Positive Long-Run Capital Taxation: Chamley-Judd Revisited

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Should we tax capital income?

- Two common rationales ...
  1. redistribution
  2. reduce labor taxes $\rightarrow$ more incentives to work
- But: efficiency costs $\rightarrow$ distorts savings decision

Chamley-Judd: *Zero* tax is optimal in steady state!

- **Ramsey approach:**
  - linear taxes on capital and labor
  - full commitment
  - restrictions on lump-sum transfers and consumption taxes
- **Judd (1985):** when used to redistribute
- **Chamley (1986):** when used to reduce labor taxes
- **Both:** *zero* taxes are optimal in the steady state!
- **Precise intuitions?**
  - No complete agreement
  - Somewhat elusive

Chamley-Judd is controversial

- Many questioned key assumptions
  - infinitely lived agent?
  - infinite elasticity of savings?
  - no uncertainty?
- Many wrote alternative models...
  - new dynamic public finance
  - models of bequest taxation
- Still, Chamley-Judd remains a **key benchmark for zero capital tax**

Mankiw et al (2009, JEP)

- *"Perhaps the most prominent result from dynamic models of optimal taxation is that the taxation of capital income ought to be avoided. This result['s ...] strong underlying logic has made it the benchmark."*

This paper: Logic not as strong...

- Revisit the Chamley-Judd results... *using their own models*
  - show proofs incomplete
  - overturn results when **intertemp. elasticity of subst. (IES)**$<$ **1**
  - **positive capital tax** in the long run
- What went wrong in a nutshell?
  - results require convergence to interior steady state for quantities *and* Lagrangian multipliers
  - $\rightarrow$ assumptions on *endogenous* objects!
  - ... not necessarily satisfied

Primal approach to optimal taxation

max objective

- subject to

resource constraint (RC)

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- Intertemporal optimality

$$\text{MRT} = \text{MRS}\,\{1 + \textbf{wedge ?}\}$$

Outline

1. Judd (1985): Capital taxation and redistribution
2. Chamley (1986): Labor and capital taxation
3. Conclusion

Judd (1985): Capital taxation and redistribution

Judd (1985): Capital taxes for redistribution

- **Capitalists**
  - own initial capital stock k_0
  - live off capital income
  - capital taxes
- **Workers**
  - only labor income, inelastic labor supply of 1
  - lump-sum transfers
  - consume hand-to-mouth
- **Policy instruments**
  - **no lump-sum taxation of capitalists**
  - no government bonds, no consumption taxes
  - full ex-ante commitment to tax policy

First: Solve capitalists' problem to get IC

- **Capitalists' problem**

$$\max_{\{C_t, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$C_t + k_{t+1} = R_t k_t$$

here: R_t after-tax interest rate

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- First order optimality

$$U'(C_{t-1}) = \beta R_t U'(C_t)$$

$$\beta^t U'(C_t) k_{t+1} \to 0$$

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here: R_t after-tax interest rate

- First order optimality + budget constraint = **IC**

$$C_t + k_{t+1} = \frac{U'(C_{t-1})}{\beta U'(C_t)} k_t$$

$$\beta^t U'(C_t) k_{t+1} \to 0$$

Second: Social planner's problem

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + \gamma U(C_t) \right\}$$

- subject to

$$c_t + C_t + g + k_{t+1} \leq f(k_t) + (1 - \delta) k_t \qquad \text{(RC)}$$

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- Want redistribution: capitalists $\longrightarrow$ workers
  - requires sufficiently low welfare weight on capitalists γ
  - for simplicity, will sometimes take the extreme: $\gamma \to 0$

First order conditions

- Assume $U(C) = C^{1-\sigma}/(1-\sigma)$, $\sigma = 1/\text{IES}$

$$\mu_0 = 0$$

$$\mu_{t+1} = \mu_t \left(\frac{\sigma - 1}{\sigma \kappa_{t+1}} + 1 \right) + \frac{1}{\beta \sigma \kappa_{t+1} \upsilon_t}$$

$$\underbrace{f'(k_{t+1}) + 1 - \delta}_{\text{MRT}} = \underbrace{\frac{U'(C_t)}{\beta U'(C_{t+1})}}_{\text{MRS}} \left\{ 1 + \underbrace{\upsilon_t^{-1}(\upsilon_{t+1} - \upsilon_t) + \beta \upsilon_{t+1}(\mu_{t+1} - \mu_t)}_{\text{wedge}} \right\}$$

$$\upsilon_t = U'(C_t)/u'(c_t)$$
$$\mu_t = \text{Lagr. multiplier on IC}$$
$$\kappa_t = k_t / C_{t-1}$$

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  $\Rightarrow$ wedge $= 0$. **Zero capital tax! ......**

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- Judd (1985) studies interior steady state for **allocation + multipliers**

  $\Rightarrow$ wedge $= 0$. **Zero capital tax! ...... or not ?**

Log case: Like a NGM

- Simple special case: $\sigma = 1$, $U(C) = \log C$
  $\Rightarrow$ constant savings rate β
- IC

$$C_t = \frac{1-\beta}{\beta} k_{t+1}$$

- IC $\rightsquigarrow$ RC in planning problem

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$c_t + \frac{1}{\beta} k_{t+1} + g \leq f(k_t) + (1-\delta) k_t$$

- **Like a neoclassical growth model, with higher cost of capital!**

Log case: Like a NGM

- Unique interior steady state
- FOC at steady state

$$\underbrace{f'(k^*) + 1 - \delta}_{\text{MRT}} = \underbrace{\frac{U'(C^*)}{\beta U'(C^*)}}_{\text{MRS}} \{1 + \underbrace{\frac{1-\beta}{\beta}}_{\text{capital tax wedge}} \}$$

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- Lansing (1999): Specific to $\sigma = 1$, “knife-edged”
- Reinhorn (2002): Because μ_t does not converge
- This paper: **Not specific to** $\sigma = 1$**!**
  Positive capital taxation for all $\sigma \geq 1$**!**

Contradiction with $\sigma > 1$

- Assume int. steady state for *quantities*

$$\mu_0 = 0$$

$$\mu_{t+1} = \mu_t \underbrace{\left(\frac{\sigma - 1}{\sigma \kappa_{t+1}} + 1\right)}_{\to \text{const} > 1} + \underbrace{\frac{1}{\beta \sigma \kappa_{t+1} \upsilon_t}}_{\to \text{const} > 0}$$

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$$\underbrace{f'(k_{t+1}) + 1 - \delta}_{\text{MRT} \to \text{ const}} = \underbrace{\frac{U'(C_t)}{\beta U'(C_{t+1})}}_{\text{MRS} \to \text{ const}} \{1 + \underbrace{\upsilon_t^{-1}(\upsilon_{t+1} - \upsilon_t) + \beta \upsilon_{t+1}(\mu_{t+1} - \mu_t)}_{\text{wedge} \to \infty}\}$$

- 1st + 2nd FOC $\Rightarrow$ μ_t explodes exponentially
- 3rd FOC $\Rightarrow$ **contradiction**

Positive long run capital taxation

- **Result #1:** If $\sigma > 1$, the optimal allocation **cannot** be converging to the zero capital tax steady state
  - ... or in fact, *any* other interior steady state

Positive long run capital taxation

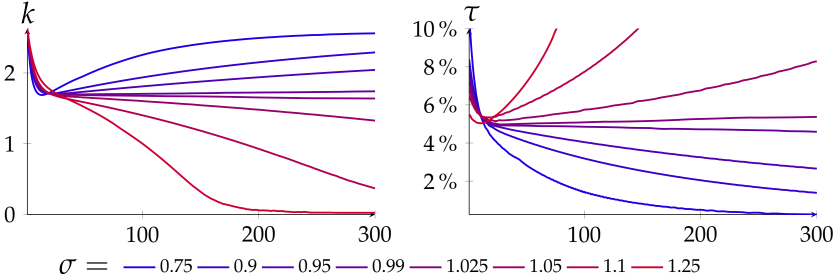
- **Result #1:** If $\sigma > 1$, the optimal allocation **cannot** be converging to the zero capital tax steady state
  - ... or in fact, *any* other interior steady state
- **Result #2:** If $\sigma > 1$, the optimal allocation satisfies

$$c_t \to 0 \qquad k_t \searrow k_g \qquad \text{tax} = 1 - \frac{R_t}{R_t^*} \to \mathcal{T}_g > 0$$

  - k_g = lowest feasible steady state capital stock, $\frac{1}{\beta}k_g + g = f(k_g) + (1-\delta)k_g$

state space

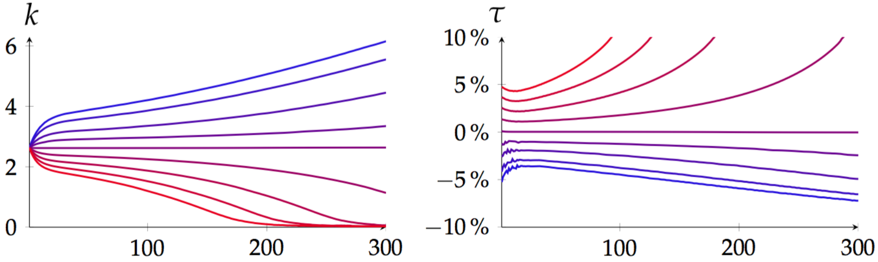
Capital and taxes for various IES's



Intuition

- Intuition: Affect capitalists' savings through **anticipatory effects**
  - start with constant tax
  - **try to raise savings temporarily**
  - $\sigma < 1 \Rightarrow$ promise low future taxes
  - $\sigma > 1 \Rightarrow$ promise **high future taxes**
- Explains the **optimal slopes** for capital taxes

Robustness: $\gamma > 0$



Robustness: General savings functions

- Capitalists save $S(I_t, R_{t+1}, R_{t+2}, \ldots)$
- Assume S weakly decreases in future interest rates (e.g. IES < 1)
- **Result #3:** Optimal tax rates **cannot** converge to zero (or anything negative)

Chamley (1986): Labor and capital taxation

Chamley (1986): Taxing capital to reduce labor taxes

- **Model overview**
  - *representative* agent, with *elastic* labor supply
  - **no lump-sum taxes, no consumption taxes**
  - bounds on capital taxes
  - unrestricted government debt
- **Chamley's (1986) main results**
  1. General recursive Koopmans utility: zero capital tax in steady state
  2. Separable isoelastic utility: same + transitional dynamics

Chamley (1986), Theorem 1: Recursive preferences

$$\max_{\{c,n\}} V(c_0, n_0, c_1, n_1, \ldots)$$

$$c_t + g + k_{t+1} \leq f(k_t, n_t) + (1-\delta)k_t \qquad \text{(RC)}$$

$$\sum_{t=0}^{\infty} \frac{1}{R_1 \cdots R_t}(c_t - w_t n_t) = R_0 k_0 + R_0^b b_0 \qquad \text{(IC)}$$

$$\frac{V_{ct}}{V_{c,t+1}} = R_t \geq 1 \qquad \text{(IC)}$$

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$$V_{ct} \geq V_{c,t+1} \qquad \text{(tax bounds)}$$

Chamley (1986), Theorem 1: Recursive preferences

- Let $V_{ct}\Lambda_t$ be the multiplier on the time t RC
- Then, FOC for capital is

$$\underbrace{f_{k_{t+1}} + (1-\delta)}_{\text{MRT}} = \underbrace{\frac{V_{c,t}}{V_{c,t+1}}}_{\text{MRS}}\{1 + \underbrace{\frac{\Lambda_t - \Lambda_{t+1}}{\Lambda_{t+1}}}_{\text{wedge}}\}$$

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- **Chamley (1986, Theorem 1):** Suppose allocation + multipliers converge to positive steady state
  - in particular: $\Lambda_t \to \Lambda > 0$
  - **Then**, tax is zero in the long run, $\tau_t \to 0$
- Similar result in Judd (1999).

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- Similar result in Judd (1999).
- Λ_t **is endogenous! ... assuming the result?**

Chamley (1986), Theorem 1: Our "Even if ..." result

- Write utility as

$$V_t = W(U_t, V_{t+1})$$

Chamley (1986), Theorem 1: Our "Even if ..." result

- Write utility in *steady state* as

$$V = W(U, V) \ \Rightarrow \ V = \overline{V}(U)$$

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- **Steady state discount factor**

$$\overline{\beta}(U) = W_V(U, \overline{V}(U))$$

Chamley (1986), Theorem 1: Our "Even if ..." result

- Write utility in *steady state* as

$$V = W(U, V) \;\Rightarrow\; V = \overline{V}(U)$$

- **Steady state discount factor**

$$\overline{\beta}(U) = W_V(U, \overline{V}(U))$$

- **Result #5:** $\overline{\beta}'(U) \neq 0$ and "everything converges", then either
    1. private assets = 0, or,
    2. labor taxes = 0
- **Symmetry between labor and capital taxes**

Chamley (1986), Theorem 2: Separable isoelastic utility

$$\max \int_0^\infty e^{-\rho t} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - \phi \frac{n^{1+\zeta}}{1+\zeta} \right\} dt$$

$$c_t + g + \dot{k}_t \leq f(k_t, n_t) - \delta k_t \qquad \text{(RC)}$$

$$\int_0^\infty e^{-\rho t} \left(u_{ct} c_t + u_{nt} n_t \right) = u_{c0} \left(k_0 + b_0 \right) \qquad \text{(IC)}$$

$$\frac{\dot{c}}{c} \geq -\frac{\rho}{\sigma} \qquad \text{(IC)}$$

Chamley (1986), Theorem 2: Separable isoelastic utility

- **Chamley (1986, Theorem 2):** $\exists\ T < \infty$
  - capital tax $\tau_t = 1$ for $t < T$
  - capital tax $\tau_t = 0$ for $t > T$
- **But:** why can T not be infinite?

Chamley (1986), Theorem 2: Separable isoelastic utility

- **Chamley (1986, Theorem 2):** $\exists\ T < \infty$
  - capital tax $\tau_t = 1$ for $t < T$
  - capital tax $\tau_t = 0$ for $t > T$
- **But:** why can T not be infinite?
  - Chamley's (1986) proof: *"the bounds cannot be binding forever or marginal utility would grow to infinity, which is absurd..."*
- **Next:** This might actually happen ...

Positive long run capital taxation for $\sigma > 1$

- **Result #5:** Take $\sigma > 1$. Pick any initial capital k_0. Then **when initial public debt is large enough, capital taxes are 1 forever**, i.e. $T = \infty$.
  - capital taxation to reduce disincentives of labor taxes
- Can construct specific analytically tractable examples (see paper)
- *Proof idea:* Essentially show that $\Lambda_t \to 0$ is possible in

$$\underbrace{f_{k_{t+1}} + (1-\delta)}_{\text{MRT}} = \underbrace{\frac{V_{c,t}}{V_{c,t+1}}}_{\text{MRS}} \{1 + \underbrace{\frac{\Lambda_t - \Lambda_{t+1}}{\Lambda_{t+1}}}_{\text{wedge}}\}$$

Proof idea

$$\max \int_0^\infty e^{-\rho t} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - \phi \frac{n^{1+\zeta}}{1+\zeta} \right\} dt$$

$$c_t + g + \dot{k}_t \leq f(k_t, n_t) - \delta k_t \qquad (\lambda_t)$$

$$\int_0^\infty e^{-\rho t} \left(u_{ct} c_t + u_{nt} n_t \right) \geq u_{c0}(k_0 + b_0) \qquad (\mu)$$

$$\dot{c}_t \geq -\frac{\rho}{\sigma} c_t \qquad (\eta_t)$$

- $b_0 \uparrow \Rightarrow$ need to tax more $\Rightarrow$ IC tighter $\Rightarrow \mu \uparrow$
- In fact: As b_0 approaches highest feasible debt level $\overline{b}$, $\mu \nearrow +\infty$
- Now pick $\sigma > 1$ and suff. high b_0 (hence high μ), and prove $T = \infty$

Proof idea (2)

- Consider FOC for consumption

$$\dot{\eta}_t - \rho\eta_t = \eta_t \frac{\rho}{\sigma} + \lambda_t - (1 - \mu(\sigma - 1))\, u_{ct}$$

where tax bound $\tau_t = \overline{\tau}$ binds if $\eta_t < 0$

- Note that if $T < \infty \Rightarrow \eta_t = \dot{\eta}_t = 0 \; \forall t > T$, implying for such t

$$\underbrace{\lambda_t}_{\geq 0} = \underbrace{(1 - \mu(\sigma - 1))}_{\text{possibly} < 0\,!} \underbrace{u_{ct}}_{>0}$$

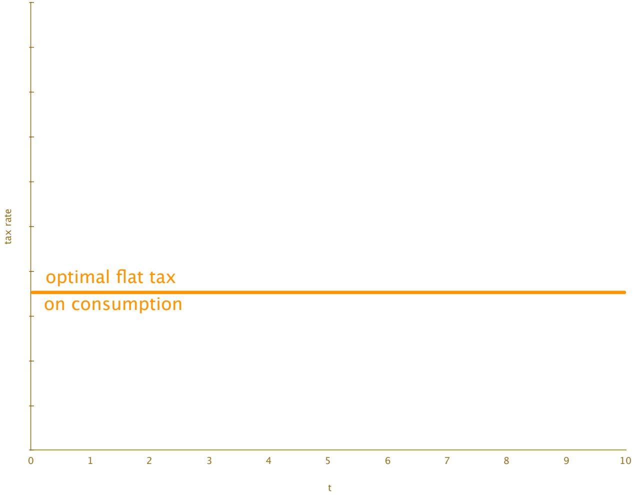
- **This is impossible if $\sigma > 1$ and μ sufficiently large!**
- Hence indefinite capital taxation, $T = \infty$, is optimal in those cases

Side note: Are long-run capital taxes "infinitely distortionary"?

- Diamond-Mirrlees: const. consumption taxes are optimal
- Here: **bounds on capital taxation** in **every period**
  - not a Diamond-Mirrlees economy for any cons. bundle $\{c_s\}_{s \geq t}$
  - ... no reason for const. consumption taxes in the long run

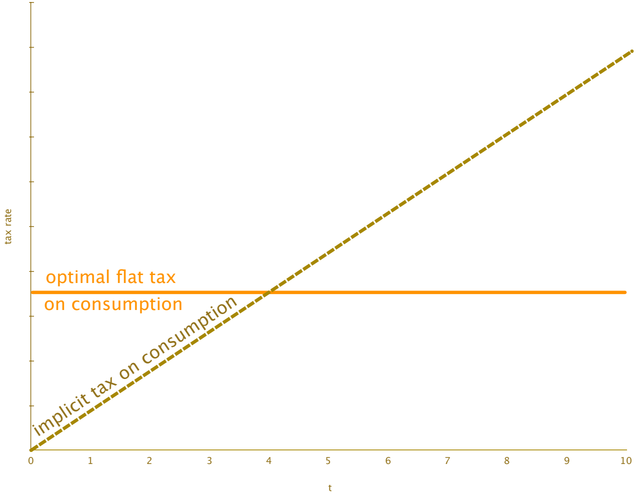
Graphical illustration

- **Flat** optimal consumption tax path without any capital tax bound



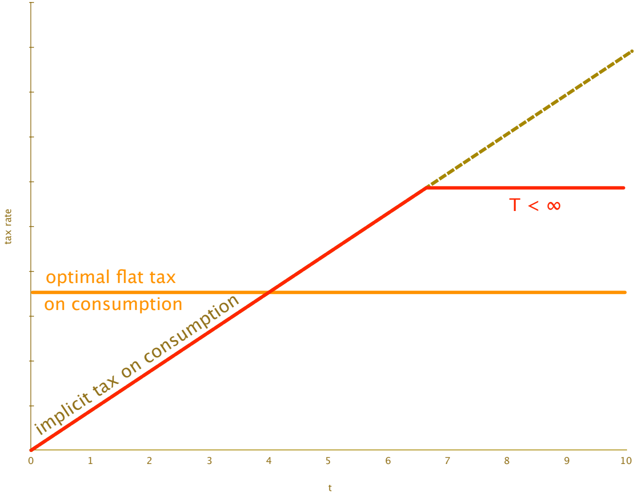
Graphical illustration

- Capital tax bound is equivalent to restriction on consumption taxes



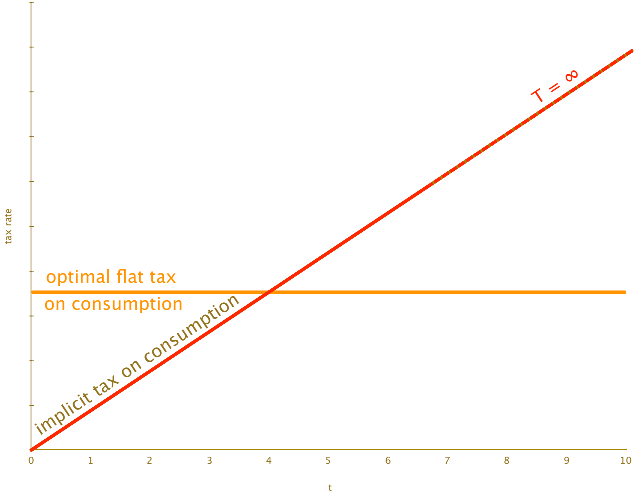
Graphical illustration

- For example, one could pick a consumption tax path like this...



Graphical illustration

- ... but it might well turn out that $T = \infty$ is actually optimal here



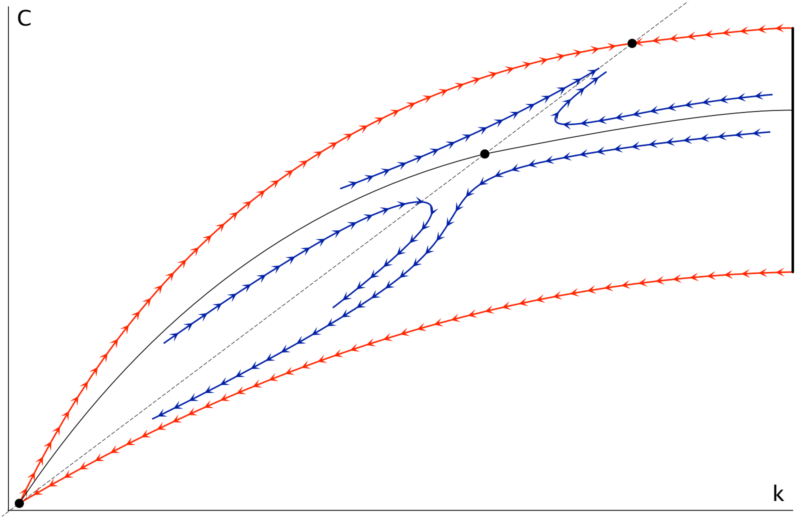
Conclusion

Takeaways

- Revisited Chamley-Judd
- IES > 1: zero long run capital tax
- IES < 1: can have **positive long-run capital taxation**
- Ever-increasing consumption taxes not nec. infinitely distortionary
- Methodological: Assumptions on endogenous multipliers not nec. valid

Backup Slides

Judd (1985) state space



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