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THE DARK SIDE OF LIQUIDITY CREATION:

LEVERAGE AND SYSTEMIC RISK

by

Viral V. Acharya* and Anjan Thakor**

* NYU Stern, CEPR & NBER. E-mail: vacharya@stern.nyu.edu
** Washington University in St. Louis & ECGI. E-mail: thakor@wustl.edu
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ABSTRACT:

This paper exposes a fundamental tension between the micro-prudential objective of subjecting banks to greater discipline through debt markets and the macro-prudential objective of containing systemic risk. We show that banks are illiquid due to the inability of bankers to credibly pre-commit to asset choices. Bank debt can reduce this illiquidity by disciplining bankers with the threat of premature liquidations. However, the liquidation of a bank's assets leads creditors of *other* banks to update their priors on common shocks affecting asset values. This can give rise to contagion and liquidations throughout the system that can be either efficient or inefficient. Thus, liquidity creation induced by the disciplining role of bank debt has the benefit of generating more information about common asset-value shocks, but it also comes at the cost of greater systemic risk, risk that is not fully internalized by banks in choosing privately optimal levels of leverage. We then show that lender of last resort (LOLR) interventions can diminish the incidence of contagion and thereby reduce systemic risk, but they also carry the misfortune of eliminating efficient liquidations. In particular, by reducing creditor incentives to intervene in banks, the LOLR can preclude the discovery of "early warnings" of a banking crisis and risk the emergence of a delayed but more severe crisis. The analysis points to the desirability of ex-ante capital requirements as an alternative to ex-post LOLR interventions.

JEL: G21, G28, G32, G35, G38

KEYWORDS: micro-prudential regulation, macro-prudential regulation, market discipline, contagion, lender of last resort, bailout, capital requirements

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"Any observed statistical regularity will tend to collapse once pressure is placed upon it for control purposes".

Goodhart (1975)

I. INTRODUCTION

In ensuring that the risk of the financial system as a whole stays at "prudent" levels, regulators are tasked to meet two forms of regulatory challenges. One is *microprudential regulation*, which needs to ensure that risk-taking at the individual bank level is not excessive. The other is *macroprudential regulation*, which seeks to contain the *systemic risk* that banks may be excessively exposed to collective failure. To date, these two forms of regulation have been typically dealt with in isolation of each other, especially in policy debates. Microprudential regulation aims to contain the distorted incentives of banks to make choices that maximize the value of bank shareholders' risk-shifting (or asset-substitution) options, especially in the presence of regulatory put options like deposit insurance.[1] Macro-prudential regulation, on the other hand, focuses on containing the risk of events like systemic capital and liquidity shortages, manifesting as fire sales and the freezing up of asset markets. Macro-prudential regulation also examines ways in which regulatory interventions like bank bailouts and lender-of-last-resort (LOLR) facilities can prevent (or engender) such occurrences and contain (or aggravate) their adverse impact. But since both forms of regulation ultimately seek to enhance financial system stability, a natural question that arises is: what are the microfoundations that possibly link these two forms of regulation? In this paper, we show that not only microprudential and macroprudential regulation affect each other, but that in fact there is a fundamental tension between the two.

[1] There is a long history of academic research on micro-prudential regulation. Merton (1977) aptly recognized the isomorphic correspondence between deposit insurance and common stock put options. An important implication was that, given deposit insurance, a bank has an economic incentive to invest in riskier assets and choose relatively low amounts of capital in its capital structure. This means regulatory monitoring of individual banks is necessary to control excessive risk taking designed to exploit deposit insurance.

Let us explain. Calomiris and Kahn (1991) and Diamond and Rajan (2001) provide a theoretical rationale for financial institutions to be highly levered – leverage increases market discipline and thereby enhances bank loan quality and/or liquidity creation.[2] This notion is also codified in bank regulation with market discipline being one of the three pillars of Basel II (the other two being regulatory monitoring and capital requirements). This argument about the market discipline of debt is concerned primarily with the attenuation of *bank-specific* risks.

The recent financial crisis has caused many to assign culpability to the leverage choices of individual financial institutions. Specifically, they argue that very high financial leverage, especially short-term leverage, induced banks to engage in illiquid and risky lending as well as securities activities that resulted in the widespread failures of these institutions (see e.g., Acharya, Schnabl and Suarez (2009), Adrian and Shin (2010), Diamond and Rajan (2009), Goel, Song and Thakor (2010), Mian and Sufi (2010), and Shleifer and Vishny (2010)). There appears to be an emerging acceptance of the fact that increases in leverage seem to increase the systemic risk, or the *collective* fragility of financial institutions insofar that financial crises are typically engendered by or associated with capital or liquidity shortages suffered by individual banks themselves, with the distress of an individual bank – determined both by its specific asset portfolio and leverage choices – coinciding with or leading to distress of other banks.

As a result, bank-specific and systemic risks, and in turn, micro-prudential and macro-prudential regulation, are rendered difficult to separate. In particular, there emerges a somewhat schizophrenic view of the role of leverage. On the one hand, higher leverage means better asset-choices by bank managers and more liquidity when banks are viewed individually. On the other hand, higher leverage also means that the system is more fragile. Faced with circumstances of possible systemic failure, regulatory

[2] Calomiris and Kahn (1991) were the first to formally argue that monitoring by uninsured depositors can result in a bank manager who is making imprudent asset choices being exposed to the threat of a bank run, and that this can induce the manager to shy away from such asset choices. Diamond and Rajan (2001) note that banks invest in assets that are inherently illiquid due to the inability of bank managers to credibly pre-commit to certain actions, and that the threat of a run by uninsured creditors can make these pre-commitments credible, thereby improving liquidity creation by banks. Acharya and Viswanathan (2011) develop this point in a model where financial intermediaries can switch to riskier assets after borrowing and short-term debt with strong control rights ensures ex-ante liquidity by containing this agency problem.

interventions can play a role in the *ex post* reduction of fragility. However, it is also precisely in these circumstances that the discipline of leverage on *ex ante* asset choices is compromised and the lines between microprudential and macroprudential regulation begin to become blurred.

The underlying linkage between leverage, *ex ante* liquidity creation, and *ex post* systemic risk raise some fundamental questions that we address in this paper.

First, what is the role of bank leverage *vis a vis* equity capital in affecting the bank's *ex ante* liquidity and portfolio risk? Second, how does maximizing *individual* bank liquidity (a micro-prudential regulation concern) affect *systemic risk* (a macro-prudential regulation concern)? Third, is there a rationale for a lender of last resort (LOLR) intervention, and if yes, under what circumstances? Fourth, how does the LOLR affect bank leverage, and what are the implications of this for microprudential regulation? That is, when does the LOLR affect (interfere) with the market discipline role of leverage and what are its (perhaps unintended) consequences?

To address these questions, we develop a model of an uninsured bank whose manager has asset-choice flexibility. The bank is *a priori* illiquid because the manager cannot credibly pre-commit to the right asset choices given his personal preference for a private-benefit project. The bank's ex ante liquidity is measured by the financing it can raise by issuing claims against its terminal cash flows. This financing can be any mix of debt and equity. We permit *both* debt and equity to discipline the bank manager to create ex ante liquidity, but this discipline is different depending upon whether it is imposed by debt or equity. Debt disciplines the bank manager by the credible threat that there will be liquidation in some interim states, conditional on interim cash-flow realizations. Equity disciplines the bank manager by providing compensation-based incentives to the manager to select the efficient project. However, since the incentives provided by equity involve payments from ex post cash flows, they reduce the ex ante liquidity of the bank relative to raising debt. In other words, debt enables the bank to discipline the manager without sacrificing cash flows that can be pledged to financiers. Offsetting this ex ante advantage is the fact that leverage leads to liquidation of the bank in some states, and this liquidation can be ex-post inefficient. The bank's privately optimal capital structure is determined in the model by the tradeoff between the *ex-ante* efficiency of leverage relative to equity in the provision of incentives to bankers and the expected ex post cost of inefficient liquidations induced by leverage.

Bank asset portfolios are then allowed to suffer systematic shocks to value that are observed by some of each bank's creditors but not commonly observed by creditors across banks. This means that the (interim) liquidation decision made by the creditors of a bank can be due to either bank-specific information or information about the systematic shock. Since not all creditors of a bank receive information about the systematic shock, but they can observe the liquidation decisions of other creditors, they learn from each other's decisions and update their beliefs about the systematic shock. Their learning is noisy, however, because of the commingling of information about idiosyncratic and systematic risks in any bank's observed liquidation. This can give rise to contagion effects as those creditors of a bank that possess *no* adverse idiosyncratic or systematic risk information about the bank, may choose nonetheless to liquidate their bank at the interim date based solely on observing the liquidations of *other* banks. Note that with the systematic asset-value shock, liquidations are *not* always ex-post inefficient since they are sometimes in response to creditors observing a negative shock to asset value that falls below liquidation values due to the shock. However, there are instances when contagion can lead to ex-post inefficient liquidations. Thus, one dark side of leverage-based liquidity creation is the attendant systemic risk (when contagious liquidations are inefficient), and the higher the leverage of banks, the greater the systemic risk.

We solve for the bank's privately-optimal capital structure in the presence of the systematic asset-value shock. We derive conditions under which the socially-optimal level of leverage for a bank is above its private optimum and the conditions under which it is below. A divergence between the social and private optima arises because, in choosing its own capital structure, an individual bank internalizes neither the valuable information about the systematic shock conveyed to other banks by its own leverage and creditor-led liquidation (a positive externality) nor the higher likelihood that another bank may be inefficiently liquidated when it is liquidated (a negative externality). We show that the privately-optimal bank leverage will be too high relative to the social optimum when the probability of the systematic shock is low enough and the bank's agency problem with the manager is severe enough. Since asset values will be higher when the likelihood of an asset impairment shock is lower, this suggests that the regulator may wish to impose ex ante (countercyclical) capital requirements on banks that are binding during high-asset-valuation periods.

Faced with the prospect of contagion arising from a bank's liquidation, a lender of last resort (LOLR) can step in with a liquidity infusion that effectively bails out the bank, prevents inefficient liquidations (and unfortunately, efficient liquidations as well), and forestalls a contagion in the form of a system-wide liquidation of banks. We consider the unintended consequences of such LOLR. We show that the presence of the LOLR can destroy all ex-ante market discipline of debt as creditors, who anticipate ex-post bailouts, and have no incentives to engage in privately-costly monitoring. This, in turn, causes a complete evaporation of the asset-value information generated by creditor liquidations. Somewhat paradoxically, capital structures of banks become irrelevant. However, since all market discipline of leverage is lost, the ex-ante liquidity of banks declines relative to that available in the absence of the LOLR. In other words, the LOLR eliminates ex-post bank liquidations and reduces system-wide fragility; the price of this ex-post liquidity created by LOLR shows up as lower *ex-ante* liquidity and the loss of valuable information about asset values, creating a tension between the micro-prudential and macro-prudential regulation of banks.

Finally, we explain another important downside of LOLR interventions, even from an ex-post standpoint. Suppose the systematic shocks to asset values consist of early-warning indicators that can help arrest asset-value declines to some extent if banks heed these warnings and take corrective actions by infusing additional equity; and also that asset-value impairment can be really severe if such recapitalization is not done. We show in this setting that LOLR can cause the financial system to ignore the early-warning indicators, implying that LOLR interventions can prevent "small" crises but risk the emergence of "big" crises as the system remains under-capitalized in response to early shocks.

In one sense, our analysis highlights Goodhart's (1975) Law in that the market discipline of debt collapses once leverage-based liquidation risk is relied upon by regulators to undertake LOLR interventions. This would – at least to some extent – not be the case if regulatory interventions were based on regulatory intelligence about bank solvency over and above market signals, rather than just based on market signals (such as creditor-led liquidations). This suggests that the LOLR may be more effective if accompanied by information generation by central banks or bank regulators that is independent of the information generated by bank creditors.[3] Our analysis can also be viewed as an illustration of the Lucas Critique (1976) in that the macroprudential effects of the LOLR cannot be predicted without accounting for how individual institutions and investors will change their behavior (monitoring by creditors and recapitalization by bank owners, in our model) in response to the change in policy resulting from the introduction of the LOLR.

The rest of the paper is organized as follows. Section II provides a brief summary of related literature. Section III develops the model. Section IV contains the analysis of the basic model and shows how leverage helps create both liquidity and systemic risk. Section V examines the role of the LOLR in controlling or aggravating systemic risk. Section VI concludes. All proofs are in the Appendix.

II. RELATED LITERATURE

Our paper is related to many strands of the literature. On the topic of micro-prudential regulation of banks, the role of leverage in imposing market discipline on banks has been recognized in numerous papers, as mentioned earlier. See, for example, Calomiris and Kahn (1991), Dewatripont and Tirole (1994)[4], Diamond and Rajan (2001), and Acharya and Viswanathan (2011). Like these papers, we also show how bank leverage, through its disciplining role, enhances bank liquidity. However, in contrast to these papers, we also allow bank equity to discipline the manager, albeit through a different channel, namely compensation. This allows us to examine the tradeoff between disciplining the bank through leverage and disciplining it through equity.

In this respect, the manner in which equity discipline works in our model is different from the way it works in various other papers where high equity capital deters asset-substitution moral hazard (see,

[3] An important exception is if the regulators require bank creditors to be segregated by priority and rely on information signals generated by non-contagious or run-prone liabilities of banks, as analyzed by Hart and Zingales (2009).

[4] Dewatripout and Tirole (1994) also discuss how shareholders can affect managerial incentives in banks.

for example, Bhattacharya, Boot and Thakor (1998) for a review of this literature). Acharya, Mehran and Thakor (2010) have recently pointed out that this role of equity in deterring asset-substitution incentives also produces a tension between having leverage and equity in a bank.

There are also various papers on systemic risk in banking that are related to our work. One strand of research focuses on effects of risk-sharing on systemic risk. Shaffer (1994) and Winton (1999) illustrated the point that "pooling (diversification) intensifies *joint* failure risk".[5] Wagner (2010) provides empirical evidence that while diversification reduces the risk of an individual bank, it increases systemic risk. Another strand of research has highlighted that the presence of the LOLR can make more likely the very state of correlated capital and liquidity shortages that the LOLR is trying to avoid ex post (see e.g., Acharya and Yorulmazer (2007), Acharya (2009), Farhi and Tirole (2009), Acharya, Mehran and Thakor (2010), and Kane (2010)). In contrast to these papers, we show that systemic risk can arise – even in the absence of regulatory safety nets – due to contagious runs upon liquidation of individual banks.[6] Our key point, however, is that the safety nets designed to address such contagion can destroy the very reason for banks to have leverage in the first place. What is more, they can in fact delay the recognition of early warnings about crises and lead to delayed but deeper crises. Finally, Diamond and Rajan (2011) also highlight that ex post forbearance reduces ex-ante liquidity creation by banks and recommend that the central bank adopt a policy that raises interest rates in good times to counter the effect of forbearance.

III: THE MODEL

This section develops the basic model and the contracting opportunities. Consider an economy in which all agents are risk-neutral and the riskless rate is zero. There are three dates $t = 0, 1, 2$. There are banks run by managers that make investments at $t=0$ that generate payoffs at $t = 1$ and $t = 2$. We will refer to banks' investments as projects. There are two types of projects, all of which require liquidity ρ at $t = 0$.

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[5] Brunnermeier and Sannikov (2010) also show that risk sharing by individual institutions within the financial sector can amplify systemic risk.

[6] Contagion can also arise due to interconnectedness rather than systematic risk exposures. Caballero and Simsek (2010), for instance, argue that increased complexity due to greater interconnectedness among individual banks can generate endogenous risk and crises.

If this liquidity need is met at $t = 0$, then the project generates a random cash flow x at $t = 1$, with density function $f(x)$ and cumulative distribution $F(x)$. The support of f is $[x_{\min}, x_{\max}]$.

A. Types of Projects

The two types of projects available to the bank are the "good" project and the "private-benefit" project.[7] The good project produces a cash flow at $t = 2$ of Hx with probability $q \in (0, 1)$, and zero with probability $1 = q$. The private-benefit project produces a cash flow at t=2 of Hx with probability $p \in (0, q)$, and zero with probability $1-p$ at $t = 2$. Both projects have the same density function of the date-1 cash flow, x. The bank's manager gets a private benefit of B per unit of x, i.e., a total benefit of Bx at $t = 2$. We assume that the private-benefit project is socially inefficient relative to the good project:

$$qH > \left[pH + B\right]. \tag{1}$$

Moreover, the good project is socially efficient:

$$qHE(x) > \rho, \tag{2}$$

where $E(x)$ is the expected value of x. The good project and the private-benefit project are mutually exclusive. Moreover, the bank manager makes an initial project choice at $t = 0$ but can costlessly switch projects at $t = 1$.

We assume that all cash flows are pledgeable but we will show that they will not always be pledged in equilibrium due to the provision of incentives to bankers. The bank can raise the required initial liquidity via debt or equity whose features we describe next.

B. Debt and Equity Contracts and Assets Portfolio Liquidity

If the bank raises debt financing at $t = 0$, it is assumed that the debt contract contains a covenant whose violation at $t = 1$ permits the creditors to take control and demand full repayment at $t = 1$. Asking for full repayment could result in the bank being liquidated at $t = 1$ if its interim cash flow is insufficient to make the repayment. Such "accelerated repayment" clauses are standard in debt contracts. If the bank is not liquidated at $t = 1$, then it will continue until $t = 2$. Another way to think about this is to view the debt

[7] The model could also potentially be written in terms of a risk-shifting or asset-substitution moral hazard, instead of a private benefit project, as in Acharya and Viswanathan (2011) or Acharya, Mehran and Thakor (2010).

contract as having a one-period maturity, so that the creditors decide at $t = 1$ whether to renew funding for an additional period after the debt matures or simply collect whatever the bank can repay at $t = 1$ and deny renewal of funding.

The face value of debt (promised repayment to creditors) is F. The date-1 cash flow is pledged to the creditors up to this face value of F, so the remaining repayment to them at date 2 (in case the bank is not liquidated at date 1) is $\max(F - x, 0)$. It is assumed that the liquidation value of the bank at t=1 is dependent on the realized date-1 cash flow x. This liquidation value is Lx. We assume that

$$qH > L > pH + B. \tag{3}$$

That is, liquidation is better than the private-benefit project under the manager, but worse than the good project.

If equity is used instead of debt, it is assumed that shareholders have no control rights at $t = 1$ other than over the cash flow x. That is, shareholders cannot "withdraw" their equity investment in the firm at $t = 1$ by liquidating the bank. We just take this feature of the equity contract relative to debt as a given. That is, our purpose is not to endogenously derive debt and equity as (constrained) efficient contracts in a security-design setting. However, if the shareholders wish, they can provide incentives for the manager to choose the value-maximizing project by giving him a suitably chosen share $\theta \in (0,1)$ of the terminal payoff at $t = 2$.[8]

We assume that a monitoring technology is available to both shareholders and creditors that would enable them to discover the bank's project choice at a cost $c > 0$. If they dislike this project choice, they can ask the bank's manager to change it. If the manager refuses, financiers have *only one* recourse—deny renewal of funding at $t = 1$ for the second period. This lack of funding renewal *may* force the bank to liquidate in order to meet its repayment obligation. Faced with this prospect, the bank manager may agree to switch project choice.

[8] Note that giving the manager a share of the date-1 payoff x is not useful for incentive purposes because the probability distribution of x is the same for both types of projects, and the manager can switch projects at $t = 1$ after x is realized.

It follows immediately that the bank's creditors will refuse to renew funding at t = 1 if they discover the manager has not chosen the project they desire. If the bank can fully pay off creditors from its cash flow at t = 1 and still continue, it will choose to do so. But if this cash flow is insufficient, then the creditors' threat of liquidation will have bite and a project change will occur. By contrast, if shareholders were to expend c and discover a project choice they did not like, they have no credible threat that they can make to force a change. This is because equity has no finite maturity and shareholders cannot ask for their investment to be returned at t = 1.

This disciplining role of debt (but not equity) is familiar from the previous work of Calomiris and Kahn (1991), and Diamond and Rajan (2001). The very nature of the debt contract facilitates monitoring-based market discipline that is not possible with equity (see also Hart and Moore (1995)).

The fact that creditors can deny renewal of funding at t = 1—through either a covenant trigger that leads to a demand for accelerated repayment on two-period debt or a denial of renewal of one-period debt—is similar to repo market funding drying up for financial institutions during the recent crises. When short-term financiers in the repo market became aware of adverse information about financial institutions, the short-term funding they had provided dried up.[9]

C. Lack of Pre-Commitment

The first-best is achieved if the manager could make a pre-commitment at t = 0 that he will invest in the good project and not switch at t = 1 to the private-benefit project. However, we assume that such commitment is in general not credible other than through incentives provided by debt and equity.

D. Systematic Shock to Asset Values

We assume that asset values are subject to a systematic shock, represented by the realized value of an underlying state variable, ξ, that is experienced at t = 1. With probability $\beta \in (0,1)$, a systematic asset-value impairment shock is realized as $\xi = \xi_t$. This shock lowers the value of H (for both types of projects) to $H^- < H$, with $qH^- < L$. With probability $1 = \beta$, the state variable is $\xi = \xi_h$, and asset value is now

[9] This aspect of wholesale funding of banks has also recently been examined by Huang and Ratnovski (2011) though they focus on its negative side-effects.

higher at $H^{+} > H$, with $\beta H^{-} + [1-\beta] H^{+} = H$. For simplicity, we also assume that $qH^{+} - L = L - H$. For a given H, these two restrictions determine H^{+} and H^{-}.

E. Observability

We assume that the interim cash flow x of each bank is not observable to any party other than the bank's shareholders, creditors and the manager. Moreover, the manager's project choice is not directly observable to anyone but the manager himself, unless financiers intervene and are able to enforce a different project choice.

The realized value of the systematic shock to a bank's asset value is not observed with certainty by the bank's creditors. The probability that the bank's creditors will observe the systematic-shock signal ξ is $\gamma \in (0,1)$. Conditional on a particular ξ, the random variable γ is independent and identically distributed across banks in the economy. The realization of the systematic shock ξ affecting a bank and the bank's cash flow x are privately observed (if at all) only by that bank's manager, creditors and shareholders. However, the liquidation of a bank by its creditors is commonly observed by all agents.

The sequence of events in the model is summarized in *Figure 1*.

IV. ANALYSIS OF THE BASIC MODEL

In this section, we present analysis of the model developed in the previous section by considering the relative advantages of debt and equity financing when there is no systematic component to asset value shock. We begin with an analysis of the events at $t = 1$, and then we turn to the events at $t = 0$.

A. Analysis of the Model without the Systematic Asset Value Shock: Managerial Incentives and Creditors' Liquidation Decision at *t* = 1

Consider first the case in which the bank finances with all equity at $t = 0$. To induce the manager to select the good project at $t = 1$, the share θ of the terminal cash flow provided to him must satisfy the incentive compatibility (IC) constraint point-wise at $t = 1$ for every realized x.[10]

$$\theta qHx \geq \theta pHx + Bx.$$

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[10] The reason why the IC constraint must hold point-wise for every x realized at *t=1* is that the manager makes his project choice at *t=1* *after* observing x.

Since this IC constraint is binding in equilibrium, we can write the optimal value of θ, call it θ^*, as:

$$\theta^* \equiv B\left[H\Delta\right]^{-1}, \qquad (4)$$

where $\Delta \equiv q - p$. That is, equity provides incentives to the manager via a payoff-contingent compensation contract.

Next we turn to the case in which the bank has financed with debt at t = 0. Having collected x from the date-1 payoff, creditors will assess their payoffs from liquidation and continuation (when they enforce the efficient project choice) as follows:

$$\text{Liquidate:} \quad \min\left(Lx, F - x\right) \qquad (5)$$

$$\text{Continue:} \quad q\min(Hx, F - x) \qquad (6)$$

We now have the following result:

Lemma 1: *In absence of the systematic shock ξ, the bank will adopt a debt contract in which the creditors set the covenant violation trigger at $F - c[\Delta]^{-1}$, so that if $x < F - c[\Delta]^{-1}$ creditors expend c to discover the manager's project choice at $t = 1$ and enforce a different project choice if they so desire. If $x \geq F - c[\Delta]^{-1}$, creditors do not expend c to investigate, and unconditionally allow the manager to continue with the project. Conditional on having expended c, creditors adopt the following liquidation/continuation policy:*

- *Continue if $x \geq \lambda F$;*
- *Liquidate If $x \in \left(\lambda F, F - \dfrac{c}{\Delta}\right)$;*

where $\lambda \equiv q[q+L]^{-1}$; All liquidations by creditors are (ex-post) inefficient. When $x > F - c[\Delta]^{-1}$, shareholders provide incentives to the manager by giving him ownership θ^* of the bank's terminal payoff, where θ^* is given by (4).

The different cases are pictorially depicted in *Figure 2*. The intuition is that when x is very high, unconditional continuation is optimal for the creditors at t = 1 because their claim is covered out of just the date-1 cash flow, and it does not matter to them what project the bank invests in. In these states in which creditors are paid off out of the date-1 cash flow, shareholders provide the necessary project-choice incentives for the continuation by awarding the manager an ownership share of the bank. That is, compensation incentives provided by equity replace the monitoring discipline provided by debt. When x is very low, creditors cannot be paid off fully from the date-1 cash flow, but they prefer to continue rather than liquidate the bank. This is because the liquidation payoff Lx is so low that it is better for creditors to take a chance on a higher payoff qHx by continuation. For intermediate values of x, liquidation is optimal for creditors because they get a sufficiently high certain payoff of Lx so that given the "risk aversion" induced by concavity of the debt contract, it does not pay for the creditors to gamble on a risky continuation payoff.

This analysis reveals the pros and cons of debt financing. With debt financing, the manager will not have to be provided a compensation incentive over the entire range of x when equity is used. These incentives need to be provided with debt only when $x > F - \frac{c}{\Delta}$. Thus, debt reduces the region over which compensation-based incentives must be provided from $[x_{\min}, x_{\max}]$ to $\left[F - \frac{c}{\Delta}, x_{\max}\right]$. This is the benefit of debt financing as compensation incentives reduce the initial maximum liquidity the firm can raise via external financing at $t = 0$. The disadvantage of debt financing is that it creates a region $\left[\lambda F, F - \frac{c}{\Delta}\right]$ over which the bank is (*ex-post*) inefficiently liquidated. In what follows, we shall assume that c is arbitrarily small by letting $c = 0$ because doing so reduces notation without affecting the qualitative analysis.

B. Analysis of the Model with the Systematic Asset-Value Shock: The Creditors' Liquidation Decision at *t = 1*

Thus far, we have examined the monitoring and liquidation decisions of creditors and the contracting decision of banks when asset values are not subject to the systematic asset-value shock. We now include this systematic shock, so that one bank's liquidation can convey information about the shock to another bank. Imagine there are two banks in the economy and conditionally on the systematic shock ξ, each bank is faced with identically and independently distributed (i.i.d.) shocks. In this two-bank economy, we also need to specify how the banks observe each other's liquidation outcomes and how this affects the liquidation decisions of creditors of each bank. We model this as a two-stage game. First, each bank's creditors simultaneously announce whether they will liquidate the bank based solely on their own information about the bank. In this state, neither bank's creditors have access to the decision of the other bank's creditors. Second, after having observed each other's first-stage liquidation outcomes, each bank's creditors decide whether they wish to liquidate in the second stage (provided they did not liquidate in the first stage). In particular, bank *i*'s creditors have *no* incentive to decide not to liquidate in stage 2 if they decided to liquidate in stage 1.[11] But they may decide to liquidate in stage 2 even if they announced in stage 1 that they would not liquidate, if they observe the creditors of the other bank liquidate. We assume (and derive condition below) such that if a bank is liquidated, the inference about ξ for the creditors of the other bank is sufficiently adverse that they liquidate too.

Note that conditional on the creditors observing $\xi = \xi_\ell$, the creditors will liquidate if

$$qH^- < L, \tag{7}$$

which we assumed earlier. To figure out the sufficient condition for our assumption that the creditors of bank *i* will liquidate in stage 2 if they observe the creditors of bank *j* liquidating, let us calculate the posterior belief of creditors of bank *i* (if they observe the creditors of bank *j* liquidating) concerning the aggregate shock being adverse. At this stage, we need to introduce some notation that is useful for examining the inference problem from the perspective of bank *i*: Define

$$\delta \equiv \Pr\left[x \in \left(\lambda F, F\right)\right] = H\left(F\right) - H\left(\lambda F\right)$$
$$\delta_1 \equiv \beta\left[1-\gamma\right]^2\left[1-\delta\right]$$
$$\delta_2 \equiv \left[1-\beta\right]\left[\gamma + \left[1-\gamma\right]\left[1-\delta\right]\right]$$

[11] There are no strategic manipulation incentives in the model. In particular, there is nothing to be gained for the creditors in one bank from liquidating or not liquidating a bank in order to strategically manipulate the behavior of creditors in another bank.

Note that δ is simply the probability that a particular bank's $x \in (\lambda F, F)$ and hence it is liquidated by its creditors solely due to the realization of its own cash flow.

Next, δ_1 is the joint probability that the adverse asset-value shock was realized (the probability that $\xi = \xi_\ell$ is β), neither bank observed the shock (probability $(1 - \gamma)^2$), and the *other* bank (which is j) did *not* liquidate due to a cash-flow realization $x \in (\lambda F, F)$ (the probability that $x \notin (\lambda F, F)$ is $1 - \delta$).

Similarly, δ_2 is the joint probability that the favorable asset-value shock was realized (probability $1-\beta$), and either it was observed by the bank (probability γ) or it was not observed (probability $1 - \gamma$) *and* the other bank (which is j) did not liquidate due to its own $x \in (\lambda F, F)$ (which has probability $1 - \delta$). Thus, $\delta_1 + \delta_2$ is the probability that bank i is *not liquidated* when its own $x \in (x_{\min}, \lambda F)$.

In calculating the posterior belief that the aggregate shock is adverse, what we are assuming is that the bank in question (bank i) did not receive the signal about the shock, decided *not* to liquidate in stage 1 but observed the other bank (bank j) liquidating in stage 1. Then bank i forms the following posterior belief about ξ:

$$\Pr\left(\xi = \xi_\ell \,\middle|\, \text{bank } j \text{ announced liquidation in stage 1}\right)$$

$$= \frac{\Pr\left(\text{bank } j \text{ liquidated in stage 1} \,\middle|\, \xi = \xi_\ell\right)\Pr\left(\xi = \xi_\ell\right)}{\Pr\left(\text{bank } j \text{ liquidated in stage 1}\right)}$$

$$= \left(\frac{[\, \gamma\, H(\lambda F) + \delta \,]\, \beta}{\left\{ [\, \gamma\, H(\lambda\, F) + \delta \,]\, \beta + [1-\beta][\delta] \right\}}\right) \equiv \hat{\beta}. \tag{8}$$

Note that this probability is positive only if bank i did not observe $\xi = \xi_h$ in the first stage. Further $\hat{\beta} > \beta$.

To understand the numerator of (8), note that, conditional on $\xi = \xi_l$, bank j can be liquidated in one of two cases: (i) if its creditors observe ξ (which has probability γ) *and* bank j's cash flow $x \leq \lambda F$ (which has probability $H(\lambda F)$), *or* (ii) bank j's cash flow realization $x \in (\lambda F, F)$ (which has probability δ). Hence, the probability of being in one of these two states is $\gamma H(\gamma F) + \delta$. This probability is multiplied with β, the probability that $\xi = \xi_l$. In the denominator, there is an additional term, $[1 - \beta]$, which is the probability that bank *j* is liquidated due to its $x \in (\lambda F,F)$ even when $\xi = \xi_h$ (which has probability $1 - \beta$

).

Also, for bank *i*'s creditors to decide to liquidate (only) after having observed bank *j*'s creditors liquidations, we need two conditions:

Condition 1: Creditors will not unconditionally liquidate before observing ξ :

$$q\left[\beta H^{-} + [1-\beta] H^{+}\right] > L$$

which implies:

$$qH > L \tag{9}$$

which we have assumed throughout.

Condition 2: It pays for the creditors of bank i to liquidate if they observe liquidation by the creditors of bank j:

$$q\left[\hat{\beta} H^{-} + \left[1-\hat{\beta}\right] H^{+}\right] < L \tag{10}$$

where $\hat{\beta}$ *is defined in (8). Define* $\bar{L}$ *as the value of L at which (10) holds as an equality, so that the inequality in (10) will hold for all* $L > \bar{L}$.

We now have:

Proposition 1: *For a given leverage of bank i, the higher the leverage of bank j, the larger is the set of exogenous parameter values (the smaller is* $\bar{L}$*) for which bank i's creditors liquidate bank i, when they observe the liquidation of bank j at* $t = 1$.

Moreover, for a given leverage of bank j, the posterior probability that the creditors of bank i will liquidate the bank upon observing the liquidation of bank j and the (unconditional) ex ante probability of liquidation of bank i are both increasing in bank i's leverage whenever bank i's leverage is sufficiently high.

The intuition can be seen by noting that $\hat{\beta}$ in (10), the posterior probability of bank *i*'s creditors that a negative systematic shock (ξ_l) has been realized when they observe the liquidation of bank *j*, is increasing in *F ceteris paribus*. And as $\hat{\beta}$ increases, it becomes easier to satisfy (12), the condition for the creditors of bank *i* to liquidate. Somewhat less technically, the intuition can also be gleaned by examining (7). The only state in which contagion-type liquidation occurs is conditional on an interim cash flow realization $x \in [x_{\min}, \lambda F]$, and the probability of this is increasing in *F*.

This proposition highlights the dark side of leverage-induced liquidity creation. As banks become more highly levered in order to raise their own (individual) liquidity, liquidation contagion that affects other banks becomes more likely but the cost this contagion imposes on other banks will not (in general) be internalized by each bank. Put a little differently, higher bank leverage to increase the liquidity of individual banks can elevate their joint failure or systemic risk. The dark side of leverage-induced liquidity creation is that this contagion effect generates liquidations that are at times inefficient. To see this, note that when $\xi = \xi_h$, bank *i*'s creditors observe that but bank *j*'s creditors do not, and thus if bank *i* is liquidated because of its realized *x*, there will be a liquidation of bank *j* even in states in which its realized *x* does not justify it. In these states, there is an *ex post* efficiency loss because the assets would have been worth more with continuation than they are with liquidation.

Recall that in the previous analysis conducted in the absence of the systematic asset-value shock ξ, *any* liquidation by a bank's creditors was ex post inefficient (see Lemma 1). That is no longer true, however, when ξ is introduced. Now there can be an ex post efficiency gain from liquidation in the case where the creditors of bank *i* do not observe ξ but liquidate based upon observing the liquidation of bank *j* whose creditors observe $\xi = \xi_l$. To the extent that leverage increases the probability of such a liquidation, it also contributes to ex post efficiency in such states. Hence, with the introduction of the systematic asset-value shock, leverage-induced liquidity creation has both a dark side and a bright side from an ex-post standpoint.[12]

[12] We have conducted this analysis for the two-bank case to convey these ideas as transparently as possible. The *n*-bank case, with n > 2, is qualitatively similar, although it permits a weakening of the restrictions on the exogenous parameters that are needed for liquidation contagion.

Note also that we have assumed that creditors in different banks are distinct. What if say a single creditor could take diversified positions in all banks? On the one hand, this will eliminate inefficient liquidations because there is no contagion to worry about. On the other hand, efficient liquidations will also decline because the probability of receiving information about the systematic shock will be limited to the probability that one bank will receive this signal.

C. The Value of the Levered Bank and Optimal Leverage at $t = 0$

We can now move to t = 0 and examine the bank's optimal capital structure decision. The following result is useful as a first step.

Lemma 2: *The total value of the levered bank at date 0 can be written as a function of its own leverage of face value F (for given leverage of the other bank) as:*

$$\begin{aligned} V(F) &= E(x) + \int_{\lambda F}^{F} Lxh(x)\mathrm{d}x + \left[1-\theta^*\right] qH \int_{F}^{x_{\max}} xh(x)\mathrm{d}x \\ &+ \left\{\delta_1 H^- + \delta_2 H^+ \right\} q \int_{x_{\min}}^{\lambda F} xh(x)\mathrm{d}x \\ &+ \left\{ 1-\delta_1-\delta_2 \right\} L \int_{x_{\min}}^{\lambda F} xh(x)\mathrm{d}x \end{aligned} \tag{11}$$

where θ^* *is given by (4). Moreover,* $V_{lev}(F)$ *is increasing in* $\delta_2[qH^+ - L]$ *and decreasing in* $\delta_1[L - qH^-]$.

To understand (11), recall that the first term, *E(x)*, is the expected value of the interim cash flow.

The second term is the liquidation by creditors that occurs when $x \in [\lambda F, F]$, and the value is *Lx*.

The third term refers to the state $x \in (F, x_{\max}]$. In this state, there is no liquidation or creditor monitoring, so the manager is incentivized by shareholders with a compensation contract that pays him a fraction of the terminal cash flow, leaving a net payoff of $(1-\theta^*)$ times the actual cash flow.

To understand the fourth term, note that $1-\delta_1-\delta_2$ is the probability of liquidation that is *not* based on this bank's realization of x, and $\delta_1+\delta_2$ is the probability of *no liquidation*, when $x \in [x_{\min}, \lambda F]$. As explained earlier, δ_1 is the joint probability of $\xi = \xi_\ell$ and no liquidation, and δ_2 is the joint probability of $\xi = \xi_h$ and no liquidation. Note also that if $\xi = \xi_\ell$ is realized, the value is qH^- per unit of x and if $\xi = \xi_h$ is realized, the value is qH^+ per unit of *x*. This explains the fourth term.

The fifth term refers to the liquidation that occurs because creditors observe the liquidation of the *other* bank, i.e., this is liquidation that is not based on the bank's own *x*. This occurs with probability $1-\delta_1-\delta_2$ when $x \in \left[x_{\min}, \lambda F\right]$, and the resulting payoff is *Lx*.

It is intuitive that bank value is increasing in $\delta_2[qH^+ - L]$ since this quantity represents the expected net gain in value from avoiding (ex post) inefficient liquidation when $\xi = \xi_h$. Moreover, bank value is decreasing in $\delta_1[L - qH^-]$ because this term represents the expected net gain from incurring (ex post) efficient liquidation when $\xi = \xi_\ell$. Finally, note that the other bank's leverage is contained in the probabilities δ_1 and δ_2 .We can now prove the following result:

Proposition 2: *The privately-optimal level of bank leverage,* F^**, at* t = 0 *that uniquely maximizes its value* V *in (11), for a given leverage of the other bank, is such that*

$$\frac{h(F^*)}{h(\lambda F^*)} = \frac{\lambda^2 \left\{ \left[\delta_1 H^- + \delta_2 H^+ \right] q - L\left[\delta_1 + \delta_2 \right] \right\}}{[1-\theta^*]\, qH - L}. \tag{12}$$

This proposition characterizes the best response of a bank in terms of its choice of leverage (given the leverage of the other bank) that maximizes the value of the bank and hence its *ex ante* liquidity. A sufficient condition for F^* to be a unique maximum is that the probability density function h has a turning point at $F \in (\lambda F^*, F^*)$ such that $h'(F) = 0, h'(F^*) > 0$, and $h'(\lambda F^*) < 0$. In the Appendix, we also provide an example of a probability density function that has the properties sufficient for the second-order condition for a unique maximum to hold. Note that each bank's leverage choice has to be examined as part of a Nash equilibrium because each bank's leverage affects the other bank's expected payoff (formally through δ_1 and δ_2). While multiple Nash equilibria are possible, banks' leverage choices in any equilibrium must satisfy (12).

Corollary 1: *Denoting as* F^0 *the (privately) optimal leverage in the base case without the asset value shock* $(\gamma = 0, \beta = 0, H^+ = H)$*, we obtain that*

$$\frac{h(F^0)}{h(\lambda F^0)} = \frac{\lambda^2 \left[qH - L\right]}{[1-\theta^*]\, qH - L}. \tag{13}$$

This corollary reveals in a transparent manner that the bank maximizes its ex ante liquidity through a combination of debt and equity. Since $h'(F^0) > 0$, the bank will choose a higher leverage when there is a decrease in the net value of equity relative to the bank's liquidation value, which is $[1-\theta^*]\, qH - L$ (per dollar of interim cash flow), since this liquidation cost is incurred more often with greater leverage. As it becomes more costly to provide the necessary incentives via compensation (θ^* increases due to an increase in B that makes the private-benefit project more attractive), banks will rely more on leverage to obtain the desired liquidity. Likewise, as creditors become more proficient in liquidating bank assets at relatively high values (higher L), banks will again rely on higher leverage.

Proposition 2 also has another interesting implication. It suggests that banks may privately choose to be more highly leveraged than is socially efficient. Since higher leverage leads to higher systemic risk in the model due to contagious liquidations at other banks[13], leverage carries a negative externality on other banks. Counteracting this is the fact that liquidations at other banks are sometimes efficient when the systematic asset-value shock is indeed adverse and this leads to efficient information updating by the creditors of a given bank. This means that higher leverage has both a bright side (more efficient liquidations) and a dark side (more inefficient liquidations). The next result, one of our key ones, highlights when the dark side of leverage – the negative externality imposed on other banks through contagious liquidations – dominates, so that the privately optimal leverage exceeds the socially optimal one.

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[13] Higher leverage means a higher probability of correlated liquidations since it increases the probability of liquidation of any given bank as well as the probability that another bank will liquidate upon observing the liquidation of any given bank.

Formally, let $V_i(F_i, F_j)$ denote the value of bank *i* given the leverage of the two banks F_i and F_j. This is given by equation (11) where all *F* in equation are replaced by F_i and F_j appears only in the probabilities δ_1 and δ_2 through its effect on δ, the probability of liquidation of bank *j*. Then the best-response of bank *i* in terms of its leverage choice $F_i^*(F_j)$ maximizes $V_i(F_i, F_j)$ for given leverage of bank *j*, F_j. This best response is as characterized in Proposition 2. Then the Nash equilibrium of private leverage choices $\left(\overline{F_i}, \overline{F_j}\right)$ satisfies the fixed-point problem $\overline{F_i} = F_i^*\left(\overline{F_j}\right)$ and $\overline{F_j} = F_j^*\left(\overline{F_i}\right)$.

Now, the socially optimal leverage choice $\left(\hat{F}_i, \hat{F}_j\right)$ maximizes the sum of the two bank values taking account of the externalities of bank leverage on other banks. That is, the social problem is to maximize $V_i(F_i, F_j) + V_j(F_i, F_j)$, whose first-order conditions are given by

$$\begin{aligned} \frac{\partial V_i}{\partial F_i} + \frac{\partial V_i}{\partial F_j} &= 0, \\ \frac{\partial V_j}{\partial F_i} + \frac{\partial V_j}{\partial F_j} &= 0. \end{aligned} \tag{14}$$

In contrast, the private optimum satisfies $\frac{\partial V_i}{\partial F_i} = 0$ and $\frac{\partial V_j}{\partial F_j} = 0$. Then, assuming a convex social optimization problem and symmetry, the socially optimal leverage choices *exceed* the privately optimal leverage choices if and only if the externality terms, $\frac{\partial V_i}{\partial F_j}$ and $\frac{\partial V_j}{\partial F_i}$, are negative. For the rest of the analysis in this section, we will assume the symmetry condition $L - qH^- = qH^+ - L$, and that $\beta \le 0.5$.

Then, we obtain the following intuitive result (focusing on the symmetric case as before):

Proposition 3: *In a Nash equilibrium involving banks i and j, the privately-optimal leverage* $\left(\overline{F_i}, \overline{F_j}\right)$ *exceeds the socially optimal leverage* $\left(\hat{F}_i, \hat{F}_j\right)$, *with* $\left(\overline{F_i}, \overline{F_j}\right)$ *and* $\left(F_i^*, F_j^*\right)$, *when* β, *the probability of the systematic asset-value shock, is sufficiently low and B, the manager's private benefit from the private benefit project, is sufficiently high.*

This proposition captures the essence of our main point highlighting the systemic risk induced by leverage-based creation of liquidity. The role of the private benefit, B, is intuitive. As B increases, disciplining the manager through compensation incentives becomes more expensive and the bank prefers to increase its leverage in its private optimum. However, as bank *i*'s privately-optimal leverage increases, its negative impact on the value of bank *j*—which derives no benefit from better discipline at bank *i*—also increases. Thus, the social optimum drops below the private optimum. When β is too low, the social value of liquidations based upon creditors observing the liquidations of other banks is also low. This is because the likelihood of the asset-value impairment shock is low, so the value of learning about this shock through another bank's liquidation is also low. Since higher bank leverage leads to a higher liquidation probability, the *social* value of higher leverage declines as the value of learning about the systematic shock through liquidations declines due to a reduction in β. The inefficiency associated with contagious liquidations remains unaffected, however. Thus, the socially-optimal leverage decreases. But since the privately-optimal leverage choice of a bank does not internalize the inefficient contagious liquidations of other banks induced by its own leverage, the bank levers up more than is socially desirable.

This is the "dark" side of leverage-based liquidity creation by individual banks. Having leverage at the bank level is a desirable objective from a micro-prudential standpoint but socially costly due to its perverse implications for macro-prudential outcomes. A natural conclusion is that when the conditions in Proposition 3 are met, so that asset-value impairment is not very likely and managerial agency problems are severe, private incentives to lever up banks may be socially inefficient, justifying a role for ex-ante capital requirements. When the conditions in Proposition 3 are not met, however, so that asset-value impairment is quite likely and managerial agency problems are not severe, then bank leverage has a bright side, both in terms of addressing bank-level agency problems as well as generating valuable information on aggregate asset-value shocks. In this case, there is a positive information externality associated with leverage that pushes the social optimum above the private optimum.

In what follows, we explore this tension between privately-optimal and socially-optimal bank leverage further by studying how ex-post regulatory interventions, such as bailouts or lender of last resort policies, can potentially reduce the dark side of bank leverage by containing contagious liquidations. However, if these interventions are conducted purely based on market information, e.g., observation of liquidation risk of banks, then they can – as an unintended consequence – interfere with the bright sides of bank leverage, namely unlocking liquidity and generating valuable information about asset-value shocks.

V. THE EFFECT OF LENDER OF LAST RESORT

Given that some of the contagious liquidations are inefficient, there may be a role for regulatory intervention in the form of lender of last resort (LOLR) actions. The LOLR could step in and bail out banks threatened by liquidation. In particular, if the LOLR observes a threatened liquidation that may trigger a contagion of liquidation on the other banks, it steps in and buys out the creditors of threatened banks. The purpose of the following discussion is to examine the (unintended) consequences of such LOLR intervention. In particular, we will argue that the desire to avoid a collapse of the industry *in all states of the world* and thereby reduce systemic risk has adverse consequences for the leverage decisions of banks and their ex ante liquidity.

A. Loss of market discipline and information roles of debt in presence of LOLR

The key assumption we make is that the LOLR operates under a serious informational constraint – it does not observe the x of any bank and hence can *not* distinguish between liquidations based on the realization of x observed by bank's own creditors and those based on the observed liquidations of *other* banks. The problem with the regulatory resolution of a threatened bank is that unless the LOLR backstops creditors, information about there being potentially an adverse asset-value shock is likely to leak out. In this case, the bailout by the LOLR serves no purpose at all since all other banks' creditors will infer that a liquidation was imminent, and contagion will set in.[14] Knowing this, creditors who can threaten liquidation at $t = 1$ will demand that the LOLR pay them $F - x$ in full after they have collected the date-1 cash flow x. This means that creditors can threaten liquidation at $t = 1$ in every state and receive $F - x$ in

[14] With dispersed creditors, it may be difficult to prevent this information from leaking out.

addition to *x*. Anticipating this, they would have no reason to engage in privately-costly monitoring, and all the market discipline of leverage is lost. Of course, the *ex ante* pricing of leverage will reflect the fact that it is riskless, so creditors will not ex ante get a "free lunch". However, the entire burden of disciplining the manager now falls on the compensation contract and the amount of bank leverage is irrelevant. The value of the bank then is:

$$V_{LOLR} = E(x) + \left[1-\theta^*\right] qH \int_{x_{\min}}^{x_{\max}} x h(x)\,\mathrm{d}x\,. \tag{13}$$

We now have the following result:

Proposition 4: *When the LOLR is present to bail out banks at* $t = 1$ *and prevent liquidations, the bank's capital structure becomes irrelevant, and for c > 0 sufficiently small, the bank's ex ante liquidity is lower with the LOLR than without.*

The intuition for capital structure irrelevance is straightforward. If we remove the disciplining role of debt and make interim liquidations impossible, then we have neither a benefit nor a cost associated with leverage. Hence, the Modigliani and Miller (1958) indifference theorem applies.[15] The reason why the LOLR causes the bank to have lower ex-ante liquidity is that the disciplining role of leverage is important for enhancing the bank's liquidity ex ante. When that role is lost, so is some ex-ante liquidity. Note, however, that this is not a total loss of liquidity as in some models where the absence of the disciplining role of leverage means that the loan financed by the bank becomes completely illiquid. The compensation-based incentives provided by equity do provide ex ante liquidity, but it is not as much as is available with the optimal combination of debt and equity in the absence of the LOLR.

This is somewhat of a paradox. The role of the LOLR is to *increase* the liquidity in the system. And yet the presence of the LOLR *reduces* the ex-ante liquidity of banks. Thus, the "price" of having a system flush with ex-post liquidity provided by the LOLR is lower ex-ante liquidity.

[15] Now, any distortion in favor of debt such as tax deductibility of interest rate payments can lead the financial sector to lever up in a significant manner. Also, as argued by Acharya and Yorulmazer (2007), Farhi and Tirole (2009), and Acharya, Mehran and Thakor (2010), there would be an incentive to increase the systematic risk of projects, if that were a choice variable in our model, and banks would prefer to fund these correlated projects with leverage to "loot" the LOLR (assuming bank equity is not bailed out in case of joint failures).

A further cost of having the LOLR intervene with bailouts is that the financial system stops generating any valuable signals about asset values that are produced via creditor liquidations absent the LOLR. Hence, the presence of the LOLR eliminates the ability to liquidate assets early and prevent further asset-value erosions from continuations when $\xi = \xi_l$. In some cases, such signals may be especially valuable because they are "early warnings" of bigger problems down the road, an issue we explore further below.

B. Lender of Last Resort and Loss of Early Warnings about Crises

In this subsection, we explore the potentially adverse impact of LOLR intervention on value-enhancing private-sector action in response to the information conveyed by creditor liquidations that occur in the absence of the LOLR. To see this, let us modify the model by introducing an additional date between date 1 and the terminal date. At t =1, the signal ξ is observed by some creditors as in the model in the previous sections, but now assume that the bank has the opportunity to enhance the bank's terminal payoff by investing an additional amount. Also, the signal is not observed by the bank's manager, shareholders or the LOLR. The only way the LOLR potentially learns about the signal is if a bank's creditors threaten to liquidate the bank and the manager seeks help of the LOLR. Then at t = 2, it becomes common knowledge whether asset-value impairment did occur, but it is too late then to do anything about it and there is bank failure if nothing was done earlier. Finally, at t = 3, the terminal cash flow is realized as in our base model. The key idea we explore in the subsequent analysis is that the payoff enhancement at t = 1—which is socially efficient whenever it is privately efficient—occurs only when the bank has precise information about the systematic state, which includes the information conveyed by creditor liquidations of other banks. Then, we show that LOLR intervention interferes with this information transmission, thereby preventing liquidations at t = 1, and precluding efficient payoff enhancements, so that there is a potentially even worse crisis at t = 2.

Specifically, at t =1, conditional on $\xi = \xi_l$, if the bank does not do anything, then the asset value will deteriorate to 0 with probability 1 at t = 2. If the bank invests xw_E to enhance the terminal payoff at

t = 2, then it can attenuate the deterioration of asset value in the $\xi = \xi_l$ state and have a cash flow of $[H^- + w + kw]x$ with probability $(w.p.)q$ and 0 $w.p. 1-q$, where xw is the amount by which the *pledgeable* cash flow xH^- increases and xkw is the increase in the *non-pledgeable* cash flow in this state. That is, xw is available to augment the repayment to the creditors, whereas xkw is available to the bank's shareholders but *cannot* be pledged to repay the creditors. It is assumed that $k \in [0, \hat{k}]$ varies in the cross-section of bank. Let $g(\cdot)$ be the density function over *k*. Each bank's manager and shareholders privately know the bank's *k*, which is not known to others. The investment of xw_E does not affect the terminal payoff xH^+ in the $\xi = \xi_h$ state. Regardless of what the bank does, this payoff is $xH^+ w.p.\ q$ and 0 $w.p. 1-q$. Moreover, if the creditors choose to liquidate the bank, the payoff enhancement is worthless in that it does not affect either the pledgeable or the not-pledgeable cash flow.

Our focus in what follows is on the fifth term in (11). This is the state in which the creditors wish to liquidate the bank and the liquidation is not based on the bank's realization of *x* i.e., $x \in [x_{\min}, \lambda F]$. Recall that, conditional on being in this liquidation state, there are two relevant states at t = 1: (a) the banks' creditors observe $\xi = \xi_l$ and threaten to liquidate; or (b) the bank's creditors threaten to liquidate because they observe another bank's creditors liquidate their bank.

We will impose a few restrictions on the exogenous parameters to focus on the cases of interest. In addition to Conditions (1) and (2) represented by equations (9) and (10) respectively, we have the following conditions:

Condition 3: *Whether they observe $\xi = \xi_l$ or they observe another bank being liquidated, the bank's creditors prefer to liquidate the bank at t = 1 rather than investing w_E themselves to allow the bank to continue. That is,*

$$q[H^- + w] < L + w_E \tag{14}$$

Condition 4: *Next, we assume that if the bank's shareholders invest* w_E *when threatened with liquidation in state (a), then the bank's creditors do not liquidate. That is,*

$$q[H^- + w] > L \tag{15}$$

We will also assume that

$$H^- + w < F < H^+ \tag{16}$$

so even with the payoff enhancement in the asset-value-impairment state, the payoff in that state is below the debt repayment and the payoff in the asset-value-increase state.

Note that, given (15), creditors will also not liquidate in state (b) when faced with the bank's shareholders investing additional equity w_E, i.e., when they do not observe $\xi = \xi_l$ but they observe another bank's creditors liquidating.

Condition 5: *The shareholders of some banks will invest* w_E *even in state (b) when faced with a liquidation threat, i.e.* $\exists k \in \left[0, \hat{k}\right]$ *for which*

$$\hat{\hat{\beta}}\{qkw\} > w_E \tag{17}$$

where $\hat{\hat{\beta}}$ *is the posterior belief that* $\xi = \xi_l$ *when another bank announces liquidation in stage 1 (i.e., this is the analog of* $\hat{\beta}$ *in (8)).*

Now let $\bar{k}_1$ be the solution to

$$q\bar{k}_1 w = w_E \tag{18}$$

and $\bar{k}_2$ be the solution to

$$\hat{\hat{\beta}} q\bar{k}_2 w = w_E \tag{19}$$

Then we see that if:

(i) $k \in [0, \bar{k}_1)$ then the bank's shareholders do not invest w_E and thus the bank is liquidated;

(ii) $k \in [\bar{k}_1, \bar{k}_2)$, then the bank's shareholders invest w_E in state (a) in which the creditors observe $\xi = \xi_l$, but not in the state (b) in which the creditors do not observe $\xi = \xi_l$ but observe another bank's creditors liquidating their bank; and

(iii) $k \in [\bar{k}_2, \hat{k}]$, then the bank's shareholders invest w_E in both states (a) and (b).

Define

$$\alpha_1 \equiv \int_0^{\bar{k}_1} g(k)\,dk \tag{20}$$

We can now write:

$$\hat{\hat{\beta}} = \frac{[\gamma H(\lambda F)\alpha_1 + \delta]\beta}{\{[\gamma H(\lambda F)\alpha_1 + \delta]\beta + [1-\beta][\delta]\}} \tag{21}$$

in a manner analogous to (8).

Condition 6: *Our final condition is that the bank's shareholders will not unconditionally invest* w_E*. That is,*

$$\beta\{qkw\} < w_E \tag{22}$$

As indicated earlier, if a bank is threatened with liquidation, it may approach the LOLR for help. We will assume that if the liquidation of a bank is prevented by LOLR intervention whereby the LOLR arranges to have the bank's creditors paid off in full (i.e., F), then no other bank learns about it. This is a strong assumption that is designed to ensure that the LOLR has the ability to prevent liquidation-contagion by paying off creditors. If the LOLR cannot do this, then bailing out a bank serves no purpose since contagion will still occur and the LOLR would need to bail out all banks.

In what follows, we shall assume that the LOLR will intervene and bail out banks only when they are threatened with liquidation by creditors, i.e. banks are not bailed out to save only the shareholders.[16] Moreover, we assume that the regulator perceives a social cost ψ that is perceived to be associated with

[16] This assumption is just to focus our discussion and can be dispensed with.

all banks failing, and that this cost is also encountered if the banks that fail are unlevered. Then we have the following result.

Proposition 5: *Consider a two-bank economy and suppose both banks are levered. Then, whenever a bank's shareholders invest additional equity at w_E at $t = 1$, this investment is socially efficient. Moreover, if β is large enough, the LOLR prefers to precommit to not intervene at $t = 1$ and bail out banks that would otherwise be liquidated at $t = 1$ because the expected future (date $t = 2$) social cost from doing so may exceed the social cost of allowing these banks to fail at $t = 1$.*

This proposition highlights another downside of LOLR intervention. By adopting an intervention policy at $t = 1$, the LOLR creates a situation in which all creditors threaten liquidation to get paid off by the LOLR. There is, therefore, no information communicated about ξ to any bank's shareholders. As a result, no bank invests additional equity w_E at $t = 1$ since unconditional payoff enhancement is not optimal (Condition 6). If the state $\xi = \xi_l$ was realized at $t = 1$, then at $t = 2$ all banks will be worthless, which is an even bigger crisis than the one averted at $t = 1$. Also note that, given private incentives to infuse equity to forestall liquidation at $t = 1$ in the absence of the LOLR, allowing some banks to be liquidated at $t = 1$ *has the benefit of generating valuable information about asset-value impairment that can be used by other banks to justify additional equity inputs that can improve asset values.*

Specifically, the benefit to the LOLR of not intervening with bailouts at $t = 1$ is that banks with $k > \bar{k}_1$, will be induced to provide additional equity inputs when faced with liquidation threats from creditors. But had the LOLR adopted an intervention policy at $t = 1$ and state $\xi = \xi_l$ had occurred, no information about asset-value impairment (a systemic risk) would have been produced, no banks would have infused equity at $t = 1$ and *all* banks would have failed at $t = 2$. In other words, the banking system can actually become stronger if some banks are allowed to fail because other banks learn from those failures and infuse equity to become stronger.[17] Moreover, pre-commitment by the LOLR to a non-

[17] The intuition is similar to the development of human immunity. When a person suffers from an illness, immunity is developed to protect that person from even more serious future illnesses.

intervention policy is necessary to avoid a situation in which all creditors mistakenly believe the LOLR will bail out banks and therefore all threaten liquidation, thereby obscuring valuable information about the asset-value impairment shock.

C. Some Remarks on Possible Private Contracting Solutions and Recent Regulatory Reform Proposals

In this section, we examine private-contracting solutions to the problem of correlated failures that would obviate the need for ex post bailouts by the LOLR. We will examine these generically, and focus on contingent capital (debt securities that are mandatorily convertible into equity under pre-specified stress conditions). Contingent capital proposals have been discussed mainly as regulatory initiatives – and some variants do involve the conversion of subordinated debt into equity to be triggered by regulatory action – but they typically do not (and in fact are proposed precisely to not) involve regulatory bailouts (e.g. see Flannery (2005) and Squam Lake (2009)). This means it would not be difficult to imagine these being implemented via private contracting, even though their design may originate with the regulator. That is, in the analysis below, we focus on schemes that are ex post privately optimal for the bank and its financiers. Of course, in practice, there may be circumstances in which social and private optima may diverge, so that regulator "activism" may be necessary to implement some schemes. We do not consider those here.

In essence, the contingent claims proposals are debt-equity swaps. We will restrict ourselves to "voluntary exchanges" in our analysis, i.e., exchanges in which the creditors are willing to participate. Mathematically, this is equivalent to the bank's shareholders being willing to buy out the creditors at a price that makes them indifferent between liquidating and not liquidating the bank. Note that, from the shareholders' perspective, this is necessary only when $x \in [x_{\min}, F]$ *and* creditors wish to liquidate. A key point is to recognize that the shareholders will be cognizant of the altered incentives of the bank manager when the creditors' claims are bought out by the shareholders and hence converted into equity. In particular, creditor monitoring disappears in those states. So, the shareholders will now have to use compensation-based incentives to motivate the manager. Whether the shareholders wish to buy out the creditors in the liquidation states now depends on a comparison of what they have to pay the creditors to buy them out (*L*) and their own *net* payoff after recalibrating managerial incentives ($[1-\theta^*]Hx$, where x is a specific realization in $\left[x_{\min}, F\right]$). Thus, we have:

Lemma 3: *If the manager's private benefit, B, is sufficiently large, then the shareholders will not wish to buy out the creditors when they threaten to liquidate the bank.*

The intuition is that converting debt into equity has a cost, namely that creditor monitoring has to be forgone. This forces reliance on compensation-based incentives that equity deploys, which diminishes the *net* terminal payoff available for shareholders. The more expensive the compensation-based incentive schemes, the lower the net terminal payoff available to shareholders, and hence the less attractive it is for shareholders to buy out the creditors. And given our discussion in earlier section on the benefit of early-warning signals, once debt is converted into equity and the discipline of bank debt is lost, there is also the loss of generation of valuable early-warning signs of asset-value deterioration. Consequently, contingent capital, like regulatory bailouts and the LOLR, may prevent a sneeze (small crisis) but increase the odds of cardiac arrest (major crisis).

Of course, if contingent capital was mandated by regulatory fiat, then the shareholders will have no choice but to accept the conversion of debt into equity. Nonetheless, the discipline of bank debt will be lost post-conversion, as will valuable early-warning signs of asset-value impairment.

D. Regulatory Minimum Capital Requirements

Our analysis shows that during times that β, the probability of a systematic asset-value impairment, is low, and the managerial agency problem is severe, privately-optimal leverage exceeds what is socially optimal. These will also be the times when bank asset values are relatively high since a low β corresponds to higher expected future asset values. This suggests that a regulatory minimum capital requirement during high asset-valuation periods may be desirable since it could make banks hold capital that is above privately-optimal levels and at or near the social optimum. When β is high and asset values are consequently low, it may be advisable to lower regulatory capital requirements, but they may not be binding because the privately-optimal leverage may be below the social optimum, in which case banks will voluntarily hold more capital than the regulatory requirement. Hence, the analysis in this paper is supportive of countercyclical capital requirements.

Clearly, such a regime would not be costless. Even countercyclical capital requirements will reduce ex ante liquidity creation during high-asset-valuation periods when they are binding. But, if the costs of systemwide failures is high enough, then the benefit of reducing systemic risk through higher capital requirements may more than offset the cost of lower ex ante liquidity. Moreover, having more capital injected into the banking system via higher capital requirements will shift more of the burden of corporate governance from debt to equity, with the attendant benefit of lower systemic risk.

One of the goals of the Dodd-Frank Act is to control systemic risk. A focus will be on identifying and monitoring Systemically Important Financial Intermediaries (SIFIs). Our analysis points out that an important complement to this would be countercyclical capital requirements, since systemic risk increases with leverage.

VI. CONCLUSION

In this paper, we re-examined the role of bank leverage as an instrument of liquidity creation. We exposed a fundamental tension between the micro-prudential goal of encouraging the market discipline of banks via greater uninsured leverage in their capital structure and the macro-prudential goal of containing systemic risk. While higher bank leverage creates stronger creditor discipline at the *individual* bank level, it leads to greater systemic risk induced by contagious runs when creditors liquidate banks. This invites ex-post LOLR intervention to bail out "failing" banks from being liquidated by creditors and thereby to maintain the interim liquidity and continuity of banks. But as a consequence, the disciplining role of bank leverage is lost, valuable information about asset-value impairment is not generated, and *ex ante* bank liquidity actually declines. And by reducing creditor incentives to intervene in banks, the LOLR can preclude the discovery of "early warnings" of a banking crisis and thereby risk the emergence of delayed but more severe crises. In fact, the more vigilant the LOLR is in reacting to early warnings by intervening with bailouts of failing banks, the worse is the problem. We also examined private-contracting alternatives to LOLR intervention such as contingent capital or debt-for-equity swaps. Such schemes can eliminate the problem of lack of private incentives to avoid contagious liquidations. But they also sacrifice the market discipline of debt and cause a loss of early-warning signals of crises.

In this context, higher regulatory minimum capital requirements that reduce systemic risk and decrease the expected social cost of complete industry collapse may be preferable to ex-post LOLR and contingent capital arrangements. While ex ante bank liquidity may be diminished by higher capital requirements, some bank failures would be tolerated ex post and the resulting early-warning signals of a crisis would be generated by bank creditors and observed by other banks that may voluntarily make equity injections to prevent a major crisis down the road. Indeed, our analysis suggests that improving the governance that can be imposed on banks by shareholders may be a more robust – even though not much discussed – intervention to reduce excessive bank reliance on leverage for liquidity.

APPENDIX

Proof of Lemma 1: First, let us consider the intervention policy. If the creditors intervene, they can force the bank manager to invest in the good project and the creditors' date-2 payoff, after having been paid x at date-1, is $q[F-x]-c$, taking into account the investigation cost. If creditors do not intervene, the manager will select the private-benefit project, and the creditors' date-2 payoff will be $p[F-x]$. Thus, intervention is optimal for the creditors if:

$$q[F-x]-c>p[F-x]$$

or

$$x<F-c[\Delta]^{-1} \tag{A-1}$$

Now, we turn to the liquidation/continuation decision after having invested *c*. We compare (5) and (6). Since $L<qH$, there are three cases to consider: (i) $F-x<Lx$; (ii) $Lx<F-x<Hx$; and (iii) $F-x>Hx$.

Case (i): $F-x<Lx$: In this region, $x>F/[1+L]$. Thus, the liquidation payoff to creditors is $F-x$. The continuation payoff is $q[F-x]<F-x$. Hence, the creditors liquidate the bank.

Case (ii): $Lx<F-x<Hx$: In this case, $Lx+x<F$, which means $x<F/[1+L]$. Moreover, $F<Hx+x$, which means $x>F/[1+H]$. So, $F/[1+H]<x<F/[1+L]$. The liquidation payoff to creditors is Lx, whereas the continuation payoff is $q[F-x]$. Creditors liquidate whenever:

$$Lx>q[F-x]$$

or

$$x>\lambda F, \text{ where } \lambda\equiv\frac{q}{q+L}.$$

Creditors continue if $x\le qF[q+L]^{-1}$.

Case (iii): $F - x > Hx$: In this region, $x < F/[1+H]$. The liquidation payoff of creditors is $\min(Lx, F-x) = Lx.$ Since $x + xH < F$ implies that $x + xL < F$. The continuation payoff of creditors is qHx. Since $qH > L$, the creditors continue. Finally, since $qH > L$, all liquidation is inefficient. ■

Proof of Proposition 1: Note that $\hat{\beta}$ (given by (8)) is the posterior probability belief of bank *i*'s creditors about $\xi = \xi_l$, conditional upon observing a liquidation by the creditors of bank *j*. We see that the creditors of bank *i* liquidate upon observing if (10) holds. Refer to the left-hand side of (10) as LHS. Then it is clear that $\frac{\partial(LHS)}{\partial\hat{\beta}} < 0.$ Thus, an increase in $\hat{\beta}$ makes it easier to satisfy (10), and it increases the set of exogenous parameters for which bank *i* liquidates after observing liquidation by the creditors of bank *j*. Now,

$$\partial\hat{\beta} / \partial F = \frac{\gamma\, \beta\, h(\lambda\, F)\, \lambda\, [1-\beta]\, \delta}{[DEN]^2} > 0$$

where $DEN \equiv [\gamma H(\lambda F) + \delta]\beta + [1-\beta]\delta$ and *F* is bank *j*'s leverage. Thus, higher leverage of bank *j* makes it more likely that the creditors of bank *i* liquidate when they observe the creditors of bank *j* liquidating.

Finally, the (unconditional) ex ante probability of liquidation by bank *i*'s creditors (see (11)) is

$$PRB \equiv H(F) - H(\lambda F)[\delta_1 + \delta_2].$$

Note that $\partial PRB / \partial F > 0$ if $h(F) > \lambda h(\lambda F)[\delta_1 + \delta_2].$

Now we will show in the proof of Proposition 2 that, at the equilibrium *F*, say F^*, $h'(F^*) > 0$ and $h'(\lambda F^*) < 0$. Thus, for F^* high enough, $h(F^*) > h(\lambda F^*)$ and hence $\frac{\partial(PRB)}{\partial F} > 0$ at that F^*, where F^* is the optimal leverage of bank *i*. ■

Proof of Lemma 2: Equation (11) is apparent from the discussion following the lemma in the text. Rewriting (11) as

$$V_{\text{lev}}(F) = E(x) + \int_{\lambda \min}^{F} Lxh(x)\mathrm{d}x + \left[1-\theta^*\right] qH \int_{F}^{x_{\max}} xh(x)\mathrm{d}x$$
$$+\left\{\delta_2[qH^+ - L] - \delta_1[L - qH^-]\right\} \int_{x_{\min}}^{\lambda F} xh(x)\mathrm{d}x$$

we see that $V_{lev}(F)$ is increasing in $\delta_2[qH^+ - L]$ and decreasing in $\delta_1[L - qH^-]$. ■

Proof of Proposition 2: The first-order condition for the optimal *F* is:

$$\partial V_{lev}(F)/\partial F = 0 \tag{A-2}$$

or

$$L\left\{ h(F^*) - \lambda^2 h(\lambda F^*) \right\} + \left[1-\theta^*\right] qH\left[-h(F^*)\right]$$
$$+\left\{ \delta_1 H^- + \delta_2 H^+ \right\} q\left[\lambda^2 h(\lambda F^*) \right]$$
$$+\left[1 - \delta_1 - \delta_2 \right] L\left[\lambda^2 h(\lambda F^*) \right] = 0$$

Rearranging, we get:

$$\frac{h(F^*)}{h(\lambda F^*)} = \frac{\lambda^2 \left\{ \left[\delta_1 H^- + \delta_2 H^+ \right] q - L\left[\delta_1 + \delta_2\right] \right\}}{\left[1-\theta^*\right] qH - L} \tag{A-3}$$

at the optimum. This is (12) in the statement of the proposition. It is clear that if F^* satisfies the conditions needed for the second-order condition to hold, namely $h'(F^*) > 0$ and $h'(\lambda F^*) < 0$ and $h'(\lambda F^*) < 0$, then $h(F^*)/h(\lambda F^*)$ is increasing F^*. Hence, an increase in $[1-\theta^*]qH - L$ decreases F^*.

To satisfy the second-order condition (SOC), we need :

$$-h'\left(F^*\right)\left\{\left[1-\theta^*\right]qH - L\right\}$$

$$+\lambda^3 h'\left(\lambda F^*\right)\left\{\left[\delta_1 H^- + \delta_2 H^+\right]q - L\left[\delta_1 + \delta_2\right]\right\} < 0. \tag{A-4}$$

This will hold, for example, if h has an inflection point at $F \in (\lambda F^*, F^*)$ so $h'(F) = 0$ and $h'\left(F^*\right) > 0, h'\left(\lambda F^*\right) < 0.$

Example: We provide here an example of a probability density function that satisfies the second-order condition in the proof of Theorem 1. Suppose $h(x) = h_1(x) = \frac{K_1}{x} + K_2 x + K_3$ for all

$x \in \left[x_{\min}, \sqrt{\frac{K_1}{K_2}} \right]$ and $h(x) = h_2(x) = -\frac{K_1}{x} - K_2 x + K_4$ for all

$x \in \left[\sqrt{\frac{K_1}{K_2}}, x_{\max} \right]$, with $K_4 - K_3 = 4\sqrt{K_1 K_2}$. Then

$$h_1'(x) < 0 \ \forall \ x < \sqrt{\frac{K_1}{K_2}}, = 0 \text{ at } x = \sqrt{\frac{K_1}{K_2}};$$
$$h_2'(x) > 0 \ \forall \ x > \sqrt{\frac{K_1}{K_2}}, = 0 \text{ at } x = \sqrt{\frac{K_1}{K_2}}$$

and (smooth pasting condition)

$$h_1(x) = h_2(x) \text{ at } x = \sqrt{\frac{K_1}{K_2}}.$$

So if $F > \sqrt{\frac{K_1}{K_2}}$ and $\lambda F < \sqrt{\frac{K_1}{K_2}}$, we can satisfy the second-order condition (A-4). ■

Proof of Lemma 3: In any state, the bank's shareholders will be willing to buy out the creditors at a price L (which makes the creditors indifferent between liquidating and not liquidating) if their *net* payoff after buying out the creditors and relying on compensation-based incentives exceeds the price, L, of buying out the creditors. That is, if

$$\left[1 - \theta^*\right] qE(H | \textit{liquidation threat}) > L \tag{A-5}$$

where $E(H | \textit{liquidation threat})$ is the expected value of H conditional on creditors threating liquidation.

Upon substituting for θ^* from (4), we get:

$$q\left[1 - B\left\{H\left[q - p\right]\right\}^{-1}\right] E(H | \text{liquidation threat}) > L \tag{A-6}$$

Clearly (A-6) will *not* hold for B sufficiently large. ■

Proof of Proposition 3: We need to evaluate $\partial V_j / \partial F_i$ at F_i^*, the private optimum for bank i. Now

$$\partial V_j / \partial F_i \bigg|_{F_i = F_i^*} = \left\{ \left[\frac{\partial \delta_1}{\partial F_i} \right] H^- + \left[\frac{\partial \delta_2}{\partial F_i} \right] H^+ \right\} q \int_{x_{\min}}^{\lambda F} x h(x) dx$$

$$- \left\{ \left[\frac{\partial \delta_1}{\partial F_i} \right] + \left[\frac{\partial \delta_2}{\partial F_i} \right] \right\} L \int_{x_{\min}}^{\lambda F} x h(x) dx$$

$$= \left\{ \left[\frac{\partial \delta_1}{\partial F_i} \right] \left[qH^- - L \right] + \left[\frac{\partial \delta_2}{\partial F_i} \right] \left[qH^+ - L \right] \right\} A_j \quad \text{(A7)}$$

where $A_j \equiv \int_{x\min}^{\lambda F} x h(x) dx$.

Recalling the definitions of δ, δ_1 and δ_2, we can write:

$$\frac{\partial \delta_1}{\partial F_i} = \beta [1-\gamma]^2 \left[\lambda h(\lambda F_i^*) - h(F_i^*) \right] \quad \text{(A8)}$$

$$\frac{\partial \delta_2}{\partial F_i} = [1-\beta][1-\gamma] \left[\lambda h(\lambda F_i^*) - h(F_i^*) \right] \quad \text{(A9)}$$

Substituting (A-8) and (A-9) into (A-7) and recognizing that $L - qH^- = qH^+ - L$, we can write:

$$\partial V_j / \partial F_i \bigg|_{F_i = F_i^*} = A_j \left[qH^+ - L \right] \left[\lambda h(\lambda F_i^*) - h(F_i^*) \right] [1-\gamma] \{ [1-\beta] - \beta [1-\gamma] \} \quad \text{(A10)}$$

Since $1 - \beta > \beta [1-\gamma] \forall \beta \leq 0.5$, we see that the sign of $\partial V_j / \partial F_i$ depends entirely on the sign of $\lambda h(\lambda F_i^*) - h(F_j^*)$.

Consider now the first-order condition for bank i's optimal choice, F_i^*. From (A-3), it follows that

$$\lambda h(\lambda F_i^*) - h(F_i^*) < 0$$

if:

$$\lambda \left\{ \delta_1 \left[qH^- - L \right] + \delta_2 \left[qH^+ - L \right] \right\} > [1-\theta^*] qH - L$$

or

$$\lambda \left[qH^+ - L \right] [\delta_2 - \delta_1] > [1-\theta^*] qH - L \quad \text{(A-11)}$$

But

$$\partial[\delta_2 - \delta_1]/\partial\beta = -\gamma - [1-\gamma][1-\delta] - [1-\gamma]^2[1-\delta]$$
$$< 0.$$

Thus, for β small enough and θ^* large enough, (A-11) will hold (it clearly holds in the limit as $\theta^* \to 1$ and thus, by continuity, it holds in a neighborhood of $\theta^* = 1$). From (4) we know that $\partial\theta^*/\partial\mathrm{B} > 0$, so we have shown that (A-11) will hold when β is small enough and B is large enough. Given (A-11), it follows from (A-10) that $\partial V_j / \partial F_i < 0$. Then it follows from the discussion in the text that the privately-optimal leverage will exceed the socially-optimal leverage. ■

Proof of Proposition 4: The proof of capital structure irrelevance is obvious (note that (13) is independent of *F*). Comparing the objective functions in (11) and (13), we see that if we ignore *c*, then (11) can be made equivalent to (13) by setting *F* = 0. But the *F* that maximizes ex ante liquidity $E_{lev}(V)$ is strictly positive. Hence, for *c* > 0 small enough, the optimized value of $V_{lev}(F)$ in (11) exceeds the V_{LOLR} in (13). ■

Proof of Proposition 5: Note that the social-efficiency condition for investing w_E at *t* = 1 is (for state (a) when the bank's creditors are liquidating in response to their own signal) is:

$$q\left[H^- + \{1+k\}w\right] - w_E > L$$

or

$$\{q[H^- + w] - L\} + \{qkw - w_E\} > 0 \tag{A-12}$$

By (15), we know that $q[H^- + w] - L > 0$, and we also know that $qkw - w_E > 0$ whenever the bank's shareholders find it privately efficient to invest w_E in the bank. Thus, (A-12) holds. A similar proof works for state (b) when the bank's creditors are liquidating based on the liquidations of other banks.

Now consider the case in which the LOLR does not intervene at *t* = 1. The probability that both banks will fail is:

$$P_1 = \delta^2 + 2\beta[1-\delta][1-\gamma]\{\gamma[1-\delta]\alpha_2\alpha_1 + \gamma\delta\alpha_2 + [1-\gamma]\delta\alpha_2\} \\ + \beta[1-\delta]\gamma^2\alpha_1^2 + 2[1-\beta][1-\delta]\delta\alpha_2 \tag{A-13}$$

where α_1 was defined in (20) and

$$\alpha_2 \equiv \int_0^{\bar{k}_2} g(k)\,dk \tag{A-14}$$

The probability that only one bank will invest w_E in arresting asset-value impairment when $\xi = \xi_l$ occurs is:

$$\mathrm{A}_1 \equiv \beta[1-\delta][1-\gamma]\{\gamma[1-\delta]\{[1-\alpha_2] + \alpha_2[1-\alpha_1]\} + \gamma\delta[1-\alpha_2] + [1-\gamma]\delta[1-\alpha_2]\} \tag{A-15}$$

and the probability that both banks will invest w_E when $\xi = \xi_l$ occurs is:

$$\mathrm{A}_2 \equiv \beta[1-\delta]\gamma^2[1-\alpha_1]^2 \tag{A-16}$$

The probability that one bank will erroneously invest w_E (based upon observing liquidation of the other bank) when $\xi = \xi_h$ is:

$$[1-\beta][1-\delta]\delta\alpha_2$$

If the LOLR intervenes by bailing out banks at t = 1, then the bank's creditors will threaten liquidation in all states to be bought out at par (since the LOLR cannot observe x or ξ). No information about ξ will be generated via liquidations. So, no investment w_E will be made in arresting asset-value impairment. Moreover, conditional on $\xi = \xi_l$, all banks will fail at t = 2. Thus, the probability that all banks will fail at t = 2 is β .

For the LOLR, not intervening at t = 1 via bailouts dominates the option to intervene if:

$$\beta\Psi > P_1\Psi - 2\mathrm{A}_1\{q[H^- + w + kw] - w_E\} - 2A_2 + 2[1-\beta][1-\delta]\delta\alpha_2\{w_E\} \tag{A-17}$$

where the term in the curly brackets multiplying A_1 on the right-hand side (RHS) is the net benefit of arresting the asset-value impairment and this is per bank so the whole expected net benefit is multiplied by 2 for two banks, and the last term on the RHS of (A-17) is the expected loss due to erroneous investment of w_E when $\xi = \xi_h$. It is straightforward to verify that the RHS of (A-17) is decreasing in β and it is increasing in δ. Thus, for β large enough and δ small enough, (A-17) will hold. ■

***Figure 1*: Sequence of Events**

0

- Bank needs liquidity ρ for project.
- Bank chooses equity or debt financing.
- Debt face value is *F*.
- All cash flows are pledgeable.
- Shareholders choose whether to give the manager a share θ of the terminal payoff and in which interim cash flow (x) states.

1

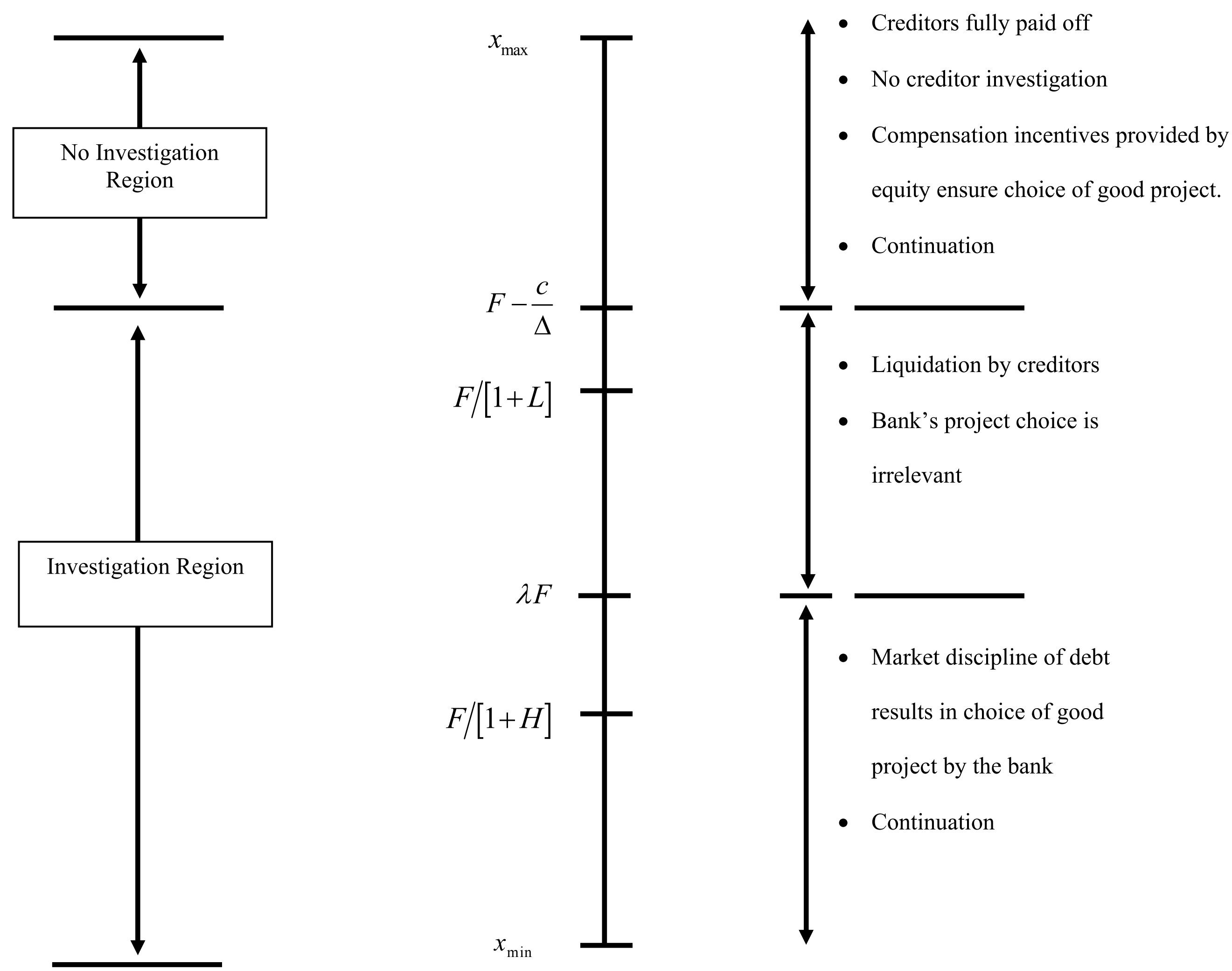
- Project throws off cash flow of x. Observable only to each bank's managers, shareholders and creditors.
- Manager makes choice of good or private-benefit project after observing x.
- Creditors may expend c to discover the manager's project choice. They can enforce a different project choice if they so desire.
- Creditors decide whether to liquidate the bank or continue after observing x and systematic liquidity shock.
- The LOLR may intervene to provide liquidity if creditors threaten to liquidate.

2

- Project yields terminal cash flow Hx with probability q and 0 with probability $1-q$ if good, and Hx with probability p and 0 with probability $1-p$ if it is the private-benefit project.
- All payments are made to managers, shareholders and (if any outstanding, to) creditors.

Figure 2

The Continuation/Liquidation Decision of Creditors



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