Optimal capital requirements over the business and financial cycles*

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PRELIMINARY AND INCOMPLETE

Abstract

I propose a simple theory of intertwined business and financial cycles, where financial regulation both optimally responds to and influences the business cycle. I build a model in which government guarantees induce excessive aggregate lending by the financial sector. In response, the regulator sets macro-prudential capital requirements to trade-off growth (expected output) with financial stability (lower probability and social cost of a banking sector collapse). This trade-off depends on the state of the economy and optimal capital requirements are therefore not constant. Because of a general equilibrium effect, optimal capital requirements increase with aggregate banking capital. A regulation that fails to take this effect into account will exacerbate economic fluctuations and result in systemic risk being created in the financial sector: aggregate bank lending will be excessive during a boom and the banking collapse that may ensue will result in an excessive credit crunch.

1 Introduction

Motivation

The recent crisis has exposed how important the interactions between the financial sector (and financial regulation) and the real side of the economy (and macroeconomic policies) can be.[1] Yet, most of the models

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[1] More generally, empirical evidence suggests that risks are built up in the financial system during good times (Borio and Drehmann (2009), and that financial booms do not just precede busts but cause them (Borio 2012). Also, the amplitude of the financial cycle is not constant, and is influenced by financial regulation regimes (Borio and Lowe 2002, Borio 2007). The model I develop here is consistent with those facts.

used by researchers and policy makers to think about these issues are separate, and there is no consensus on an integrated approach.[2] I develop here a simple theory of intertwined business and financial cycles, where financial regulation both optimally responds to and influences the business cycle.

The questions I seek to address are**:**

- How does bank capital requirements affect the interactions between the business cycle and the financial cycle?
- Should bank capital requirements be higher in "good times" and reduced in "bad times"?
- What macroeconomic variables are key for determining the optimal stringency of capital requirements?

To study these questions, I build a model where government guarantees induce excessive aggregate lending by the financial sector. In response, the regulator sets macro-prudential capital requirements to trade-off growth (expected output) with financial stability (lower probability and social cost of a banking sector collapse). This trade-off depends on the state of the economy and optimal capital requirements are therefore not constant. Although other tools could be used by the regulator of to correct the market failure in the model, the focus on capital requirements is simply motivated by the current policy debate on the "pro-cyclicality" of bank capital regulation (Repullo and Suarez (2012)).

Cyclically adjusted capital requirements have been used in Spain for a while (Jimenez et al. (2012)) and in September 2012, amid credit crunch concerns, the UK Financial Service Authority has decided to soften bank capital requirement for new lending. More generally, the introduction of "counter-cyclical capital buffers" is an explicit recommendation of the Basel Committee on Banking Supervision. While no explicit reference to an analytical framework is provided in the supporting document, the main logic seems the following: If "high" capital requirements are contractionary[3], such a cost has to be balanced with the benefits in terms of financial stability, or of tax-payer exposure to systemic financial crisis (Kashyap and Stein (2004)). If these costs and benefits are dependent on the state of the economy, optimal capital requirements may vary over the cycle.

Description of the model's main features

It is an overlapping generation model with savers and bankers. Bankers wealth is assumed to be initially scarce. They are protected by limited liability, they collects deposit and lend to competitive firms, which operate a constant-return-to-scale risky production function. Bank lending is the only source of firm funding. Firms always make zero profits, and bankers are in effect the residual claimants of the production. Note that the decreasing marginal productivity of physical capital translate into decreasing marginal return to lending. Or, from a reverse perspective, aggregate bank lending affects the marginal productivity of physical capital. This general equilibrium effect is a key feature of the model.

Deposits are insured. If the banks are not able to repay the deposits in full, the regulator compensates for the shortfall and levies lump-sum taxes to break even.[4] As a result, the interest rate paid on deposits is insensitive to the risk taken by the banks. This friction is the root of the market failure in the model, and correcting this market failure is the rationale for regulation. Deposit insurance is taken in the model as an existing institutional feature. It is a particular and explicit form of government guarantees on banks' debt, but one could alternatively

[2] See for instance Goodhart (2010); Woodford (2010); Galati and Moessner (2011).

[3] Which is actually disputed by some prominent economists (see Admati et al. (2010) and Hellwig (2010)).

[4] Note that whether deposit insurance is funded by tax payers ex post or through an ex-ante premium does not matter as long as the premium does not fully reflect the full social cost of the risks taken by the bank. I show in the Appendix how such a "fair" premium relates to the optimal capital requirements. (to do)

build a model in which implicit guarantees would endogenously arise due to the inability of the government to fully commit not to bailout bank creditors. It would generate the same distortion and lead to similar results.[5]

Finally, there is a regulator that seeks to maximize expected social welfare, a weighted sum of successive generation agents' expected utility. To do so, the regulator can constrain bank lending. In practice, it can choose capital requirements at the beginning of each period to solve the following trade-off: allowing more lending increases expected output (up to a certain point), but it also increases the probability of a banking sector collapse, and the severity thereof.

Description of the model's main mechanisms

The interest rate paid on deposits is insensitive to the risk taken by the banks. As a result, banks have an incentive to leverage up to levels which are socially too high. Although this is not an asset-substitution mechanism, this is essentially reminiscent of the classic result of bank risk-shifting behavior when deposit are insured (Kareken and Wallace (1978)).

Moreover, the guarantees act as a subsidy and banks do not internalize the full expected cost of borrowing; there is a wedge in their expected marginal cost, which makes them willing to issue loans with excessively low expected return. In fact, in a competitive equilibrium, this results in aggregate over-lending, and in the marginal loan being of negative NPV. This deteriorates bank balance sheets and increases both the probability of a banking sector collapse and the severity there-of. It is important to note that such increase in risk is originated in the financial system. It is different than "fundamental" risk, which corresponds to the intrinsic riskiness of the production function: up to a point, an increase in risk also correspond to an increase in expected output. In the case of negative NPV investment, the economy is past that point, and risk is excessive. It can thus be interpreted as systemic risk.

Results and insights

A first set of results comes from a comparative statics exercise. I find that optimal capital requirements are:

- decreasing in expected productivity;
- increasing in aggregate bank capital.

The first is very intuitive since an increase in expected productivity makes the marginal investment in the economy more profitable. Therefore, it makes the marginal loan more profitable since the probability of default of the borrowers go down (this channel is essentially the rationale behind the Basel II regulation). Second, since savers end up footing the bill in case of a bailout, their marginal utility determines the marginal cost of bank failure. All other things equal, regulation should therefore be more stringent when their marginal utility is high. The third part is perhaps less intuitive. On the one hand, more banking capital means that the banking sector can absorb more losses, which also weighs in for more lending. But, on the other hand, there is a general equilibrium effect that dominates the loss absorbing effect for any reasonable level of risk aversion.

To see the intuition behind the general equilibrium effect, first consider a single (atomistic) bank that doubles the size of its initial capital. It should simply be allowed to double the size of its assets. However, if all banks in the economy double the size of their initial capital, and if they are allowed to double the size of their assets, this will double aggregate lending in the economy. Given the decreasing marginal return to physical capital on the real side of the economy, this implies a substantially lower marginal return to lending. In fact. the banking

[5] Note that deposit insurance is observed in all most advanced economies, although coverage may be different. Coverage, in terms of maximum amount per person (or account) has generally be extended during the 2008 crisis. In some cases, it has been fully extended ex-post, including to other types of debt. More recently however, Cyprus ex-ante un-insured depositors have been excluded ex-post.

sector should be allowed to expand, but less than proportionally, which corresponds to an increase in capital requirements.

To highlight this general equilibrium effect is a first contribution of the paper. If this effect is overlook by the regulator, it exacerbates economic fluctuations and result in systemic risk being created in the financial sector: aggregate bank lending will be excessive during a boom and the banking collapse that may ensue will result in an excessive credit crunch. Such a prediction that risks are being piled up by the banking sector during good times finds empirical support (see Borio and Drehmann, 2009 for instance).

The dynamics of the model provide a simple theory of intertwined business and financial cycles that goes as follows: productivity shocks affect output and therefore aggregate banking capital accumulation (directly through banks retained profits, and indirectly through the wealth that new bankers will be able to inject as fresh capital in the sector). Productivity shocks also affect the socially optimal level of aggregate lending at the next period (through persistence in productivity). This socially optimal level of lending, which will be implemented through the optimal capital requirements, also depends on the level of aggregate banking capital. In turn, the optimal capital requirements affect physical capital accumulation and therefore output. They also affect the exposure of the banking sector to productivity shocks and therefore influence banking capital accumulation.

These dynamics deliver periods of "good times", when productivity, consumption, investment, physical and banking capital are high, and periods of "bad times", when they are all low. Looking at the comparative statics results tells us that, in good times, high productivity and consumption plead for lower capital requirements, but that the general equilibrium effect of aggregate bank capital goes in the other direction. It turns out that the latter dominates and optimal capital requirements are higher in good times. The intuition is that, in this model, aggregate banking capital is relatively more procyclical than the optimal level of aggregate lending.

This result (which resonates with the notion of "counter-cyclical capital buffers" of Basel 3) is more specific to the model since the dynamics for aggregate banking capital are largely driven by demographics. However, it conveys an important and more general policy insight: if aggregate banking capital varies more over the cycle than the "desired" level of aggregate lending, than optimal capital requirements should be higher in good times, and conversely.

Related literature

This paper belongs to strand of literature that builds on Kareken and Wallace (1978) and Dewatripont and Tirole (1994) and sees prudential regulation of banks as a response to excessive risk taking by banks induced by the government guarantees.

The most closely related paper is Repullo and Suarez (2012), which studies optimal bank capital regulation over the cycle and compares them to regulations that resemble Basel I, II, and III. They find that "counter-cyclical buffers" help to mitigate the "pro-cyclical" effects of regulation such as Basel II. In their set-up, optimal capital requirements are always higher in bad times than in good times. An important feature of their model is that the production function is linear in investment, which does not leave much room for interaction between aggregate banking capital and marginal productivity in the real sector. In other words, this assumption restricts interactions between the financial and the real business cycle. I therefore generalize their approach in that dimension.

The paper is also related to the literature on macro-prudential regulation.

Martinez-Miera and Suarez (2012) propose a macroeconomic model of endogenous systemic risk-taking in which correlated risk-shifting by some banks give an incentive to other banks to play it safe, because the more banks fail at the same time, the higher the scarcity rent after a crisis. Still, the competitive outcome is inefficient, and the optimal (constant) capital requirement trades off output at steady state with the severity of financial crisis (the time it takes to go back to steady state). They do not consider business cycle dynamics but, in their model, reducing capital requirement after a banking crisis reduces the rent earned ex-post by the “last banks standing” and induces thus more systemic risk-taking ex-ante. In contrast, in Dewatripont and Tirole (2012) incentives to gamble for resurrection are stronger after a negative macro-economic shock. Related to this, Morrison and White (2005) study optimal capital requirements in a model with both moral hazard and adverse selection. They find that the appropriate policy response to a crisis of confidence may be to tighten capital requirements. This happens when the regulator’s ability to alleviate adverse selection through banking supervision is relatively low. In my model, there is no asset substitution (or effort) problem, and the primary role of banking capital is to act as a buffer to absorb losses (and therefore protect the tax payer) rather than to correct incentives.

Most of the micro-funded models of macro-prudential regulation that are currently burgeoning are developed around pecuniary externalities mechanisms, such as fire-sale externalities for instance (Bianchi (2011), Brunnermeier and Sannikov (2010), Korinek (2011), Jeanne and Korinek (2010), Jeanne and Korinek (2013), Gersbach and Rochet, 2011, Stein (2012) are good examples; see Hanson et al. (2011) for an overview). More generally, these models build on debt deflation (Fisher (1933)) and financial accelerator mechanisms (Bernanke and Gertler (1989); Kiyotaki and Moore (1997)). In these papers, borrowing constraints, generally justified by moral hazard concerns, are imposed by lenders. One of the main message from this approach is that the equilibrium level of lending in the economy may not be efficient because investors fail to internalize the social cost of a binding borrowing constraint in presence of a pecuniary externality. Another strand of papers provides a rationale for a systemic approach to regulation based on financial institution incentives to choose correlated exposure (Farhi and Tirole (2012), Acharya (2009)) or on network externalities (Allen et al. (2011)).

My approach differs from these latter approaches in that I set aside incentive problems and maturity mismatch (or liquidity) issues (which can lead to fire-sale externalities) to focus on distortions induced by government guarantees (which lead to solvency problems) and their general equilibrium interactions with the real business cycle.

Other papers that study the distortions caused ex-ante by government guarantees include Merton (1977), Kareken and Wallace (1978), Keeley (1990), Pennacchi (2006), and Gomes et al. (2010), for instance, attempts to quantify the distortions that arise ex-post, when taxes need to be raised to finance the bailouts.

More generally, the paper relates to the literature on bank capital regulation (see Santos (2001) for a survey) and in particular to that on the costs and/or benefits of bank capital requirements. These include Van den Heuvel (2008), Admati et al. (2010), Hellwig (2010), and Morrison and White (2005)).

2 The model

2.1 The environment

There is an infinity of periods, indexed by $t = 0, 1, 2...$, in which generations of agents born at different dates overlap.

The young generations. There is a measure 1 of agents born at the beginning of each period. These agents are endowed with one unit of labor, which they supply inelastically during the first period of their life for a wage

w_t. At the end of the period, these agents incur an "ability" shock:

- A share $\eta \ll 1$ is endowed with "banking ability". They will be able to set up a bank at date $t+1$ and use their wage w_t as equity.
- A share $1-\eta$ has no more working ability and retire. I refer to them as savers.

The firms. Each period, there is a continuum of penniless firms that operate a constant-return-to-scale risky production function:

$$F(A_t, k_t) = A_t k_t^{\alpha},$$

where k_t is physical capital per worker, $0 < \alpha < 1$, and A_t is a random variable, distributed over $\mathbb{R}_0^+$ with density function $f(A_t)$, that captures aggregate risk.

Firms competitively hire workers and borrow from banks to invest in physical capital, which fully depreciates in the production process. They pay a competitive wage w_t, and they borrow capital from the bankers in exchange of the repayment of R_t per unit of capital.

Production is risky in the sense that A_t only realizes at the end of the production process. Hence, the realized marginal productivity of physical capital and labor is not known when equilibrium quantities are determined. For now, let me assume that the realized prices correspond to realized marginal productivity. In that case, the equilibrium conditions are the usual:

$$\begin{cases} w_t = (1-\alpha) A_t k_t^{\alpha} \\ R_t = \alpha A_t k_t^{\alpha-1} \end{cases},$$

and they ensure that firms always make 0 profit.

The old generations of savers. During the second period of their life,savers can choose between depositing their income from the previous period at the bank or use a storage technology. The rate of return to storage is normalized to 0. Deposits are insured by the government. I focus on cases where deposits are in excess supply at equilibrium[6], so that they pay the same rate of return as storage. At the end of their second period of life, they receive their deposit back (either from the bank or the government), pay taxes, if any, and consume (one can consider that they die at the end of that period). They derive utility from their consumption (see more details below).

The old generation(s) of bankers. The measure η of agents that receive "banking ability" can use the wage they earned during their first period as equity to set-up a new bank under the protection of limited liability. It is assumed that a bank cannot operate without a strictly positive value of equity.

Bankers raise deposits and compete to lend to firms. Banks are the only source of fund to firms, which are penniless and act competitively. Therefore, the realized marginal return to lending is the realized marginal return to capital $R_t(A_t) = \alpha A_t k_t^{\alpha-1}$. Bankers take the distribution of marginal return to lending as given.

Let v_{it} denote the book value of bank i's equity (the net worth of the bank) at the end of period t:

[6] This happens when η is small enough so that banking capital is scarce.

$$v_{it}(A_t) \quad \equiv \quad R_t(A_t)l_{it} - (l_{it} - e_{it})$$

where l_{it} is the total lending of bank i in period t, and e_{it} is its banking capital (or equity) at the beginning of the period. It corresponds to the realized value of the lending portfolio $(R_t(A_t)l_{it})$, minus the repayment on deposits $(l_{it} - e_{it})$.

The realized (or private) value of banking equity is:

$$v_{it}^{+}(A_t) \equiv [R_t(A_t)l_{it} - l_{it} + e_{it}]^{+}$$

The difference between the book value and the private value comes from limited liability, which gives bankers the option to walk away with 0 when $v_{it} < 0$.

After deposit repayment takes place, bankers may loose their banking ability. It happens to them with probability δ, irrespective of their tenure. In that case, they retire and consume v_{it}^{+}.[7]

If they keep their banking ability (which happens with probability $(1 - \delta)$), there are two cases:

- If $v_{it}^{+} = 0$, they have no other choice than becoming inactive (they die, or leave the economy).
- If $v_{it}^{+} > 0$, they can choose between consuming and reinvesting.[8]

Bankers are risk neutral and derive utility from the sum of their lifetime consumption. That is, the utility of a banker that was born at date t is: $u_t^b \equiv E\left[\sum_{s=t+1}^{\infty} c_{ts}^b\right]$, where c_{ts}^b denotes their consumption at date s.

The regulator. There is a regulator whose role is to correct market failures. This role is exposed in details in the next section.

Summary of intraperiod timeline.

- Relevant predetermined variables:
  - A_{t-1}, which will be assumed to be a sufficient statistic for the parameters of A_t's distribution.
  - aggregate banking capital: e_t.
- The regulator decides upon x_t, a capital requirement imposed on bankers.
- Market activity takes place. Simultaneously:
  - Bankers borrow from savers and lend to firms;
  - Firms competitively hire workers, borrow from bankers, and invest in physical capital.
- Production takes place, A_t realizes, wages are paid, and the share of capital goes to the bankers.

[7] One can either think that they die at the end of the period, or remain alive, but have no way to earn income from capital. Since they face the risk to be taxed in the future, it is then optimal for them to consume immediately.

[8] To make it simple, I do not allow them to "wait". That is, to store or deposit at another bank, and re-enter the banking sector later on. Such strategy may be profitable in some cases. Indeed, if the realization of A_t turns out to be very low, the banking sector will go broke and banking capital will earn large scarcity rent at the next period (see Martinez-Miera and Suarez 2012). Note, however, that a banker choosing such strategy would face the (un-modeled) risk of being taxed, and may also loose its banking ability. Hence, if δ is high, this assumption should not be restrictive.

- If possible, bankers repay deposits. If not, they walk away with nothing, and the regulator compensates depositors for the shortfall, imposing a break-even lump-sum tax on savers.
- Consumption takes place.

3 Market failure and need for regulation

3.1 Market failure

The banker problem.

LEMMA 1:

- *If banking capital earns scarcity rent, it is inefficient for bankers to consume as long as they keep their banking ability. Therefore:* $e_{i,t+1} = v_{it}^{+}$.
- *Since bankers are risk-neutral, at any level of beginning of period capital (equity), it is individually optimal for them to maximize the end-of-period expected (private) value of equity.*

Proof: see appendix.

Bailout and over-investment. The shortfall in bank i value to repay depositors is given by:

$$v_{it}^{-} \equiv [R_t l_{it} - l_{it} + e_{it}]^{-}$$

From Lemma 1, a banker i chooses $l_{it} \geq 0$ so as to maximize $E\left[v_{it}^{+}\right]$, which can be rewritten:

$$E\left[v_{it} + v_{it}^{-}\right].$$

The first order condition (with respect to l_{it}) for an interior solution is

$$E[R_t - 1] + E\left[\frac{\partial v_{it}^{-}}{\partial l_{it}}\right] = 0$$

First note that the second expectation term would not appear if the banker were to maximize the social value of equity $E\left[v_{it}\right]$ instead of the private one $E\left[v_{it}^{+}\right]$.
LEMMA 2:

- $\frac{\partial v_{it}^{-}}{\partial l_{it}} \geq 0$.
- If $0 < e_{it} < l_{it}$, there exists an $A_0 > 0$ such that $\forall A_t < A_0$, $v_{it} < 0$, and $\frac{\partial v_{it}^{-}}{\partial l_{it}} > 0$.

Proof: straightforward.

This lemma captures the impact of the implicit subsidy from government guarantees: they introduce a wedge in the first order condition of the banker. Indeed, banks do not internalize to full marginal cost of lending (and therefore the cost of marginal risk taking), which gives them an incentive to increase leverage.

PROPOSITION 1: *The unregulated market outcome is inefficient: it exhibits negative NPV investment.*

Proof: Denote R^m the equilibrium realized return to lending. First, note that if $E[R_t^m] > 1$, bankers want to expand (Lemma 2, first part), this can therefore not be an equilibrium. If $E[R_t^m] = 1$, this can be an equilibrium only if A has a degenerated distribution. Otherwise, there exists a sufficiently high l_{it} for which $E\left[v_{it}^-\right] > 0$ and bankers have incentives to leverage to infinity (alternatively, one could consider that bankers desiring to lend a given amount l_{it}, will first consume most of their initial wage and only put a fraction $\epsilon \to 0$ of it as equity, which akin to infinite leverage). In a laisser-faire equilibrium, $E[R_t^m]$ must then be smaller than 1, which is necessarily inefficient since the return to storage is 1. Discounted at the return to storage, the marginal investment in the economy has a negative NPV. This is a first rationale for macro-prudential regulation in the model.

COROLLARY 1: *At the social optimum, capital requirements are binding.*

3.2 A two-stage view of macro-prudential regulation: systemic risk and socially excessive risk-taking.

Systemic risk. Consider the hypothetical case of a single bank representing the banking sector. The end-of-period t net-worth of the bank (the banking sector) is $v_t \equiv \alpha A_t l_t^\alpha - l_t + e_t$, where l_t represent aggregate lending and e_t the aggregate beginning-of-period banking capital.

Assume that the bank leverages up to the point where $E[R_t^m] = 1$. The corresponding level of aggregate lending is the one that maximizes the expected output. It is given by:

$$\alpha E[A_t]\bar{l}_t^{\alpha-1} = 1 \;\Leftrightarrow\; \bar{l}_t = (\alpha E[A_t])^{\frac{1}{1-\alpha}} \,. \tag{1}$$

At this level of lending, v_t will be strictly negative if the realization for A_t is such that:

$$\frac{A_t}{E[A_t]} < \frac{\bar{l} - e_t}{\bar{l}_t} \tag{2}$$

This risk of financial sector failure reflects the fundamental riskiness of the production process. Note that the right-hand side is a direct measure of the financial sector leverage. Hence, the risk can de decreased by reducing leverage, but it comes at a cost of a lower expected output.

But, as I have shown above, the implicit subsidy from government guarantees gives financial institutions an incentive to increase leverage beyond the level that corresponds to $\bar{l}$. In such a case, the threshold for the realization of A_t from which the banking sector does not fail goes up. The probability of a banking sector collapse therefore goes up too, and so does the severity of a crisis, should it happen (which is a direct application of the second part of *Lemma 2*). It is important to note that such increase in risk is originated in the financial system as a consequence of distorted incentives. It is different than “fundamental” risk, which corresponds to the intrinsic riskiness of the production function: up to a point, an increase in risk also correspond to an increase in expected output. In the case of negative NPV investment, the economy is past that point, and risk is excessive. Given its origination within the system, it can be interpreted as systemic risk.

Socially excessive risk-taking. The second stage view of macro-prudential regulation I propose here is to consider that the regulator's objective is to maximize a social welfare function, which is a discounted weighted sum of each generation two representative agents utility:

$$\sum_{t=0}^{\infty} \beta^t E\left[\eta u_t^b + (1-\eta)u_t\right],$$

where u_t denotes the utility of savers born at date t, and $\beta \in]0,1]$ is a discount factor. The weights correspond to the class of agent's respective measure.

Under such an approach, and as we will see below, the level of lending $\bar{l}$ defined above is likely to be too large if savers (which end-up footing the bill in case of a bailout) are risk-averse. In that case, the regulator should go beyond the elimination of systemic risk and the optimal capital requirements solve a trade-off between expected output and the risk borne by these agents.

Capital requirements. The regulatory tool I consider in both stages is a capital requirement $x \in [0,1]$, which constraints bank lending to a multiple of its equity:

$$e_i \geq x l_i$$

Alternatively, one could consider a tax on leverage and obtain similar results. However, focusing on capital requirements is justified by several reasons. First of all, capital adequacy ratios are one of the main tools use for prudential purpose. And, indeed, they are the regulatory tools that are being used (see the introduction) or considered to deal with its macro-prudential aspects (Basel 3). Second, they enable the regulator, in such a simple context (there is for instance no asymmetry of information), to implement the social optimum. Last but not least, they simplify computation, they make aggregation straightforward and economize on notation, and they are easy to interpret.

Aggregate banking capital.

DEFINITION 1 (scarcity of bank capital): *Aggregate bank capital at date s (e_s) is scarce if there exist a possible sequence of realization of A_s, A_{s+1}... such that condition (2) is satisfied for some $t \geq s$.*

For what follows, I assume that aggregate banking capital is scarce. Given the structure of the model, this is not very restrictive. I essentially just rule out the uninteresting cases were the banking capital is plentiful and is always sufficient to cover all possible level of losses.

Since optimal capital requirement are binding at the social optimum (*Corollary 1*), $l_i = \frac{e_i}{x}$ for all banks, and $k = \frac{e}{x}$ in the aggregate, with e denoting aggregate banking capital. Capital requirements thus can be used to control aggregate bank lending, and therefore aggregate physical capital.

Let me define the end-of-period net-worth of the aggregate banking sector $v_t \equiv \alpha A_t k_t^\alpha - k_t + e_t$, and the aggregate private value of bank equity: $v_t^+ \equiv [\alpha A_t k_t^\alpha - k_t + e_t]^+$.

Assuming a full reinvestment policy[9], only bankers that lose their banking ability consume at the end of the period. Hence, aggregate bankers consumption at date t is given by is δv_t^+. Accordingly, the law of motion for banking capital is:

[9]This corresponds to the interesting case in which banking capital is scarce.

$$e_{t+1} = \underbrace{(1-\delta)v_t^+}_{\substack{reinvested \\ profits}} + \underbrace{\eta w_t}_{\substack{fresh \\ capital}} .$$

Bailout tax. The shortfall in aggregate banking sector net worth to repay depositors is $v_t^- \equiv [\alpha A_t k_t^\alpha - k_t + e_t]^-$. Hence, the per capita bailout tax b_t that the regulator has to impose on consumers in order to break even is:

$$b_t = \frac{v_t^-}{1-\eta}.$$

Remark: for simplicity, I assume here that the full burden of the bailout falls on the saver. An interesting extension would be to study the possibility of splitting the burden of taxation between the current generations. This would improve inter-generational risk-sharing but would impair banking capital accumulation.

3.3 The regulator's problem

The regulator chooses $\{x_t\}_{t=0}^{\infty}$ so as to maximize the social welfare function:

$$\sum_{t=1}^{\infty} \beta^t E\left[\eta u_t^b + (1-\eta)u_t\right], \tag{3}$$

subject to the following laws of motion:

$$st: \begin{cases} k_t & = \frac{e_t}{x_t} \\ e_{t+1} & = (1-\delta)v_t^+ + \eta w_t \\ A_t & \sim f(A_t \mid A_{t-1}) \end{cases}$$

3.3.1 The intertwined business and financial cycles

The optimal capital requirement x_t^* that solves this program, together with the laws of motion for k_t, e_{t+1}, and A_t provides a theory of intertwined business and financial cycles.

Productivity shocks affect output and therefore aggregate banking capital accumulation (directly through banks retained profits, and indirectly through the wealth that new bankers will be able to inject as fresh capital in the sector). Productivity shocks also affect the socially optimal level of aggregate lending at the next period (through persistence in productivity). This socially optimal level of lending, which will be implemented through the optimal capital requirements, also depends on the level of aggregate banking capital. In turn, the optimal capital requirements affect physical capital accumulation and therefore output. They also affect the exposure of the banking sector to productivity shocks and therefore influence banking capital accumulation.

4 Optimal capital requirements over the business and financial cycles

In this section, I first analyze the case where savers are risk neutral, and then, the more general one where they are risk averse.

It is useful to analyze this risk-neutral case because it delivers easy to interpret analytical results. It also enables to focus on the systemic risk prevention stage of macro-prudential regulation. The risk-averse case is more general, but has to be solved numerically. It, by and large, confirms the intuition from the risk-neutral case, but also delivers additional insights on the second stage of macro-prudential regulation: the trade-off between expected output and tax-payer exposure to banking sector collapses.

4.1 The risk-neutral case

4.1.1 Specific assumptions

Preferences. In this section, I assume that savers are risk neutral: they derive linear utility from their consumption, which corresponds to their wage net of taxes:

$$u_t = w_t - b_{t+1}.$$

I also set $\beta = 1$, which considerably simplifies expressions.[10]

Productivity. I assume that A_t follows a Markov process with 2 possible states $A_t \in \{A_L, A_H\}$, with $A_L < A_H$, and a transition matrix:

$$\Omega = \begin{pmatrix} \omega_L & 1-\omega_L \\ 1-\omega_H & \omega_H \end{pmatrix},$$

where $0 < \omega_L < 1$ and $0 < \omega_H < 1$, $E[A_t] = 1$, and I denote the conditional expectation: $\bar{A}_L \equiv E[A_{t+1} \mid A_t = A_L]$, and impose $\bar{A}_L < 1 < \bar{A}_H$. This modeling strategy captures the ideas of persistence in productivity: after a good draw (A_H) economic prospects are better, that is, expected productivity is high ($\bar{A}_H > \bar{A}_L$). Note that this implies:

$$\frac{A_H}{A_L} > \frac{\bar{A}_H}{\bar{A}_L}$$

.

4.1.2 Solving for the optimal capital requirements.

LEMMA 3: *when savers are risk-neutral and $\beta = 1$, maximizing the welfare function (3) boils down to maximizing expected output at each period.*

Proof: see Appendix.

The intuition is quite simple: when savers are risk-neutral, as defined above, their marginal utility is always the same as that of bankers. A bailout out has therefore no ex-post effect on welfare. If $\beta = 1$, the social

[10] This does not affect the qualitative results. In fact, $\beta < 1$ is only required in the general case so that the numerical solution converges.

intertemporal marginal rate of substitution is always 1. Therefore, maximizing expected output at each date must maximize social welfare.

Hence, the relevant program is:

$$\max_{x_t} E\left[A_t k_t^{\alpha} - k_t\right]$$

$$s.t.: \begin{cases} k_t = \frac{e_t}{x_t} \\ e_t = (1-\delta)v_{t-1}^{+} + \eta(1-\alpha)A_{t-1}k_{t-1}^{\alpha} \end{cases}$$

The first constraint reflects that capital requirements are binding (which will be verified later). The second is the law of motion for aggregate banking capital, under a full reinvestment policy (which will be optimal as long as banking capital is scarce).

The level of physical capital that maximizes expected output is:

$$k_t^* = \left(\alpha \bar{A}_{t-1}\right)^{\frac{1}{1-\alpha}}, \tag{4}$$

Note that it does not depend on e_t (see discussion below).

The optimal capital requirement is then:

$$x_t^* = \frac{e_t}{\left(\alpha \bar{A}_{t-1}\right)^{\frac{1}{1-\alpha}}}. \tag{5}$$

PROPOSITION 3: *In the risk-neutral case, optimal capital requirements x_t^* are decreasing in expected productivity $\bar{A}_{t-1}$ and increasing in aggregate banking capital e_t.*

Proof: straightforward.

The intuition goes as follows: First, x_t^* decreases with expected productivity simply because higher productivity makes marginal lending more productive. The second part may be a bit counter intuitive as, after all, more banking capital sounds like good news. However, in this risk-neutral set-up, banking capital plays no role (from a welfare point of view) as a buffer against losses. This is why it does not affect the optimal level of aggregate lending. In a sense, it is simply the numerator of what determines the optimal capital requirement ratio.

In the risk-averse case (or in a version of the model that would incorporate deadweight losses form bank failure), banking capital does play such a role. Therefore, it does affect the optimal level of lending (more capital means a larger loss absorbing buffer which, all other things equal, calls for an increase in optimal capital requirements). See the next subsection for the full discussion.

4.1.3 Pseudo steady states (PSS)

Let me define a Pseudo Steady State (PSS) j as the state the economy would converge to (under the optimal capital requirements derived above) if state j would always occur. That is, for any variable a of the model :

$$a_j \equiv \lim_{t\to\infty} a_t \mid_{\{A_t = A_j\}_{t=0}^{\infty}}$$

One can easily characterize the pseudo-steady state value of the two key state variables. I denote these values e_j and k_j^*:

$$\begin{cases} k_j^* = \left(\alpha \bar{A}_j\right)^{\frac{1}{1-\alpha}} \\ e_j = (1-\delta)\left[\alpha A_j k_j^{*^{\alpha}} - k_j^* + e_j\right]^+ + \eta(1-\alpha) A_j k_j^{*^{\alpha}} \end{cases} . \tag{6}$$

Good times and bad times. It is easy to show that PSS H (that is at the PSS corresponding to an infinite sequence of realized productivity A_H) is associated with higher levels of output, consumption, and investment (and of course physical and banking capital) than PSS L. I therefore interpret and refer to PSS H as "good times" and PSS L as "bad times". They can respectively be interpreted as the peak of a boom and the through of a recession.

4.1.4 Optimal requirements in good and bad times

To analyze the properties of optimal capital requirements at the "business/financial cycle frequency", I propose here a comparative statics exercise at PSS.

PROPOSITION 4: *Optimal macro-prudential capital requirements are higher in good times than in bad times. Formally:*

$$x_H^* > x_L^*$$

Proof: First, note that v_H^+ is always positive since $\bar{A}_H > 1$. If $v_L^+ > 0$, for $j = L, H$:

$$x_j^* = \frac{1}{\delta}\left[\left(\alpha(1-\delta) + \eta(1-\alpha)\right) A_j \left(\alpha \bar{A}_j\right)^{-1} - (1-\delta)\right].$$

Hence, I have that:

$$x_H^* > x_L^* \iff \frac{A_H}{A_L} > \frac{\bar{A}_H}{\bar{A}_L},$$

which is always satisfied since shocks are not fully persistent ($\omega_L, \omega_L < 1$).

If $v_L^+ = 0$, I have:

$$x_L^* = \eta(1-\alpha) A_L k_L^{*^{\alpha-1}},$$

and I need to show that:

$$\left(\alpha(1-\delta) + \eta(1-\alpha)\right) A_H k_H^{*^{\alpha-1}} - (1-\delta) > \delta\eta(1-\alpha) A_L k_L^{*^{\alpha-1}}$$

which can be rewritten:

$$(1-\delta)\underbrace{\left(\alpha A_H k_H^{*^{\alpha-1}} - 1\right)}_{>0} + \eta(1-\alpha) A_H k_H^{*^{\alpha-1}} > \delta\eta(1-\alpha) A_L k_L^{*^{\alpha-1}}$$

and is satisfied because the definition of k^*, k_j^*, and $\bar{A}_L < 1 < \bar{A}_H$ imply:

$$\alpha A_H {k_H^*}^{\alpha-1} > 1 > \alpha A_L {k_L^*}^{\alpha-1}.\square$$

Elasticity interpretation. The general expression for the optimal capital requirement is given by:

$$x_t^* = \frac{e_t}{k_t^*}.$$

Both the numerator and the denominator depend (positively) on A_{t-1}. Thus, optimally, x_t should increase with A_{t-1} if the numerator increases faster than the denominator following an increase in A_{t-1}. In term of elasticities, we have that:

$$\varepsilon_{xA} = \varepsilon_{eA} - \varepsilon_{kA}, \tag{7}$$

where the three terms respectively denote the point elasticity of x_t, e_t, and k_t^* with respect to A_{t-1} (and where I have dropped the t subscripts for readability).

These elasticities, evaluated at PSS j, are given by:

$$\begin{cases} \varepsilon_{eA}^j \equiv \frac{\partial e_j(A_j)/e_j(A_j)}{\partial A_j/A_j} \\ \varepsilon_{kA}^j \equiv \frac{\partial k_j^*(A_j)/k_j^*(A_j)}{\partial A_j/A_j} \end{cases}.$$

They can be compared, and indeed, it turns out that $\varepsilon_{kA}^j < \varepsilon_{eA}^j$, which implies that PSS optimal requirements are increasing in PSS productivity.

To grasp the intuition, first note that

$$\varepsilon_{kA}^j = \frac{\varepsilon_{\bar{A}A}^j}{1-\alpha},$$

where $\varepsilon_{\bar{A}A}^j \equiv \frac{\partial \bar{A}_j/\bar{A}_j}{\partial A_j/A_j} = \frac{\omega_j A_j}{\omega_j A_j + (1-\omega_j)A_{-j}}$ captures the proportional impact on expected productivity ($\bar{A}_j$) of an infinitesimal change in A_j.

Then, consider ε_{eA}^j, and focus on the simple case where $\delta = 1$, in this case, we have that $e_j = \eta(1-\alpha)A_j {k_j^*}^{\alpha}$, which gives:

$$\varepsilon_{eA}^j = 1 + \alpha \varepsilon_{kA}^j.$$

It is then straightforward to show that since $\varepsilon_{\bar{A}A}^j < 1$, it must be the case that $\varepsilon_{kA}^j < \varepsilon_{eA}^j$. Therefore, the comparative statics tell us that an increase in PSS productivity affects aggregate banking capital proportionally more than the optimal stock of physical capital. Such increase should thus be associated with higher capital requirements, which confirms Proposition 1.

The reason why e_j is more reactive to A than k_j has a neo-classical interpretation. New capital comes from wages, and PSS wages increase with A for two reasons. First, A directly increases labor productivity, and, second, it fosters physical capital accumulation which also increases labor productivity.

The same general logic applies to the case where $\delta = 1$, even though the PSS elasticity of e with respect to A is now:

$$\varepsilon^j_{eA} = \lambda(1 + \alpha\varepsilon^j_{kA}) - (1 - \lambda)\varepsilon^j_{kA},$$

where $\lambda \in [0, 1]$ is the share of retained profits in the PSS level of aggregate banking capital. (see the appendix for the details.)

4.1.5 Short-term capital requirement adjustments

I consider here the optimal short-term adjustments in capital requirement. In particular, I look at one period responses to deviations from PSS, which I abusively call "short-run productivity elasticities":

$$\triangle^j_{eA} \equiv \frac{\frac{e_{t+1}(A_i,e_j,k_j)-e_j}{e_j}}{\frac{A_i-A_j}{A_j}}, \quad \text{and} \quad \triangle^j_{kA} \equiv \frac{\frac{k_{t+1}(A_i,e_j,k_j)-k_j}{k_j}}{\frac{A_i-A_j}{A_j}}.$$

The optimal adjustment in capital requirement is then:

$$\triangle^j_{xA} \simeq \triangle^j_{eA} - \triangle^j_{kA}.$$

It is easy to show that:

$$\begin{cases} \triangle^L_{eA} = \triangle^H_{eA} = 1 & ; \delta = 1 \\ \triangle^L_{eA} > 1 & ; \delta < 1 \\ \triangle^H_{eA} > 0 & ; \delta < 1 \end{cases}$$

$$\triangle^j_{kA} \simeq \frac{\varepsilon^j_{\bar{A}A}}{1 - \alpha}$$

PROPOSITION 5: *The sign of $\triangle^j_{xA}$, the optimal short-term adjustment that follow a productivity shock, depends on parameter values.*

Proof: see Appendix.

This result complements Proposition 4. Whereas the elasticity of e with respect to A is larger than that of k^* in the long-run, it may be the opposite in the short run. This depends on three elements:

- Technology: Parameter α and the persistence of the shock (ω_{jj}) both tend to increase $\triangle_{kA}$. Note the limit cases:
  - if $\varepsilon_{\bar{A}A} = 1$ (permanent shocks), $\triangle_{kA} = \frac{1}{1-\alpha}$, as in the static model.
  - if $\varepsilon_{\bar{A}A} = 0$ (iid shocks), $\triangle_{kA} = 0$, the optimal physical capital stock is constant.
- Banking capital accumulation, which is affected by banker exit rate (δ). The higher it is, the lower $\triangle_{eA}$.
- Whether banks go bust when a bad shock occur at PSS H. In this case,$\triangle^H_{eA}$ is affected by limited liability (the drop in e is less sharp than it would if negative equity would be transferred to the next period). I expose and interpret the different cases in the next subsection in the context of the risk-averse case.

4.2 The risk-averse case

In the risk-neutral case, the loss-absorbing power of banking capital is irrelevant to social welfare. It therefore does not affect k^*, the socially optimal level of physical capital. In this subsection, I consider the case where savers are risk-averse and, therefore, where the level of aggregate banking capital affects k^*.

Now, assuming that k_t^* can be expressed as a function of both $e_t(\bar{A}_{t-1})$ and $\bar{A}_{t-1}$, I have:

$$\varepsilon_{xA} = \varepsilon_{eA} - (\varepsilon_{kA} + \varepsilon_{ke}\varepsilon_{eA}),$$

where the second term in the parenthesis captures the idea that the size of the banking capital buffer affects the socially optimal level of aggregate lending. If ε_{ke} is positive, a larger buffer allows for an expansion of aggregate lending. On the other hand, a smaller buffer puts the tax-payers more at risk, and calls therefore for some banking sector deleveraging, which has a negative impact on expected output.

The purpose of this section is the following:

- Exploring the trade-off between tax-payer exposure and expected output.
- Test the robustness of the risk-neutral case predictions regarding the cyclical properties of optimal capital requirements. In particular, I am interested in seeing whether
  - the "loss absorbing channel" I have just described more than offsets the difference between ε_{eA} and ε_{kA}.
  - there are dynamic effects that overturn the results.

4.2.1 Specific assumptions

Preferences. I consider the following function for savers utility:

$$u(c_{t+1}) = w_t + \frac{(c_{t+1} - w_t + 1)^{1-\rho} - 1}{1-\rho},$$

where $\rho > 0$ [11]. This formulation implies that, given their wage, consumer utility is concave in consumption. It corresponds to a normalization, so that marginal utility and its curvature is the same for all generation when no "bailout" occurs. The main reasons for doing this is that it rules out situations in which the regulator would like the banks to fund negative NPV investment for redistributive purpose.[12]

REMARKS:

1. Substituting $c_{t+1} = w_t - b_{t+1}$, one gets:

$$u(w_t, b_{t+1}) = w_t + \frac{(1 - b_{t+1})^{1-\rho} - 1}{1-\rho}.$$

Savers utility is linear in their wage, but they incur a convex utility cost of taxation. This formulation has, hence, the second advantage that it also corresponds to a model where savers are risk-neutral, but there are

[11] If $\rho = 1$, one should substitute the logarithmic function to the power one: $u(w, b) = w + \ln(1 - b)$.

[12] Over-investment in capital generates higher expected wages, which is a way to transfer resources from bankers to workers. I study this case in an extension. (To do).

convex distortion costs of taxation. Obviously, the qualitative results would also be the same in a model where taxation is not distortive, but bank failures have convex deadweight costs.[13]

2. When $\rho = 0$, we are in the risk-neutral case.

Technology. A_t follows the same Markov process as in the risk-neutral case.

4.2.2 Methodology

I solve the risk-aversion case numerically through value function iterations (see Appendix for a description of the solution algorithm).

4.2.3 Main results

- The results of the risk-neutral case are confirmed for any reasonable level of risk aversion.
- Risk-aversion affects the average level of optimal capital requirements, but has little impact on their cyclical properties.

4.3 Examples

In this subsection, I present a series of examples that:

1. Confirm and illustrate the insights of the risk-neutral case in presence of risk-aversion.
2. Confirm the result of the static model that except when both risk and risk aversion are very large, they do not significantly alter the cyclical behavior of optimal capital requirement (to do).

I first give three examples for different values of δ. The baseline value for the other parameters are: $\rho = 2$, $\omega_{HH} = \omega_{LL} = 0.95$, $\beta = 0.98$, $\eta = 0.05$, $A_H = 1.09$, $A_L = 0.91$. Persistence is high (0.95) so that it is easy to visually identify Pseudo Steady States.

4.3.1 Case 1: higher requirements in good times with gradual short-term adjustment

Description. Figure 1 depicts a typical simulation result for $\delta = 0.94$. One can clearly see that capital requirements are higher in good times (when A is high), and in particular they are higher at the high PSS than at the low one (this corresponds to Proposition 3).

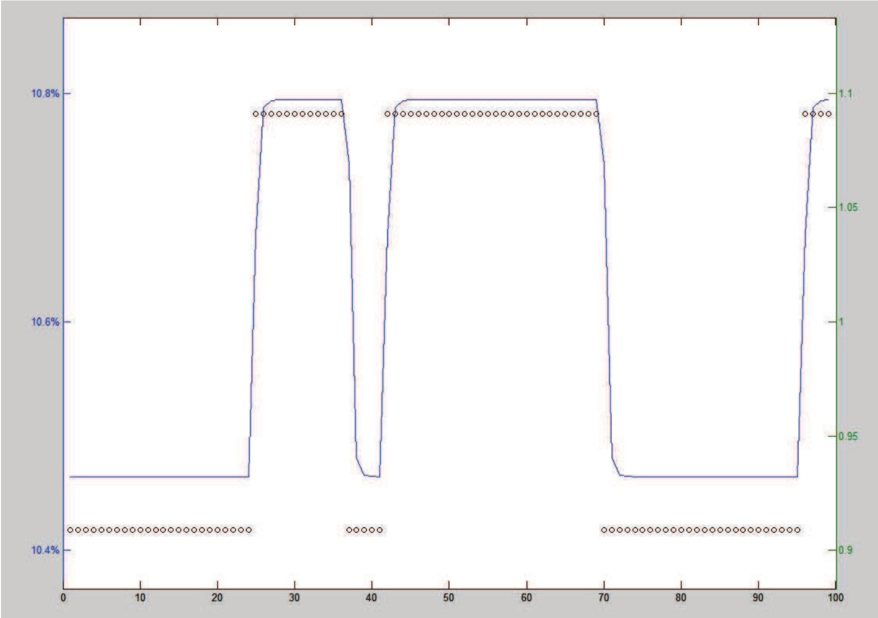
In this particular case, transition dynamics are smooth and gradual. It means that the short-run productivity elasticity of e is larger than that of k^* but lower than its own long-run elasticity. It corresponds therefore to:

$$\varepsilon_{kA}^{j} < \triangle_{eA}^{j} < \varepsilon_{eA}^{j} \tag{8}$$

[13] For instance if z dollar of tax would be needed per dollar of shortfall of bank value, with:

$$z = \frac{(1 - v^{-})^{1-\rho} - 1}{1 - \rho}.$$

Figure 1: Higher requirements in good times ($\delta = 0.94$)



This figure depicts a typical simulation result (100 periods) for the considered set of parameters. The solid line corresponds to $x^*(A_{t-1}, e_t)$, whose values are on the left scale. Small circles give the realizations of A_{t-1}, which determines $\bar{A}_t$, and whose values are on the right scale. Clearly, PSS requirements are higher in good times, and transition in x^* is gradual.

Interpretation. This is the most intuitive case. For instance, starting from the low PSS, a positive shock directly shifts k^* to its high PSS value k^*_H. Banking capital, however, takes a few periods to gradually reach its new PSS level. Still, the initial percentage increase in e is larger than that of k^* (the first inequality in (8)), which explains why x^* should go up. To sum up: economic prospects are better (which calls for a decrease in x), but aggregate banking capital has increased, and more than compensates.

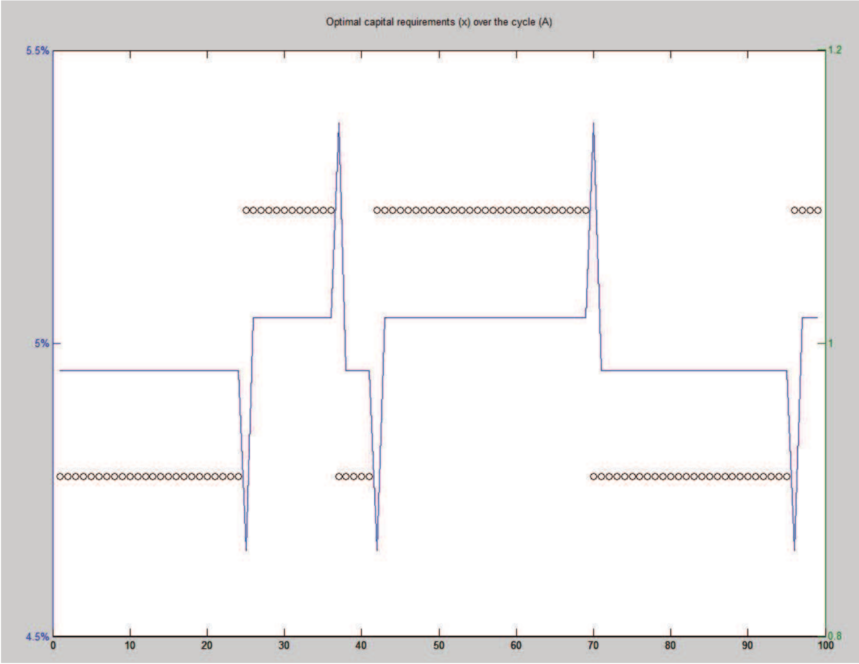
4.3.2 Case 2: opposite short-term adjustment

Description. Figure 2 depicts the limit case when $\delta = 1$. A higher δ means a lower productivity-elasticity of bank capital. In fact, the increase in banking capital is proportionally smaller than that of short-term optimal physical capital, and the optimal policy response is hence to first decrease capital requirements, before to hike them up to the high PSS level. It corresponds therefore to:

$$\triangle^j_{eA} < \varepsilon^j_{kA} < \varepsilon^j_{eA}$$

Interpretation In this case, the only source of banking capital is fresh capital that comes from the wages of future bankers. From a low state, wages increase in response to a positive shock (wages are "risky"). However, the capital stock that determines labor marginal productivity is the one of the low PSS. Wages are therefore still below their new PSS level (wage transition takes in fact two periods). Since persistence is high (0.95), optimal physical capital stock elasticity is high, and the optimal response to the shock is hence to decrease

Figure 2: Opposite short-term adjustments ($\delta = 1$)



In this case again, PSS requirements are higher in good times. The short-run optimal response to a (positive) shock is however to first decrease (increase) capital requirements. This is due to the low productivity-elasticity of banking capital.

capital requirements for one period. Note that this decrease lasts for one period only, because wage transition takes two periods and there is no other source of banking capital.

4.3.3 Case 3: overshooting short-term adjustment

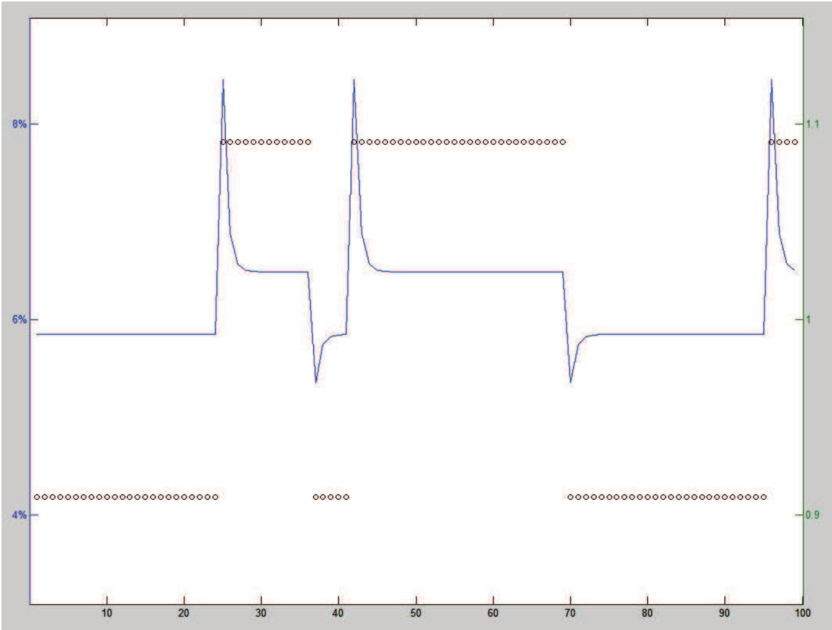
Description Figure 3 depicts a case ($\delta = 0.8$) where optimality requires to "overshoot the capital requirements". In fact, for this value of δ, the short-run productivity-elasticity of banking capital is actually larger than its long-run elasticity. Since the optimal physical capital short- and long-run elasticities are identical, it implies that capital requirements should increase more in the short-run.

$$\varepsilon^{j}_{kA} < \varepsilon^{j}_{eA} < \triangle^{j}_{eA}$$

Interpretation What happens is that there is fewer exit in the banking sector, which increases substantially the productivity-elasticity of banking capital. Wages still take two periods to adjust, but a significant fraction of banking capital comes from retained profits.

From a low state, after a positive shock, profits are unusually high. They are higher than at the high PSS because the stock of physical capital has not yet adjusted. The relatively low wages reflect in fact the scarcity rent earned by the owner of capital). After a period, physical capital adjusts, and factors' marginal productivity will be at their PSS levels. However, accumulated past profits still make aggregate banking capital larger than

Figure 3: Overshooting short-term adjustments ($\delta = 0.8$)



In this case, the short-run optimal response to a (positive) shock is however to overshoot the capital requirements. This is due to the high productivity-elasticity of banking capital implied by the relatively low value of δ.

its new PSS level for a while. This explain the gradual downward adjustment after the overshooting.

After a negative shock, the whole banking sector is wiped out, and the only source of banking capital in the first following period comes from the wage of new bankers. This time wages will be higher than PSS level because they will reflect high marginal productivity due to (ex-post) excessive capital. Losses are however capped by limited liability, which mitigates the mechanism after a crash and explains the asymmetry (note that crashes generate here bailouts). If the accumulation channel is really important, then banking capital will be lower than at the low steady state. Capital requirements should then go down (remember that physical capital takes one period only to adjust[14]).

5 Discussion and policy insights

INSIGHT 1: *Optimal capital requirements are increasing in aggregate banking capital.*

The key driver to this result is that decreasing marginal return to physical capital translates into decreasing marginal return to lending. To see the intuition on how the general equilibrium effect operates, first consider a single (atomistic) bank that doubles the size of its initial capital. It should simply be allowed to double the size of its assets. However, if all banks in the economy double the size of their initial capital, and if they are allowed to double the size of their assets, this will double aggregate lending in the economy. Given the decreasing marginal return to physical capital on the real side of the economy, this implies a substantially lower marginal return to lending. In fact. the banking sector should be allowed to expand, but less than proportionally, which

[14] An interesting extension would of course be to consider adjustment costs in physical. This is outside the scope of this paper.

corresponds to an increase in capital requirements.

If this general equilibrium effect is overlooked by the regulator, for instance if capital requirement adjust to expected productivity but not to aggregate banking capital (as is for instance the case of Basel II), regulation will magnify the business and financial cycles. There will be over-investment (negative NPV) during booms, and systemic risk will build up (the economy will be more fragile to productivity shocks). Then, if the realization of the shock is not good enough, the banking sector collapses and the credit crunch that ensues is inefficiently severe.

In the model, the indicators of systemic risk are a low expected quality of the marginal loan but high profits in the financial sector (relative to the rest of the economy).

INSIGHT 2: *The joint dynamics of aggregate banking capital and the socially optimal level of aggregate lending are key to financial stability.*

The model delivers optimal capital requirements that are higher in good times. This need not be the case in reality as it hinges on the assumptions that determine the law of motion for e and k^*. However, that these two variables are key to the stance of capital regulation is more general and would extend to other models were banks enjoy government guarantees, banking capital is scarce, and bank activity as an impact on the quality of the marginal loan in the economy.

INSIGHT 3: *Regulatory versus economic capital*

In the model, there is no asymmetry of information, and all loans last 1 period only. There is therefore no difference between economic and regulatory capital. However, in reality, there can be differences as the book value of equity (as computed for regulatory purpose) depends on how long term loans are valued. In particular, loan provisions play a huge role. In terms of the model, one can use the following equation to think about the issue:

$$\varepsilon_{xA} = \varepsilon_{eA} - (\varepsilon_{kA} + \varepsilon_{ke}\varepsilon_{eA})$$

The term ε_{eA} reflects regulatory capital: this comes from the numerator to which x will apply. On the other hand, $\varepsilon_{ke}\varepsilon_{eA}$ reflects economic capital. It captures the "loss absorbing" effect of bank capital on the socially optimal level of aggregate lending.

6 Extensions

6.1 A comparison with "through the cycle" capital requirements (to update)

I present here a comparison with a constant (or through-the-cycle) capital requirement.

6.1.1 Exercise

The exercise is the following: I restrict the regulator to the choice of a single number x^{fixed}, which can therefore not be contingent on e, w, and A. To find the "optimal" x^{fixed}, I simulate the economy for different values of this number and let the regulator pick the one that maximizes the average utility of 10000 generations.

Remark:

When lending is very high, each banker has still an incentive to increase leverage, but the sector becomes unprofitable at some point, so that we no longer are in a full-reinvestment equilibrium. In such a case, some bankers simply exit and chose to consume (or store if they just became potential bankers). The bankers that do not exit still find it optimal to leverage to the max. The measure of active bankers is undetermined, but what matters is "active banking capital", which is pined down by the condition that bankers leveraging up to x^{fixed} make no profit in expectation. Thus bankers are indifferent whether to exit or not.

6.1.2 Results

Description Quite intuitively, the best constant capital requirement seems close to an average of the contingent PSS optimal requirements.

First, note the first two following results:

- Lending is more volatile under constant capital requirement, which illustrates that such regulation can have excessive "pro-cyclical" effects (in the sense that it exacerbates the cycle).
- Negative present value (when discounted at the rare of return to storage) loans are issued during booms. This is what I call "extremely excessive credit expansion". "Excessive credit expansion" would correspond to too much lending with respect to the social optimum, but the loans still being positive net present value when discounted at the rate of return to storage.

6.2 Outside equity (sketch)

- I consider the possibility for bankers to issue outside equity (sold to depositors), and I show that it does not affect the qualitative results (comparative statics and cyclical properties of optimal requirements).
- The intuition is the following:
  - When buying a share of bank equity, depositor acquire a risky asset, whose risk is positively correlated with their marginal utility (which is obvious: equity is wiped out precisely in the states where taxation is positive).
  - At the social optimum, bank equity is more risky in bad times. Therefore, it is intuitive that depositors will not buy relatively more equity in bad times, and aggregate banking capital is not likely to become less pro-cyclical.
- The extension I consider is the following:
  - At the beginning of the period, the regulator announces a capital requirement (the regulator anticipates what the constrained competitive outcome will then be).
  - Bankers competitively issue outside equity.
- Remarks
  - At a given level of (their own individual) lending, bankers have no incentive to raise equity, which would strictly dilute their claims (all the more that buyers are risk-averse). Therefore, raising equity is done in order to scale up.

– Bankers do not internalize the effect it has on the marginal return to lending.

– The sequence of the events above could be reversed. This is because depositors and bankers act competitively, they would therefore not internalize the effect of their decision on capital requirement if the requirement was announced ex-post.

– The statement that bank equity is more risky in bad times as to be understood at the social optimum. It means that, conditional on the optimal capital requirement, a marginal increase in aggregate bank lending commands a higher premium in bad times than in good times.

– Depositors buying equity do not internalize the effect on future taxation. The competitive equilibrium with outside equity does therefore also imply over-investment.

– From a welfare point of view, the direct effect of equity issuance is not positive. For a given level of aggregate investment, swapping deposits with equity increases expected consumption of depositors. However, this extra consumption materializes in states where there is no bailout tax, and depositors marginal utility of consumption becomes strictly smaller than 1. Bankers claims are diluted by exactly as much as the increase in depositor consumption, and their marginal utility is constant and equal to 1.

6.3 Prices versus quantities (sketch)

- There is no information asymmetry in the model.The social optimum can therefore be equivalently implemented by many different tools. As explained in the introduction, the focus on capital requirements is mainly motivated by the current policy debate. In this subsection I show that:

  – The social optimum could also for instance be implemented through

    * an increasing (and time varying) tax on leverage (which becomes steep enough at the inverse of the capital requirement).

    * a flat (and time varying) tax on lending (or deposit insurance premium) together with a subsidy on bank equity.

  – The second example, however, does only work assuming a symmetric equilibrium. Otherwise, the policy should be complemented with a leverage cap (or a capital requirement) that rules out the following deviation: a banker consuming all its initial equity (she then forgoes the subsidy), but choosing a very high level of lending. At a given level of expected marginal return (provided that there is a state in which realized return is positive), there will be a level of leverage from which the option value of equity will more than compensate the loss of subsidy). The needed leverage cap need not be time-varying.

References

Acharya, Viral, "A theory of systemic risk and design of prudential bank regulation," *Journal of Financial Stability*, **2009,** *5* **(3), 224–255.**

Admati, A.R., P.M. DeMarzo, M.F. Hellwig, and P. Pfleiderer, "Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is Not Expensive," 2010.

Allen, Franklin, Ana Babus, and Elena Carletti, "Asset commonality, debt maturity and systemic risk," *Journal of Financial Economics*, 2011.

Bernanke, Ben and Mark Gertler, "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, March 1989, *79* (1), 14–31.

Bianchi, J., "Overborrowing and systemic externalities in the business cycle," *American Economic Review*, 2011, *101* (7), 3400–3426.

Borio, C. and M. Drehmann, "Assessing the risk of banking crises–revisited," *BIS Quarterly Review*, 2009, pp. 29–46.

Brunnermeier, M. and Y. Sannikov, "A macroeconomic model with a financial sector," *Unpublished manuscript, Princeton University*, 2010.

den Heuvel, Skander J Van, "The welfare cost of bank capital requirements," *Journal of Monetary Economics*, 2008, *55* (2), 298–320.

Dewatripont, M. and J. Tirole, "Macroeconomic Shocks and Banking Regulation," *Journal of Money, Credit and Banking*, 2012. Forthcoming.

Dewatripont, Mathias and Jean Tirole, *The Prudential Regulation of Banks*, MIT Press, 1994.

Farhi, Emmanuel and Jean Tirole, "Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts," *American Economic Review*, 2012, *102*, 60–93.

Fisher, I., "The debt-deflation theory of great depressions," *Econometrica*, 1933, pp. 337–357.

Galati, G. and R. Moessner, "Macroprudential policy-a literature review," 2011.

Gersbach, H. and J.C. Rochet, "Aggregate investment externalities and macroprudential regulation," Technical Report, mimeo 2011.

Goodhart, Charles, "The role of macro-prudential supervision," in "presented at the Financial Markets Conference of the Federal Reserve Bank of Atlanta, Georgia" 2010.

Hanson, S., A. Kashyap, and J. Stein, "A macroprudential approach to financial regulation," *Journal of Economic Perspectives*, Winter 2011, *25* (1), 3–28.

Hellwig, M., "Capital Regulation after the Crisis: Business as Usual?," *Preprints fo the Max Planck Institute for Research on Collective Goods, Bonn*, 2010, *31.*

Jeanne, Olivier and Anton Korinek, "Managing credit booms and busts: A Pigouvian taxation approach," 2010.

— and — , "Macroprudential regulation versus mopping up after the crash," Technical Report, National Bureau of Economic Research 2013.

Jimenez, Gabriel, Seven Ongena, Jesus Saurina, and Jose-Luis Peydro, "Macroprudential Policy, Countercyclical Bank Capital Buffers and Credit Supply: Evidence from the Spanish Dynamic Provisioning Experiments," Technical Report 2012.

Kareken, J.H. and N. Wallace, "Deposit insurance and bank regulation: A partial-equilibrium exposition," *Journal of Business*, **1978, pp. 413–438.**

Kashyap, A.K. and J.C. Stein, "Cyclical implications of the Basel II capital standards," *ECONOMIC PERSPECTIVES-FEDERAL RESERVE BANK OF CHICAGO*, **2004,** *28* **(1), 18–33.**

Keeley, Michael C, "Deposit insurance, risk, and market power in banking," *The American Economic Review*, **1990, pp. 1183–1200.**

Kiyotaki, Nobuhiro and John Moore, "Credit Cycles," *Journal of Political Economy*, **April 1997,** *105* **(2), 211–248.**

Korinek, Anton, "Systemic risk-taking: Amplification effects, externalities, and regulatory responses," 2011. ECB Working paper No. 1345, University of Maryland.

Martinez-Miera, D. and J. Suarez, "A Macroeconomic Model of Endogenous Systemic Risk Taking," 2012. mimeo, CEMFI.

Merton, Robert, "An analytic derivation of the cost of deposit insurance and loan guarantees An application of modern option pricing theory," *Journal of Banking and Finance*, **1977,** *1*, **3–11.**

Morrison, Alan D and Lucy White, "Crises and capital requirements in banking," *American Economic Review*, **2005, pp. 1548–1572.**

Pennacchi, George, "Deposit insurance, bank regulation, and financial system risks," *Journal of Monetary Economics*, **2006,** *53* **(1), 1–30.**

Repullo, R. and J. Suarez, "The procyclical effects of Bank Capital Regulation," *Review of Financial Studies*, **2012. Forthcoming.**

Santos, Joao AC, "Bank capital regulation in contemporary banking theory: A review of the literature," *Financial Markets, Institutions & Instruments*, **2001,** *10* **(2), 41–84.**

Stein, Jeremy, "Monetary policy as financial-stability regulation," *Quarterly Journal of Economics*, **2012,** *127*, **57–95.**

Woodford, Michael, "Financial intermediation and macroeconomic analysis," *The Journal of Economic Perspectives*, **2010,** *24* **(4), 21–44.**

A Proofs

A.1 Proof of Lemma 3

I show here that maximizing the welfare function

$$E\left[\sum_{t=1}^{\infty}\eta u_t^b + (1-\eta)u_t\right], \tag{9}$$

with respect to $\{x_t\}_{t=0}^{\infty}$ (or equivalently to $\{k_t\}_{t=0}^{\infty}$) boils down to maximizing expected output at each period.

We have: $u_t = w_{t-1} - v_t^- /(1-\eta)$ and $u_t^b = \sum_{s=1}^{\infty} c_{t,t+s}^b$.

Step 1: Note that $u_t = (1-\eta)w_{t-1} - v_t^-$.

Step 2: Note that $\sum_{t=1}^{\infty}\sum_{s=1}^{\infty} c_{t,t+s}^b = \sum_{t=1}^{\infty} C_t^b$, where C_t^b is the aggregate consumption of bankers in t, that is: $C_t^b = \delta v_t^+ = \eta u_t^b$ (there is a measure η/δ of bankers, δ of them die, which is a measure η; therefore, $u_t^b = \delta v_t^+/\eta$).

Hence, the welfare function becomes:

$$E\left[\sum_{t=1}^{\infty}\delta v_t^+ + (1-\eta)w_{t-1} - v_t^-\right]$$

or

$$E\left[\sum_{t=1}^{\infty}\delta\left(\alpha\theta_t k_t^{\alpha} - k_t + e_t\right)^+ + (1-\eta)w_{t-1} - \left(\alpha\theta_t k_t^{\alpha} - k_t + e_t\right)^-\right]$$

Step 3:Assume that date s is the first date (after $t-1$) at which v_t is negative. That is $v^- = 0$ for all $t < s$, and $v^+ = 0$ for $t = s$.

Let me write down all the terms in this sum that depend on k_t (directly, and through e_{t+1}):

$$e_{t+1} = (1-\delta)\left(\alpha\theta_t k_t^{\alpha} - k_t + e_t\right) + \eta(1-\alpha)\theta_t k_t^{\alpha}$$

		dividend	wage	taxe
t		$\delta\left(\alpha\theta_t k_t^{\alpha} - k_t + e_t\right)$	...	0
$t+1$		$\delta(... + e_{t+1})$	$(1-\eta)(1-\alpha)\theta_t k_t^{\alpha}$	0
$t+2$		$\delta(... + \underbrace{(1-\delta)\left(\alpha\theta_{t+1}k_{t+1}^{\alpha} - k_{t+1} + e_{t+1}\right) + ...}_{e_{t+2}})$	...	0
...		...	...	0
$t+n$		$\delta(1-\delta)^{n-1}\left(e_{t+1}\right)$	...	0
$t+s$		0	...	$-(... - (1-\delta)^{s-1}e_{t+1})$

Summing them gives:

$$\delta\left(\alpha\theta_t k_t^{\alpha} - k_t + \sum_{n=0}^{s-1}(1-\delta)^n e_{t+1}\right) + (1-\eta)(1-\alpha)\theta_t k_t^{\alpha} + (1-\delta)^{s-1}e_{t+1}$$

Step 4: Noting that

$$\sum_{n=0}^{s-1}(1-\delta)^n = \sum_{n=0}^{\infty}(1-\delta)^n - \sum_{n=s-1}^{\infty}(1-\delta)^n = \frac{1}{\delta} - \frac{(1-\delta)^{s-1}}{\delta},$$

it appears that the coefficients of the terms in e_{t+1} sum to 1. Then, using

$$e_{t+1} = (1-\delta)\left(\alpha\theta_t k_t^{\alpha} - k_t + e_t\right) + \eta(1-\alpha)\theta_t k_t^{\alpha}$$

and rearranging, one finds easilly that:

$$e_{t+1} + \theta_t k_t^{\alpha}\left(\alpha\delta + (1-\eta)(1-\alpha)\right) - \delta k_t = \theta_t k_t^{\alpha} - k_t \quad \square$$