The Maturity Rat Race*

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Abstract

Why do some firms, especially financial institutions, finance themselves so short-term? We develop an equilibrium model of maturity structure and show that extreme reliance on short-term financing may be the outcome of a *maturity rat race*: an individual creditor can have an incentive to shorten the maturity of his loan, allowing him to adjust his financing terms or pull out before other creditors can. This, in turn, causes all other creditors to shorten their maturity as well. This dynamic toward short maturities is present whenever interim information is mostly about the probability of default rather than the recovery in default. For borrowers that cannot commit to a maturity structure, most importantly financial institutions, financing is inefficiently short-term.

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One of the central lessons of the financial crisis of 2007-09 is the importance of maturity structure for financial stability: The crisis vividly exposed the vulnerability of institutions with strong maturity mismatch—those who finance themselves short-term and invest long-term—to disruptions in their funding liquidity. This raises the question of why firms and, in particular, financial institutions, use so much short-term financing, even if this exposes them to significant rollover risk?

In this paper we argue that excessive reliance on short-term financing may be the outcome of an inefficient dynamic that we call the *maturity rat race*. To demonstrate this point, we develop a model of the equilibrium maturity structure for a borrower who borrows from multiple creditors to finance long-term investments. When this borrower cannot commit to an aggregate maturity structure, the equilibrium maturity structure may be inefficiently short-term—leading to excessive maturity mismatch, unnecessary rollover risk, and inefficient creditor runs. This dynamic may be particularly hard to counteract for financial institutions, for which it is especially difficult and often privately undesirable to commit to a maturity structure in order to prevent this dynamic.

The intuition for our result is as follows: A borrower who cannot commit to an aggregate maturity structure may have an incentive to approach one of its creditors and suggest switching from a long-term to a short-term (rollover) debt contract. This dilutes the remaining long-term creditors: If negative information comes out at the rollover date, the short-term creditor increases his face value. This reduces the payoff to the long-term creditors in case of ultimate default, whose relative claim on the firm is diminished. On the other hand, if positive information is revealed at the rollover date, rolling over short-term debt is cheap. While this benefits the remaining long-term debtholders in case the borrower defaults, typically bankruptcy is less likely after positive news than after negative news. Hence, in expectation the long-term creditors are worse off—they suffer a negative externality. This means that the borrower has an incentive to shorten its maturity whenever interim information received at rollover dates is mostly about the *probability of default.* Whenever this is the case, rollover financing is the unique equilibrium, even though it leads to inefficient rollover risk. In contrast, when interim information is mostly about the *recovery given default*, long-term financing can be sustained.

The same logic extends to settings in which short-term credit can be rolled over multiple times before an investment pays off. In fact, when multiple rollover dates are possible the contractual externality between short-term and long-term debt can lead to a successive unraveling of the maturity structure: If everyone's debt matures at time T, the financial institution has an incentive to start shortening an individual creditor's maturity, until everyone's maturity is of length $T-1$. Yet, once everyone's maturity is of length $T-1$, there would be an incentive to give some creditors a maturity of $T-2$. Under certain conditions, the maturity structure thus successively unravels to the very short end—a *maturity rat race*.

This incentive to shorten the maturity structure can emerge whenever a borrower deals with multiple creditors: The key friction in the model is the borrower's inability to commit to an aggregate maturity structure when dealing. It is the nature of this friction that makes our model apply particularly to financial institutions, rather than to corporates in general: When offering debt contracts to its creditors, it is almost impossible for a financial institution to commit to an aggregate maturity structure. While corporates that tap capital markets only occasionally may be able to do this through covenants or seniority restrictions, committing to a maturity structure is much more difficult (and potentially undesirable) for financial institutions. Frequent funding needs, opaque balance sheets, and their continuous activity in the commercial paper market makes committing to a particular maturity structure virtually impossible.

While in our baseline model we simply rule out covenants, we develop an extension of the model in which firms can prevent the rat race by adopting covenants. However, covenants are costly in the sense that they require monitoring by creditors and reduce the firm's financial flexibility. In this extension, firms for which the cost of self-imposed covenants (corporates) are low eliminate the rat race, while firms with high private costs of covenant adoption (financial institutions) choose not adopt covenants and expose themselves to the rollover risk generated by maturity shortening through the rat race. This is consistent with the empirical observation that financial institutions rarely use covenants, and also echoes the arguments in Flannery (1994). This extension also shows that even when financial institutions have the option of eliminating the rat race through covenants, a role for intervention is likely to remain. In particular, a regulatory intervention is warranted whenever a financial institution's private incentives to adopt covenants differs from the social incentives, which, for example, is the case in the presence of fire-sale externalities. We also discuss why seniority restrictions (i.e., giving seniority to long-term debt) can generally do not eliminate the rat race. Essentially, while long-term creditors would be senior *de jure*, by taking out their funding early short-term creditors may still be paid off first, giving them *de facto* seniority.

Moreover, as with covenants, even in cases where seniority could eliminate the rat race, financial institutions may choose not to do so if they attach a private high value to financial flexibility.

The maturity rat race is inefficient: It leads to excessive rollover risk and causes inefficient liquidation of the long-term investment project after negative interim information. Moreover, because creditors anticipate the costly liquidations that occur when rolling over short-term debt is not possible, some positive NPV projects do not get started in the first place. This inefficiency stands in contrast to some of the leading existing theories of maturity mismatch. For example, Diamond and Dybvig (1983) highlight the role of maturity mismatch in facilitating long-term investment projects while serving investors' liquidity needs. Calomiris and Kahn (1991) and Diamond and Rajan (2001) demonstrate the role of short-term financing and the resulting maturity mismatch as a disciplining device for bank managers.

Relative to this literature, the implication of our paper is that even when short-term debt serves a beneficial role in the sense of allowing liquidity provision of disciplining managers, in equilibrium financial institutions may choose maturity structures that are too short-from a social perspective. Hence, to the extent that maturity mismatch results from our 'rat race' mechanism, a regulator may want to impose restrictions on short-term financing to preserve financial stability and reduce rollover risk. In this respect, our paper thus complements Diamond (1991) and Stein (2005) in arguing that financing may be excessively short-term. However, while the driving force in these models is asymmetric information about the borrower's type, a mechanism that is also highlighted in Flannery (1986), our model emphasizes the importance of contractual externalities among creditors of different maturities.

Our paper relates to a number of recent papers on short-term debt and rollover risk. Brunnermeier and Yogo (2009) provide a model of liquidity risk management in the presence of rollover risk. Their analysis shows that liquidity risk management does not necessarily coincide managing duration risk. Acharya, Gale, and Yorulmazer (2010) show how rollover risk can reduce a security's collateral value. In contrast to our paper, they take short-term financing of assets as given, while we focus on why short-term financing emerges in the first place. In He and Xiong (2009) coordination problems among creditors with debt contracts of random maturity can lead to the liquidation of financially sound firms. Given a fixed expected rollover frequency, they show that each creditor has an incentive to raise his individual rollover threshold, inducing others to raise theirs as well. Unlike their dynamic global games setting, in which interest rates and maturity structure are exogenous, we focus on the choice of maturity with endogenous interest rates. Farhi and Tirole (2009) show how excessive maturity mismatch emerges through a collective moral hazard that anticipates untargeted ex-post monetary policy intervention during systemic crises. Unlike their paper, in our model shows that excessive maturity mismatch may arise even in the absence of an anticipated ex-post intervention by the central bank.

The paper is also related to the literature on debt dilution, either by issuing senior debt to dilute existing debt (see, e.g., Fama and Miller, 1972), by borrowing from more lenders (White, 1980; Bizer and DeMarzo, 1992; Parlour and Rajan, 2001), or by preferentially pledging collateral to some creditors. While our paper shares the focus on dilution, the mechanism (maturity structure) is different. First, as our model shows, shortening the maturity of a subset of creditors is not equivalent to granting seniority, and only works in the favor of the financial institution under certain conditions. Second, in contrast to borrowers in the sequential banking papers by Bizer and DeMarzo (1992) and Parlour and Rajan (2001), the financial institution in our model can commit to the aggregate amount borrowed, it just cannot commit to whether the amount borrowed is financed through long-term or rollover debt.

The remainder of the paper is structured as follows. We describe the setup of our model in Section 1. In Section 2 we then characterize the equilibrium maturity structure and show that the inability to commit to an aggregate maturity structure can lead to excessive short-term financing. In Section 3 we discuss the implications of our model. We show that the maturity rat race leads to excessive rollover risk and underinvestment, contrast our results to the banking literature, and develop an extension of the model that explicitly allows for covenants. We also discuss the role of seniority and leverage, and develop some further empirical predictions. Section 4 concludes.

1 Model Setup

Consider a risk-neutral borrower who can undertake a long-term investment opportunity. The investment opportunity arises at $t = 0$, is of fixed scale, and we normalize the required $t = 0$ investment cost to 1. At time T, the investment pays off a random non-negative amount θ, distributed according to a distribution function $F(\cdot)$ on the interval $[0, \infty)$. Seen from $t = 0$, the unconditional expected payoff from investing in the long-term project is $E[\theta] = \int_0^\infty \theta dF(\theta)$, and its net present value is positive when $E[\theta] > 1$. For simplicity we assume that there is no time discounting.

Once the project has been undertaken, over time more information is learned about its profitability. At any interim date $t = 1, ..., T-1$, a signal s_t realizes. We assume that for any history of signals $\{s_1, .., s_t\}$, there is a sufficient statistic S_t, conditional on which the distribution of the project's payoff is given by $F(\theta|S_t)$, and its expected value accordingly by $E(\theta|S_t)$. For the remainder of the text, we will loosely refer to S_t, the sufficient statistic for the signal history, as the signal at time t. We assume that $F(\cdot)$ satisfies the strict monotone likelihood ratio property with respect to the signal S_t. This implies that when $S_t^A > S_t^B$, the updated distribution function $F(\theta|S_t^A)$ dominates $F(\theta|S_t^B)$ in the first-order stochastic dominance sense (Milgrom, 1981). The signal S_t is distributed according to the distribution function $G_t(\cdot)$. We refer to the highest possible signal at time t as S_t^H, and the lowest possible signal as S_t^L.

The long-term investment can be liquidated prematurely at time $t < T$ with a continuous liquidation technology that allows to liquidate all or only part of the project. However, early liquidation yields only a fraction of the conditional expectation of the project's payoff, $\lambda E(\theta|S_t)$, where $\lambda < 1$. This implies that early liquidation is always inefficient—no matter how bad the signal realization S_t turns out to be, in expectation it always pays more to continue the project rather than to liquidate it early. These liquidation costs reflect the deadweight costs generated by shutting down the project early (technological illiquidity), or the lower valuation of a second-best owner, who may purchase the project at an interim date (market illiquidity).[1]

The borrower has no initial capital and needs to raise the financing for the long-term project from a competitive capital market populated by a continuum of risk-neutral lenders. Each lender has limited capital, such that the borrower has to raise financing from multiple creditors to undertake the investment. Financing takes the form of (unsecured) debt contracts. We take debt contracts as given and do not derive their optimality from a security design perspective.

We assume that the borrower simultaneously offers debt contracts to a continuum of creditors.

[1] Of course, in practice early liquidation must not always be inefficient. In this case, if the financial institution may want to continue inefficient projects because of private benefits or empire building motives, some amount of short-term financing may be desirable, because it may help force liquidation in states where this is efficient (see, e.g., Eisenbach, 2010). We intentionally rule out this possibility for the remainder of the paper in order to restrict the analysis to situations in which short-term debt has no inherent advantage, and then show that under reasonable assumptions short-term financing will nevertheless emerge as the equilibrium outcome.

Debt contracts with differing maturities are available, such that the borrower has to make a choice about its maturity structure when financing the project. A debt contract specifies a face value and a maturity date at which that face value is due. We refer to a debt contract with maturity T as a long-term debt contract. This long-term debt contract matches the maturity of the borrower's assets and liabilities and has a face value of $D_{0,T}$ to be paid back at time T. By definition, long-term debt contracts do not have to be rolled over before maturity, which means that long-term debtholders cannot adjust their financing terms in response to the signals observed at the interim dates $t < T$.

In addition to long-term debt, the borrower can also issue debt with shorter maturity, which has to be rolled over at some time $t < T$. A short-term debt contract written at date 0 specifies a face value $D_{0,t}$ that comes due at date t, at which point this face value has to be repaid or rolled over. When short-term debt is rolled over at t, the outstanding face value is adjusted to reflect the new information contained in the signal S_t. In terms of notation, if debt is rolled over from time t to time $t + \tau$, we denote the rollover face value due at $t + \tau$ by $D_{t,t+\tau}(S_t)$.

Short-term debtholders are atomistic and make uncoordinated rollover decisions at the rollover date. If short-term debtholders refuse to roll over their obligations at date t, some or even all of the long-term investment project may have to be liquidated early to meet the repayment obligations to the short-term debtholders. If the borrower cannot repay rollover creditors by offering new rollover debt contracts or repaying them by liquidating part or all of the long-term investment, the borrower defaults. In the case of default at time $t \leq T$, long-term debt is accelerated[2] and that there is equal priority among all debtholders. Consistent with U.S. bankruptcy procedures, we do not draw a distinction between principal and accrued interest in the case of default. Equal priority then implies that in the case of default the liquidation proceeds are split among all creditors, proportionally to the face values (principal plus matured interest). Holders of non-matured debt do not have a claim on interest that has not accrued yet.

We now describe the main friction in our model: the borrower cannot commit to an aggregate maturity structure when dealing with its creditors (for example by promising to issue only long-term debt contracts with maturity T). It is this non-commitment assumption that makes our

[2] Note that the acceleration of long-term debt in default makes dilution of long-term debt harder. This means that the race to short maturities would be even stronger in the absence of acceleration.

model particularly relevant for financial institutions. The reason is that, relative to corporates, financial institutions have much more opaque maturity structures. For example, while corporates raise financing only occasionally, financial institutions more or less constantly finance and refinance themselves in the commercial paper and repo markets. This makes it much more difficult for a financial institution to commit to a particular maturity structure. Moreover, even if such commitment were possible, for example through covenants, financial institutions may not find it privately optimal to bind themselves to a particular maturity structure in order to keep financial flexibility.

In our baseline model we simply assume that commitment to a maturity structure is not feasible (i.e., we treat it as a technological constraint). Specifically, we assume that when raising financing at date 0, the borrower simultaneously offers debt contracts to a continuum of individual creditors. Individual creditors can only observe the financing terms offered to them, but they cannot observe the financing terms and maturities offered to other creditors, nor can they observe the borrower's aggregate maturity structure.[3] In Section 3.3 we then develop a generalized in which firms can use covenants in order to commit not to shorten the maturity. This extension will show that financial institutions will often not choose to bind themselves to a maturity structure even if they could. Because the limited commitment assumption that underlies our model is particularly relevant for financial institutions, in the remainder of the paper we will refer to the borrower as a financial institution.

2 The Equilibrium Maturity Structure

Given our setup, two conditions must be met for a maturity structure to constitute an equilibrium.[4] First, since capital markets are competitive, a zero profit condition applies, such that in any equilibrium maturity structure all creditors must just break even in expectation.[5] Given that all creditors just break even, the financial institution thus has to issue a combination of debt contracts of potentially different maturities that have an aggregate expected payoff equal to the initial cost

[3]We derive our equilibrium assuming that each creditor can only observe his own contract, but not the contracts offered to other other creditors at date 0. However, the driving assumption behind our results is not this unobservability, but the financial institution's inability to commit to a maturity structure. For example, even when contracts are observable at date 0, our results would go through if the financial institution approached creditors sequentially, along the lines of Bizer and DeMarzo (1992).

[4]For a formal definition of our equilibrium concept see Definition 1 below.

[5]In Parlour and Rajan (2001) lenders make positive profits in competitive equilibrium. This is due to a moral hazard problem that is not present in our setting.

of undertaking the investment.

However, while creditor breakeven is a necessary condition for equilibrium, it is not sufficient. A second condition arises because the financial institution deals bilaterally with multiple creditors and cannot commit to an aggregate maturity structure when entering individual debt contracts. Hence, for a maturity structure to be an equilibrium, at the breakeven face values the financial institution must have no incentive to deviate by forming a coalition with an individual creditor (or a subset of creditors) and changing this creditor's maturity, while holding fixed everybody else's financing terms and beliefs about the institution's aggregate maturity structure.

To illustrate this second requirement, consider for example a conjectured equilibrium in which all creditors expect financing to be in the form of long-term debt that matures at T. The 'no-deviation' requirement is violated when the financial institution has an incentive to move one of the long-term creditors to a shorter maturity contract, given that all remaining creditors anticipate financing to be purely long-term and set their financing terms such that they would just break even under all long-term financing. If this deviation is profitable, the all long-term financing outcome cannot be an equilibrium maturity structure.

We now examine the break-even and no-deviation conditions in turn. For simplicity, in what follows we will initially focus on the financial institution's maturity structure choice when there is only one potential rollover date t. Later on we will show that the analysis can be extended to accommodate multiple rollover dates.

2.1 Creditor Break-Even Conditions

Assume for now that there is only one rollover date, $t < T$. Consider first the rollover decision of creditors whose debt matures at the rollover date t, and denote by α the fraction of creditors that has entered such rollover contracts. In order to roll over the maturing short-term debt at time t, the financial institution has to issue new debt of face value $D_{t,T}(S_t)$, which, conditional on the signal S_t, must have the same value as the amount due to a matured rollover creditor, $D_{0,t}$. This means that the rollover face value must be set such that

$$\int_0^{\bar{D}_T(S_t)} \frac{D_{t,T}(S_t)}{\bar{D}_T(S_t)} \theta dF(\theta|S_t) + D_{t,T}(S_t) \int_{\bar{D}_T(S_t)}^{\infty} dF(\theta|S_t) = D_{0,t}, \tag{1}$$

where $\bar{D}_T(S_t) = \alpha D_{t,T}(S_t) + (1-\alpha) D_{0,T}$ denotes the aggregate face value due at time T.

The interpretation of equation (1) is as follows. If default occurs at time T, the creditors rolling over at t receive a proportional share of the projects cash flows, $\frac{D_{t,T}(S_t)}{\bar{D}_T(S_t)}\theta$. When the financial institution does not default, the entire face value $D_{t,T}(S_t)$ is repaid. Equation (1) thus says that for rollover to occur, $D_{t,T}(S_t)$ must be set such that in expectation the creditors receive their outstanding face value $D_{0,t}$.[6]

Short-term debt can be rolled over as long as the project's future cash flows are high enough such that the financial institution can find a face value $D_{t,T}(S_t)$ for which (1) holds. Given equal priority at time T, the maximum the financial institution can pledge to the short-term creditors at time t is the entire expected future cash flow from the project. This is done by setting $D_{t,T}(S_t)$ to infinity, in which case rollover creditors effectively become equity holders and long-term debtholders are wiped out. Hence, rolling over short-term debt becomes impossible when the expected future cash flows conditional on the signal S_t are smaller than the maturing face value $\alpha D_{0,t}$ owed to the short-term creditors at t. This is the case when

$$\alpha D_{0,t} > E[\theta | S_t]. \tag{2}$$

First-order stochastic dominance implies that the amount of pledgeable cash flow the financial institution has at its disposal to roll over debt at time t is increasing in the signal realization S_t. Hence, there is a critical signal $\tilde{S}_t(\alpha)$ for which (2) holds with equality:

$$\alpha D_{0,t} = E\left[\theta | \tilde{S}_t(\alpha)\right] \tag{3}$$

When the signal realization S_t is below $\tilde{S}_t(\alpha)$, the financial institution cannot roll over its short-term obligations. This is because the bank's dispersed creditors make their rollover decision in an uncoordinated fashion. They will thus find it individually rational to pull out their funding when $S_t < \tilde{S}_t(\alpha)$ in a 'fundamental bank run:' When the financial institution cannot offer short-term creditors full repayment via rollover, each individual creditor will prefer to take out his money in order to be fully repaid that way. In this case, the financial institution has to liquidate the entire

[6] Note that both $\bar{D}_T(S_t)$ and $D_{t,T}(S_t)$ are also functions of α, the fraction of creditors with debt contracts that need to be rolled over at time t. For ease of notation we will generally suppress this dependence in the text.

project and defaults. Note that the critical signal realization below which the project is liquidated is a monotonically increasing in α, the fraction of overall debt that has been financed short-term.

The above argument implicitly assumes that short-term debt cannot be restructured at the rollover date, such that uncoordinated rollover decisions lead to inefficient liquidation at the rollover date. This assumption reflects the fact that in the presence of multiple creditors such debt restructurings are often difficult or even impossible to achieve, mainly because of the well-known holdout problem that arises in debt restructuring. Essentially, since the Trust Indenture Act prohibits changing the timing or the payment amounts of public debt, debt restructuring must take the form of exchange offers, which usually require consent of a specified fraction of debtholders to go through. If each debtholder is small, he will not take into account the effect of his individual tender decision on the outcome of the exchange offer. This means that assuming that a sufficient number of other creditors accept the restructuring, an individual creditor prefers not to accept in order to be paid out in full.[7]

Anticipating potential early liquidation that arises when the financial institution cannot roll over its short-term obligations, the rollover face value from 0 to t must satisfy

$$\int_{S_t^L}^{\tilde{S}(\alpha)} \lambda E\left[\theta|S_t\right] dG\left(S_t\right) + \left[1 - G\left(\tilde{S}_t\left(\alpha\right)\right)\right] D_{0,t} = 1. \tag{4}$$

The interpretation of (4) is as follows. When $S_t < \tilde{S}\left(\alpha\right)$, the short-term creditors withdraw their funding at the rollover date and the financial institution defaults. Long-term debt is accelerated, and each rollover creditor receives $\lambda E\left[\theta|S_t\right] = \lambda \int_0^\infty \theta dF\left(\theta|S_t\right)$.[8] When $S_t \geq \tilde{S}\left(\alpha\right)$, short-term creditors roll over, in which case they are promised a new face value $D_{t,T}\left(S_t\right)$, which in expectation has to be worth $D_{0,t}$. Together, these two terms must be equal to the setup cost for rollover creditors to break even.

Now turn to the break-even condition for the long-term creditors. Since long-term creditors enter their debt contracts at $t = 0$ and cannot change their financing terms after that, they must break even taking an expectation across all signal realizations at the rollover date. When $S_t < \tilde{S}_t\left(\alpha\right)$, the

[7] The holdout problem in debt restructuring is analyzed in more detail in Gertner and Scharfstein (1991). See also the parallel discussion on takeovers in Tirole (2006).

[8] Since long-term debtholders do not have a claim on non-matured interest, when default occurs at date 1, all creditors are treated symmetrically in bankruptcy and the cash flow from liquidation is split equally among all creditors.

project is liquidated at time t, long-term debt is accelerated, and the long-term creditors receive their share of the liquidation proceeds, $\lambda E[\theta|S_t] = \lambda \int_0^\infty \theta dF(\theta|S_t)$. When $S_t \geq \tilde{S}_t(\alpha)$ the project is not liquidated at time t, and the long-term creditors receive either their proportional share of the cash flow $\frac{D_{0,T}}{\bar{D}_T(S_t)}\theta$ if the financial institution defaults at time T, or they are paid back their entire face value $D_{0,T}$. Taking an expectation across all signal realizations at the rollover date t, this leads to the long-term break-even condition

$$\int_{S_t^L}^{\tilde{S}_t(\alpha)} \lambda E[\theta|S_t]\, dG(S_t) + \int_{\tilde{S}_t(\alpha)}^{S_t^H} \left[\int_0^{\bar{D}(S_t)} \frac{D_{0,T}}{\bar{D}_T(S_t)} \theta dF(\theta|S_t) + D_{0,T} \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t)\right] dG(S_t) = 1. \tag{5}$$

2.2 Profit to the Financial Institution and No-Deviation Condition

Consider the expected profit for the financial institution. As the residual claimant, the financial institution receives a positive cash flow at time T if two conditions are met. First, the project must not be liquidated at t, which means that the financial institution only receives a positive cash flow when $S_t \geq \tilde{S}_t(\alpha)$. Second, conditional on survival until T, the realized cash flow θ must exceed the aggregate face value owed to the creditors of different maturities, $\bar{D}_T(S_t)$. This means that we can write the expected profit the to the financial institution as

$$\Pi = \int_{\tilde{S}_t(\alpha)}^{S_t^H} \int_{\bar{D}_T(S_t)}^{\infty} \left[\theta - \bar{D}_T(S_t)\right] dF(\theta|S_t)\, dG(S_t). \tag{6}$$

The inner integral of this expression is the expected cash flow to the institution given a particular signal realization S_t. The outer integral takes the expectation of this expression over signal realizations for which the project is not liquidated at time t.

To find the no-deviation condition, we now calculate the payoff to the financial institution of moving one additional creditor from a long-term debt contract to a short-term debt contract. Following McAfee and Schwartz (1994), when observing this out-of-equilibrium contract offer, the creditor keeps his beliefs about all other contract offers by the financial institution unchanged.[9] The

[9]This *passive beliefs* restriction proposed by McAfee and Schwartz (1994) is the standard refinement used in games with unobservable bilateral contracts (see, for example, Chapter 13 in Bolton and Dewatripont (2005)). In essence, it means that when observing an out-of-equilibrium contract, a creditor believes that all other contracts have remained unchanged.

deviation condition payoff can then be calculated by differentiating the financial institution's profit (6) with respect to the fraction of rollover debt α, keeping in mind that $\bar{D}_T(S_t) = \alpha D_{t,T}(S_t) + (1-\alpha) D_{0,t}$. This yields

$$\frac{\partial \Pi}{\partial \alpha} = \int_{\tilde{S}_t(\alpha)}^{S_t^H} \int_{\bar{D}_T(S_t)}^{\infty} \left[D_{0,T} - D_{t,T}(S_t) \right] dF(\theta|S_t)\, dG(S_t)\,. \tag{7}$$

The intuitive interpretation for this expression is as follows. On the margin, the gain from moving one long-term creditor to a rollover contract is given by the differences of the marginal cost of long-term and short-term debt. Because there is one less long-term creditor, the financial institution saves $D_{0,T}$ in states in which it is the residual claimant, i.e. when $S_t \geq \tilde{S}_t(\alpha)$ and $\theta > \bar{D}_T(S_t)$. This gain has to be weighed against the marginal cost of short-term credit in those states, which is given by $D_{t,T}(S_t)$. Note that in deriving this expression we made use of the fact that the derivatives with respect to the lower integral boundaries drop out, since in both cases the term inside the integral equals zero when evaluated at the boundary. This is an advantage of setting up the model using a continuous (rather than discrete) state space.

If starting from any conjectured equilibrium maturity structure, in which all creditors just break even, we have

$$\frac{\partial \Pi}{\partial \alpha} > 0, \tag{8}$$

the financial institution has an incentive to move an additional creditor from long-term financing to a shorter maturity, keeping everybody else's financing terms fixed. The no-deviation condition thus implies that an equilibrium maturity structure will either be characterized by $\frac{\partial \Pi}{\partial \alpha} = 0$ (with the appropriate second order condition holding), or it will be an extreme maturity structure, either with all long-term debt ($\alpha = 0$ and $\frac{\partial \Pi}{\partial \alpha}\big|_{\alpha=0} \leq 0$), or all short-term rollover debt ($\alpha = 1$ and $\frac{\partial \Pi}{\partial \alpha}\big|_{\alpha=1} \geq 0$).[10]

Definition 1 *An equilibrium maturity structure is characterized by a fraction of rollover debt* α^* *and face values* $\{D_{0,T}(\alpha^*), D_{0,t}(\alpha^*), D_{t,T}(S_t, \alpha^*)\}$ *such that the following conditions are satisfied:*

[10] Note that the discussion in the text focuses on local deviations. In a 'local deviation' the financial institution deviates by moving a single creditor from a long-term to a short-term debt contract. This differs from a 'global deviation,' in which the financial institution deviates by approaching multiple creditors simultaneously. We can focus exclusively on local deviations, since in our setup local deviations are profitable if and only if global deviations are profitable. We discuss this in more detail in the proof of Proposition 1 in the appendix.

1. *Creditors correctly conjecture the fraction of rollover debt α^*.*

2. *Face values $D_{0,T}(\alpha^*)$, $D_{0,t}(\alpha^*)$ and $D_{t,T}(S_t, \alpha^*)$ are set such that all creditor's break even.*

3. *The financial institution has no incentive to deviate from α^* by changing the maturity of one (or a subset of) individual creditors.*

2.3 Interim Information and Maturity Shortening

Before stating our result in the general setup outlined above, we present two simple examples to build intuition. The first example illustrates the mechanism that leads to the unraveling of short-term financing: short-term debt imposes a negative contractual externality on the remaining long-term creditors and, hence, long-term financing cannot be an equilibrium. The second example highlights that not any type of interim information leads to an incentive to shorten the maturity structure. In particular, when information at the rollover date is exclusively about the recovery in default, but does not affect the default probability, there is no incentive for the financial institution to deviate from long-term financing. Hence, for maturity shortening to be privately optimal for the financial institution, the signal at the rollover date must thus contain sufficient information about the financial institution's default probability, a restriction that we will make more precise when we discuss the general case in Section 2.4.

2.3.1 Example 1: Information about Default Probability

Consider a setting in which the final cash flow θ can only take two values, θ^H and θ^L. Assume that the high cash flow is sufficiently large to repay all debt at time T, whereas the low cash flow realization leads to default (i.e., $\theta^L < 1$). The probability of default at date T is thus equal to the probability of the low cash flow. At the rollover date, additional information is revealed about this probability of default: Seen from date 0, the probability of the high cash flow realization is given by p_0, and the probability of default by $1 - p_0$. At the rollover date t the probability of the high cash flow realization is updated to p_t.

For this example, we focus on the initial deviation from all long-term financing (i.e., from a conjectured equilibrium in which the fraction of short-term financing is given by $\alpha = 0$). If all financing is long-term, the break-even condition for the long-term creditors (5) can be rewritten as

$$(1-p_0)\,\theta^L + p_0 D_{0,T} = 1, \tag{9}$$

which implies a face value for long-term debt of $D_{0,T} = \frac{1-(1-p_0)\theta^L}{p_0}$.

But is financing with all long-term debt an equilibrium maturity structure? To determine this, we need to check the no-deviation condition derived above. Since $\theta^L > 0$, the first short-term creditor can always be rolled over at time t, which implies that $D_{0,t} = 1$. From (1), the time-t rollover condition for the first rollover creditor is given by

$$(1-p_t)\,\frac{D_{t,T}}{D_{0,T}}\theta^L + p_t D_{t,T} = 1, \tag{10}$$

which implies a rollover face value of $D_{t,T} = \frac{1-(1-p_0)\theta^L}{\theta^L p_0 + \left(1-\theta^L\right)p_t}$.

The financial institution has an incentive to deviate from all long-term financing when

$$\frac{\partial \Pi}{\partial \alpha} = p_0 D_{0,T} - E\left[p_t D_{t,T}\right] > 0. \tag{11}$$

Using the face values calculated above we can rewrite (11) as

$$1 > E\left[\frac{p_t}{\theta^L p_0 + \left(1-\theta^L\right)p_t}\right]. \tag{12}$$

A simple application of Jensen's inequality shows that this condition is satisfied for any $\theta^L \in (0,1)$.[11] All long-term financing can thus *not* be an equilibrium outcome—starting from a conjectured equilibrium in which financing is all long-term, the financial institution has an incentive to shorten the maturity structure.

The financial institution's incentive to deviate from all long-term financing is illustrated in Figure 1. Panel A shows the face value of long-term debt, and the rollover face value, as a function of the interim signal p_t. As the figure shows, the rollover face value is a convex function of the

[11] The expression inside the expectation is strictly concave in p_t when $\theta^L \in (0,1)$. From Jensen's inequality we thus know that

$$E\left[\frac{p_t}{\theta^L p_0 + \left(1-\theta^L\right)p_t}\right] < \frac{E\left[p_t\right]}{\theta^L p_0 + \left(1-\theta^L\right)E\left[p_t\right]} = 1,$$

where the final equality uses the fact that $E\left[p_t\right] = p_0$.

realization of p_t, which means that, unconditionally, the expected rollover face value is actually higher than the face value of long-term debt.

The intuition for this convexity is straightforward: Assume for a moment that the rollover creditor had the same face value as the long-term creditors, $D_{0,T}$. Then, when probability of success is updated from p_0 to $p_t = p_0 + \sigma$, the rollover creditor would make money on this contract, while when $p_t = p_0 - \sigma$, the rollover creditor would lose money. The amount he would make after good news is equal to the amount he would lose after bad news. Of course, the rollover creditor has to just break even conditional on the realization of p_t, such that his face value has to be lower than $D_{0,T}$ in the first case, and higher than $D_{0,T}$ in the second case. However, a marginal change in the face value of the rollover debt contract has a larger effect on the value of the debt contract after good news than after bad news.[12] This implies that the rollover creditor lowers his face value less after good news than he has to raise his face value after bad news, resulting in the convexity depicted in Figure 1.

The financial institution, however, does not care about face values per se, but about the marginal cost of financing from the equityholder's perspective, i.e., the face values multiplied by the probability of being the residual claimant. This is depicted in Panel B. Crucially for the financial institution's incentive to deviate, once we multiply the face values by the survival probability p_t, the marginal cost of rollover financing conditional on the realization of p_t is a concave function. Taking an expectation over all possible realizations of p_t (and using that $E[p_t] = p_0$) we see that from the equityholder's perspective rollover financing is cheaper than long-term financing, which makes the deviation profitable. In other words, even though unconditionally the expected face value of rollover debt is higher than the face value of long-term debt, the financial institution has an incentive to use rollover debt, since it anticipates that it will repay its debt less often when the rollover face value is high.[13]

One important implication of this analysis is that he incentive to deviate from all long-term

[12] This can easily be checked by differentiating (10) with respect to $D_{t,T}$.

[13] Note that this type of dilution can affect other claims than just long-term debt. For example, assume that the financial institution (borrower) arranges for a credit line up to an amount that just allows it to repay all rollover creditors, if needed. The credit line can be drawn at a fixed, pre-determined interest rate, and the borrower pays a commitment fee such that the credit line provider breaks even in expectation. But now, with the credit line in place, the borrower may have an incentive to increase the amount of short-term debt. As a result, the borrower can now only replace part of its maturing short-term debt through the credit line. The additional short-term creditors dilute the provider of the credit line by demanding higher face values, or by completely pulling their financing at the rollover date.

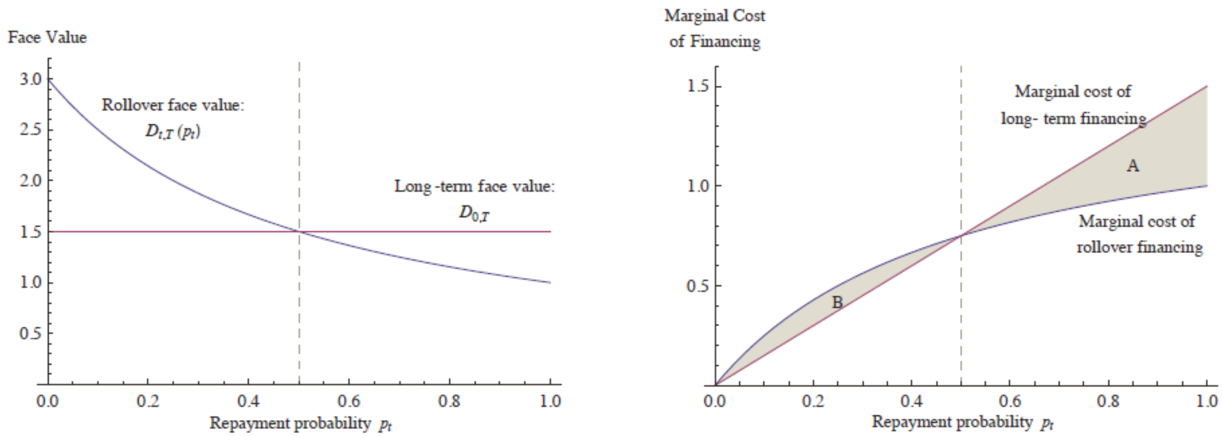

Figure 1: **Illustration: News about Default Probability.** The left panel shows the long-term face value $D_{0,T}$ and the rollover face value $D_{t,T}(p_t)$ as a function of p_t, the probability that the firm will repay its debt given all information available at the rollover date. While the long-term face value is fixed, the face value charged by the first rollover creditor is a convex function of the repayment probability p_t. The right panel shows that, even though the expected rollover face value exceeds the long-term face value, the marginal cost of rollover finance is, in expectation, less than the marginal cost of long-term financing. Hence, an incentive to shorten the maturity structure arises. For this illustration we set $p_0 = 0.5$.

financing is increasing in volatility: The higher the variance of the signal p_t the stronger the incentive to shorten the maturity. To see this, assume that p_t can only take two values, $p_0 + \sigma$ or $p_0 - \sigma$. The concavity of the marginal cost of rollover financing depicted in the right panel of Figure 1 implies that the deviation from long-term financing becomes more profitable when σ increases. This means that in this example the incentive to shorten the maturity structure depends on the amount of interim updating of the default probability. This is consistent with the maturity shortening during financial crisis. For example, Krishnamurthy (2010) shows that maturities in the commercial paper market shortened substantially in September 2008, when, in the aftermath of Lehman's default, investors were expecting to learn which other institutions might also default.

Finally, it is instructive to look at the two polar cases when either $\theta^L = 0$ or when $\theta^L = 1$. It turns out that in either of these cases, the deviation ceases to be profitable. When $\theta^L = 0$, there is nothing to be distributed among the creditors in the default state. Thus, the rollover creditor cannot gain at the expense of the long-term creditors by adjusting his face value at the rollover date when default is more likely. When $\theta^L = 1$, on the other hand, all debt becomes safe. In this case, default will never occur, again preventing the rollover creditor from diluting the existing long-term creditors by increasing his face value. These polar cases illustrate that it is the rollover creditor's ability to increase his face value in states when default is more likely in order to appropriate more of the bankruptcy mass θ^L that makes the deviation profitable.

2.3.2 Example 2: Information about Recovery Value

We now present a second example, in which long-term financing can be sustained as an equilibrium. In contrast to the example in Section 2.3.1, in which information released at the rollover date was exclusively about the probability of default, in this second example, interim information only affects the recovery in default, but not the default probability.

Again, assume that the final cash flow can take two values, θ^H or θ^L. However, this time we keep the probability of the high cash flow fixed at p, whereas the value of the low cash flow θ^L is random seen from date 0, and its realization is revealed at the rollover date t. Assume that θ^L is always smaller than one, such that the financial institution defaults when the low cash flow realizes, regardless of what value θ^L takes. Information revealed at date t is thus exclusively about the recovery in default.

The face value of long-term debt, assuming all long-term financing, is determined by rewriting (5) as

$$(1-p)\, E\left[\theta^L\right] + pD_{0,T} = 1, \tag{13}$$

which implies that $D_{0,T} = \frac{1-(1-p)E\left[\theta^L\right]}{p}$. The face value the first rollover creditor would charge can be determined by rewriting the breakeven condition for rollover creditors (1) as

$$(1-p)\,\frac{D_{t,T}}{D_{0,T}}\theta^L + pD_{t,T} = 1, \tag{14}$$

which, after substituting in for $D_{0,T}$, yields $D_{t,T}\left(\theta^L\right) = \frac{1-(1-p)E\left[\theta^L\right]}{p+p(1-p)\left(\theta^L-E\left[\theta^L\right]\right)}$.

Given these face values, we can now check whether the financial institution has an incentive to deviate from all long-term financing by checking the no-deviation condition. In contrast to the example above, when all information is about the recovery in default the financial institution has

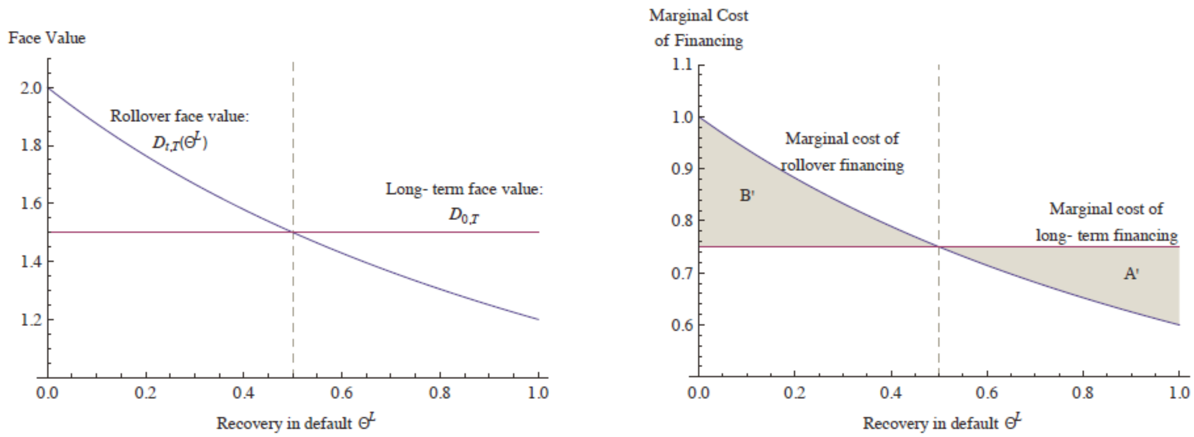

Figure 2: **Illustration: News about Recovery in Default.** The left panel shows the long-term face value $D_{0,T}$ and the rollover face value $D_{t,T}\left(\theta^L\right)$ as a function of θ^L, the recovery value in default. The long-term face value is fixed, while the rollover face value is a convex, decreasing function of the realized recovery in default θ^L. Since the probability of default is independent of the realization of θ^L, the expected marginal cost of rollover finance is higher than the expected marginal cost of long-term financing (right panel). For this illustration we set $p = 0.5$.

no incentive to shorten the maturity structure. This is because

$$\frac{\partial \Pi}{\partial \alpha} = pD_{0,T} - pE\left[D_{t,T}\left(\theta^L\right)\right] < 0 \tag{15}$$

As before, this follows from a simple application of Jensen's inequality. In contrast to the earlier example, when p is fixed the marginal cost of rollover financing $pD_{t,T}\left(\theta^L\right)$ is a convex, decreasing function in the realized recovery value θ^L, which implies that $E\left[pD_{t,T}\left(\theta^L\right)\right] > pD_{t,T}\left(E\left[\theta^L\right]\right) = pD_{0,T}$. This is illustrated in Figure **??**. Hence, from the financial institution's perspective the marginal cost of rollover financing now exceeds the marginal cost of long-term financing, such that the deviation from all long-term financing is unprofitable. This shows that when interim information is purely about the recovery value in default, all long-term financing is an equilibrium.

This second example shows that the introduction of rollover debt does not always dilute remaining long-term debt. In fact, in this example the remaining long-term creditors are *better off* after the introduction of a rollover creditor. The intuition for this result is as follows. By the same reasoning as in the prior example, the rollover face value is convex in the realization of the interim signal. This means that, as before, unconditionally the expected rollover face value is larger than the face value of long-term debt. In the prior example, however, whenever the rollover face value was high, it was likely that the financial institution would default anyway, such that the costs of potential increases in the cost of rollover debt were disproportionately borne by the remaining long-term creditors, who were diluted. In this second example, on the other hand, the probability of default is held fixed because information learned at the rollover date is purely about the recovery in default. The financial institution thus fully bears the expected cost of rollover debt, which due to the convexity in the face value of rollover debt is larger than the cost of long-term debt. Long-term creditors, on the other hand, profit from the reduction in the rollover creditor's face value after positive signals more than they lose when the rollover creditor increases his face value after a negative signal.[14] They thus receive a subsidy.

More broadly, these two examples show that the incentive to shorten the maturity structure depends on the type of information that is revealed by the rollover date. More precisely, the signal at the rollover date must contain sufficient information about the probability of default, as opposed to the recovery in default, a notion we will make more precise in Section 2.4.

2.4 Maturity Structure Shortening: The General Case

Of course, both examples given above are special cases. First, the final cash flow was restricted to only take two values, and interim information was either purely about the probability of default or purely about the recovery in default. Below, we allow the final cash flow to follow a general distribution function and the interim signal to affect both probability of default and recovery. Second, in both examples we only considered the initial deviation starting from a conjectured equilibrium with all long-term financing. Below we generalize the analysis to conjectured equilibrium maturity structures with both long-term debt and rollover debt. This will allow us to characterize the equilibrium maturity structure and determine under which conditions the maturity structure unravels to all short-term rollover debt.

To show this, we need to demonstrate that under certain conditions there is a profitable deviation for the financial institution starting from any maturity structure that involves some amount of long-term debt ($\alpha < 1$). The unique equilibrium maturity structure then exhibits all rollover financing ($\alpha = 1$): All creditors receive short-term contracts and roll over at time t. When this is the case,

[14] Loosely speaking, the reduction in face value profits them more because it is applied to the sharing of a larger realization of θ^L. When rollover creditors raise their face value, on the other hand, θ^L is smaller, such that long-term creditors are less affected: given a low θ^L, it is harder to dilute them.

the equilibrium maturity structure leads to strictly positive rollover risk, such that the long-term project has to be liquidated at the rollover date with positive probability. The financial institution's incentive to shorten the maturity structure thus leads to a real inefficiency.

To extend the intuition gained through the two examples above, we can use the relation $E[XY] = E[X]E[Y] + cov[X,Y]$ to to rewrite the deviation payoff (7). This shows the financial institution has an incentive to shorten the maturity structure whenever

$$E_s\left[D_{0,T} - D_{t,T}(S_t)\,|S_t \geq \widetilde{S}_t(\alpha)\right] E_s\left[\int_{\bar{D}_T(S_t)}^{\infty} dF(\theta|S_t)\,|S_t \geq \widetilde{S}_t(\alpha)\right]$$
$$-cov\left(D_{t,T}(S_t), \int_{\bar{D}_T(S_t)}^{\infty} dF(\theta|S_t)\,|S_t \geq \widetilde{S}_t(\alpha)\right) > 0. \tag{16}$$

From the breakeven conditions, we can be show that $E_s\left[D_{t,T}(s)\,|S_t \geq \widetilde{S}_t(\alpha)\right] > D_{0,T}$.[15] This implies that, conditional on rollover at t, the expected promised yield for rollover debt is *higher* than the promised yield for long-term debt. The first term in (16) is thus strictly negative. This is the case because the rollover face value is convex in the signal S_t—i.e. it increases more after bad signals than it decreases after good signals (as illustrated in Figures 1 and **??**). However, as the residual claimant the financial institution cares not about the face value conditional on rollover, but the face value conditional on rollover in states where the financial institution does not default.

This is captured by the covariance term in (16): the financial institution has an incentive to shorten its maturity provided that the covariance between the rollover face value $D_{t,T}(S_t)$ and the survival probability $\int_{\bar{D}_T(S_t)}^{\infty} dF(\theta|S_t)$ is sufficiently negative. In other words, the deviation is profitable if after bad signals and a correspondingly high rollover face value, it is unlikely that the financial institution will be the residual claimant. Hence, equation (16) shows that in the general setup the deviation to shorten the maturity structure is profitable if the signal received at the rollover date contains sufficient information regarding the probability of default, rather than just the recovery given default, confirming the intuition gained from the examples above. In the first example, all interim information was about the probability of default, thus maximizing the negative covariance between the rollover face value and the survival probability. In the second example, the covariance between the rollover face value and the survival probability was zero, such that (16) is

[15] See Lemma 1 in the appendix for a proof. Essentially, this statement extends our finding, from the examples in 2.3.1 and 2.3.2, that unconditionally the rollover face value is convex in the signal realization.

not satisfied.

We now provide a simple and economically motivated condition on the signal structure that guarantees that interim information contains sufficient information about the probability of default. (Recall that up to this point we have only assumed that the signal S_t orders the updated distribution according to first-order stochastic dominance.)

Condition 1 $D_{t,T}(S_t)\int_{\tilde{D}_T(S_t)}^{\infty} dF(\theta|S_t)$ *is weakly increasing in* S_t *on the interval* $S_t \geq \tilde{S}_t(\alpha)$.

Condition 1 restricts the distribution function $F(\cdot)$ to be such that, whenever rollover is possible, the fraction of expected compensation that rollover creditors receive through full repayment rather than through default is weakly increasing in the signal realization. In other words, under Condition 1 a positive signal is defined as one that increases the amount that creditors expect to receive through full repayment at maturity, as opposed to repayment through recovery in default. This condition is satisfied whenever positive information is mostly about the *probability of default*, rather than about the expected *recovery in default*.

The condition thus directly relates to the intuition gained through the two examples above: Condition 1 is satisfied in the first example, in which all interim information is about the probability of default ($p_t D_{t,T}(p_t)$ is increasing in the realization of p_t), but violated in the second example, in which all interim information was about the recovery in default ($pD_{t,T}(\theta^L)$ is decreasing in θ^L). Condition 1 thus makes the intuition gained from the two examples precise: When Condition 1 holds, the signal received at the rollover date contains sufficient information about the default probability, as opposed to the recovery in default, such that the financial institution has an incentive to shorten the maturity structure starting from any conjectured equilibrium that involves some long-term debt. This allows us to state the following general proposition.

Proposition 1 ***Equilibrium Maturity Structure (A).*** *Suppose that Condition 1 holds. Then in any conjectured equilibrium maturity structure with some amount of long-term financing,* $\alpha \in [0,1)$*, the financial institution has an incentive to increase the amount of short-term financing by switching one additional creditor from maturity* T *to the shorter maturity* $t < T$*. The unique equilibrium maturity structure involves all short-term financing.*

Why is the financial institution unable to sustain a maturity structure in which it enters into long-term debt contracts with all (or even just some) creditors? To see this, consider what happens when the institution moves one creditor from a long-term contract to a shorter maturity while keeping the remaining long-term creditors' financing terms fixed. The difference between long-term and short-term debt is that the face value of the short-term contract reacts to the signal observed at time t. When the signal is positive, rolling over the maturing short-term debt contract at time t is cheap. When, on the other hand, the signal is negative, rolling over the maturing short-term debt at t is costly or even impossible.

The reason why the deviation to short-term financing is profitable for the financial institution is that under Condition 1 rolling over short-term financing is cheap exactly in those states in which the financial institution is likely to be the residual claimant. This means that benefits from an additional unit of short-term financing accrue disproportionately to the financial institution. On the other hand, the signal realizations for which rolling over short-term debt is costly or even impossible are the states in which the financial institution is less likely to be the residual claimant. The costs that arise from an additional unit of short-term financing are thus disproportionately borne by the existing long-term debtholders.

Note that when the financial institution moves an additional creditor to a short-term contract, the remaining long-term creditors do not lose on a state by state basis. However, under Condition 1 existing long-term creditors are worse off when the financial institution moves an additional creditor to short-term contract. This is because rollover creditors raise their face value whenever default is likely, while they lower their face value only in states when default is less likely.

Proposition 1 also shows that this rationale is not limited just to the initial deviation from a conjectured equilibrium in which all financing is through long-term debt, as in the examples above. Rather, under Condition 1 *any* maturity structure that involves some amount of long-term debt cannot be an equilibrium. Starting from any conjectured equilibrium that involves some amount of long-term debt, an additional rollover creditor imposes a negative contractual externality on the value of long-term debt, such that the financial institution gains from moving an additional creditor from a long-term to a short-term debt contract. The financial institution's maturity structure thus unravels to all short-term financing.[16]

[16] The derivation of the deviation payoff uses the fact that the deviation is not observed by other creditors, both long-term and short-term. An alternative assumption would be that rollover creditors notice the deviation when they roll over their debt at t. In this case, the deviation payoff would have an additional term that captures the 'infra-

Reversing Condition 1, on the other hand, provides a sufficient condition under which long-term financing is the unique equilibrium. This generalizes the intuition gained through our second example, in which all interim information was about the recovery rate. More specifically, when the amount that rollover creditors expect to receive through full repayment at time T is decreasing in the signal S_t, then the financial institution has no incentive to deviate from long-term financing. Moreover, starting from any conjectured equilibrium with some amount of short-term debt, the financial institution would have an incentive to increase the fraction of long-term debt, such that long-term financing is the unique equilibrium. This shows that when most interim information is about recovery in default, as opposed to the probability of default, long-term financing is the unique equilibrium.

Proposition 2 ***Equilibrium Maturity Structure (B).*** *Suppose that Condition 1 is reversed, i.e., $D_{t,T}(S_t)\int_{\tilde{D}_T(S_t)}^{\infty} dF(\theta|S_t)$ is weakly decreasing in S_t on the interval $S_t \geq \tilde{S}_t(\alpha)$. Then the unique equilibrium maturity structure involves all long-term debt.*

2.5 Successive Unraveling of the Maturity Structure

Up to now we have focused our analysis on a situation with just one possible rollover date t. In this section we show how in a setup with multiple rollover dates the deviation illustrated above can be applied repeatedly, successively unraveling the maturity structure to the very short end.

Successive unraveling of the maturity structure is illustrated in Figure 3. Consider starting in a conjectured equilibrium in which all debt is long-term, i.e., all debt matures at time T. From our analysis with just one rollover date, we know that if everyone's debt matures at time T, under Condition 1 the financial institution has an incentive to start shortening some creditor's maturity until everyone's maturity is only of length $T-1$. But now consider the same deviation again, but from a conjectured equilibrium in which everyone's maturity is $T-1$. Then, under a condition analogous to Condition 1 the financial institution has an incentive to shorten the maturity of some

marginal' effect of an extra unit of rollover financing on the cost of rolling over existing short-term debt. While it is possible to incorporate this into the model, the analysis becomes significantly less tractable, without much additional economic insight. The general effect of letting existing short-term creditors react to a deviation is a slight reduction in the incentive to shorten the maturity structure further when there is already some existing short-term debt. To quantify this effect, one has to revert to numerical analysis. Rollover financing remains the unique equilibrium when negative interim information is sufficiently correlated with increases in the financial institution's probability of default, but the required amount of correlation increases relative to the setup in the paper.

creditors to $T-2$. The financial institution would do this until all creditors have an initial maturity of $T-2$, after which the whole process would repeat again, in an analogous manner. This implies that in a model with multiple rollover dates, the maturity structure can unravel all the way to the extremely short end—the financial institution writes debt contracts of the shortest possible maturity with all creditors and rolls over its entire debt every period.

To state this more formally, we now generalize Condition 1. Condition 2 is the natural extension of Condition 1 to multiple rollover dates.

Condition 2 $D_{t-1,t}\left(S_{t-1}\right)\int_{\tilde{S}_t}^{\infty} dG\left(S_t|S_{t-1}\right)$ *is increasing in* S_{t-1} *on the interval* $S_{t-1}\geq \widetilde{S}_{t-1}$.

Recall that $\widetilde{S}_{t-1}$ is the signal below which rollover fails at date $t-1$, while $\tilde{S}_t$ is the signal below which rollover fails at date t given successful rollover at date $t-1$. Hence, in the spirit of Condition 1, Condition 2 states that the amount that a creditor who is rolling over at $t-1$ expects to receive through successful rollover at the next rollover date t is increasing in the signal at $t-1$. Condition 2 thus directly extends Condition 1's notion of what constitutes a positive signal to a framework with multiple rollover dates.

Proposition 3 ***Successive Unraveling of the Maturity Structure.*** *Assume that Condition 2 holds. When many rollover dates are possible, successive application of the one-step deviation principle results in a complete unraveling of the maturity structure to the minimum rollover interval.*

Intuitively, this successive unraveling of the maturity structure is a direct extension of the one-step deviation principle stated in Proposition 1. Starting from any time τ at which all creditors roll over for the first time, if Condition 2 holds, it is a profitable deviation for the financial institution to move a creditor to a shorter maturity contract, keeping all other creditors' financing terms fixed. While in the original one-step deviation this increases the financial institution's expected payoff at time T, in this case the deviation increases the financial institution's expected continuation value at the rollover date τ. Save for this adjustment, the proof of sequential unraveling of the maturity structure is similar to the proof of the one-step deviation in Proposition 1.

Conceptually, Proposition 3 demonstrates the power of the simple contractual externality that arises when a financial institution cannot commit to an aggregate maturity structure. Not only does it result in a shortening of the maturity structure, it can result in a successive shortening to

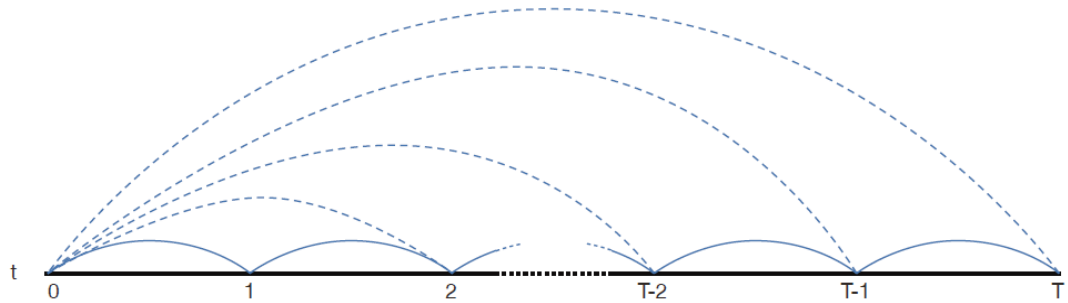

Figure 3: **Illustration of the Maturity Rat Race.** Start in a conjectured equilibrium in which all financing has maturity T (dashed line). In that case it is a profitable deviation for the financial institution to move some creditors to an initial maturity of $T-1$ and then roll over from $T-1$ to T. However, once all creditors' initial maturity is $T-1$, there is an incentive to move some creditors to an initial maturity of $T-2$. The process repeats until all financing has the shortest possible maturity and is rolled over from period to period.

the very short end of the maturity structure. This successive unraveling maximizes rollover risk and the possibility of inefficient liquidation of the long-term project.

3 Implications

In this section we discuss the economic implications that result from the maturity rat race. In Section 3.1 we show that, in the context of our model, the maturity rat race leads to rollover risk that is excessive from a social perspective and highlight that, if anticipated by the market, this can lead to underinvestment relative to first-best. In Section 3.2 we then contrast our findings to those of the classic banking literature. In particular, we argue that the maturity rat race may lead to inefficiencies even when we recognize the benefits of short-term debt that the banking literature has emphasized. In Section 3.3 we then reexamine the efficiency implications of our model in a more general setting where firms may choose to counteract the maturity rat race by adopting covenants. This extension shows that an inefficiency is likely to remain even when firms can counteract the rat race through covenants. It also sharpens the distinction between financial institutions and non-financial firms in the context of our model. Section 3.4 discusses the effect of seniority restrictions, while Section 3.5 discusses the role of leverage. Finally, Section 3.6 discusses some further empirical implications of our model.

3.1 Excessive Rollover Risk and Underinvestment

The maturity mismatch that arises in our model is inefficient. In the context of our model, matching maturities by financing the long-term project via long-term debt is *always* efficient, whereas the short-term debt is inefficient because it leads to rollover risk and inefficient early liquidation, but provides no benefit.[17] Hence, the equilibrium maturity structure in our model is inefficiently short-term whenever Condition 1 is satisfied. This excessive reliance on short-term financing leads to inefficient rollover risk and underinvestment, which is stated more formally in the following two corollaries. For simplicity, we state the two corollaries for the case with only one rollover date.

Corollary 1 ***Excessive rollover risk.*** *When Condition 1 holds, the equilibrium maturity structure ($\alpha = 1$) exhibits excessive rollover risk when, conditional on the worst interim signal, the expected cash flow of the project is less than the initial investment 1, i.e.* $\int_0^\infty \theta dF\left(\theta|S_t^L\right) < 1$.

Corollary 2 ***Some positive NPV projects will not get financed.*** *When Condition 1 holds, as a result of the maturity rat race, some positive NPV projects will not get financed. Only projects for which the NPV exceeds* $(1-\lambda)\int_{S_t^L}^{\tilde{S}_t(1)}\int_0^\infty \theta dF\left(\theta|S_t\right)dG\left(S_t\right)$ *will be financed in equilibrium.*

Corollary 1 states that the maturity rat race leads to a positive amount of rollover risk when, conditional on the worst signal, rolling over short-term debt fails at date t. This leads to inefficient liquidation with positive probability. Corollary 2 states that this rollover risk in turn can make projects that have positive NPV in absence of early liquidation unprofitable. To illustrate the intuition behind Corollary 2, consider a positive NPV project with expected cash flow $E[\theta] > 1$. When the project is finance entirely through short-term debt, the project will be liquidated at date t for any signal realization $S_t < \tilde{S}_t(1)$, since the uncoordinated rollover decision of the short-term creditors makes continuation of the project infeasible. Given this positive probability of liquidation at time t, the pledgeable worth of the project is given by the expected cash flows minus expected

[17] We discuss the implications of our model in the presence of benefits to short-term debt when we relate our model to the banking literature in Section 3.2.

liquidation costs,

$$\underbrace{E[\theta]}_{\text{Expected cash flow}} \underbrace{-(1-\lambda)\int_{S_t^L}^{\tilde{S}_t(1)}\int_0^\infty \theta dF(\theta|S_t)\,dG(S_t)}_{\text{Value destruction from early liquidation}}. \tag{17}$$

In equilibrium creditors will correctly anticipate these liquidation costs, such that in order to receive financing the project's expected cash flows must exceed its setup cost plus the expected liquidation costs. This means that as a result of the maturity rat race and the resulting rollover risk, some positive NPV projects will not be financed in equilibrium.

Corollaries 1 and 2 show that the rat race leads to inefficiencies whenever early liquidation of the project is costly (i.e., when $\lambda < 1$). This shows that maturity mismatch in itself is only a necessary but not a sufficient condition for inefficiencies that may warrant regulatory intervention: In addition to maturity mismatch, there has to be liquidity mismatch of assets and liabilities (i.e., assets with limited technological or market liquidity financed by short-term liabilities) at the rollover date.

Corollaries 1 and 2 raise the question why the financial institution does not internalize the rollover risk and resulting inefficiency of shortening the maturity structure. After all, in equilibrium all creditors just break even, such that ultimately the cost of the inefficient rollover risk is borne by the financial institution. The reason why the financial institution nevertheless has an incentive to shorten its maturity structure is that starting from any conjectured equilibrium with some amount of long-term debt, moving one more creditor to a rollover contract results in a first-order gain, while the increased rollover risk only causes a second-order loss to the financial institution. The first-order gain results because, under Condition 1, the deviation increases the expected profit to the financial institution in states where it survives. The increase in rollover risk, on the other hand, slightly decreases the probability that the financial institution will survive. However, the additional states in which the financial institution defaults are states in which the financial institution would make (next to) zero profits even in absence of the deviation, which means that these losses are of second order compared to the gains from the deviation.[18]

[18] Formally, this can be seen from equation (7). Increasing the fraction of rollover debt α increases the probability that rollover fails by raising $\tilde{S}_t(\alpha)$, the lowest signal for which rollover is possible at the interim date. However, evaluated at the critical signal $\tilde{S}_t(\alpha)$, the payoff to the financial institution is zero. Hence, a small increase in $\tilde{S}_t(\alpha)$ only leads to a second order loss to the financial institution.

3.2 Relation to the Banking Literature

In our model, short-term debt has no benefits. Maturity mismatch does not help serve investors' interim liquidity needs, as in Diamond and Dybvig (1983). Nor does maturity mismatch serve a beneficial role by disciplining bank managers, as in Calomiris and Kahn (1991) or Diamond and Rajan (2001). However, the point of our paper is not to argue that short-term debt has no benefits. Rather, the aim is document a novel mechanism which, despite benefits associated with short-term debt, can lead to *excessive* amounts of short-term financing in equilibrium.

In order isolate this mechanism, we have assumed in our model that short-term debt provides no benefits in terms of liquidity provision or as a disciplining device. A logical question is thus how the implications of our model might change if we allowed for some benefits of short-term debt. To this end, consider an extension of our model in which short-term debt serves a beneficial role in disciplining managers, as has been argued by Calomiris and Kahn (1991) and Diamond and Rajan (2001). Clearly, in this situation, the optimal thing to do is to trade off the costs and benefits of short-term debt. In fact, a number of papers in the banking literature have proposed just this approach (see, e.g., Diamond and Rajan, 2000; Cheng and Milbradt, 2010; Eisenbach, 2010).

The general implication of these models is that there is an optimal fraction of short-term debt that just balances the costs and benefits of short-term debt in the right way. Relative to these papers, the implication of our model is that, in equilibrium, financial institutions may adopt maturity structures that go above and beyond this optimal fraction of short-term debt. For example, while a judicious choice of maturity structure that trades off costs and benefits of short-term debt will most likely combine long-term and short-term debt, the maturity rat race may induce financial institutions to nevertheless move to a maturity structure that is entirely short-term debt. Hence, the broader implication of our paper is that even in the presence of benefits of short-term debt, the maturity rat race will lead to inefficiencies by financial institutions adopt maturity structures that are excessively short-term relative to first best.

3.3 Covenants

Covenants may restrict the ability of short-term creditors to impose externalities on long-term creditors (see, e.g., Smith and Warner, 1979). For simplicity, in the main part of the paper we assumed that the financial institution cannot counteract the maturity rat race by writing covenants. Up to now, we have treated this assumption as a technological constraint: because of their opaqueness, financial institutions cannot write (or enforce) covenants. As pointed out in section 3.1, when covenants are not possible the resulting equilibrium amount of short-term debt is inefficient. In this section we discuss how this result holds up when we allow for covenants. Moreover, introducing covenants will allow us to sharpen the distinction between financial and non-financial firms in the context of our model.

Consider a generalization of our model in which covenants are possible. At date 0, firms can include a covenant in their debt contracts that restricts the firm's ability to issue short-term debt. However, these covenants are not without costs. First, in order to be effective, covenants have to be monitored by the lenders, as emphasized, e.g., by Bjerre (1999) and Ayotte and Bolton (2011).[19] In equilibrium, these monitoring costs are borne by the firm.[20] Second, in addition to monitoring costs, covenants are costly because they reduce ex-post financial flexibility. In particular, should a firm face an additional liquidity need at a future date, covenants make it harder for the firm to raise additional funds to meet this need.

These costs of covenants are likely to be higher for financial institutions than for non-financials. First, because of the complexity and opaqueness of financial institutions, monitoring costs for covenants are likely to be substantially higher for financial institutions. Because in equilibrium these monitoring costs are borne by the borrower, this suggests that financial institutions are less likely to adopt covenants to counteract the rat race. Second, because financial institutions have frequent and unpredictable liquidity needs, financial flexibility is particularly important to financial institutions (relative to non-financials). Hence, also from a financial flexibility perspective financial

[19] The need to monitor covenants is also well recognized among practitioners. For example, as pointed out by the accounting firm Clayton&McKervey (2005): "The most important thing to remember about loan covenants is this: An unmonitored loan covenant is more dangerous than having no loan covenant at all. Therefore, you must put in place a framework for monitoring and enforcing your covenants. While you don't necessarily have to call a loan or declare it in default if covenants are violated, you do need to notify the borrower of the violation. Failure to do so establishes a course of dealing that could adversely affect your ability to exercise your remedies in the future, should you decide to do so."

[20] Each creditor has to monitor the covenant and notify the firm in the case that a covenant is breached. This changes the creditor's breakeven condition. If creditors expect only long-term debt to be issued (because of the covenant), the breakeven condition with covenants is given by $\int_0^{D^c_{0,T}} \theta dF(\theta) + D^c_{0,T} \int_{D^c_{0,T}}^{\infty} dF(\theta|S_t) = 1 + c^M$, where the right hand side reflects the higher expected return that is required to compensate the creditor for the cost of monitoring the covenant, c^M. Hence, in equilibrium, the cost of monitoring covenants will be borne by the financial institution.

institutions are less likely to use covenants to eliminate the rat race, even if they could.[21]

To see this more formally, consider again the one-rollover-date model, but now assume that at date 0 firms have the choice to offer regular debt contracts (as before), or debt contracts that contain covenants. We assume for simplicity that firms can either offer debt without covenants to all creditors, or include covenants into all of their debt contracts. The equilibrium without covenants is as described in the main part of the paper: Under Condition 1 the equilibrium maturity structure is all short-term. Hence, a firm that does not prevent the rat race through covenants has to liquidate whenever the interim signal lies below $\tilde{S}_t(1)$, the threshold signal that triggers a run when all debt is short-term, i.e., $\alpha = 1$.

Now consider the case in which the firm offers debt contracts that include covenants which prevent the rat race to short-term debt. To capture the heterogeneity in the cost of covenants discussed above, assume that, across the population of firms, this cost c, which captures both required monitoring costs and the reduction in financial flexibility, is uniformly distributed on the interval $[\underline{c}, \overline{c}]$. Firms for which monitoring covenants is cheap and for which financial flexibility plays less of a role have relatively low c. In our interpretation, these firms will be predominantly regular (non-financial) corporates. On the other hand, firms that attach a high value to financial flexibility and for which monitoring covenants is relatively costly have high c. This is likely the relevant case for financial institutions. More generally, c captures the cost to a particular firm to undo the commitment problem that leads to maturity shortening.

Because under covenants that implement all long-term debt financing there is no rollover risk, the expected payoff to a firm that adopts covenants is equal to the NPV of the investment, minus the private costs of covenants (monitoring and reduced financial flexibility). It is thus privately optimal for firms to adopt covenants whenever their private cost of adopting covenants lies below some critical value $\widetilde{c}_{po}$, where

$$\widetilde{c}_{po} = \underbrace{(1-\lambda) \int_{S_t^L}^{\tilde{S}_t(1)} \int_0^\infty \theta dF(\theta|S_t)\, dG(S_t)}_{\text{rollover costs without covenants}}. \tag{18}$$

[21] Along similar lines, Flannery (1994), argues that it is usually hard or even undesirable for financial institutions to use covenants. In fact, this is what we observe in practice: in contrast to corporates, most debt financing used by financial institutions does not contain covenants.

Hence, firms for which the costs of covenants are sufficiently low will choose to eliminate the rat race by adopting covenants. Most likely, this is the relevant case for non-financials, for which the costs of covenants, both in terms of monitoring and reduced financial flexibility, are likely lower than for financial institutions. For financial institutions, on the other hand, the private cost of covenants is more likely to outweigh the private benefit because of higher monitoring costs and a higher value of financial flexibility. Hence, when we allow for covenants, our model makes the cross-sectional prediction that regular corporates eliminate the temptation to shorten the maturity through the adoptions of covenants, while financial institutions, for which covenants are particularly costly, will not find it in their interest to do so. The maturity structures of financial institutions and other firms for which it is costly to eliminate the commitment problem are likely to be short-term because, in equilibrium, financial institutions choose expose themselves to rollover risk rather than paying the cost of covenants to eliminate the rat race.

We now turn to the efficiency implications of our model when covenants are possible. To do this, we consider whether a simple policy intervention can raise welfare over the private contracting outcome. For the sake of simplicity, assume that the intervention takes the extremely simple form of prohibiting short-term debt. (While this is optimal in this simple example, more generally one could think of a tax that implements the optimal level of short-term debt.) The first reason why such an intervention may be welfare-enhancing is that the regulator's cost of enforcing the intervention is likely to be lower than the joint monitoring costs required of creditors to make covenants effective. In particular, while a financial institution may choose not to bind itself through covenants at cost c and thus incurs the rollover cost, the policy intervention can help the financial institution bind itself not to use excessive short-term debt, thus eliminating the expected rollover cost. This raises welfare whenever the cost of implementing the intervention is lower than expected rollover costs. More formally, if enforcing this regulation comes at a cost c^R to the regulator, intervention leads to a potential Pareto improvement whenever $c^R < c^M$. Note that it is natural to assume that the regulator's cost of enforcing the intervention lies below the sum of all creditors' monitoring costs. First, relative to creditors, the regulator may have special legal authority in gathering the relevant information. Second, the regulator avoids the duplication of effort that is inherent in monitoring by creditors. In effect, when covenants are costly, the law (or appropriate regulation) can thus help financial institutions to commit not to use short-term financing.

In addition, even in the absence of monitoring costs (or in cases where the regulator has no comparative advantage in enforcing financing with long-term debt), there is still room for regulatory intervention if a financial institution's decision to adopt covenants has externalities on other financial institutions, such that the private and social incentives to adopt covenants differ. For example, an externality of this type arises if the liquidation discount λ is endogenous, in the sense that it depends on the number of financial institutions that need to liquidate assets at the interim date because they cannot role over their short-term financing. To capture the endogeneity of λ we thus write $\lambda(\widetilde{c})$, where, as before, $\widetilde{c}$ denotes the critical value in the costs of covenants below which firms adopt covenants to eliminate the rat race. The liquidation discount at the rollover date is lower the more firms adopt covenants and thus prevent liquidation, such that $\lambda'(\widetilde{c}) > 0$.

As before, it is privately optimal for firms to adopt covenants whenever the private cost of failing to roll over the debt at date t exceeds the cost of reduced financial flexibility c, which is the case whenever c lies below $\widetilde{c}_{po}$, where the only change we need to make to (18) is to replace λ by $\lambda(\widetilde{c}_{po})$, to denote the endogeneity of the liquidation value.

In this equilibrium, the number of firms that adopts covenants is less than the social optimum. This is because when the liquidation (or fire sale) discount λ is endogenous, firms' privately optimal decision to adopt covenants ignores the external effect on other financial institutions. Because of this externality, too few firms (financial institutions) adopt covenants. Formally, the socially optimal cutoff for covenant adoption c_{so} solves

$$\begin{aligned}\widetilde{c}_{so} \;=\;& [1-\lambda(\widetilde{c}_{so})]\int_{S_t^L}^{\tilde{S}_t(1)}\int_0^{\infty}\theta dF(\theta|S_t)\,dG(S_t) \\ &+(\overline{c}-\widetilde{c}_{so})\left.\frac{d\lambda}{dc}\right|_{c=\widetilde{c}_{so}}\left[\int_{S_t^L}^{\tilde{S}_t(1)}\int_0^{\infty}\theta dF(\theta|S_t)\,dG(S_t)\right], \end{aligned} \tag{19}$$

where the second term on the right hand side reflects the positive externality of covenant adoption on other firms.

3.4 Seniority

Similar to covenants, seniority restrictions also limit the ability of short-term debt to dilute long-term debt. Hence, to some extent, seniority can reduce the short-term debtholders' ability to exploit the long-term debtholders by raising their face value in response to negative information that arrives at rollover dates. This is because if default occurs at time T and long-term debtholders are senior (in contrast to the equal priority assumption we have made throughout the paper), short-term debtholders will not receive a larger share of the liquidation mass, even if they have raised their face values at rollover. However, making long-term debtholders senior will generally *not* eliminate the incentive to shorten the maturity structure. This is because of the well-known difference between *de facto* and *de jure* seniority that arises with short and long-term debt.

In particular, anticipating their junior status at date T, under seniority for long-term debt rollover creditors may decide to pull out their financing in response to negative news at the rollover date t. But by pulling their funding at the rollover date, short-term creditors get repaid fully even in some states where the long-term debtholders end up making losses (because the financial institution defaults at date T).[22] Hence, even in the presence of seniority for long-term debt, rollover debt can impose a negative externality on long-term debt.

In addition, even in cases where seniority restrictions can counteract the rat race, it is not clear that financial institutions would choose to impose such restrictions on themselves. In particular, as demonstrated in Diamond (1993) (for corporate debt) and Bolton and Jeanne (2009) (for sovereign debt), allowing for some for of ex-post dilution may be optimal in order to overcome debt overhang problems that may arise once initial financing is in place.

3.5 Leverage

How does leverage affect the maturity rat race? One may think that as leverage increases, investors expect more default-relevant information at rollover dates, such that the incentive to shorten the maturity structure is stronger for more highly levered institutions. However, our model shows that this relationship is somewhat more subtle.

This is best illustrated through a simple example. Suppose a firm is financed by all long-term debt with face value D. The final cash flow of the firm can take three values: 4, 6 or 10. News at the rollover date results in updating of the probabilities of these three cash flow outcomes. Assume first that the firm has very low leverage, $D = 3$. In this case, the firm's debt is safe and there is no

[22] This type of dilution is possible even when long-term debt is accelerated in the case of default at date t, since acceleration only protects long-term creditors when the decision of short-term creditors to pull their funding leads to default at the rollover date.

news about the default probability, nor is there news about the recovery in default. In this case, the maturity rat race would not occur. Now consider the same firm with more leverage, $D = 5$. Now all news received at the rollover date is about the probability of default (since the recovery in default is fixed at 4). Hence, as in the example in Section 2.3.1, the maturity rat race would occur. However, once if we continue to increase the leverage of the firm, say $D = 7$, information at the rollover date is both about the probability of default and the recovery in default. Here the rat race may occur, but it need not occur—this will depend on how much of the interim information is about recovery vs. default probability. Hence, while some leverage makes the rat race more likely, at some point of extreme leverage, this effect may be non-monotonic.

However, leverage is clearly an important determinant of the real consequences that result from the maturity rat race. In particular, the higher the leverage of the financial institution, the more likely it is that the short-term maturity structure that results from the rat race will lead to inefficient liquidation at the rollover date. The reason is that the higher leverage, the less of an equity cushion there is to protect the financial institution from creditor runs.

3.6 Empirical Implications

In this section we briefly sketch out a number of further empirical implications of our model. First, the analysis in Section 3.3 predicts that corporates will counteract the rat race through covenants. This implies that corporate debt should be relatively covenant rich and have longer maturity. Financial institutions, on the other hand, are less likely to counteract the rat race through covenants. Hence, debt of financial institutions, according to our model, should contain few covenants and should be short-term. Both of these empirical predictions are consistent with the stylized facts.

Given that financial institutions do not use covenants to prevent the rat race, our model predicts a shortening of the maturity structure of financial institutions whenever Condition 1 holds. As discussed above, this is the case whenever creditors expect to receive sufficient default-relevant information at the rollover date. To the extent that default relevant information is more prevalent during crises, our model would thus predict a shortening of the maturity structure during those times. Moreover, recall from Section 2.3.1 that when Condition 1 is satisfied, the incentive to shorten the maturity structure is increasing in volatility. Hence, under Condition 1 higher volatility increases the financial institution's incentive to shorten the maturity of its debt. Note, however, that times of high volatility are exactly the times in which, after the maturity structure has unraveled, equilibrium rollover costs resulting from the rat race are particularly high. This link of maturity shortening to volatility is consistent with Krishnamurthy (2010), who shows that maturities in the commercial paper market shortened substantially in September 2008, when, in the aftermath of Lehman's default, investors were expecting to learn which other institutions might also default.

In addition, if the value of financial flexibility increases during crises, our model predicts that financial institutions (and firms) will be less willing to counteract the rat race through covenants during those times. Essentially, when the right-hand side in (18) increases, the adoption of covenants by financial institutions becomes less likely. On the other hand, to the extent that the liquidation discount rises during crises (i.e., λ decreases), *ceteris paribus* firms have more of an incentive to use covenants to prevent the rat race. A priori, it is not clear which of these two effects dominates.

However, as we can see from (19), any privately optimal change from covenant adoption that results from the direct effects of the value of financial flexibility of the liquidation discount would also be made by a social planner. Whether or not the wedge between privately and socially optimal covenant adoption increases during crises depends on what happens to the sensitivity of the liquidation discount λ to further sales during a crisis (as captured by the additional term in (19)). In particular, if the fire sale externality that results from additional sales at the rollover date is particularly large during crisis times, for example because liquidity is already disrupted or because potential outside buyers have lost funding, then the wedge between privately and socially optimal covenant adoption should be particularly pronounced during financial crises. Intervention to prevent the shortening of maturities may thus be particularly desirable during those times.

4 Conclusion

We provide a model of equilibrium maturity structure for borrowers that deal with multiple creditors. Our analysis shows that a contractual externality between long-term and short-term debtholders can lead to an inefficient shortening of the maturity structure when borrowers deal with creditors on a bilateral basis and cannot commit to an aggregate maturity structure. This limited commitment assumption is likely to be particularly relevant for financial institutions. Our model predicts that whenever interim information is mostly about the probability of default, rather than the recovery in default, all short-term financing is the unique equilibrium. This also implies that incentive to shorten the maturity structure is particularly strong during periods of high volatility, such as financial crises, when investors expect substantial default-relevant interim information. The resulting maturity mismatch is inefficient, which stands in contrast to a number of other existing theories of maturity mismatch. Hence, to the extent that maturity mismatch is driven by the forces outlined in this paper, our model suggests future financial regulation should limit maturity in the financial system.

5 Appendix

Proof of Proposition 1: To prove the claim we need to show that starting from any conjectured equilibrium involving any amount of long-term debt, i.e. for all $\alpha \in [0,1)$, in expectation the financial institution is better off by moving an additional creditor to a rollover contract. From (7) we know that this is the case when

$$E\left[\left(D_{0,T}-D_{t,T}\right)\int_{\bar{D}(S_t)}^{\infty} dF\left(\theta|S_t\right)|S_t \geq \widetilde{S}_t\left(\alpha\right)\right] > 0. \tag{20}$$

Implicit in equation (20) is that as the equityholder, the financial institution only gets paid when rollover succeeds at time t (i.e. $S_t \geq \widetilde{S}_t(\alpha)$) and when the project's cash flow θ exceeds the total value of debt that is to be repaid at time T, $\bar{D}(S_t)$.

Before proving that (20) holds for any $\alpha \in [0,1)$ under the Condition 1, we first establish a lemma that will be useful in the proof.

Lemma 1 $E\left[\frac{1}{D_{t,T}(S_t)} - \frac{1}{D_{0,T}}|S_t \geq \widetilde{S}_t\left(\alpha\right)\right] = 0.$

Proof. Using (1) and (4), we can write the rollover breakeven constraint as

$$\int_0^{\bar{D}(S_t)} \frac{D_{t,T}\left(S_t\right)}{\bar{D}\left(S_t\right)}\theta dF\left(\theta|S_t\right) + D_{t,T}\left(S_t\right)\int_{\bar{D}(S_t)}^{\infty} dF\left(\theta|S_t\right) = K, \tag{21}$$

where we define

$$K = \frac{1-\int_{S_L}^{\widetilde{S}_t(\alpha)} \lambda E\left[\theta|S\right] dG\left(S_t\right)}{\int_{\widetilde{S}_t(\alpha)}^{S^H} dG\left(S_t\right)}. \tag{22}$$

In similar fashion, we can rewrite the long-term breakeven constraint (5) as

$$E\left[\int_0^{\bar{D}(S_t)} \frac{D_{0,T}}{\bar{D}(S_t)} \theta dF(\theta|S_t) + D_{0,T} \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t) | S_t \geq \widetilde{S}_t(\alpha)\right] = K. \tag{23}$$

To show that $E\left[\frac{1}{D_{t,T}(S_t)} - \frac{1}{D_{0,T}} | S_t \geq \widetilde{S}_t(\alpha)\right] = 0$, note that from (21) we know that

$$\frac{1}{D_{t,T}(S_t)} = \frac{1}{K}\left[\int_0^{\bar{D}(S_t)} \frac{1}{\bar{D}(S_t)} \theta dF(\theta|S_t) + \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t)\right],$$

and from (23) it follows that

$$\frac{1}{D_{0,T}} = \frac{1}{K} E\left[\int_0^{\bar{D}(S_t)} \frac{1}{\bar{D}(S_t)} \theta dF(\theta|S_t) + \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t) | S_t \geq \widetilde{S}_t(\alpha)\right].$$

This implies that

$$\frac{1}{D_{0,T}} = E\left[\frac{1}{D_{t,T}(S_t)} | S_t \geq \widetilde{S}_t(\alpha)\right].$$

Note that by Jensen's inequality this also implies that $E\left[D_{t,T}(S_t) | S_t \geq \widetilde{S}_t(\alpha)\right] > D_{0,T}$. ■

We now proceed to prove that for any maturity structure that involves any amount of long-term debt, the financial institution has an incentive to shorten its maturity.

Proof. Assume that Condition 1 holds. In order to prove the assertion, we rewrite (20) as

$$E\left[(D_{0,T} - D_{t,T}(S_t)) \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t) | S_t \geq \widetilde{S}_t(\alpha)\right] \tag{24}$$

$$= E\left[\left(\frac{1}{D_{t,T}(S_t)} - \frac{1}{D_{0,T}}\right) D_{t,T}(S_t) D_{0,T} \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t) | S_t \geq \widetilde{S}_t(\alpha)\right] \tag{25}$$

$$= \underbrace{E\left[\frac{1}{D_{t,T}(S_t)} - \frac{1}{D_{0,T}} | S_t \geq \widetilde{S}_t(\alpha)\right]}_{=0} E\left[D_{t,T}(S_t) D_{0,T} \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t) | S_t \geq \widetilde{S}_t(\alpha)\right]$$

$$+ cov\left(\frac{1}{D_{t,T}(S_t)} - \frac{1}{D_{0,T}}, D_{t,T}(S_t) D_{0T} \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t) | S_t \geq \widetilde{S}_t(\alpha)\right) \tag{26}$$

Using Lemma 1 and dividing by the constant term D_{0T}, we see that (20) holds whenever

$$cov\left(\frac{1}{D_{t,T}(S_t)} - \frac{1}{D_{0,T}}, D_{t,T}(S_t) \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t) | S_t \geq \widetilde{S}_t(\alpha)\right) > 0. \tag{27}$$

We know that for all $S_t \geq \widetilde{S}_t(\alpha)$, $D_{t,T}(S_t)$ is decreasing in S_t. This follows from stochastic dominance. This implies that $\frac{1}{D_{t,T}(S_t)} - \frac{1}{D_{0,T}}$is increasing in S_t. Moreover, from Condition (1) we know that on the interval $S_t \geq \widetilde{S}_t(\alpha)$, $D_{t,T}(S_t) \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t)$ is weakly increasing in S_t. We also know that $D_{t,T}(S_t) \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t)$ must be strictly increasing on some interval (when $S_t = \widetilde{S}_t(\alpha)$ the expression is zero, while it is strictly positive for positive realizations of S_t). This implies that the covariance of these two terms is indeed positive, which establishes that (20) indeed holds for any $\alpha \in [0,1]$.

Finally, it remains to establish that $\alpha = 1$ is indeed an equilibrium. The above analysis establishes that there is no profitable local deviation from $\alpha = 1$, since (20) holds at $\alpha = 1$, which means that moving one creditor from a short-term do a long-term contract is strictly unprofitable. It thus remains to check a global deviation, in which the financial institution deviates from a conjectured equilibrium with $\alpha = 1$ by offering long-term contracts to multiple creditors. In this situation, each creditor, only observing his own contract, will assume that all other creditors' contracts remain unchanged (this follows from the concept of 'passive beliefs,' introduced by McAfee and Schwartz (1994): when observing an out-of-equilibrium contract in a game with unobservable offers, a player assumes that all other offers remain unchanged). This implies that also a global deviation from $\alpha = 1$ cannot be profitable, because the payoff from a global deviation is just equal to the payoff from a local deviation scaled by the mass of creditors moved from short-term to long-term debt contracts. As we saw above, the payoff from the local deviation from $\alpha = 1$ is negative. ■

Proof of Proposition 2: The proof follows the same steps as the proof of Proposition **1.** The only change is that the direction of the inequality in (27) is reversed. Following the same argument, all long-term financing is then the unique equilibrium when $D_{t,T}(S_t) \int_{\bar{D}(S_t)}^{\infty} dF(\theta|S_t)$ is weakly decreasing in S_t on the interval $S_t \geq \widetilde{S}_t(\alpha)$.

Proof of Proposition 3: Assume that the first date at which all creditors roll over is date $t \leq T$. We want to consider a deviation from a conjectured equilibrium in which all creditors first roll over at time t, and then roll over every period after that until T. Of course, when $t = T$, the project is financed entirely through long-term debt and the proof of Proposition 1 implies that there is an incentive to shorten the maturity structure to $T-1$. When $t < T$, on the other hand, we need to extend the proof of Proposition 1. Intuitively, rather than showing that the deviation raises the expected time T payoff of the financial institution, we now show that it raises the expected time t continuation value.

Let V_t be the time-t continuation value for the financial institution. This continuation value is a function of three state variables. The first is the face value of debt that has to be rolled over at time t. Consistent with our earlier notation, we denote the aggregate face value maturing at time t by $\bar{D}_t$. The aggregate face value that needs to be rolled over at time t is the sum of the face value issued at time 0 and at the potential earlier rollover date $t-1$, i.e. $\bar{D}_t = \alpha D_{t-1,t}(S_{t-1}) + (1-\alpha) D_{0,t}$. The second state variable is the time-t distribution of the final cash flow. A sufficient statistic for this distribution is the time t signal S_t. The third state variable is the remaining time to maturity, $T-t$ (which is also equal to the number of the remaining rollover dates). Together this implies that, conditional on all the information released up to time t, we can write the time t continuation value for the financial institution as

$$V_t\left(\bar{D}_t, S_t, T-t\right). \tag{28}$$

Seen from $t=0$, the expected continuation value for the entrepreneur at time t is then given by

$$\int_{\tilde{S}_{t-1}}^{\infty}\int_{\tilde{S}_t}^{\infty} V_t\left(\bar{D}_t, S_t, T-t\right) dG\left(S_t|S_{t-1}\right) dG\left(S_{t-1}\right), \tag{29}$$

where $\tilde{S}_{t-1}$and $\tilde{S}_t$ are the signals below which the project is liquidated at times t and $t-1$, respectively, because rollover fails. Note that because the face value of the debt that is rolled over at $t-1$ depends on the signal at $t-1$, we have to take an expectation over the S_{t-1} when calculating the expected continuation value at time t.

Now take the derivative of (29) with respect to α. This yields

$$\int_{\tilde{S}_{t-1}}^{\infty}\int_{\tilde{S}_t}^{\infty} \frac{\partial V_t}{\partial \bar{D}_t}\frac{d\bar{D}_t}{d\alpha} dG\left(S_t|S_{t-1}\right) dG\left(S_{t-1}\right). \tag{30}$$

To prove that there is a profitable deviation from a conjectured equilibrium in which all creditors roll over for the first time at time t, we need to show that this expression is positive. From the definition $\bar{D}_t = \alpha D_{t-1,t}(S_{t-1}) + (1-\alpha) D_{0,t}$ we know that $\frac{d\bar{D}_t}{d\alpha} = D_{t-1,t}(S_{t-1}) - D_{0,t}$. This means that we need to show that

$$\int_{\tilde{S}_{t-1}}^{\infty}\int_{\tilde{S}_t}^{\infty}\frac{\partial V_t}{\partial \bar{D}_t}\left[D_{t-1,t}\left(S_{t-1}\right)-D_{0,t}\right]dG\left(S_t|S_{t-1}\right)dG\left(S_{t-1}\right)>0. \tag{31}$$

Before we proceed with the proof, we now extend Lemma 1 to the multiperiod setting.

Lemma 2 $E\left[\frac{1}{D_{t-1,t}(S_{t-1})}-\frac{1}{D_{0,t}}|S_{t-1}\geq \widetilde{S}_{t-1}\left(\alpha\right)\right]=0.$

Proof. Proceeding analogously to the steps in the proof of Lemma 1, we can write the rollover breakeven constraint from $t-1$ to t as

$$\int_{S_t^L}^{\widetilde{S}_t(\alpha)}\frac{D_{t-1,t}\left(S_{t-1}\right)}{\bar{D}_t\left(S_{t-1}\right)}\lambda E\left[\theta|S_t\right]dG\left(S_t|S_{t-1}\right)+D_{t-1,t}\left(S_{t-1}\right)\int_{\widetilde{S}_t(\alpha)}^{S_t^H}dG\left(S_t|S_{t-1}\right)=K, \tag{32}$$

where we define

$$K=\frac{1-\int_{S_{t-1}^L}^{\widetilde{S}_{t-1}(\alpha)}\lambda E\left[\theta|S_{t-1}\right]dG\left(S_{t-1}\right)}{\int_{\widetilde{S}_{t-1}(\alpha)}^{S_{t-1}^H}dG\left(S_{t-1}\right)}. \tag{33}$$

In similar fashion, we can rewrite the breakeven constraint for creditors that lend from 0 to t as

$$E\left[\int_{S_t^L}^{\widetilde{S}_t(\alpha)}\frac{D_{0,t}}{\bar{D}_t\left(S_{t-1}\right)}\theta dG\left(S_t|S_{t-1}\right)+D_{0,t}\int_{\widetilde{S}_t(\alpha)}^{S_t^H}dG\left(S_t|S_{t-1}\right)|S_{t-1}\geq \widetilde{S}_{t-1}\left(\alpha\right)\right]=K. \tag{34}$$

To show that $E\left[\frac{1}{D_{t,T}(S_t)}-\frac{1}{D_{0,T}}|S_t\geq \widetilde{S}_t\left(\alpha\right)\right]=0$, note that from (32) we know that

$$\frac{1}{D_{t-1,t}\left(S_{t-1}\right)}=\frac{1}{K}\left[\int_{S_t^L}^{\widetilde{S}_t(\alpha)}\frac{1}{\bar{D}_t\left(S_{t-1}\right)}\theta dG\left(S_t|S_{t-1}\right)+\int_{\widetilde{S}_t(\alpha)}^{S_t^H}dG\left(S_t|S_{t-1}\right)\right], \tag{35}$$

and from (34) it follows that

$$\frac{1}{D_{0,t}}=\frac{1}{K}E\left[\int_{S_t^L}^{\widetilde{S}_t(\alpha)}\frac{1}{\bar{D}_t\left(S_{t-1}\right)}\theta dG\left(S_t|S_{t-1}\right)+\int_{\widetilde{S}_t(\alpha)}^{S_t^H}dG\left(S_t|S_{t-1}\right)|S_{t-1}\geq \widetilde{S}_{t-1}\left(\alpha\right)\right]. \tag{36}$$

This implies that

$$\frac{1}{D_{0,t}}=E\left[\frac{1}{D_{t-1,t}\left(S_{t-1}\right)}|S_{t-1}\geq \widetilde{S}_{t-1}\left(\alpha\right)\right]. \tag{37}$$

■

We now proceed analogously to Proposition 1 to rewrite (31) in as a covariance. Following the same steps as in the proof of Proposition 1 and applying Lemma 2, we find that the deviation is profitable when

$$cov\left(\frac{1}{D_{t-1,t}\left(S_{t-1}\right)}-\frac{1}{D_{0,t}},D_{t-1,t}\left(S_{t-1}\right)\int_{\tilde{S}_t}^{\infty}-\frac{\partial V_t}{\partial \bar{D}_t}dG\left(S_t|S_{t-1}\right)|S_{t-1}\geq \widetilde{S}_{t-1}\left(\alpha\right)\right)>0. \tag{38}$$

This condition corresponds to equation (27) in the proof of Proposition 1.

As before, we know that $\frac{1}{D_{t-1,t}(S_{t-1})}-\frac{1}{D_{0,t}}$ is increasing in S_{t-1}. Hence, a sufficient condition for the deviation to be profitable is that

$$D_{t-1,t}\left(S_{t-1}\right)\int_{\tilde{S}_t}^{\infty}-\frac{\partial V_t}{\partial \bar{D}_t}dG\left(S_t|S_{t-1}\right) \tag{39}$$

is increasing in S_{t-1} when $S_{t-1}\geq \widetilde{S}_{t-1}$. Recall that from Condition 2 we know that

$$D_{t-1,t}\left(S_{t-1}\right)\int_{\tilde{S}_t}^{\infty}dG\left(S_t|S_{t-1}\right) \tag{40}$$

is increasing in S_{t-1} when $S_{t-1}\geq \widetilde{S}_{t-1}$. We now show that if Condition 2 holds, then is has to be the case that (39) is increasing in S_{t-1} such that (38) holds.

Proof. To build intuition, consider first what happens if $-\frac{\partial V_t}{\partial \bar{D}_t}$ were independent of S_{t-1} and S_t. If this were the case, (39) would be equal to (40) multiplied by a constant, such that (40) would immediately imply (39). Of course, $-\frac{\partial V_t}{\partial \bar{D}_t}$ is not a constant and depends both on S_{t-1} and S_t. However, we now show that this dependence works in favor of the proof. In other words, if (40) implies (39) when $-\frac{\partial V_t}{\partial \bar{D}_t}$ is a constant, it also implies (39) when we allow for $-\frac{\partial V_t}{\partial \bar{D}_t}$ to depend on S_t and S_{t-1}. In order to see this, it is useful to think of the continuation value V_t as an option on the final payoff, and use the result that the value of an option is convex in its moneyness. When the signal S_{t-1}, is higher, the amount to be rolled over at date t, $\bar{D}_t$, is lower. But when $\bar{D}_t$ is lower, this means that for any realization of S_t, the financial institution's option on the final payoff is further in the money. When the option is further in the money, the 'option delta,' $-\frac{\partial V_t}{\partial \bar{D}_t}$, is larger, because the value of the convexity of the option value. Hence, $-\frac{\partial V_t}{\partial \bar{D}_t}$ is increasing in S_{t-1}. Similarly, when S_t is high, the probability that the option will be in the money. Again, this increases the delta of the option value. However, this means that if we know that $D_{t-1,t}\left(S_{t-1}\right)\int_{\tilde{S}_t}^{\infty}dG\left(S_t|S_{t-1}\right)$

is increasing in S_{t-1}, we know *a fortiori* that $D_{t-1,t}(S_{t-1}) \int_{\tilde{S}_t}^{\infty} -\frac{\partial V_t}{\partial D_t} dG(S_t|S_{t-1})$ is increasing in S_{t-1}, which completes the proof. ■

Proof of Corollary 1: Since early liquidation is always inefficient in this model, the socially optimal level of rollover risk is zero. Any positive probability of liquidation means that there is excessive rollover risk. The unraveling of the maturity structure to all short-term financing leads to positive rollover risk when conditional on the worst interim signal the expected cash flow is less than 1, i.e.

$$\int_0^\infty \theta dF\left(\theta|S_t^L\right) < 1. \tag{41}$$

Proof of Corollary 2: Proof follows directly from the discussion in the main text.

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