Sovereign Default: The Role of Expectations by Ayres, Navarro, Nicolini, and Teles

Fernando Alvarez

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- Multiple equilibria in models of sovereign debt/default.
- Important topic w/ normative & positive implications.
- Paper full of interesting results.
- Clarifies role of order of play borrowers/lenders.

- Time protocol on otherwise “standard" model of sovereign default.
- Order of play and multiplicity.
- If atomistic investors move first, then multiple equilibrium is possible.
- Loan supply can be downward slopping: high rate due to expected defaults, itself rationalized high probability of defaults.
- Analytics: two period version.
- Multiple equilibrium, even using a refinement.
- Choice of current debt vs debt maturity irrelevant.

Supply and Demand: Laffer Curve case multiple equilibrium “refined" away - - -

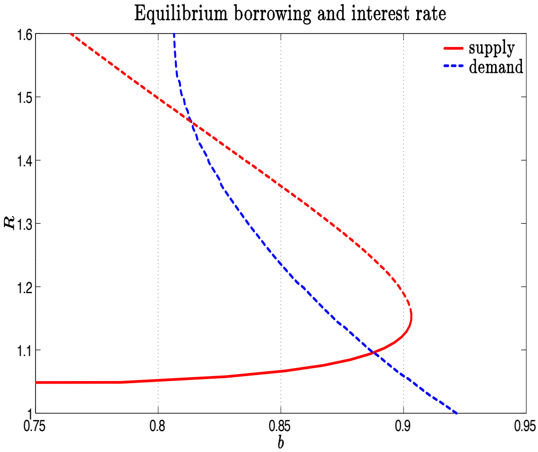

Figure 3: Supply and demand curves

Bimodal case, preferred by authors

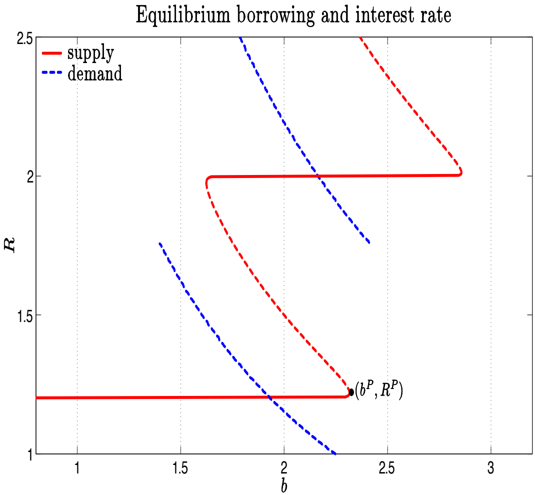

Figure 5: Supply and demand for the bimodal distribution

Benchmark Two-period Model

- Default occurs iff second period consumption below one: $y - bR < 1$

- Borrower problem, given R solves

$$b^d(R) = \arg\max_{b \leq \bar{b}} U(1+b) + \beta \int_1^Y \max\left\{U(1)\,,\; U\left(y - bR\right)\right\} dF(y) \qquad (1)$$

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- Atomistic lender i problem, given R and b solve:

$$b_i(R,b) = \arg\max_{b_i \leq \bar{b}_i} -b_i + b_i\, R\left[1 - F(1+bR)\right]/R^* \quad (2)$$

- Supply of funds:

$$R^* = R\left[1 - F\left(1 + b^s(R)R\right)\right] \quad (3)$$

- Equilibrium (b, R) such that: $b^d(R) = b^s(R)$.

“Local" Refinement (skip)

- Take contract (R, b)
- Borrower make offer to coalition $\alpha > 0$ of lenders.
- Offer occurs with (small) probability π
- Offer has (small) departure of interest rate to $R - \delta$
- Limit as $\delta, \pi \to 0$.
- Thus, argument is “local".

Borrower's problem

- Borrower problem, given R, maximizes

$$J(b) \equiv U(1+b) + \beta \int_1^Y \max\left\{U(1)\,,\ U\left(y - bR\right)\right\} dF(y)$$

- If $U(\cdot)$ linear $\implies$ convex objective function $\implies$ corner solution.
- If $U'' < 0$, objective not concave, envelope of concave functions.
- Optimal $b^d(R)$ piece-wise decreasing, discontinuous w/upward jumps.

Discrete Case

- Assume that $y \in \{y_1, y_2\}$ with $1 < y_1 < y_2$ with $\Pr\{y = y_i\} = p_i$

- Borrower objective function of b for given R

$$J(b) = \begin{cases} U(1+b) + \beta \sum_{i=1,2} U(y_i - bR)\, p_i & \text{if } b \leq \frac{y_1 - 1}{R} \\ U(1+b) + \beta\, U(1)\, p_1 + \beta U(y_2 - bR)\, p_2 & \text{if } \frac{y_1-1}{R} < b \leq \frac{y_2-1}{R} \\ U(1+b) + \beta U(1) & \text{if } b > \frac{y_2-1}{R} \end{cases}$$

- First order condition: $J_b\left(b^d(R)\right) = 0$:

$$J_b(b) = \begin{cases} U'(1+b) - \beta\, R \sum_{i=1,2} U'(y_i - bR)\, p_i & \text{if } b \leq \frac{y_1 - 1}{R} \\ U'(1+b) - \beta\, R\, U'(y_2 - bR)\, p_2 & \text{if } \frac{y_1-1}{R} < b \leq \frac{y_2-1}{R} \\ U'(1+b) & \text{if } b > \frac{y_2-1}{R} \end{cases}$$

- Derivative of objective function $J_b(b)$:
  - Decreasing in each segment
  - Jumps at at $R\, b = y_1 - 1$.

Demand curve: blue discontinuous lime

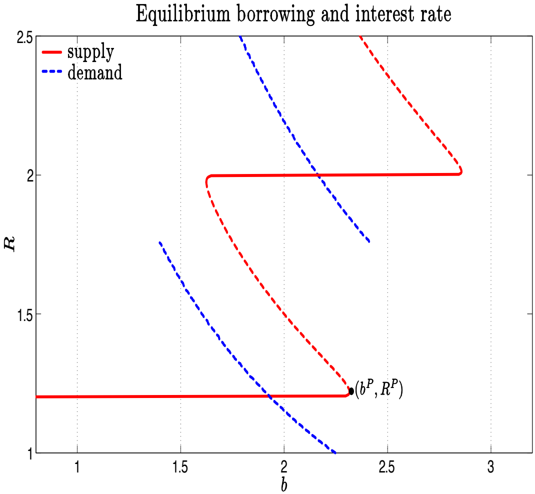


Figure 5: Supply and demand for the bimodal distribution

Continuous Case (skip)

- Assume that $y \in [1, Y]$ with density f and CDF F.
- Borrower objective function of b given R:

$$J(b) = U(1+b) + \beta F(1+bR)\; U(1) + \beta \int_{1+bR}^{Y} U(y-bR)\; f(y)\, dy$$

- First order condition $J_b\left(b^d(R)\right) = 0$:

$$J_b(b) = U'(1+b) - \beta R \int_{1+bR}^{Y} U'(y-bR)\; f(y)\, dy$$

- Second derivative objective function J :

$$J_{bb}(b) = U''(1+b) + \beta R^2 \int_{1+bR}^{Y} U''(y-bR)\; f(y)\, dy + \beta R^2 U'(1)\; f(1+bR)$$

- Derivative optimal decision rule: $b'(R) = -\frac{J_{bR}}{-J_{bb}(b)}$
  - Optimality requires $J_{bb} < 0$ at solution.
  - Income & substitution effect same direction (borrower) so $J_{bR} < 0$.

Supply of Funds: discrete case

- Aggregating indifference of lenders w/borrower borrows b

$$R^* = R\ [1 - F(1 + bR)]$$

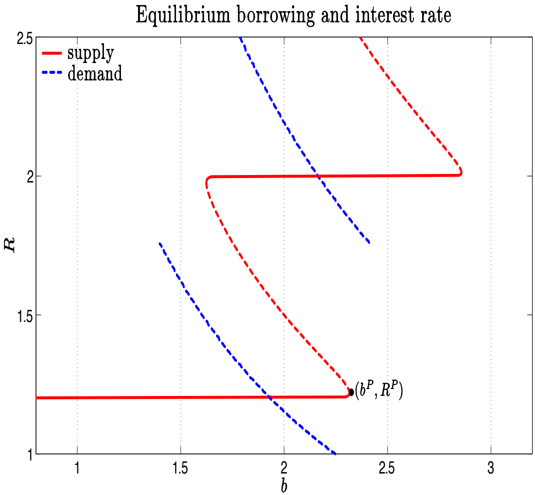
- Discrete case has vertical segments of $b^s(R)$ at discrete values of R.

- Example: $1 < y_1 < y_2$ with $\Pr\{y = y_1\} = p$, inverse supply R^s:

$$R^s(b) = \begin{cases} R^* & \text{if } b \leq \frac{y_1 - 1}{R^*} \\ \frac{R^*}{1-p} & \text{if } (1-p)\ \frac{y_1-1}{R^*} \leq b \leq (1-p)\ \frac{y_2-1}{R^*} \\ \infty & \text{if } b \geq (1-p)\ \frac{y_2-1}{R^*} \end{cases}$$

- Range $b \in \left((1-p)\ \frac{y_1-1}{R^*}, \frac{y_1-1}{R^*}\right)$ supports two interest rates R^* and $\frac{R^*}{1-p}$.

Supply curve: two (solid) flat red segments



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Equilibrum in binomial case

- Two equilibrium w/rates R^* and $R^*/(1-p)$
- Eqbm with risk-less rate & no default:

$$U'(1+b) = \beta R^* \left[U'(y_1 - bR^*)\, p + U'(y_2 - bR^*)\,(1-p)\right]$$

- Eqbm w/ risky rate & default:

$$U'(1+b) = \beta R^* \, U'\left(y_2 - b\frac{R^*}{1-p}\right)$$

- RHS can be higher or lower than risky-case:

$U'\left(y_2 - b\,\frac{R^*}{1-p}\right)$ vs $\mathbb{E}\left[U'(y - b\,R^*)\right]$

Continuous supply case

- Assume $F' > 0$ all y

$$R^* = R \; [1 - F(1 + bR)]$$

  solve for unique $b^s(R)$ for each R

- Depending on shape F, supply $b^s(R)$ can be hump-shaped (similar to Laffer curve).

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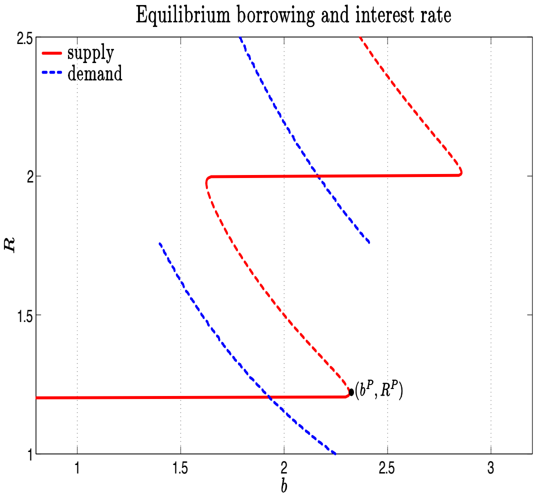
- slope of supply:

$$\frac{\partial b^s(R)}{\partial R} = \frac{1 - \text{hazard}\, bR}{\text{hazard}\, R^2} = \frac{1 - \frac{F'(1+bR)}{1-F(1+bR)}\ bR}{\frac{F'(1+bR)}{1-F(1+bR)}\ R^2}$$

Bimodel approximate binomial

- F mixture of two distributions:
- prob p_i dist. mean y_i and small and variance σ^2, each w/smooth density.
- Smooth version with two peaks.
- Around each peak, decreasing and increasing branch.
- Interestingly: local refinenment "works well":
  - Flat segments has to be join by decreasing segments
  - Refinement t discards decreasing segments!

Supply and Demand: Bimodal case



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Discussion

- Atomistic lenders moving first
  - Multiple equilibrium
  - Local refinement: no equilibrium in downward slopping segment.
  - Still multiple equilibrium in "lumpy (discrete) case".
- Atomistic lenders moving second:
  - Multiple equilibrium depending on current debt vs debt at maturity
  - each equilibrium correspond to a selection of supply correrspondence.
- Important topic: inefficient default and potential policy solution.
- Which case is more reasonable?