A Model of Secular Stagnation

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Secular Stagnation Hypothesis

> *I wonder if a set of older ideas . . . under the phrase secular stagnation are not profoundly important in understanding Japan's experience, and may not be without relevance to America's experience — Lawrence Summers*

Original hypothesis:

- Alvin Hansen (1938)
- Reduction in population growth and investment opportunities
- Concerns about insufficient demand ended with WWII and subsequent baby boom

Secular stagnation resurrected:

- Lawrence Summers (2013)
- Highly persistent decline in the natural rate of interest
- Chronically binding zero lower bound

Why are we so confident interest rates will rise soon?

Interest rates in the US during the Great Depression:

- Started falling in 1929
- Reached zero in 1933
- Interest rates only started increasing in 1947

Started dropping in Japan in 1994:

- Remains at zero today

Why are we so confident interest rates are increasing in the next few years?
Wanted: A model that allows for long-lasting slumps.

Shortcomings of Some Existing Models

Representative agent models:

$$r_{ss} = \frac{1}{\beta}$$

- Real interest rate must be positive in steady state
- Households problem not well defined if $\beta \geq 1$
- ZLB driven by temporary shocks to discount rate (Eggertsson and Woodford (2003))

Patient-impatient agent models:

- Steady state typically pinned down by the discount factor of the representative saver (Eggertsson and Krugman (2012))
- Deleveraging only has temporary effect

Questions

Question 1

- Can we formalize the idea of secular stagnation?

Question 2

- "What facts, exactly, is this meant to capture"
- Answer:
- (i) Resources are underutilized ("unemployment").
- (ii) Short-term risk-free nominal interest rates are at zero and the CB want to cut them more.
- (iii) This situation can last for an arbitrarily long time.

Outline for Presentation

1. Model
   (1958) OLG endowment economy without capital – Negative short-term real interest rate can be triggered by:
      - Deleveraging shock
      - Slowdown in population growth
      - Increase in income inequality
      - Fall in relative price of investment
   - Endogenous production–Stagnation steady state
      - Permanently binding zero lower bound
      - Low inflation or deflation
      - Permanent shortfall in output from potential
2. Policy options
3. Capital
4. Conclusions

Economic Environment

Endowment economy

- Time: $t = 0, 1, 2, ...$
- Goods: consumption good (c)
- Agents: 3-generations: $i\epsilon \{y, m, o\}$
- Assets: riskless bonds (B^i)
- Technology: exogenous borrowing constraint D

Households

Objective function:

$$\max_{C^y_{t,},C^m_{t+1},C^o_{t+2}} U = \mathbb{E}_t \left\{ \log\left(C^y_t\right) + \beta \log\left(C^m_{t+1}\right) + \beta^2 \log\left(C^o_{t+2}\right) \right\}$$

Budget constraints:

$$\begin{aligned}
C^y_t &= B^y_t \\
C^m_{t+1} &= Y^m_{t+1} - (1+r_t)B^y_t + B^m_{t+1} \\
C^o_{t+2} &= Y^o_{t+2} - (1+r_{t+1})B^m_{t+1} \\
(1+r_t)B^i_t &\leq D_t
\end{aligned}$$

Consumption and Saving

Credit-constrained youngest generation:

$$C_t^y = B_t^y = \frac{D_t}{1 + r_t}$$

Saving by the middle generation:

$$\frac{1}{C_t^m} = \beta \mathbb{E}_t \frac{1 + r_t}{C_{t+1}^o}$$

Spending by the old:

$$C_t^o = Y_t^o - (1 + r_{t-1}) B_{t-1}^m$$

Determination of the Real Interest Rate

Asset market equilibrium:

$$
\begin{aligned}
N_t B_t^y &= -N_{t-1} B_t^m \\
(1+g_t)\, B_t^y &= -B_t^m
\end{aligned}
$$

Demand and supply of loans:

$$
\begin{aligned}
L_t^d &= \frac{1+g_t}{1+r_t} D_t \\
L_t^s &= \frac{\beta}{1+\beta}\left(Y_t^m - D_{t-1}\right) - \frac{1}{1+\beta}\frac{Y_{t+1}^o}{1+r_t}
\end{aligned}
$$

Determination of the Real Interest Rate

Expression for the real interest rate (perfect foresight):

$$1 + r_t = \frac{1+\beta}{\beta} \frac{(1+g_t)D_t}{Y_t^m - D_{t-1}} + \frac{1}{\beta} \frac{Y_{t+1}^o}{Y_t^m - D_{t-1}}$$

Determinants of the real interest rate:

- Tighter collateral constraint reduces the real interest rate
- Lower rate of population growth reduces the real interest rate
- Higher middle age income reduces real interest rate
- Higher old income increases real interest rate

Effect of a Deleveraging Shock

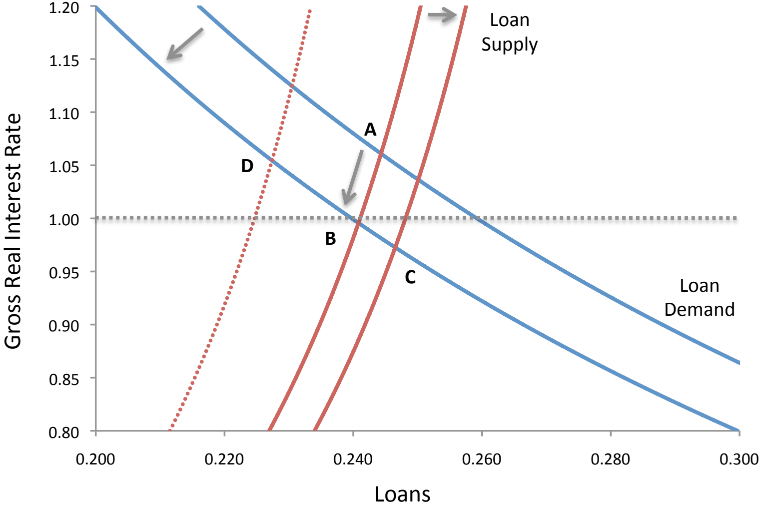
Impact effect:

- Collateral constraint tightens from D_h to D_l
- Reduction in the loan demand and fall in real rate
- Akin to Eggertsson and Krugman (2012)

Delayed effect:

- Next period, a shift out in loan supply
- Further reduction in real interest rate
- Novel effect from Eggertsson and Krugman (2012)
- Potentially powerful propagation mechanism

Effect of a Deleveraging Shock



Income Inequality

Does inequality affect the real interest rate?

- Our result due to generational inequality that triggers borrowing and lending
- What about inequality within a given cohort?

Generalization of endowment process:

- High-type households with high income in middle period
- Low-type households with low income in middle period
- Both types receive same income in last period

INCOME INEQUALITY AND REAL INTEREST RATE

Credit constrained middle income:

- Fraction η_s of middle income households are credit constrained
- True for low enough income in middle generation and high enough income in retirement
- Fraction $1-\eta_s$ lend to both young and constrained middle-generation households

Expression for the real interest rate:

$$1+r_t = \frac{1+\beta}{\beta}\frac{(1+g_t+\eta_s)\,D_t}{(1-\eta_s)\left(Y_t^{m,h}-D_{t-1}\right)} + \frac{1}{\beta\,(1-\eta_s)}\frac{Y_{t+1}^{o}}{\left(Y_t^{m,h}-D_{t-1}\right)}$$

Price Level Determination: Woodford's cashless limit

Euler equation for nominal bonds:

$$\frac{1}{C_t^m} = \beta \mathbb{E}_t \frac{1}{C_{t+1}^o}(1+i_t)\frac{P_t}{P_{t+1}}$$
$$i_t \geq 0$$

The ZLB implies a bound on steady state inflation:

$$\bar{\Pi} \geq \frac{1}{1+r}$$

- If steady state real rate is negative then steady state inflation must be positive
- No steady state with zero inflation
- But what happens when prices are NOT flexible and the central bank does not tolerate inflation?

Outline for Presentation

1. **Model**
   - **Endowment economy**
   - Endogenous production
2. Monetary and fiscal policy
3. Capital
4. Conclusions

Endogenous Production

There are N_{t-1} firms with production function

$$Y_t = L_t^\alpha$$

- Labor only factor of production (capital coming up)
- Firms take prices and wages as given

Labor supply:

- Constant inelastic labor supply from households
- Assume only middle-generation household supplies labor
- Possibility of unemployment due to wage rigidity

Aggregate Supply - Full Employment

Output and labor demand:

$$Y_t = L_t^{\alpha}$$
$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1}$$

Labor supply:

$$L_t = \bar{L}$$

- Implies a constant market clearing real wage $\bar{W} = \alpha \bar{L}^{\alpha-1}$
- Implies a constant full-employment level of output: $Y^f = \bar{L}^{\alpha}$
- Again, analogous to the endowment economy, steady state has to be consistent $\bar{\Pi} \geq \frac{1}{1+r}$.

Downward Nominal Wage Rigidity

Partial wage adjustment:

$$W_t = \max\left\{\tilde{W}_t, W^{flex}\right\}$$

$$\text{where } \; \tilde{W}_t = \gamma W_{t-1} + (1-\gamma)W^{flex}$$

Wage rigidity and unemployment:

- $\tilde{W}_t$ is a wage norm
- $W^{flex} = P_t \alpha \bar{L}^{\alpha-1}$ is the market clearing wage.
- If real wages exceed market clearing level, employment is rationed
- Unemployment: $U_t = \bar{L} - L_t$
- Similar assumption in Kocherlakota (2013) and Schmitt-Grohe and Uribe (2013)

The government

Government sets inflation at $\Pi = \Pi^*$. It can always achieve this target except if it implies negative i_t.

$$\Pi_t = \Pi^* \text{ and } i_t \geq 0.$$

If $\Pi_t = \Pi^*$ imples $i_t < 0$ then

$$i_t = 0 \text{ and } \Pi_t < \Pi^*$$

Implementation?

$$1 + i_t = \max(1, (1 + i^*)(\frac{\Pi_t}{\Pi^*})^{\phi_\pi})$$

Analyzing the model

Will analyze the steady state of the model

- A constant solution for (Π, Y, i, r) that solves the equations of the model
- Reflect a permanent recession (or not).
- Will look suspiciously similar to a old fashion IS/LM model. Bug, feature?
- Key weakness: Wage setting is reduced form. Have done Calvo prices, and other variations. Most important thing: Long-run tradeoff between inflation and output.

Proposition 1: Characterization

The steady state of the model is four numbers (Y, Π, i, r) that satisfy:

$$Y = \frac{(1+\beta)(1+g)}{\beta} \frac{D}{1+r} + D \quad (1)$$

$$1+r = \frac{1+i}{\Pi} \quad (2)$$

$$\Pi = \Pi^* \text{ or } i = 0 \text{ AND } \Pi < \Pi^* \quad (3)$$

$$Y = \begin{cases} Y^f & \text{if } \Pi \geq 1 \\ Y^f \left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} & \text{otherwise} \end{cases} \quad (4)$$

DEFINITIONS

Definition The natural level of output is (Friedman (1968))

$$Y^f \equiv \bar{L}^\alpha$$

Definition The natural rate of interest is (Wicksell (1998))

$$1 + r^f \equiv \frac{(1+\beta)(1+g)}{\beta} \frac{D}{Y^f - D}$$

Assumption $\Pi^* \geq 1$

Lemma on Possibilities

Given our assumed policy commitment: There are three possibilities, label them Cases I, II and III.

- Case I (normal equilibrium) $\Pi = \Pi^* \; i > 0$.
- Case II (full employment ZLB) $\Pi \neq \Pi^*, \Pi^* > \Pi > 1, i = 0$.
- Case III (secular stagnation) $\Pi \neq \Pi^*, \Pi < 1, i = 0$.

Lemma on Characterization

- In Case I: $\Pi = \Pi^*$

$$Y = Y^f, 1 + r = 1 + r^f, 1 + i = (1 + r^f)\Pi^*$$

- In Case II: $\Pi^* > \Pi > 1$

$$Y = Y^f, 1 + r = 1 + r^f, \Pi = (1 + r^f)^{-1}, i = 0$$

- In Case III: $\Pi < 1$.

$$Y - Y^f = \psi\left(\frac{1}{1+r} - \frac{1}{1+r^f}\right), i = 0$$

$$Y = Y^f\left(\frac{1 - \frac{\gamma}{\Pi}}{1 - \gamma}\right)^{\frac{\alpha}{1-\alpha}}$$

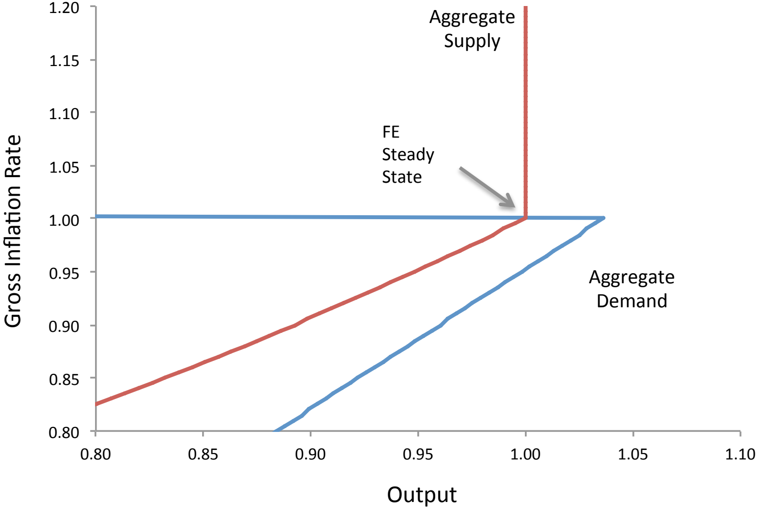
$$1 + r = \frac{1}{\Pi}$$

where $\psi \equiv \frac{(1+\beta)(1+g)}{\beta} D > 0$

Propositions

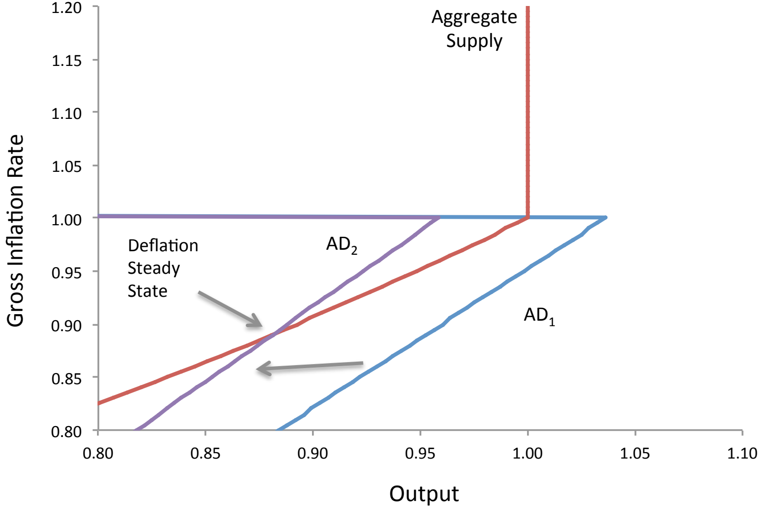
- Prop 1: Suppose $r^f > 0, \Pi^* = 1$. Then, there exists a normal equilibrium (Case I) with $\Pi = 1, r = i = r^f, Y = Y^f$.
- Prop 2: Suppose $r^f < 0, \Pi^* = 1$. Then a normal equilibrium does not exist.
- Prop 3: Suppose $r^f < 0$ and $\Pi^* = 1$. Then there exists a secular stagnation equilibrium (Case III). This is the unique equilibrium in this case.

Full Employment Steady State



Parameter Values

Effect of a D shock



Properties of the Stagnation Steady State

Long slump:

- Binding zero lower bound so long as natural rate is negative
- Deflation raises real wages above market-clearing level
- Output persistently below full-employment level

Existence and stability:

- Secular stagnation steady state exists so long as $\gamma > 0$
- Prop 4: If $\Pi^* = 1$, secular stagnation steady state is determinate. (There is a unique bounded solution local to ss)
- Contrast to deflation steady state emphasized in Benhabib, Schmitt-Grohe and Uribe (2001)
- Can do comparative statics!

Mechanism of adjustment back to full employment

Financial shock

- Reverts back to original level, you get back to where you started
- Observation: Policy been going into the opposite direction.

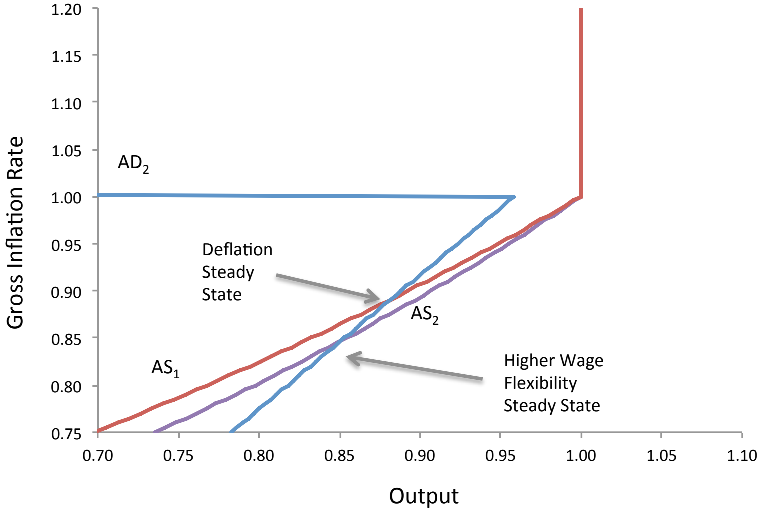
Wages become more flexible

- Works in the wrong direction: Paradox of flexibility
- Output drops by more as wages become more flexible.
- Same result if you add more forward looking behavior in wage setting: More deflation, bigger drop in output.

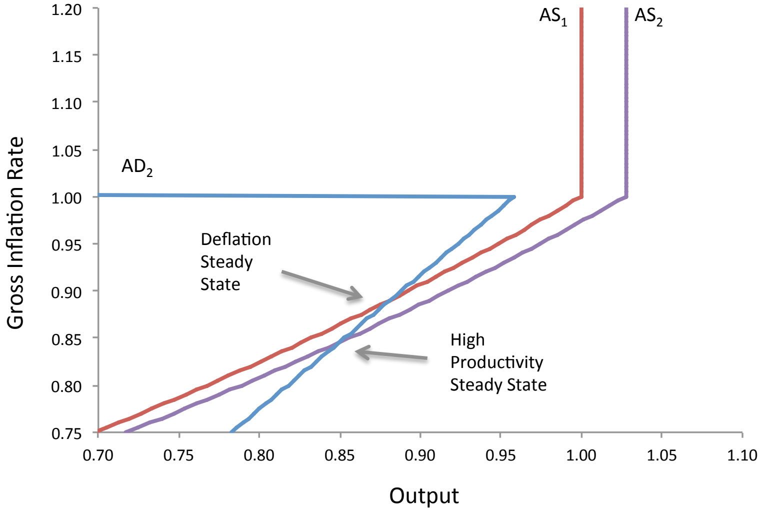
Labor participation decreases

- Reduces W^{flex} and increases output.
- But reduces Y^f: Paradox of toil.

INCREASING WAGE FLEXIBILITY



REDUCTION IN LABOR SUPPLY (HYSTERESIS)



Monetary Policy Responses

Forward guidance:

- Extended commitment to keep nominal rates low?
- Ineffective if households/firms expect rates to remain low indefinitely

Raising the inflation target:

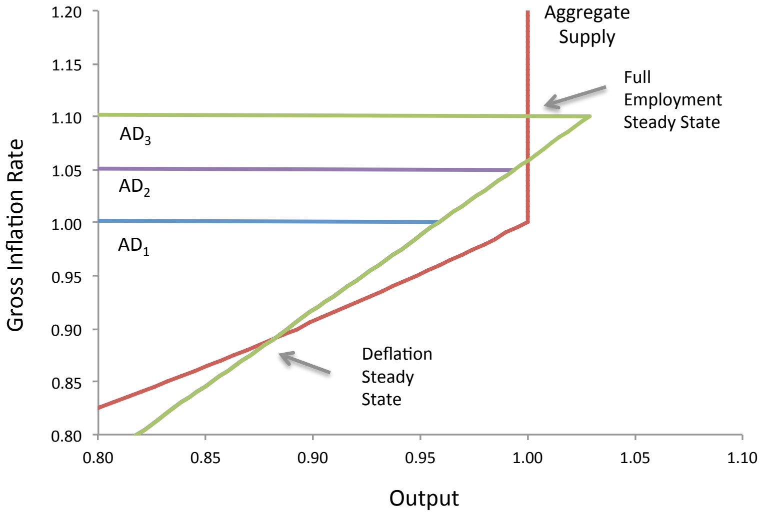
- For sufficiently high inflation target, full employment steady state exists
- Timidity trap (Krugman (2014))
- Multiple determinate steady states
- Monetary policy not as powerful as in earlier models because no way to exclude secular stagnation

Raising the Inflation Target

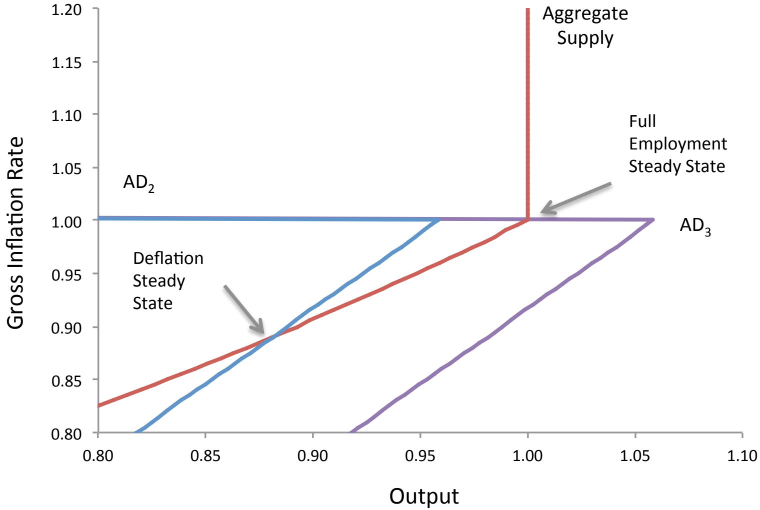
Proposition

Suppose $r^f < 0$ and $\Pi^* > \frac{1}{1+r^f}$. Then three equilibria in the model are possible - all three cases from the Lemma on possibilities.

Raising the Inflation Target



Expansionary Fiscal Policy



FISCAL POLICY

Fiscal policy and the real interest rate:

$$L_t^d = \frac{1+g_t}{1+r_t} D_t + B_t^g$$

$$L_t^s = \frac{\beta}{1+\beta}\left(Y_t^m - D_{t-1} - T_t^m\right) - \frac{1}{1+\beta}\frac{Y_{t+1}^o - T_{t+1}^o}{1+r_t}$$

Government budget constraint:

$$B_t^g + T_t^y\left(1+g_t\right) + T_t^m + \frac{1}{1+g_{t-1}} T_t^o = G_t + \frac{1+r_t}{1+g_{t-1}} B_{t-1}^g$$

Fiscal instruments:

$$G_t, B_t^g, T_t^y, T_t^m, T_t^o$$

Temporary Increase in Public Debt

Under constant population and set $G_t = T_t^y = B_{t-1}^g = 0$:

$$T_t^m = -B_t^g$$
$$T_{t+1}^o = (1 + r_t)\, B_t^g$$

Implications for natural rate:

- Loan demand and loan supply effects cancel out
- Temporary increases in public debt ineffective in raising real rate
- Temporary monetary expansion equivalent to temporary expansion in public debt at the zero lower bound
- Effect of an increase in public debt depends on beliefs about future fiscal policy

PERMANENT INCREASE IN PUBLIC DEBT

Consider steady state following fiscal rule:

$$T^o = \beta\,(1+r)\,T^m$$

$$L^d = \frac{1+g}{1+r}D + B^g$$

$$L^s = \frac{\beta}{1+\beta}\,(Y^m - D) - \frac{1}{1+\beta}\frac{Y^o}{1+r}$$

Implications for natural rate:

- Changes in taxation have no effects on loan supply
- Permanent rise in public debt always raises the real rate
- Equivalent to helicopter drop at the zero lower bound
- We have not modeled here possible tradeoff: Default premia, probability of exiting ss and face higher interest rates, etc

Government Purchases Multiplier

Slope of the AD and AS curves:

$$\psi = \frac{1+\beta}{\beta}(1+g)D$$
$$\kappa = \frac{1-\alpha}{\alpha}\frac{1-\gamma}{\gamma}$$

Purchases multiplier at the zero lower bound:

Financing	Multiplier	Value
Increase in public debt	$\frac{1+\beta}{\beta}\frac{1}{1-\kappa\psi}$	> 2
Tax on young generation	0	0
Tax on middle generation	$\frac{1}{1-\kappa\psi}$	> 1
Tax on old generation	$-\frac{1+g}{\beta}\frac{1}{1-\kappa\psi}$	< 0

HOUSEHOLDS

Objective function:

$$\max_{C_{t,}^{y},C_{t+1}^{m},C_{t+2}^{o}} U = \mathbb{E}_t \left\{ \log\left(C_t^y\right) + \beta \log\left(C_{t+1}^m\right) + \beta^2 \log\left(C_{t+2}^o\right) \right\}$$

Budget constraints:

$$\begin{aligned} C_t^y &= B_t^y \\ C_{t+1}^m + p_{t+1}^k K_{t+1} + (1+r_t) B_t^y &= w_{t+1} L_{t+1} + r_{t+1}^k K_{t+1} + B_{t+1}^m \\ C_{t+2}^o + (1+r_{t+1}) B_{t+1}^m &= p_{t+2}^k (1-\delta) K_{t+1} \end{aligned}$$

Dynamic Efficiency

Characterization

Capital supply (perfect foresight):

$$\left(p_t^k - r_t^k\right)\frac{1}{C_t^m} = \beta p_{t+1}^k\left(1-\delta\right)\frac{1}{C_{t+1}^o}$$

Loan supply and demand:

$$L_t^d = \frac{1+g_t}{1+r_t}D_t$$

$$L_t^s = \frac{\beta}{1+\beta}\left(Y_t - D_{t-1}\right) - \frac{\beta}{1+\beta}\left(p_t^k + p_{t+1}^k\frac{1-\delta}{\beta\left(1+r_t\right)}\right)K_t$$

Capital and Secular Stagnation

Rental rate and real interest rate:

$$r_t^k = p_t^k - p_{t+1}^k \frac{1-\delta}{1+r_t} \geq 0$$

$$r_{ss} \geq -\delta$$

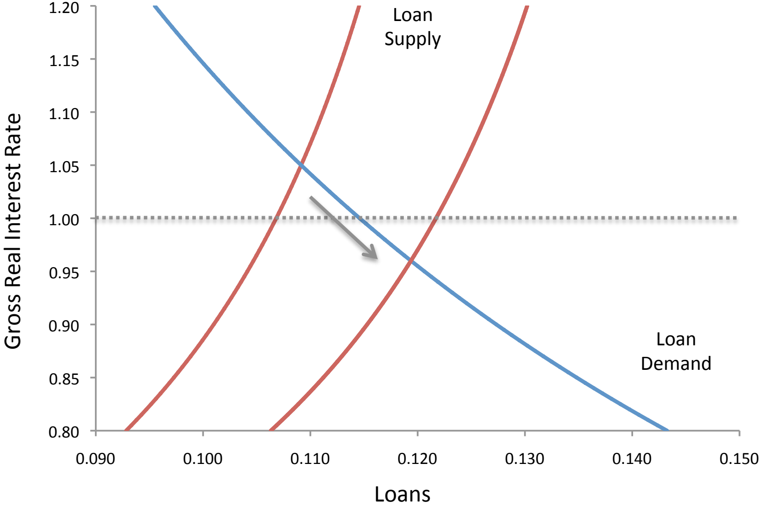
- Negative real rate now constrained by fact that rental rate must be positive

Relative price of capital goods:

- Decline in relative price of capital goods
- Need less savings to build the same capital stock
- –> downward pressure on the real interest rate.
- Global decline in price of capital goods (Karabarbounis and Neiman, 2014)

Land

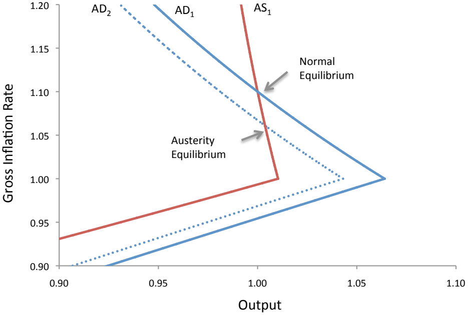
Effect of a Shock to Price of Capital Goods



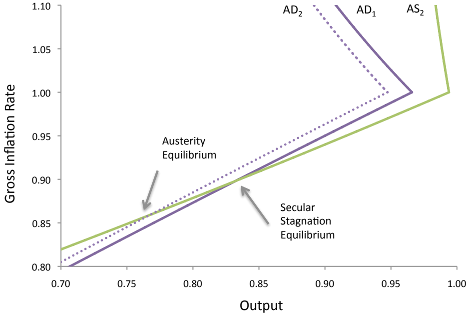
Paradox of Thrift

Effect of a discount rate shock

Positive natural rate



Negative natural rate



Conclusions

Policy implications:

- Higher inflation target needed
- Limits to forward guidance
- Role for fiscal policy
- In absence of policy, not an obvious mechanism for adjustment.
- Pay as you go social security, increase retirement age

Key takeaways:

- NOT that we will stay in a slump forever
- Slump of arbitrary duration
- OLG framework to model interest rates