# Intermediation and Voluntary Exposure to Counterparty Risk *

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**Abstract**

I develop a model of the financial sector in which endogenous intermediation among debt financed banks generates excessive systemic risk. Financial institutions have incentives to capture intermediation spreads through strategic borrowing and lending decisions. By doing so, they tilt the division of surplus along an intermediation chain in their favor, while at the same time reducing aggregate surplus. I show that a *core-periphery* network – few highly interconnected and many sparsely connected banks – endogenously emerges in my model. The network is inefficient relative to a constrained efficient benchmark since banks who make risky investments "overconnect", exposing themselves to excessive counterparty risk, while banks who mainly provide funding end up with too few connections. The predictions of the model are consistent with empirical evidence in the literature.

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# 1 Introduction

The years following the financial crisis resulted in an intense scrutiny of the architecture of financial markets. Many prominent economists have argued that the existing financial structure was socially suboptimal due to high systemic risk that emerged from excessive interconnectedness between financial intermediaries.[1] A relatively new, but fast growing, body of work tries to understand the optimal regulatory response to such financial structure. This literature mostly takes the financial structure as given, and assesses appropriate policy responses which minimize the systemic risk.[2] However, any policy which is implemented to mitigate the risk in the current financial architecture could feedback into bank decisions and influence the choice of inter-linkages. Alternative policy should account for endogenous changes to the financial structure. In this paper I develop a new model where the bilateral exposures of financial institutions emerge endogenously from their profit maximizing decisions. In doing so, I generate the underpinnings of interconnectedness in the financial sector, which allows me to evaluate formally the efficiency of the current financial architecture.

I develop a model of the financial sector in which endogenous intermediation among debt financed banks generates excessive systemic risk, which is measured as the distribution of total value lost due to bank failures. Financial institutions have incentives to capture intermediation spreads through strategic borrowing and lending decisions. By so doing, they tilt the division of surplus along an intermediation chain in their favor, while at the same time reducing aggregate surplus. I show that a *core-periphery network* – few highly interconnected and many sparsely connected banks – endogenously emerges in my model. In other words, my model predicts that there is a small number of very interconnected banks that trade with many other banks and a large number of banks that trade with a small number of counterparties.

There is overwhelming recent evidence that interbank markets exhibit a core periphery structure.[3] Moreover, banks at the core has high gross exposures and low net exposures

[1]A high degree of interconnectedness among financial institutions has been frequently recognized by policy makers. Federal Reserve chairman Ben Bernanke and undersecretary of finance Robert Steel, in their senate testimony on April 3, 2008, alluded to potential risk of system wide failure due to mutual interconnections of financial institutions in defending Bear Stearns bailout.

[2]Notable examples are stress tests designed by the Fed. See Fed (2012), Fed (2013) for more detail.

[3] See Bech and Atalay (2010), Allen and Saunders (1986),Afonso and Lagos (2014) and Afonso et al. (2011) for evidence on federal funds market, Boss et al. (2004), Chang et al. (2008), Craig and Von Peter (2014) and van Lelyveld and in 't Veld (2014) for interbank market in other countries, Peltonen et al. (2014) and Vuillemey and Breton (2013) for OTC derivatives and Di Maggio et al. (2015) for corporate bond inter-dealer market.

among themselves. My model not only provides a theoretical framework that jointly explains these empirical stylized facts; its main contribution is to do so by explicit modeling of intermediation among banks and its frictions.

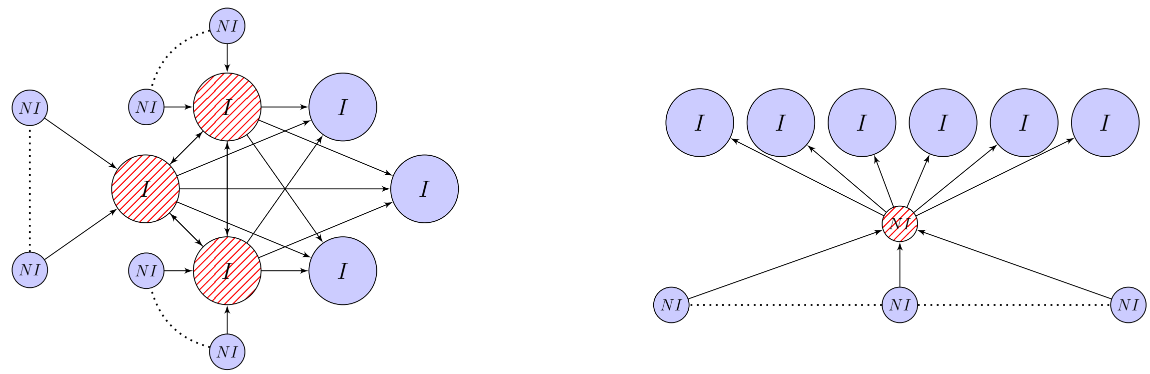
In the model, the financial network consists of banks and their lending decisions. Banks need to raise resources for investment either from households or from other banks. My model endogenously generates indirect lending and borrowing in the interbank market, which is a prominent feature of both the federal funds market and over-the-counter market for derivatives.[4] If the investment fails and the borrowing bank does not have sufficient funds to pay back her lender(s), it fails and potentially triggers a cascade of failures to the lenders, lenders of lenders and so on.

Banks are profit maximizers. There are two groups of banks in the model: those who have access to a risky investment opportunity, and those who do not. Each bank chooses its lending and borrowing relationships to get the highest expected possible rate on the funding it lends out and the investment it undertakes, net of cost of failure. When there are positive intermediation rents in the system, profit maximization creates private incentives to provide intermediation, which in turn leads to a particular structure for the equilibrium network. Since intermediation is profitable per-se, in equilibrium, competition implies that the banks who are able to offer the highest expected returns become intermediaries. These banks are exactly the ones who have access to the risky investment technology. On the other hand, a bank who is not an intermediator still wants to earn the highest possible returns, thus opting for the shortest connecting path to investing banks to avoid paying intermediation spread as often as possible. These two forces give rise to a core-periphery equilibrium network in which (a subset of) banks with risky investment opportunities constitute the core (Theorem 1, section 5).

The interbank network generated by the model is socially inefficient. Banks who make risky investments "overconnect", exposing themselves to excessive counterparty risk, while banks who mainly provide funding end up with too few connections.[5] In other words, when default is costly, efficiency requires reaching the optimal scale of investment while minimizing the loss of failure, which leads to a different structure from the one which arises in equilibrium (Theorem 2, section 5). This is in contrast to Gale and Kariv (2007) and Blume et al. (2009) who suggest that the financial architecture does not matter for efficiency. The main driving force behind this difference is the presence of intermediation rents which prevent social and private incentives from being aligned.

[4]Bech and Atalay (2010), Gofman (2012) and Di Maggio et al. (2015).

[5]The socially optimal structure is the one which maximizes the equally weighted sum of all bank expected profits.




(a) Equilibrium

(b) Efficient

Figure 1: Equilibrium versus efficient structure of the interbank network[6]

## 1.1 Model Implications

The model predicts that multiple banks can be at the core of the financial system, with high gross and low net exposures among core banks. Consistent with this prediction, there is direct evidence from the financial crisis on substantial exposure among large financial institutions, which entailed runs and subsequent failure of one entity following its counterparty's failure.[7]

Equilibrium intermediaries are exposed to excessive risk since they do not contribute to the investment except through intermediation. The social planner prefers leaving such intermediaries out of the chain, replacing them with intermediaries who take minimal extra risk by intermediating. This minimizes the systemic risk without hurting the scale of investment. Thus social planner balances the net gain from investment with the expected loss of default. In contrast, private incentives compare rents, partially in the form of intermediation spreads, with the cost of default. The cost of default is a real cost while intermediation spreads are a mere redistribution of surplus. Consequently, I illustrate that the social and private incentives diverge in several situations. The intuition can be obtained by focusing on Figure 1 that compares the equilibrium interbank network with the efficient one. Banks at the core are hatched in red in each structure.

[6]The labels $I$ and $NI$ refer to banks with and without potential risky investment, the latter solely raising funds from households and intermediating them to investing banks. See the model for the detail. The dots represent more $NI$ banks.

[7]A prominent example, as reported in the FCIC report on the financial crisis, is the immediate run on holders of Lehman unsecured Commercial Paper (CP) and lenders to Lehman in tri-party repo, such as Wachovia's Evergreens Investment and Reserve Management Company's Reserve Primary Fund, after Lehman failed on September 15, 2008. The first wave of runs was followed by a second wave of withdrawal from Lehman OTC counterparties, most notably UBS and Deutche Bank. Fore more details please see FCIC (2011).

One can also interpret the implications of the model in terms of *contagion*. In the model, investment and funding opportunities arise at different banks, which requires funding to be channeled from banks with liquidity surplus to the ones with investment opportunities. This decentralized distribution of resources and investment opportunities gives rise to endogenous interbank intermediation. Moreover, the return to risky investment is not contractible, so all the bank liabilities are in the form of debt, which leads to failure if obligations are not met. As a result, lenders and intermediators are exposed to counterparty risk. Because investment is positive NPV, there is an *optimal level* of contagion, due to counterparty risk exposure, in order to provide funding for the projects. In other words, even the financial structure chosen by a social planner involves a certain level of contagion when risky investment fails. The important prediction of the model is that the equilibrium interbank network involves *excessive* contagion, more than what is necessary to support the optimal level of investment.

The core-periphery structure implies that many banks are connected to each other only indirectly, a similar notion to weak ties as defined in Granovetter (1973). In the context of the model, the weak ties are intermediator's borrowing and lending relationships. As these relationships are associated with rents, every bank prefers to have many weak ties. In equilibrium, banks who are able to pledge the highest return to their investors have many weak ties and are in the core.

The model not only provides predictions on the global structure of the interbank network, but also has implications about the bilateral interbank rates. Consistent with findings of Di Maggio et al. (2015) who empirically study the inter-dealer market for corporate bonds, my model predicts that core dealers charge higher average prices to the peripheral dealers than to other core ones. I also explore diversification incentives of banks in equilibrium, which uncovers a different channel for inefficiency, due to under-provision of insurance in the network (similar to Zawadowski (2013)).

Finally, I use the model to shed light on several policies related to the architecture of the financial networks. The model provides a framework to assess bailouts, as well as policy proposals to impose a cap on the number of counterparties and swaps. Moreover, it provides a new rationale for introduction of a Central Clearing Party (CCP) (section 6).

## 1.2 Literature Review

As a model of interbank networks, my paper is closely related to application of networks in economics (three early seminal papers are Jackson and Wolinsky (1996), Bala and Goyal (2000) and Aumann and Myerson (1988)). Jackson (2005), Jackson (2010) and Allen and

Babus (2009) provide excellent reviews of the existing work.

There is also a fast growing literature on contagion and systemic risk in financial networks, started by the seminal work of Allen and Gale (2000) who studies the propagation of negative shocks in simple financial networks. A large part of this literature either focuses on properties of large networks, or take the structure of the network as given.[8] More recent work in this area focuses on strategic link formation among financial institutions.[9] Acemoglu et al. (2014), by locating banks on a ring, predicts that the equilibrium network can exhibit both under and over connection. Zawadowski (2013) uses the same ring network to provide a rational for under-insurance due to the high market price of insurance. Related to this literature is Kiyotaki and Moore (1997), who is one of the first papers that look at the formation of credit networks. Although the modeling assumptions of this paper are more closely related to supply chain networks, the implications for contagion and under-insurance can be interpreted in the context of financial networks.

Most relevant to my paper are Hojman and Szeidl (2006), Hojman and Szeidl (2008) and Babus (2012), which predict minimally connected star equilibrium structures, based on costly link formation.[10] Moreover, unlike mine, these papers focus on undirected networks which is less suitable to model interbank, often asymmetric, relationships. My model contributes to this literature by providing rich predictions consistent with stylized facts about global structure of interbank networks missing from the previous work, and does that by underpinning a microfoundation for endogenous cost and benefit of interbank relationships.

There is also an emerging literature on bargaining and intermediation in (financial) networks (Gale and Kariv (2007), Manea (2013), Gofman (2011) and Babus (2012)). In all of these models except Babus (2012) intermediaries are determined exogenously. In my model, certain agents endogenously assume the role of intermediaries, which can lead to welfare losses in equilibrium.

Finally, my paper is also related to the literature which studies the role of banks as intermediaries, their balance sheet structure and issues related to insolvency.[11] In this literature, banks are intermediaries between investors and entrepreneurs. I add to this

[8]See Acemoglu et al. (2015), Eisenberg and Noe (2001), Elliott et al. (2012), Gofman (2011), Gai and Kapadia (2010) and Caballero and Simsek (2013).

[9]See Acemoglu et al. (2014), Blume et al. (2011), Babus (2013), Allen et al. (2012), Moore (2011), Rotemberg (2008), Zawadowski (2011), Zawadowski (2013),Bluhm et al. (2013) and Cabrales et al. (2012).

[10]Babus (2012) can have an equilibrium which is an interlinked star network as well.

[11]An incomplete list includes Diamond (1984), Rochet and Tirole (1996), Kiyotaki and Moore (1997), Moore (2011), Lagunoff and Schreft (2001), Leitner (2005), Cifuentes et al. (2005), Dang et al. (2010), Dasgupta (2004), Acharya et al. (2012), Acharya and Yorulmazer (2008) Bhattacharya and Gale (1987), Bolton and Scharfstein (1996), Diamond and Rajan (2005), Farhi and Tirole (2013) and Gorton and Metrick (2012).

literature by specifically modeling the role of banks as intermediaries among each other, and study the corresponding implications for the structure and efficiency of financial sector, as well as systemic risk.

The rest of the paper is organized as follows. Section 2 lays out the basic environment. Section 3 provides a simplified version of the economy with four banks and solves for the equilibrium and constraint efficient structure. Section 4 specifies the detail of the lending contracts for general network structures. Section 5 provides the general results. Section 6 discusses policy implications of the model. Section 7 concludes.

# 2 Model

The model has three periods, $t = 0, 1, 2$, and one good, which I refer to as funding. There are two types of agents: banks and households. There are $K$ banks in the economy: banks that randomly get risky investment opportunities (type $I$) and banks that do not (type $NI$). Let $\mathbb{I}$ and $\mathbb{NI}$ denote the set of $I$ and $NI$ banks, respectively, and let $\mathbb{N} = \mathbb{I} \cup \mathbb{NI}$. There are $k_I$ banks of type $I$ and $k_{NI}$ banks of type $NI$. Throughout the paper assume $k_{NI} \geq k_I$.

The financial system consists of banks and their bilateral exposures. The bilateral exposures represent lending and borrowing relationships among banks. Note that bilateral exposures among banks are quite complex in reality. Banks can be exposed to each other through multiple channels: secured and unsecured lending, derivative contracts, and similar asset holding. For the purpose of this paper, I will restrict interbank relationships to exposures through debt contracts (lending). Bank $i$ who lends to bank $j$ through a debt contract is exposed to bank $j$ since if bank $j$ fails, it will not be able to pay bank $i$ back, which affects the balance sheet of bank $i$ and might cause $i$ to fail.

The investment opportunity is a risky asset. Each bank $I$ receives the opportunity to invest in the risky asset with probability $q$, which is iid across all $I$ banks. Let $\tilde{\mathbb{I}}_R$ denote the random variable corresponding to the subset of $\mathbb{I}$ that receive the opportunity, and let $\mathbb{I}_R$ be the realization of such subset. [12]

Let $\tilde{R}_i \in [0, \bar{R}]$ denote the (per-unit) random return of bank $i$'s investment in the risky asset, which is iid across banks.[13] The investment is linearly scalable. I assume the support of the asset return distribution has two mass points: the project succeeds with probability

[12]Throughout the paper, I will use the following convention: $\tilde{x}$ denotes a random variable, and $x$ denotes the realization of that random variable.

[13]iid assumption leads to maximum room for diversification. I will later discuss how changing this assumption to having correlated project returns strengthens my results.

$p$ and returns $R$, and fails with probability $1-p$ and returns 0:[14]

$$\tilde{R}_i = \begin{cases} R & \text{with probability } p \\ 0 & \text{otherwise.} \end{cases}$$

Besides the risky investment opportunity, each bank $i$ (of type $I$ or $NI$) has a value $V_i$, which is the value of the other businesses, assets, and services the bank provides. If the bank fails for any reason, this value is lost.[15][16] For simplicity, I will assume $V_i = V_I$ for every $i \in \mathbb{I}$ and $V_j = V_{NI}$ for every $j \in \mathbb{NI}$.

Bankers do not have any wealth. They can raise funding from two sources in the form of debt. At $t = 0$, each bank $NI_j$ raises resources from a continuum of households $hh_j$, of measure one. Each household is endowed with one unit of funding. Because each set of households is a continuum, they are competitive and they lend their endowment to their corresponding bank if they break even. [17][18] Second, a bank can borrow from other banks at $t = 1$. To do so, at $t = 0$, it must have established a *potential borrowing relationship* with them.

I model the financial system as a network. The financial network is a directed graph $G = (\mathbb{N}, E)$, where $\mathbb{N} = \{1, 2, \cdots, K\}$ is the set of nodes and $E = \{e_{ij}\}_{i,j \in V}$ is the set of edges. Each node is a bank, and edge $e_{ij} \in E$ is a potential lending relationship from bank $i$ to bank $j$. $e_{ij} \in E$ only if at $t = 1$ funding is lent along this potential lending relationship with non zero probability. Otherwise $e_{ij}$ is removed from $E$.

Each bank chooses its potential borrowing and lending relationships, that is, links over which he can borrow or lend, to maximize its expected profit net of failure cost.

The timing of the model is as follows: At $t = 0$, the banks saise funding from households and the potential lending and borrowing relationships are formed. A link $e_{ij}$ means bank $j$ *can* borrow from $i$ in the period that follows. At $t = 1$, investment opportunities are realized and actual lending happens only along (some of) the links formed at $t = 0$. At $t = 2$ random returns are realized, and banks that are not able to pay back their creditors

[14] The binomial nature of project return is purely for simplicity and is inconsequential for the results.

[15] This value accrues to the banker himself. This model is isomorphic to one with bankruptcy costs that are borne by the bankers in the event of failure.

[16] James (1991) finds that losses due to bank failure are substantial, losses on assets and direct expenses averaging 30% and 10% of the failed bank's assets, respectively.

[17] I assume only $NI$ banks raise funding from households to get stark normative predictions. The positive predictions are invariant to whether $I$ banks also raise funding from households. The normative results remain the same under appropriate parameter restrictions. I will elaborate further on this in section 5.1.

[18] Households breaking even (i.e., zero rate of return) is a normalization. Any constant positive rate of return would work as well.

fail. Holding precautionary liquidity is ruled out, so banks lend or invest as much resources as they are able to raise.

# 3 Economy with Four Banks

Starting with a simplified version of the model before formal description of the contracts is useful. Here I characterize all the equilibria in an economy with four banks, which illustrates the main forces of the model. In section 4 I will provide a description of the contracts under which the same intuition carries over to the general case.

Assume there are two $I$ and two $NI$ banks, $\mathbb{I} = \{I_1, I_2\}$ and $\mathbb{NI} = \{NI_1, NI_2\}$. Recall that each $NI$ bank raises one unit from a continuum of households, while $I$ banks do not raise any outside funding. As a result, each bank $I$ needs to secure funding on the interbank market at $t = 0$ to be able to invest in its project later, at $t = 1$, if it gets an investment opportunity.

As explained in the previous section, to borrow on the interbank market at date $t = 1$, banks need to enter potential agreements at $t = 0$. Potential agreements are similar to credit lines, except that they do not have a limit. I will use the two terms interchangeably. Each agreement (established at $t = 0$) is a promise by the lender to deliver at least one unit (at $t = 1$) if the borrower receives an investment opportunity, or if the borrower has a credit line to another bank that has received an investment opportunity.

**Definition 1.** **[Eligibility]** *Any potential borrower who has a direct or indirect access to a realized investment opportunity is eligible to draw on his credit line.*

With some abuse of language, I will use *eligible* for potential relationships as well. I assume a bank cannot default on its eligible promises, i.e., for any realization of investment opportunities, a potential lender must have sufficient funds to lend each eligible potential borrower one unit.

For a concrete example, consider Figure 2: in 2a, $NI_1$ has the unit it has raised from households, but no other source of funding. In particular, credit line $NI_2 \to NI_1$ does not exist. Moreover, $NI_1$ has a credit line to each $I$ bank, so both $NI_1 \to I_1$ and $NI_1 \to I_2$ exist. In 2b, $NI_1$ has two units pledged to it, one from households and one through credit line $NI_2 \to NI_1$. Now consider the $t = 1$ state where both $I$ banks receive investment opportunities. In both structures, $NI_1$ has promised one unit to each $I$ bank. However, in 2a, it will not be able to keep its promise. 2a is not a feasible structure, and is ruled out. This restriction is formalized in the following assumption.


(a) Infeasible set of relationships (b) Feasible set of relationships

Figure 2: Two possible sets of potential relationships between two $NI$ banks and two $I$ banks.

**Assumption 1. [Feasibility]** *Each realized lending has a minimum size, normalized to one unit. Each potential lender must satisfy his eligible potential lending promises.*

This assumption implies an opportunity cost for forming potential lending relationships, and puts an endogenous limit on the number of potential relationships a bank can establish. At $t = 0$ a bank can enter into as many potential lending relationships as he chooses, as long as, for each realization of uncertainty at $t = 1$, he is able to raise sufficient funding either from households ($t = 0$) or on the interbank market ($t = 1$), through his potential borrowing contracts, to service them.[19]

There is an exogenous division of expected net surplus that allocates a strictly positive share to every bank involved in an intermediation chain, for each realization of investment opportunities. When bank $i$ raises funding from households and lends directly to bank $j$ who makes the investment ($i \to j$), $j$ and $i$ receive in expectation a share $1 - \alpha$ and $\alpha$ of expected net surplus of the project, respectively. Alternatively, if $i$ raises the funding, lends to $k$ who in turn lends to $j$ who invests ($i \to k \to j$), then $j$, $k$, and $i$ receive $1 - \alpha$, $\alpha(1 - \alpha)$, and $\alpha^2$ shares, respectively.[20]

All the contracts are bilateral. The final return of the project at $t = 2$ is not contractible, so all the contracts are in the form of debt. However, the contract can be written contingent on all date $t = 0$ and $t = 1$ outcomes, specifically on the network structure, as well as the realization of investment opportunities. So bilateral contracts are *contingent debt* in which the face value of debt is set such that given the network and the realization of investment

[19]This assumption can be justified by an appropriately chosen upfront fixed or declining cost of link formation. The cost should be such that with $j$ units of available funds, the expected marginal gains from $j + 1^{\text{th}}$ potential lending relationship is below the cost, while it covers the cost with $j + 1$ units. However, the motivation for this assumption is not to capture a fixed cost.

[20]Multiple papers provide evidence for existance of intermediation rents. Examples includes Di Maggio et al. (2015) and Li and Schürhoff (2014) who document intermediation spreads charged by dealers in the dealer network for corporate bonds and municipal bonds, respectively.

opportunities, each bank along the intermediation chain receives its appropriate share, as described above.

A direct lending relationship between an $I$ and $NI$ is socially desirable if

$$pR - 1 > (1-p)(V_I + V_{NI}). \tag{1}$$

That is, the net expected return from the project should cover the expected cost, which is the expected loss of outside value of the two banks, because both the lender and borrower fail if the project fails (borrower cannot pay the lender and lender cannot pay the households from which he borrowed). The participation constraints for the lender and borrower are the following, respectively:

$$(1-\alpha)(pR-1) > (1-p)V_I \tag{2}$$

$$\alpha(pR-1) > (1-p)V_{NI}. \tag{3}$$

Note that 2 and 3 imply 1. I also assume that lending via one intermediator is viable.

$$\alpha^2(pR-1) > (1-p)V_{NI} \tag{4}$$

Throughout the paper, for more general rules for division of surplus, I assume conditions analogous to 2, 3 and 4 are satisfied, so that direct single lending-borrowing as well one-hop intermediated lending-borrowing is individually rational for all parties.[21]

As noted above, with only two banks individual rationality is sufficient for efficiency, but not necessary. As a result, with only one lender and one borrower, equilibrium can only exhibit under-investment, in the form of under-lending.[22] Remarkably, I will show that with more banks and the possibility of multiple investment opportunities, the equilibrium involves over-lending among a certain group of banks.

I analyze the model under the following assumption, and later relax it in section 5.2.

**Assumption 2.** *If a bank $i$ has potential lending relationships to multiple $I$ banks with realized investment opportunities, all of its funding is allocated randomly with equal probability to exactly one of them. An $I$ bank that receives an investment opportunity invests all of its funds in its own project.*

I make the above assumption for two reasons: first, it simplifies the exposition. More

[21]The exact parametric restriction would depend on the rule for division of surplus.

[22]Or equivalently, under-exposure to banks that have investment opportunities. This is similar to trade break down due to high outside options, as in Gofman (2011).

importantly, it allows me to analyze pure intermediation and diversification separately, as explained next.

Consider a bank with sufficient funds. He has an incentive to establish potential relationships to multiple banks with potential investment opportunities, in order to channel funds to different points of the financial system where investment opportunities arise, increase the scale of investment, and capture profits. In other words, banks have an incentive to *intermediate* as often as possible through having many potential relationship, and this incentive is not affected by the degree of correlation across success of different investment opportunities.

Alternatively, a bank can *diversify* his portfolio by investing in multiple projects, through lending to multiple banks with realized investment opportunities, to whom he had already established potential lending relationships. As such, depending on the parameters, even if only some of the projects are successful, the lender bank would be able to service all of his liabilities and avoid default. As a result, a bank with sufficient funds has incentives to establish potential lending relationships to multiple banks in order to benefit from diversification when many of them have (direct or indirect) access to different realized investment opportunities simultaneously. The more correlated the projects' success across different banks are, diversification incentives are weaker as the gains to diversification deteriorates.

Assumption 2 disables diversification, as defined above, and allows me to focus on intermediation. Since diversification is relatively well-studied in a number of different contexts, I choose to abstract away from it in order to focus on the novel point of the model. I will re-introduce diversification in section 5.2 and show how it interacts with the main intermediation mechanism introduced in this paper.[23]

## 3.1 Equilibrium

I borrow the equilibrium concept, group stability, from the matching literature, as defined in Roth and Sotomayor (1990). It is a generalization of pairwise stability defined in Jackson and Wolinsky (1996), generalized to allow for any number of banks to participate in the deviation.[24]

[23]Random allocation of all funds is not sufficient to kill diversification at an $I$ bank because $I$ banks can have cross-lendings. I need to make an assumption that implies each $I$ bank is involved in at most one project, which is achieved through the second clause of the above assumption.

[24]Two points are worth mentioning here: first, a subtlety exists in adopting a concept from matching to my framework. In most matching models, the utility of each agent is only own-match dependent and does not depend on the rest of the matching. However, in my model, utility of the blocking coalition can depend on the rest of the network, so what banks outside the coalition do matters. I assume they don't change their actions. This equilibrium concept is different from the one in which the blocking coalition

**Definition 2.** *A network structure $G$ is blocked by a coalition $B$ of banks if there exists another (feasible, individually rational) network structure $G'$ and a coalition $B$ such that*

*(a) $G'$ can be reached from $G$ by a set of bilateral deviations by $b, b' \in B$ and unilateral deviations by $b \in B$.*

*(b) Every bank $b \in B$ is strictly better off in $G'$ than in $G$.*[25]

**Definition 3.** *A* **group stable** *network is one that is not blocked by any coalition of banks.*

Note that I have adapted the possible group deviations to the structure of my model. The only viable group deviations are those in which the resulting network $G'$ is feasible, and every $e_{ij} \in G'$ is traversed with positive probability at $t = 1$. If $\exists e_{ij} \in G'$ such that lending over $e_{ij}$ always violates either the lender or borrower individual rationality, and never happens in equilibrium, then $e_{ij}$ is removed from $G'$ and the corresponding $B$ is not a blocking deviation.

Before moving to equilibrium characterization, consider the following two lending arrangements: in the first arrangement bank $i$ lends one unit, at face value $D$, directly to $j$ who invests the unit. In the second arrangement $i$ lends the one unit to $k$ at face value $D_1$, who in turns lends the unit, at face value $D_2$, to $j$ who invests the unit.

$$D_1 = \frac{\alpha^2(pR - 1) + 1}{p} < D_2 = D = \frac{\alpha(pR - 1) + 1}{p}$$

$D_2 - D_1$ represents the *intermediation spread*. Simply put, the face value of debt is set to ensure that in expectation, each party (including the intermediator) receives its share of expected net surplus. However, that inability to repay debt obligation entails costly default, i.e. rents come at a cost.

Now consider the network structures in Figure 3, which are the possible equilibria of the economy with four banks. To study the individual incentives in equilibrium, let us focus on two specific equilibria in Figure 3, namely, 3a and 3d. Intuitively, the main difference among the two structures 3a and 3d is the following: in 3d, regardless of which bank(s) have the investment opportunity, all the banks are involved as either investor, intermediator, or final lender in every investment, whereas in 3a, if only one $I$ invests, the other $I$ bank is

goes to autarky, which is referred to as the $\beta$-core in the network literature.

Second, the reason I use this more complex equilibrium concept is that it is better suited to think about the core mechanism of the model. Interbank intermediation generically involves more than two banks, so pairwise stability is not the appropriate notion for addressing the relevant deviations.

[25] The strict notion is equivalent to weak notion with an $\epsilon \to 0$ cost of deviation.

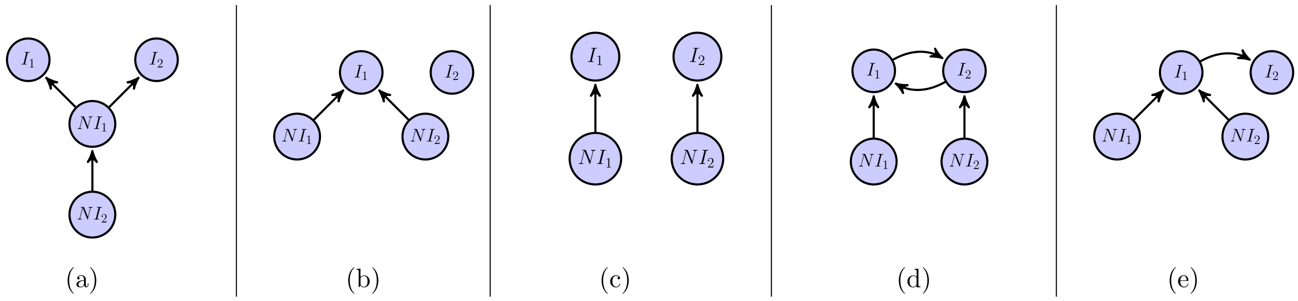



Figure 3: Possible equilibria for an economy with two $I$ and two $NI$ banks

not exposed to the risk of investment failure. With constant return to scale, the expected return in a financial structure only depends on the scale of investment and not on how it is distributed among investors. Because in both 3a and 3d, both units are always invested, the net expected return of the risky investment is the same and the two structures only differ in cost of default, which I argued is lower in 3a.

Assume we are in 3a and consider the joint deviation by $\{I_1, I_2, NI_2\}$ leading to 3d, as depicted in 4. Observe that when only $I_2$ receives the investment opportunity, $I_1$ serves as the intermediator for $NI_1$ and captures the intermediation rents. However, it fails if $I_2$ fails. So the incremental cost of default in 3d (compared to 3a) is born by $I$ banks, that is, precisely the banks that can choose to be out of the chain of intermediation (as in 3a). However, if the intermediation spread $(D_2 - D_1)$ is high enough, $I$ banks would want to deviate from 3a to 3d in order to earn the spread. In other words, $I$ banks intentionally choose to expose themselves to this incremental cost, which is counterintuitive. The insight is that the financial system contains rents that can be captured only through voluntary exposure to counterpart risk, and if these rents are high enough, banks would choose to incur the additional risk in order to capture them.

Finally, $NI_2$ must benefit from joining the coalition. Each bank chooses to lend to counterparties that offer the highest rate of return. Given that intermediation spreads exists, being "close" to the banks who invest translates into higher returns, and in 3a, $NI_2$ is always far from the banks who invest. As a result, $NI_2$it also has an incentive to join a deviation which leads to a structure in which it sometimes avoid paying intermediation spread.

In sum, if the ratio of intermediation rent associated with one unit of funding, relative to expected cost of default, is higher than a certain threshold, 3a ceases to be an equilibrium and 3d becomes an equilibrium instead.

Alternatively, assume we are in 3e. Now whenever $I_1$ does not get an investment opportunity but $I_2$ does, $I_1$ receives an intermediation spread for two units, whereas his cost


(a) Unstable

(b) Stable

Figure 4: Joint deviation by $\{I_1, I_2, NI_2\}$ leading from structure 3a to 3d.

of default stays the same as in 3d. So the per-unit intermediation rent he would require to maintain the link $e_{I_1 I_2}$ is lower compared to 3d. Nevertheless, if the intermediation rents are too low, $I_1$ would unilaterally deviate and stop lending to $I_2$, and 3e would not be an equilibrium anymore.

Formally, let $X = pR - 1$ be the net expected return of one unit investment in the project. Also, let $\kappa = \frac{\alpha(1-\alpha)X}{(1-p)V_I}$, which is the ratio of the intermediation spread per unit intermediated over the expected cost of default due to intermediation for an $I$ bank. Note that the participation constraints imply $\kappa > \tilde{\kappa} = \max\{\alpha, (1-\alpha)\frac{V_{NI}}{V_I}\}$, which is the range that I focus on in this paper. Finally, let $\bar{\kappa} = 1 + \frac{q}{2(1-q)} > 1$, and $\underline{\kappa} = \frac{(1-q)(1-\alpha)}{\alpha(\alpha(2-q)-1)}\frac{V_{NI}}{V_I}$. The next proposition characterizes the range of parameters for which each financial structure is an equilibrium.

**Proposition 1.** *For every set of parameter values $(q, p, R, \alpha, V_I, V_{NI})$, the following conditions characterize all the equilibria:*

*(a) 3a is an equilibrium when $\kappa \leq \bar{\kappa}$,*

*(b) 3b and 3c are an equilibrium when $\kappa \leq \frac{1}{2}$,*[26]

*(c) 3d is an equilibrium when $\kappa \geq 1$,*

*(d) 3e is an equilibrium when $\kappa \geq \frac{1}{2}$.*

Figure 5 depicts the range of equilibria as a function of $\kappa$.[27]

Which network is constraint efficient, i.e. maximizes the total surplus subject to feasibility and individual rationality? Under (1), maximizing scale of investment is efficient

[26] The range for 3c depends on $q$. There is a $\bar{q} > \frac{1}{2}$, where for $q < \bar{q}$ it is $\kappa \leq \frac{1}{2}$ and for $q > \bar{q}$ it is $\kappa \leq 1$. I focus on the former, because it corresponds to the case where banks do not get a project too often.

[27] Depending on choice of parameters the region of 3b and 3c might not exist.

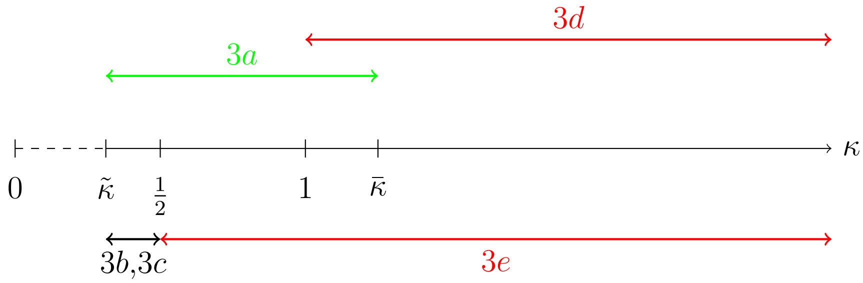



Figure 5: Equilibria of the economy with four banks as a function of $\kappa$, the ratio of per-unit intermediation spread over the expected cost of default for $I$. The green equilibrium is efficient. The two red ones are inefficient due to overconnection, and the black one is inefficient due to underconnection. The range for $4c$ assumes $q < \bar{q}$, where $\bar{q} > \frac{1}{2}$ as defined in the text.

because the return on the asset exhibits constant return to scale. So the social planner's problem reduces to minimizing expected loss of default due to failure of project(s). Note that when a project fails, *contagion* also occurs: $I$ will not be able to pay its lenders back, and based on the lenders' portfolios, they sometimes fail as well. Consequently, the solution to the social planner's problem is to have one $NI$ bank be the intermediary, borrow from the other $NI$, and lend to both $I$s (3a).

This result is quite intuitive: the social planner's objective is to maximize total net return from the projects minus the expected loss, and he does not care about the division of surplus. Assume $NI_1$ is the $NI$ bank chosen as the "intermediator." Given that maximizing the scale of invest requires that $NI_1$ lends to $I_1$ and $I_2$ when either of them has an investment opportunity, it can as well intermediate the funding raised by $NI_2$, and this intermediation does *not* expose $NI_1$ to any extra risk. As such, the scale of investment is maximized and the cost in the event of failure is minimized. A similar intuition goes through in the general case.

Knowing the efficient network provides more insight on the structure of the equilibria depicted in 3. Note that there are multiple equilibria. If $\kappa < \bar{\kappa}$ efficient and inefficient equilibria coexist. Otherwise, both equilibria are inefficient.

Region $1 < \kappa < \bar{\kappa}$ is interesting: in this region, there is an inefficient equilibrium in which the banks with risky investment opportunities are in fact willing to decrease their (inefficient) exposure to counterparty risk and deviate to a more efficient equilibrium. However, they are not able to convince their lender banks to agree to a lower rate and keep funding them, so they are *stuck* in the bad, high-risk equilibrium.

Now consider the financial networks 3b and 3c. They have the same feature of under-investment due to under-lending. In 3b if intermediation spreads are high enough to cover the expected cost of exposure to counterpart risk borne by the *well-funded* $I$ bank (here, bank $I_1$), $I_1$ and $I_2$ will deviate by adding $e_{I_1 I_2}$, and the economy switches from one inefficient equilibrium to the other. This scenario happens if $\kappa \geq \frac{1}{2}$. Let $\bar{q} = \left(2 - \frac{(1-p)V_I}{(1-\alpha)(pR-1)}\right)^{-1} > \frac{1}{2}$. When $q < \bar{q}$ same happens in 3c, only $NI_2$ is also part of the deviating coalition.[28] Another perceivable joint deviation would to 3a, if being intermediated does not dramatically decrease the rate the lender receives (weighted by how often it gets it). Such deviation would be possible for $\kappa > \underline{\kappa}$ if $\alpha > \frac{1}{2-q}$. I show that this never happens. In other words, to get a range where the efficient structure 3a is unique, there must be a non empty range $(\underline{\kappa}, \frac{1}{2})$, where $\tilde{\kappa} \leq \underline{\kappa}$ and $\frac{1}{2-q} < \alpha$. This never happens since $\frac{1}{2-q} > \frac{1}{2}$, so whenever $\tilde{\kappa} \leq \frac{1}{2}$, $\underline{\kappa} < 0$. There are always multiple equilibira: for low intermediation rents there are both efficient and inefficient equilibria, and when the intermediation rents become sufficiently large, $\kappa > \bar{\kappa}$, all the equilibria become inefficient.

Finally, note the role of the particular division of surplus used here: given that a borrower (final and intermediate) does not care about the source of the funding. For him, keeping constant the expected net surplus in other states where he does not intermediate, the only gain to a change in the intermediation chain is if he becomes an intermediary and is able to absorb intermediation rent.

# 4 General Specification

Because few constraints are imposed on the structure of the interbank network, complex structures can form. In particular, multiple intermediation chains might exist between two banks. As a result, I need a rich set of contracts to specify how the funds flow in the network given a network structure and a realization of investment opportunities.

## 4.1 Lending Contracts

Lending contracts are formed before banks receive their investment opportunities. $e_{ij}$ represents bank $i$'s potential lending relationship to bank $j$, subject to a generalized notion of feasibility explained shortly. In this sense, lending contracts are *conditional credit lines.*

There is perfect information: every bank knows the set $\mathbb{I}$ and $\mathbb{NI}$, the structure of the formed lending contracts, the realization of the investment opportunities and the realization of final returns. However, as mentioned earlier, markets are incomplete: first, the realization

[28] Otherwise 3c is an equilibrium for $\kappa < 1$.

of returns are not contractible, so all the contracts are of the form of debt. Second, the potential lending contracts are formed before investment opportunities are realized.

Given that the only restriction on lending relationships is 1, financial network $G$ can be quite complex. The following definitions are useful to explain the contracts.

**Definition 4.** *Given financial network $G$, a "path" from bank $i$ to bank $j$ is a sequence of banks $\{i_1, \cdots, i_m\}$ such that $e_{i_d i_{d+1}} \in E$ for $\forall d = 1, \cdots, m-1$.*

*A "cycle" is a closed path; that is, $i_m = i_1$.*

*A "leaf" bank is a bank that only lends to other banks and does not borrow.*

*Bank $i$ is "connected" to bank $j$ if a path exists from bank $i$ to bank $j$.*

*For every unit of money raised from households at bank $i$, invested by bank $j$, and intermediated through a number of intermediaries $\{i_1, \cdots, i_m\}$, the sequence of banks involved $\{i, i_1, \cdots, i_m, j\}$ (or any subsequence of it) is called an "intermediation chain" (or simply a "chain"). Banks $\{i_1, \cdots, i_m\}$ are "intermediators" along the chain.*

*The "shortest path" from bank $i$ to $j$, $SP(i,j)$, is the (set of) path(s) that involves the minimum number of intermediaries. With some abuse of notation, I use $SP(i, \mathbb{J})$ to denote the collection of shortest paths of $i$ to every bank $j \in \mathbb{J}$, $SP(i, \mathbb{J}) = \{SP(i,j)\}_{j \in \mathbb{J}}$.*

*The "distance" from bank $i$ to $j$ is the number of edges along the shortest path between $i$ and $j$, denoted by $dist(i,j)$.*

Banks are not competitive. For each set of investment opportunities, $\mathbb{I}_R$, and set of lending contracts, $E$, a subset of potential lendings will be realized. There is a fixed distribution of expected total surplus over all the banks involved in raising, intermediating, and investing the funds, denoted by $\mathcal{L}(G, \mathbb{I}_R)$, which is a primitive of the model. With a slight abuse of notation, let $\mathcal{L}(i; G, \mathbb{I}_R)$ denote the share of bank $i$.

$\mathcal{L}(.)$ satisfies the following (sufficient but not necessary) properties. First, the rule is anonymous, and the net expected surplus from each unit of investment is divided only among the banks in the corresponding intermediation chain, as a function of length of the chain and bank position. Second, for every unit of funds, every member of the corresponding *intermediation chain* receives strictly positive shares of net surplus generated by that unit. Third, eliminating an intermediator from an intermediation chain weakly increase the share of every other bank along the chain, and strictly increases the share of the initial lender. Moreover, renegotiation and side payments are ruled out.[29]

[29]The model rules out the possibility that after investment opportunities arrive, banks change their lending decisions. The current static model is a reduced form model for a dynamic game, to be explained shortly. When funding and investment opportunities arrive at different times, and the cost of finding, verifying, and matching with borrowers is sufficiently high, a lender prefers to be intermediated through its

Let $B(i;G) = \{j|e_{ij} \text{ exists}\}$ and $C(i;G) = \{j|e_{ji} \text{ exists}\}$ denote the set of *relationship borrowers* and *relationship creditors (lenders)* of bank $i$ in interbank network $G$, respectively. Note that for every realization of $\mathbb{I}_R$, $i$ can be connected to each $I \in \mathbb{I}_R$ through multiple intermediation chains of different lengths. So we need to generalize the concept of *eligible* potential borrower from section 3.

**Definition 5. [Eligibility Revisited]** *Given the interbank network and each realization of investment opportunities, each borrower of $i$ that is on at least one of $i$'s shortest paths to the set of banks with realized investment opportunities receives at least one unit from $i$.*

(5) $\forall\, \mathbb{I}_R,\ \forall\, j \in B(i;G)$ *if* $\exists I \in \mathbb{I}_R$ *s.t.* $j \in SP(i,I) \Rightarrow i$ *lends* $j$ *at least one unit.*

The financial network has to satisfy feasibility defined in assumption 1, incorporating the above more general notion of eligibility.[30]

This notion of eligibility ensures that in the interim period, if $i$'s fund is (directly or indirectly) lent to $I \in \mathbb{I}_R$, it is intermediated through $i$'s shortest path to $I$; that is, minimum intermediation rents are paid. The intuition is that when bank $i$ can lend to a bank with an investment opportunity through multiple intermediaton paths, at $t = 1$, it chooses the option that provides it with the highest possible rate. What the lender is not able to do in the interim period is to add a new lending. After the investment opportunities are realized, if $i$ wants to be able to borrow from $j$, link $e_{ji}$ needs to exist in $G$. Moreover, only lending contracts along the shortest paths are realized at $t = 1$.

Intermediator $j \in SP(i, \mathbb{I}_R)$, who receives the unit raised at $i$, must lend the unit along (one of the) $SP(i, \mathbb{I}_R)$ paths on which it lies. Within $SP(i, \mathbb{I}_R)$ , $j$ has discretion to allocate $i$'s unit so that $j$ satisfies the minimum size constraint over all its realized lendings. The unit $j$ has raised from outsiders receives equal treatment. Starting from leaf banks, at every bank, units are lent accordingly to satisfy the minimum size constraint. Any excess unit is divided equally among all the corresponding shortest paths. The process is done recursively starting from the leaf nodes until either all the units are allocated to investment

current connections to a bank that has an investment opportunity, as opposed to searching and switching every period. Moreover, long term relationships among banks is frequently documented, as in Afonso et al. (2011) and Di Maggio et al. (2015). In this sense, renegotiation is not a big issue. In addition, because investment happens at $t = 1$ and non-contractible return is realized at $t = 2$, the borrower cannot commit to pay the lender a side payment above and beyond the face value of debt enforceable by the contract. Note that in the period during which actual lending happens, no extra funding is available to make an early side payment. As a result, ruling out side payments is also a reasonable assumption.

[30]The existence of link $e_{ij}$ does not necessarily mean at least one unit of funds is pledged to $j$ *for every realization of investment opportunities* to whome $j$ is connected, because the shortest path of $i$ to some of $I$ banks might not go through $j$.

opportunities, or no credit line exists along which a unit can be lent.[31]

Finally, the face value of the debt is contingent on the network $G$ as well as the realization of $\mathbb{I}_R$, which means it is contingent on all the realized lendings. It is set such that in expectation (over realizations of random returns $\{\tilde{R}_k\}_{k\in\mathbb{I}_R}$), each bank $i$ receives $\mathcal{L}(i; G, \mathbb{I}_R)$.[32]

Recall that the set of lending commitments along with the banks themselves constitute $G$. This particular choice of $G$ warrants some explanation, because it does not refer to the *realized* financial network. This representation captures a reduced form for the dynamic game played among banks, such as the one described in Moore (2011). In the full dynamic game of Moore (2011) in each period, some banks receive funding and some receive investment opportunities, and lending happens each period.[33] In the subsequent periods, borrower banks find it optimal not to immediately pay back their debt and instead lend their levered-up resources to other banks that have an investment opportunity. The simplification I have made abstracts away from the detailed dynamics. Abstracting away from dynamics of network formation process allows me to instead focus on characterizing the properties of the equilibrium network.

At $t = 1$, given the equilibrium network $G$ and each realization of investment opportunities, $\mathbb{I}_R$, the contracts determine the number of units lent along each potential lending agreement, as well as the face value of debt corresponding to this realized lending. Let $m_{ij} = m(i, j; G, \mathbb{I}_R)$ denote the size of the loan from bank $i$ to $j$, and let $D_{ji} = D(j, i; G, \mathbb{I}_R)$ denote the per-unit face value corresponding to this loan. Moreover, let $D_i^h = D(i; G, \mathbb{I}_R)$ be the face value of debt from $i$ to households.

The first proposition provides bounds on the flow of funds at date $t = 1$ given the realization of investment opportunities. The following definition is useful for understanding the proposition.

**Definition 6.** *A "cut" is a partition of the nodes of a graph into two disjoint subsets that are joined by at least one edge.*

[31] This detail can be specified differently without altering the results as long as 5 is satisfied. The reason is that contracts can be written on what happens at date $t = 1$. At $t = 0$, banks correctly forecast the expected rates they will be pledged, as well as their expected probability of default given any set of rules and adjust their connections accordingly. This particular choice helps explain the deviations.

[32] I have chosen contingent debt to avoid any additional market incompleteness except that each bank can only chooses a (limited) set of counterparties and then has to execute all of its trades through these established counterparties. This structure is consistent with the costly establishment of relationship lending (information, trust, etc.), as well as the observation that hedge funds, even large ones, typically maintained only one or two prime brokerage relationships and did not frequently switch. (https://www.wellsfargo.com/downloads/pdf/com/securities/hedge-fund-risk.pdf).

[33] Moore (2011) assumes a bank cannot invest in its own project.

*The "cut-set" of the cut is the set of edges whose end points are in different subsets of the partition. Edges are said to be "crossing" the cut if they are in its cut-set.*

*In a flow network, an "s-t cut" is a cut that requires the source and the sink to be in different subsets, and its cut-set only consists of edges going from the source's side to the sink's side.*

*In a weighted graph, the "size" of a cut is the sum of the weights of the edges crossing the cut.*

Now construct the following auxiliary graph $\hat{G}$ from $G$, given the realization of $\mathbb{I}_R$: remove all edges among $I$ banks. Moreover, remove $\mathbb{I} \setminus \mathbb{I}_R$ and all the remaining edges incident on them from $G$. Define the weight of edge $e_{ij}$ to be $m_{ij}$. Finally, reverse the direction of all edges. I say $i_1$ is $i_2$'s parent if $e_{i_1 i_2}$ exists in $\hat{G}$.

**Proposition 2.** *For every subset $\hat{\mathbb{I}}_R \subset \mathbb{I}_R$, let $\hat{\mathbb{I}}_R$ be the source(s) and let (different subsets of) leaf bank(s) be the sink(s). Consider each s-t cut $C(\hat{\mathbb{I}}_R)$ with the following property: if $b$ is on the source side of the cut, all parents of $b$ are also on the source side. Let Size$(C)$ denote the size of the cut; that is, Size$(C) = \sum_{e_{ij} \in C} m_{ij}$; and $X_S(C)$ denote the number of banks on the sink side of the cut. Moreover, let $C_o(\hat{\mathbb{I}}_R)$ be the cut with the above property that only has $\hat{\mathbb{I}}_R$ on the source side of the cut. Finally, let Count$(C)$ be the number of edges in the cut set, Count$(C) = \sum_{e_{ij} \in C} 1$.*

$$\begin{cases} \textit{Size}(C) \leq X_S(C) & \forall \hat{\mathbb{I}}_R \ \forall C(\hat{\mathbb{I}}_R) \\ \textit{Count}(C_o) \leq \textit{Size}(C_o) & \forall \hat{\mathbb{I}}_R \end{cases}$$

*where the first inequality hold with equality when $C$ is such that only leaf nodes are on the sink side.*

The main intuition is that each bank in $\mathbb{I}_R$ is entitled to at least one unit from each of its lenders, which gives the lower bound. These lenders will then draw their credit lines from their own lenders, and so on. As a result, the amount of money that flows into each set of banks cannot be more than the amount of money that their lenders (direct and indirect) have. Note that these bounds do not necessarily uniquely determine each $m_{ij}$.[34]

At $t = 2$, given any realization of project returns $\{R_k\}_{k \in \mathbb{I}_R}$, the borrower may or may not be able to pay lenders back in full. Let $d_{ji} = d(j, i; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R})$ and $d_j^h = d(j; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R})$ denote the per-unit repayment bank $j$ makes to lender bank $i$ and

[34] Recall that extra funds are randomly allocated over corresponding shortest paths. The reason is exactly to resolve the above indeterminacy in a way that is not consequential to results, but allows me to abstract away from bank choices when the flow of funds is not uniquely determined.

households, respectively. As a convention, $D_j^h = d_j^h = 0$ if $j$ has not borrowed from households. By construction, $d_{ji} \in [0, D_{ji}]$. Finally, let $L(i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R})$ and $A(i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R})$ denote the total liabilities and assets of bank $i$ at date 2 when all the uncertainty is resolved:

$$L_i = L(i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) = \sum_{j\in\mathbb{N}} m_{ji} d_{ij} + s_i d_i^h$$
$$A_i = A(i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) = \mathbb{1}[i \in \mathbb{I}_R]\Big(\tilde{R}_i\big(\sum_{j\in\mathbb{N}}(m_{ji} - m_{ij})\big)\Big) + \sum_{j\in\mathbb{N}} m_{ij} d_{ji},$$

where $\mathbb{1}[i \in \mathbb{I}_R]$ is the indicator function that takes value one if $i$ has access to an investment opportunity. Consequently, the per-unit (partial) repayment from $j$ to $i$ in each state of the world can be written as

$$d_{ji}(j, i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) = \max\big\{0, \min\{D_{ji}, D_{ji}\frac{A_j}{L_j}\}\big\}, \tag{6}$$

and a similar expression holds for $d_j^h$. The above expression simply means that if a borrower does not have sufficient funds to repay its lenders, each lender will be paid back pro-rata, and there is limited liability.[35]

Given the solution to the system of (partial) debt repayments at $t = 2$, specified by (6), using backward induction, the face value of each debt contract at date $t = 1$ is set such that in expectation, each bank $i$ receives its share of surplus according to $\mathcal{L}(i; G, \mathbb{I}_R)$. This completes the specification of contracts.

## 4.2 Bank Optimization Problem

Let $S(i; G, \mathbb{I}_R, \{R_i\}_{i\in\mathbb{I}_R})$ denote the ex-post profit of bank $i$, which can be written as

$$S(i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) = A(i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) - L(i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}).$$

Let $\mathbb{1}[i \text{ survives}; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}]$ be the indicator function that is equal to one if bank $i$ survives at $t = 2$ and zero otherwise. With this notation, banker $i$'s optimization problem

[35] This definition implies that all debt is pari passu. Junior household debt can be interpreted as capital and be used to study the effect of capital requirements.

at $t = 0$ can be written as:

$$\text{(7)} \qquad \max_{\{e_{im}, e_{mi}\}_{m\in\mathbb{N}, m\neq i}} \mathcal{V}_i\big(\{e_{im}, e_{mi}\}; G\big) = \mathbb{E}\Big[S(i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}) + \mathbb{1}[j \text{ survives}; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}]V_i\Big]$$

$$\text{s.t.} \qquad \text{Feasibility (1) ;} \qquad \text{Participation Constraint.}$$

where the expectation is taken over both realization of investment opportunity (at date $t = 1$), which determines $\mathbb{I}_R$, and realization of project returns (at date $t = 2$), $\{R_k\}_{k\in\mathbb{I}_R}$. The choices of other banks are reflected in $G$. As I explained in section 3.1, the notion of equilibrium here is not pairwise stability of Jackson and Wolinsky (1996), which only allows for unilateral (breaking links) or bilateral (adding a link) deviations. Rather, group stability is used, which allows for joint deviations.

Note that $V_i$ is a constant, so one can write the expectation of the indicator function for bank $i$ survival as a probability function. Let $P(i; G)$ denote the probability that bank $i$ survives given the financial network $G$ formed at $t = 0$:

$$P(i; G) = \mathbb{E}\Big[\mathbb{1}[i \text{ survives}; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R}]\Big].$$

With this notation, (7) is simplified to:

$$\max_{\{e_{im}, e_{mi}\}_{m\in\mathbb{N}, m\neq i}} \mathbb{E}\Big[S(i; G, \mathbb{I}_R, \{R_k\}_{k\in\mathbb{I}_R})\Big] + P(i; G)V_i$$

$$\text{s.t.} \qquad \text{Feasibility (1) ;} \qquad \text{Participation Constraint.}$$

The intuition for the bank's optimization problem is the following: consider the first term in the above objective function. Because the banking sector is non-competitive and each player gets part of the surplus, each bank would like to use the structure of its connections to extract more rents. Each bank balances the costs and benefits of exposure to more risk via intermediation and chooses the set of lending and borrowing relationships that maximizes its total expected profit.

Note that constant return to scale in identical projects implies that for any given aggregate scale of investment, total expected net surplus is independent of the distribution of investment among banks. As a result, as long as the identity of intermediators does not change the scale of the project, the rent-seeking activity translates into a change in the division of surplus in favor of intermediators, without any aggregate welfare implications. However, this surplus redistribution is not the only effect of a change in the identity of the intermediator. Intuitively, all banks along the path of intermediation are exposed to the risk of failure if the investment fails, so a change in the set of banks that do the intermediation also changes the cost of default. As a result, the identity and characteristics of the intermediaries does not merely have a redistribution effect.

### 4.3 Lending Structure and Division of Surplus

In this section, I specify a highly tractable rule for surplus division, $\alpha$-*rule*, which is an extension of what I used in section 3. I use $\alpha$-rule throughout the paper, and show in Theorem 3 that they hold for any fixed surplus division $\mathcal{L}$ that satisfies the properties of section 4.

Consider an intermediation chain of infinite length, and one unit of funding intermediated along the chain. The share of net surplus received by each bank along the chain, starting from the final borrower, falls at rate $\alpha$, so that the initial lender (who is infinitely far away) receives a negligible share of net surplus and only breaks even. In other words, the initial lender only receives the cost of initial investment. Because the sum of the shares should add up to one, the final borrower receives share $(1-\alpha)$, the immediate intermediator receives $(1-\alpha)\alpha$, and the intermediator at distance $d$ receives $(1-\alpha)\alpha^d$. Now suppose the initial lender is at distance $k$ (instead of being infinitely far away). It receives the cumulative share of all *hypothetical* intermediators at distance $k$ and further, so it gets $\alpha^k$ share of net surplus plus the cost of initial investment. This particular division implies the lender bears all the cost of intermediation, so the face value of unit of debt payable to each lender can be calculated in a straight-forward manner regardless of the source of each unit.[36]

## 5 Results

In this section I provide results for general networks. To abstract away from diversification I need a generalized version of 2 to work with complex networks.[37]

[36] $\alpha$-rule might look too restrictive since it does not incorporate the expected default costs. In section 8.2.3 I solve the model with $\alpha$-rule augmented to incorporate expected default costs and show that the same results hold.

[37] This assumption is relaxed in section 5.2.

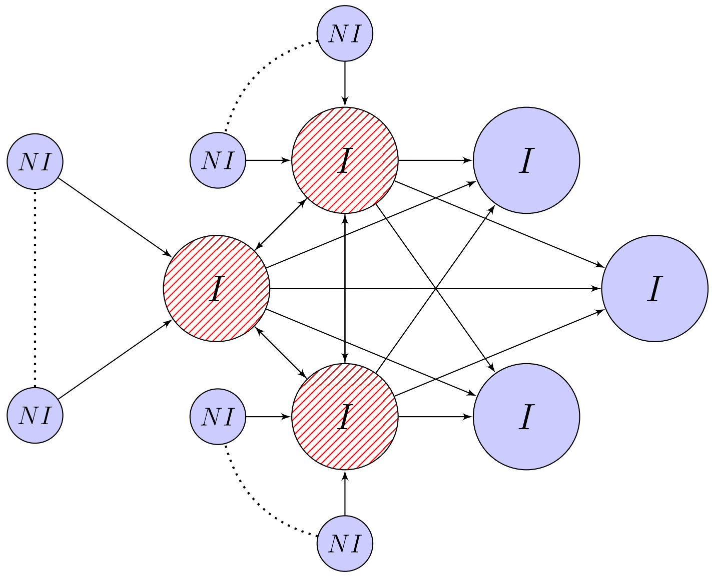



Figure 6: Equilibrium interbank lending structure with sufficiently many $NI$ banks. The hatched red banks are banks $I \in \mathbb{C}$ in Theorem 1.

**Assumption 3.** [**Assumption 2 Revisited**] *If a bank $i$ owes funds to multiple banks, all of its funding is randomly assigned to exactly one of them such that in expectation, each borrower receives the amount determined by $\mathcal{L}$. An $I$ bank that receives an investment opportunity invests all of its funds in its own project.*

The first result addresses the length of intermediation chains, and characterizes an endogenous maximum length for any intermediation chain.

**Lemma 1.** *There is no intermediation chain of length more than $l_{max}$, such that $\alpha^{l_{max}} X \geq (1-p)V_{NI}$ and $\alpha^{l_{max}+1} X < (1-p)V_{NI}$.*

This lemma is intuitive: the share of each bank along the chain falls as the length of the chain grows, whereas the expected cost of default is constant. Under assumption 2, each bank $j$ fails if the project at the single $I$ bank to which $j$ has (directly or indirectly) lent fails, so the expected cost of default is $(1-p)V_{NI}$. The trade-off between a benefit that geometrically decreases in distance, and a constant cost, determines the endogenous maximum length of the intermediation chain.

The next theorem presents the main result of the paper.

**Theorem 1.** [**Core-Periphery Equilibrium**] *Assume $k_{NI} > k_I$, and surplus is divided via $\alpha$-rule. For $\kappa > M$, and a properly chosen constant $M$, a family of equilibria exists with the following structure: choose a subset $\mathbb{C} \in \mathbb{I}$, referred to as "core." $\mathbb{C}$ is a complete* *digraph. Each $NI$ bank lends to exactly one $I \in \mathbb{C}$, such that at least $k_I$ $NI$ banks lend to each $I \in \mathbb{C}$. Every $I \in \mathbb{C}$ lends to every other $I$ bank, and every $I \notin \mathbb{C}$ does not lend to any bank. This family of equilibria is inefficient.*

*Moreover, let $s = |\mathbb{C}|$ be the size of the set of intermediating $I$ banks. Then there exist a sequence of strictly increasing constants $\{M_s\}_{s=1,\cdots,k_I}$, $M = M_{k_I}$, where with $k_{NI} \geq sk_I$, the financial structure with core size $s$ is an equilibrium iff $\kappa > M_s$.*

The above family of equilibria is depicted in Figure 6. The main idea of the proof is the following: if the project is profitable enough, $I$ banks will be able to cover their cost of default using the intermediated rents. Moreover, they will be able to connect to every other $I$ bank if they have enough peripheral lenders. On the other hand, each $NI$ bank would want to get as high a return as possible (be as close as possible to an $I$ bank), as well as receive positive returns as often as possible (be connected directly or indirectly to as many $I$ banks as possible). Because sufficiently many $NI$ banks exist, there are configurations in which each $I$ bank is able to be connected to every other $I$ bank (a *well-connected $I$* bank). As a result, any subset of well-connected $I$ banks can act as intermediators in a stable structure.

Clearly, there are multiple equilibria, but all of them share the same properties. The degree of inefficiency varies among the equilibria, and equilibria with smaller $\mathbb{C}$ (i.e., fewer $I$ banks as intermediators) are less inefficient. The second part of the theorem implies that financial structures with smaller core sizes are equilibria for a wider range of parameters. This result is intuitive as well: if the core is smaller, each $I$ bank in the core can receive funding from more $NI$ banks and absorb more intermediation rents, which in turn cover a higher expected cost of default.

Recall that the rule for division of surplus is exogenous, in the sense that banks can affect their return only through strategically positioning themselves in the network. Nevertheless, the pricing implications of the model is consistent with recent empirical evidence. Di Maggio et al. (2015) empirically investigates the inter-dealer market for corporate bonds and documents that it exhibits a clear core-periphery structure. Moreover, they show that core dealers on average charge higher prices to the peripherals than to other core dealers, which is precisely what my model predict.

The next proposition provides an existence result.

**Proposition 3.** *An equilibrium exists.*

The proof is in two steps. First I show that taking network structure as given, for any resolution of uncertainty, the system of equations for interbank repayments has a unique solution (a la Acemoglu et al. (2015)). Then I provide a constructive proof of the equilibrium structure in the network formation stage.

The next theorem provides the constraint efficient benchmark, which maximizes total net surplus subject to feasibility and participation constraint.

**Theorem 2.** *Assume $k_{NI} > k_I$. Solution to the social planner's problem is an $NI$ bank that borrows from every other $NI$ bank, directly or indirectly, and lends to every $I$ bank. Let $NI_c$ denote the $NI$ bank who borrows from every $NI$ banks. Every socially efficient structure is a tiered network: all the paths from each bank $NI_j$ to $NI_c$ has the same length, $dist(NI_j, NI_c)$.*

*Moreover, there are constant $\bar{M}$ and $\bar{K}$ where for $\kappa > \bar{M}$ and $\frac{k_{NI}}{k_I} > \bar{K}$, no efficient equilibrium exists.*

The defining feature of a constraint efficient structure is that no $I$ bank intermediates in it, to avoid excessive unnecessary defaults of Theorem 1; and all the funding is allocated via the same $NI$ so that maximum concentration is achieved. Let $NI$-star be the constraint efficient structure in which one $NI$ directly borrows from every other $NI$. Figure 1b depicts network $NI$-star. $NI_c$, the hatched red $NI$, can be interpreted as a central clearing house in that all of the lending goes through this particular bank. Recall that diversification is assumed away in this section, so what makes the existence of the central clearing party (CCP) optimal is not the gains to diversification. Rather, the CCP is an entity that is able to channel the funding to all the investment opportunities optimally without being exposed to *excessive* risk.

To extend the results beyond $\alpha$-rule consider any rule for division of surplus which satisfies the properties of section 4.1, and Let $\mathcal{L}(k; K, X)$ denote the share of expected net surplus from a unit of investment intermediated through a chain of length $K$, accrued to the bank in position $k$, with an $I$ bank as the final borrower at position $k = 1$ who has received an investment opportunity and have made an investment. The following result generalizes Theorem 1 and 2. The basic intuition is that the $\alpha$-rule has two special characteristics: the shares are decreasing along an intermediation chain, and final borrower share is invariant to the length of the chain. None are crucial for the above results.

**Theorem 3.** *Redefine $\kappa = \frac{\mathcal{L}(2;3,X)}{(1-p)V_I}$. Then Theorem 1 and 2 hold for any $\mathcal{L}(.)$ satisfying conditions of section 4.1.*

## 5.1 Discussion

In this paper, I have provided a setting to study the role of intermediation in interbank networks. Intermediation enhances welfare by allocating funds from parts of financial sector with excess liquidity to parts with profitable investment opportunities, but it can also destroy welfare if banks expose themselves to excessive counterparty risk to capture intermediation spreads. The model demonstrates that each bank's motive to absorb intermediation spreads, along with differential abilities of different banks in offering rates of return on their borrowing, leads to a specific equilibrium structure, a core-periphery network. As such, my model provides a novel explanation of why banks choose to expose themselves to counterparty risk, which differ from the existing explanations such as bailouts and ignoring tail risk. Any of these other explanations amplifies banks' incentives, but I argue that neither is necessary to explain excessive exposure to counterparty risk.

Moreover, the same rent-seeking behavior mis-aligns private and social incentives. Since contracts are debt and default is costly, efficiency requires maximum scale of investment while minimizing the cost of failure, which would imply that for any investment, every bank in the intermediation chain should either provide sufficient funds from outsiders or make the investment (or both). This turns out not to be the case when intermediation spreads are high, which leads to excessive exposure to counterparty risk. The main source of inefficiency is that the gains from intermediation are purely redistributional, whereas the loss is incremental. This is an important insight from the model, missing from the existing literature on interbank interconnectedness, which emphasizes a *downside externality*, contagion, as the main externality in the interbank network: a bank does not take into account that his creditors fail if he fails and does not pay them back. Here, although contagion happens ex-post, it is due to an *upside externality*, i.e. rent-seeking: banks choose to expose *themselves* to an excessive probability of failure in order to be able to absorb intermediation rent. So the additional loss borne by the system is failure of this particular bank, above and beyond any contagion from this bank to other banks. In the next section I show that the two externalities can coexist.

Note that net expected return to investment can be large if $p$ is large; that is, when probability of project failure is low. This means, not only the intermediation spreads are high but also the expected cost of default is fairly low, although $V_I$ can be very large. Consequently, it is highly likely that the more inefficient equilibria arise; that is, the equilibria with many $I$ banks at the core. In this scenario, even though the expected cost of default for each bank is small, the ex-post realized losses can be arbitrarily large. This interpretation rationalizes the high degree of interconnectedness among large financial institutions during the run-up to the financial crisis of 2008, as well as the enormous losses once the financial sector collapsed.

Furthermore, when the funding is dispersed throughout the financial sector, i.e. there are many small institutions with access to funding, this provides an opportunity for many banks with investment opportunities to attract sufficient lenders and capture enough intermediation spreads to cover their expected cost of default due to counterparty risk exposure. The size of the core grows and the ex-post realized losses are magnified.

Finally, let me briefly discuss the key assumptions of the paper. The two main assumptions of the models are existence of intermediation spreads, and market incompleteness. Intermediation spreads have been documented in different interbank markets.[38] Requiring banks to have exante relationships in order to lend, along with feasibility, implements market incompleteness and implies that banks cannot spontaneously reallocate their funding, or borrow on the interbank market, so intermediation is necessary to allocate resources within the financial sector. There is ample evidence that banks interacts through long term relationships. Afonso et al. (2011) documents that in the federal funds market, approximately 60% of the funds an individual bank borrows in one month persistently come from the same lender. Di Maggio et al. (2015) finds that in the inter-dealer market, banks with longer term relationships get access to better terms.

I have also assumed that $I$ bank raise no funds from households. Clearly, the equilibrium structure is insensitive to this assumption. In terms of efficiency, what is important is that $I$ bank's contribution to scale of investment should not be sufficient to justify its risk-taking behavior. To be more precise, assume $I_1$ has raised $\epsilon < 1$ funds from households. Recall that without intermediation, the participation constraint of a direct lender ($I_1$ in this case) to $I_2$ requires that $\epsilon\alpha(pR-1) \geq (1-p)V_I$. Let $\hat{\epsilon}$ be the amount of funds for which the above inequality holds with equality. Then for any $\epsilon < \hat{\epsilon}$, it is more efficient that an $NI$ bank with one unit raised from households do the intermediation as opposed to $I_1$.

Lastly, I have assumed lenders cannot default on their promises. As a result the contagion in my model spreads only from borrowers to lenders. An interesting extension would be to allow lenders to promise liquidity to several borrowers, with lenders defaulting on their contingent promises if several borrowers demanded liquidity at once. This extension would enrich the model and open the possibility of contagion from lenders to borrowers. Moreover, how financial institution restructure the interbank network in the face of failure of some banks is an important avenue for future research.

[38]Di Maggio et al. (2015), Li and Schürhoff (2014), Adrian (2011) and Bech and Atalay (2010) provide evidence for intermediation and positive intermediation spreads in different markets.

## 5.2 Diversification

In this section I relax assumption 3 to allow banks to hold diversified portfolios, and study the equilibrium structures. I find that the same structure of equilibria emerges, albeit with a twist. I focus on an economy with two $I$ banks, $k_I = 2$, and $k_{NI}$ $NI$ banks, $k_{NI} > 4$. Restricting the number of $I$ banks keeps the problem tractable while incorporating the main intuition associated with diversification.

I make the following bargaining assumption:

**Assumption 4.** *Consider a realization of $\mathbb{I}_R$. If bank $b$ has access to multiple $I \in \mathbb{I}_R$ through intermediation chains of different lengths, it can use the shortest chain to bargain its share in other chains up to what he gets in the shortest one. $b$'s (direct and indirect) borrowers in each longer chain divide the remaining share pro-rata.*

Consider the following simple structure. $NI_0 \to NI_1 \to I_1$, and $NI_1 \to NI_2 \to I_2$. When both $I$ banks have investment opportunities, $NI_1$ has direct access to one and indirect access to the other. The above assumption says that $NI_1$ can bargain up its share in the chain $NI_1 \to NI_2 \to I_2$ to $\alpha$. $I_2$ and $NI_2$ divide the remaining $(1-\alpha)$ share with proportions $\frac{1}{1+\alpha}$ and $\frac{\alpha}{1+\alpha}$, respectively.[39]

The above assumption has an important implication for behavior of banks. It implies that all else equal, between two intermediators, $i$ cannot be worse off if the intermediator to which it lends is connected to an extra $I$ banks, even if through longer chains. The following lemma formalizes this intuition.

**Lemma 2.** **[Dominance]** *Consider two banks $j_1$ and $j_2$. Let $SPL_i = \{l^i_1, l^i_2 \cdots, l^i_{z_i}\}$ be the set whose elements are lengths of paths in $SP(j_i, \mathbb{I})$, $i = 1, 2$. Assume elements of each set are sorted in increasing order. Also, without loss of generality, assume $j_1$ has more shortest paths to $\mathbb{I}_R$, $z_1 > z_2$. A leaf bank $b$ prefers to lend to $j_1$ if*

$$\forall\, k \leq z_2 :\ l^1_k \leq l^2_k$$

*independent of $l^1_k$ for $k > z_2$.*

Assume parameters are such that, absent diversification, an $I$ bank chooses to intermediate (even) with a single peripheral lender. Consider the 2-$I$ core-periphery structure that is an equilibrium without diversification. Assume each $I_i$ has credit lines from $Y_i$ of $NI$ banks, where $Y_1 + Y_2 = k_{NI}$.

[39] I will restrict the analysis to parameters where individual rationaliy is maintained.

| Assets | Liabilities |
|---|---|
| $\frac{Y_1+Y_2}{2}\tilde{R}$ | $Y_1 D_{11}$ |
| $\frac{Y_1-Y_2}{2}D_{21}$ | |

(a) Net Lender ($I_1$)

| Assets | Liabilities |
|---|---|
| $\frac{Y_1+Y_2}{2}\tilde{R}$ | $Y_2 D_{22}$ |
| | $\frac{Y_1-Y_2}{2}D_{21}$ |

(b) Net Borrower ($I_2$)

Figure 7: Balance sheet of banks $I_1$ and $I_2$ when banks net out their payments. There are two $I$ banks and $k_{NI}$ $NI$ banks. $Y_i$ $NI$ banks lend to $I_i$ such that $Y_1 > Y_2$, so $I_1$ is the net lender and $I_2$ is the net borrower. In equilibrium, $D_{21} = R$ and $D_{22} = (1 + \alpha X)/p$.

Consider the date $t = 1$ event where both $I_1$ and $I_2$ have investment opportunities (probability $q^2$). As described in section 4, $I_i$ lends $\frac{Y_i}{2}$ to $I_j$. Let $D_{ii}$ denote the face value of debt promised by $I_i$ to each of its $NI$ lenders. Moreover, let $D_{ij}$ denote the face value of the debt payable to $I_j$ by $I_i$.

I assume banks net out their payments at date $t = 2$. As a result, when $\frac{Y_i}{2}D_{ji} > \frac{Y_j}{2}D_{ij}$, $j$ owes $i$ the difference, namely, $\frac{Y_i}{2}D_{ji} - \frac{Y_j}{2}D_{ij}$.[40] So $I_j$ is the *net borrower* and $I_i$ is the *net lender*.

Without loss of generality, let $i = 1$ and $j = 2$ in the above discussion, so that $I_1$ is the net lender. Assumption 4 is extremely useful in determining $D_{12}$ and $D_{21}$. Each $I_i$ has access to two investment opportunities: its own investment, which provides it with all the return (out of which he has to pay his lenders); as well as $I_j$ investment opportunity. By assumption 4, each $I_i$ receives all the return from investment for each unit it lends to $I_j$.[41] This argument pins down both inter-$I$ face values to be exactly $R$, $D_{12} = D_{21} = R$. So $I_1$ being the net lender implies $Y_1 > Y_2$. Consequently, at $t = 2$, bank $I_2$ owes $I_1$ a net payment of $\frac{Y_1-Y_2}{2}R$.

The balance sheets of $I_1$ and $I_2$ are depicted in Figure 7. The critical observation is that survival of the net borrower solely depends on its own investment, while for the net lender, it also depends on whether the net borrower pays back. As a result, when both $I$ banks invest, the net borrower survives exactly with probability $p$, whereas net lender's survival probability depends on other parameters of the model as well as the structure of the network, and is determined in equilibrium. Here I provide the main ingredients of the argument, and exact details are provided in the appendix.

[40] Although both banks lend to each other, and face values of debt are determined in equilibrium.

[41] Note that $I_j$ accepts as long as it has funding pledged to it directly by $NI$ banks and the share of that investment covers its expected cost of default.

I show that depending on the value of $R$, there can be two cases, as depicted in Figure 8. Panel 8a and 8b correspond to high and levels of return, respectively. In each plot, the horizontal axis is $\alpha$, the share of surplus that goes to a direct lender in a chain of length two, and the vertical axis is the ratio of the number of peripheries of the net borrower to the net lender, $y = \frac{Y_2}{Y_1}$. Note that $0 \leq y \leq 1$ and $0 \leq \alpha \leq 1$, so only the unit square in the first quadrant is relevant. Within this area, below the solid red line (yellow region), the liabilities of $I_1$ are low, so having more peripheries increases the gain to diversification, and $I_1$ survives with probability $1 - (1-p)^2$. That is when $\alpha < \bar{\alpha}$. The reverse situation happens below the dashed blue line (green region). Here the liabilities are so high that $I_1$ fails unless all of his assets pay, so having many direct lenders increases his liabilities and leads to a higher probability of default, and $I_1$ survives only with probability $p^2$ . In the intermediate region, above both lines, $I_1$ survives exactly when its investment survives and fails exactly when its investment fails; that is, with probability $p$. On the horizontal axis, $y = 0$, $I_1$ fails with probability $p$. So incentives of bank $I_1$ depends on level of $\alpha$, which governs the amount of his liabilities.

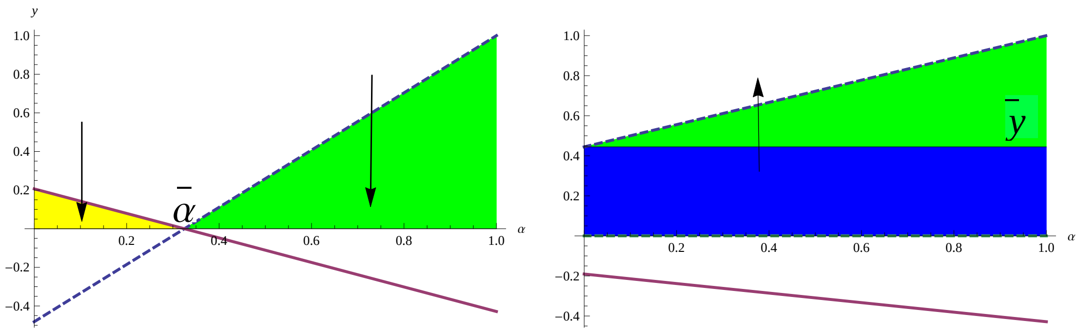
Incentives of $NI$ banks are more complicated. First note that they are purely driven by minimizing the probability of default, and default probability of $NI$ banks who are peripheral to the net borrower $I_2$ is $p$. The complexity stems from the fact that $NI$ bank liabilities are independent of $\alpha$, and consequently its default probability is determined at $\alpha = 0$. Here is the relevant intuition for 8a: the reason $I_1$ fails more often in certain regions compared to others, with the same successful assets, is that its liabilities are higher, i.e. $\alpha$ is high. However, $NI$ banks have to pay the households only one unit in expectation, regardless of what $\alpha$ is. As a result, $\alpha$ is not relevant in determining failure probability of the $NI$ banks. As a result in 8a all $NI$ banks *migrate* and lend to $I_1$, although at $\alpha > \bar{\alpha}$ this increases $I_1$'s probability of default.

Given the above discussion, the next proposition characterize the equilibrium.

**Proposition 4.** *Let $y$ denote the ratio of the number of $NI$ peripheries of net borrower to net lender $I$ bank. There is a constant $\bar{R}$ such that*

- *When $R > \bar{R}$, there are two core-periphery equilibria with $I$ banks at the core: $y = 0$ with $I_1$ at the core, and $y = \frac{1}{k_{NI}-1}$ with both $I_1$ and $I_2$ at the core.*
- *When $R < \bar{R}$, the single-core equilibrium is still an equilibrium. There are multiple two-core equilibria, one for each $y > \bar{y}$, where $\bar{y} = \frac{2}{p^2 R} - \frac{2-p}{p}$.*

*Moreover, there are constants $\bar{\alpha}_l$, $\hat{\alpha}_l < \hat{\alpha}_h$ and $\hat{q} < \bar{q}$, all in $(0, 1)$, such that*




(a) $R > \bar{R}$

(b) $R < \bar{R}$

Figure 8: Possible Equilibira with two $I$ banks and $k_{NI}$ $NI$ banks and diversification. The $x$-axis is the share of expected net surplus that goes to the lender in a direct lending, $\alpha$, and the $y$-axis is the ratio of the number of $NI$ peripheries of $I_2$ to $I_1$, $y$. The arrows show the direction of the deviation of the $NI$ banks.

- *$R > \bar{R}$ and $\hat{\alpha}_l < \alpha < \hat{\alpha}_h$: 2-I core-periphery equilibrium is inefficient when $\alpha > \bar{\alpha}_l$.*

  *It is also inefficient when $\alpha < \bar{\alpha}_l$ and $q < \bar{q}$.*

- *$R < \bar{R}$: 2-I core-periphery equilibrium is inefficient when $q < \hat{q}$.*

A detailed argument is provided in the appendix, but there are a few points worth mentioning here. First, diversification creates coordination problems between lenders an borrowers, which can in turn lead to inefficiencies in the financial network. In 8b, for equilibria with $y$ between the $y = \bar{y}$ and the dashed blue line, there are two sources of inefficiency: first, $I_1$ is exposed to the risk of default of $I_2$ when he only intermediates. Second, $I_1$ is not diversified in the best possible way when he also invests.

Second, this proposition shows that adding diversification does not alter the incentives to intermediates. Even when the gains from diversification are larger in the core-periphery network with two $I$ banks at the core compared to the $NI$-star network, they can be dwarfed by the extra cost of $I$ banks' failure due to excessive exposure to counterparty risk, and the core-periphery structure remains inefficient. Note that I have used the $NI$-star network to find sufficient conditions under which the 2-$I$ core-periphery structure is not efficient, but these conditions are not necessary. There are more parameter regions where the above equilibria are dominated by $NI$-stare. Moreover, with diversification, the efficiency benchmark, which is necessary to characterize the necessary conditions, is more complicated to compute, and is left for future research.

Finally, adding diversification enables me to study the interesting question of under-insurance in the context of the model. Consider the $y = 0$ equilibrium, and assume $R > \bar{R}$

and $\alpha < \bar{\alpha}$. Imagine $I_1$ was able to offer the following deal to $I_2$ when both have investment opportunities: $I_1$ lends half of its funds to $I_2$ in order to fully diversify, and it pays $I_2$ exactly enough to cover $I_2$'s expected cost of default, $(1-p)V_I$. Such an offer increases $I_1$ and all of $NI$'s probability of survival from $p$ to $1-(1-p)^2$, whereas it imposes some extra cost of default (that of $I_2$) on the economy. One can show that if $k_{NI} > \frac{V_I}{V_{NI}}\frac{(1-p)}{p}$, the above strategy improves welfare. However, $I_1$ would not make such an offer even if it could, because its individual gain to diversification, $p(1-p)V_I$, is lower than the price that it has to pay, $(1-p)V_I$. This means that $I_1$ does not internalize the positive externality of it buying insurance on its lenders. In other words, the price of insurance is too high for $I_1$, which leads to voluntary under-insurance and contagion.

# 6 Policy Implications

Multiple policies targeting the structure of financial networks can be studied in the context of the model. First, the model provides a new rationale for introduction of a Central Clearing Party (CCP). Designating a non-investing bank as the CCP and enforcing all the lendings to go through CCP prevents excessive bilateral exposure among banks with investment opportunities, and can enhance welfare particularly as investment returns become more correlated. Note that this effect is different from roles identified by Duffie and Zhu (2011) and Bond (2004). The model predicts that such structure is not an equilibrium when intermediation rents are sufficiently high, so intervention is necessary to implement it.

Second, the model can be used to study bailouts and how they affect the equilibrium structure. A natural way to incorporate bailouts in the model is as a wedge between true $V_i$ and the loss to the bank if it is in default. The interpretation is that when a bank fails, there are disruptions and the bank bears some of the cost, such as a stream of interim forgone profits. However, it is bailed out and resumes functioning and generating surplus. So if the social cost of bank failure is $V_i$, the bank only bears $\hat{V}_i = \beta V_i$, for some $\beta < 1$, and the difference $(1-\beta)V_i$ is borne by the government. Let $\mathbb{C}$ denote the core of the equilibrium interbank network, as defined in Theroem 1. One is tempted to compute the ex-post cost of bailout as $\sum_{I_i \in \mathbb{C}}(1-\beta)V_{I_i}$. However, such calculation misses the critical point of this paper: implementing a bailout policy not only has an ex-post cost at the current financial architecture, but also feeds back into bank decisions and affect the interbank network. As bailout decreases the cost of default borne by investing banks, it expands the core in two ways: first, more banks at the previous level of *true* default costs would be willing to intermediate with the same amount of intermediation spread, as they each need fewer spreads to cover their *subjective* cost of failure, which is now lower than than the true (social) cost. Moreover, banks with larger default costs will now be willing to expose themselves to excessive counterparty risk to capture intermediation spreads. The crucial observation is that these banks, absent bailout, would not take this risk because their opportunity cost was too big. Bailouts decreased this opportunity cost by shifting it to the government.

Therefore, incorporating the effect of network formation into evaluation of bailout policies uncovers an important amplification effect. Expectation of a bailout not only makes the the highly interconnected core of the financial sector larger, but also banks with larger default consequence join the core. More large financial institutions default, and need to be saved, due to exposure to counterparty risk, which deepens the financial crisis. Of course the ex-post bailout cost needs to be probability weighted, but correctly estimating the ex-post cost is essential for policy evaluation. To the extent that projects are relatively correlated, and most of the interbank exposure is due to incentives to capture spreads, a larger core do not substantially decrease the contagion probability, and bailouts increase the expected cost of systemic failure.

Third, part of Title VII of the Dodd-Frank Wall Street Reform and Consumer Protection Act was a proposed cap on the number of counterparties and swaps, which was later eliminated from the finalized rules.[42] Gofman (2012) calibrates a model of bank bilateral-exposures using federal funds data to empirically study the welfare effects of this policy, and suggests such policy can entail potentially large welfare losses because banks would not be able to effectively channel funding to profitable investment opportunities.

The current paper provides sharp theoretical predictions about such a policy: it either leads to under-investment, or more inefficient equilibria with larger cores. In the context of the model, financial structures exist that would allow the optimal scale of investment without entailing an excessive risk of failure, but such efficient allocation of funding requires intermediaries with many connections. Moreover there is an adverse effect on the equilibrium family. With a cap $\bar{Z}$ on number of bank connections ($\bar{Z} < k_I$), no bank would be able to lend to every $I$ bank. When multiple core-periphery equilibria are possible, this policy shifts the family of equilibria toward larger cores, so that every core bank is directly or indirectly connected to as many $I$ banks as possible, such that in expectation all core banks offer the same rate of return, as high as possible. A larger core increases the cost in the event of failure, and is particularly costly when investment outcomes are highly

[42]See CFTC/SEC (2012) and Stroock Special Bulletin (http://www.stroock.com/SiteFiles/Pub1201.pdf) for more detail.

correlated. This is particularly relevant for crisis of 2007-08.

# 7 Conclusion

I develop a model of the financial sector in which endogenous intermediation among debt financed banks generates excessive systemic risk. The central feature of the model is that financial institutions have incentives to capture intermediation spreads through strategic borrowing and lending decisions. By doing so, they tilt the division of surplus along an intermediation chain in their favor, while at the same time reducing aggregate surplus. I show that a core-periphery network – few highly interconnected and many sparsely connected banks – endogenously emerges in my model. The network is inefficient relative to a constrained efficient benchmark since banks who make risky investments "overconnect", exposing themselves to excessive counterparty risk, while banks who mainly provide funding end up with too few connections.

This paper suggests that explicitly modeling how banks' incentives to capture higher returns interacts with intermediation, which is a necessary mechanism to allocate liquidity within the financial system, is able to jointly explain the stylized facts about global structure of interbank network, interbank interconnectedness, and gross and net exposures among financial institutions. Moreover, by providing sharp predictions about sources of inefficiency in interbank relationships, the model contributes to the heated policy debate on how to regulate the financial market.

Finally, the model can be extended to incorporate other interesting aspect of the financial system. A fruitful avenue of future research is to incorporate liquidity risk, and how it can lead to "unraveling" of the interbank network. Moreover, one can think of specialization by intermediaries in the context of the model. Whether banks should specialize, and if so in which activities, has long been an issue of debate among economists. The current model cannot answer this question because it takes the existence of different types of banks (and their numbers) as given. Assessing how positive and normative trade-offs change, when specialization is allowed to be a bank's choice variable, is worth further investigation.

# 8 Appendix

## 8.1 Proofs

will present all the proofs using $\alpha$-rule as it greatly simplifies the exposition of the proof. It is easy to verify that all the proofs go through with any rule for surplus division which satisfies the properties of section 4.1. The general proofs are available upon request.

I will first show that the our structures in 9 are the only possible equilibria of the economy with four banks and then prove proposition 1.

**Lemma 3.** *Network structures depicted in Figure 3 are the only possible equilibria with four banks.*

*Proof.* Any structure in which an $NI$ does not lend to any other bank is trivially not an equilibrium. Aside from those, all the feasible structures with four banks are depicted in 9. Each structure consists of the four banks and credit lines among them depicted in black.

Finally, the deviations which rule out the other structures (9d, 9g and 9h) are depicted as red or crossed out edges. For instance in 9h, $NI_1$ has two units pledged to him but is only lending to a single $I$ bank. $NI_1$ and $I_2$ strictly prefer to jointly deviate together. $NI_1$ saves on the intermediation rent payed to $I_1$ when only $I_2$ has an investment opportunity, while post deviation $I_2$ gets to invest 50% of time when both $I_1$ and $I_2$ get the investment opportunity and prior to deviation $I_2$ would not invest.[43] $e_{I_1I_2}$ is removed since nothing is ever lent over that credit line and we move from 9h to 9a.

In 9c, adding the $e_{I_1I_2}$ and $e_{I_2I_1}$ is not always a viable deviation because if $\alpha(1-\alpha)X < (1-p)V_I$, in the resulting network, lending over $e_{I_iI_j}$ always violates the participation constraint of $I_i$, so it would happen with probability zero. So this is not a valid coalitional deviation and 9c is a possible equilibrium. □

*Proposition 1.* As mentioned in the text, I will focus on the space of parameters in which a single $NI$ to $I$ lending is both socially desirable and individually rational, i.e. 1, 2 and 3 are satisfied. Following the notation in the text, let $\kappa = \frac{\alpha(1-\alpha)}{(1-p)V_I}$ and $\tilde{\kappa} = \max\{\alpha, (1-\alpha)\frac{V_{NI}}{V_I}\}$. 2 and 3 imply $\kappa \geq \tilde{\kappa}$ is the relevant range of parameters. Assume the economy is in 3d. The face values of debt are set as explained in section 3.1. In expectation, an $I$ bank and $NI_2$

[43] If the bargaining rule is such that both final lender and initial borrower save on intermediation rents when an intermediator is removed the second part of argument is redundant as $I_2$ also saves on intermediation rents when only he gets the investment opportunity and lending goes through $I_1$. However, in $\alpha$-rule borrower does not care for the source of funds so the second part of argument is necessary.

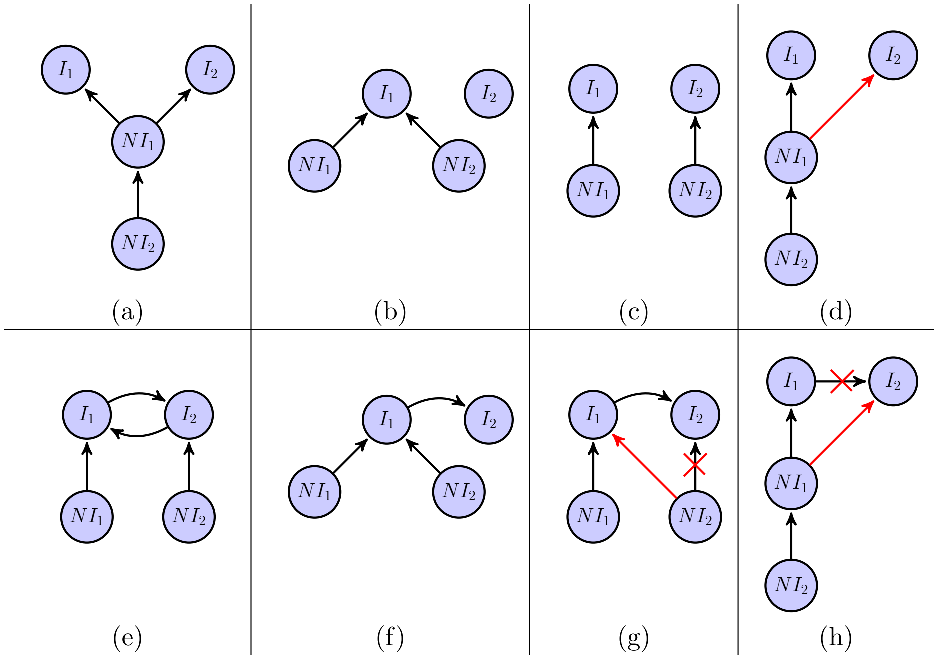



Figure 9: Feasible lending structures for an economy with two $I$ and two $NI$ banks. The black edges are in the feasible structure. The red and crossed-out edges are the deviations which rule out each particular structure as an equilibrium.

get the following, respectively:

$$\mathcal{V}_I^d = (1-q)^2 V_I + q^2[p(V_I + R - D)] + q(1-q)[p(V_I + 2(R-D))] + (1-q)q[p(V_I + D - D_1)]$$
$$\mathcal{V}_{NI_2}^d = (1-q)^2 V_{NI} + q^2[p(V_{NI} + D) - 1] + q(1-q)[p(V_{NI} + D) - 1] + q(1-q)[p(V_{NI} + D_1) - 1]$$

Now consider 3a:

$$\mathcal{V}_I^a = (1-q)^2 V_I + q^2\big[\frac{1}{2}V_I + \frac{1}{2}[p(V_I + 2(R-D))]\big] + q(1-q)[p(V_I + 2(R-D))] + (1-q)qV_I$$
$$\mathcal{V}_{NI_2}^a = (1-q)^2 V_{NI} + q^2[p(V_{NI} + D_1)) - 1] + 2(1-q)q[p(V_{NI} + D_1) - 1]$$

where I have substituted $D$ for $D_2$. Note that $D - D_1 = \alpha(1-\alpha)X$ and it represents the intermediation spread. Substitute $D$ and $D_1$ and compare what either bank gets in 3a and 3d to see that $NI_2$ always prefers to deviate to 3d while $I$ bank would deviate if:

$$\frac{\alpha(1-\alpha)X}{(1-p)V_I} > 1 + \frac{q}{2(1-q)}.$$

Let $\bar{\kappa} = 1 + \frac{q}{2(1-q)}$ and take the joint deviation of the two $I$ banks along with $NI_2$ to see that 3a is not an equilibrium if $\kappa > \bar{\kappa}$.

Now is 3d an equilibrium when $\kappa < \bar{\kappa}$? Counter intuitively, the answer is yes. Although both $I$ bank prefer to deviate back to 3a, they need both $NI$ banks to join the deviation and no $NI$ bank agrees to be a leaf who is always intermediated, when in the current structure he gets to lend anytime there is an investment opportunity and with positive probability he gets un-intermediated rent. 3d ceases to be an equilibrium when intermediation rents do not cover the cost of default anymore and each $I_i$ would prefer to unilaterally break $e_{I_iI_j}$ link. This happens when $\kappa < 1$. Finally, is 3d an equilibrium if $\kappa > \bar{\kappa}$? Yes since none of the $I$ bank can improve on what either $NI$ bank gets in this structure, so there is no way to convince $NI$ banks to join any deviation.

Now assume the economy is in 3e. $I_1$, $I_2$ and each $NI$ bank receive:

$$\begin{aligned}
\mathcal{V}^e_{I_1} &= (1-q)^2V_I + q^2[p(V_I + 2(R-D))] + q(1-q)[p(V_I + 2(R-D))] \\
&\quad + (1-q)q[p(V_I + 2(D-D_1))] \\
\mathcal{V}^e_{I_2} &= (1-q(1-q))V_I + q(1-q)[p(V_I + 2(R-D))] \\
\mathcal{V}^e_{NI} &= (1-q)^2V_{NI} + (q^2 + q(1-q)[p(V_{NI} + D_1)) - 1] + (1-q)q[p(V_{NI} + D_1) - 1]
\end{aligned}$$

In 3b $I_1$and each $NI$ get:

$$\begin{aligned}
\mathcal{V}^b_{I_1} &= (1-q)V_I + q[p(V_I + 2(R-D))] \\
\mathcal{V}^b_{NI} &= (1-q)V_{NI} + q[p(V_{NI} + D) - 1]
\end{aligned}$$

Although $NI$ does not want to deviate from 3b to 3e but $I_1$ will unilaterally deviate and break $e_{I_1I_2}$ link if that increases his expected profit, which happens if $\kappa < \frac{1}{2}$.

Next consider 3b. Two type of deviations is perceivable: first, the two $I$ banks jointly deviate and add $e_{I_1I_2}$, which happens when $\kappa > \frac{1}{2}$.[44] Second, one might think that even if the above deviation cannot happen i.e. $\kappa < \frac{1}{2}$, the two $NI$ banks can jointly deviate with $I_2$ to go to 3a. This latter deviation requires $\alpha > \frac{1}{2-q}$ and $\kappa > \underline{\kappa} \geq \tilde{\kappa} > 0$. However, we know that $\alpha < \kappa < \frac{1}{2}$ and $\frac{1}{2-q} > \frac{1}{2}$, so this deviation is never feasible. In other words, when $\tilde{\kappa} < \frac{1}{2}$, $\underline{\kappa} < 0$. Finally, note that for 3b to ever be an equilibrium it should be that $\tilde{\kappa} < \frac{1}{2}$, so $\alpha < \frac{1}{2}$ and $(1-\alpha)\frac{V_{NI}}{V_I} < \frac{1}{2}$ which implies $V_{NI} < V_I$. Finally, consider 3c. The difference with 3b is that now both banks lose scale when they have an investment opportunity, and they would ex-ante be better off adding $e_{I_1I_2}$ and $e_{I_2I_1}$ even when $\kappa < 1$. However, this is not a viable deviation when $\kappa < 1$, because in the interim period, when only investment opportunity $i$

[44] Deviating to 3d is also possible but the former deviation is viable whenever the latter is, so there is no need to consider the latter.

is realized, lending over $e_{I_j I_i}$ violated the participation constraint of $I_j$ and will not happen, so $e_{I_j I_i}$ is never traversed and the above is not a viable deviation when $\kappa < 1$. The second candidate deviation in case of 3b is ruled out by the same argument as above. There is a third possible deviation: $NI_2$, $I_2$ and $I_1$ jointly deviate, break $e_{NI_2 I_2}$, and add $e_{NI_2 I_1}$ and $e_{I_1 I_2}$. The first necessary condition is that adding $e_{I_1 I_2}$ must be a viable deviation, which requires $\kappa < \frac{1}{2}$. If so, $I_1$ and $NI_2$ gain. $I_2$ incentives are ambiguous because in 3a he does not get to invest when $I_1$ get an opportunity, but gets to invest 2 units when $I_1$ does not.

$$\mathcal{V}^c_{I_2} = (1-q)V_I + q[p(V_I + (R-D))]$$

For the latter deviation to happen we need $\mathcal{V}^c_{I_2} < \mathcal{V}^e_{I_2}$, which holds if $q < \bar{q} = \left(2 - \frac{(1-p)V_I}{(1-\alpha)X}\right)^{-1}$. Note that $\frac{1}{2} < \bar{q} < 1$. □

*Proposition 2.* For every cut $C$, parents of node $b$ in $\hat{G}$ are exactly the banks to whom $b$ is lending to $G$. By construction of $C$, these parents are all included on the source side of $C$. So and node who is on the sink side of $C$ only lends to banks on the source side. The total amount of funding which flows into any set of nodes cannot be more that total funding raised by their direct and indirect lenders. The total flow is by construction Size($C$) and total funding raised at direct and direct lenders is $X_S(C)$, which is the number of banks on the sink side of $C$. So Size($C$) $< X_S(C)$. When only leaf nodes are on the sink side, every edge in the cut set on a shortest path, and each leaf node has exactly one unit of funding, so the inequality holds with equality.

For the second inequality, note that every edge with one end in $I in \mathbb{I}_R$ and the other in $\mathbb{NI}$ is on the shortest path of some $NI$ to $\mathbb{I}_R$,[45] so there is at least one unit lent over such edge in $G$. By construction the sum of flows of funding on such edges is Size($C_o$) which I just argued is at least as large as the number o such edges. □

*Lemma 1.* Consider a bank $b$ who lends along a longest chain of length $l_{max}$ with probability non zero.[46] There is no diversifiation so if the ultimate borrower $I$ fails every bank who has lent to him through any chain fails. As a result when bank $NI$ lends directly or to indirectly to a bank $I$ then he fails with probability $(1-p)$ regardless of the length of the intermediation chain. However, when he lends through his longest chain of length $l_{max}$ in expectation he gets $\alpha^{l_{max}}X$. As a result $l_{max}$ is the largest number for which $b$'s participation constraint is not violated, which means $\alpha^{l_{max}}X \geq (1-p)V_{NI}$ and $\alpha^{l_{max}+1}X < (1-p)V_{NI}$. □

[45] It is certainly on the shortest path of the $NI$ at the end point, and maybe on the shortest path of others.

[46] Note that $b$ can lend over shorter paths to other banks $I$ as well.

*Theorem 1.* I will show that there is no feasible deviation for the relevant set of parameters. Let $\mathbb{C}(G)$ and $s$ denote the core and the size of the core, respectively, so there are $s$ $I$ banks in $\mathbb{C}(G)$ and $k_I - s$ out of the core. First consider the unilateral deviation of $I_1 \in \mathbb{C}(G)$. Note that with sufficiently many peripheries (as described in the statement of the theorem), if an $I$ lends to one other $I$ he would lend to as many $I$'s as he can, since everything is linear; and similarly if he drops a lending he drops every lending. So $I_1$'s relevant unilateral deviation is to drop all of his links to $I$ banks and stop intermediating. That is the case if intermediation rents that $I_1$ captures is not sufficient to cover his cost of default. With a core of size $s$, the division of peripheries which maximizes the profit of the worst-off member of the core is the equal division of $NI$ peripheries, so that each $I \in \mathbb{C}(G)$ gets $\frac{k_{NI}}{s}$ lending to him. So $I_1$ deviates if $\frac{k_{NI}}{s}\alpha(1-\alpha)X < (1-p)V_I$ which determines a lower bound on $\kappa$: $M_s = \frac{s}{k_{NI}}$.

Next, consider other possible deviations. The first coalition consists of only $I \in \mathbb{C}(G)$. Each $I$ who is in the core has maximum possible lending relationships so $I$'s at the core can not form a blocking coalition alone. Second, there can be a coalition of a (proper) subset of $I$'s in the core $\mathbb{C}_D$, and $NI$ banks lending to $I \in \mathbb{C}(G) \setminus \mathbb{C}_D$. In the current network, every $NI$ gets an expected return of $\alpha X$ with probability $q$ and $\alpha^2 X$ with probability $(1-q)(1-(1-q)^{k_I-1})$, and every single lending generates positive expected net profits (net of cost of default), so this is the maximum possible expected profit any bank can get without having any funds pledged from the interbank network. Simply becoming a periphery to a different core bank does not increase this payoff, so this is not a valid blocking deviation either.

Third, can a combination of $I$'s outside the core and $NI$'s form a profitable deviation? With the exact same argument as the last paragraph there is no such feasible deviation because it is not possible to make any $NI_j$ better off than what they are without making some $NI_k$ worse off (peripheral to $NI_j$). In this case, it is not even possible to make them as well of as before because the $I \in \mathbb{C}(G)$ bank(s) whose peripheral $NI$'s are part of the suggested deviation never agree to join the deviation and add links to borrow from the $I$ banks who are part of the suggested deviation (currently out of the core). So $NI$ banks who join such deviation would get intermediated spreads strictly less often that current structure (and the exact same unintermediated spreads), so they would be strictly worse off.

Forth, can $I \notin \mathbb{C}(G)$ deviate alone? It cannot add any links, and only loses by severing links, so there is no such deviation either.

Finally, can (a subset of) $NI$'s jointly deviate without any $I$'s in the coalition? Again the answer is no, for the following reason: Any such deviation implies that there is some $NI$ at distance 2 to his closest $I$ bank without any improvement in probability of being involved in the investment opportunity, which will be rejected by that $NI$.

The converse is simple. Assume $\kappa < M_s$. Then $\frac{k_{NI}}{s}\alpha(1-\alpha)X < (1-p)V_I$. Moreover, in any $s$-core network, at least one of $I \in \mathbb{C}(G)$ has $\frac{k_{NI}}{s}$ or less peripheries. This $I$ bank would unilaterally deviate and severe all his potential lending contracts to all other $I$ banks and strictly increase his expected surplus. □

*Proposition 3.* The proof is done in two steps. First I show that given any network $G$, realizations $\mathbb{I}_R$, and $\{R_k\}_{k\in\mathbb{I}_R}$, and face values of debt $\{D_{ij}\}_{i,j\in\mathbb{N}}$ and $\{D_i^h\}_{i\in\mathbb{N}_F}$ set at date $t = 1$, the system of interbank repayments (6) has a unique solution. This part of the proof is very similar to that of Acemoglu et al. (2015), proposition 1. The proof proceeds in multiple steps. First define the total liabilities of bank $i$ to bank $j$ by multiplying the per-unit payment by number of units lent and then define the share of each bank $j$ in bank $i$ liabilities. Then I define an appropriate mapping function $\Phi(.)$ which maps the min of partial and full payments to itself. It is straight forward to show that this mapping is a contraction which maps a convex and compact subset of Euclidean space to tself. As a result by Brouwer fixed point theorem, this contraction mapping has a fixed point which is the set of feasible interbank face values of debt and their relevant partial payments. For detail of generic uniqueness see Acemoglu et al. (2015).

Next, I focus on network formation stage, and show that (at least) one of following three networks is an equilibrium for any parameter set: smallest member of the core-periphery family (single-$I$-core network), the star structure with an $NI$ core ($NI$-star network), or a structure where every $NI$ banks lend to a (potentially multiple) $I$ bank(s) but $I$ banks are not connected to each other (island network). Assume the $NI$-star is not an equilibrium. Either $k_{NI}$ times intermediation spread is larger than $(1-p)V_I$ (case 1) or it is smaller (case 2). The single-$I$-core is an equilibrium in case 1 (proof of theorem 1). Now consider case 2. Since $NI$-star is not an equilibrium, there is a coalitional deviation to block it. The deviation cannot be only breaking links since every banks is getting strictly positive expected net surplus from every transaction at $t = 1$, and solely breaking the link gives it zero net surplus. So the deviation involves adding links. For a peripheral $NI$ to deviate, he needs to get strictly closer than one intermediary away, to at least one $I$ bank, as in $NI$-star he is one-intermediary away from every $I$. So any deviation requires (at least) adding a link between a peripheral $NI_j$ and one of the $I$ banks, $I_i$.

Consider a potential deviation which is only $NI_j$ breaking his link from the core $NI$ and adding a link to $I_i$. In this deviation, $NI_j$ trades off the spread he had to always pay the core $NI$ with the lower probability of getting it only when $I_i$ receives an investment opportunity. There are two possible cases: when this deviation is profitable for $NI_j$ (case 2-1) and when it is not (case 2-2). First consider the former. Assume we start in the island network where every $NI$ bank lends to $I_i$ (single island). As we are in case 2-1, $NI$ banks have no incentive to deviate and become peripheral to one of the $NI$s, and (at best) create the $NI$-star network in order to get the lower, intermediated rate of return, more often. $I_i$ has no incentive to start intermediating as we are in case 2. The only remaining deviation is if (a subset of) other $I$'s deviate with (a subset of) $NI$'s and create a multi-core structure where $I_i$ is completely left out. Note that $I_i$ would not agree to be part of any deviating coalition. In the current structure he gets all the funding when he has a project and he is not willing to intermediate, so he cannot be better off than what he is in any other network). This structure is preferred by $NI$'s because they get the same high rate that they get in the single island, plus they sometimes get an intermediated rate of return, so they would be willing to join such deviation. However, any link between two $I$ banks, $e_{I_l I_k}$, will never be traversed because it is not individually rational for an $I$ bank ($I_j$) to intermediate, which rules out this latter class of deviations. So the single-island network is an equilibrium in case 2-1.

Finally consider case 2-2. $NI_j$ is only willing to deviate if he becomes peripheral to $I_i$ who himself has a potential lending relationship to at lease some other $I_j$. However by the exact same argument as above, such deviations are rules out because traversing any link $e_{I_i I_k}$ violates individual rationality of $I_i$ with probability one, so such links cannot be added in a coalitional deviation. So no $NI_j$ bank would ever join a coalition, case 2-2 never happens, and $NI$-star is an equilibrium itself, which completes the existence proof.

□

*Theorem 2.*

**Efficiency.** First note that in this structure feasibility as well as the participation constraint of every bank are satisfied. Regardless of which bank receives the investment opportunity, all the funding will be channeled to some investment opportunity. Moreover, since every $NI$ bank is lending to all $I$ banks only through the same common intermediator, maximal concentration of risk is achieved. In other words, when multiple $I$ banks receive investment opportunities, one and only one of them invests, which given the no diversification assumption 3 is welfare enhancing since it concentrates risk as much as possible and saves on expected cost of default of some $I$'s, while reaching the same scale of investment. Finally, for any realization of investment opportunities, aside form the single $I$ bank who does the investment, every other bank with a realized lending and/or borrowing relationship provides funding for the investment, so removing him from the set of active lenders decreases the scale of investment by one while also decreasing the expected cost of default by $(1-p)V_{NI}$. By condition 1 the former is larger, so this removal will be welfare destroying.

**Tiered Network.** For any $(NI_j, I_i)$, every path from $NI_j$ to $I_i$ goes through $NI_c$, so finding $SP(NI_j, I_i)$ reduces to finding $SP(NI_j, NI_c)$. As a result, any lending that $NI_j$ does happens over his shortest path(s) to $NI_c$, and any potential lending of $NI_j$ who is not part of (one of) $SP(NI_j, NI_c)$ is traversed with probability zero, and removed from the graph. So all the paths from $NI_j$ to $NI_c$ have the same length, $dist(i, j)$. So the socially efficient network is a tiered network.

**Efficient Network Not Equilibrium.** I will choose appropriate bounds $\bar{K}$ and $\bar{M}$, and an appropriate group of $NI$ banks, $\mathbb{L}'$ such that $\mathbb{L}'$ and $\mathbb{I}$ form a blocking coalition.

Let $\hat{G} = (\hat{V}, \hat{E})$ be the subgraph consisting of all the $NI$ banks and the edges among them in any efficient network $G$. Also, let $\mathbb{L}$ denote the collection of leaf nodes in $G$. By Lemma 1, the length of the longest intermediation chain in $\hat{G}$ is $l_{max} - 1$. Let $\tilde{G} = (\tilde{V}, \tilde{E})$, $\tilde{V} = \hat{V} \setminus NI_C$ and $\tilde{E} = \hat{E} \setminus \{e_{iNI_c}\}$ $\forall i \in \tilde{V}$ , and $\tilde{G}$ is undirected by removing all the directions of edges in $E$. $\tilde{G}$ is a collection of connected components. For any fixed number of leafs $z_l$, there is only finitely many possible connected component which has $z_l$ leaf nodes (because the length of each chain is limited). As a result, fixing $k_I$, for sufficiently large $k_{NI}$, $k_{NI} > \bar{K}_1$, there will be at least $k_I^2$ leafs in $G$, i.e. either there are many connected components in $\tilde{G}$ with few leafs or some connected components with many leafs, or a combination of both. Let $\bar{K} = \bar{K}_1 + k_I$.

Let

$$\bar{M} = \frac{1}{k_I}\Big(1 + \frac{q}{1-q}\frac{k_I - 1}{k_I}\Big) \tag{8}$$

Choose $k_I^2$ of leaf $NI$ banks ($NI \in \mathbb{L}$) , and call it $\mathbb{L}'$. Then there is a joint deviation by all $NI \in \mathbb{L}'$ and all the $I$ banks: groups of $k_I$ of $NI \in \mathbb{L}'$ banks lend to each $I$ bank, and all $I$ banks start lending and borrowing from each other. The deviating $NI$ banks are clearly better off.

Next consider the $I$ banks. For any realization of investment opportunities, part of the surplus is generated by resources provided by $NI \in \mathbb{L}'$ and the rest by $NI \notin \mathbb{L}'$. Each $I$ bank increases his share of surplus of the former part, which is insured by the first term on the right hand side of 8, i.e. the spreads that and $I$ captures by intermediating $k_I$ banks when he does not have an investment opportunity, compensates him for his extra cost of default. As for the latter part, by definition of $\bar{K}$ after the deviation there are still more than $k_I$ $NI$ banks in $G$, so $NI_c$ keeps all of his edges to all $I$ banks. When $NI \in \mathbb{L}'$ deviates, his pre-deviation potential borrower, $\hat{N}I$ might lose one potential lending due to feasibility, but since all of his intermediation chains to $\mathbb{I}$ have the same length, there is no change in division of surplus generated by the remaining units, i.e. pre and post $dist(\hat{N}I, \mathbb{I})$ are the same.

Finally, we need to consider change in default cost of $I$ banks. In $NI$-star network all the lending is done through $NI_c$, which corresponds to the highest level of concentration as argued previously: When more than one investment opportunity is realized only one $I$ bank invests but at a high scale. Since no diversification is allowed this enhances each $I$ bank's expected surplus because his expected investment remains the same while he fails less often. Maximum gains to concentration is attained when all $I$ banks receive an investment opportunity, in which case each only incur a $\frac{1}{k_I}(1-p)V_I$ default cost for investing, as opposed to $(1-p)V_I$. This put a upper-bound of $\frac{k_I-1}{k_I}(1-p)V_I$ on how much an $I$ bank would loose in default costs, by joining the deviation, when he does receive an investment opportunity. An $I$ bank receives an investment opportunity with probability $q$ and receives intermediation spreads with probability $(1-q)$, so we should have $(1-q)k_I\alpha(1-\alpha)X > q\frac{k_I-1}{k_I}(1-p)V_I$, which is insured by the second term on the right hand side of 8. So the proposed coalition is a blocking coalition and $G$ is not an equilibrium.

□

*Theorem 3.*

**Theorem 1.** Note that the only place in proof of Theorem 1 where the value of the spreads are used is in determining the cut-offs. Analogous to that proof, in an $s$-core core-periphery network, the min expected profit of a core bank $I$ who intermediates is $\frac{k_{NI}}{s}\mathcal{L}(2;3,X)$, which should cover his cost of default $(1-p)V_I$, which in turn determines the lower bound $M_s = \frac{s}{k_{NI}}$.

Every other deviational argument goes through directly from the properties of the rule for division of surplus defined in section 4.1: Adding to the chain weakly decreases the share of every member of the chain and strictly decreases the share of initial lender, which implies that everything else equal, every $NI$ bank unambiguously prefers to be lending to $I$ bank through as few intermediaries as possible. Moreover, the rule is anonymous and holding the surplus fixed, does not depend on the identity of banks in the chain.

**Theorem 2**. Similar to the previous part, the only place where the level of intermediation spreads are referenced are exactly the same as Theorem 1, so the same argument as above applies. There can be an extra force here from the borrower, when borrower share of surplus also decreases as a function of the length of the intermediation chain. Let $\mathbb{NI}_i$ denote the set of $k_I$ $NI$ banks who lend to $I_i$ in the coalitional deviation by $\mathbb{L}'$ and $\mathbb{I}$. $I_i$ bank requires even less intermediation spreads to cover his extra expected cost of default due to intermediation, because he receives some extra share of surplus of own investment, by having $\mathbb{NI}_i$ lending directly to him rather than indirectly, through $NI_c$. Moreover, the intermediation chain to $I_i$ from any other $NI \in \mathbb{L}' \setminus \mathbb{NI}_i$ is at least as short as before: they are one intermediary away post deviation and they were at least one intermediary away pre-deviation. For $NI \notin \mathbb{L}'$ the proof is the same as 2, as the socially efficient network is tiered.

The above arguments all rely on anonymity, i.e. the division of surplus does not depend on the identity of banks in the chain. Appendix 8.2.3 provides an example where the rule does depend on the identities by incorporating the default costs into $\mathcal{L}$, and proves similar results.

□

*Lemma 2.* $j_1$ is connected to at least $z_2$ of $I \in \mathbb{I}$, through "pointwise" weakly shorter paths, as defined in the lemma. Call this set $\mathbb{I}_{j_2}^{z_2}$. When any $I \in \mathbb{I}_{j_2}^{z_2}$ is in $\mathbb{I}_R$, the expected rate that $j_1$ (and consequently any lender to $j_1$) receives on their (indirect) lending is independent from distance of any $I \notin \mathbb{I}_{j_2}^{z_2}$ but $I \in \mathbb{I}_R$ to whom $j_1$ is connected. As a result the expected return that $j_1$ (and his lenders) receive conditional on realization of an investment opportunity at $I \in \mathbb{I}_{j_2}^{z_2}$ is larger that what $j_2$ (and his lenders) receive when what of the $I$ banks $j_2$ is connected to is in $\mathbb{I}_R$. The above two events happen with exactly same probability (equal to at least one out of $z_2$ binomial random variables being one). Conditional the former event not happening $j_1$ still earns positive rents when $I \in \mathbb{I}$
$\mathbb{I}_{j_2}^{z_2}$ is in $\mathbb{I}_R$ which more than covers his expected cost of default[47], while $j_1$ earns no rents. So in expectation over all realizations of investment opportunities, $j_1$ and his lenders are better off than $j_2$ and his lenders, respectively. □

*Proposition 4.*

**Equilibrium.**

All the references to figures in this proof are to Figure 8.

First, solve for the face values payable to $NI$ peripheries, $D_{11}$ and $D_{22}$. Failure probability of $I_2$ determines the face value payable to its $NI$ peripheries to be $D_{22} = \frac{1+\alpha X}{p}$. As a result, the only remaining equilibrium object is $D_{11}$. $D_{11}$ depends on the share of surplus

[47]Because I assume participation constraint must be satisfied for each realization of lending.

that goes to a direct lender, the endogenous probability of (partial) repayment by $I_1$, as well as $Y_1$ and $Y_2$.

The structure of equilibrium and the face value of debt from $I_1$ to his $NI$ peripheries are jointly determined in equilibrium, based on which of the following regions the total liabilities of the net lender $I_1$ lies in:

$$\begin{cases} Y_1 D_{11} \geq \frac{Y_1+Y_2}{2}R & I_1 \text{ survives with probability } p^2 \\ \frac{Y_1-Y_2}{2}R \leq Y_1 D_{11} < \frac{Y_1+Y_2}{2}R & I_1 \text{ survives with probability } p \\ Y_1 D_{11} < \frac{Y_1-Y_2}{2}R & I_1 \text{ survives with probability } 1-(1-p)^2 \end{cases}$$

First note that liabilities can be high for two reasons: either $\alpha$ is high so that a large share of surplus goes to the lenders, or default probability of borrower is high. In the first region above liabilities are so high that unless both assets pay, $I_1$ fails. In the middle region $I_1$ fails if his asset investment fails and survives otherwise, and in the last region $I_1$ survives unless both assets fail. In the first two regions there will be partial payments. Let $\hat{D} = D_{22} = \frac{1+\alpha X}{p}$, which is the face value of debt which corresponds to the case where a bank fails exactly when his own investment fails.

**Region One ($Y_1 D_{11} > \frac{Y_1+Y_2}{2}R$).**

$$p^2 D_{11} + p(1-p)\frac{Y_1+Y_2}{2Y_1}R + (1-p)p\frac{Y_1-Y_2}{2Y_1}R = \alpha X + 1$$
$$D_{11} = \frac{1}{p}(\hat{D} - (1-p)R)$$

In order for the total liabilities with the above face value to be in region one it must be that

$$\frac{Y_2}{Y_1} < \frac{2}{pR}\hat{D} - \frac{2-p}{p}$$

**Region Two ($\frac{Y_1-Y_2}{2}R \leq Y_1 D_{11} < \frac{Y_1+Y_2}{2}R$).**

$$pD_{11} + (1-p)p\frac{Y_1-Y_2}{2Y_1}R = \alpha X + 1$$
$$D_{11} = \hat{D} - (1-p)\frac{R}{2}(1 - \frac{Y_2}{Y_1})$$

In order for the total liabilities with the above face value to be in region two it must be that

$$\frac{Y_2}{Y_1} > \frac{2}{pR}\hat{D} - \frac{2-p}{p} \tag{9}$$

$$\frac{Y_2}{Y_1} > 1 - \frac{2}{R(2-p)}\hat{D} \tag{10}$$

**Region Three ($Y_1 D_{11} < \frac{Y_1 - Y_2}{2} R$).**

$$(1-(1-p)^2)D_{11} = \alpha X + 1$$
$$D_{11} = \frac{1}{2-p}\hat{D}$$

In order for the total liabilities with the above face value to be in region two it must be that

$$\frac{Y_2}{Y_1} < 1 - \frac{2}{R(2-p)}\hat{D}$$

Let $y = \frac{Y_2}{Y_1} \leq 1$ denote the ratio of the $NI$ peripheries of $I_2$ to $I_1$. The inequality holds because $I_1$ is assumed to have more peripheries. The two inequalities defined in 9 characterize the three regions in which $I_1$ fails with different probabilities; where each region characterizes the set of $(\alpha, y)$ for which the probability of $I_1$ failure is the same.

The two lines cross each other and zero, if they do so, at $(\bar{\alpha}, 0)$ such that

$$1 = \frac{2}{R(2-p)} \frac{1+\bar{\alpha}X}{p}$$

However, the two lines will not cross zero (and each other) at any $\alpha \geq 0$ if even at $\alpha = 0$ $I_1$'s own investment must survive for him to survive. This happens if

$$\frac{2}{pR}\frac{1}{p} - \frac{2-p}{p} > 0$$

Let $\bar{R} = \frac{2}{p(2-p)}$. The above inequality holds if

$$R < \bar{R} \tag{11}$$

This happens in panel 8b. Recall that $R > \frac{1}{p}$ for the project to be positive NPV. The intuition is that if the project is positive NPV but the upside is not sufficiently high, $I_1$ fails if its own project, i.e. its larger asset, does not pay off. In other words, there are different combinations of $(p, R)$ with the same NPV, that is, constant $pR$. $I_1$ prefers the combinations with higher $R$ because it provides $I_1$ with sufficient resources to be able to pay its lenders, even if only $I_1$'s smaller asset pays back. In this case $\bar{\alpha} < 0$.

In the left panel, 8a, $\bar{\alpha} > 0$. When $0 \leq \alpha < \bar{\alpha}$, $I_1$ bank prefers to have many peripheries to lie below the red line, which would imply an unbalanced core-periphery structure, while for $\bar{\alpha} < \alpha \leq 1$ it prefers to have similar number of peripheries as $I_2$ has, which will be a more balanced core-periphery structure.

So the equilibria in the two case defined by 11 should be studied separately. For now ignore the constraint that $\alpha$ should be such that intermediation rents are high enough so that either one or both of the $I$ banks agree to intermediate, i.e. ignore the participation constraint.[48]

When 11 does not hold, the two lines defines in 9 cross at $\alpha = \bar{\alpha}$ in 8a. Recall that peripheries of net borrower fail with probability $p$ and we need to consider incentives of $NI$s peripheral to net lender. These incentives are not necessarily aligned with that of the $I$ banks. $NI$ incentives about which $I$ bank to lend to is purely driven by their default probability, and are determined at $\alpha = 0$, as explained in the text. Here at $\alpha = 0$ there is a range of positive $y$ for which $NI$ banks (and $I_i$) survive as often as possible, i.e. unless both projects fail. So at those $y$'s $NI$'s will survive at higher values of $\alpha$ as well, since the (partial) payments they receive from $I_i$ only increases in $\alpha$.

To see this, consider two different economies; $L$ and $H$, with two different levels of $\alpha$; $\alpha_L = 0$ and $\alpha_H > \bar{\alpha}$.[49] Denote the $NI$ banks in economy $L$ and $H$, $NI_L$ and $NI_H$, respectively. First consider economy $L$ and assume $Y_1$ and $Y_2$ are such that $y$ lies below the solid red line. For this level of $y$, if at least one of the assets held by $I_1$ pays back (probability $(1 - (1 - p)^2)$), $NI_L$ peripheries of $I_1$ are payed back in full. They pay all of what they get to households[50], and they survive with probability $(1 - (1 - p)^2)$, the same probability as $I_1$ survives.

Now consider economy $H$. Here $I_1$ survives only if both of its assets pay back, that is, if both investments are successful, because its liabilities are too high. This happens with probability $p^2$. However, when $I_1$ fails it makes partial payments if either of his assets pay back. As a result, for every state of the world, what each $NI_H$ bank gets in the $H$ economy,

[48]Note that I have assumed participation constraint must be satisfied case by case. When only one bank get the investment opportunity diversification does not come in, so this argument does not affect the range of $\alpha$ for which either one or both $I$s are willing to intermediate. The final equilibria are the ones which are consistent with both sets of conditions.

[49]This example is purely for illustration, so ignore the fact that $NI_L$'s participation constraint is violated at $\alpha = 0$.

[50]Because $\alpha = 0$.

is at least as high as what each $NI_L$ bank gets in the $L$ economy. As $NI_L$ and $NI_H$ banks have the same expected liabilities, $NI_H$ cannot fail more often than $NI_L$. This implies that for each $(p, R, V_I, V_{NI})$, and each level of $y$, the probability of default for an $NI$ periphery of $I_1$, for any $\alpha$, is the same as probability of default of an $NI$ with $\alpha = 0$.

For $\alpha < \bar{\alpha}$, every $NI$ lenders of $I_2$ prefers to instead lend to $I_1$ and save on the expected cost of default. $I_1$ likes that too. So every $NI$ periphery of $I_2$ deviates to $I_1$ as long as $I_2$ has one periphery. If $I_2$ loses its last periphery, when both $I$ banks have an investment opportunity, even if $I_1$ lends to $I_2$ and $I_2$ invests, $I_2$ does not receive a share of his own investment's net surplus, because $I_1$ absorbs all the returns. However, $I_2$ still incurs the expected cost of default. As a result, participation constraint of $I_2$ is violated and $I_1 \rightarrow I_2$ will not happen when both banks have the investment opportunity. Consequently, $I_1$'s probability of default would rise to $p$, and $I_2$'s last periphery would be indifferent between deviating or not, which by definition of equilibrium implies it does not deviate.[51]

On the other hand, when $\alpha > \bar{\alpha}$, $I_1$ fails more often below the dashed blue line while $NI$ lenders to $I_1$ still fail less often. As a result, $NI$ peripheries of $I_2$ want to deviate and lend to $I_1$. Interestingly, $I_1$ does agree to this deviation although it increases its probability of default. The reason is that the return it gets from investing this extra unit, more than covers the incremental cost of default, $\alpha(1-\alpha)X > (1-p)V_I > p(1-p)V_I$.

The above argument requires a minor adjustment. Note that the 2-$I$ core-periphery equilibrium never features $y = 0$, instead $y = \frac{1}{k_{NI}-1}$, which must be in the Region Three at $\alpha = 0$ for the above argument to work. As a result $\bar{R}$ needs to be updated to adjust for this:

$$\bar{R} = \frac{2}{p(2-p)} z \tag{12}$$

where $z = \frac{k_{NI}-1}{k_{NI}-2}$. Note that $\bar{R} \rightarrow \frac{2}{p(2-p)}$ as $k_{NI} \rightarrow \infty$. Moreover, instead of $\bar{\alpha}$ there are two relevant thresholds, $\bar{\alpha}_l$ and $\bar{\alpha}_h$, one on each line defining the borders of the three regions, which replace $\alpha$

$$\bar{\alpha}_l = \Big(\frac{p(2-p)R}{2}(1 - \frac{1}{k_{NI}-1}) - 1\Big)(pR-1)^{-1}$$
$$\bar{\alpha}_h = \Big(\frac{pR}{2}(\frac{p}{k_{NI}-1} + 2 - p) - 1\Big)(pR-1)^{-1}$$

[51]The fact that $I_2$ remains with a single $NI$ periphery is simply because I assumed intermediation rents are high enough so that intermediating a single unit of funding covers $I$'s extra cost of default. If intermediating $c$ units is necessary to keep $I_2$ intermediating, then it will end up with $c$ peripheries.

Note that as $k_{NI} \to \infty$, $\bar{\alpha}_l \to \alpha$ and $\bar{\alpha}_h \to \alpha$.

In the region where 11 does not hold(with adjusted $\bar{R}$ defined in 12), if $\alpha < \bar{\alpha}_l$, then $I_1$ survives with probability $1-(1-p)^2$. If $\bar{\alpha}_l < \alpha < \bar{\alpha}_h$, then $I_1$ survives with probability $p$. If $\alpha > \bar{\alpha}_h$, then $I_1$ survives with probability $p^2$. So a small region is added in the middle for $I_1$. All $NI$s who lend to $I_1$ still survive with probability $1-(1-p)^2$.

Next consider the case where 11 holds. As a result, Region Three disappears. Here the realized return of the project, $R$, is so low that even at $\alpha = 0$, regardless of level of $y$, $I_1$ fails if its larger asset, namely, its own investment, does not pay back. However, depending on the level of $y$ and $\alpha$, $I_1$ may need its second asset to also pay back in order to survive. Specifically, if $\alpha$ is high $I_1$ survives only if both assets pay back.

Now consider default probability of $NI$ banks who are peripheral to $I_1$. Again, the relevant range of the parameters for $NI$ peripheries, to prefer one borrower to the other, is determined only at $\alpha = 0$, but for different reasons. First note that the highest (partial) payments that an $NI$ bank receives is at $\alpha = 1$, where $NI$ receives $R$ for the proportion of his portfolio invested in the successful project(s), and has to pay lenders who only have to break even, i.e. they in turn have $\alpha = 0$ effectively. This is the exact same problem that $I_1$ faces when his $NI$ lenders have $\alpha = 0$.

Two different scenarios must be considered separately. First, can $NI$ fail only with probability $1-(1-p)^2$, given that we know this is not possible for $I_1$? As I argued, the best an $NI$ can do is at $\alpha = 1$, and for him to survive unless the two projects fail we should have

$$\frac{1}{Y_1}\frac{Y_1 - Y_2}{2}R > \frac{1}{1-(1-p)^2}$$

which boils down to the boundary of Region three at $\alpha = 0$ as argued above, which we know is negative when 11 holds. So this case never happens (regardless of how often $I_1$ survives).

When $I_2$ survives with probability $\pi$, $NI$ does also survive with probability at least as high as $\pi$. So the only remaining case is when $I_1$ survives with probability $p^2$ but his peripheries survive with probability $p$.

Let $D_{1h}$ denote the face value of debt payable to households lending to an $NI$ bank peripheral to $I_1$. The trick is to realize that when $I_1$ fails, he pays all the proceeds from his project as partial payment, *as if* $NI$ has $\alpha = 1$, and when $NI$ fails himself he pays all of those proceeds to his households. As a result the equation which defines $D_{1h}$ boils down to the same equation which defines $D_{11}$ in Region two, at $\alpha = 0$:

$$pD_{1h} + (1-p)p\frac{Y_1 - Y_2}{2Y_1}R = 1$$

Which in turns implies that the boundary for this case is the same as the boundary in Region two at $\alpha = 0$, $\bar{y}$ in 8b. So in this case when $y > \bar{y} = \frac{2}{p^2 R} - \frac{2-p}{p}$, $NI$ peripheries of either $I$ bank are indifferent between moving around since they have no room to improve on their default probability. However, when $y < \bar{y}$, $NI$ peripheries of $I_1$ deviate to $I_2$ until $y \geq \bar{y}$. Such deviation pushes $y$ up and above $\bar{y}$. Any $y > \bar{y}$ is an equilibrium because $NI$ peripheries of $I_1$ has no incentive to deviate to $I_2$, because they fail with the same probability in both places.

Finally, one should consider $y = 0$, where only $I_1$ lends to $I_2$, separately. As long as intermediation rents are sufficiently high, $y = 0$ is also an equilibrium. The reason is that $NI$s would not benefit from any joint deviation with $I_2$ unless $I_1$ agrees to the deviation and adds the $e_{I_2 I_1}$ potential relationship, which would require $I_1$ to lose at least one of its peripheries to $I_2$, and $I_1$ does not agree to be part of such deviation even if it improves his survival probability, as explained above.

**Efficiency.**

I will show that in the range provided in the proposition, the 2-$I$ core periphery equilibrium is dominated by $NI$-star, and cannot be efficient. This does not necessarily means $NI$-star itself is efficient.

Consider $NI$-star, and let $NI_c$ be the $NI$ who lends to all $I$ banks. $NI_c$ survives either with probability $p^2$ or $1-(1-p)^2$ because his two assets are symmetric. Assume $NI_c$ fails only of both projects fail. So if each of his assets pay back, he must be able to pay his liabilities in full

$$\frac{k_{NI}}{2}\frac{\alpha X + 1}{p} \geq (k_{NI} - 1)\frac{\alpha^2 X + 1}{p(2-p)} + \frac{1}{p(2-p)}$$

The first term on right hand side is his total liabilities from other $NI$s assuming he pays back with probability $1-(1-p)^2$, and the second term to his households. With some algebra we get

$$k_{NI}\Big[\frac{2-p}{2}(\alpha X + 1) - (\alpha^2 X + 1)\Big] > -\alpha^2 X$$

This is a similar condition to what $I_1$ faces, with a few adjustments. Unlike $I_1$, total assets available to $NI_c$ when an investment survives is lower than full value, $R$. His liabilities are also lower, and are not fully symmetric. A sufficient condition for the above inequality is

$$\alpha^2 X + 1 < \frac{2-p}{2}(\alpha X + 1) \tag{13}$$

This is now very similar to the condition for $I_1$, except that both assets and liabilities decrease with $\alpha$, so for instance at $\alpha = 0$, $NI_c$ fail: his liabilities are low, but the same with his assets. It does not hold at $\alpha = 1$ either. So the corresponding quadratic equation has two roots, $0 < \hat{\alpha_l} < \hat{\alpha_h} < 1$, and the above inequality holds if $\hat{\alpha_l} < \alpha < \hat{\alpha_h}$.

In this case, every $NI$ survives with probability $1-(1-p)^2$. $NI_c$ diversifies the risk that $NI$s face very well, but not the risk that $I_1$ faces.

Note that

$$\alpha^2 X + 1 < \frac{p(2-p)}{2}\frac{(\alpha X + 1)}{p} < \frac{p(2-p)}{2}R$$

where the last inequality holds simply because face value paid to $NI_c$ is less than $R$ as final borrower gets positive share of surplus. As a result when 11 holds, the total assets are too low and $NI_c$ survives only if both asset pay.

When $\alpha > \hat{\alpha}_l$, all peripheral $NI$s can still survive with probability $1-(1-p)^2$ if their partial payment, when only one project pays off, is sufficiently large

$$\frac{\alpha^2 X + 1}{(k_{NI}-1)(\alpha^2 X + 1) + 1}\frac{k_{NI}}{2}\frac{\alpha X + 1}{p} > \frac{1}{p(2-p)}$$

The left hand side is increasing in $\alpha$, so there is a constant $\tilde{\alpha}$ such that for $\alpha > \tilde{\alpha}$ it holds.

Next I compare the difference between the expected loss in $NI$-star and core-periphery equilibria. Let $\Delta$ denote the difference, so $\Delta > 0$ implies that the core-periphery network is inefficient (but not the reverse).

- $R > \bar{R}$, $\hat{\alpha_l} < \alpha < \hat{\alpha_h}$, $\alpha > \bar{\alpha}_l$

$$\Delta = q^2[(1-p)pV_{NI} + \mathbb{1}[\alpha > \bar{\alpha}_h]p(1-p)V_I] + 2q(1-q)(1-p)V_I > 0$$

  The first term is when there are two investment opportunities. In $NI$-star, all $NI$s and one $I$ survive if only one project pays off. In the core-periphery if only project of $I_1$ pays off, $I_2$ and his periphery fail. If $\alpha > \bar{\alpha}_h$, $I_1$ fails unless both projects payoff. The last term corresponds to states where only one $I$ get the investment. So the $NI$-star is strictly better.

- $R > \bar{R}$, $\hat{\alpha_l} < \alpha < \hat{\alpha_h}$, $\alpha < \bar{\alpha}_l$

$$\Delta = q^2[(1-p)pV_{NI} - p(1-p)V_I] + 2q(1-q)(1-p)V_I$$

  Here if only $I_1$'s project survive there is a gain of one extra $NI$ being saved in $NI$-star, but if only $I_2$'s project survive there is a cost of $I_1$ failing in $NI$-star. Intermediation costs are the same. If $q < \hat{q} = \frac{V_I}{V_I + 0.5p(V_I - V_{NI})}$, $\Delta > 0$.

- $R < \bar{R}$: Here project payoff in case of success is low, so $NI$-star does poorly in terms of diversification.

$$\Delta > q^2[-p(1-p)k_{NI}V_{NI}] + 2q(1-q)(1-p)V_I$$

  the first inequality comes from the fact that there are equilibria here where $I_1$ fail unless both projects payoff, in which case $NI$-star saves on that. However, no matter which project fail all $NI$s fail, which is not the case in the core-periphery equilibrium as $NI$ banks reorganize themselves to improve on survival probability. This is the first term on right hand side. A sufficient condition for the above is

$$q < \hat{q} = \frac{V_I}{V_I + 0.5\frac{p}{1-p}k_{NI}V_{NI}}$$

□

## 8.2 Extensions and Examples

### 8.2.1 Numerical Example for Economy with Four banks

Let $q = 0.5$, $\alpha = 0.4$, $V_I = 10$, and $V_{NI} = 5$. We have $\bar{\kappa} = 1.5$ and $\alpha < \frac{1}{2-q}$. Let $p = 0.8$ and $R = 20$. Then $\kappa = 1.8 > 1.5$, so both 3d and 3e are equilibria. Note that in 3d, either $I$ bank would like to deviate jointly with the $NI$ periphery of the other $I$ bank and deviate to 3e, but because $NI$ banks are equally well-off in the two structures, this deviation is not valid, so the two equilibria coexist.

Now consider $R = 15$, where $1 < \kappa = 1.32 < 1.5$. In this range intermediation rents are sufficiently low that $I$ banks prefer 3a to 3d, so 3a is an equilibrium. However, they are not able to jointly deviate back to 3a if they start from 3d. As a result 3d is an equilibrium as well. Finally, intermediation rents are sufficiently high that an $I$ bank is willing to be exposed to counterparty risk if he intermediates two units, so 3e is also an equilibrium.

Take an even lower return, $R = 10$, for which $0.5 < \kappa = .84 < 1$. In this range, intermediation rent associated with one unit is not sufficient to cover the incremental cost of default due to exposure to counterparty risk for an $I$ bank, so 3d ceases to exist. Finally, note that in order to have participation constraint of $I$ bank satisfied and $\kappa < \frac{1}{2}$ it is necessary that $\alpha < 0.5$. Now consider an even lower $R = 6$, $\kappa = .456 < 0.5\bar{\kappa}$ and 3e is not an equilibrium either.

### 8.2.2 Perfectly Correlated Project Outcomes

Here I solve an extension of the model where lenders lend to all eligible borrowers, i.e. assumption 3 is relaxed[52], and the project outcomes are perfectly correlated across banks. This exhibit the extreme opposite case of having iid return realizations for projects, and shows how social planner and individual incentives to intermediate and diversify, vary as a function of this degree of correlation.

Perfectly correlated project returns implies that there is no room for diversification. All active investment opportunities fail or succeed together. However, as 3 is relaxed a lender has to lend at least one unit to each of his eligible borrowers. The first implication is that from the social planner's perspective, gains from lending to one extra $I$ bank is decreasing while the cost is constant. To see this, assume a bank is lending to $x$ $I$ banks. The net benefit from lending to the $x + 1^{\text{th}}$ bank is that the lender is now able to lend as much funds as he is able to raise, when bank $x+1$ receives an investment opportunity while none of the first $x$ banks did, net of cost of default of the borrower and lenders. However, there is an extra cost. Everything else equal, when any (subset) of the first $x$ $I$ banks, as well as bank $x + 1$ receive an investment opportunity, the scale of investment remains fixed, but bank $x + 1$ also invests and is now exposed to failure (of his own project). In other words, with multiple realized investment opportunities there is gain to *concentrating* the risk, which is lost here. Let $Z(x; K)$ denote the total net surplus from an $NI$ bank, with $K$ unites of funds (raised from his households and $K - 1$ other $NI$ banks), lending to $x$ $I$ banks ($K > x$).

$$Z(x; K) = (1 - (1 - q)^x)K\Big((pR - 1) - (1 - p)V_{NI}\Big) - (1 - p)qV_I x$$
$$\Delta(x; K) = Z(x + 1; K) - Z(x; K) = q\Big[(1 - q)^x K\Big((pR - 1) - (1 - p)V_{NI}\Big) - (1 - p)V_I\Big]$$

Let $c = V_I\Big(K(\frac{pR-1}{1-p} - V_{NI})\Big)^{-1}$. Note that from the assumption that one unit $NI \to I$ is

[52] Assumption 4 is maintained.

efficient we know $c < 1$. The marginal gain turns negative when

$$x > x^* = \frac{\log(c)}{\log(1-q)}$$

First assume $k_I < x^*$, so the social planner prefers to lend to every $I$ bank. The efficient solution requires investing every unit of funding whenever there is at least one realized investment opportunity, i.e. there should be a path from every $NI$ bank to every $I$ bank. Note that there is no room for concentration as Assumption 3 is relaxed. Moreover, all the intermediation must be done by $NI$ banks, so no $I$ bank lends.

In terms of equilibrium structure, the analogue of Theorem 1 holds here, with the exact same proof. This is the case because as long as there are no diversification effects, a lender only cares about level of rents, not where (or from how many borrowers) they come from, or what risk is undertaken to generate them. Moreover, the efficient structure is not an equilibrium when intermediation spreads are sufficiently high.

More interestingly, assume $k_I > x^*$. Now the social planner prefer to keep some investment opportunities unfunded because the marginal benefit is too small. In other words, reaching optimal scale of investment in one low-probability state requires destroying surplus in many states. This is the case when $q$ is large, while $k_{NI}$ is not too large. However, the same family of equilibria as defined in 1 still exist. A lender and/or intermediator wants to get as high a return as possible, as often as possible, so he prefers to be connected (directly or indirectly) to as many $I$ banks as possible. Each $I$ bank wants to invest as often as he gets an investment opportunity, so he would want to be connected to all units of funding. In this case, not only redistributional effects within a state are not internalized by individual players, but also redistributional effect across states are ignored.

This appendix manifests that incentives of banks to intermediate are the same with and without Assumption 3. In the extreme case where project returns are perfectly correlated, the core-periphery equilibrium remains inefficient because there is no gain to diversification. As section 5.2 shows, even with iid projects the core-periphery structure is inefficient under certain parameter restrictions. As the correlation across project returns rises,[53] the gain to diversification falls but gain to intermediate remains the same, so the space of parameters for which the core-periphery equilibrium is inefficient grows.[54]

[53]keeping project expectations the same.

[54]Solving for the most efficient structure with interim levels of return correlation is not straightforward, and is left for future work.

### 8.2.3 Incorporating Cost of Default into Rule for Division of Surplus

Here I solve the 4 bank model of section 3 with a variation of $\alpha$-rule which incorporates the default cost of banks along the intermediation chain. In this variation, the net surplus divided between the members of an intermediation chain is net of expected cost of default, and each agent receives his expected default cost plus his share. Let $L$, $B$, and $In$ denote lender, borrower and intermediator respectively and let $V_k$ be the cost of default of agent $k \in \{L, B, In\}$. Let $X_k$ be the expected net surplus associated with a unit of investment intermediated through a chain of length $k$.[55]

$$X_1(V_B, V_L, V_{In}) = X - (1-p)(V_B + V_L)$$
$$X_2(V_B, V_L, V_{In}) = X - (1-p)(V_B + V_L + V_{In})$$

I suppress arguments to simplify the notation to $X_1$ and $X_2(V_{In})$, as the rest of the arguments do not change ($V_B = V_I$ and $V_L = V_{NI}$. Note that in each chain, each agent is compensated for the risk he takes as if this unit was the only unit he is involved in. This rule does not satisfy anonymity. Nevertheless, considering it reveals more insight from the model.

The new rule implies that agents are always compensated for the risk that they take (and maybe over-compensated). Now consider the deviation analogous to the one depicted in Figure 4. Let $\hat{x}$ denote variables in the right panel, i.e. the core-periphery structure.

$$\hat{\mathcal{V}}_{NI_2} = q\alpha X_1 + (1-q)q\alpha^2 X_2(V_I) + V_{NI}$$
$$\hat{\mathcal{V}}_I = q^2(1-\alpha)X_1 + q(1-q)(1-\alpha)[X_1 + X_2(V_I)] + q(1-q)\alpha(1-\alpha)X_2(V_I) + V_I$$

while

$$\mathcal{V}_{NI_2} = (1-(1-q)^2)\alpha X_2(V_{NI}) + V_{NI}$$
$$\mathcal{V}_I = (q(1-q) + \frac{1}{2}q^2)2(1-\alpha)X_2(V_{NI}) + V_I$$

Let $\Delta\mathcal{V}_j = \hat{\mathcal{V}}_j - \mathcal{V}_j$, $j = I, NI$. With some algebra we get

$$\Delta\mathcal{V}_{NI} = q\alpha^2\Big[\frac{1-\alpha}{\alpha}X_1 + (1-p)V_{NI}\Big] + q(1-q)\alpha^2(1-p)(V_{NI} - V_I)$$
$$\Delta\mathcal{V}_I = q^2(1-\alpha)(1-p)V_{NI} + q(1-q)(1-\alpha)\alpha X_2(V_I) + q(1-q)(1-\alpha)(1-p)(2V_{NI} - V_I)$$

[55] $k$ is the number of edges along the chain.

The sign of the last term in both expressions is ambiguous. The first observation is that if $V_I = V_{NI}$, both the peripheral lender and the $I$ banks want to *unconditionally* deviate: $I$ bank is now compensated for the excessive risk that he can take, and the cost is born by $NI_1$ (recall that the expected length of chains is *the same* in both network structures). Moreover, $\forall V_I\ \exists \bar{C}$ such that for $X > \bar{C}$, both $\Delta\mathcal{V}_{NI} > 0$ and $\Delta\mathcal{V}_I > 0$ even if $V_I > 2V_{NI}$. This condition is similar to what we have in section 3: if surplus of a unit investment is sufficiently large, the share of it which used to go to the $NI$ intermediator before the deviation, and post deviation is divided between the peripheral $NI$ and the new intermediators, $I$ banks, is sufficiently large to cover the extra cost that they hove to each bear by deviating. The higher cost is due to the fact that a *costlier* $I$ banks intermediates in the new network, which is directly incorporated in the rule of division of surplus.

Now let me make an even more extreme assumption, and assume $V_{NI} = 0$, so if an $NI$ intermediates it is costless. Then we have

$$\Delta\mathcal{V}_{NI} = q\alpha^2\Big[\frac{1-\alpha}{\alpha}X_1\Big] - q(1-q)\alpha^2(1-p)V_I$$
$$\Delta\mathcal{V}_I = q(1-q)(1-\alpha)\Big[\alpha X_2(V_I) - (1-p)V_I\Big]$$

comparing the two pair of expressions, it is clear that it is more difficult to satisfy the latter two. However, still $\exists \ddot{C} > \bar{C}$ for which the same argument goes through.

This appendix shows that the intuition for role of intermediation in formation of financial networks is quite general and beyond the sufficient conditions provided in section 4.1, leading to a core-periphery interbank equilibrium. The crucial assumption is that there are positive intermediation spreads, and longer intermediation chains are associated with lower spreads per bank involved.

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