

# Discussion of Straub & Werning's Positive Long Run Capital Taxation: Chamley-Judd Revisited

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# What This Paper Could Be Interpreted to Say

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- Judd is wrong!

# How I Read the Paper

- In a standard macro model,
  - With well-functioning markets
  - With only taxes on labor and capital income
  - With an upper bound on capital taxes
- Optimal taxation implies either
  - Capital taxes are zero after a transition phase at the upper bound, or
  - Capital taxes are at their upper bound forever.
- Which occurs depends on the size of the initial debt.

# How I Read the Paper

- Very nice technical contribution
- Would like to see some quantitative exercises
- Not really a justification for current tax systems

# Static Public Finance Environment

- I households
- Commodities  $x = (c_1, \dots, c_N, n_1, \dots, n_M, K)$ 
  - $c_i$ : consumption goods
  - $n_j$ : types of labor
  - $K$ : primary inputs (land or initial capital)
- Technology:

$$f(c_1 + g_1, \dots, c_N + g_N, n_1, \dots, n_M, K) = 0 \quad [RF]$$

- Preferences:  $u^i(x^i)$

# Competitive Equilibrium

- Policy:  $\pi = (\tau_{c_i}, \tau_{n_j}, \tau_K)$
- Household problem (HH):

$$\max u^i(x^i)$$

s.t.

$$\sum_{i=1}^N p_i(1 + \tau_{c_i})c_i \leq \sum_{j=1}^M w_j(1 - \tau_{n_j})n_j + r(1 - \tau_K)K$$

- CE is allocation, prices, policies that solves HH and is resource feasible.



# Complete and Incomplete Tax Systems

- CE allocation must satisfy IC

$$\sum_{i=1}^N c_i u_{c_i} + \sum_{j=1}^M n_j u_{n_j} = (1 - \tau_K) F_K K \quad (1)$$

- If no restrictions on taxes, any CE allocation satisfying IC and RF can be implemented as CE.
- Will call such tax systems complete
- Will call tax systems without this property incomplete

# The Ramsey Problem

- Choose  $x, \tau_K$  to solve

$$\max \sum \lambda^i u^i(x^i)$$

$$\sum_{i=1}^N c_i u_{c_i} + \sum_{j=1}^M n_j u_{n_j} = (1 - \tau_K) F_k K$$

$$f(c_1 + g_1, \dots, c_N + g_N, n_1, \dots, n_M, K) = 0$$

- Clearly optimal to set  $\tau_K = 1$ .

# The Intermediate Goods Result

- With a complete tax system, Ramsey problem is equivalent to

$$\max \sum \lambda^i u^i(x^i)$$

s.t.

$$IC$$

$$p \cdot y \leq p \cdot y^*$$

- That is, given a price vector  $p$ , production efficiency is optimal.

# Uniform Commodity Taxation

- If preferences are homothetic over a subset of consumption goods and separable from all labor,
  - Optimal to tax subset at uniform rate
- Corollary of intermediate goods result.

## Applying to Macro Models

- Assume taxes on consumption  $\eta_t$ , labor  $\tau_{n_t}$ , and capital income  $\tau_{k_t}$ .
- Initial tax rates are given.
- Preferences:  $\sum \beta^t [u(c_t) - v(n_t)]$ , where  $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$
- IC now becomes

$$\sum \beta^t [c_t u'(c_t) - n_t v'(n_t)] = \frac{u'(c_0)}{1 + \eta_0} [(1 - \tau_{k_0})(F_{k_0} - \delta) + b_0]$$

- $w(c_t, n_t) = u(c_t) - v(n_t) + \lambda [c_t u'(c_t) - n_t v'(n_t)]$  for  $t \geq 1$ .

# Applying to Macro Models

- Ramsey problem implies

$$w_{c_t} = \beta w_{c_{t+1}} [F_{k_{t+1}} - \delta + 1]$$

- Since  $w_{c_t}$  proportional to  $u_{c_t}$
- No distortion from period 1 on
- May require high taxes on capital income or high consumption taxes in period 1.

# Where Did Chamley Go Wrong

- Assume no consumption taxes
- Assume  $\tau_{k_t} \leq 1$  for all  $t$ .
- Then get additional constraint

$$u_{c_t} \geq \beta u_{c_{t+1}}, \quad \forall t$$

# Chamley's Problem

$$V(b_0) = \max \int_0^{\infty} e^{-\rho t} [u(c_t) - v(n_t)]$$

s.t.

$$\dot{c}_t \geq -\frac{\rho}{\sigma} c_t$$

$$c_t + g_t + \dot{k}_t = f(k_t, n_t) - \delta k_t$$

$$\int_0^{\infty} e^{-\rho t} [c_t u'(c_t) - n_t v'(n_t)] \geq u'(c_0)(k_0 + b_0)$$



# Key First Order Conditions

- $\dot{\eta}_t - \rho\eta_t = \eta_t \frac{\rho}{\sigma} + \lambda_t - (1 + \mu(1 - \delta))u'(c_t)$
- $\dot{\lambda}_t = (\rho - (F_k - \delta))\lambda_t.$
- Claim: Suppose  $\eta_t = 0$ ,  $t \in [T, T + \epsilon)$ , then  $\eta_t = 0$ ,  $\forall t \geq T$ .
  - $\lambda_t = (1 + \mu(1 - \delta))u'(c_t)$ ,  $t \in [T, T + \epsilon)$ .
  - So  $\dot{\eta}_t = 0$  at  $t = T + \epsilon$ .
- So  $\eta_t = 0$ ,  $\forall t \geq T$ .

## Straub & Werning's First Insight

- Suppose  $\sigma > 1$ .
- Then if  $T < \infty$ ,  $1 + \mu(1 - \sigma) > 0$ , so  $\mu$  bounded from above by  $\frac{1}{\sigma-1}$ .
- So  $V'(b_0) = -\mu u'(c_0)$  is bounded below.

## Straub & Werning's Second Insight

- Let  $V_\infty(b_0)$  be utility associated with solution assuming constraint binds forever, so

$$c_t = c_0 e^{-\frac{\rho}{\sigma} t}.$$

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$$V_\infty(b_0) = \max_{c_0} u(c_0) \frac{\sigma}{\rho} - \tilde{V}(c_0)$$

s.t.

$$c_0 \frac{\sigma}{\rho} - c_0^\sigma (1 + \tau_c) \tilde{V}(c_0) \geq k_0 + b_0$$

- Can show  $V'_\infty(b_0) \rightarrow -\infty$
- So for large  $b_0$ ,  $V'(b_0) \rightarrow -\infty$
- Contradiction!

# Judd and All That

- Capitalists and workers
- Suppose workers have no capital, but can save or borrow.
- Two IC constraints

$$\sum \beta^t [c_t^w u'_w(c_t^w) - n_t^w v'_w(n_t^w)] = 0$$

$$\sum \beta^t [c_t^c u'_c(c_t^c) - n_t^c v'_c(n_t^c)] = u'(c_0)(k_t + b_t)$$

- Problem essentially identical to Chamley

# Judd and All That

- Suppose workers cannot save or borrow.
- Now get countable infinity of IC

$$c_t^w u'_w(c_t^w) - n_t^w v'_w(n_t^w) = 0, \forall t$$

- Now we have to worry about whether multipliers converge.

# Bottomline

- Theory tells us: if markets work well and consumption, labor, and capital taxes are available,
  - Do not distort production efficiency;
  - That is, do not distort intertemporal margins.
- Developed economies have access to all three.
- So good policy advice is: do not distort intertemporal margins.