing Eqs. (29) and (32) illustrates that the planner has the same direct benefit from building residential capital, $1-\delta^h$, as the household with the competitive equilibrium. However, the planner perceives a lower cost of building, $1-\tau_0 < 1$, which leads to the following result.

Proposition 4 (Slowing Down Disinvestment)**.** *Consider the equilibrium characterized in Lemma 2. There exists* $\tilde{b}_0^{sep} > \overline{b}_0^{sep}(k_0)$ *such that the planner's choice of residential capital exceeds the target level,* $h_{1,pl} > h^*$*, if and only if* $b_0 > \tilde{b}_0^{sep}$*.*

Put differently, if the overbuilding is sufficiently severe, then the planner optimally stimulates residential investment, thereby postponing some of the decumulation of residential capital to date 1. Intuitively, the planner recognizes that the opportunity cost of building at date 0 is low due to the demand shortage and the (inefficiently) low employment. The strength of this effect is captured by the labor wedge, τ_0. When the overbuilding is sufficiently large, the labor wedge is also large. Then, the planner finds it optimal to increase residential investment beyond the target level, h^*.

An equivalent intuition is provided by *aggregate demand externalities*. Residential investment increases aggregate demand and output, which in turn increases other agents' employment, wages, profits, and so on. Since the economy is experiencing a demand shortage, these effects are socially beneficial. Individual households that choose residential investment do not take these positive externalities into account, which leads to too little residential investment in the competitive equilibrium.

Eq. (32) illustrates an additional benefit of residential investment, captured by the term $\frac{dc_0}{dh_1}\tau_0$. Appendix B.3 shows that this term is positive, which also induces the planner to choose a high level of h_1. Intuitively, bringing more residential capital to date 1 creates a future wealth effect that raises consumption not only at date 1, but also at date 0 (see the Euler equation in problem (30)). Since there is a demand shortage at date 0, raising consumption at date 0 is socially valuable. This channel is reminiscent of the forward guidance policies that create a similar wealth effect by committing to low interest rates in the future. In fact, increasing h_1 also lowers the future interest rate, r_2, in our setting. Note, however, that future output remains efficient in our model, $y_1 = s(k_1)$, whereas it exceeds the efficient level in environments with forward guidance (Werning, 2012).

To isolate the relevant trade-offs, we have focused on a planner who can only interfere with residential investment decisions. In practice, the policymakers can use various other tools to fight a liquidity trap. Eq. (32) that characterizes the planner's trade-off would also apply in variants of the model in which the planner optimally utilizes some other policy tools. In those variants, the equation would imply that the planner should interfere in residential investment as long as not doing so would result in a large and positive labor wedge, $\tau_0 > 0$. Our model, thus, suggests that the planner should slow down the decumulation of residential capital whenever she cannot substantially mitigate the liquidity trap using other available policies. This prediction is arguably applicable to various developed economies in recent years, e.g., Japan, the US, and Europe, that have featured zero nominal interest rates with low employment and output.

5.2 Ex-ante policies: Restricting investment

We next analyze whether the planner can improve welfare via ex-ante interventions. To this end, consider the baseline model with an ex-ante period, date -1. Suppose also that the economy can be in one of two states at date 0, denoted by $s \in \{H, L\}$. State L is a low-demand state in which the target level of housing capital is h^* as before (and the planner has no tools for ex-post intervention). State H is a high-demand state in which the utility function in (2) is modified so that the target level of housing capital is $\left(1+\lambda^H\right) h^*$ for some $\lambda^H > 0$. Suppose the economy starts with $h_{-1} = \left(1+\lambda^H\right) h^*$ and $k_{-1} = k^*$, and let $\pi^H \in (0,1)$ denote the ex-ante probability of the high-demand state at date 0.

We think of state L as representing the normal or "historical" state of affairs. The model captures a situation in which the housing demand has recently increased relative to its historical level, and the economy has already adjusted to this new level. However, there is a possibility that the current level of demand will not be sustainable, in which case the housing demand will revert back to its historical average.[18] We also envision that π^H is large, so that the representative household believes the high-demand state is likely to persist, but also that $\pi^H < 1$ so that there is room for precautionary policies. We first characterize the competitive equilibrium, and then investigate its constrained efficiency.

First consider the choice of residential capital at date -1. The preferences in (2) imply that the opportunity cost of consuming housing services below target is very large (since u^h is assumed to be very large). Consequently, households invest in residential capital according to their demand in state H, that is,

$$h_0 = \left(1+\lambda^H\right) h^*, \text{ which also implies } i_{-1}^h = \delta^h \left(1+\lambda^H\right) h^*.$$

This feature of the model is extreme. However, a similar outcome would also obtain in less extreme versions of the model as long as the ex-ante probability of state H is sufficiently high.

Next consider the choice of nonresidential capital, k_0, which is determined by a

[18] As we discuss in Section 2, an alternative interpretation is to think of state H as capturing the historical housing demand. In this case, state L represents a "new normal" in which housing demand permanently declines.

standard optimality condition,

$$u'(c_{-1}) = \beta \left(\begin{array}{c} \pi^H \left(R_0^H + 1 - \delta^k\right) u'\left(c_0^H\right) \\ + \left(1 - \pi^H\right) \left(R_0^L + 1 - \delta^k\right) u'\left(c_0^L\right) \end{array} \right). \tag{33}$$

The following result establishes sufficient conditions under which the pair (h_0, k_0), triggers a liquidity trap if state L is realized at date 0.

Lemma 3. *Consider the modified model with the ex-ante date* -1*, with the initial conditions,* $h_{-1} = h^* \left(1 + \lambda^H\right)$ *and* $k_{-1} = k^*$*. Suppose the demand for housing in state* H *is sufficiently large so that* $\lambda^H > \bar{b}_0^{sep}(k^*)$*. Then, there there exists* $\overline{\pi} < 1$ *such that if the ex-ante probability of state* H *is sufficiently high, so that* $\pi^H \in (\overline{\pi}, 1)$*, then the equilibrium features a liquidity trap in state* L *of date* 0 *(but not in any other dates or states).*

The equilibrium path starting the high-demand state H of date 0 is straightforward. It solves the neoclassical planning problem (11) with a steady level of residential investment given by, $i_t^h = \delta \left(1 + \lambda^H\right) h^*$ for each $t \geq 0$. The zero lower bound does not bind and the rental rate of capital is given by $R_0^H = s'(k_0)$.

The equilibrium path starting the low-demand state L of date 0 is characterized as in Lemma 2. In particular, note that $\lambda^H = b_0$, so that overbuilding is now endogenously determined by the demand for housing in the counterfactual state H. As long as $\lambda^H > \bar{b}_0^{sep}(k_0)$, which holds under the assumptions of Lemma 3, the economy features a liquidity trap with inefficiently low level of employment and output.

In this context, consider a constrained planner that can determine households' date -1 allocations, including the choice of k_0, h_0 at date -1, but cannot interfere with equilibrium allocations starting date 0. Like households, the planner also optimally chooses $h_{0,pl} = h_0 = \left(1 + \lambda^H\right) h^*$ in view of the large opportunity cost of not consuming housing in state H. However, the planner's choice of nonresidential capital, $k_{0,pl}$, is potentially different.

To characterize the capital choice, let $V_0^H(k_0, h_0)$ and $V_0^L(k_0, h_0)$ denote the welfare of the representative household in respectively states H and L of date 0. The ex-ante constrained planning problem can then be written as,

$$\max_{c_{-1},k_0} u\left(c_{-1}\right)+\beta\left(\pi^H V_0^H\left(k_0,h_0\right)+\left(1-\pi^H\right)V_0^L\left(k_0,h_0\right)\right), \tag{34}$$
$$\text{s.t.} \qquad c_{-1}+k_0+h_{0,pl}=s\left(k_{-1}\right)+\left(1-\delta^k\right)k_{-1}+\left(1-\delta^h\right)h_{-1}.$$

In particular, the planner optimally trades off the ex-ante consumption, c_{-1}, with investment in capital, k_0, evaluating the benefits of the latter in the competitive equilibrium that will obtain in each state.

The appendix characterizes the marginal value of capital in each state, $\frac{dV_0^H(k_0,h_0)}{dk_0}$ and $\frac{dV^L(k_0,h_0)}{dk_0}$, and derives the planner's optimality condition,

$$u'\left(c_{-1}\right)=\beta\left(\begin{array}{c}\pi^H\left(R_0^H+1-\delta^k\right)u'\left(c_0^H\right)\\+\left(1-\pi^H\right)\left(R_0^L+\left(1-\tau_0\right)\left(1-\delta^k\right)\right)u'\left(c_0^L\right)\end{array}\right). \tag{35}$$

Comparing Eqs. (33) and (35) shows that the planner perceives the same benefit from investment in state H as the competitive equilibrium. The planner also perceives the same benefit from using the capital in production in state L, as captured by the identical rental rates in both equations. However, the planner discounts the value of the nondepreciated part of the capital in this state since $1-\tau_0<1$. This leads to the following result.

Proposition 5 (Restricting Ex-ante Investment)**.** *Consider the setup described in Lemma 3. The planner chooses a lower level of investment compared to the competitive equilibrium,* $k_{0,pl}<k_0$*. By doing so, the planner alleviates but does not completely eliminate the demand-driven recession in state* L*, that is,* $\tau_0>\tau_{0,pl}>0$*.*

Intuitively, given the demand shortage in state L, the planner perceives the opportunity cost of building new capital in state L to be low. Hence, she optimally builds less capital ex-ante, so as to postpone some building to the demand deficient state with low cost. An equivalent intuition is provided by the aggregate demand externalities. Having more capital at date 0 leaves less room for new investment and reduces aggregate demand [cf. Eq. (31)]. Since the economy has a demand shortage, reducing demand is socially costly. The private agents do not internalize these negative externalities and invest too much from a social point of view.

In our stylized model, the constrained planner does not interfere with households' ex-ante residential investment, because the extreme preferences in (2) imply a corner solution. In alternative formulations with somewhat elastic housing demand, the planner would optimally restrict ex-ante investment in both types of capital. In fact, Eq. (35) suggests that the aggregate demand externalities are particularly strong for more durable types of capital, such as residential capital or durable goods, because the inefficiency is driven by entirely the nondepreciated part of the capital stock.

Our analysis of ex-ante policies is reminiscent of the results in a recent literature, e.g., Korinek and Simsek (2014) and Farhi and Werning (2013), which investigate the implications of aggregate demand externalities for ex-ante macroprudential policies in financial markets. For instance, Korinek and Simsek (2014) show that, in the run-up to liquidity traps, private agents take on too much debt, because they do not internalize that leverage reduces aggregate demand. We complement this analysis by showing that similar aggregate demand externalities also create inefficiencies for ex-ante *physical* investment decisions. Our model highlights a distinct mechanism by which ex-ante decisions affect aggregate demand. In fact, we work with a representative household whereas heterogeneity is central for the results in Korinek and Simsek (2014) or Farhi and Werning (2013). Our model also generates policy implications that are different than the macroprudential policies typically emphasized in this literature. We provide a rationale for restricting ex-ante investment (or slowing down the accumulation of capital) regardless of whether investment is financed by debt or other means.

6 Conclusion

We have presented a model of investment hangover in the Great Recession that combines both Austrian and Keynesian features. On the Austrian side, the recession is precipitated by overbuilding in the residential sector, which necessitates a reallocation of resources to other sectors. The reallocation problem is exacerbated by the durability of residential capital, which prevents depreciation from naturally erasing the overbuilt capital. On the Keynesian side, a lower bound on interest rates slows down reallocation and creates an aggregate demand shortage. The demand shortage can also reduce investment in sectors that are not overbuilt, leading to a severe recession. Eventually, nonresidential investment recovers, but the slump in the residential sector continues for a long time.

The model yields predictions that are consistent with the broad trends of GDP, residential investment, and investment in the Great Recession. In particular, the model explains why housing investment collapsed and has not really recovered, but also why other types of investment also declined initially, but then recovered much more robustly. We need both Keynesian and Austrian features to obtain these empirically accurate predictions.

The model also features aggregate demand externalities, with several policy implications for investment. Ex-post, once the economy is in the liquidity trap, welfare can be improved by policies that slow down the disinvestment of residential capital. Ex-ante, before the economy enters the liquidity trap, welfare can be improved by policies that restrict the accumulation of capital. The optimal ex-post and ex-ante policies share the broad principle that they intertemporally substitute investment from periods that feature efficient outcomes to periods (or states) that feature deficient demand.

Although we have focused on the Great Recession, the model is more widely applicable. Perhaps the most straightforward extension is to overbuilding in sectors other than housing. In the 1930s, when both Hayek and Keynes wrote, speculative overbuilding was seen as a critical impetus to recessions, but the focus was more on railroads and perhaps industrial plant than on housing. In our model, such extensions would require only a relabeling of variables.

Less obvious is the extension to other forms of restrictions on interest rates, such as currency unions, which also slow down the Austrian reallocation of resources from the overbuilt sector to others. In the recent European context, such restrictions may have played a critical role, and generated Keynesian aggregate demand effects along the lines suggested by our model. We leave an elaboration of these mechanisms to future work.

References

Abraham, Katharine G., and Lawrence F. Katz (1986), "Cyclical Unemployment: Sectoral Shifts or Aggregate Disturbances?" *Journal of Political Economy*, 94(3), p.367-388.

Adam, Klaus, and Roberto M. Billi (2006), "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates," *Journal of Money, Credit & Banking*, 38(7), p.1877-1905.

Auerbach, Alan J., and Maurice Obstfeld (2005), "The Case for Open-Market Purchases in a Liquidity Trap," *American Economic Review*, p.110-137.

Beaudry, Paul, and Franck Portier (2004), "An Exploration into Pigou's Theory of Cycles," *Journal of Monetary Economics*, 51, p.1183–1216.

Beaudry, Paul, and Franck Portier (2013), "News Driven Business Cycles: Insights and Challenges," *NBER Working Paper* no. 19411.

Beaudry, Paul, Dana Galizia, and Franck Portier (2014), "Reconciling Hayek's and Keynes Views of Recessions," *NBER Working Paper* no. 20101.

Blanchard, Olivier, and Peter Diamond (1989), "The Beveridge Curve," *Brookings Papers on Economic Activity*, 1989(1), p.1-60.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry (2012), "Really Uncertain Business Cycles," *NBER Working Paper* no. 18245.

Brunnermeier, Markus (2009), "Deciphering the Liquidity and Credit Crunch of 2007–08," *Journal of Economic Perspectives*, 23, p.77–100.

Brunnermeier, Markus, Thomas Eisenbach, and Yuliy Sannikov (2013), "Macroeconomics With Financial Frictions: A Survey," *Advances In Economics And Econometrics, Tenth World Congress Of The Econometric Society*, New York: Cambridge University Press.

Caballero, Ricardo J. (1999), "Aggregate Investment," in Taylor, J. B. and M. Woodford, eds., *Handbook of Macroeconomics,* Amsterdam: North Holland.

Caballero, Ricardo, and Emmanuel Farhi (2014), "The Safety Trap," *NBER Working Paper* no. 19927.

Caballero, Ricardo, and Mohamad Hammour (1996), "On the Ills of Adjustment," *Journal of Development Economics*, 51(1), p.161 192.

Campbell, Jeffrey R. and Zvi Hercowitz (2005), "The Role of Collateralized Household Debt in Macroeconomic Stabilization," *NBER working paper* no. 11330.

Campello, Murillo, John Graham, and Campbell Harvey (2010), "The Real Effects of Financial Constraints: Evidence from a Financial Crisis," *Journal of Financial Economics*, 97, p.470-487.

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo (2011), "When Is the Government Spending Multiplier Large?" *Journal of Political Economy*, 119, p.78-121.

Christiano, Lawrence, Martin Eichenbaum, and Mathias Trabandt (2014), "Understanding the Great Recession," *NBER Working Paper* no. 20040.

Chodorow-Reich, Gabriel (2014), "The Employment Effects of Credit Market Disruptions: Firm-level Evidence from the 2008-09 Financial Crisis," *Quarterly Journal of Economics,* 129, p.1-59.

Clark, J. Maurice (1917), "Business Acceleration and the Law of Demand: a Technical Factor in Economic Cycles," *Journal of Political Economy,* 25, p.217-235.

Correia, Isabel, Emmanuel Farhi, Juan Pablo Nicolini and Pedro Teles (2013), "Unconventional Fiscal Policy at the Zero Bound," *American Economic Review*, 103(4), p.1172-1211.

Davila, Eduardo (2015), "Dissecting Fire Sale Externalities," NYU Working Paper.

Davis, Morris A. and Jonathan Heathcote (2005), "Housing and the Business Cycle," *International Economic Review*, 46, p.751–784.

Davis, Steven J. and John Haltiwanger (1990), "Gross job creation and destruction: microeconolnic evidence and macroeconomic implications," *NBER Macroeconomics Annual*, 5, p.123-168.

DeLong, Bradford (1990), "'Liquidation' Cycles: Old-Fashioned Real Business Cycle Theory and the Great Depression," *NBER Working paper* no. 3546.

Eckaus, Rirchard S. (1953), "The Acceleration Principle Reconsidered," *Quarterly Journal of Economics*, 67, p.209-230.

Eggertsson, Gauti, and Paul Krugman (2012), "Debt, Deleveraging, and the Liquidity Trap," *Quarterly Journal of Economics*, 127(3), p.1469-1513.

Eggertsson, Gauti, and Neil Mehrotra (2014), "A Model of Secular Stagnation," *NBER Working Paper* no. 20574.

Eggertsson, Gauti, and Michael Woodford (2003), "The Zero Bound on Interest Rates and Optimal Monetary Policy", *Brookings Papers on Economic Activity,* 2003(1), p.139-233.

Farhi, Emmanuel and Iván Werning (2013), "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities", *NBER Working Paper* no. 19313.

Fisher, J.D.M. (2007), "Why Does Household Investment Lead Business Investment Over the Business Cycle? *Journal of Political Economy*, 115, p.141–168.

Gertler, Mark, and Peter Karadi (2011), "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 58, p.17-34.

Gertler, Mark, and Nobuhiro Kiyotaki (2011), "Financial Intermediation and Credit Policy in Business Cycle," in *Handbook of Monetary Economics*, North Holland: Elsevier.

Gervais, Martin (2002), "Housing Taxation and Capital Accumulation," *Journal of Monetary Economics*, 49.7, p.1461-1489.

Greenwood, Jeremy and Zvi Hercowitz (1991), "The Allocation of Capital and Time Over the Business Cycle," *Journal of Political Economy*, 99, p.1188–1214.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman (1988), "Investment, Capacity Utilisation and the Real business Cycle," *American Economic Review*, 78, p. 402-417.

Guerrieri, Veronica and Guido Lorenzoni (2011), "Credit Crises, Precautionary Savings, and the Liquidity Trap," *NBER Working Paper* no. 17583.

Hall, Robert E. (2011), "The Long Slump," *American Economic Review* 101(2), p.431–469.

Hart, Oliver, and Luigi Zingales (2011), "Inefficient Provision of Liquidity," *NBER Working Paper* no. 17299.

Hayek, Friedrich (1931), *Prices and Production*, London: Routledge & Kegan Paul.

He, Zhiguo, and Péter Kondor (2014), "Inefficient Investment Waves," *NBER Working Paper* no. 18217.

He, Zhiguo, and Arvind Krishnamurthy (2014), "A Macroeconomic Framework for Quantifying Systemic Risk," *NBER Working Paper* no. 19885.

Iacoviello, Matteo (2005), "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review*, 95, p.739-764.

Iacoviello, Matteo, and Marina Pavan (2013), "Housing and Debt Over the Life Cycle and Over the Business Cycle," *Journal of Monetary Economics*, 60, p.221-238.

Jaimovich, Nir and Sergio Rebelo (2009), "Can News about the Future Drive the Business Cycle?" *American Economic Review*, 99, p.1097-1118.

Jappelli, Tullio, and Luigi Pistafferi (2010), "The Consumption Response to Income Changes," *Annual Review of Economics*, 2, p.479–506.

Jeanne, Olivier, and Lars E. O. Svensson (2007), "Credible Commitment to Optimal Escape From a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank," *American Economic Review*, p.474-490.

Jermann, Urban, and Vincenzo Quadrini (2012), "Macroeconomic Effects of Financial Shocks," *American Economic Review*, 102(1), p.238–71.

Kaplan, Greg, and Giovanni L. Violante (2014), "A Model of the Consumption Response to Fiscal Stimulus Payments," *Econometrica*, 82, p.1199-1239.

Kiyotaki, Nobuhiro, Alexander Michaelides, and Kalin Nikolov (2011), "Winners and Losers in Housing Markets," *Journal of Money, Credit and Banking*, 43, p.255–296.

Korinek, Anton and Alp Simsek (2014), "Liquidity Trap and Excessive Leverage," NBER Working Paper no. 19970.

Krugman, Paul (1998), "The Hangover Theory," *Slate Magazine*, available at: http://www.slate.com/id/9593. (Accessed: Jul 5, 2014).

Krugman, Paul (1998), "It's Baaack: Japan's Slump and the Return of the Liquidity Trap," *Brookings Papers on Economic Activity*, p.137-205.

Leamer, Edward E. (2007), "Housing is the Business Cycle," *NBER Working Paper* no. 13428.

Lilien, David (1980), "The Cyclical Pattern of Temporary Layoffs in United States Manufacturing," *Review of Economics and Statistics*, 62, p.24-31.

Lorenzoni, Guido (2008), "Inefficient Credit Booms," *Review of Economic Studies*, 75(3), p.809-833.

Metzler, Lloyd (1941), "The Nature and Stability of Inventory Cycles," *Review of Economics and Statistics*, 23, p.113-129.

Mian, Atif and Amir Sufi (2014), *The House of Debt*, Chicago: University of Chicago Press.

Mian, Atif and Amir Sufi (2015), "What Explains the 2007-2009 Drop in Employment?", *Econometrica*, forthcoming.

Mian, Atif, Kamelesh Rao, and Amir Sufi (2013), "Household Balance Sheets, Consumption, and the Economic Slump," *Quarterly Journal of Economics,* 128(4), *p*.1687-1726.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel (2007), "Housing, Consumption and Asset Pricing," *Journal of Financial Economics*, 83(3), p.531-569.

Samuelson, Paul (1939), "Interactions Between the Multiplier Analysis and the Principle of Acceleration," *Review of Economic Statistics*, 21(2), p.75–78

Schmitt-Grohé, Stephanie, and Martín Uribe (2012), "The Making of a Great Contraction with a Liquidity Trap and a Jobless Recovery," *NBER Working Paper* no. 18544.

Summers, Lawrence (2013), "Why Stagnation Might Prove to be the New Normal," The Financial Times.

Stein, Jeremy C. (2012), "Monetary Policy as Financial Stability Regulation," *Quarterly Journal of Economics*, 127, p.57–95.

Werning, Iván (2012), "Managing a Liquidity Trap: Monetary and Fiscal Policy", *NBER Working Paper* no. 17344.

A Appendix: Omitted extensions

A.1 Consumption response and the Keynesian multiplier

The baseline model features a representative household whose consumption satisfies the Euler equation. However, the Euler equation cannot fully capture the behavior of consumption in response to income changes in the data (see Jappelli and Pistaferri, 2010). We next modify the model by introducing constrained agents that have high MPCs out of income. We show that, unlike the baseline model, this version of the model can account for the drop in consumption earlier in the recession. The model also features a Keynesian income multiplier, which exacerbates the recession and reinforces the investment accelerator mechanism.

Suppose, in addition to the representative household analyzed earlier, there is an additional mass l^{tr} of households which we refer to as *income-trackers*. These agents are excluded from financial markets so that they consume all of their income, that is, their MPC is equal to 1 (for simplicity). Each income-tracker inelastically supplies 1 unit of labor in a competitive market for a wage level w_t^{tr}, which provides her only source of income. Consequently, total consumption is now given by $c_t + w_t^{tr} l^{tr}$, where c_t is the consumption of the representative household and $w_t^{tr} l^{tr}$ denotes the consumption of income-trackers.

The aggregate production function can generally be written as $\tilde{F}(k_t, l_t, l^{tr})$, where l_t is the labor supply by the representative household and l^{tr} is the total labor supply by income-trackers. To simplify the analysis, we focus on the special case

$$\tilde{F}\left(k_t, l_t, l^{tr}\right) = F\left(k_t, l_t\right) + \eta^{tr} l^{tr},$$

where F is a neoclassical production function and $\eta^{tr} > 0$ is a scalar. We continue to use the notation $y_t = F(k_t, l_t) - v(l_t)$ to refer to net output excluding the supply of income-trackers. Total net output is given by $y_t + \eta^{tr} l^{tr}$. The rest of the model is the same as in the previous section.

In view of these assumptions, the economy is subject to the resource constraint,

$$c_t + i_t^k + i_t^h + w_t^{tr} l^{tr} = y_t + \eta^{tr} l^{tr} \leq s(k_t) + \eta^{tr} l^{tr}. \tag{A.1}$$

Lemma 4 in Appendix B characterizes the income-trackers' wage level as

$$w_t^{tr} = \psi(k_t, y_t)\, \eta^{tr}. \tag{A.2}$$

Here, $\psi(k_t, y_t) \in [0,1]$ is a measure of efficient resource utilization (more specifically, $\psi = 1 - \tau$ where τ is the labor wedge). It is an increasing function of y_t and satisfies $\psi = 1$ when the output is at its efficient level, $y_t = s(k_t)$. Intuitively, the demand shortage lowers factor returns, including the wages of income trackers.

Combining Eqs. $(A.1)$ and $(A.2)$ implies

$$y_t = c_t + k_{t+1} - (1-\delta) k_t + i_t^h + (\psi(k_t, y_t) - 1)\, \eta^{tr} l^{tr}, \tag{A.3}$$

for each $t \in \{0, 1, .., T-1\}$. This expression illustrates a *Keynesian cross* as well as a *Keynesian income multiplier* in our setting. The equilibrium obtains when the actual and demanded net outputs are equal, as in a typical the Keynesian cross. Moreover, total demand depends on the output y_t through income-trackers' consumption, illustrating the multiplier. Consider, for instance, a shock to aggregate demand that lowers net output. This lowers income-trackers' income and their consumption, which in turn induces a second round reduction in aggregate demand and output, and so on.

Next consider a residential investment shock that lasts T periods as in the previous section. We conjecture an equilibrium with a liquidity trap at all dates $t \in \{0, 1, .., T-1\}$. As before, the break-even condition (25) holds. Eqs. $(A.3)$ and (25) can then be solved backwards starting with $k_T = \overline{k}$. The next result establishes the existence of an equilibrium, and characterizes the behavior of consumption in equilibrium.

Proposition 6 (Consumption Response)**.** *Consider the model with mass l^{tr} of income-trackers and the adjustment length $T \geq 1$. Suppose Assumptions 1-2 and Assumption 3^{tr} in Appendix B hold.*

(i) There exists $i^{h,1}$ such that if $i^h < i^{h,1}$, then there is an equilibrium path $\{k_t, y_{t-1}\}_{t=1}^T$, which solves Eqs. (25) *and* $(A.3)$ *along with $k_T = \overline{k}$. Any equilibrium features a liquidity trap at each date $t \in \{0, .., T-1\}$ with $r_{t+1} = 0$ and $y_t < s(k_t)$.*

(ii) There exists l_1^{tr} such that if $l^{tr} > l_1^{tr}$, then total consumption at date 0 *(in any*

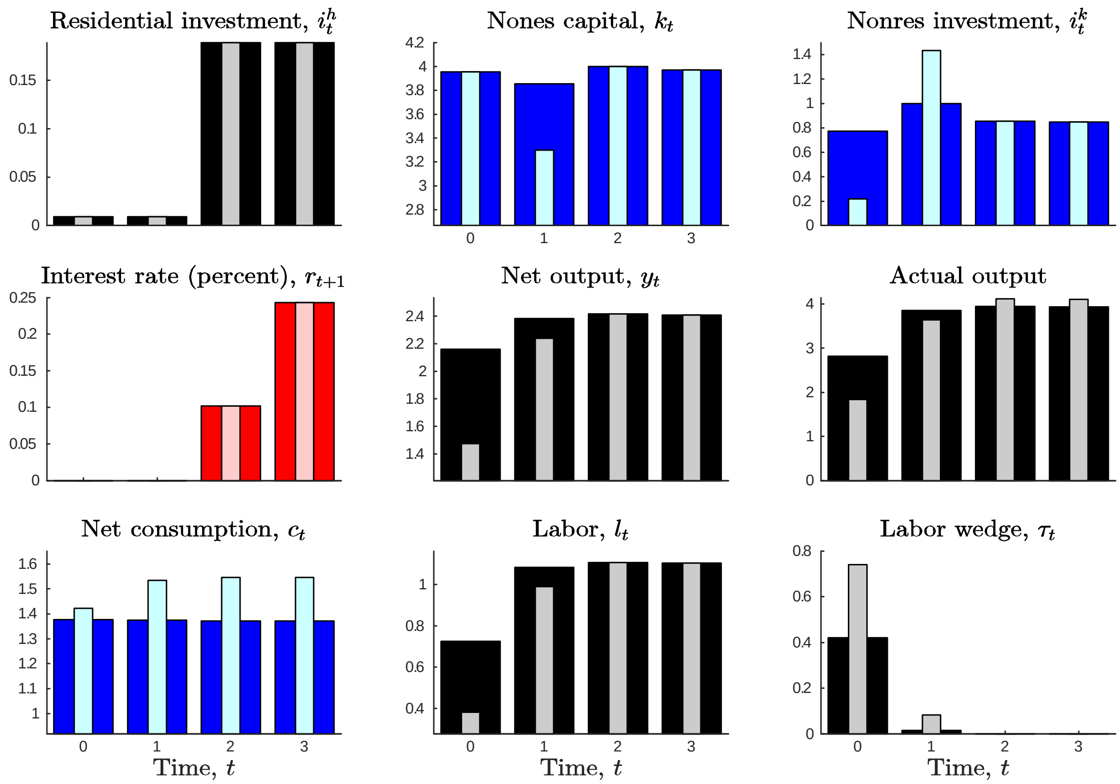

Figure 7: The dynamic equilibrium with income-trackers (light bars) compared to the equilibrium without income-trackers (dark bars).

equilibrium) is below its steady-state level, that is

$$c_0 + w_0^{tr} l^{tr} < c^* + \eta^{tr} l^{tr}.$$

The main result of this section is the second part, which establishes conditions under which overbuilding also lowers total consumption at date 0 in any equilibrium.[19] When the economy is in a liquidity trap, output falls due to the demand shortage, which lowers income-trackers' consumption. With sufficiently many income-trackers, this also reduces total consumption in contrast to the baseline model. The light bars in Figure 7 illustrate this result by plotting the dynamic equilibrium using the same parameters as before except for the new parameters $\eta^{tr}, l^{tr} > 0$. Consumption declines early in the recession, and it recovers later in the recession due to the recovery

[19]The equilibrium is unique in all of our numerical simulations. However, there could in principle be multiple equilibria because Eq. $(A.3)$ represents an intersection of two increasing curves in Y_t.

in output. Hence, this version of the model can generate a nonmonotonic response in consumption, similar to the nonmonotonic response of investment identified in Proposition 3.

It follows that this version of the model can explain the asymmetric recovery from the Great Recession depicted in Figure 1. In the first phase of the recession, consumption as well as nonresidential investment simultaneously fall, triggering a deep recession. In the second phase, the boom in nonresidential investment increases output, which also increases consumption. Hence, the second phase is a partial and asymmetric recovery in which the residential sector is left behind, as in the aftermath of the Great Recession.

Figure 7 also contrasts this equilibrium with the earlier equilibrium without income-trackers, which is plotted with dark bars. Note that the equilibrium in this section features a greater drop in output and employment, as well as a greater labor wedge. Intuitively, the Keynesian income multiplier aggravates the recession. Perhaps less obviously, the equilibrium also features a more severe drop in investment at date 0, followed by a stronger recovery at date 1. Intuitively, a more severe recession implies a lower return to capital, which in turn lowers investment at date 0. Put differently, the Keynesian income multiplier exacerbates the investment accelerator mechanism. The decline in investment at date 0 further lowers net output and consumption, aggravating the Keynesian income multiplier. In this sense, the multiplier and the accelerator mechanisms reinforce each other.

B Appendix: Omitted proofs

B.1 Proofs for the baseline model

This section presents the proofs of the results for the baseline model and its variants analyzed in Sections 2, 3 and 4.

Characterization of the Efficient Benchmark. Consider a planner that maximizes households' welfare starting date t onwards, given the initial state h_t, k_t, and the feasibility constraints of the economy. In view of the preferences (2), the planner chooses the same level of i_t^h as the representative household given by Eq. (3). The planner's problem can then be written as,

$$\max_{\left\{\hat{c}_{\tilde{t}}, l_{\tilde{t}}, k_{\tilde{t}+1}, [l_{\tilde{t}}(\nu), k_{\tilde{t}}(\nu)]_{\nu}\right\}_{\tilde{t}=t}^{\infty}} \sum_{\tilde{t}=t}^{\infty} \beta^{\tilde{t}} u\left(\hat{c}_{\tilde{t}} - \nu\left(l_{\tilde{t}}\right)\right),$$

$$\text{s.t. } \hat{c}_{\tilde{t}} + k_{\tilde{t}+1} + i_{\tilde{t}}^{h} \leq \hat{y}_{\tilde{t}}, \text{ where}$$

$$\hat{y}_{\tilde{t}} = \left(\int_0^1 \left(F\left(k_{\tilde{t}}(\nu), l_{\tilde{t}}(\nu)\right)\right)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\varepsilon/(\varepsilon-1)}, k_{\tilde{t}} = \int k_{\tilde{t}}(\nu)\, d\nu, \text{ and } l_{\tilde{t}} = \int l_{\tilde{t}}(\nu)\, d\nu.$$

By concavity, the planner chooses $k_{\tilde{t}}(\nu) = k_{\tilde{t}}$ and $l_{\tilde{t}}(\nu) = l_{\tilde{t}}$ for each $\tilde{t}$. The optimality condition for labor then implies Eq. (12). In view of these observations, the planner's problem reduces to the neoclassical problem (11). The solution $\{c_{\tilde{t}}, k_{\tilde{t}+1}\}_{\tilde{t}=t}^{\infty}$ is characterized as the solution to the neoclassical system of equations,

$$\begin{aligned} c_{\tilde{t}} + k_{\tilde{t}+1} + i_{\tilde{t}}^{h} &= s\left(k_{\tilde{t}}\right) + \left(1 - \delta^{k}\right) k_{\tilde{t}}, \\ u'\left(c_{\tilde{t}}\right) &= \beta\left(1 + s'\left(k_{\tilde{t}}\right) - \delta^{k}\right) u'\left(c_{\tilde{t}+1}\right), \end{aligned} \tag{B.1}$$

along with a transversality condition. The steady-state to this system is characterized by,

$$\beta\left(1 - \delta^{k} + s'\left(k^{*}\right)\right) = 1 \text{ and } c^{*} = s\left(k^{*}\right) - \delta k^{*} - \delta h^{*}.$$

Starting with any k_t, h_t, there is an interior solution to the system in $(B.1)$ that converges to the steady state (as long as the residential investment is feasible at all dates, which we assume). Since the planner's problem is strictly concave, this is also the unique solution, completing the characterization of the efficient benchmark. □

Proof of Lemma 1. First consider the case $r_{t+1} > 0$. In this case, the monetary policy implements the efficient allocation with $l_t = l_t^{*}$ and $y_t = s(k_t)$. In addition, the first order conditions for problems (12) and (4) further imply,

$$F_l\left(k_t, l_t^{*}\right) = v'\left(l_t^{*}\right) = w_t.$$

Combining this with Eq. (15) implies that the labor wedge is zero, $\tau_t = 0$. Combining Eqs. (15) and (12) then imply the rental rate of capital is given by $F_k\left(k_t, l_t^{*}\right) = s'\left(k_t\right)$, completing the proof for the first part.

Next consider the case $r_{t+1} = 0$. In this case, Eq. (15) implies $F_l\left(k_t, l_t\right) \geq v'\left(l_t\right)$.

This in turn implies that $l_t \in [0, l_t^*]$. By feasibility, net output satisfies

$$y_t = c_t + i_t^h + i_t^h = F(k_t, l_t) - v(l_t).$$

This right hand side is strictly increasing in l_t over the range $[0, l_t^*]$. The minimum and the maximum are respectively given by 0 and $s(k_t)$, which implies $y_t \in [0, s(k_t)]$. Moreover, given y_t that satisfies these resource constraints, there is a unique solution to problem (14), which we denote by $L(k_t, y_t)$. Combining this with Eq. (15), we further obtain the labor wedge as,

$$1 - \tau_t = \frac{v'(l_t)}{F_l(k_t, l_t)} = \frac{v'(L(k_t, y_t))}{F_l(k_t, L(k_t, y_t))}.$$

Plugging this into Eq. (15) for capital, we obtain the rental rate of capital as,

$$R_t = \frac{v'(L(k_t, y_t))}{F_l(k_t, L(k_t, y_t))} F_k(k_t, L(k_t, y_t)) \equiv R(k_t, y_t),$$

where the last equality defines the function $R(\cdot)$. Note that $R(k_t, y_t) \leq s'(k_t)$ since the labor wedge is nonnegative. It can also be checked that $R_k < 0$ and $R_y > 0$, completing the proof. □

Proof of Proposition 1. We first claim that the equilibrium at date 1 starting with $h_1 = h^*$ and $k_1 \leq \overline{k}$ is the same as the efficient benchmark. Recall that the efficient benchmark is characterized as the solution to the neoclassical system in $(B.1)$. Since $i_t^h = \delta h^*$ for each t, the neoclassical system is stationary. It can then be seen that the capital stock in the efficient benchmark converges monotonically to $k^* < \overline{k}$ starting with $k_1 \leq \overline{k}$. In particular, the capital stock remains weakly below $\overline{k}$ at all dates. This implies $r_{t+1} = s'(k_{t+1}) - \delta^k \geq s'\left(\overline{k}\right) - \delta^k \geq 0$ for each $t \geq 1$. That is, the zero lower bound constraint does not bind at any date $t \geq 1$. This in turn implies that the monetary policy in (13) exactly replicates the efficient benchmark starting date 1, proving our claim. Note also that consumption at date 1 is given by $c_1 = C(k_1)$, where $C(\cdot)$ is an increasing function that describes the solution to the neoclassical system $(B.1)$.

To characterize the equilibrium at date 0, we define $K_1(r_0)$ for each $r_0 \geq 0$ as the solution to

$$s'\left(K_1\left(r_0\right)\right) - \delta^k = r_0.$$

Note that $K_1(r_0)$ is decreasing in the interest rate, with $K_1(0) = \overline{k}$ and $\lim_{r_0 \to \infty} K_1(r_0) = 0$. Similarly, define the function $C_0(r_0)$ as the solution to the Euler equation

$$u'\left(C_0\left(r_0\right)\right) = \beta\left(1 + r_0\right) u'\left(C\left(K_1\left(r_0\right)\right)\right).$$

Note that $C_0(r_0)$ is decreasing in the interest rate, with $C_0(0) = \overline{c}_0$ and $\lim_{r_0 \to \infty} C_0(r_0) = 0$. Finally, define the aggregate demand function

$$Y_0\left(r_0\right) = C_0\left(r_0\right) + K_1\left(r_0\right) - \left(1 - \delta^k\right) k_0 + i_0^h.$$

Note that $Y_0(r_0)$ is also decreasing in the interest rate, with

$$Y_0(0) = \overline{y}_0 \text{ and } \lim_{r_0 \to \infty} Y_0\left(r_0\right) = i_0^h - \left(1 - \delta^k\right) k_0.$$

Next consider the date 0 equilibrium for the case $b_0 \leq \overline{b}_0$. Note that this implies $s(k_0) \leq \overline{y}_0 = Y_0(0)$, and that we also have $\lim_{r_0 \to \infty} Y_0(r_0) < s(k_0)$ (since we assume residential investment is feasible). By the intermediate value theorem, there is a unique equilibrium interest rate $r_0 \in [0, \infty)$ such that $Y_0(r_0) = s(k_0)$. The equilibrium features $c_0 = C_0(r_0)$ and $K_1(r_0) = k_1$, along with $y_0 = s(k_0)$ and $l_0 = l_0^*$.

Next consider the date 0 equilibrium for the case $b_0 > \overline{b}_0$. In this case, $Y_0(0) < s(k_0)$. Thus, the unique equilibrium features $r_0 = 0$ and $y_0 = \overline{y}_0 < s(k_0)$. Consumption and investment are given by $c_0 = \overline{c}_0$ and $k_1 = \overline{k}_1$. Labor supply l_0 is determined as the unique solution to (14) over the range $l_0 \in (0, l_0^*)$. Finally, Eq. (21) implies the equilibrium output, $y_0 = \overline{y}_0$, is declining in the initial overbuilding b_0.

In either case, it can also be checked that the economy reaches date 1 with $h_1 = h^*$ and $k_1 \geq \min(k_0, k^*)$. Thus, the continuation equilibrium is characterized as described above, completing the proof. □

Proof of Proposition 2. Note that the recession is triggered if $\overline{y}_0 < s(k_0)$, where $\overline{y}_0$ is given by Eq. (23). Since $1 - \delta^{h^d} > 1 - \delta^{h^n}$, increasing b_0^d (while keeping $b_0 = \left(b_0^d + b_0^n\right)/2$ constant) reduces $\overline{y}_0$, proving the result. □

Proposition 3 also requires the following assumption.

Assumption 3. (i) $i^h \in [-\bar{c}_T, s(\bar{k}) - \delta\bar{k} - \bar{c}_0)$ and (ii) $R(k_0, \tilde{y}_0) < \delta^k$, where $\tilde{y}_0 = \bar{c}_0 - \bar{c}_T + \bar{k} - (1-\delta^k) k_0$

Part (i) ensures that i^h is not too low to induce zero aggregate demand, but also not too high so that a liquidity trap is possible. Part (ii) ensures that the worst possible shock $i^h = -\bar{c}_T$ is sufficient to induce a liquidity trap at date 0.

Proof of Proposition 3. We first claim that the solution to Eq. (25) can be written as $k_t = K(y_t)$, where $K(\cdot)$ is an increasing function over $(0, s(\bar{k}))$. To this end, consider some $y \in (0, s(\bar{k}))$. Let $\tilde{k} < \bar{k}$ denote the unique capital level such that $y = s(\tilde{k})$. Note that

$$R(\tilde{k}, y) = s'(\tilde{k}) > \delta^k \text{ and } R(\bar{k}, y) < s'(\bar{k}) = \delta^k.$$

Here, the former inequality follows since $\tilde{k} < \bar{k}$, and the latter inequality follows from Lemma 1 since $y < s(\bar{k})$. Since $R_k < 0$, there exists a unique $K(y) \in (\tilde{k}, \bar{k})$ such that $R(K(Y), Y) - \delta^k = 0$. Thus, the function $K(\cdot)$ is well defined. Note also that $K(\cdot)$ is continuous and strictly increasing. Note also that $\lim_{y\to 0} K(y) = 0$ and $K(s(\bar{k})) = \bar{k}$.

Given the function $K(\cdot)$, the path of capital can be written as the solution to the system,

$$\begin{aligned} k_t &= K(y_t) \\ \text{and } y_t &= Y_t(k_t) \equiv \bar{c}_t + k_{t+1} + i^h - (1-\delta^k) k_t. \end{aligned} \tag{B.2}$$

Here, the second equation defines the function $Y_t(k_t)$, which is strictly decreasing in k_t. Hence, the current level of output and capital are satisfied as the intersection of a strictly increasing and a strictly decreasing relation. We next claim that, given $\bar{c}_t$ and $k_{t+1} \in (0, \bar{k}]$, there is a unique solution to the system in $(B.2)$. To see this, first note that the boundary conditions at $y_t = 0$ and $k_t = 0$ respectively satisfy,

$$\lim_{y\to 0} K(y) = 0,\ Y_t(0) \geq 0,$$

where the inequality follows since Assumption 3(i) implies $\bar{c}_t + i^h \geq 0$ for each t. Next note the following boundary conditions at $y_t = s(\overline{k})$ and $k_t = \overline{k}$,

$$K\left(s\left(\overline{k}\right)\right) = \overline{k} \text{ and } Y_t\left(\overline{k}\right) \leq \overline{c}_t + \delta\overline{k} + i^h < s\left(\overline{k}\right),$$

where the strict inequality follows by using Assumption 3(i) together with $\overline{c}_t < \overline{c}_0$. It follows that there is a unique solution to the system $(B.2)$ which also satisfies $k_t \in \left(0, \overline{k}\right)$ and $y_t \in \left(0, s\left(\overline{k}\right)\right)$. It can also be seen that k_t and y_t are both strictly increasing in i^h.

Since the solution satisfies $k_t < \overline{k}$, we can reiterate the same analysis to solve for $k_{t-1} \in \left(0, \overline{k}\right)$ and $y_{t-1} \in \left(0, s\left(\overline{k}\right)\right)$. By induction, we obtain a unique equilibrium path $\{k_1, y_1\}_{t=0}^{T-1}$. Note also that k_0 is given, and output at date 0 is determined by

$$y_0 = Y_0(k_0) = \overline{c}_0 + k_1 + i^h - \left(1 - \delta^k\right) k_0.$$

Note that the path $\{k_1, y_1\}_{t=0}^{T-1}$ as well as the initial output y_0 are strictly increasing in i^h.

We next claim there is a liquidity trap at date 0, $y_0 < s(k_0)$, as long as the residential investment is below a threshold. Note that $y_0 < s(k_0)$ if and only if $R(k_0, y_0) < \delta^k$. First consider the claim for the worst possible shock, $i^h = -\overline{c}_T$. In this case, the output at date 0 satisfies,

$$y_0 \leq \overline{c}_0 - \overline{c}_T + \overline{k} - \left(1 - \delta^k\right) k_0.$$

Combining this with Assumption 3(ii), we obtain $R(k_0, y_0) < \delta^k$, proving the claim. Since y_0 is strictly increasing in i^h, there exists $i_1^h > -\overline{c}_T$ such that $y_0 = s(k_0)$. It follows that $y_0 < s(k_0)$ whenever $i^h < i^{h,1}$, proving the first part of the proposition.

Similarly, we claim that the worst allowed shock $i^h = -\overline{c}_T$ induces $k_1 < k_0$. To see this, consider the output at date 1 given by,

$$y_1 = \overline{c}_1 - \overline{c}_T + k_2 - \left(1 - \delta^k\right) k_1 \leq \overline{c}_0 - \overline{c}_T + \overline{k} - \left(1 - \delta^k\right) k_1.$$

Combining this with Assumption 3(ii), we obtain $R(k_0, y_1) < \delta^k$. This in turn implies $k_1 = K(y_1) < k_0$, proving the claim. Since k_1 is strictly increasing in i^h, there exists $i^{h,2} \leq i^{h,1}$ and $i^{h,2} > -\overline{c}_T$ such that $k_1 = k_0$. It follows that $k_1 < k_0$ whenever $i^h < i^{h,2}$, completing the proof. $\square$

B.2 Proofs for the extension with income-trackers

Lemma 4. *The income-trackers' wage level is given by Eq.* $(A.2)$ *for some function* $\psi(k_t, y_t)$*, which has the following properties:*

(i) $\psi(k_t, y_t) = 1 - \tau_t = \frac{v'(l_t)}{F_l(k_t, l_t)}$,

(ii) $\psi(k_t, y_t) = 1$ *if* $r_{t+1} > 0$*, and* $\psi(k_t, y_t) \in [0,1]$ *if* $r_{t+1} = 0$,

(iii) $\psi(k_t, y_t)$ *is strictly decreasing in* k_t*, and strictly increasing in* y_t.

Proof. As in the proof of Lemma 1, let $L(k, y)$ denote the labor supply corresponding to capital level $k \in (0, \bar{k}]$ and output $y \in (0, s(k)]$. Next consider the analogue of Problem (10) that also includes firms' demand for hand-to-mouth labor. The firm's optimization in this case implies

$$w^{tr}(k_t, y_t) = (1 - \tau_t)\,\eta^{tr},$$

where $\tau_t \geq 0$ is the Lagrange multiplier on the demand constraint. As before, the same problem also implies that τ_t is equal to the labor wedge, that is:

$$1 - \tau_t = \frac{v'(L(k_t, y_t))}{F_l(k_t, L(k_t, y_t))} \equiv \psi(k_t, y_t).$$

Here, the last line defines the function $\psi(k_t, y_t)$. Combining these expressions proves the first part. Recall that the labor wedge satisfies $\tau_t = 0$ if $r_{t+1} = 0$, and $\tau_t \in [0,1]$ if $r_{t+1} > 0$, proving the second part. It can also be checked that $\psi_k < 0$ and $\psi_y > 0$, completing the proof. □

Proposition **6** requires a strengthening of Assumption 3(i). Assumption 3(ii) remains unchanged.

Assumption 3$^{\text{tr}}$.(i) $i^h \in [-(\bar{c}_T - \eta^{tr} l^{tr}), s(\bar{k}) - \delta\bar{k} - \bar{c}_0)$.

Proof of Proposition 6. Let $K(y)$ denote the function defined in the proof of Proposition 3 that describes the break-even capital level $k_t = K(y_t)$ given output y_t. Eqs. (25) and $(A.3)$can then be written as, $y_t = f(y_t)$ for each $t \geq 1$, where

$$f(y_t) \equiv \bar{c}_t + k_{t+1} - \left(1 - \delta^k\right) K(y_t) + i^h + (\psi(K(y_t), y_t) - 1)\,\eta^{tr} l^{tr}. \qquad \text{(B.3)}$$

The output at date 0 is separately characterized as the solution to Eq. $(A.3)$ with the initial k_0 (as opposed to $K(y_0)$).

We next claim that, given $k_{t+1} \in (0, \overline{k}]$, there exists a solution to $(B.3)$ over the range $y_t \in \left(0, s\left(\overline{k}\right)\right)$. To see this, note that

$$\lim_{y_t \to 0} f\left(y_t\right) > \overline{c}_T + i^h - \eta^{tr} l^{tr} \geq 0,$$

where the first inequality uses $\overline{c}_t \geq \overline{c}_T, k_{t+1} > 0$ and $\psi \geq 0$, and the second inequality uses Assumption 3^{tr}(i). Next note that

$$f\left(s\left(\overline{k}\right)\right) \leq \overline{c}_0 + \overline{k} - \left(1 - \delta^k\right) k_0 + i^h < s\left(\overline{k}\right),$$

where the first inequality uses $\overline{c}_t \leq \overline{c}_0, k_{t+1} \leq \overline{k}$ and $\psi \leq 1$, and the second inequality uses Assumption 3^{tr}(ii). Combining the last two inequalities implies the existence of a solution $y_t \in \left(0, s\left(\overline{k}\right)\right)$ along with $k_t = K\left(y_t\right) \in \left(0, \overline{k}\right)$. Applying the same argument recursively, we obtain the path $\{k_t, y_t\}_{t=1}^{T-1}$. By the same argument, there exists $y_0 \in \left(0, s\left(\overline{k}\right)\right)$ that solves Eq. $(A.3)$ with the initial k_0.

Note that there could be multiple solutions to Eq. $(B.3)$ [and Eq. $(A.3)$ for date 0], which could generate multiple equilibria. We establish the desired results for the "best" equilibrium that has the highest capital and net output, which also implies the results for any other equilibrium. To this end, let y_t^b denote the supremum over all y_t's that solve Eq. $(B.3)$ [and Eq. $(A.3)$ for date 0] given k_{t+1}^b. Then let $k_t^b = K\left(y_t^b\right)$. By induction, we obtain a particular solution to Eq. $(B.3)$ [and Eq. $(A.3)$ for date 0]. It is easy to show that this is the "best" solution in the sense that any other solution satisfies $k_t \leq k_t^b$ and $y_t \leq y_t^b$ for each t.

We next claim that there is a liquidity trap at date 0, $y_0^b < s\left(k_0\right)$, or equivalently $R\left(k_0, y_0^b\right) < \delta^k$, as long as i^h is below a threshold. First consider the claim for the worst possible shock $i^h = -\left(\overline{c}_T - \eta^{tr} l^{tr}\right)$. In this case, output at time 0 satisfies,

$$\begin{aligned} y_0^b &= \overline{c}_0 - \overline{c}_T + k_1^b - \left(1 - \delta^k\right) k_0 + \left(\psi - 1\right) \eta^{tr} l^{tr} \\ &\leq \overline{c}_0 - \overline{c}_T + \overline{k} - \left(1 - \delta^k\right) k_0. \end{aligned}$$

Combining this with Assumption 3(ii), we obtain $R\left(k_0, y_0^b\right) < \delta^k$, proving the claim for the worst possible shock. As in the proof of Proposition 3, this further implies that there exists $i^{h,1} > -\left(\overline{c}_T - \eta^{tr} l^{tr}\right)$ such that there is a liquidity trap at date 0 if and only if $i^h < i^{h,1}$. This completes the proof of the first part of the proposition.

To prove the second part, first note that $y_t^b < s\left(k_t^b\right)$ also implies $\psi_t\left(k_t^b, y_t^b\right) < 1$ for each $t \in \{0, .., T-1\}$. Eqs. $(B.3)$ and $(A.3)$ then imply that y_t^b is strictly decreasing in l^{tr} for each $t \in \{0, .., T-1\}$. Next note that the required inequality can be rewritten as,

$$\bar{c}_0 - c^* < \left(1 - \psi\left(k_0, y_0^b\right)\right)\eta^{tr} l^{tr}. \tag{B.4}$$

Since y_0^b is strictly decreasing in l^{tr}, so is the expression $\psi\left(k_0^b, y_0^b\right)$. Thus, there exists l_1^{tr} such that $(B.4)$ holds for the best equilibrium path, $\left\{k_t^b, y_{t-1}^b\right\}_{t=0}^{T-1}$ if and only if $l^{tr} > l_1^{tr}$. Note also that any other equilibrium features $y_0 \leq y_0^b$, and thus $\psi\left(k_0, y_0\right) \leq \psi\left(k_0, y_0^b\right)$. It follows that, as long as $l^{tr} > l_1^{tr}$, the inequality in $(B.4)$ holds for any equilibrium, completing the proof. □

B.3 Proofs for the policy analysis in Section 5

Proof of Lemma 2. We conjecture and verify that the economy experiences a liquidity trap at date 0. Recall that the model is unchanged starting date 1. Then, given $r_1 = 0$ and $R_1 = s'\left(k_1\right)$, the investment at date 0 is at its maximum level, $k_1 = \bar{k}$. Likewise, consumption at date 0 is also at its maximum level, $\bar{c}_0$, characterized in Section 3. Finally, the residential investment part of the model is unchanged, and thus, i_0^h is given by Eq. (18). Combining these ingredients implies that the output in the conjectured equilibrium is at its maximum level, $y_0 = \bar{y}_0$, where $\bar{y}_0$ is given by Eq. (21).

Next consider the efficient level of employment at date 0. The efficiency implies the household's intratemporal condition holds, $w_0 u'\left(c_0\right) = v'\left(l_0\right)$, and the equilibrium wage level is determined by the labor's marginal product, $w_0 = F_l\left(k_0, l_0\right)$. Combining these conditions is equivalent to setting the labor wedge in (28) to zero. Thus, the efficient level of labor supply is characterized by the condition, $\frac{v_0'\left(l_0^{*,sep}\right)}{F_l\left(k_0, l_0^{*,sep}\right)u'\left(\bar{c}_0\right)} = 1$. This also implies an efficient level of output, $y_0^{*,sep} = F\left(k_0, l_0^{*,sep}\right)$. Define the threshold level of overbuilding to ensure $\bar{y}_0 = y_0^{*,sep}$, that is,

$$\bar{b}_0^{sep} = \frac{\bar{k} - \left(1 - \delta^k\right)k_0 + \bar{c}_0 + \delta^h h^* - y_0^{*,sep}}{\left(1 - \delta^h\right)h^*}. \tag{B.5}$$

Since $\overline{y}_0$ is decreasing in b_0, it follows that the economy experiences a liquidity trap when $b_0 > \overline{b}_0^{sep}$, verifying our conjecture.

The employment level during a liquidity trap satisfies $l_0 < l_0^{*,sep}$, and it is characterized by solving, $y_0 = \overline{y}_0 = F(k_0, l_0)$. The labor wedge is characterized by solving, $1 - \tau_0 = \frac{v_0'(l_0)}{F_l(k_0,l_0)u'(\overline{c}_0)}$, and it satisfies $\tau_0 > 0$. The rental rate of capital is given by $R_0 = (1 - \tau_0) F_k(k_0, l_0)$, and it is below the efficient benchmark, $R_0 < F_k(k_0, l_0^{*,sep})$. This completes the proof.

For future reference, note that the employment level can also be written as a function, $L(k_0, y_0)$, as in the proof of Lemma 1. This also implies the labor wedge can be written as a similar function,

$$\tau(k_0, y_0) = 1 - \frac{v_0'(L(k_0, y_0))}{F_l(k_0, L(k_0, y_0))\, u'(\overline{c}_0)}.$$

Finally, the rental rate of capital can also be written as a function,

$$R_0 = R_0(k_0, y_0) \equiv (1 - \tau(k_0, y_0))\, F_k(k_0, L(k_0, y_0)).$$

It can be checked that functions, $1 - \tau(\cdot)$, and $R_0(\cdot)$, are both decreasing in k_0 and increasing in y_0. □

Proof of Proposition 4. We first derive the household's optimality condition (29). The analogue of problem (5) can be written as,

$$\begin{aligned} W_0(h_1) &= \max_{\{c_t, a_{t+1}\}_t} \sum_{t=0}^{\infty} \beta^t u(c_t) \\ \text{s.t.} \quad & c_t + a_{t+1} + h_{t+1} = e_t + a_t(1 + r_t) + \Pi_t + \left(1 - \delta^h\right) h_t \\ \text{given } h_0 &\geq h^*, h_1 \geq h^* \text{ and } h_t = h^* \text{ for each } t \geq 2. \end{aligned}$$

Using the envelope theorem, we obtain,

$$\frac{dW_0(h_1)}{dh_1}|_{h_1 = h^*} = \beta u'(c_1)\left(1 - \delta^h\right) - u'(c_0).$$

Combining this with the Euler equation, $u'(c_0) = \beta(1 + r_1)\, u'(c_1)$, establishes Eq. (29).

We next derive the planner's optimality condition (32). When h_1 is in a neighborhood of h^*, the planner's allocation solves problem (30). This problem implies that the capital stocks k_0 and $k_1 = \overline{k}$ are constant, and that the remaining variables, $c_0(h_1), y_0(h_1), l_0(h_1)$, are determined as implicit functions of h_1. Implicitly differentiating the aggregate demand constraint (31) with respect to h_1, we obtain,

$$\frac{dl_0}{dh_1} = \frac{1 + \frac{dc_0}{dh_1}}{F_l(k_0, l_0)} = \left(1 + \frac{dc_0}{dh_1}\right) \frac{(1-\tau_0) u'(c_0)}{v'(l_0)}.$$

Here, the second equality substitutes the labor wedge from Eq. (28). Using problem (11) along with the envelope theorem, we also obtain,

$$\frac{dV_1(k_1, h_1)}{dh_1} = \left(1 - \delta^h\right) u'(c_1) = \left(1 - \delta^h\right) \frac{u'(c_0)}{\beta}.$$

Here, the second equality uses the Euler equation. Differentiating the objective function with respect to h_1, and using these expressions, we obtain,

$$\begin{aligned} \frac{dW_{0,pl}(h_1)}{dh_1} &= u'(c_0) \frac{dc_0}{dh_1} - v_0'(l_0) \frac{dl_0}{dh_1} + \beta \frac{dV_1(k_1, h_1)}{dh_1}, \\ &= u'(c_0) \left(\frac{dc_0}{dh_1} - \left(1 + \frac{dc_0}{dh_1}\right)(1-\tau_0) + 1 - \delta^h \right). \end{aligned}$$

Rearranging terms establishes Eq. (32). Comparing this expression with Eq. (29) also implies that the household's and the planner's marginal utilities are equated when $\tau_0 = 0$.

We next show that the planner's marginal utility, $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*}$, is increasing in the labor wedge, τ_0. Note that the Euler equation in problem (30) implies,

$$\frac{dc_0}{dh_1}|_{h_1=h^*} = \frac{\beta u''(C(h^*))}{u''(\overline{c}_0)} C'(h^*) > 0.$$

Here, the inequality follows because the solution to the neoclassical problem (11) implies $C'(h^*) > 0$. Note also that the derivative $\frac{dc_0}{dh_1}|_{h_1=h^*}$ is independent of b_0 or τ_0. Combining this with Eq. (32) proves that $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*}$ is increasing τ_0.

Next note from the proof of **2** that the labor wedge, τ_0, is strictly decreasing in aggregate demand, $y_0 = \overline{y}_0$. Since the maximum demand, $\overline{y}_0$, in Eq. (21) is strictly decreasing in overbuilding, b_0, this implies that the labor wedge is strictly increasing in overbuilding, b_0. This in turn implies that the planner's marginal utility, $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*}$, is strictly increasing in b_0. It can also be checked that $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*} > 0$ for sufficiently high levels of b_0. Let $\tilde{b}_0^{sep} > \overline{b}_0^{sep}$ denote the level of overbuilding such that $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*} = 0$. It follows that, $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*} > 0$ if and only if $b_0 > \tilde{b}_0^{sep}$. This also implies $h_{1,pl} > h^*$ if and only if $b_0 > \tilde{b}_0^{sep}$, completing the proof. □

Proof of Lemma 3. First consider the limiting case with $\pi^H = 1$. In this case, given the initial conditions, the economy is at an efficient steady-state with,

$$h_t = h^* \left(1 + \lambda^H\right), k_t = k^* \text{ and } c^* = s\left(k^*\right) - \delta^h \left(1 + \lambda^H\right) h^* - \delta^k k^*.$$

In particular, the competitive equilibrium features $k_0 = k^*$. In this equilibrium, the economy does not feature a liquidity trap at date 0 or state H of date 1. In fact, we have $r_1 = r_2^H = 1/\beta > 0$. However, since $\lambda^H > \overline{b}_0^{sep}\left(k^*\right)$, the economy features a liquidity trap in the (zero probability) state L.

Next note that the capital choice in competitive equilibrium is a continuous function of the probability of the high state, $k_0\left(\pi^H\right)$. By Eq. $(B.5)$, $\overline{b}_0^{sep}\left(k_0\right)$ is also a continuous function of k_0. It follows that there exists $\overline{\pi}^1$ (which could also be $\overline{\pi}^1 = 0$) such that $\lambda^H > \overline{b}_0^{sep}\left(k^*\right)$ if and only if $\pi^H > \overline{\pi}^1$. Similarly, note that the interest rates r_1 and r_2^H are also continuous functions of π^H. Using continuity once again, there exists $\overline{\pi}^2 < 1$ (which could also be $\overline{\pi}^2 = 0$) such that the economy does not feature a liquidity trap at date 0 or at state H if and only if $\pi^H > \overline{\pi}^2$. Taking $\overline{\pi} = \max\left(\overline{\pi}^1, \overline{\pi}^2\right)$ proves the statement. □

Proof of Proposition 5. We first derive $\frac{dV_0^H(k_0,h_0)}{dk_0}$ and $\frac{dV_0^L(k_0,h_0)}{dk_0}$. If state H is realized, then the equilibrium solves the analogue of problem (11) (with appropriate modifications to capture the higher target level, $\left(1 + \lambda^H\right) h^*$). Then, using the envelope theorem implies

$$\frac{dV_0^H\left(k_0, h_0\right)}{dk_0} = \left(s'\left(k_0\right) + 1 - \delta^k\right) u'\left(c_0^H\right).$$

Suppose instead state L is realized. We conjecture (and verify below) that the planner's allocation also features a liquidity trap in this state. The equilibrium is characterized by Lemma 2. It solves problem (30) with $h_1 = h^*$ (since we rule out ex-post policies). This problem implies that the following variables are constant, $k_1 = \overline{k}, c_0 = \overline{c}_0, h_1 = h^*$ (and thus, the continuation value V_1 is also constant). In contrast, output and employment, $y_0(k_0), l_0(k_0)$, are determined as implicit functions of k_0. Implicitly differentiating the aggregate demand constraint (31) with respect to k_0, we obtain,

$$\frac{dl_0}{dk_0} = -\frac{F_k(k_0, l_0) + (1 - \delta^k)}{F_l(k_0, l_0)} = -\left(F_k(k_0, l_0) + (1 - \delta^k)\right) \frac{(1 - \tau_0) u'(\overline{c}_0)}{v'(l_0)}.$$

Here, the second equality substitutes the labor wedge from Eq. (28). Differentiating the objective function with respect to k_0, and using this expression, we further obtain,

$$\frac{dV_0^L(k_0, h_0)}{dk_0} = -v_0'(l_0) \frac{dl_0}{dh_1} = (1 - \tau_0)\left(F_k(k_0, l_0) + (1 - \delta^k)\right) u'(\overline{c}_0).$$

Plugging in $R_0^L = (1 - \tau_0) F_k(k_0, l_0)$ from Lemma 2 implies,

$$\frac{dV_0^L(k_0, h_0)}{dk_0} = \left(R_0^k + (1 - \tau_0)(1 - \delta^k)\right) u'(\overline{c}_0).$$

Next note that combining $\frac{dV_0^H(k_0,h_0)}{dk_0}$ and $\frac{dV_0^L(k_0,h_0)}{dk_0}$ implies the planner's optimality condition (35). The analysis in the main text shows that the solution to this problem features $k_{0,pl} < k_0$. From the proof of Lemma 2**,** this also implies $\tau_{0,pl} < \tau_0$, that is, the planner mitigates the demand shortage in state L. However, the planner does not fully alleviate the demand shortage, $\tau_{0,pl} > 0$. This is because setting $\tau_{0,pl} = 0$ makes the planner's optimality condition (33) identical to the competitive equilibrium condition (35), which contradicts the result that the planner chooses a different allocation, $k_{0,pl} < k_0$. This completes the proof. □