Discussion of "Inattentive Valuation and Reference-Dependent Choice" by Mike Woodford

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The Shaw & Shaw Experiment

The Experiment

- a letter $\{T, V\}$ is flashed on the screen in 2 locations $\{H, L\}$
- example:

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$$V$$

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Formalization of Experiment

- states of the world $x_{ij} \in X$
- nature
    - draws location $i \in \{H, L\}$ from dist (π_H, π_L)
    - draws letter $j \in \{T, V\}$ from dist. $(1/2, 1/2)$

Formalization of Experiment

- states of the world $x_{ij} \in X$
- nature
  - draws location $i \in \{H, L\}$ from dist (π_H, π_L)
  - draws letter $j \in \{T, V\}$ from dist. $(1/2, 1/2)$
- agents are asked to correctly identify the letter

$$\min \sum \mathbb{I}\left(k \neq j\right)$$

where $k \in \{T, V\}$ is agent's response

Shaw & Shaw Results

- $(\pi_H, \pi_L) = (.5, .5)$
  - same fraction of errors made in both locations

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- $(\pi_H, \pi_L) = (.5, .5)$
  - same fraction of errors made in both locations
- $(\pi_H, \pi_L) = (.9, .1)$
  - fraction of errors different across locations
  - greater fraction of errors made in L location

Rational Inattention

Rational Inattention: General Setup (Sims)

- primitives
  - $x \in X$ states of the world
  - $\pi(x)$ prior
  - agent's payoffs (preferences)

Rational Inattention: General Setup (Sims)

- primitives
  - $x \in X$ states of the world
  - $\pi(x)$ prior
  - agent's payoffs (preferences)
- agent chooses information channel s.t. constraint
  - $s \in S$ signals (perceived states)
  - $p(s|x)$ conditional probabilities

Rational Inattention: the Shaw & Shaw Experiment

- for simplicity, assume agents can perfectly observe location i
- agents observe some signal $k \in \{T, V\}$ of letter j, but signal has error:

$$
\begin{aligned}
p\left(ij|ij\right) &= 1 - e_i \\
p\left(ik|ij\right) &= e_i \qquad\qquad \text{for } k \neq j
\end{aligned}
$$

- agent optimally chooses probability of error $\mathbf{e} = \{e_H, e_L\}$

Rational Inattention Problem

- objective

$$\min_{\mathbf{e}} \sum_i \pi_i e_i + \theta I\left(\mathbf{e}\right)$$

Rational Inattention Problem

- objective

$$\min_{\mathbf{e}} \sum_i \pi_i e_i + \theta I(\mathbf{e})$$

- mutual information

$$I(\mathbf{e}) \equiv H(x) - H(x|k)$$

is reduction in entropy (measure of uncertainty)

$$H(x) \equiv - \sum_{x \in X} p(x) \log p(x)$$

Rational Inattention Problem

- mutual info

$$I = \underbrace{-\left(\sum_i \pi_i \log \pi_i + \log \frac{1}{2}\right)}_{H(x)} - \underbrace{\sum_i \pi_i h\left(e_i\right)}_{H(x|k)}$$

- $h\left(e_i\right)$ is conditional entropy within location i

$$h\left(e_i\right) = -\left(1 - e_i\right) \log\left(1 - e_i\right) - e_i \log e_i$$

Rational Inattention Solution

- objective

$$\min_{\mathbf{e}} \sum_i \pi_i e_i + \theta \left(H(x) - \sum_i \pi_i h(e_i) \right)$$

- FOC

$$\pi_i = \theta \pi_i h'(e_i)$$

Rational Inattention Solution

- objective

$$\min_{\mathbf{e}} \sum_i \pi_i e_i + \theta \left(H(x) - \sum_i \pi_i h(e_i) \right)$$

- FOC

$$1 = \theta h'(e_i)$$

Rational Inattention Solution

- FOC $1 = \theta h' \left(e_i \right)$ implies

$$e_i = \bar{e}, \qquad \forall i$$

Rational Inattention Solution

- FOC $1 = \theta h'(e_i)$ implies

$$e_i = \bar{e}, \qquad \forall i$$

- solution incompatible with Shaw & Shaw results
- why? letter j is the only payoff relevant variable
  - cost/benefit same across locations
  - no need to add additional info of location

Woodford's Alternative Formulation

Alternative Formulation

- rational inattention problem

$$\min_{\mathbf{e}} \sum_i \pi_i e_i + \theta I(\mathbf{e})$$

Alternative Formulation

- rational inattention problem

$$\min_{\mathbf{e}} \sum_{i} \pi_i e_i + \theta I\left(\mathbf{e}\right)$$

- alternative problem (Woodford)

$$\min_{\mathbf{e}} \sum \pi_i e_i + \theta C\left(\mathbf{e}\right)$$

$$C\left(\mathbf{e}\right) \equiv \max_{\boldsymbol{\pi}} I\left(\mathbf{e}; \boldsymbol{\pi}\right)$$

Woodford's Cost Function

- for given prior, $I\left(\mathbf{e}\right)$ is *actual* reduction in entropy

Woodford's Cost Function

- for given prior, $I(\mathbf{e})$ is *actual* reduction in entropy
- $C(\mathbf{e})$ is *potential* reduction in entropy over any possible prior

$$C(\mathbf{e}) \equiv \max_{\boldsymbol{\pi}} I(\mathbf{e}; \boldsymbol{\pi})$$

  - given channel $\mathbf{e}$, choose prior to maximize reduction in entropy

Alternative Formulation

- rational inattention problem

$$\min_{\mathbf{e}} \sum_i \pi_i e_i + \theta I(\mathbf{e})$$

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Alternative Formulation

- rational inattention problem

$$\min_{\mathbf{e}} \sum_i \pi_i e_i - \theta \sum_i \pi_i h(e_i)$$

- alternative problem (Woodford)

$$\min_{\mathbf{e}} \sum \pi_i e_i + \theta C(\mathbf{e})$$

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Alternative Formulation

- rational inattention problem

$$\min_{\mathbf{e}} \sum_i \pi_i e_i - \theta \sum_i \pi_i h\left(e_i\right)$$

- alternative problem (Woodford)

$$\min_{\mathbf{e}} \sum_i \pi_i e_i + \theta \log \left(\sum_i \exp\left(-h\left(e\right)\right) \right)$$

Results of Woodford's formulation

- FOC
$$\pi_i = \tilde{\theta} \exp\left(-h\left(e_i\right)\right) h'\left(e_i\right)$$
RHS decreasing in e_i
- thus, e_i^* inversely related to π_i
$$e_i^* = g\left(\pi_i\right) \qquad g' < 0$$
- compatible with Shaw & Shaw results!
  - in fact, Woodford calibrates θ and predictions fit quite well

Comments

Interpreting Woodford's Cost Function

- Woodford's new cost function

$$C(\mathbf{e}) \equiv \max_{\boldsymbol{\pi}} I(\mathbf{e}; \boldsymbol{\pi})$$

represents greatest *potential* entropy reduction from channel $\mathbf{e}$

- why max? could have prior μ over priors

$$C(\mathbf{e}) = \int I(\mathbf{e}; \boldsymbol{\pi}) \, \mu(\boldsymbol{\pi}) \, d\boldsymbol{\pi}$$

Interpreting Woodford's Cost Function

- need: cost/benefit to differ in π_i
  - isomorphic to rational inattention problem with θ_i
  - could have chosen arbitrary cost function C

Interpreting Woodford's Cost Function

- need: cost/benefit to differ in π_i
  - isomorphic to rational inattention problem with θ_i
  - could have chosen arbitrary cost function C
- however, Woodford's formulation more elegant
  - retains properties of Shannon measure of info
  - more restrictive than just choosing θ_i or C_i
  - explains experimental anomomalies (stochastic choice, focusing effects, choice-set effects, reference dependence, etc.)

Interpreting Woodford's Cost Function

$$C(\mathbf{e}) \equiv \max_{\boldsymbol{\pi}} I(\mathbf{e}; \boldsymbol{\pi})$$

- possible interpretation: sequential game
  - first stage. agent chooses channel e
  - second stage. evil nature chooses prior π to max $I(\mathbf{e}; \boldsymbol{\pi})$
- possible interpretation: robust control

$$\min_{\mathbf{e}} \sum \pi_i e_i + \theta \left\{ \max_{\boldsymbol{\pi}} I(\mathbf{e}; \boldsymbol{\pi}) \right\}$$

Exogenous Coding/Channel matters

- rational inattention $\longrightarrow$ only payoff relevant variables matter
  - underlying shocks out in the world
  - we choose our perception of them
  - depends only on: curvature of payoffs, volatility of shocks
- but exogenous information channels matter
  - newspapers, own sales, competitors' prices, word-of-mouth