Understanding the Equity-premium and Correlation Puzzles

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June 2012

The correlation puzzle

- The covariance and correlation between stock returns and measurable fundamentals, especially consumption, is weak at the 1, 5, and 10 year horizons.
- This fact underlies virtually all modern asset-pricing puzzles.
    - The equity premium puzzle, Hansen-Singleton-style rejection of asset pricing models, Shiller's excess volatility of stock prices, etc.
- Hansen and Cochrane (1992) and Cochrane and Campbell (1999) call this phenomenon the "correlation puzzle."

Asset prices and economic fundamentals

- Classic asset pricing models load all uncertainty onto the supply-side of the economy.
  - Stochastic process for the endowment in Lucas-tree models.
  - Stochastic process for productivity in production economies.
- These models abstract from shocks to the demand for assets.
- It's not surprising that models with only supply shocks can't simultaneously account for the equity premium puzzle and correlation puzzles.

Fundamental shocks

- What's the other shock?

- We explore the possibility that it's a shock to the demand for assets.

Shocks to the demand for assets

- We model the shock to the demand for assets in the simplest possible way: time-preference shocks.

- Macro literature on zero lower bound suggests these shocks are a useful way to model changes in household savings behavior.
    - e.g. Eggertsson and Woodford (2003).

- These shocks also capture effects of changes in the demographics of stock market participants or other institutional changes that affect savings behavior.

Key results

- The model accounts for the equity premium and the correlation puzzle (taking sampling uncertainty into account).
  - It also accounts for the level and volatility of the risk free rate.

- The model's estimated risk aversion coefficient is very low (close to one).

- This finding is consistent with Lucas' conjecture about fruitful avenues to resolve the equity premium puzzle.

"It would be good to have the equity premium resolved, but I think we need to look beyond high estimates of risk aversion to do it."
Robert Lucas, Jr., "Macroeconomic Priorities," American Economic Review, 2003.

Empirical counterpart of time-preference shocks

- Our model implies that preference shock is a scaled version of the risk free rate.
- In our benchmark model, the estimated variance of the preference shock process equals the variance of the risk free rate (taking sampling uncertainty into account).
- An augmented version of the model also matches the persistence of the risk-free rate.

Key results

- Model with Epstein-Zin preferences and no time-preference shocks
    - Can't account for the equity premium or the correlation puzzle.
- CRRA preferences with or without time-preference shocks.
    - Can't account for the equity premium or the correlation puzzle.
- Bansal, Kiku and Yaron (2011)
    - Can account for the equity premium puzzle with a risk aversion coefficient of 10.
    - Can't account for the correlation puzzle.

Trade-offs

- On the one hand, we introduce a new source of shocks into the model.
- On the other hand, our model is simpler than many alternatives.
- We assume that consumption and dividends are a random walk with a homoskedastic error term.
- We don't need:
  - Habit formation, long-run risk, time-varying endowment volatility, model ambiguity.
  - Any of these features could be added.
- Straightforward to modify DSGE models to allow for these shocks.

The importance of Epstein-Zin preferences

- Just introducing time-preference shocks isn't sufficient to generate an equity premium.
- For time-preference shocks to improve the model's performance, it's critical that agents have Epstein-Zin preferences and that risk aversion is larger than the inverse of the EIS.
- Introducing time-preference shocks in a model with CRRA preferences is counterproductive.
- In the CRRA case, the equity premium is a *decreasing* function of the variance of time-preference shocks.

The correlation puzzle

- We use data for 17 OECD countries and 7 non-OECD countries, covering the period 1871-2006.

- Correlations between stock returns and consumption, as well as correlations between stock returns and output are low at all time horizons.
  - The correlation puzzle for consumption is even *worse* if we restrict ourself to the post-1929 period.

- The correlation between stock returns and dividend growth is substantially higher for horizons greater than 10 years, but it's similar to that of consumption at shorter horizons.

Historical data

- Sample: 1871-2006.
- Nakamura, Steinsson, Barro, and Ursúa (2011) for stock returns.
- Barro and Ursúa (2008) for consumption expenditures and real per capita GDP.
- Shiller for real S&P500 earnings and dividends.

Historical data

- We use realized real stock returns.
- As in Mehra and Prescott (1985) and the associated literature, we measure the risk free rate using realized real returns on nominal, one-year Treasury Bills.
- This measure is far from perfect because there is inflation risk, which can be substantial.

The correlation puzzle

Correlation between real stock market returns and the growth rate of fundamentals, United States

	Full sample, 1871-2006				1930-2006			
	Consumption	Output	Dividends	Earnings	Consumption	Output	Dividends	Earnings
1 year	0.090	0.136	-0.039	0.126	-0.017	0.073	-0.087	0.100
	(0.089)	(0.101)	(0.0956)	(0.1038)	(0.120)	(0.127)	(0.155)	(0.157)
5 years	0.397	0.249	0.382	0.436	-0.093	-0.047	0.277	0.197
	(0.177)	(0.137)	(0.148)	(0.179)	(0.111)	(0.125)	(0.122)	(0.140)
10 years	0.248	-0.001	0.642	0.406	-0.416	-0.270	0.722	0.287
	(0.184)	(0.113)	(0.173)	(0.125)	(0.181)	(0.192)	(0.190)	(0.109)

Using NIPA measures of consumption, 1952-2006

Horizon	Durables	Non-durables	Services
1 year	0.133 (0.089)	0.289 (0.124)	0.035 (0.010)
5 years	0.185 (0.098)	0.249 (0.140)	−0.141 (0.179)
10 years	0.127 (0.201)	0.106 (0.230)	−0.497 (0.134)

-

The correlation puzzle

Correlation between real stock market returns and growth rate of fundamentals
G7 and non G7 countries

	Full sample, 1871-2006				1930-2006			
	G7 countries		Non G7 countries		G7 countries		Non G7 countries	
	Consumption	Output	Consumption	Output	Consumption	Output	Consumption	Output
1 year	0.008	0.182	0.050	0.089	-0.020	0.196	0.039	0.076
	(0.062)	(0.081)	(0.027)	(0.031)	(0.077)	(0.102)	(0.030)	(0.034)
5 years	0.189	0.355	0.087	0.157	0.121	0.338	0.064	0.125
	(0.105)	(0.092)	(0.069)	(0.074)	(0.141)	(0.119)	(0.077)	(0.082)
10 years	0.277	0.394	0.027	0.098	0.252	0.402	-0.003	0.063
	(0.132)	(0.119)	(0.122)	(0.130)	(0.179)	(0.151)	(0.135)	(0.143)

A model with time-preference shocks

- Epstein-Zin preferences
  - Life-time utility is a CES of utility today and the certainty equivalent of future utility, U^*_{t+1}.

$$U_t = \max_{C_t} \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U^*_{t+1} \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}$$

- λ_t determines how agents trade off current versus future utility, isomorphic to a time-preference shock.

- ψ is the elasticity of intertemporal substitution.

A model with time-preference shocks

$$U_t = \max_{C_t} \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U_{t+1}^* \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}$$

- The certainty equivalent of future utility is the sure value of $t+1$ lifetime utility, U_{t+1}^* such that:

$$\left(U_{t+1}^* \right)^{1-\gamma} = E_t \left(U_{t+1}^{1-\gamma} \right)$$

$$U_{t+1}^* = \left[E_t \left(U_{t+1}^{1-\gamma} \right) \right]^{1/(1-\gamma)}$$

- γ is the coefficient of relative risk aversion.

Special case: CRRA

$$U_t = \max_{C_t} \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U_{t+1}^* \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}$$

- When $\gamma = 1/\psi$, preferences reduce to CRRA with a time-varying rate of time preference.

  $$V_t = E_t \sum_{i=0}^{\infty} \delta^i \lambda_{t+i} C_{t+i}^{1-\gamma},$$

  where $V_t = U_t^{1-\gamma}$.

- Case considered by Garber and King (1983) and Campbell (1986).

Stochastic processes

- Consumption follows a random walk

$$
\begin{aligned}
\log(C_{t+1}) &= \log(C_t) + \mu + \eta^c_{t+1} \\
\eta^c_{t+1} &\sim N(0, \sigma^2_c)
\end{aligned}
$$

- Process for dividends:

$$
\begin{aligned}
\log(D_{t+1}) &= \log(D_t) + \mu + \pi\eta^c_{t+1} + \eta^d_{t+1} \\
\eta^d_{t+1} &\sim N(0, \sigma^2_d)
\end{aligned}
$$

Stochastic processes

- Time-preference shock:

$$\log\left(\lambda_{t+1}/\lambda_t\right) = \rho \log\left(\lambda_t/\lambda_{t-1}\right) + \varepsilon_{t+1}$$
$$\varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2)$$

- We assume that agents know λ_{t+1} at time t.

- What matters for agents' decisions is the growth rate of λ_t, which we assume is highly persistent but stationary (ρ is very close to one).

- The idea is to capture, in a parsimonious way, persistent changes in agents' attitudes towards savings.

- We assume that ε_{t+1} is uncorrelated with η_{t+1}^c and η_{t+1}^d.
    - This assumption is reasonable for an endowment economy but not for a production economy.

Solving the model

- Returns to the stock market are defined as returns to claim on dividend process:
  - Standard assumption in asset-pricing literature (Abel (1999)).
- Realized gross stock-market return:

$$R^d_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}.$$

- Define:

$$\begin{aligned} r_{d,t+1} &= \log(R^d_{t+1}), \\ z_{dt} &= \log(P_t / D_t). \end{aligned}$$

Solving the model

- Realized gross return to a claim on the endowment process:

$$R^c_{t+1} = \frac{P^c_{t+1} + C_{t+1}}{P^c_t}.$$

- Define:

$$\begin{aligned} r_{c,t+1} &= \log(R^c_{t+1}), \\ z_{ct} &= \log(P^c_t / C_t). \end{aligned}$$

Solving the model

- Using a log-linear Taylor expansion:

$$\begin{aligned} r_{d,t+1} &= \kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1}, \\ r_{c,t+1} &= \kappa_{c0} + \kappa_{c1} z_{ct+1} - z_{ct} + \Delta c_{t+1}, \end{aligned}$$

$$\begin{aligned} \kappa_{d0} &= \log\left[1 + \exp(z_d)\right] - \kappa_{1d} z_d, \\ \kappa_{c0} &= \log\left[1 + \exp(z_c)\right] - \kappa_{1c} z_c, \end{aligned}$$

$$\kappa_{d1} = \frac{\exp(z_d)}{1 + \exp(z_d)}, \quad \kappa_{c1} = \frac{\exp(z_c)}{1 + \exp(z_c)}.$$

- z_d and z_c are the unconditional mean values of z_{dt} and z_{ct}.

Solving the model

- The log-SDF is:

$$m_{t+1} = \theta \log(\delta) + \theta \log(\lambda_{t+1}/\lambda_t) - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1},$$

$$\theta = \frac{1-\gamma}{1-1/\psi}.$$

- $r_{c,t+1}$ is the log return to a claim on the endowment,

$$r_{c,t+1} = \log(R_{t+1}) = \log\left(\frac{P_{t+1} + C_{t+1}}{P_t}\right).$$

- Euler equation:

$$E_t\left[\exp\left(m_{t+1} + r_{d,t+1}\right)\right] = 1$$

Solving the model

- Use Euler equation:

$$E_t \left[\exp\left(m_{t+1} + r_{d,t+1}\right)\right] = 1.$$

- Replace m_{t+1} and $r_{d,t+1}$ using equations:

$$m_{t+1} = \theta \log\left(\delta\right) + \theta \log\left(\lambda_{t+1}/\lambda_t\right) - \frac{\theta}{\psi}\Delta c_{t+1} + \left(\theta - 1\right) r_{c,t+1},$$

$$r_{d,t+1} = \kappa_{d0} + \kappa_{d1} z_{dt+1} - z_{dt} + \Delta d_{t+1}.$$

- Replace $r_{c,t+1}$ with:

$$r_{c,t+1} = \kappa_{c0} + \kappa_{c1} z_{ct+1} - z_{ct} + \Delta c_{t+1}.$$

Solving the model

- Guess and verify that the equilibrium solutions for z_{dt} and z_{ct} take the form:

$$\begin{aligned} z_{dt} &= A_{d0} + A_{d1} \log\left(\lambda_{t+1}/\lambda_t\right), \\ z_{ct} &= A_{c0} + A_{c1} \log\left(\lambda_{t+1}/\lambda_t\right). \end{aligned}$$

- Since consumption is a martingale, price-dividend ratios are constant absent movements in λ_t.

- In calculating conditional expectations use properties of lognormal distribution.

- Use method of indeterminate coefficients to compute A_{d0}, A_{d1}, A_{c0}, and A_{c1}.

The risk-free rate

$$
\begin{aligned}
r^f_{t+1} &= -\log(\delta) - \log(\lambda_{t+1}/\lambda_t) + \mu/\psi - (1-\theta)\kappa^2_{c1}A^2_{c1}\sigma^2_{\varepsilon}/2 \\
&\quad + \left[\frac{(1-\theta)}{\theta}(1-\gamma)^2 - \gamma^2\right]\sigma^2_c/2,
\end{aligned}
$$

$$
\theta = \frac{1-\gamma}{1-1/\psi}.
$$

- $\theta = 1$ when preferences are CRRA.
- The risk-free rate is a decreasing function of $\log(\lambda_{t+1}/\lambda_t)$.
  - If agents value the future more, relative to the present, they want to save more. Since aggregate savings can't increase, the risk-free rate has to fall.

Equity premium

$$
\begin{aligned}
r_{t+1}^{f} &= -\log(\delta) - \log(\lambda_{t+1}/\lambda_t) + \mu/\psi - (1-\theta)\kappa_{c1}^2 A_{c1}^2 \sigma_\varepsilon^2/2 \\
&\quad + \left[\frac{(1-\theta)}{\theta}(1-\gamma)^2 - \gamma^2\right]\sigma_c^2/2.
\end{aligned}
$$

$$
\begin{aligned}
E_t(r_{d,t+1}) - r_{t+1}^{f} &= \pi\sigma_c^2(2\gamma-\pi)/2 - \sigma_d^2/2 \\
&\quad + \kappa_{d1}A_{d1}\left[2(1-\theta)A_{c1}\kappa_{c1} - \kappa_{d1}A_{d1}\right]\sigma_\varepsilon^2/2.
\end{aligned}
$$

- It's cumbersome to do comparative statics exercises because κ_{c1} and κ_{d1} are functions of the parameters of the model.

Equity premium: CRRA case

- Suppose that $\theta = 1$:

$$r^f_{t+1} = -\log(\delta) - \log(\lambda_{t+1}/\lambda_t) + \mu/\psi - \gamma^2\sigma_c^2/2.$$

$$E_t(r_{d,t+1}) - r^f_{t+1} = \pi\sigma_c^2(2\gamma - \pi)/2 - \sigma_d^2/2 - \kappa_{d1}^2 A_{d1}^2 \sigma_\varepsilon^2/2.$$

- Including time-preference shocks in a model with CRRA utility lowers the equity premium!

Equity premium: CRRA case

- To get some intuition consider the case where the stock market is a claim to consumption ($\pi = 1$, $\sigma_d^2 = 0$) and $\log(\lambda_{t+1}/\lambda_t)$ is a random walk ($\rho = 1$).

- The Euler equation can be written as

$$\begin{aligned} \frac{P_t}{C_t} &= \alpha \exp(\sigma_\varepsilon \varepsilon_{t+1}) \left[E_t \left(\frac{P_{t+1}}{C_{t+1}} \right) + 1 \right] \\ \alpha &= \delta \exp \left[(1-\gamma)\, \mu + (1-\gamma)^2 \sigma_c^2 / 2 \right] \end{aligned}$$

- We used the fact that ε_{t+1} is known at time t and P_{t+1}/C_{t+1} depends only on $\log(\lambda_{t+2}/\lambda_{t+1})$.

Equity premium: CRRA case

- Recursing on P_t / C_t:

$$\frac{P_t}{C_t} = \alpha \exp(\sigma_\varepsilon \varepsilon_{t+1}) E_t \left[\begin{array}{c} 1 + \alpha \exp(\sigma_\varepsilon \varepsilon_{t+2}) \\ +\alpha^2 \exp(\sigma_\varepsilon \varepsilon_{t+2}) \exp(\sigma_\varepsilon \varepsilon_{t+3}) + \ldots \end{array} \right]$$

- Computing expectations:

$$\frac{P_t}{C_t} = \alpha \exp(\sigma_\varepsilon \varepsilon_{t+1}) \left[1 + \alpha \exp(\sigma_\varepsilon^2 / 2) + \alpha^2 \left[\exp(\sigma_\varepsilon^2 / 2) \right]^2 + \ldots \right]$$

- Assume that $\alpha \exp(\sigma_\varepsilon^2 / 2) < 1$ so price is finite.

Equity premium: CRRA case

$$\frac{P_t}{C_t} = \alpha \exp(\sigma_\varepsilon \varepsilon_{t+1}) \left[1 + \alpha \exp(\sigma_\varepsilon^2/2) + \alpha^2 \left[\exp(\sigma_\varepsilon^2/2)\right]^2 + \ldots\right]$$

- The price-consumption ratio is an increasing function of σ_ε^2.
  - This variance enters because the mean of a lognormal variable is increasing in the variance.
- An increase in σ_ε^2 raises the expected value of $\lambda_{t+1+j}/\lambda_{t+j}$, $j \geq 1$, so agents want to delay consumption.
- Expected returns have to fall to induce them to hold the tree.
- The risk-free rate is unaffected because agents know λ_{t+1} at time t.
- The last two observations imply that the equity premium is decreasing in σ_ε^2.

Equity premium: Epstein-Zin

$$
\begin{aligned}
E_t\left(r_{d,t+1}\right) - r^f_{t+1} &= \pi\sigma_c^2(2\gamma-\pi)/2 - \sigma_d^2/2 \\
&\quad + \kappa_{d1}A_{d1}\left[2\left(1-\theta\right)A_{c1}\kappa_{c1} - \kappa_{d1}A_{d1}\right]\sigma_\varepsilon^2/2.
\end{aligned}
$$

- Recall that:

$$
\begin{aligned}
r_{d,t+1} &= \kappa_{d0} + \kappa_{d1}z_{dt+1} - z_{dt} + \Delta d_{t+1}, \quad \kappa_{d1} = \frac{\exp(z_d)}{1+\exp(z_d)} \\
r_{c,t+1} &= \kappa_{c0} + \kappa_{c1}z_{ct+1} - z_{ct} + \Delta c_{t+1}, \quad \kappa_{c1} = \frac{\exp(z_c)}{1+\exp(z_c)}
\end{aligned}
$$

- Necessary condition for time-preference shocks to help explain the equity premium: $\theta < 1$ $(\gamma > 1/\psi)$.

- This condition is more likely to be satisfied for higher risk aversion, higher IES.

Estimating the parameters of the model

- We estimate the model using GMM.
- We find the parameter vector $\hat{\Phi}$ that minimizes the distance between the empirical, Ψ_D, and model population moments, $\Psi(\hat{\Phi})$,

$$L(\hat{\Phi}) = \min_{\Phi} \left[\Psi(\Phi) - \Psi_D\right]' \Omega_D^{-1} \left[\Psi(\Phi) - \Psi_D\right].$$

- Ω_D is an estimate of the variance-covariance matrix of the empirical moments.

The moments used in GMM

- The vector Ψ_D includes the following 14 moments:
  - Consumption growth: mean and standard deviation;
  - Dividend growth: mean, standard deviation;
  - Correlation between growth rate of dividends and growth rate of consumption;
  - Real stock returns: mean and standard deviation;
  - Real risk free rate: mean and standard deviation;
  - Correlation between stock returns and consumption growth (1, 5 and 10 years);
  - Correlation between stock returns and dividend growth (1, 5 and 10 years).

GMM estimation

- We constrain the growth rate of dividends and consumption to be the same.
- In estimating Ψ_D, we used a standard 2-step efficient GMM estimator
  - We use a Newey-West weighting matrix with 10 lags.
- Our procedure yields an estimate of Ω_D.

Estimated parameters

- Agents make decisions on a monthly basis. We compute moments at an annual frequency.

- The parameter vector, Φ, includes the 8 parameters:
    - γ: coefficient of relative risk aversion;
    - ψ: elasticity of intertemporal substitution;
    - δ: rate of time preference;
    - σ_c: volatility of innovation to consumption growth;
    - π: parameter that controls correlation between consumption and dividend shocks;
    - σ_d: volatility of dividend shocks;
    - ρ: persistence of time-preference shocks;
    - σ_ε: volatility of innovation to time-preference shocks.

- We set the growth rate of consumption and dividends to the first-stage GMM point estimate.

Parameter estimates, benchmark model, 1871-2006

Parameter	Estimates	Parameter	Estimates
γ	1.22 (0.04)	σ_d	0.019 (0.0007)
ψ	1.26 (0.67)	π	0.47 (0.08)
δ	0.999 (0.003)	σ_ε	0.00012 (0.00028)
σ_c	0.008 (0.0003)	ρ	0.9996 (0.002)
μ	0.00133 (0.00026)		

Moments, annual data and model

Moments	Data	Model
Std (Δd_t)	8.98 (1.33)	6.55
Std (Δc_t)	3.35 (0.40)	2.75
Corr$(\Delta c_t, \Delta d_t)$	0.21 (0.12)	0.20

Moments (annual), data and model

Moments	Data	Model	Moments	Data	Model
$E(R_t^d)$	6.36 (1.19)	4.17	$\mathrm{Std}(R_t^d)$	18.40 (1.61)	17.99
$E(R_t^f)$	1.53 (0.64)	0.52	$\mathrm{Std}(R_t^f)$	4.30 (0.70)	4.84
$E(R_t^d) - E(R_t^f)$	4.83 (1.64)	3.65			

Annual correlations between fundamentals and real stock returns

Consumption	Data	Model
1 year	0.09 (0.07)	0.07
5 year	0.29 (0.13)	0.06
10 year	0.26 (0.17)	0.06

Dividends	Data	Model
1 year	−0.03 (0.11)	0.36
5 year	0.37 (0.10)	0.32
10 year	0.62 (0.09)	0.30

The importance of the correlation puzzle

- Since corr$(\Delta d_t, R_t^d)$ and corr$(\Delta c_t, R_t^d)$ are estimated with more precision than average rates of return, the estimation criterion gives them more weight.

- If we drop the 5 and 10 year correlations from the criterion, the parameters move to a region where the equity premium is larger.

- The value of $\theta = (1-\gamma)/(1-1/\psi)$ goes from -1.06 to -2.03 , which is why the equity premium implied by the model rises.

	Data	Benchmark	Benchmark No corr. with returns in criterion
γ	-	1.22 (0.04)	0.21
ψ	-	1.26 (0.67)	0.72
$E(R_t^d)$	6.36 (1.19)	4.17	5.16
$E(R_f)$	1.53 (0.64)	0.52	0.90
$E(R_t^d) - R_f$	4.83 (1.64)	3.65	4.26
$\mathrm{corr}(\Delta d_t, R_t^d)$	-0.03 (0.11)	0.36	0.34
$\mathrm{corr}(\Delta c_t, R_t^d)$	0.09 (0.07)	0.07	0.02

Model without time-preference shocks

- The model cannot generate an equity premium.
- It also cannot account for the correlation puzzle
  - $\text{corr}(\Delta d_t, R_t^d) = 1$, $\text{corr}(\Delta c_t, R_t^d) = 0.38$.

	Data	Benchmark	Benchmark No time pref.shocks
γ	-	1.22 (0.04)	1.70
ψ	-	1.26 (0.67)	4.6
$E(R_t^d)$	6.36 (1.19)	4.17	5.65
$E(R_f)$	1.53 (0.64)	0.52	5.65
$E(R_t^d) - R_f$	4.83 (1.64)	3.65	0.0
$\mathrm{corr}(\Delta d_t, R_t^d)$	-0.03 (0.11)	0.36	1.00
$\mathrm{corr}(\Delta c_t, R_t^d)$	0.09 (0.07)	0.07	0.38

CRRA Preferences

- With preference shocks, the CRRA model generates a *negative* equity premium and does poorly on the consumption correlation.
- Without preference shocks, the CRRA model doesn't generate an equity premium and does very poorly on the correlation puzzle.

	Data	Benchmark	CRRA	CRRA No time pref. shocks
γ	-	1.22 (0.04)	2.15	1.53
ψ	-	1.26 (0.67)	1/2.15	1/1.53
$E(R_t^d)$	6.36 (1.19)	4.17	3.18	5.65
$E(R_f)$	1.53 (0.64)	0.52	4.18	5.65
$E(R_t^d) - R_f$	4.83 (1.64)	3.65	-1.00	0.00
$\text{corr}(\Delta d_t, R_t^d)$	-0.03 (0.11)	0.36	0.35	1.0
$\text{corr}(\Delta c_t, R_t^d)$	0.09 (0.07)	0.07	0.40	0.38

Measuring the shocks

- According to the model

$$
\begin{aligned}
r^f_{t+1} &= -\log(\delta) - \log(\lambda_{t+1}/\lambda_t) + \mu/\psi - (1-\theta)\,\kappa^2_{c1}A^2_{c1}\sigma^2_{\varepsilon}/2 \\
&\quad + \left[\frac{(1-\theta)}{\theta}(1-\gamma)^2 - \gamma^2\right]\sigma^2_c/2,
\end{aligned}
$$

- So

$$
\log(\lambda_{t+1}/\lambda_t) = \chi - r^f_{t+1}
$$

$$
\begin{aligned}
\chi &= -\log(\delta) + \mu/\psi - (1-\theta)\,\kappa^2_{c1}A^2_{c1}\sigma^2_{\varepsilon}/2 \\
&\quad + \left[\frac{(1-\theta)}{\theta}(1-\gamma)^2 - \gamma^2\right]\sigma^2_c/2,
\end{aligned}
$$

- So, up to a constant, we can measure the preference shock as minus the risk-free rate.

Persistence in the risk-free rate

- The previous observations imply that $\log\left(\lambda_{t+1}/\lambda_t\right)$ should be as persistent as the risk-free rate.
- In our estimated model,

$$\log\left(\lambda_{t+1}/\lambda_t\right) = 0.9996 \log\left(\lambda_t/\lambda_{t-1}\right) + \varepsilon_{t+1}.$$

- If we regress the demeaned risk-free rate on one lag we obtain an AR coefficient of 0.64, with a standard error of 0.05.

Getting persistence right

- Fixing the persistence problem is straightforward and doesn't have a major effect on other aspects of the models' performance.

- Suppose that:

$$\log(\lambda_{t+1}/\lambda_t) = x_{t+1} + \sigma_\varepsilon \varepsilon_{t+1}.$$

$$x_{t+1} = \rho_x x_t + \sigma_x \xi_{t+1}$$

  where ε_{t+1} is i.i.d.

- So the time preference shock is the sum of a persistent shock and an i.i.d. shock.

Getting persistence right

- Solving the model we obtain:

$$r_{t+1}^f = -\left(\begin{array}{c} \log(\delta) + \log\left(\frac{\lambda_{t+1}}{\lambda_t}\right) - \frac{1}{\psi}\mu + \left(\gamma^2 - \frac{\theta-1}{\theta}(1-\gamma)^2\right)\sigma_\eta^2/2 \\ -(\theta-1)(\kappa_1 A_1 \sigma_x)^2/2 - (\theta-1)(\kappa_1 A_2)^2/2 \end{array} \right)$$

and

$$\begin{array}{rcl} E_t(r_{d,t+1}) - r_t^f & = & \pi(2\gamma - \pi)\sigma_\eta^2/2 - \varphi_d^2\sigma_\eta^2/2 \\ & & +\kappa_{d1}A_{d1}(2(1-\theta)\kappa_1 A_1 - \kappa_{d1}A_{d1})\sigma_x^2/2 \\ & & +\kappa_{d1}(2(1-\theta)\kappa_1 - \kappa_{d1})\sigma_\varepsilon^2/2 \end{array}$$

Estimating the augmented model

- We re-estimate the model
  - Include σ_ε as a new parameter.
  - Add the AR coefficient, τ, of the risk-free rate to our specification of Ψ_D.
- Main finding: we can match the persistence of the risk-free rate with relatively minor changes in the properties of the model.

Parameter estimates, augmented model, 1871-2006

Parameter	Estimates	Parameter	Estimates
γ	0.95 (0.24)	σ_d	0.019 (0.0006)
ψ	0.93 (0.12)	π	0.57 (0.074)
δ	0.999 (0.001)	σ_λ	0.00008 (0.00009)
σ_c	0.009 (0.0002)	ρ	0.9997 (0.0006)
μ	0.00133 (0.00026)	σ_ε	0.00009 (0.000006)

Properties of the augmented model

	Data	Benchmark	Augmented Model
$E(R_t^d)$	6.36 (1.19)	4.17	3.33
$E(R_t^f)$	1.53 (0.64)	0.52	0.62
$E(R_t^d) - R_f$	4.83 (1.64)	3.65	2.71
$\text{corr}(\Delta d_t, R_t^d)$	-0.03 (0.11)	0.36	0.38
$\text{corr}(\Delta c_t, R_t^d)$	0.09 (0.07)	0.07	0.09
τ	0.64 (0.05)	.9995	0.61

Bansal, Kiku and Yaron (2011)

- Originally, they emphasized importance of long run risk.
- More recently they emphasized the importance of movements in volatility.

$$U_t = \max_{C_t} \left[\lambda_t C_t^{1-1/\psi} + \delta \left(U_{t+1}^* \right)^{1-1/\psi} \right]^{1/(1-1/\psi)}$$

$$U_{t+1}^* = \left[E_t \left(U_{t+1}^{1-\gamma} \right) \right]^{1/(1-\gamma)}$$

$$\begin{aligned} g_t &= \mu + x_{t-1} + \sigma_{t-1} \eta_t, \\ x_t &= \rho_x x_{t-1} + \phi_e \sigma_{t-1} e_t, \\ \sigma_t^2 &= \sigma^2 (1 - \nu) + \nu \sigma_{t-1}^2 + \sigma_w^2 w_t. \end{aligned}$$

BKY parameters

Parameter	BKY	Parameter	BKY
γ	10	σ	0.0072
ψ	1.5	ν	0.999
δ	0.9989	σ_w	0.28×10^{-5}
μ	0.0015	ϕ	2.5
ρ_x	0.975	π	2.6
ϕ_e	0.038	φ	5.96

Parameter estimates, benchmark model, 1930-2006

Parameter	Estimates	Parameter	Estimates
γ	2.05	σ_d	0.019
ψ	3.68	π	-0.17
δ	0.998	σ_λ	0.0002
σ_c	0.005	ρ	.998
μ	0.00185 (0.0002)		

Model performance, 1930-2006

1930-2006	Data	Benchmark	BKY
$E(R_t^d)$	8.47 (1.55)	4.47	8.75
$\text{std}(R_t^d)$	16.25 (1.17)	17.93	23.37
$E(R_f)$	0.42 (0.89)	-0.31	1.05
$\text{std}(R_t^f)$	3.47 (0.90)	3.46	1.22
$E(R_t^d) - R_f$	8.05	4.78	7.70

Correlation between stock returns and consumption growth, 1930-2006

- Correlation puzzle is stronger in the short sample than in the full sample.

1930-2006	Data	Bench.	BKY
1 year	-0.13 (0.13)	-0.02	0.66
5 year	0.06 (0.13)	-0.02	0.88
10 year	-0.42 (0.15)	-0.02	0.92

Correlation between stock returns and dividend growth, 1930-2006

1930-2006	Data	Bench.	BKY
1 year	0.20 (0.07)	0.37	0.66
5 year	0.36 (0.10)	0.35	0.90
10 year	0.72 (0.11)	0.34	0.93

Bansal, Kiku and Yaron (2011)

- The BKY model does a very good job at accounting for the equity premium and the average risk free rate.
- Problem: correlations between stock market returns and fundamentals (consumption or dividend growth) are close to one.
- Our benchmark model understates the long-term correlation between equity returns and dividend growth.

Parameter estimates, augmented model benchmark model, 1930-2006

Parameter	Estimates	Parameter	Estimates
γ	1.48	σ_d	0.016
ψ	2.38	π	-0.19
δ	0.999	σ_λ	0.00009
σ_c	0.005	ρ	.999
μ	0.00185 (0.0002)		

Model performance, augmented model, 1930-2006

1930-2006	Data	Benchmark
$E(R_t^d)$	8.47 (1.55)	3.37
$\text{std}(R_t^d)$	16.25 (1.17)	17.24
$E(R_f)$	0.42 (0.89)	0.22
$\text{std}(R_t^f)$	3.47 (0.90)	3.26
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Correlation between stock returns and consumption growth, augmented model, 1930-2006

1930-2006	Data	Augmented Model
1 year	-0.13 (0.13)	-0.02
5 year	0.06 (0.13)	-0.02
10 year	-0.42 (0.15)	-0.02

Correlation between stock returns and dividend growth, augmented model, 1930-2006

1930-2006	Data	Bench.
1 year	0.20 (0.07)	0.37
5 year	0.36 (0.10)	0.35
10 year	0.72 (0.11)	0.34

Implications for the term premium

- According to Beeler and Campbell (2012) the real yield on long-term bonds has always been positive and is usually above 2 percent.
- The BKY model implies a 10-year yield of -0.43 percent.
  - Long term bonds are a hedge against long-run risk (Piazzesi and Schneider (2006)).
  - So long-term bonds command a negative risk premium.
- Our augmented model implies a 10-year yield of 1.36 percent.
  - Since there is uncertainty about how agents will value consumption in 10 years, 10-year bonds command a positive risk premium.

Conclusion

- We propose a simple model that accounts for the level and volatility of the equity premium and of the risk free rate.
- The model is broadly consistent with the correlations between stock market returns and fundamentals, consumption and dividend growth.
- Key features of the model
  - Consumption and dividends follow a random walk;
  - Epstein-Zin utility;
  - Stochastic rate of time preference.
- The model accounts for the equity premium with low levels of risk aversion.

Robustness check using global financial statistics data, 1930-2006

Parameter	Estimates	Parameter	Estimates
γ	1.5628	σ_d	0.011378
ψ	2.8138	π	1.3756
δ	0.99801	σ_λ	4.0278×10^{-5}
σ_c	0.0043003	ρ	0.99984
μ	0.035658 (0.0029309)		

Moments, annual data and model

Moments	Data	Model
Std (Δd_t)	4.4424	6.55
Std (Δc_t)	1.4897	2.75
Corr$(\Delta c_t, \Delta d_t)$	0.46129	0.20

Moments (annual), data and model

Moments	Data	Model	Moments	Data	Model
$E(R_t^d)$	4.522	4.17	$\text{Std}(R_t^d)$	15.4995	17.99
$E(R_t^f)$	1.7422	0.52	$\text{Std}(R_t^f)$	2.7186	4.84
$E(R_t^d) - E(R_t^f)$	2.8798	3.65			

Annual correlations between fundamentals and real stock returns

Consumption	Data	Model	Dividends	Data	Model
1 year	0.2266	0.13221	1 year	0.071877	0.28662
5 year	0.048778	0.1252	5 year	0.25978	0.27142
10 year	−0.36584	0.11795	10 year	0.38509	0.25569