A Macroeconomic Framework for Quantifying Systemic Risk

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Systemic Risk

- Systemic risk: risk (probability) of a state where
  - financial intermediation is disrupted
  - small fundamental shocks to financial intermediaries can have quantitatively large effects on macro economy
- Goal: Write down a non-linear macro model to assess systemic risk
  - much of the time the link between financial intermediation and macro economy is small
  - but in (crisis) states the effects are greatly amplified

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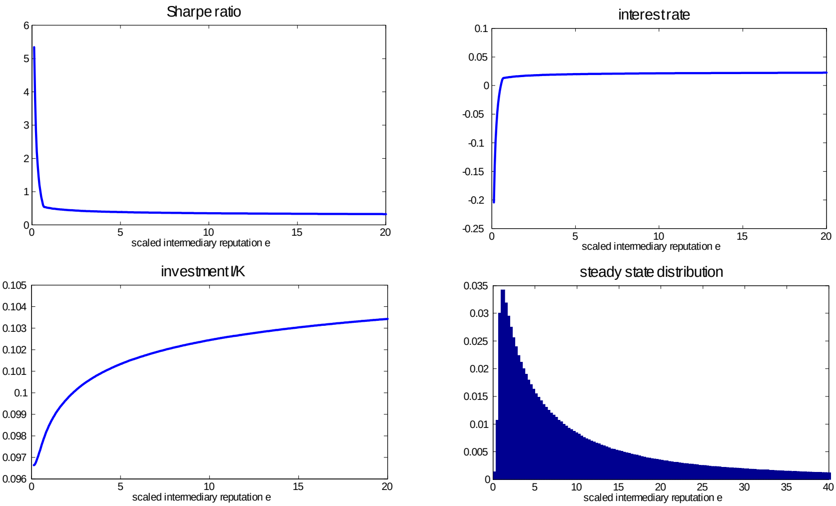
- Goal: Write down a non-linear macro model to assess systemic risk
  - much of the time the link between financial intermediation and macro economy is small
  - but in (crisis) states the effects are greatly amplified
- How well does the model match asymmetry (i.e. occasional effects of financial intermediation) in the data?
- How well can an intermediary shock channel explain patterns in 2007-2009?
- How likely is the economy, say unconditionally, to enter a systemic risk episode?

Innovation Relative to Much of Literature

- We study a model with occasionally binding financial constraint
- Typical models (e.g., Kiyotaki-Moore (1997),...) linearize around steady state where constraint binds.
  - Cannot talk about 1) likelihood that intermediation is disrupted (its always disrupted...) and 2) how severely it is disrupted
- Our model solution has stochastic steady state, with fully solved equilibrium prices and policies
  - Main drawback: need to reduce state variables
  - Have to leave out some common DSGE elements
- Similar methodology to Mendoza (2010) and Brunnermeier-Sannikov (2011)
- Model elements adopted from He-Krishnamurthy (2012), with real investment and housing

Preview of model result



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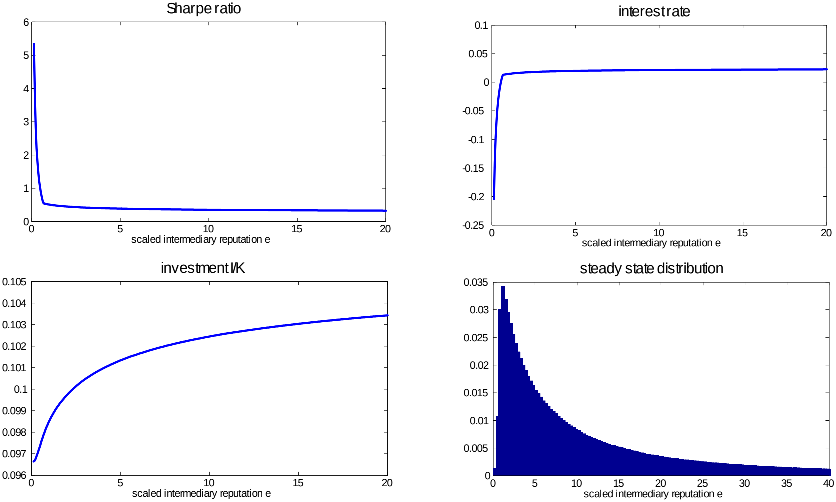


- **Crisis:** $e_{crisis} = 0.65$, binding capital constraint
- **Distress:** $e_{distress} = 4$ so that $\Pr\left(e \leq e_{distress}\right) = 33\%$ as in data

Strategy

- Crises are rare. How do we quantify model?
- Even if economy is currently not in a crisis state, the anticipation of a crisis affects decisions.
  1. We match data on "distress" (33% of data) and "non-distress" periods (67% of data).
  2. We extrapolate to a crisis and ask how well the model can match patterns from 2007-2009.
  3. We compute conditional probabilities of triggering a crisis (measuring "systemic risk" probabilities).

Evidence of Non-Linearity

- **Excess bond premium** (EBP): the risk premium part of credit spread (removing default part), Gilchrist and Zakrajsek (2010). Correlates with measures of intermediary health.
- Use EBP to classify distress periods (33%) and non-distress periods (the rest)

Distress Periods	NBER Recessions
1973Q1 - 1975Q3	11/73 - 3/75
1982Q2 - 1982Q4	7/81 - 11/82
1985Q4 - 1987Q3	
1988Q4 - 1990Q1	7/90 - 3/91
1992Q4 - 1993Q2	
2001Q2 - 2003Q1	3/01 - 11/01
2007Q3 - 2009Q3	12/07 - 6/09

State-Dependent Covariances (1)

- Equity = Total market value of equity of finance, insurance and real estate sectors. (works as well if only include banks + broker/dealers)
- All variables are growth, except Sharpe ratio constructed from EBP

	Distress		Non Distress	
	Cov	Corr	Cov	Corr
Equity, Investment	1.31%	51.48	0.07	5.79
Equity, Consumption	0.25%	45.85	0.03	14.74
Equity, Sharpe	-6.81%	-35.96	-0.14	-0.06
Equity, Landprice	4.06%	60.65	0.12	0.07

State-Dependent Covariances (2)

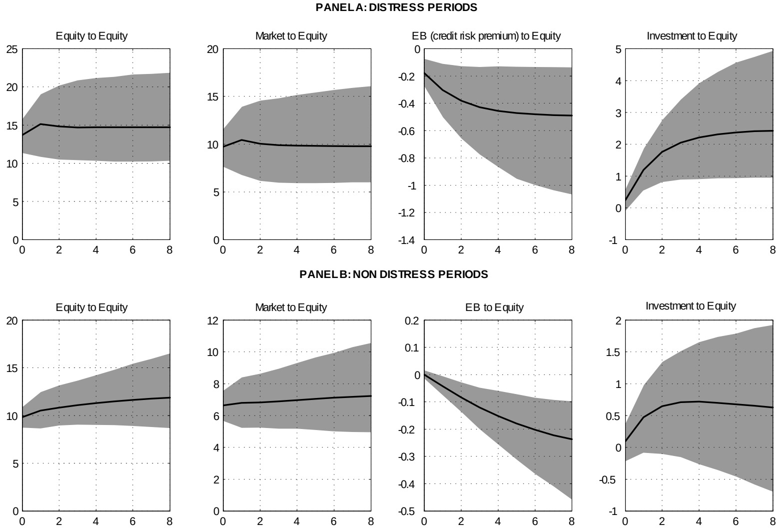
All variables are growth, except Sharpe ratio constructed from EBP

	Distress		Non Distress	
	NBER+2	Excl-Crisis	NBER+2	Excl-Crisis
Equity, *Investment*	0.84%	0.37	-0.06	0.03
Equity, *Consumption*	0.13%	0.04	0.01	0.03
Equity, *Sharpe*	-7.57%	-2.12	-0.78	-0.19
Equity, *Landprice*	4.39%	-0.63	-0.31	-0.01

Note: Similar numbers if only use NBER dates, but distress sample is only 20% of observations.

VAR Evidence of Non-Linearity (3)

VAR order: [intermediary equity, aggregate stock market, EBP, investment]. Coefficients depend on distress/non-distress state. Quarterly growth rates.



Road Map of the Rest of Talk

- Model, mechanism, and solution
- Calibration
  - Baseline parameters
  - Prices and polices, comparative statics
- Matching data on distress and non-distress
- Systemic crisis
  - Extrapolate to crisis state
  - Uncover fundamental shocks in the recent crisis
  - How likely are crises?

Agents and Technology

- Two classes of agents: households and bankers
  - Households own the entire economy, but subject to frictions related to bankers who control intermediaries (next slide)
- Two types of capital: productive capital K_t and housing capital H. Fixed supply of housing $H \equiv 1$
  - Price of capital q_t and price of housing P_t determined in equilibrium

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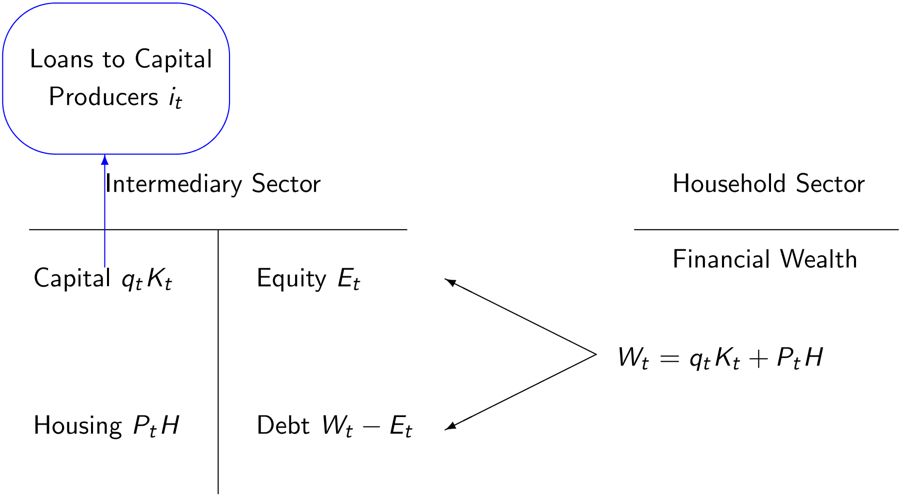
- Two types of capital: productive capital K_t and housing capital H. Fixed supply of housing $H \equiv 1$
  - Price of capital q_t and price of housing P_t determined in equilibrium
- Production $Y = AK_t$, with A being constant
- Fundamental shocks: stochastic capital quality shock dZ_t

$$\frac{dK_t}{K_t} = i_t dt - \delta dt + \sigma dZ_t$$

- Investment/Capital i_t, quadratic adjustment cost

$$\Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2} (i_t - \delta)^2 K_t$$

Aggregate Balance Sheet



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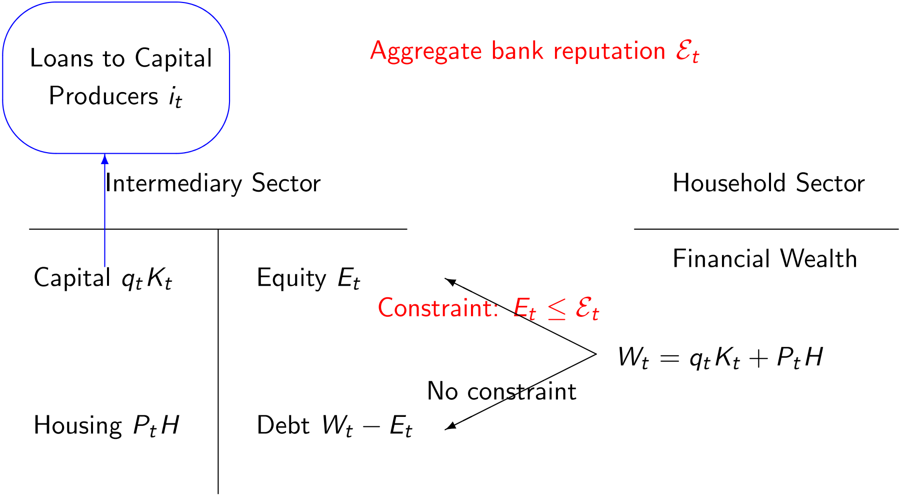

Single Bank/Banker

Assets	Liabilities
Capital $q_t k_t$	Equity e_t
Housing $P_t h_t$	Debt d_t

Portfolio share in capital: $\alpha_t^k = \frac{q_t k_t}{e_t}$

Portfolio share in housing : $\alpha_t^h = \frac{P_t h_t}{e_t}$

Borrowing (no constraint): $d_t = q_t k_t + P_t h_t - e_t = (\alpha_t^k + \alpha_t^h - 1)e_t$

Return on bank equity: $d\tilde{R}_t = \alpha_t^k dR_t^k + \alpha_t^h dR_t^h - (\alpha_t^k + \alpha_t^h - 1)r_t dt$

Banker (log preference) solves: $\max_{\alpha_t^k, \alpha_t^h} E[d\tilde{R}_t - r_t dt] - \frac{m}{2} Var_t[d\tilde{R}_t]$

Single Bank/Banker

Assets	Liabilities
Capital $q_t k_t$	Equity e_t
Housing $P_t h_t$	Debt d_t

Properties

- (k, h) scales with e
- (k, h) increasing in $E_t[dR - r]$
- (k, h) decreasing in $Var[dR]$

Portfolio share in capital: $\alpha_t^k = \frac{q_t k_t}{e_t}$

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General Equilibrium (1)

Intermediary Sector			Household Sector
Capital $q_t K_t$	Equity E_t	Constraint: $E_t \leq \mathcal{E}_t$	Financial Wealth
Housing $p_t H$	Debt $W_t - E_t$		$W_t = q_t K_t + p_t H$

Portfolio share in capital: $\alpha_t^k = \frac{q_t K_t}{E_t}$

Portfolio share in housing: $\alpha_t^h = \frac{P_t H}{E_t}$

- Given a particular state $(K_t, \mathcal{E}_t)$, the portfolio shares are pinned down by GE
- Portfolio shares must also be optimally chosen by banks

$$\max_{\alpha_t^k, \alpha_t^h} \; \mathbb{E}_t[d\tilde{R}_t - r_t dt] - \frac{m}{2} Var_t[d\tilde{R}_t]$$

General Equilibrium (2)

Intermediary Sector		Household Sector
Capital $q_t K_t$	Equity E_t	Financial Wealth
Housing $p_t H$	Debt $W_t - E_t$	$W_t = q_t K_t + p_t H$

Constraint: $E_t \leq \mathcal{E}_t$

Portfolio share in capital: $\alpha_t^k = \frac{q_t K_t}{E_t}$

Portfolio share in housing: $\alpha_t^h = \frac{P_t}{E_t}$

- Prices (returns) have to adjust for optimality:
  - $\mathbb{E}_t[dR_t^h - r_t dt], \mathbb{E}_t[dR_t^k - r_t dt] \Rightarrow$ equations for $\mathbb{E}_t[dP_t], \mathbb{E}_t[dq_t]$
- Rewrite to get ODEs for $P(K, \mathcal{E})$ and $q(K, \mathcal{E})$
- Scale invariance: Define $e \equiv \mathcal{E}/K$; then $P = Kp(e)$ and $q(e)$

Capital Producers and Investment

- Capital goods producers (owned by households) undertake real investment
- Producers must sell the capital stock to intermediaries at price q_t
  - Risk averse intermediaries bear aggregate fundamental shocks
  - Real investment is affected by financial condition of intermediaries to capture "credit crunch"
- Possible interpretations:
  - Entrepreneurs raise capital from VC/PE at the price of q_t
  - Commercial banks makes collateralized loans
- Investment decision

$$\max_{i_t} \; q_t i_t K_t - \Phi(i_t, K_t) \;\Rightarrow\; i_t = \delta + \frac{q_t - 1}{\kappa}$$

Capital Constraint

- Single bank has reputation ϵ_t linked to intermediary performance (constant m)
$$\frac{d\epsilon_t}{\epsilon_t} = m\tilde{R}_t.$$
  - Poor past returns reduce reputation
- Households invest a maximum of ϵ_t dollars of equity capital with this banker

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- Death rate η, and entry $d\psi_t > 0$ of new bankers in extreme states (modeled later)
- $\mathcal{E}_t$: aggregate reputation. Identical banks, aggregate dynamics of $\mathcal{E}_t$

$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = md\tilde{R}_t - \eta dt + d\psi_t$$

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$$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m d\tilde{R}_t - \eta dt + d\psi_t$$

- Note: $\mathcal{E}_t$ is like "net worth" in many other models.

Households' Problem (1)

- Choose consumption c_t^y and housing c_t^h to maximize

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\left((1-\phi)\ln c_t^y + \phi \ln c_t^h\right) dt\right]$$

- Equilibrium rental price D_t (housing asset dividend), FOC $\frac{c_t^h D_t}{\phi} = \frac{c_t^y}{1-\phi}$. In equilibrium ($C_t^h = H = 1$)

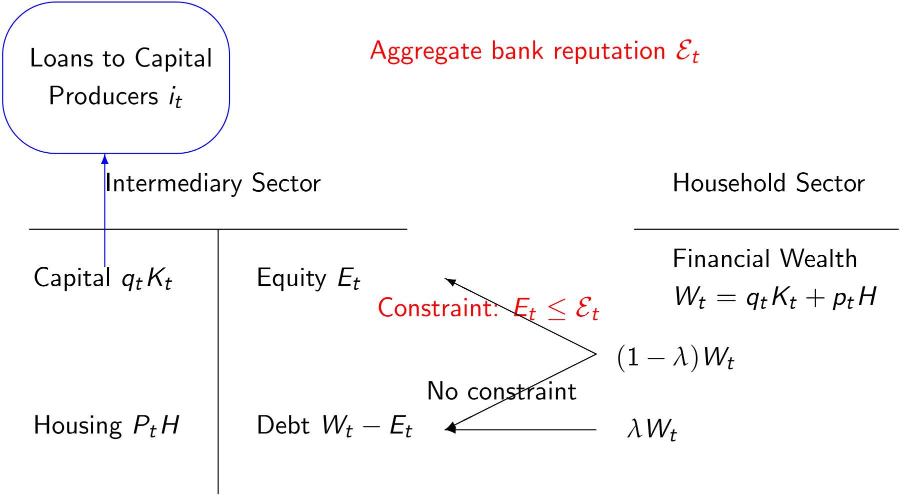
$$D_t = \frac{\phi}{1-\phi} C_t^y$$

  - ϕ: expenditure share in housing, or the relative size of housing sector
- Households free to trade short-term debt.
- Interest rate $r_t = \rho + \mathbb{E}_t\left[dC_t^y / C_t^y\right] - Var_t\left[dC_t^y / C_t^y\right]$

Households' Problem (2)

- Representative household enters time t with financial wealth W_t
- The household splits wealth: $(1-\lambda)\, W_t$ to "equity households," λW_t to "bond households"
  - Equity households invest their portion of wealth as equity of intermediaries, **subject to capital frictions**
  - Bond households invest in riskless bonds
- Once returns are realized, both members pool their wealth again (as in Lucas 1990)
- The only role of bond households (i.e. parameter λ) is to introduce intermediary's leverage in normal time

Debt/Equity Ratio



Equity Capital Constraint

- Unconstrained capital structure: λW_t of Debt, $(1-\lambda)W_t$ of Equity.
- Intermediary equity capital E_t is given by

$$E_t = \min\left[\mathcal{E}_t, (1-\lambda)W_t\right]$$

- How can capital constraint come to bind, beginning in a state where $\mathcal{E}_t > (1-\lambda)W_t$?
- Suppose a -10% shock to real estate and price of capital, so that $W_t \downarrow 10\%$ (Household wealth = aggregate wealth)
- Reputation follows $\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m d\tilde{R}_t + \ldots$ Two forces make $\mathcal{E}_t \downarrow$ more than 10%:
  - Equity is levered claim on assets: Return on equity $= d\tilde{R}_t < -10\%$
  - $m > 1$ in our calibration.

Boundary Conditions

- When $e = \infty$, $\mathcal{E}_t > (1-\lambda) W_t$ frictionless economy
  - We solve for $p(\infty), q(\infty)$ analytically
- As $e \to 0$, intermediaries' portfolio volatility, i.e. Sharpe ratio, rises
- New bankers enter if $e = \underline{e}$ (Sharpe ratio hits γ, exogenous constant)
  - Entry increases aggregate $\mathcal{E}$ but requires physical capital K at conversion rate of β
  - $\underline{e}$ is a reflecting boundary
- Boundary conditions at the entry point $\underline{e}$

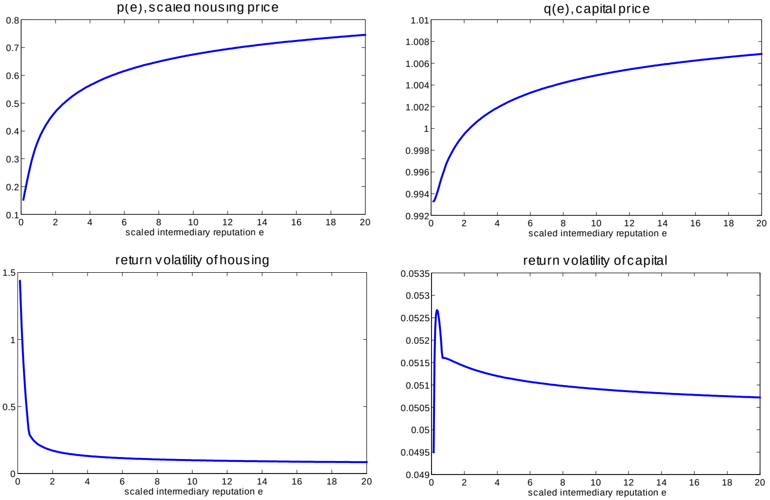
$$q'\left(\underline{e}\right) = 0,\ p'\left(\underline{e}\right) = \frac{p\left(\underline{e}\right)\beta}{1+\underline{e}\beta},\ \text{and } Sharpe\_Ratio\left(\underline{e}\right) = \gamma$$

Calibration: Baseline Parameters

	Parameter	Choice	Target
Panel A: Intermediation			
m	Performance sensitivity	2.5	Average Sharpe ratio (38%)
λ	Debt ratio	0.5	Average intermediary leverage
η	Banker exit rate	13%	Good model dynamics
γ	Entry trigger	5.5	Highest Sharpe ratio
β	Entry cost	2.35	Land price volatility
Panel B: Technology			
σ	Capital quality shock	5%	Investment and Consumption volatilities
δ	Depreciation rate	10%	Literature
κ	Adjustment cost	2	Literature
A	Productivity	0.14	Investment-to-capital ratio
Panel C: Others			
ρ	Time discount rate	2%	Literature
ϕ	Housing share	0.5	Housing-to-wealth ratio

Equilibrium Prices and Policies (1)

- $e_{crisis} = 0.65$: binding capital constraint
- $e_{distress} = 4$ so that $\Pr\left(e \leq e_{distress}\right) = 33\%$ as in data



Equilibrium Prices and Policies (2)

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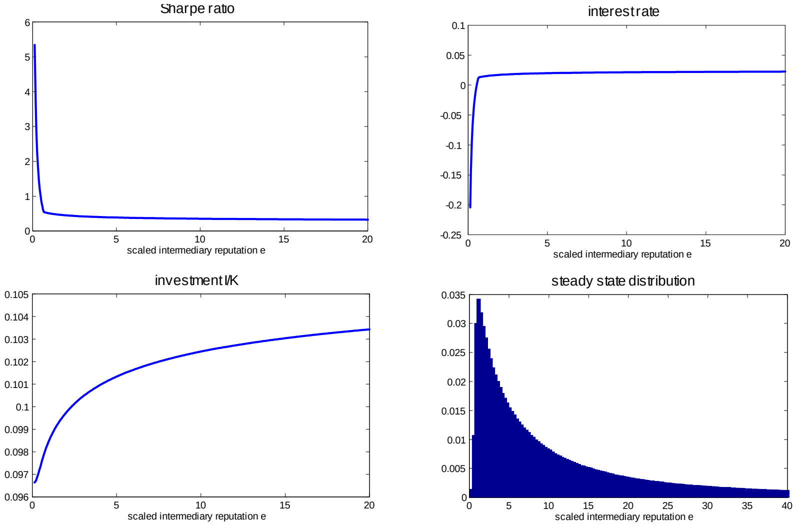
- $e_{distress} = 4$ so that $\Pr\left(e \le e_{distress}\right) = 33\%$ as in data



Matching State-Dependent Covariances: Baseline

	Distress		Non Distress	
	Data	Baseline	Data	Baseline
$vol\,(Eq)$	31.48%	26.01	17.54	6.77
$vol\,(I)$	8.05%	**5.73**	6.61	**5.39**
$vol\,(C)$	1.71%	**3.29**	1.28	**3.94**
$vol\,(LP)$	21.26%	**22.87**	9.79	9.38
$vol\,(EB)$	60.14%	49.96	12.72	6.32
$cov\,(Eq, I)$	1.31%	0.80	0.07	0.36
$cov\,(Eq, C)$	0.25%	0.34	0.03	0.26
$cov\,(Eq, LP)$	4.06%	4.56	0.12	0.63
$cov\,(Eq, EB)$	-6.81%	-6.69	-0.14	-0.09

Matching State-Dependent Covariances: lower σ

	Distress			Non Distress		
	Data	Baseline	$\sigma = 4\%$	Data	Baseline	$\sigma = 4\%$
$vol\,(Eq)$	31.48%	26.01	20.57	17.54	6.77	5.09
$vol\,(I)$	8.05%	5.73	4.40	6.61	5.39	4.18
$vol\,(C)$	1.71%	3.29	2.67	1.28	3.94	3.36
$vol\,(LP)$	21.24%	21.26	12.91	9.79	9.38	6.41
$vol\,(EB)$	60.14%	48.96	36.52	12.72	6.32	4.33
$cov\,(Eq, I)$	1.31%	0.81	0.47	0.07	0.37	0.22
$cov\,(Eq, C)$	0.25%	0.35	0.22	0.03	0.26	0.17
$cov\,(Eq, LP)$	4.06%	4.56	2.03	0.12	0.63	0.34
$cov\,(Eq, EB)$	-6.81%	-6.69	-3.58	-0.14	-0.09	-0.04

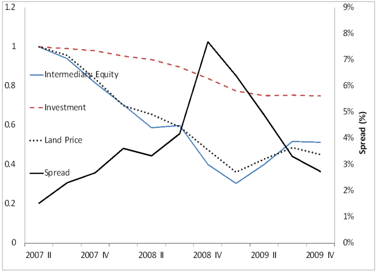
Matching State-Dependent Covariances: No Housing

	Distress			Non Distress		
	Data	Baseline	$\phi = 0$	Data	Baseline	$\phi = 0$
$vol\,(Eq)$	31.48%	26.09	14.62	17.54	6.77	5.00
$vol\,(I)$	8.05%	5.73	5.14	6.61	5.40	5.01
$vol\,(C)$	1.71%	3.29	4.52	1.28	3.92	4.94
$vol\,(LP)$	21.24%	21.26		9.79	9.38	
$vol\,(EB)$	60.14%	48.96	9.42	12.72	6.32	0.03
$cov\,(Eq, I)$	1.31%	0.81	0.53	0.07	0.37	0.25
$cov\,(Eq, C)$	0.25%	0.35	0.44	0.03	0.26	0.25
$cov\,(Eq, LP)$	4.06%	4.56		0.12	0.63	
$cov\,(Eq, EB)$	-6.81%	-6.69	-0.33	-0.14	-0.09	-0.00

Uncovering Shocks in the Recent Crisis

Data

Model



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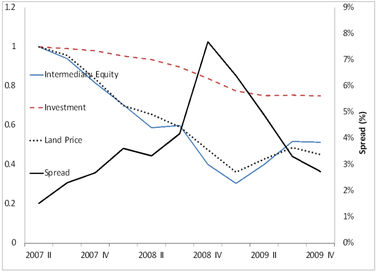

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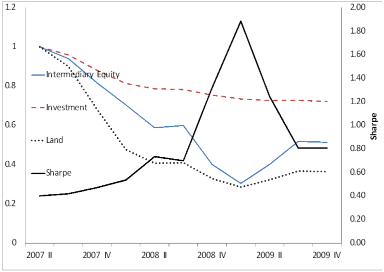


- Based on realized equity return we uncover fundamental shocks to K

07QIII	07QIV	08QI	08QII	08QIII	08QIV	09QI	09QII	09QIII	09QIV
-3.77%	-7.24	-6.62	-2.85	-0.48	-3.10	-2.32	-1.15	-0.04	-0.77

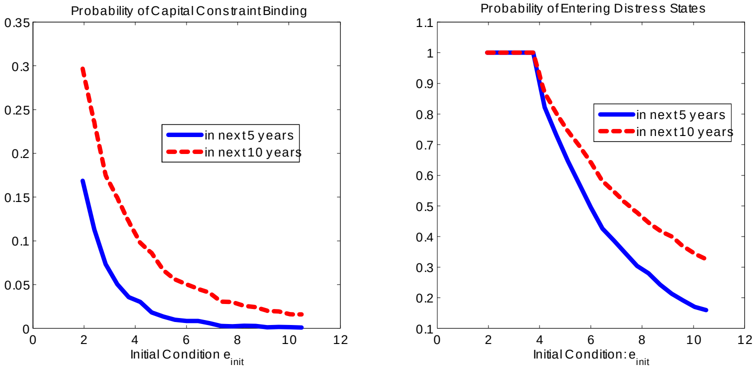
- Total -25%. Capital constraint binds after 08QII—systemic crisis
  - In the model (data), land price fall by 71% (55%)

Probability of Crisis

- 2007Q2, Prob(crisis occurs in the next 2 years)=0.09%, Prob(5 years) = 2.62%, Prob (10 years) = 10.05%

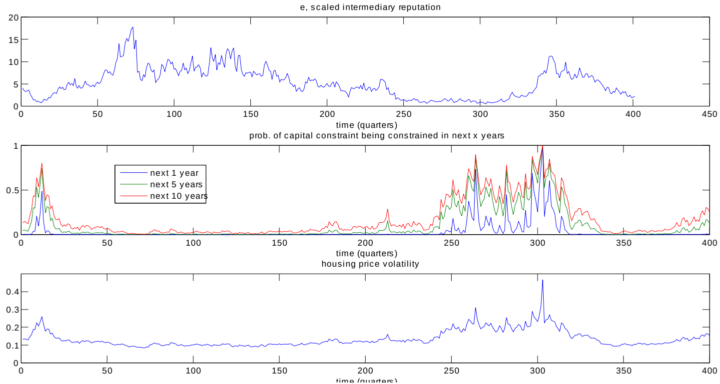
Probability of Crisis

- 2007Q2, Prob(crisis occurs in the next 2 years)=0.09%, Prob(5 years) = 2.62%, Prob (10 years) = 10.05%
- Conditional probability of hitting crisis (left) or distress (right)



Note: Probabilities are low, which suggests improved capital buffers would have limited effects.

VIX and Systemic Risk



- Volatility in our model rises most sharply when the constraint binds
- Coincident indicator and no predictive content.
- What might work better? A VIX spread: Long-maturity VIX minus short-maturity VIX
- Other indicators...

Conclusion

- We develop a fully stochastic model of systemic crisis, with two major frictions:
  - Equity capital constraint on intermediary sector
  - Intermediaries have substantial holdings in real assets (physical capital or housing)
- We find that the model
  - not only qualitatively delivers the nonlinearity observed in the data
  - but also quantitatively matches the differential comovements in distress and non-distress periods
- Recent 07/08 crisis requires a cumulative negative shock around -25%
- Things we are working on: more on model-based measure of systemic risk