Discussion: Understanding the Equity-Premium and Correlation Puzzle

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Kreps-Porteus Model

$$V_t = \left[(\zeta\lambda_t C_t)^{1-\psi} + \exp(-\delta)\left[\mathcal{R}_t(V_{t+1})\right]^{1-\psi}\right]^{\frac{1}{1-\psi}}. \tag{1}$$

where $\zeta > 0$ is a scale factor that does not alter preferences, and

$$\mathcal{R}_t\left(V_{t+1}\right) = \left(E\left[(V_{t+1})^{1-\gamma}|\mathcal{F}_t\right]\right)^{\frac{1}{1-\gamma}}$$

adjusts the continuation value V_{t+1} for risk. With these preferences, $\frac{1}{\psi}$ is the elasticity of intertemporal substitution and δ is a subjective discount rate. The process λ is an unobserved (to an econometrician) preference shifter.

Exogenous stochastic evolution

$$\lambda_{t+1} - \lambda_t = \mu_\lambda + X_t$$
$$X_{t+1} = \rho X_t + \sigma_x \cdot W_{t+1}$$

where W_{t+1} is a three-dimensional standard normal shock and $|\rho| < 1$.

- preference shifter is locally predictable
- $\sigma_x \cdot W_{t+1}$ independent of shocks to cash flow and consumption
- $\log C$ and $\log D$ are correlated random walks with drift. No predictability for mean or volatility. Independent of the preference shifter process.
- X_t is the single state variable.

Recursion reconsidered

Exploit homogeneity and transform the recursion. Instead of:

$$V_t = \left[(\zeta \lambda_t C_t)^{1-\psi} + \exp(-\delta) \left[\mathcal{R}_t(V_{t+1}) \right]^{1-\psi} \right]^{\frac{1}{1-\psi}} .$$

Construct a recursion for

$$W_t = \frac{V_t}{\lambda_t C_t} = \frac{V_t}{C_t^*}$$

where $C_t^* = C_t \lambda_t$

$$W_t = \left[(\zeta)^{1-\psi} + \exp(-\delta) \left(\mathcal{R}_t \left[W_{t+1} \left(\frac{C_{t+1}^*}{C_t^*} \right) \right] \right)^{1-\psi} \right]^{\frac{1}{1-\psi}} .$$

This recursion has a solution of the form:

$$W_t = f(X_t)$$

Recursion Revisited

$$W_t = \left[(\zeta)^{1-\psi} + \exp(-\delta) \left(\mathcal{R}_t \left[W_{t+1} \left(\frac{C^*_{t+1}}{C^*_t} \right) \right] \right)^{1-\psi} \right]^{\frac{1}{1-\psi}} .$$

- This is the *same recursion* as for a “long-run” risk model but with a change in numeraire.
- $C^* = C\lambda$ evolves just as C in Bansal and Yaron (abstracting from stochastic volatility) and is a restricted version of the specification used by Hansen, Heaton and Li. The λ risk becomes the “growth-rate risk” for C^*.

Two stochastic discount factors

- Transformed specification:

$$\frac{S^*_{t+1}}{S^*_t} = \exp(-\delta)\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}\left[\frac{V_{t+1}/C^*_{t+1}}{\mathcal{R}_t\left(V_{t+1}/C^*_t\right)}\right]^{\psi-\gamma}.$$

- Original numeraire

$$\begin{aligned}\frac{S_{t+1}}{S_t} &= \exp(-\delta)\left(\frac{\lambda_{t+1}}{\lambda_t}\right)^{1-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[\frac{V_{t+1}/C^*_{t+1}}{\mathcal{R}_t(V_{t+1}/C^*_t)}\right]^{\psi-\gamma} \\ &= \exp(-\delta)\exp\left[(1-\gamma)X_t\right]\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[\frac{f(X_{t+1})}{g(X_t)}\right]^{\psi-\gamma}\end{aligned}$$

Approximate f and g as log-linear in the realized state x. Approximate continuation values.

Limit approximation and Existence

Hansen (Econometrica) and Hansen and Scheinkman (PNAS)

Solve

$$E\left[\left(\frac{C^*_{t+1}}{C_t}^*\right)^{1-\gamma} e(X_{t+1})|X_t = x\right] = \exp(\eta)e(x).$$

where

$$e(x) = \exp(\beta x).$$

Under log-normality this equation has a log-linear solution with coefficient

$$\beta = \frac{1-\gamma}{1-\rho}$$

Use this as the starting point for showing existence of the infinite-horizon value function. The implied value of η restricts the range of admissible parameters.

What happens at the boundary?

- At the boundary, the forward-looking channel is featured to the greatest extent possible.

$$\frac{S_{t+1}}{S_t} \approx \exp(-\eta)\exp[(1-\gamma)z_t]\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left[\frac{h(X_{t+1})}{h(X_t)}\right]. \quad (2)$$

where

$$h(x) = e(x)^{\frac{\psi-\gamma}{1-\gamma}} = \exp\left[\left(\frac{\psi-\gamma}{1-\rho}\right)x\right]$$

- The implied S process is log-normal.

$$\log S_{t+1} - \log S_t = \mu_s + \alpha X_t + \sigma_s \cdot W_{t+1}$$
$$X_{t+1} = \rho X_t + \sigma_x \cdot W_{t+1}$$

Why not just start here? Interesting restrictions implied by the model?

Cash flow pricing

- Equity is a composite of "dividend strips".

$$\frac{P_t}{D_t} = \sum_{j=1}^{\infty} E\left[\frac{S_{t+j}D_{t+j}}{S_tD_t}|X_t\right].$$

- The component terms:

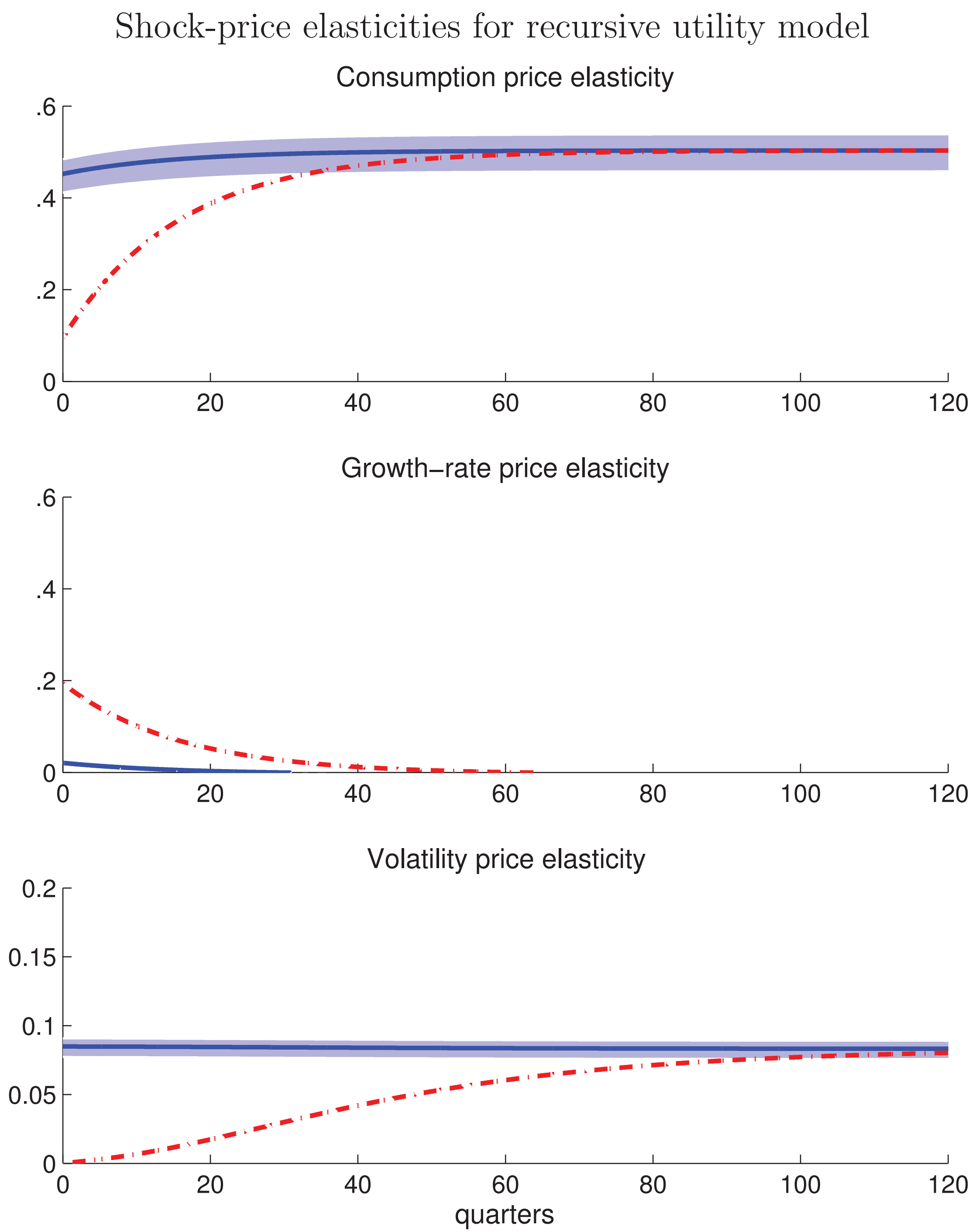
$$E\left[\frac{S_{t+j}D_{t+j}}{S_tD_t}|X_t\right]$$

  are the outcome of jointly compounding the stochastic discount factor the cash flow growth. Risk prices are encoded in S and risk exposures in D.
- See interesting recent work by van Binsbergen, Brandt and Koijen (AER) on using information from derivative claims to extract prices of dividend strips.
- All of the dependence on the state variable comes through the stochastic discount factor channel S.

Pricing shock exposures

"Building blocks" of multi period valuation.

- Assign "prices" to shocks. Pricing counterpart to impulse response functions. Horizon dependent term structure of risk prices.
- For log-normal models "shock price elasticities" are revealed by the impulse response functions for $\log S$. Direct extensions to more general nonlinear Markov models. Exposure to which shocks require the largest compensation and how this changes for different payoff or investment horizons.
- In this paper there is apparently only weak correlations between S and D. One-period equity returns are presumably explosed to shocks to the preference process via the capital gains channel.


Shock-price elasticities for recursive utility model
Consumption price elasticity
Growth−rate price elasticity
Volatility price elasticity
quarters
0
20
40
60
80
100
120
.6
.4
.2
0.2
0.15
0.1
0.05

Summary

- What is the role for consumption in this analysis? What happens if you simply drop consumption from the analysis and posit a process for λ that is not locally predictable and has a predictable mean component but independent of the cash flow process?
- Why independence? This paper imposes independence between the preference process on the one hand and the consumption and cash flow process on the other hand. Challenging to defend.
- How do we interpret this preference shock process? Authors mention low frequency demographic changes in demographics. Observable? Alternative, sentiments or animal spirits. Answering this may lead us to put more structure on how we model taste changes.
- State dependence in risk premia? Not in this model. Empirical evidence suggests that risk premia are bigger in bad times than good ones. Open question as to what is driving this, say risk exposures or risk prices? If prices, why? What might this line of research contribute?