Involuntary Unemployment and the Business Cycle*

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Abstract

We propose a monetary model in which the unemployed satisfy the official US definition of unemployment: they are people without jobs who are (i) currently making concrete efforts to find work and (ii) willing and able to work. In addition, our model has the property that people searching for jobs are better off if they find a job than if they do not (i.e., unemployment is 'involuntary'). We integrate our model of involuntary unemployment into the simple New Keynesian framework with no capital and use the resulting model to discuss the concept of the 'non-accelerating inflation rate of unemployment'. We then integrate the model into a medium sized DSGE model with capital and show that the resulting model does as well as existing models at accounting for the response of standard macroeconomic variables to monetary policy shocks and two technology shocks. In addition, the model does well at accounting for the response of the labor force and unemployment rate to the three shocks.

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1. Introduction

The unemployment rate is a key variable of interest to policy makers. A shortcoming of standard monetary dynamic stochastic general equilibrium (DSGE) models is that they are silent about this important variable. Work has begun recently on the task of introducing unemployment into DSGE models. However, the approaches taken to date assume the existence of perfect consumption insurance against labor market outcomes, so that consumption is the same for employed and non-employed households. With this kind of insurance, a household is delighted to be unemployed because it is an opportunity to enjoy leisure without a drop in consumption.[1] In contrast, the theory of unemployment developed here has the implication that the unemployed are worse off than the employed. Our approach follows the work of Hopenhayn and Nicolini (1997) and others, in which finding a job requires exerting a privately observed effort.[2] In this type of environment, the higher utility enjoyed by employed households is necessary for people to have the incentive to search for and keep jobs.[3]

We define unemployment the way it is defined by the agencies that collect the data. To be officially 'unemployed' a person must assert that she (i) has recently taken concrete steps to secure employment and (ii) is currently available for work.[4] To capture (i) we assume that people who wish to be employed must undertake a costly effort. Our model has the implication that a person who asserts (i) and (ii) enjoys more utility if she finds a job than if she does not, i.e., unemployment is 'involuntary'. Empirical evidence appears to be consistent with the notion that unemployment is in practice more of a burden than a blessing.[5] For example, Chetty and Looney (2006) and Gruber (1997) find that US households suffer roughly a 10 percent drop in consumption when they lose their job. Also, there is a substantial literature which purports to find evidence that insurance against labor market outcomes

[1] The drop in utility reflects that models typically assume preferences that are additively separable in consumption and labor or that have the King, Plosser, Rebelo (1988) form. Examples include Blanchard and Gali (2009), Christiano, Ilut, Motto and Rostagno (2008), Christiano, Trabandt and Walentin (2009,2009a), Christoffel, Costain, de Walque, Kuester, Linzert, Millard, and Pierrard (2009), Christoffel, and Kuester (2008), Christoffel, Kuester and Linzert (2009), den Haan, Ramey and Watson (2000), Gali (2009), Gali, Smets and Wouters (2010), Gertler, Sala and Trigari (2009), Groshenny (2009), Krause, Lopez-Salido and Lubik (2008), Lechthaler, Merkl and Snower (2009), Sala, Soderstrom and Trigari (2008), Sveen and Weinke (2008, 2009), Thomas (2008), Trigari (2009) and Walsh (2005).

[2] An early paper that considers unobserved effort is Shavell and Weiss (1979). Our approach is also closely related to the efficiency wage literature, as in Alexopoulos (2004).

[3] Lack of perfect insurance in practice probably reflects other factors too, such as adverse selection. Alternatively, Kocherlakota (1996) explores lack of commitment as a rationale for incomplete insurance. Lack of perfect insurance is not necessary for the unemployed to be worse off than the employed (see Rogerson and Wright, 1988).

[4] See the Bureau of Labor Statistics website, http://www.bls.gov/cps/cps_htgm.htm#unemployed, for an extended discussion of the definition of unemployment, including the survey questions used to determine a household's employment status.

[5] There is a substantial sociological literature that associates unemployment with an increased likelihood of suicide and domestic violence.

is imperfect. An early example is Cochrane (1991). These observations motivate our third defining characteristic of unemployment: (iii) a person looking for work is worse off if they fail to find a job than if they find one.[6]

To highlight the mechanisms in our model, we introduce it into the simplest possible DSGE framework, the model presented by Clarida, Gali and Gertler (1999) (CGG). The CGG model has frictions in the setting of prices, but it has no capital accumulation and no wage-setting frictions. In our model, households gather into families for the purpose of partially ensuring themselves against bad labor market outcomes. Each household experiences a privately observed shock that determines its aversion to work. Households that experience a sufficiently high aversion to work stay out of the labor force. The other households join the labor force and are employed with a probability that is an increasing function of a privately observed effort. The only thing about a household that is observed is whether or not it is employed. Although consumption insurance is desirable in our environment, perfect insurance is not feasible because everyone would claim high work aversion and stay out of the labor force. We view the family as a stand-in for the various market and non-market arrangements that actual households have for dealing with idiosyncratic labor market outcomes. Accordingly, households are assumed to have no access to loan markets, while families have access to complete markets.

In principle, in an environment like ours the wage would be set through a bargaining mechanism. Instead, for simplicity we suppose the wage rate is determined competitively so that firms and families take the wage rate as given.[7] Firms face no search frictions and hire workers up to the point where marginal costs and benefits are equated. Although individual households face uncertainty as to who will work and who will not, families are sufficiently large that there is no uncertainty at the family level. Once the family sets incentives by allocating more consumption to employed households than to non-employed households, it knows exactly how many households will find work. The family takes the wage rate as given and adjusts employment incentives until the marginal cost (in terms of foregone leisure and

[6] Although all the monetary DSGE models that we know of fail (iii), they do not fail (ii). In these models there are workers who are not employed and who would say 'yes' in response to the question, 'are you currently available for work?'. Although such people in effect declare their willingness to take an action that reduces utility, they would in fact do so. This is because they are members of a large family insurance pool. They obey the family's instruction that they value a job according to the value assigned by the family, not themselves. In these models everything about the individual household is observable to the family, and it is implicitly assumed that the family has the technology necessary to enforce verifiable behavior. In our environment - and we suspect this is true in practice - the presence of private information makes it impossible to enforce a labor market allocation that does not completely reflect the preferences of the individual household. (For further discussion, see Christiano, Trabandt and Walentin, 2009, 2009a).

[7] One interpretation of our environment is that job markets occur on Lucas-Phelps-Prescott type 'islands'. Effort is required to reach those islands, but a person who finds the island finds a perfectly competitive labor market. For recent work that uses a metaphor of this type, see Veracierto (2007).

reduced consumption insurance) of additional market work equals the marginal benefit. The firm and family first order necessary conditions of optimization are sufficient to determine the equilibrium wage rate.

Our environment has a simple representative agent formulation, in which the representative agent has an indirect utility function that is a function only of market consumption and labor. As a result, our model is observationally equivalent to the CGG model when only the data addressed by CGG are considered. In particular, our model implies the three equilibrium conditions of the New Keynesian model: an IS curve, a Phillips curve and a monetary policy rule. The conditions can be written in the usual way, in terms of the 'output gap'. The output gap is the difference between actual output and output in the 'efficient equilibrium': the equilibrium in which there are no price setting frictions and distortions from monopoly power are extinguished. In our model there is a simple relation between the output gap and the 'unemployment gap': the difference between actual and efficient unemployment.[8] The presence of this gap in our model allows us to discuss the microeconomic foundations of the non-accelerating inflation rate of unemployment (NAIRU). The NAIRU plays a prominent role in public discussions about the inflation outlook, as well as in discussions of monetary and labor market policies. In practice, these discussions leave the formal economic foundations of the NAIRU unspecified. This paper, in effect takes a step towards integrating the NAIRU into the formal quantitative apparatus of monetary DSGE models.[9]

Next, we introduce our model of unemployment into a medium-sized monetary DSGE model that has been fit to actual data. In particular, we work with a version of the model proposed in Christiano, Eichenbaum and Evans (2005) (CEE). In this model there is monopoly power in the setting of wages, there are wage setting frictions, capital accumulation and other features.[10] We estimate and evaluate our model using the Bayesian version of the impulse response matching procedure proposed in Christiano, Trabandt and Walentin (2009a) (CTW). The impulse response methodology has proved useful in the basic model formulation stage of model construction, and this is why we use it here. The three shocks we consider are the ones considered in Altig, Christiano, Eichenbaum and Linde (2004) (ACEL). In particular, we consider VAR-based estimates of the impulse responses of macroeconomic variables to a monetary policy shock, a neutral technology shock and an investment-specific technology shock. Our model can match the impulse responses of standard variables as well as the standard model. However, our model also does a good job matching the responses of the labor force and unemployment to the three shocks.

The next two sections lay out our model in the context of the CGG and CEE models,

[8] This relationship is a formalization of the widely discussed 'Okun's law'.

[9] For another approach, see Gali (2010).

[10] The model of wage setting in the standard DSGE model is the one proposed in Erceg, Henderson and Levin (2000).

respectively. After that, we estimate the parameters of medium-sized model and report our results. The paper ends with concluding remarks. In those remarks we draw attention to some microeconomic implications of our model. We describe evidence that provides tentative support for the model.

2. An Unemployment-based Phillips Curve

To highlight the mechanisms in our model of unemployment, we embed it into the framework with price setting frictions, flexible wages and no capital analyzed in CGG. The agents in our model are heterogeneous, some households are in the labor force and some are out. Moreover, of those who are in the labor force, some are employed and some are unemployed. Despite this heterogeneity, the model has a representative agent representation. As a result, the linearized equilibrium conditions of the model can be written in the same form as those in CGG. Indeed, relative to a standard macroeconomic data set that includes consumption, employment, inflation and the interest rate, but not unemployment and the labor force, our model and CGG are observationally equivalent.[11]

In our environment, the output gap is proportional to what we call the unemployment gap, the difference between the actual and efficient rates of unemployment. As a result, the Phillips curve can also be expressed in terms of the unemployment gap. We discuss the implications of the theory developed here for the NAIRU and for the problem of forecasting inflation.

2.1. Families, Households and the Labor Market

The economy is populated by a large number of identical families. The representative family's optimization problem is:

$$\max_{\{C_t,h_t,B_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, h_t\right), \ \beta \in (0,1), \tag{2.1}$$

subject to

$$P_t C_t + B_{t+1} \leq B_t R_{t-1} + W_t h_t + \text{Transfers and profits}_t. \tag{2.2}$$

[11] We found that there is a certain sense in which the welfare implications of the CGG model and our model are also equivalent. In the technical appendix to this paper we display examples in which data are generated from our model of involuntary unemployment and provided to an econometrician who estimates the CGG model using a data set that does not include unemployment and the labor force. Consistent with the observational equivalence result, the econometrician's misspecified model fits the data as well as the true model (i.e., our involuntary unemployment model). To our suprise, when the econometrician computes the welfare cost of business cycles, he finds that they coincide, to 11 digits after the decimal, with the true cost of business cycles. Thus, our model suggests that studies such as Lucas (1987), which abstract from imperfections in labor market insurance, do not understate the welfare cost of business cycles. This finding is consistent with similar findings reported by Imrohoroğlu (1989) and Atkeson and Phelan (1994).

Here, C_t, h_t denote family consumption and market work, respectively. In addition, B_{t+1} denotes the quantity of a nominal bond purchased by the family in period t. Also, R_t denotes the one-period gross nominal rate of interest on a bond purchased in period t. Finally, W_t denotes the competitively determined nominal wage rate. The family takes W_t as given and makes arrangements to set h_t so that the relevant marginal conditions are satisfied.

The representative family is composed of a large number of ex ante identical households. The households band together into families for the purpose of insuring themselves as best they can against idiosyncratic labor market outcomes. Individual households have no access to credit or insurance markets other than through their arrangements with the family. In part, we view the family construct as a stand-in for the market and non-market arrangements that actual households use to insure against idiosyncratic labor market experiences. In part, we are following Andolfatto (1996) and Merz (1995), in using the family construct as a technical device to prevent the appearance of difficult-to-model wealth dispersion among households. We emphasize that, although there is no dispersion in household wealth in our model, there is dispersion in consumption.

The family utility function, $u(\cdot,\cdot)$ in (2.1), is the utility attained by the solution to an efficient risk sharing problem subject to incentive constraints, for given values of C_t and h_t. Our simplifying assumptions guarantee that $u(\cdot,\cdot)$ has a simple analytic representation. An important simplifying assumption is that consumption allocations across households within the family are contingent only upon a household's current employment status, and not on its employment history.[12]

The representative family is composed of a unit measure of households. We follow Hansen (1985) and Rogerson (1988) in supposing that household employment is indivisible. A household can either supply one unit of labor, or none at all.[13] This assumption is consistent with the fact that most variation in total hours worked over the business cycle reflects variations in numbers of people employed, rather than in hours per person.

At the start of the period, each household in the family draws a privately observed idiosyncratic shock, l, from a uniform distribution with support, $[0,1]$.[14] The random variable, l, determines the household's utility cost of working:

$$F + \varsigma_t (1 + \sigma_L) l^{\sigma_L}. \tag{2.3}$$

The parameters, $\varsigma_t, \sigma_L \geq 0$ and F are common to all households. The object ς_t is potentially

[12]The analysis of Atkeson and Lucas (1995) and Hopenhayn and Nicolini (1997) suggests that ex ante utility would be greater if consumption allocations could be made contingent on a household's reports of its past labor market outcomes.

[13]The indivisible labor assumption has attracted substantial attention recently. See, for example, Mulligan (2001), and Krusell, Mukoyama, Rogerson, and Sahin (2008, 2009).

[14]A recent paper which emphasizes a richer pattern of idiosyncracies at the individual firm and household level is Brown, Merkl and Snower (2009).

stochastic. It is one shock, among several, that is included in the analysis in order to document what happens when the NAIRU is stochastic. After drawing l, a household decides whether or not to participate in the labor market. A household that chooses to participate must choose a privately observed job search effort, e.[15] The larger is e, the greater is the household's chance of finding a job.

Consider a household which has drawn an idiosyncratic work aversion shock, l, and chooses to participate in the labor market. This household has utility given by:[16]

$$p(e_t) \overbrace{\left[\log\left(c_t^w\right) - F - \varsigma_t \left(1+\sigma_L\right) l^{\sigma_L} - \frac{1}{2} e_t^2\right]}^{\text{ex post utility of household that joins labor force and finds a job}} \tag{2.4}$$
$$+\left(1-p(e_t)\right) \overbrace{\left[\log\left(c_t^{nw}\right) - \frac{1}{2} e_t^2\right]}^{\text{ex post utility of household that joins labor force and fails to find a job}}.$$

Here, c_t^w and c_t^{nw} denote the consumption of employed and non-employed households, respectively. An individual household's consumption can only be dependent on its employment status and labor type because these are the only household characteristics that are publicly observed. In (2.4), $p(e_t)$ denotes the probability that a household which participates in the labor market and exerts effort, e_t, finds a job. This probability is the following linear function of $e_t \geq 0$:

$$p(e_t) = \eta + a e_t, \; \eta, a \geq 0. \tag{2.5}$$

The only admissible model parameterizations are those that imply $0 \leq p(e_t) \leq 1$ in equilibrium.[17] The object $e_t^2/2$ is the utility cost associated with effort. In (2.4) we have structured the utility cost of employment so that σ_L affects its variance in the cross section and not its mean.[18]

A household which participates in the labor force and has idiosyncratic work aversion, l, selects search effort $e_{l,t} \geq 0$ to maximize (2.4). This leads to the following necessary and

[15] In principle, we would still have a model of 'involuntary unemployment' if we just made effort unobservable and allowed the household aversion to work, l, be observable. The manuscript focuses on the symmetric case where both e and l are not observed, and it would be interesting to explore the other case.

[16] The utility function of the household is assumed to be additively separable, as is the case in most of the DSGE literature. In the technical appendix, we show how to implement the anlaysis when the utility function is non-separable.

[17] The specification of $p(e)$ in (2.5) allows for probabilities greater than unity. We could alternatively specify the probability function to be $min\{\eta + a e_t, 1\}$. This would complicate some of the notation and the corner would have to be ignored anyway given the solution strategy that we pursue.

[18] To see this, note:

$$\int_0^1 \left(1+\sigma_L\right) l^{\sigma_L} dl = 1, \; \int_0^1 \left[\left(1+\sigma_L\right) l^{\sigma_L} - 1\right]^2 dl = \frac{\sigma_L^2}{1+2\sigma_L}.$$

sufficient condition:

$$e_{l,t} = \max\left\{a\left(\log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t\left(1+\sigma_L\right)l^{\sigma_L}\right), 0\right\}.$$

The corresponding probability of finding a job is:

$$p\left(e_{l,t}\right) = \eta + a^2 \max\left\{\log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t\left(1+\sigma_L\right)l^{\sigma_L}, 0\right\}. \tag{2.6}$$

Collect the terms in $p(e_t)$ in (2.4) and then substitute out for $p(e_t)$ using $p(e_{l,t})$ in (2.6). We then find that the utility of a household that draws work aversion index, l, and chooses to participate in the labor force is:

$$\begin{aligned} &\left[\eta + a^2 \max\left\{\log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t\left(1+\sigma_L\right)l^{\sigma_L}, 0\right\}\right] \\ &\times\left[\log\left(\frac{c_t^w}{c_t^{nw}}\right) - F - \varsigma_t\left(1+\sigma_L\right)l^{\sigma_L}\right] + \log\left(c_t^{nw}\right) \\ &-\frac{1}{2}\left[\max\left\{a\left(\log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t\left(1+\sigma_L\right)l^{\sigma_L}\right), 0\right\}\right]^2. \end{aligned} \tag{2.7}$$

The utility of households which do not participate in the labor force is simply:

$$\log\left(c_t^{nw}\right). \tag{2.8}$$

Let m_t denote the value of l for which a household is just indifferent between participating and not participating in the labor force (i.e., (2.7) is equal to (2.8)):

$$\log\left[\frac{c_t^w}{c_t^{nw}}\right] = F + \varsigma_t\left(1+\sigma_L\right)m_t^{\sigma_L}. \tag{2.9}$$

For households with $1 \geq l \geq m_t$, (2.7) is smaller than (2.8). They choose to be out of the labor force. For households with $0 \leq l < m_t$ (2.7) is greater than (2.8), and they strictly prefer to be in the labor force. By setting c_t^w and c_t^{nw} according to (2.9) the family incentivizes the m_t households with the least work aversion to participate in the labor force. Imposing (2.9) on (2.7), we find that the ex ante utility of households which draw $l \leq m_t$ is:

$$\eta\varsigma_t\left(1+\sigma_L\right)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) + \frac{1}{2}a^2\varsigma_t^2\left(1+\sigma_L\right)^2\left(m_t^{\sigma_L} - l^{\sigma_L}\right)^2 + \log\left(c_t^{nw}\right). \tag{2.10}$$

If households with work aversion index $l \in [0, m_t]$ participate in the labor force, then the number of employed households, h_t, is:

$$h_t = \int_0^{m_t} p\left(e_{l,t}\right) dl, \tag{2.11}$$

or, after making use of (2.6) and (2.9) and rearranging,

$$h_t = m_t \eta + a^2 \varsigma_t \sigma_L m_t^{\sigma_L + 1}. \tag{2.12}$$

Note that the right side is equal to zero for $m_t = 0$. In addition, the right side of (2.12) is unbounded above and monotonically increasing in m_t. As a result, for any value of $h_t \geq 0$ there exists a unique value of $m_t \geq 0$ that satisfies (2.12), which we express as follows:

$$m_t = f(h_t, \varsigma_t), \tag{2.13}$$

where f is monotonically increasing in h_t.

Let $\bar{p}_t$ denote the largest value of $p(e_{t,l})$. Evidently, $\bar{p}_t$ is the probability associated with the household having the least aversion to work, $l = 0$. Setting $l = 0$ in (2.6) and imposing (2.9):

$$\bar{p}_t = \eta + \varsigma_t a^2 (1 + \sigma_L) m_t^{\sigma_L}. \tag{2.14}$$

We require

$$\bar{p}_t \leq 1, \tag{2.15}$$

for all t. We assume that model parameters have been chosen to guarantee this condition holds.

From (2.11) and the fact that $p(e_{l,t})$ is strictly decreasing in l, we see that

$$h_t < m_t \bar{p}_t.$$

It then follows from (2.15) that $h_t < m_t$, so that the unemployment rate, u_t,

$$u_t \equiv \frac{m_t - h_t}{m_t}, \tag{2.16}$$

is strictly positive. We gain insight into the determinants of the unemployment rate in the model, by substituting out h_t in (2.16) using (2.12):

$$u_t = 1 - \eta - a^2 \varsigma_t \sigma_L m_t^{\sigma_L}. \tag{2.17}$$

According to (2.17), a rise in the labor force is associated with a proportionately greater rise in employment, so that the unemployment rate falls. This greater rise in employment reflects that an increase in the labor force requires raising employment incentives, and this simultaneously generates an increase in search intensity. From (2.11) we see that h_t is linear in m_t if search intensity is held constant, but that h_t/m_t increases with m_t if search intensity increases with m_t. That search intensity indeed does increase in m_t can be seen by substituting (2.9) into (2.6). It is important to note that the theory developed here does not imply that the empirical scatter plot of the unemployment rate against the labor force lies rigidly on a negatively sloped line. Equation (2.17) shows that disturbances in ς_t (or in the parameters of the search technology, (2.5)) would make the scatter of u_t versus m_t resemble a shotgun blast rather than a line. A similar observation can be made about the relationship between h_t and m_t in the context of (2.12).

Consider a household with aversion to work, l, which participates in the labor force. For such a household the ex post utility of finding work minus the ex post utility of not finding work is:

$$\Delta(l) = \log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t(1+\sigma_L)\, l^{\sigma_L}.$$

Condition (2.9) guarantees that, with one exception, $\Delta(l) > 0$. That is, among households that participate in the labor force, those that find work are strictly better off than those that do not. The exceptional case is the marginal household with $m = l$, which sets search effort to zero and finds a job with probability η. The ex post utility enjoyed by the marginal household is the same, whether its job search is successful or not.

In addition to the incentive constraint, the allocation of consumption across employed and non-employed households must also satisfy the following resource constraint:

$$h_t c_t^w + (1-h_t)\, c_t^{nw} = C_t. \tag{2.18}$$

Here, C_t is the aggregate consumption of the family and h_t is the fraction of households that is employed. Solving (2.18) and (2.9), for c_t^{nw}:

$$c_t^{nw} = \frac{C_t}{h_t\left(e^{F+\varsigma_t(1+\sigma_L)m_t^{\sigma_L}} - 1\right) + 1}. \tag{2.19}$$

Integrating the utility, (2.10), of the m_t households in the labor force and the utility, (2.8), of the $1-m_t$ households not in the labor force, we obtain:

$$\int_0^{m_t}\left[\eta\varsigma_t(1+\sigma_L)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) + \frac{1}{2}a^2\varsigma_t^2(1+\sigma_L)^2\left(m_t^{\sigma_L} - l^{\sigma_L}\right)^2\right] dl + \log\left(c_t^{nw}\right). \tag{2.20}$$

Evaluating the integral, and making use of (2.13) and (2.19), we obtain

$$u(C_t, h_t,) = \log(C_t) - z(h_t, \varsigma_t), \tag{2.21}$$

where

$$\begin{aligned} z(h_t,\varsigma_t) &= \log\left[h_t\left(e^{F+\varsigma_t(1+\sigma_L)f(h_t,\varsigma_t)^{\sigma_L}} - 1\right) + 1\right] \\ &\quad - \frac{a^2\varsigma_t^2(1+\sigma_L)\sigma_L^2}{2\sigma_L+1} f(h_t,\varsigma_t)^{2\sigma_L+1} - \eta\varsigma_t\sigma_L f(h_t,\varsigma_t)^{\sigma_L+1}. \end{aligned} \tag{2.22}$$

In (2.22) the function, f, is defined in (2.13).

We now briefly discuss expression (2.21). First, note that the derivation of the utility function, (2.21), involves no maximization problem by the family. This is because the family incentive and resource constraints, (2.9) and (2.18), are sufficient to determine c_t^w and c_t^{nw} conditional on h_t and C_t. In general, the constraints would not be sufficient to determine the household consumption allocations, and the family problem would involve non-trivial optimization. Second, we can see from (2.21) that our model is likely to be characterized by a particular observational equivalence property. To see this, note that although the agents in our model are in fact heterogeneous, C_t and h_t are chosen as if the economy were populated by a representative agent with the utility function specified in (2.21). A model such as CGG, which specifies representative agent utility as the sum of the log of consumption and a constant elasticity disutility of labor is indistinguishable from our model, as long as data on the labor force and unemployment are not used. This is particularly obvious if, as is the case here, we only study the linearized dynamics of the model about steady state. In this case, the only properties of a model's utility function that are used are its second order derivative properties in nonstochastic steady state. This observational equivalence result reflects our simplifying assumptions. These assumptions are primarily driven by the desire for analytic tractability, so that the economics of the environment are as transparent as possible. Presumably, a careful analysis of microeconomic data would lead to different functional forms and the resulting model would then not be observationally equivalent to the standard model.

Our model and the standard CGG model are distinguished by two features. First, our model addresses a larger set of time series than the standard model does. Second, in our model the representative agent's utility function is a reduced form object. Its properties are determined by details of the technology of job search, and by cross-sectional variation in preferences with regard to attitudes about market work. As a result, the basic structure of the utility function in our model can in principle be informed by time use surveys and studies of job search.[19]

With the representative family's utility function in hand, we are in a position to state the necessary conditions for optimization by the representative family:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\pi_{t+1}} \tag{2.23}$$

$$C_t z_h \left(h_t, \varsigma_t\right) = \frac{W_t}{P_t}. \tag{2.24}$$

[19]A similar point was made by Benhabib, Rogerson and Wright (1991). They argue that a representative agent utility function of consumption and labor should be interpreted as a reduced form object, after non-market consumption and labor activities have been maximized out. From this perspective, construction of the representative agent's utility function can in principle be guided by surveys of how time in the home is used.

Here, π_{t+1} is the gross rate of inflation from t to $t+1$. The expression to the left of the equality in (2.24) is the family's marginal cost in consumption units of providing an extra unit of market employment. This marginal cost takes into account the need for the family to provide appropriate incentives to increase employment. A cost of the incentives, which involves increasing the consumption differential between employed and non-employed households, is that consumption insurance to family members is reduced.

2.2. Goods Production and Price Setting

Production is standard in our model. Accordingly, we suppose that a final good, Y_t, is produced using a continuum of inputs as follows:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f}, \; 1 \leq \lambda_f < \infty. \tag{2.25}$$

The good is produced by a competitive, representative firm which takes the price of output, P_t, and the price of inputs, $P_{i,t}$, as given. The first order necessary condition associated with optimization is:

$$\left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} Y_t = Y_{i,t}. \tag{2.26}$$

A useful result is obtained by substituting out for Y_{it} in (2.25) from (2.26):

$$P_t = \left[\int_0^1 (P_{i,t})^{\frac{-1}{\lambda_f - 1}} di \right]^{-(\lambda_f - 1)}. \tag{2.27}$$

Each intermediate good is produced by a monopolist using the following production function:

$$Y_{i,t} = A_t h_{i,t},$$

where A_t is an exogenous stochastic process whose growth rate,

$$g_{A,t} \equiv \frac{A_t}{A_{t-1}},$$

is stationary. The marginal cost of the i^{th} firm is, after dividing by P_t:

$$s_t = (1 - \nu) \frac{W_t}{A_t P_t} = (1 - \nu) \frac{C_t z_h (h_t, \varsigma_t)}{A_t}, \tag{2.28}$$

after using (2.24) to substitute out for W_t/P_t. Here, ν is a subsidy designed to remove the effects, in steady state, of monopoly power. To this end, we set

$$1 - \nu = \frac{1}{\lambda_f}. \tag{2.29}$$

Monopolists are subject to Calvo price frictions. In particular, a fraction ξ_p of intermediate good firms cannot change price:

$$P_{i,t} = P_{i,t-1}, \tag{2.30}$$

and the complementary fraction, $1 - \xi_p$, set their price optimally:

$$P_{i,t} = \tilde{P}_t.$$

The i^{th} monopolist that has the opportunity to reoptimize its price in the current period is only concerned about future histories in which it cannot reoptimize its price. This leads to the following problem:

$$\max_{\tilde{P}_t} E_t \sum_{j=0}^{\infty} \left(\xi_p \beta\right)^j \upsilon_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j}\right], \tag{2.31}$$

subject to (2.26). In (2.31), υ_t is the multiplier on the representative family's time t flow budget constraint, (2.2), in the Lagrangian representation of its problem. Intermediate good firms take υ_{t+j} as given. The nature of the family's preferences, (2.21), implies:

$$\upsilon_{t+j} = \frac{1}{P_{t+j} C_{t+j}}.$$

2.3. Market Clearing, Aggregate Resources and Equilibrium

Clearing in the loan market requires $B_{t+1} = 0$. Clearing in the market for final goods requires:

$$C_t + G_t = Y_t, \tag{2.32}$$

where G_t denotes government consumption. We model G_t as follows:

$$G_t = g_t N_t, \tag{2.33}$$

where $\log g_t$ is a stationary stochastic process independent of any other shocks in the system, such as A_t. The variable, N_t, appears in (2.33) in order to ensure that the model exhibits balanced growth, and has the following representation:

$$N_t = A_t^{\gamma} N_{t-1}^{1-\gamma}, \ 0 < \gamma \leq 1.$$

In the extreme case, $\gamma = 1$, (2.33) reduces to the model of G_t proposed in Christiano and Eichenbaum (1992). That model implies, implausibly, that G_t responds immediately to a shock in A_t. With γ close to zero, G_t is proportional to a long average of past values of

A_t, and the immediate impact of a disturbance in A_t on G_t is arbitrarily small. For any admissible value of γ,

$$n_t \equiv \frac{N_t}{A_t}, \tag{2.34}$$

converges in nonstochastic steady state. The law of motion of n_t is:

$$n_t = \left(\frac{n_{t-1}}{g_{A,t}}\right)^{1-\gamma}.$$

The relationship between aggregate output of the final good, Y_t, and aggregate employment, h_t, is given by (see Yun, 1996):

$$Y_t = p_t^* A_t h_t, \tag{2.35}$$

where

$$p_t^* \equiv \left(\frac{P_t^*}{P_t}\right)^{\frac{\lambda_f}{\lambda_f - 1}}, \; P_t^* = \left[\int_0^1 P_{i,t}^{\frac{\lambda_f}{1-\lambda_f}} di\right]^{\frac{1-\lambda_f}{\lambda_f}}. \tag{2.36}$$

The model is closed once we specify how monetary policy is conducted and time series representations for the shocks. A sequence of markets equilibrium is a stochastic process for prices and quantities which satisfies market clearing and optimality conditions for the agents in the model.

2.4. Log-Linearizing the Private Sector Equilibrium Conditions

It is convenient to express the equilibrium conditions in linearized form relative to the 'efficient' equilibrium. We define the efficient equilibrium as the one in which $\pi_t = 1$ for all t, monopoly power does not distort the level of employment, and there are no price frictions. We refer to the equilibrium in our market economy with sticky prices as simply the 'equilibrium', or the 'actual equilibrium' when clarity requires special emphasis.

2.4.1. The Efficient Equilibrium

In the efficient equilibrium, the marginal cost of labor and the marginal product of labor are equated:

$$C_t z_h (h_t, \varsigma_t) = A_t.$$

The resource constraint in the efficient equilibrium is $C_t + G_t = A_t h_t$, which, when substituted into the previous expression implies:

$$(h_t^* - g_t n_t) \, z_h (h_t^*, \varsigma_t) = 1, \tag{2.37}$$

where the '$*$' indicates an endogenous variable in the efficient equilibrium. Evidently, the efficient level of employment, h_t^*, fluctuates in response to disturbances in g_t and ς_t. It also responds to disturbances in $g_{A,t}$ in the plausible case, $\gamma < 1$. The level of work in the nonstochastic steady state of the efficient equilibrium coincides with the level of work in the nonstochastic steady state of the actual equilibrium. This object is denoted by h in both cases. The values of all variables in nonstochastic steady state coincide across actual and efficient equilibria.

Linearizing (2.37) about steady state,

$$\hat{h}_t^* = \frac{\frac{\eta_g}{1-\eta_g}\left(\hat{g}_t + \hat{n}_t\right) - \sigma_\varsigma \hat{\varsigma}_t}{\frac{1}{1-\eta_g} + \sigma_z} = \frac{\eta_g}{1 + \left(1-\eta_g\right)\sigma_z}\left(\hat{g}_t + \hat{n}_t\right) - \frac{1-\eta_g}{1 + \left(1-\eta_g\right)\sigma_z}\sigma_\varsigma \hat{\varsigma}_t, \quad (2.38)$$

where

$$\sigma_z \equiv \frac{z_{hh}h}{z_h},\ \sigma_\varsigma \equiv \frac{z_{h\varsigma}\varsigma}{z_h}, \quad (2.39)$$

η_g denotes the steady state value of G_t/Y_t, and

$$\hat{n}_t = (1-\gamma)\left(\hat{n}_{t-1} - \hat{g}_{A,t}\right). \quad (2.40)$$

In (2.39), z_{ij} denotes the cross derivative of z with respect to i and j $(i, j = h, \varsigma)$, evaluated in steady state and z_h denotes the derivative of z with respect to h, evaluated in steady state. We follow the convention that a hat over a variable denotes percent deviation from its steady state value.

The object, σ_z, is a measure of the curvature of the function, z, in the neighborhood of steady state. Also, $1/\sigma_z$ is a consumption-compensated elasticity of family labor supply in steady state. Although $1/\sigma_z$ bears a formal similarity to the Frisch elasticity of labor supply, there is an important distinction. In practice the Frisch elasticity refers to a household's willingness to change its labor supply on the intensive margin in response to a wage change. In our environment, all changes in labor supply occur on the extensive margin.

The efficient rate of interest, R_t^*, is derived from (2.23) with consumption and inflation set at their efficient rates:

$$R_t^* = \frac{1}{\beta E_t\left[\frac{h_t^* - g_t n_t}{g_{A,t+1}\left(h_{t+1}^* - g_{t+1}n_{t+1}\right)}\right]}.$$

Linearizing the efficient rate of interest expression about steady state, we obtain:

$$\hat{R}_t^* = E_t \hat{g}_{A,t+1} + \frac{1}{1-\eta_g}E_t\left(\hat{h}_{t+1}^* - \hat{h}_t^*\right) - \frac{\eta_g}{1-\eta_g}E_t\left[\left(\hat{g}_{t+1} + \hat{n}_{t+1}\right) - \left(\hat{g}_t + \hat{n}_t\right)\right], \quad (2.41)$$

where $\hat{h}_{t+1}^*$, $\hat{h}_t^*$ are defined in (2.38).

2.4.2. The Actual Equilibrium

We turn now to the linearized equilibrium conditions in the actual equilibrium. The monetary policy rule (displayed below) ensures that inflation and, hence, price dispersion, is zero in the steady state. Yun (1996) showed that under these circumstances, p_t^* in (2.35) is unity to first order, so that

$$\frac{C_t}{A_t} = \frac{h_t A_t - g_t n_t A_t}{A_t} = h_t - g_t n_t. \tag{2.42}$$

Linearizing (2.28) about the non-stochastic steady state equilibrium and using (2.42), we obtain:

$$\hat{s}_t = \left(\frac{1}{1-\eta_g} + \sigma_z\right)\hat{h}_t - \left[\frac{\eta_g}{1-\eta_g}\left(\hat{g}_t + \hat{n}_t\right) - \sigma_\varsigma \hat{\varsigma}_t\right] = \left(\frac{1}{1-\eta_g} + \sigma_z\right)\left(\hat{h}_t - \hat{h}_t^*\right),$$

using (2.38). Then,

$$\hat{s}_t = \left(\frac{1}{1-\eta_g} + \sigma_z\right)\hat{x}_t, \tag{2.43}$$

where $\hat{x}_t$ denotes the 'output gap', the percent deviation of actual output from its value in the efficient equilibrium:

$$\hat{x}_t \equiv \hat{h}_t - \hat{h}_t^*. \tag{2.44}$$

Condition (2.27), together with the necessary conditions associated with (2.31) leads (after linearization about a zero inflation steady state) to:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\left(\frac{1}{1-\eta_g} + \sigma_z\right)\hat{x}_t. \tag{2.45}$$

The derivation of (2.45) is standard, but is included in appendix A in this paper's technical appendix for completeness.

The family's intertemporal Euler equation, (2.23), after using (2.42), can be expressed as follows:

$$1 = \beta E_t \frac{h_t - g_t n_t}{\left(h_{t+1} - g_{t+1} n_{t+1}\right) g_{A,t+1}} \frac{R_t}{\pi_{t+1}}.$$

Linearize this around steady state, to obtain:

$$\hat{h}_t = \eta_g\left[\left(\hat{g}_t + \hat{n}_t\right) - \left(\hat{g}_{t+1} + \hat{n}_{t+1}\right)\right] + \hat{h}_{t+1} + \left(1-\eta_g\right)\hat{g}_{A,t+1} - \left(1-\eta_g\right)\left(\hat{R}_t - \hat{\pi}_{t+1}\right).$$

Use (2.41) to solve out for $\left(1-\eta_g\right)\hat{g}_{A,t+1}$ in the preceding expression, to obtain:

$$\hat{x}_t = E_t \hat{x}_{t+1} - \left(1-\eta_g\right)E_t\left(\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}_t^*\right). \tag{2.46}$$

Expression (2.46) is the standard representation of the 'New Keynesian IS' curve, expressed in terms of the output gap, $\hat{x}_t$, and the efficient rate of interest, $\hat{R}_t^*$.

The model is closed with the assumption that monetary policy follows a Taylor rule of the following form:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[r_\pi \hat{\pi}_t + r_y \hat{x}_t\right] + \varepsilon_t, \tag{2.47}$$

where ε_t is an iid monetary policy shock. The equilibrium conditions of the log-linearized system are (2.38), (2.40), (2.41), (2.45), (2.46), and (2.47). These conditions determine the equilibrium stochastic processes, $\hat{h}_t^*$, $\hat{n}_t$, $\hat{R}_t^*$, $\hat{R}_t$, $\hat{\pi}_t$ and $\hat{x}_t$ as a function of the exogenous stochastic processes, $\hat{g}_{A,t}$, $\hat{g}_t$, $\hat{\varsigma}_t$ and ε_t. The first three stochastic processes enter the system via the efficient rate of interest and employment as indicated in (2.38) and (2.41), and the monetary policy shock enters via (2.47). The variables, $\hat{h}_t$, can be solved using (2.38) and (2.44).

The model parameters that enter the equilibrium conditions are γ, η_g, σ_z, $\sigma_\varsigma \hat{\varsigma}_t$, ξ_p and β. Consistent with the observational equivalence discussion after (2.21), there is no way, absent observations on unemployment and the labor force, to tell whether these parameters are the ones associated with CGG or with our involuntary unemployment model. Thus, relative to time series on the six variables, $\hat{R}_t^*$, $\hat{R}_t$, $\hat{\pi}_t$, $\hat{x}_t$, $\hat{h}_t$, and $\hat{h}_t^*$, our model and the standard CGG model are observationally equivalent.

2.5. The NAIRU

We can solve for the labor force and unemployment from (2.16) and (2.12). Linearizing (2.12) about steady state, we obtain

$$\hat{m}_t = \frac{1-u}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}}\hat{h}_t - \delta_\varsigma \hat{\varsigma}_t, \tag{2.48}$$

where[20]

$$\delta_\varsigma \equiv \frac{\eta}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}} > 0. \tag{2.49}$$

Linearizing (2.17):

$$du_t = -a^2\varsigma\sigma_L m^{\sigma_L}\left[\sigma_L \hat{m}_t + \hat{\varsigma}_t\right],$$

where

$$du_t \equiv u_t - u,$$

[20] To see this, note from (2.12):

$$\begin{aligned} h\hat{h}_t &= \eta m \hat{m}_t + a^2\varsigma\sigma_L m^{\sigma_L+1}\left[(\sigma_L+1)\hat{m}_t + \hat{\varsigma}_t\right] \\ &= \left[\eta m + (h-m\eta)(\sigma_L+1)\right]\hat{m}_t + (h-m\eta)\hat{\varsigma}_t. \end{aligned}$$

Then, divide by h and rearrange using the identity, $u = 1 - h/m$. Finally, replace $1-\eta-u$ in this expression with $a^2\varsigma\sigma_L m^{\sigma_L}$ using the steady state version of (2.17) in the text.

and u_t is a small deviation from steady state unemployment, u. Substituting from (2.48),

$$u_t = u - \kappa^{okun}\hat{h}_t - a^2\varsigma\sigma_L m^{\sigma_L}\left(1-\sigma_L\delta_\varsigma\right)\hat{\varsigma}_t, \tag{2.50}$$

where

$$\kappa^{okun} = \frac{a^2\varsigma\sigma_L^2 m^{\sigma_L}\left(1-u\right)}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}} > 0.$$

The analogous equation holds in the efficient equilibrium, with $\hat{h}_t$ replaced by $\hat{h}_t^*$:

$$u_t^* = u - \kappa^{okun}\hat{h}_t^* - a^2\varsigma\sigma_L m^{\sigma_L}\left(1-\sigma_L\delta_\varsigma\right)\hat{\varsigma}_t. \tag{2.51}$$

Here, the notation reflects that the steady states in the actual and efficient equilibria coincide. In (2.51), u_t^* denotes unemployment in the efficient equilibrium, i.e., the efficient rate of unemployment. The coefficients on $\hat{\varsigma}_t$ in (2.50) and (2.51) are positive, because $\sigma_L\delta_\varsigma < 1$.[21]

Let u_t^g denote the 'unemployment gap'. Subtracting (2.51) from (2.50), we obtain:

$$u_t^g \equiv u_t - u_t^* = -\kappa^{okun}\hat{x}_t. \tag{2.52}$$

Note that the unemployment gap is the level deviation of the unemployment rate in the actual equilibrium from the efficient rate. The notation is chosen to emphasize that (2.52) represents the model's implication for Okun's law. In particular, a one percentage point rise in the unemployment rate above the efficient rate is associated with a $1/\kappa^{okun}$ percent fall in output relative to its efficient level. The general view is that $1/\kappa^{okun}$ is somewhere in the range, 2 to 3.

The model can be rewritten in terms of the unemployment gap instead of the output gap. Substituting (2.52) into (2.45), (2.46) and (2.47), respectively, we obtain:

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} - \kappa u_t^g \tag{2.53}$$

$$u_t^g = \kappa^{okun}E_t u_{t+1}^g + \kappa^{okun}\left(\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}_t^*\right) \tag{2.54}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + \left(1-\rho_R\right)\left[r_\pi\hat{\pi}_t - \frac{r_y}{\kappa^{okun}}u_t^g\right] + \varepsilon_t \tag{2.55}$$

where

$$\kappa \equiv \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\frac{1+\sigma_z}{\kappa^{okun}}$$

This is the expression Stock and Watson (1999) refer to as the unemployment rate Phillips curve.

[21] To see this, note

$$\frac{\eta\sigma_L}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}} = \frac{\left(1-u-a^2\varsigma_t\sigma_L m_t^{\sigma_L}\right)\sigma_L}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}} < 1$$

We can relate the theory derived here to the idea of a non-accelerating inflation rate of unemployment (NAIRU). One interpretation of the NAIRU focuses on the first difference of inflation. Under this interpretation, the NAIRU is a level of unemployment such that whenever the actual unemployment rate lies below it, inflation is predicted to accelerate and whenever the actual unemployment rate is above it, inflation is predicted to decelerate. The efficient level of unemployment, u_t^*, does not in general satisfy this definition of the NAIRU. From (2.53) it is evident that a negative value of u_t^g does not predict an acceleration of inflation in the sense of predicting a positive value for

$$\beta E_t \hat{\pi}_{t+1} - \hat{\pi}_t. \tag{2.56}$$

On the contrary, according to the unemployment rate Phillips curve, (2.53), a negative value of u_t^g creates an anticipated *deceleration* in inflation.[22] Testing this implication of the data empirically is difficult, because u_t^* is not an observed variable. However, some insight can be gained if one places upper and lower bounds on u_t^*. For example, suppose $u_t^* \in (4, 8)$. That is, the efficient unemployment rate in the postwar US was never below 4 percent or above 8 percent. In the 593 months between February 1960 and July 2009, the unemployment rate was below 4 percent in 52 months and above 8 percent in 42 months. Of the months in which unemployment was above its upper threshold, the change in inflation from that month to three months later was negative 79 percent of the time. Of the months in which unemployment was below the 4 percent lower threshold, the corresponding change in inflation was positive 67 percent of the time. If one accepts our assumption about the bounds on u_t^*, these results lend empirical support to the proposition that there exists a NAIRU in the first difference sense. They also represent evidence against the model developed here.[23]

An alternative interpretation of the NAIRU focuses on the level of inflation, rather than its change. Under this interpretation, u_t^* in the theory developed here is a NAIRU.[24] To see this, one must take into account that the theory (sensibly) implies that inflation returns to steady state after a shock that causes u_t^g to drop has disappeared. That is, the eventual

[22] In their discussion of the NAIRU, Ball and Mankiw (2002) implicitly reject (2.53) as a foundation for the notion that u_t^* is a NAIRU. Their discussion begins under a slightly different version of (2.53), with $\beta E_t \hat{\pi}_{t+1}$ replaced by $E_{t-1}\hat{\pi}_t$. They take the position that u_t^* in this framework is a NAIRU only when monetary policy generates the random walk outcome, $E_{t-1}\hat{\pi}_t = \hat{\pi}_{t-1}$. In this case, a negative value of u_t^g is associated with a deceleration of current inflation relative to what it was in the previous period. Ball and Mankiw argue that the random walk case is actually the relevant one for the US in recent decades.

[23] Our bounds test follows the one implemented in Stiglitz (1997) and was executed as follows. Monthly observations on the unemployment and the consumer price index were taken from the Federal Reserve Bank of St. Louis' online data base, FRED. We worked with the raw unemployment rate. The consumer price index was logged, and we computed a year-over-year rate of inflation rate, π_t. The percentages reported in the text represent the fraction of times that $u_t < 4$ and $\pi_{t+3} - \pi_t > 0$, and the fraction of times that $u_t > 8$ and $\pi_{t+3} - \pi_t < 0$.

[24] In his discussion of the NAIRU, Stiglitz (1997) appears to be open to either the first difference or level interpretation of the NAIRU.

effect on inflation of a negative shock to u_t^g must be zero. That a negative shock to u_t^g also creates the expectation of a deceleration in inflation then implies that inflation converges back to steady state from above after a negative shock to u_t^g. That is, a shock that drives u_t below u_t^* is expected to be followed by a higher level of inflation and a shock that drives u_t above u_t^* is expected to be followed by a lower level of inflation.[25]

Thus, u_t^* in the theory derived here is a NAIRU if one adopts the level interpretation of the NAIRU and not if one adopts the first difference interpretation. Interestingly, u_t^* is a NAIRU under the first difference interpretation if one adopts the price indexation scheme proposed in CEE, in which (2.30) is replaced by

$$P_{i,t} = \pi_{t-1} P_{i,t-1}.$$

In this case, $\hat{\pi}_t$ and $\hat{\pi}_{t+1}$ in (2.53) are replaced by their first differences. Retracing the logic of the previous two paragraphs establishes that with price indexation, u_t^* is a NAIRU in the first difference sense. Under our assumptions about the bounds on u_t^*, price indexation also improves the empirical performance of the model on the dimensions emphasized here.

It is instructive to consider the implications of the theory for the regression of the period $t+1$ inflation rate on the period t unemployment and inflation rates. In the very special case that u_t^* is a constant, the regression coefficient on u_t would be κ and other variables would not add to the forecast.[26] However, these predictions depend crucially on the assumption that u_t^* is constant. If it is stochastic, then u_t^* is part of the error term. Since u_t^* is expected to be correlated with all other variables in the model, then adding these variables to the forecast equation is predicted to improve fit.

[25] A quick way to formally verify the convergence properties just described is to consider the following example. Suppose the monetary policy shock, ε_t, is an iid stochastic process. Let the response of the endogenous variables to ε_t be given by

$$u_t^g = u_\varepsilon \varepsilon_t, \; \hat{R}_t = R_\varepsilon \varepsilon_t, \; \hat{\pi}_t = \pi_\varepsilon \varepsilon_t,$$

where u_ε, R_ε and π_ε are undetermined coefficients to be solved for. Substituting these into the equations that characterize equilibrium and imposing that the equations must be satisfied for every realization of ε_t, we find:

$$u_\varepsilon = \frac{\kappa^{okun}}{1 + \kappa^{okun} \kappa r_\pi + r_y}, \; \pi_\varepsilon = -\kappa u_\varepsilon, \; R_\varepsilon = \frac{1}{\kappa^{okun}} u_\varepsilon.$$

According to these expressions, a monetary policy shock drives u_t^g and R_t in the same direction. Thus, a monetary policy shock that drives the interest rate down also drives the unemployment gap down. The same shock drives current inflation up.

[26] In our model, u_t^* is constant only under very special circumstances. For example, it is constant if government spending is zero and the labor preference shock, ς_t, is constant. However, as explained after (2.37), u_t^* is a function of all three shocks when government spending is positive and $\gamma < 1$.

3. Integrating Unemployment into a Medium-Sized DSGE Model

Our representation of the 'standard DSGE model' is a version of the medium-sized DSGE model in CEE or Smets and Wouters (2003, 2007). The first section below describes how we introduce our model of involuntary unemployment into the standard model. The last section derives the standard model as a special case of our model.

3.1. Final and Intermediate Goods

A final good is produced by competitive firms using (2.25). The i^{th} intermediate good is produced by a monopolist with the following production function:

$$Y_{i,t} = \left(z_t H_{i,t}\right)^{1-\alpha} K_{i,t}^{\alpha} - z_t^{+}\phi, \tag{3.1}$$

where $K_{i,t}$ denotes capital services used for production by the i^{th} intermediate good producer. Also, $\log(z_t)$ is a technology shock whose first difference has a positive mean and ϕ denotes a fixed production cost. The economy has two sources of growth: the positive drift in $\log(z_t)$ and a positive drift in $\log(\Psi_t)$, where Ψ_t is the state of an investment-specific technology shock discussed below. The object, z_t^{+}, in (3.1) is defined as follows:

$$z_t^{+} = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

Along a non-stochastic steady state growth path, Y_t/z_t^{+} and $Y_{i,t}/z_t^{+}$ converge to constants. The two shocks, z_t and Ψ_t, are specified to be unit root processes in order to be consistent with the assumptions we use in our VAR analysis to identify the dynamic response of the economy to neutral and capital-embodied technology shocks. The two shocks have the following time series representations:

$$\Delta \log z_t = \mu_z + \varepsilon_t^{n},\ E\left(\varepsilon_t^{n}\right)^2 = \left(\sigma_n\right)^2 \tag{3.2}$$

$$\Delta \log \Psi_t = \mu_\psi + \rho_\psi \Delta \log \Psi_{t-1} + \varepsilon_t^{\psi},\ E\left(\varepsilon_t^{\psi}\right)^2 = \left(\sigma_\psi\right)^2. \tag{3.3}$$

Our assumption that the neutral technology shock follows a random walk with drift matches closely the finding in Smets and Wouters (2007) who estimate $\log z_t$ to be highly autocorrelated. The direct empirical analysis of Prescott (1986) also supports the notion that $\log z_t$ is a random walk with drift.

In (3.1), $H_{i,t}$ denotes homogeneous labor services hired by the i^{th} intermediate good producer. Intermediate good firms must borrow the wage bill in advance of production, so that one unit of labor costs is given by

$$W_t R_t,$$

where R_t denotes the gross nominal rate of interest. Intermediate good firms are subject to Calvo price-setting frictions. With probability ξ_p the intermediate good firm cannot reoptimize its price, in which case it is assumed to set its price according to the following rule:

$$P_{i,t} = \bar{\pi} P_{i,t-1}, \tag{3.4}$$

where $\bar{\pi}$ is the steady state inflation rate. With probability $1 - \xi_p$ the intermediate good firm can reoptimize its price. Apart from the fixed cost, the i^{th} intermediate good producer's profits are:

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} \{P_{i,t+j} Y_{i,t+j} - s_{t+j} P_{t+j} Y_{i,t+j}\},$$

where s_t denotes the marginal cost of production, denominated in units of the homogeneous good. The object, s_t, is a function only of the costs of capital and labor, and is described in the technical appendix, section E. In the firm's discounted profits, $\beta^j \upsilon_{t+j}$ is the multiplier on the family's nominal period $t + j$ budget constraint. The equilibrium conditions associated with this optimization problem are reported in section E of the technical appendix.

We suppose that the homogeneous labor hired by intermediate good producers is itself 'produced' by competitive labor contractors. Labor contractors produce homogeneous labor by aggregating different types of specialized labor, $j \in (0, 1)$, as follows:

$$H_t = \left[\int_0^1 (h_{t,j})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w}, \ 1 \leq \lambda_w < \infty. \tag{3.5}$$

Labor contractors take the wage rate of H_t and $h_{t,j}$ as given and equal to W_t and $W_{t,j}$, respectively. Profit maximization by labor contractors leads to the following first order necessary condition:

$$W_{j,t} = W_t \left(\frac{H_t}{h_{t,j}} \right)^{\frac{\lambda_w - 1}{\lambda_w}}. \tag{3.6}$$

Equation (3.6) is the demand curve for the j^{th} type of labor.

3.2. Family and Household Preferences

We integrate the model of unemployment in the previous section into the Erceg, Henderson and Levin (2000) (EHL) model of sticky wages used in the standard DSGE model. Each type, $j \in [0, 1]$, of labor is assumed to be supplied by a particular family of households. The j^{th} family resembles the single representative family in the previous section, with one exception. The exception is that the unit measure of households in the j^{th} family is only able to supply the j^{th} type of labor service. Each household in the j^{th} family has the utility cost of working, (2.3), and the technology for job search, (2.5). The five parameters of these functions are

$$F, \varsigma_t, \sigma_L, a, \eta,$$

where the first three pertain to the cost of working and the last two pertain to job search. In the analysis of the empirical model, the preference shock, ς_t, is constant. We assume that these parameters are identical across families. In order that the representative family in the current section have habit persistence in consumption, we change the way consumption enters the additive utility function of the household. In particular, we replace $\log\left(c_t^{nw}\right)$ and $\log\left(c_t^w\right)$ everywhere in the previous section with

$$\log\left(c_{j,t}^{nw} - bC_{t-1}\right), \; \log\left(c_{j,t}^{w} - bC_{t-1}\right),$$

respectively. Here, C_{t-1} denotes the family's previous period's level of consumption. When the parameter, b, is positive, then each household in the family has habit in consumption. Also, $c_{j,t}^{nw}$ and $c_{j,t}^{w}$ denote the consumption levels allocated by the j^{th} family to non-employed and employed households within the family. Although families all enjoy the same level of consumption, C_t, for reasons described momentarily each family experiences a different level of employment, $h_{j,t}$. Because employment across families is different, each type j family chooses a different way to balance the trade-off between the need for consumption insurance and the need to provide work incentives. For the j^{th} type of family with high $h_{j,t}$, the premium of consumption for working households to non-working households must be high. It is easy to verify that the incentive constraint in the version of the model considered here is the analog of (2.9):

$$\log\left[\frac{c_{j,t}^{w} - bC_{t-1}}{c_{j,t}^{nw} - bC_{t-1}}\right] = F + \varsigma\left(1+\sigma_L\right) m_{j,t}^{\sigma_L},$$

where $m_{j,t}$ solves the analog of (2.12):

$$h_{j,t} = m_{j,t}\eta + a^2 \varsigma \sigma_L m_{j,t}^{\sigma_L+1}. \tag{3.7}$$

Consider the j^{th} family that enjoys a level of family consumption and employment, C_t and $h_{j,t}$, respectively. It is readily verified that the utility of this family, after it efficiently allocates consumption across its member households subject to the private information constraints, is given by:

$$u\left(C_t - bC_{t-1}, h_{j,t}\right) = \log\left(C_t - bC_{t-1}\right) - z\left(h_{j,t}\right), \tag{3.8}$$

where the z function in (3.8) is defined in (2.22) with ς_t replaced by ς. The j^{th} family's discounted utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t - bC_{t-1}, h_{j,t}\right). \tag{3.9}$$

Note that this utility function is additively separable, like the utility functions assumed for the households. Additive separability is convenient because perfect consumption insurance at the level of families implies that consumption is not indexed by labor type, j. As we show later, this simplification appears not to have come at a cost in terms of accounting for aggregate data. Still, it would be interesting to explore the implications of non-separable utility. The technical appendix to this paper derives (3.8) for two non-separable specifications of utility for households. Moreover, Guerron-Quintana (2008) shows how to handle the fact that family consumption is now indexed by j.

3.3. The Family Problem

The j^{th} family is the monopoly supplier of the j^{th} type of labor service. The family understands that when it arranges work incentives for its households so that employment is $h_{j,t}$, then $W_{j,t}$ takes on the value implied by the demand for its type of labor, (3.6). The family therefore faces the standard monopoly problem of selecting $W_{j,t}$ to optimize the welfare, (3.9), of its member households. It does so, subject to the requirement that it satisfy the demand for labor, (3.6), in each period. We follow EHL in supposing that the family experiences Calvo-style frictions in its choice of $W_{j,t}$. In particular, with probability $1-\xi_w$ the j^{th} family has the opportunity to reoptimize its wage rate. With the complementary probability, the family must set its wage rate according to the following rule:

$$W_{j,t} = \tilde{\pi}_{w,t} W_{j,t-1} \tag{3.10}$$

$$\tilde{\pi}_{w,t} = (\pi_{t-1})^{\kappa_w} (\bar{\pi})^{(1-\kappa_w)} \mu_{z^+}, \tag{3.11}$$

where $\kappa_w \in (0,1)$. Note that in a non-stochastic steady state, non-optimizing families raise their real wage at the rate of growth of the economy. Because optimizing families also do this in steady state, it follows that in the steady state, the wage of each type of family is the same.

In principle, the presence of wage setting frictions implies that families have idiosyncratic levels of wealth and, hence, consumption. However, we follow EHL in supposing that each family has access to perfect consumption insurance. At the level of the family, there is no private information about consumption or employment. The private information and associated incentive problems all exist among the households inside a family. Because of the additive separability of the family utility function, perfect consumption insurance at the level of families implies equal consumption across families. We have used this property of the equilibrium to simplify our notation and not include a subscript, j, on the j^{th} family's consumption. Of course, we hasten to add that although consumption is equated across families, it is not constant across households.

The j^{th} family's period t budget constraint is as follows:

$$P_t \left(C_t + \frac{1}{\Psi_t} I_t \right) + B_{t+1} \leq W_{t,j} h_{t,j} + X_t^k \bar{K}_t + R_{t-1} B_t + a_{jt}. \tag{3.12}$$

Here, B_{t+1} denotes the quantity of risk-free bonds purchased by the household, R_t denotes the gross nominal interest rate on bonds purchased in period $t-1$ which pay off in period t, and a_{jt} denotes the payments and receipts associated with the insurance on the timing of wage reoptimization. Also, P_t denotes the aggregate price level and I_t denotes the quantity of investment goods purchased for augmenting the beginning-of-period $t+1$ stock of physical capital, $\bar{K}_{t+1}$. The price of investment goods is P_t/Ψ_t, where Ψ_t is the unit root process with positive drift specified in (3.3). This is our way of capturing the trend decline in the relative price of investment goods.[27]

The family owns the economy's physical stock of capital, $\bar{K}_t$, sets the utilization rate of capital and rents the services of capital in a competitive market. The family accumulates capital using the following technology:

$$\bar{K}_{t+1} = (1-\delta)\,\bar{K}_t + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t. \tag{3.13}$$

Here, S is a convex function, with S and S' equal to zero on a steady state growth path. The function, S, is defined in section H in the technical appendix. The function has one free parameter, its second derivative in the neighborhood of steady state, which we denote simply by S''.

For each unit of $\bar{K}_{t+1}$ acquired in period t, the family receives X_{t+1}^k in net cash payments in period $t+1$,

$$X_{t+1}^k = u_{t+1}^k P_{t+1} r_{t+1}^k - \frac{P_{t+1}}{\Psi_{t+1}} a(u_{t+1}^k). \tag{3.14}$$

where u_t^k denotes the rate of utilization of capital. The first term in (3.14) is the gross nominal period $t+1$ rental income from a unit of $\bar{K}_{t+1}$. The family supply of capital services in period $t+1$ is:

$$K_{t+1} = u_{t+1}^k \bar{K}_{t+1}.$$

It is the services of capital that intermediate good producers rent and use in their production functions, (3.1). The second term to the right of the equality in (3.14) represents the cost of capital utilization, $a(u_{t+1}^k) P_{t+1}/\Psi_{t+1}$. See section H in the technical appendix for the functional form of the capital utilization cost function. This function is constructed so the

[27] We suppose that there is an underlying technology for converting final goods, Y_t, one-to-one into C_t and one to Ψ_t into investment goods. These technologies are operated by competitive firms which equate price to marginal cost. The marginal cost of C_t with this technology is P_t and the marginal cost of I_t is P_t/Ψ_t. We avoid a full description of this environment so as to not clutter the presentation, and simply impose these properties of equilibrium on the family budget constraint.

steady state value of utilization is unity, and $u(1) = u'(1) = 0$. The function has one free parameter, which we denote by σ_a. Here, $\sigma_a = a''(1)/a'$ and corresponds to the curvature of u in steady state.

The family's problem is to select sequences, $\left\{C_t, I_t, u_t^k, W_{j,t}, B_{t+1}, \bar{K}_{t+1}\right\}$, to maximize (3.9) subject to (3.6), (3.10), (3.11), (3.12), (3.13), (3.14) and the mechanism determining when wages can be reoptimized. The equilibrium conditions associated with this maximization problem are standard, and so appear in section E of the technical appendix.

3.4. Aggregate Resource Constraint, Monetary Policy and Equilibrium

Goods market clearing dictates that the homogeneous output good is allocated among alternative uses as follows:

$$Y_t = G_t + C_t + \tilde{I}_t. \tag{3.15}$$

Here, C_t denotes family consumption, G_t denotes exogenous government consumption and $\tilde{I}_t$ is a homogenous investment good which is defined as follows:

$$\tilde{I}_t = \frac{1}{\Psi_t}\left(I_t + a\left(u_t^k\right)\bar{K}_t\right). \tag{3.16}$$

As discussed above, the investment goods, I_t, are used by the families to add to the physical stock of capital, $\bar{K}_t$, according to (3.13). The remaining investment goods are used to cover maintenance costs, $a\left(u_t^k\right)\bar{K}_t$, arising from capital utilization, u_t^k. Finally, Ψ_t in (3.16) denotes the unit root investment specific technology shock with positive drift discussed after (3.1).

We suppose that monetary policy follows a Taylor rule of the following form:

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[r_\pi \log\left(\frac{\pi_{t+1}}{\pi}\right) + r_y \log\left(\frac{gdp_t}{gdp}\right)\right] + \frac{\varepsilon_{R,t}}{4R}, \tag{3.17}$$

where $\varepsilon_{R,t}$ is an iid monetary policy shock. As in CEE and ACEL, we assume that period t realizations of ε_R are not included in the period t information set of households and firms. Further, gdp_t denotes scaled real GDP defined as:

$$gdp_t = \frac{G_t + C_t + I_t/\Psi_t}{z_t^+}, \tag{3.18}$$

and gdp denotes the nonstochastic steady state value of gdp_t. We adopt the model of government spending suggested in Christiano and Eichenbaum (1992):

$$G_t = g z_t^+.$$

Lump-sum transfers are assumed to balance the government budget.

An equilibrium is a stochastic process for the prices and quantities having the property that the family and firm problems are satisfied, and goods and labor markets clear.

3.5. Aggregate Labor Force and Unemployment in Our Model

We now derive our model's implications for unemployment and the labor market. At the level of the j^{th} family, unemployment and the labor force are defined in the same way as in the previous section, except that the endogenous variables now have a j subscript (the parameters and shocks are the same across families). Thus, the j^{th} family's labor force, $m_{j,t}$, and total employment, $h_{j,t}$, are related by (2.12) (or, (3.7)). We linearize the latter expression as in (2.48):

$$\hat{m}_{j,t} = \frac{1-u}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}}\hat{h}_{j,t}, \tag{3.19}$$

though we ignore $\hat{\varsigma}_t$. Also, u and m denote the steady state values of unemployment and the labor force in the j^{th} family. Because we have made assumptions which guarantee that each family is identical in steady state, we drop the j subscripts from all steady state labor market variables (see the discussion after (3.10)).

Aggregate household hours and the labor force are defined as follows:

$$h_t \equiv \int_0^1 h_{j,t}dj,\ m_t \equiv \int_0^1 m_{j,t}dj.$$

Totally differentiating,

$$\hat{h}_t = \int_0^1 \hat{h}_{j,t}dj,\ \hat{m}_t \equiv \int_0^1 \hat{m}_{j,t}dj.$$

Using the fact that, to first order, type j wage deviations from the aggregate wage cancel, we obtain:

$$\hat{h}_t = \hat{H}_t. \tag{3.20}$$

See section G in the technical appendix for a derivation. That is, to a first order approximation, the percent deviation of aggregate household hours from steady state coincides with the percent deviation of aggregate homogeneous hours from steady state. Integrating (3.19) over all j :

$$\hat{m}_t = \int_0^1 \hat{m}_{j,t}dj = \frac{1-u}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}}\hat{H}_t.$$

Aggregate unemployment is defined as follows:

$$u_t \equiv \frac{m_t - h_t}{m_t},$$

so that

$$du_t = \frac{h}{m}\left(\hat{m}_t - \hat{h}_t\right).$$

Here, du_t denotes he deviation of unemployment from its steady state value, not the percent deviation.

3.6. The Standard Model

We derive the utility function used in the standard model as a special case of the family utility function in our involuntary unemployment model. In part, we do this to ensure consistency across models. In part, we do this as a way of emphasizing that we interpret the labor input in the utility function in the standard model as corresponding to the number of people working, not, say, the hours worked of a representative person. With our interpretation, the curvature of the labor disutility function corresponds to the (consumption compensated) elasticity with which people enter or leave the labor force in response to a change in the wage rate. In particular, this curvature does not correspond to the elasticity with which the typical person adjusts the quantity of hours worked in response to a wage change. Empirically, the latter elasticity is estimated to be small and it is fixed at zero in the model.

Another advantage of deriving the standard model from ours is that it puts us in position to exploit an insight by Gali (2010). In particular, Gali (2010) shows that the standard model already has a theory of unemployment implicit in it. The monopoly power assumed by EHL has the consequence that wages are on average higher than what they would be under competition. The number of workers for which the wage is greater than the cost of work exceeds the number of people employed. Gali suggests defining this excess of workers as 'unemployed'. The implied unemployment rate and labor force represent a natural benchmark to compare with our model.

Notably, deriving an unemployment rate and labor force in the standard model does not introduce any new parameters. Moreover, there is no change in the equilibrium conditions that determine non-labor market variables. Gali's insight in effect simply adds a block recursive system of two equations to the standard DSGE model which determine the size of the labor force and unemployment. Although the unemployment rate derived in this way does not satisfy all the criteria for unemployment that we described in the introduction, it nevertheless provides a natural benchmark for comparison with our model. An extensive comparison of the economics of our approach to unemployment versus the approach implicit in the standard model appears in section F in the technical appendix to this paper.

We suppose that the family has full information about its member households and that households which join the labor force automatically receive a job without having to expend any effort. As in the previous subsections, we suppose that corresponding to each type j of labor, there is a unit measure of households which gather together into a family. At the beginning of each period, each household draws a random variable, l, from a uniform distribution with support, $[0, 1]$. The random variable, l, determines a household's aversion to work according to (2.3), with $F = 0$. The fact that no effort is needed to find a job implies $m_{t,j} = h_{t,j}$. Households with $l \leq h_{t,j}$ work and households with $h_{t,j} \leq l \leq 1$ take leisure.

The type j family allocation problem is to maximize the utility of its member households with respect to consumption for non-working households, $c_{t,j}^{nw}$, and consumption of working households, $c_{t,j}^{w}$, subject to (2.18), and the given values of $h_{t,j}$ and C_t. In Lagrangian form, the problem is:

$$u\left(C_t - bC_{t-1}, h_{j,t}\right) = \max_{c_{t,j}^{w}, c_{t,j}^{nw}} \int_0^{h_{t,j}} \left[\log\left(c_{t,j}^{w} - bC_{t-1}\right) - \varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right] dl$$
$$+ \int_{h_{t,j}}^{1} \log\left(c_{t,j}^{nw} - bC_{t-1}\right) dl + \lambda_{j,t}\left[C_t - h_{t,j}c_{t,j}^{w} - \left(1-h_{t,j}\right) c_{t,j}^{nw}\right].$$

Here, $\lambda_{j,t} > 0$ denotes the multiplier on the resource constraint. The first order conditions imply $c_{t,j}^{w} = c_{t,j}^{nw} = C_t$. Imposing this result and evaluating the integral, we find:

$$u\left(C_t - bC_{t-1}, h_{j,t}\right) = \log\left(C_t - bC_{t-1}\right) - \varsigma h_{t,j}^{1+\sigma_L}. \tag{3.21}$$

The problem of the family is identical to what it is in section 3.3, with the sole exception that the utility function, (3.8), is replaced by (3.21).

A type j household that draws work aversion index l is defined to be unemployed if the following two conditions are satisfied:

$$(a)\ l > h_{j,t},\ \ (b)\ \upsilon_t W_{j,t} > \varsigma l^{\sigma_L}. \tag{3.22}$$

Here, υ_t denotes the multiplier on the budget constraint, (3.12), in the Lagrangian representation of the family optimization problem. Expression (a) in (3.22) simply says that to be unemployed, the household must not be employed. Expression (b) in (3.22) determines whether a non-employed household is unemployed or not in the labor force. The object on the left of the inequality in (b) is the value assigned by the family to the wage, $W_{j,t}$. The object on the right of (b) is the fixed cost of going to work for the l^{th} household. Gali (2010) suggests defining households with l satisfying (3.22) as unemployed. This approach to unemployment does not satisfy properties (i) and (iii) in the introduction. The approach does not meet the official definition of unemployment because no one is exercising effort to find a job. In addition, the existence of perfect consumption insurance implies that unemployed workers enjoy higher utility that employed workers.

We use (3.22) to define the labor force, l_t^*, in the standard model. With l_t^* and aggregate employment, h_t, we obtain unemployment as follows

$$u_t = \frac{l_t^* - h_t}{l_t^*},$$

or, after linearization about steady state:

$$du_t = \frac{h}{l^*}\left(\hat{l}_t^* - \hat{h}_t\right).$$

Here, $h < l^*$ because of the presence of monopoly power. The object, $\hat{h}_t$ may be obtained from (3.20) and the solution to the standard model. We now discuss the computation of the aggregate labor force, l_t^*. We have

$$l_t^* \equiv \int_0^1 l_{j,t}^* dj,$$

where $l_{j,t}^*$ is the labor force associated with the j^{th} type of labor and is defined by enforcing (b) in (3.22) at equality. After linearization,

$$\hat{l}_t^* \equiv \int_0^1 \hat{l}_{j,t}^* dj.$$

We compute $\hat{l}_{j,t}^*$ by linearizing the equation that defines $l_{j,t}^*$. After scaling that equation, we obtain

$$\psi_{z^+,t}\bar{w}_t\mathring{w}_{j,t} = \varsigma\left(l_{j,t}^*\right)^{\sigma_L}, \tag{3.23}$$

where

$$\psi_{z^+,t} \equiv \upsilon_t P_t z_t^+,\ \bar{w}_t \equiv \frac{W_t}{z_t^+ P_t},\ \mathring{w}_{j,t} \equiv \frac{W_{j,t}}{W_t}.$$

Linearizing (3.23) about steady state and integrating the result over all $j \in (0,1)$:

$$\hat{\psi}_{z^+,t} + \widehat{\bar{w}}_t + \int_0^1 \widehat{\mathring{w}}_{j,t} dj = \sigma_L \hat{l}_t^*.$$

From the result in section G in the technical appendix, the integral in the above expression is zero, so that:

$$\hat{l}_t^* = \frac{\hat{\psi}_{z^+,t} + \widehat{\bar{w}}_t}{\sigma_L}.$$

4. Estimation Strategy

We estimate the parameters of the model in the previous section using the impulse response matching approach applied by Rotemberg and Woodford (1997), CEE, ACEL and other papers. We apply the Bayesian version of that method proposed in CTW. To promote comparability of results across the two papers and to simplify the discussion here, we use the impulse response functions and associated probability intervals estimated using the 14 variable, 2 lag vector autoregression (VAR) estimated in CTW. Here, we consider the response of 11 variables to three shocks: the monetary policy shock, $\varepsilon_{R,t}$ in equation (3.17), the neutral technology shock, ε_t in equation (3.2), and the investment specific shock, ε_t^{Ψ} in equation (3.3).[28] Nine of the eleven variables whose responses we consider are the standard macroeconomic variables displayed in Figures 1-3. The other two variables are the unemployment

[28]The VAR in CTW also includes data on vacancies, job findings and job separations, but these variables do not appear in the models in this paper and so we do not include their impulse responses in the analysis.

rate and the labor force which shown in Figure 4. The VAR is estimated using quarterly, seasonally adjusted data covering the period 1952Q1 to 2008Q4.

The assumptions that allow us to identify the effects of our three shocks are the ones implemented in ACEL. To identify the monetary policy shock we suppose all variables aside from the nominal rate of interest are unaffected contemporaneously by the policy shock. We make two assumptions to identify the dynamic response to the technology shocks: (i) the only shocks that affect labor productivity in the long run are the two technology shocks and (ii) the only shock that affects the price of investment relative to consumption is the innovation to the investment specific shock. All these identification assumptions are satisfied in our model. Details of our strategy for computing impulse response functions imposing the shock identification are discussed in ACEL.

Let $\hat{\psi}$ denote the vector of impulse responses used in the analysis here. Since we consider 15 lags in the impulses, there are in principle 3 (i.e., the number of shocks) times 11 (number of variables) times 15 (number of lags) = 495 elements in $\hat{\psi}$. However, we do not include in $\hat{\psi}$ the 10 contemporaneous responses to the monetary policy shock that are required to be zero by our monetary policy identifying assumption. Taking the latter into account, the vector $\hat{\psi}$ has 485 elements. To conduct a Bayesian analysis, we require a likelihood function for our 'data', $\hat{\psi}$. For this, we use an approximation based on asymptotic sampling theory. In particular, when the number of observations, T, is large, we have

$$\sqrt{T}\left(\hat{\psi} - \psi\left(\theta_0\right)\right) \overset{a}{\sim} N\left(0, W\left(\theta_0, \zeta_0\right)\right). \tag{4.1}$$

Here, θ_0 and ζ_0 are the parameters of the model that generated the data, evaluated at their true values. The parameter vector, θ_0, is the set of parameters that is explicit in our model, while ζ_0 contains the parameters of stochastic processes not included in the analysis. In (4.1), $W\left(\theta_0, \zeta_0\right)$ is the asymptotic sampling variance of $\hat{\psi}$, which - as indicated by the notation - is a function of all model parameters. We find it convenient to express (4.1) in the following form:

$$\hat{\psi} \overset{a}{\sim} N\left(\psi\left(\theta_0\right), V\left(\theta_0, \zeta_0, T\right)\right), \tag{4.2}$$

where

$$V\left(\theta_0, \zeta_0, T\right) \equiv \frac{W\left(\theta_0, \zeta_0\right)}{T}.$$

We treat $V\left(\theta_0, \zeta_0, T\right)$ as though it were known. In practice, we work with a consistent estimator of $V\left(\theta_0, \zeta_0, T\right)$ in our analysis (for details, see CTW). That estimator is a diagonal matrix with only the variances along the diagonal. An advantage of this diagonality property is that our estimator has a simple graphical representation.

We treat the following object as the likelihood of the 'data', $\hat{\psi}$, conditional on the model parameters, θ :

$$f\left(\hat{\psi}|\theta, V(\theta_0, \zeta_0, T)\right) = \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} |V(\theta_0, \zeta_0, T)|^{-\frac{1}{2}} \times \exp\left[-\frac{1}{2}\left(\hat{\psi} - \psi(\theta)\right)' V(\theta_0, \zeta_0, T)^{-1}\left(\hat{\psi} - \psi(\theta)\right)\right]. \quad (4.3)$$

The Bayesian posterior of θ conditional on $\hat{\psi}$ and $V(\theta_0, \zeta_0, T)$ is:

$$f\left(\theta|\hat{\psi}, V(\theta_0, \zeta_0, T)\right) = \frac{f\left(\hat{\psi}|\theta, V(\theta_0, \zeta_0, T)\right) p(\theta)}{f\left(\hat{\psi}|V(\theta_0, \zeta_0, T)\right)}, \quad (4.4)$$

where $p(\theta)$ denotes the priors on θ and $f\left(\hat{\psi}|V(\theta_0, \zeta_0, T)\right)$ denotes the marginal density of $\hat{\psi}$:

$$f\left(\hat{\psi}|V(\theta_0, \zeta_0, T)\right) = \int f\left(\hat{\psi}|\theta, V(\theta_0, \zeta_0, T)\right) p(\theta)\, d\theta.$$

As usual, the mode of the posterior distribution of θ can be computed by simply maximizing the value of the numerator in (4.4), since the denominator is not a function of θ. The marginal density of $\hat{\psi}$ is required when we want an overall measure of the fit of our model and when we want to report the shape of the posterior marginal distribution of individual elements in θ. We do this using the MCMC algorithm.

5. Estimation Results for Medium-sized Model

The first section discusses model parameter values. We then show that our model of involuntary unemployment does well at accounting for the dynamics of unemployment and the labor force. Fortunately, the model is able to do this without compromising its ability to ability to account for the dynamics of standard macroeconomic variables.

5.1. Parameters

Parameters whose values are set a priori are listed in Table 1. We found that when we estimated the parameters, κ_w and λ_w, the estimator drove them to their boundaries. This is why we simply set λ_w to a value near unity and we set $\kappa_w = 1$. The steady state value of inflation (a parameter in the monetary policy rule and the price and wage updating equations), the steady state government consumption to output ratio, and the growth rate of the investment-specific technology were chosen to coincide with their corresponding sample means in our data set.[29] The growth rate of neutral technology was chosen so that, conditional on the growth rate of investment-specific technology, the steady state growth rate of

[29] In our model, the relative price of investment goods represents a direct observation of the technology shock for producing investment goods.

output in the model coincides with the corresponding sample average in the data. We set $\xi_w = 0.75$, so that the model implies wages are reoptimized once a year on average. We did not estimate this parameter because we found that it is difficult to separately identify the value of ξ_w and the curvature of family labor disutility. Finally, to ensure that we only consider parameterizations that imply an admissible probability function, $p(e)$, we simply fix the maximal value of this probability in steady state, $\bar{p}$, to 0.97 (see (2.14).)

The parameters for which we report priors and posteriors are listed in Table 2. We report results for two estimation exercises. In the first exercise we estimate the standard DSGE model discussed in section 3.6. In this exercise we only use the impulse responses of standard macroeconomic variables in the likelihood criterion, (4.3). In particular, we do not include the impulse responses of the unemployment rate or the labor force when we estimate the standard DSGE model. Results based on this exercise appear under the heading, 'standard model'. In the second exercise we estimate our model with involuntary unemployment and we report those results under the heading, 'involuntary unemployment model'.

We make several observations about the parameters listed in Table 2. First, the results in the last two columns are similar. This reflects that the two models (i) are observationally equivalent relative to the impulse responses of standard macroeconomic variables and (ii) no substantial adjustments to the parameters are required for the involuntary unemployment model to fit the unemployment and labor force data.

Second, the list of household parameters contains one endogenous parameter, the curvature of utility, σ_z, defined in (2.39). Moreover, the list seems to be missing the structural parameters of the search technology and disutility of labor. We begin by explaining this in the context of the involuntary unemployment model. Throughout the estimation, we fix the steady state unemployment rate, u, at its sample average, 0.056, and the steady state labor force participation rate, m, at a value of 2/3. For given values of the four objects, σ_z, $\bar{p}$, u and m, we can uniquely compute values for:[30]

$$F, \varsigma, a, \eta. \tag{5.1}$$

This is why σ_z is included in the list of estimated parameters in Table 2, while the four parameters in (5.1) are not. The parameter, σ_L, appears in Table 2 because it is distinct from σ_z and separately identifiable. We apply an analogous treatment to household parameter values in the case of the standard model. In particular, throughout estimation we fix the steady state level of hours worked, h, to the value implicit in the u and m used for the involuntary unemployment model. We choose the value of ς so that conditional on the other standard model parameter values, steady state hours worked coincides with h. Since $\sigma_z = \sigma_L$ in the standard model (see (3.21)), we only report estimation results for σ_z in Table 2.

[30]For details, see section E.4.2 in the technical appendix to this paper.

Turning to the parameter values themselves, note first that the degree of price stickiness, ξ_p, is modest. The implied time between price reoptimizations is a little less than 3 quarters. The amount of information in the likelihood, (4.3), about the value of ξ_p is reasonably large. The posterior standard deviation is roughly an order of magnitude smaller than the prior standard deviation and the posterior probability interval is half the length of the prior probability interval. Generally, the amount of information in the likelihood about all the parameters is large in this sense. An exception to this pattern is the coefficient on inflation in the Taylor rule, r_π. There appears to be relatively little information about this parameter in the likelihood. Note that σ_z is estimated to be quite small, implying a consumption-compensated labor supply elasticity for the family of around 8. Such a high elasticity would be regarded as empirically implausible if it were interpreted as the elasticity of supply of hours by a representative agent. However, as discussed above, this is not our interpretation.

Table 3 reports steady state properties of the two models, evaluated at the posterior mean of the parameters. According to the results, the capital output ratio is a little lower than the empirical value of 12 typically reported in the real business cycle literature. The replacement ratio, c^{nw}/c^w, is a novel feature of our model, that does not appear in standard monetary DSGE models. The replacement ratio is estimated to be roughly 80 percent. This is a somewhat lower replacement ratio than the 90 percent number reported in the introduction. It is higher than the number reported for developed countries in OECD (2006). However, those replacement ratios pertain to income, rather than consumption.[31] So, they are likely to underestimate the consumption concept relevant for us.

Not surprisingly, our model's implications for the consumption replacement ratio is very sensitive to the habit persistence parameter, b. If we set the value of that parameter to zero, then our model's steady state replacement ratio drops to 20 percent.

5.2. Impulse Response Functions of Non-labor Market Variables

Figures 1-3 display the results of the indicated macroeconomic variables to our three shocks. In each case, the solid black line is the point estimate of the dynamic response generated by our estimated VAR. The grey area is an estimate of the corresponding 95% probability interval.[32] Our estimation strategy selects a model parameterization that places the model-implied impulse response functions as close as possible to the center of the grey area, while

[31] The income replacement ratio for the US is reported to be 54 percent in Table 3.2, which can be found at http://www.oecd.org/dataoecd/28/9/36965805.pdf.

[32] We compute the probability interval as follows. We simulate 2,500 sets of impulse response functions by generating an equal number of artificial data sets, each of length T, using the VAR estimated from the data. Here, T denotes the number of observations in our actual data set. We compute the standard deviations of the artificial impulse response functions. The grey areas in Figures 1-5 are the estimated impulse response functions plus and minus 1.96 times the corresponding standard deviation.

not suffering too much of a penalty from the priors. The estimation criterion is less concerned about reproducing VAR-based impulse response functions where the grey areas are the widest.

The thick solid line and the line with solid squares in the figures display the impulse responses of the standard model and the involuntary unemployment models, respectively, at the posterior mode of the parameters. Note in Figures 1-3 that in many cases only one of these two lines is visible. Moreover, in cases where a distinction between the two lines can be discerned, they are nevertheless very close. This reflects that the two models account roughly equally well for the impulse responses to the three shocks. This is a key result. Expanding the standard model to include unemployment and the labor force does not produce a deterioration in the model's ability to account for the estimated dynamic responses of standard macroeconomic variables to monetary policy and technology shocks.

Consider Figure 1, which displays the response of standard macroeconomic variables to a monetary policy shock. Note how the model captures the slow response of inflation. Indeed, the model even captures the 'price puzzle' phenomenon, according to which inflation moves in the 'wrong' direction initially. This apparently perverse initial response of inflation is interpreted by the model as reflecting the reduction in labor costs associated with the cut in the nominal rate of interest.[33] It is interesting that the slow response of inflation is accounted for with a fairly modest degree of wage and price-setting frictions. The model captures the response of output and consumption to a monetary policy shock reasonably well. However, there is a substantial miss on capacity utilization. Also, the model apparently does not have the flexibility to capture the relatively sharp rise and fall in the investment response, although the model responses lie inside the grey area. The relatively large estimate of the curvature in the investment adjustment cost function, S'', reflects that to allow a greater response of investment to a monetary policy shock would cause the model's prediction of investment to lie outside the grey area in the initial and later quarters. These findings for monetary policy shocks are broadly similar to those reported in CEE, ACEL and CTW.

Figure 2 displays the response of standard macroeconomic variables to a neutral technology shock. Note that the models do reasonably well at reproducing the empirically estimated responses. The dynamic response of inflation is particularly notable. The estimation results in ACEL suggest that the sharp and precisely estimated drop in inflation in response to a neutral technology shock is difficult to reproduce in a model like the standard monetary DSGE model. In describing this problem for their model, ACEL express a concern that the failure reflects a deeper problem with sticky price models. Perhaps the emphasis on price and wage setting frictions, largely motivated by the inertial response of inflation to a

[33]For a defense, based on firm-level data, of the existence of this 'working capital' channel of monetary policy, see Barth and Ramey (2001).

monetary shock, is shown to be misguided by the evidence that inflation responds rapidly to technology shocks.[34] Our results suggest a far more mundane possibility. There are two differences between our model and the one in ACEL which allow it to reproduce the response of inflation to a technology shock more or less exactly without hampering its ability to account for the slow response of inflation to a monetary policy shock. First, as discussed above (see (3.4)), in our model there is no indexation of prices to lagged inflation. ACEL follows CEE in supposing that when firms cannot optimize their price, they index it fully to lagged aggregate inflation. The position of our model on price indexation is a key reason why we can account for the rapid fall in inflation after a neutral technology shock while ACEL cannot. We suspect that our way of treating indexation is a step in the right direction from the point of view of the microeconomic data. Micro observations suggest that individual prices do not change for extended periods of time. A second distinction between our model and the one in ACEL is that we specify the neutral technology shock to be a random walk (see (3.2)), while in ACEL the growth rate of the estimated technology shock is highly autocorrelated. In ACEL, a technology shock triggers a strong wealth effect which stimulates a surge in demand that places upward pressure on marginal cost and thus inflation.[35]

Figure 3 displays dynamic responses of macroeconomic variables to an investment-specific shock. The evidence indicates that the two models, parameterized at their posterior means, do well in accounting for these responses.

5.3. Impulse Response Functions of Unemployment and the Labor Force

Figure 4 displays the response of unemployment and the labor force to our three shocks. The key thing to note is that the model has no difficulty accounting for the pattern of responses. The probability bands are large, but the point estimates suggest that unemployment falls about 0.2 percentage points and the labor force rises a small amount after an expansionary monetary policy shock. The model roughly reproduces this pattern. In the case of each response, the model generates opposing movements in the labor force and the unemployment rate. This appears to be consistent with the evidence.

As discussed in section 3.6 above, Gali (2010) points out that the standard model has implicit in it a theory of unemployment and the labor force. Figure 5 adds the implications

[34] The concern is reinforced by the fact that an alternative approach, one based on information imperfections and minimal price/wage setting frictions, seems like a natural one for explaining the puzzle of the slow response of inflation to monetary policy shocks and the quick response to technology shocks (see Maćkowiak and Wiederholt, 2009, Mendes, 2009, and Paciello, 2009). Dupor, Han and Tsai (2009) suggest more modest changes in the model structure to accommodate the inflation puzzle.

[35] An additional, important, factor accounting for the damped response of inflation to a monetary policy shock (indeed, the perverse initial 'price puzzle' phenomenon) is the assumption that firms must borrow in advance to pay for their variable production costs. But, this model feature is present in both our model and ACEL as well as CEE.

of the standard model for these variables to the impulses displayed in Figure 4. Note that the impulses implied by the standard model are so large that they distort the scale in Figure 5. Consider, for example, the first panel of graphs in the figure, which pertain to the monetary policy shock. The standard model predicts a massive fall in the labor force after an expansionary monetary policy shock. The reason is that the rise in aggregate consumption (see Figure 1) reduces the value of work by reducing υ_t in (3.22). The resulting sharp drop in labor supply strongly contradicts our VAR-based evidence which suggests a small rise. Given the standard model's prediction for the labor force, it is not surprising that the model massively over-predicts the fall in the unemployment rate after a monetary expansion.

The failure of the standard model raises a puzzle. Why does our involuntary unemployment model do so well at accounting for the unemployment rate and the labor force? The puzzle is interesting because the two models share essentially the same utility function at the level of the household. One might imagine that our model would have the same problem with wealth effects. In fact, it does not have the same problem because there is a connection in our model between the labor force and employment that does not exist in the standard model. In our model, the increased incentive to work hard that occurs in response to an expansionary monetary policy shock simultaneously encourages households to search for work more intensely, and to substitute into the labor force.

The standard model's prediction for the response of the unemployment rate and the labor force to neutral and investment-specific technology shocks is also strongly counterfactual. The problem is always the same, and reflects the operation of wealth effects on labor supply.

The problems in Figure 5 with the standard model motivate Gali (2010), Gali, Smets and Wouters (2010) and CTW to modify the household utility function in the standard model in ways that reduce wealth effects on labor. In effect, our involuntary unemployment model represents an alternative strategy for dealing with these wealth effects. Our model has the added advantage of being consistent with all three characteristics (i)-(iii) of unemployment described in the introduction.

6. Concluding Remarks

We constructed a model in which households must make an effort to find work. Because effort is privately observed, perfect insurance against labor market outcomes is not feasible. To ensure that people have an incentive to find work, workers that find jobs must be better off than people who do not work. With additively separable utility, this translates into the proposition that employed workers have higher consumption than the non-employed. We integrate our model of unemployment into a standard monetary DSGE model and find that the model's ability to account for standard macroeconomic variables is not diminished. At the same time, the new model appears to account well for the dynamics of variables like unemployment and the labor force.

We leave it to future research to quantify the various ways in which the new model may contribute to policy analysis. In part, we hope that the model is useful simply because labor market data are of interest in their own right. But, we expect the model to be useful even when labor market data are not the central variables of concern. An important input into policy analysis is the estimation of 'latent variables' such as the output gap and the efficient, or 'natural', rate of interest. Other important inputs into policy analysis are forecasts of inflation and output. By allowing one to systematically integrate labor market information into the usual macroeconomic dataset, our model can be expected to provide more precise forecasts, as well as better estimates of latent variables.[36]

Our model of unemployment has several interesting microeconomic implications that deserve closer attention. The model implies that the consumption premium of employed workers over the non-employed, c_t^w/c_t^{nw}, is procyclical. Although Chetty and Looney (2006) and Gruber (1997) report that there is a premium on average, we cannot infer anything about the cyclicality of the premium from the evidence they present. Studies of the cross section variance of log household consumption are a potential source of evidence on the cyclical behavior of the premium. To see this, let V_t denote the variance of log household consumption in the period t cross section in our model:[37]

$$V_t = (1 - h_t)\, h_t \left(\log \left(\frac{c_t^w}{c_t^{nw}} \right) \right)^2 .$$

According to this expression, the model posits two countervailing forces on the cross-sectional dispersion of consumption, V_t, in a recession. First, for a given distribution of the population across employed and non-employed households (i.e., holding h_t fixed), a decrease in the consumption premium leads to a decrease in consumption dispersion in a recession. Second, holding the consumption premium fixed, consumption dispersion increases as people move

[36] For an elaboration on this point, see Basistha and Startz (2004).

[37] Strictly speaking, this formula is correct only for the model in the second section of this paper. The relevant formula is more complicated for the model with capital because that requires a non-trivial aggregation across households that supply different types of labor services. To see how we derived the formula in the text, note that the cross sectional mean of log household consumption is:

$$E_t = h_t \log\left(c_t^w\right) + (1 - h_t) \log\left(c_t^u\right),$$

so that

$$\begin{aligned} V_t &= h_t \left(\log c_t^w - E_t\right)^2 + (1 - h_t)\left(\log c_t^u - E_t\right)^2 \\ &= h_t (1 - h_t) \left(\log c_t^w - \log c_t^u\right)^2 . \end{aligned}$$

from employment to non-employment with the fall in h_t.[38] These observations suggest that (i) if V_t is observed to drop in recessions, this is evidence in favor of the model's prediction that the consumption premium is procyclical and (ii) if V_t is observed to stay constant or rise in recessions then we cannot conclude anything about the cyclicality of the consumption premium. Evidence in Heathcote, Perri and Violante (2010) suggests that the US was in case (i) in three of the previous five recessions.[39] In particular, they show that the dispersion in log household non-durable consumption decreased in the 1980, 2001 and 2007 recessions.[40] We conclude tentatively that the observed cross sectional dispersion of consumption across households lends support to our model's implication that the consumption premium is procyclical.

Another interesting implication of the model is its prediction that high unemployment in recessions reflects the procyclicality of effort in job search. There is some evidence that supports this implication of the model. The Bureau of Labor Statistics (2009) constructs a measure of the number of 'discouraged workers'. These are people who are available to work and have looked for work in the past 12 months, but are not currently looking because they believe no jobs are available. This statistic has only been gathered since 1994, and so it covers just two recessions. However, in both the recessions for which we have data, the number of discouraged workers increased substantially. For example, the number of discouraged workers jumped 70 percent from 2008Q1 to 2009Q1. In fact, the number of discouraged workers is only a tiny fraction of the labor force. However, to the extent that the sentiments of discouraged workers are shared by workers more generally, a jump in the number of discouraged workers could be a signal of a general decline in job search intensity in recessions. But, this is an issue that demands a more careful investigation.

[38] This statement assumes that the empirically relevant case, $h_t > 1/2$.

[39] Of course, we cannot rule out that the drop in V_t in recessions has nothing to do with the mechanism in our model but rather reflects some other source of heterogeneity in the data.

[40] A similar observation was made about the 2007 recession in Parker and Vissing-Jorgensen (2009).

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Table 1: Non-Estimated Parameters in Medium-sized Model

Parameter	Value	Description
α	0.25	Capital share
δ	0.025	Depreciation rate
β	0.999	Discount factor
π	1.0083	Gross inflation rate
η_g	0.2	Government consumption to GDP ratio
κ_w	1	Wage indexation to π_{t-1}
λ_w	1.01	Wage markup
ξ_w	0.75	Wage stickiness
$\overline{p}$	0.97	Max, $p(e)$
μ_n	1.0041	Gross neutral tech. growth
μ_ψ	1.0018	Gross invest. tech. growth

Table 3: Medium-sized Model Steady State at Posterior Mean for Parameters

Variable	Standard Model	Involuntary Unemp. Model	Description
$p_{k'}k/y$	7.73	7.73	Capital to GDP ratio (quarterly)
c/y	0.56	0.56	Consumption to GDP ratio
i/y	0.24	0.24	Investment to GDP ratio
$H = h$	0.63	0.63	Steady state labor input
c^{nw}/c^{w}	1.0	0.81	Replacement ratio
R	1.014	1.014	Gross nom. int. rate (quarterly)
R^{real}	1.006	1.006	Gross real int. rate (quarterly)
r^k	0.033	0.033	Capital rental rate (quarterly)
u	0.077	0.056	Unemployment rate
m	-	0.67	Labor force (involuntary unemployment model)
l^*	0.68	-	Labor force (standard model)
ς	1.98	1.95	Slope, labor disutility
F	-	0.75	Intercept, labor disutility
a	-	0.52	Slope, $p(e)$
η	-	0.75	Intercept, $p(e)$

Table 2: Priors and Posteriors of Parameters for Medium-sized Model

Parameter		Prior		Posterior[a]	
		Distribution [bounds]	Mean, Std.Dev. [5% and 95%]	Mean, Std.Dev. [5% and 95%] **Standard Model**	**Involuntary Unemp. Model**
Price setting parameters					
Price Stickiness	ξ_p	Beta	0.50, 0.15	0.63, 0.04	0.64, 0.04
		[0, 0.8]	[0.23, 0.72]	[0.57, 0.70]	[0.58, 0.70]
Price Markup	λ_f	Gamma	1.20, 0.15	1.15, 0.07	1.36, 0.09
		[1.01, ∞]	[1.04, 1.50]	[1.03, 1.26]	[1.21, 1.50]
Monetary authority parameters					
Taylor Rule: Int. Smoothing	ρ_R	Beta	0.80, 0.10	0.87, 0.02	0.89, 0.01
		[0, 1]	[0.62, 0.94]	[0.85, 0.90]	[0.87, 0.91]
Taylor Rule: Inflation Coef.	r_π	Gamma	1.60, 0.15	1.49, 0.11	1.47, 0.11
		[1.01, 4]	[1.38, 1.87]	[1.30, 1.66]	[1.30, 1.65]
Taylor Rule: GDP Coef.	r_y	Gamma	0.20, 0.10	0.06, 0.03	0.06, 0.02
		[0, 2]	[0.07, 0.39]	[0.02, 0.10]	[0.02, 0.09]
Household parameters					
Consumption Habit	b	Beta	0.75, 0.15	0.76, 0.02	0.79, 0.02
		[0, 1]	[0.47, 0.95]	[0.73, 0.79]	[0.76, 0.81]
Power, labor disutility[b]	σ_L	Uniform	10.0, 5.77	–	7.40, 0.47
		[0, 20]	[1.00, 19.0]	–	[6.61, 8.14]
Inverse labor supply elast.[b]	σ_z	Gamma	0.30, 0.20	0.13, 0.03	0.13, 0.02
		[0, ∞]	[0.06, 0.69]	[0.08, 0.17]	[0.09, 0.17]
Capacity Adj. Costs Curv.	σ_a	Gamma	1.00, 0.75	0.34, 0.09	0.30, 0.09
		[0, ∞]	[0.15, 2.46]	[0.18, 0.48]	[0.15, 0.44]
Inv. Adj. Costs Curv.	S''	Gamma	12.00, 8.00	15.63, 3.28	20.26, 4.06
		[0, ∞]	[2.45, 27.43]	[10.5, 20.7]	[14.0, 26.6]
Shocks					
Autocorr. Invest. Tech.	ρ_ψ	Uniform	0.50, 0.29	0.60, 0.08	0.59, 0.08
		[0, 1]	[0.05, 0.95]	[0.48, 0.72]	[0.47, 0.71]
Std.Dev. Neutral Tech. Shock	σ_n	Inv. Gamma	0.20, 0.10	0.22, 0.02	0.22, 0.02
		[0, ∞]	[0.10, 0.37]	[0.19, 0.25]	[0.19, 0.24]
Std.Dev. Invest. Tech. Shock	σ_ψ	Inv. Gamma	0.20, 0.10	0.16, 0.02	0.16, 0.02
		[0, ∞]	[0.10, 0.37]	[0.12, 0.19]	[0.12, 0.20]
Std.Dev. Monetary Shock	σ_R	Inv. Gamma	0.40, 0.20	0.45, 0.03	0.43, 0.03
		[0, ∞]	[0.21, 0.74]	[0.39, 0.50]	[0.38, 0.48]

[a] Based on standard random walk metropolis algorithm. 150 000 draws, 30 000 for burn-in, acceptance rate 26%.

[b] In the case of the baseline model, σ_z and σ_L coincide. In the case of the involuntary unemployment model these two parameters are different.

Figure 1: Dynamic Responses of Non−Labor Market Variables to a Monetary Policy Shock

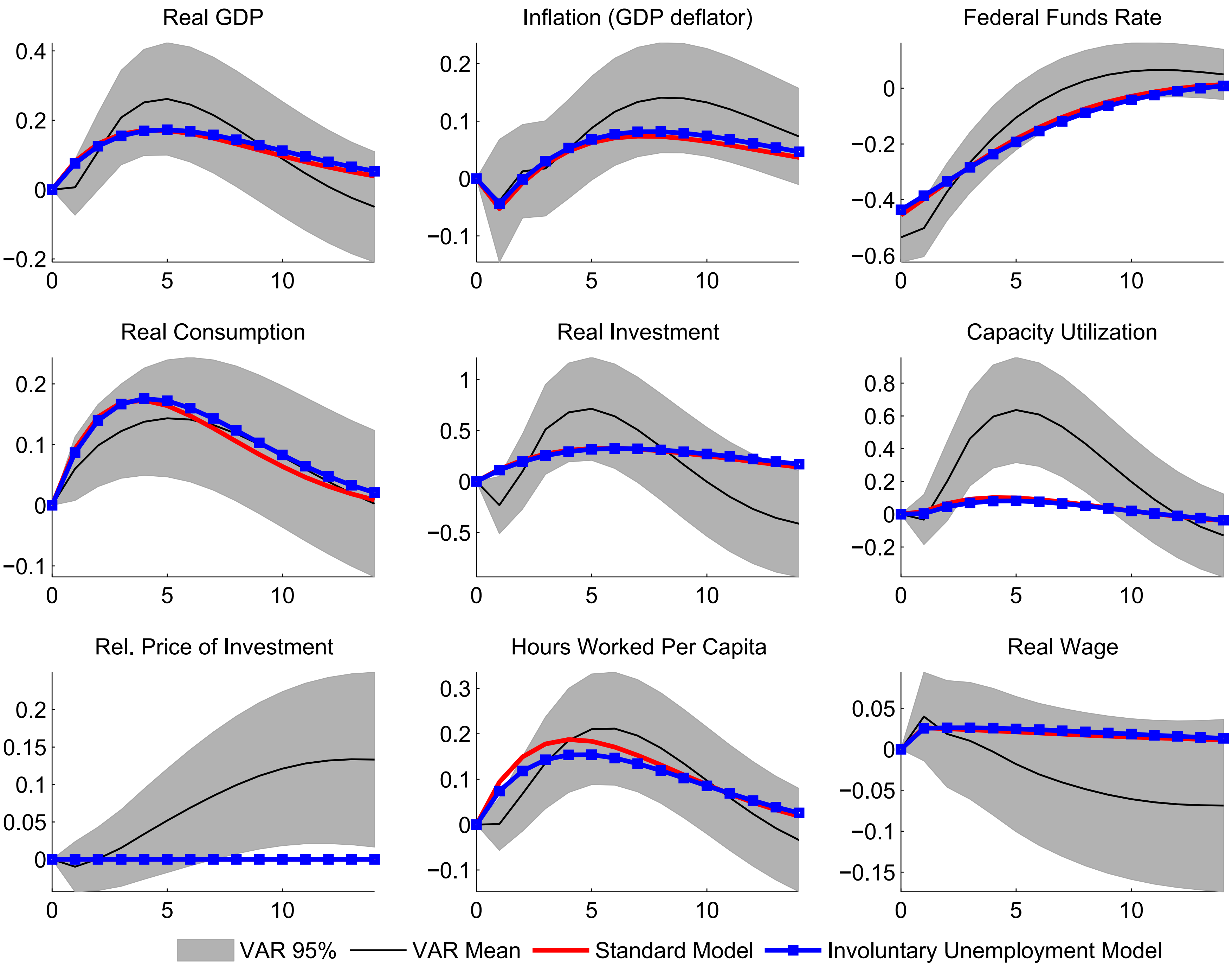


Figure 2: Dynamic Responses of Non−Labor Market Variables to a Neutral Technology Shock

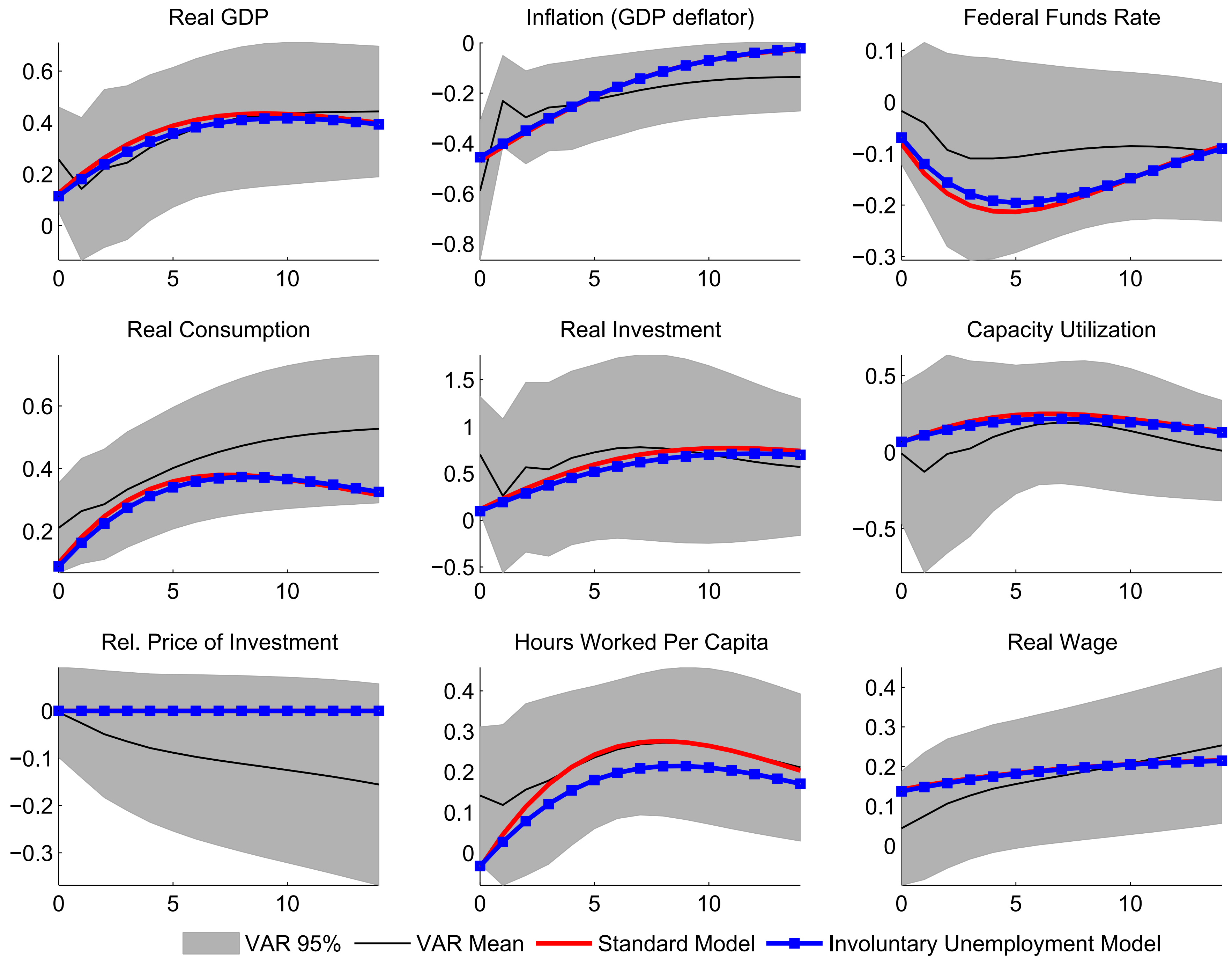


Figure 3: Dynamic Responses of Non−Labor Market Variables to an Investment Specific Technology Shock

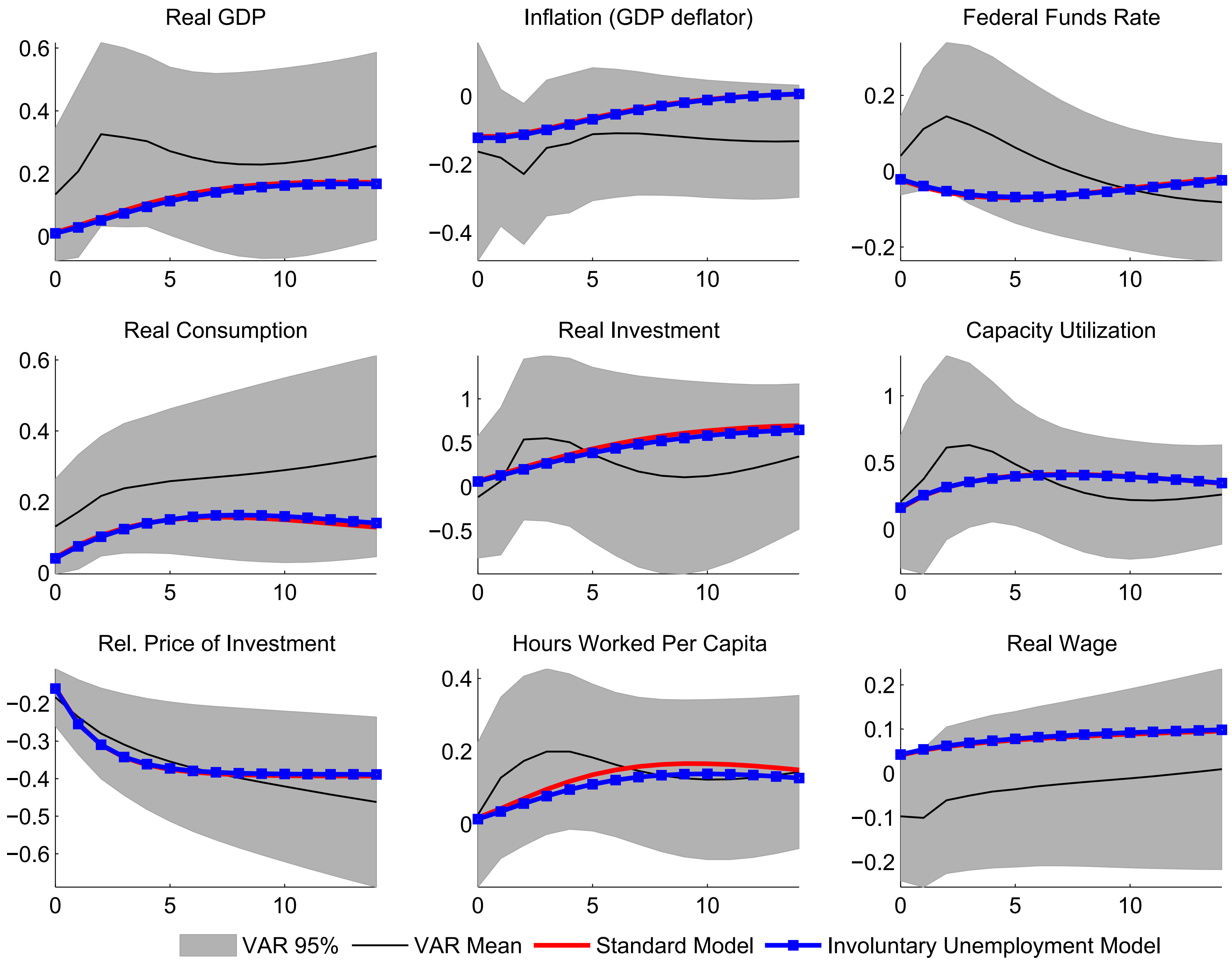


Figure 4: Dynamic Responses of Labor Market Variables to Three Shocks

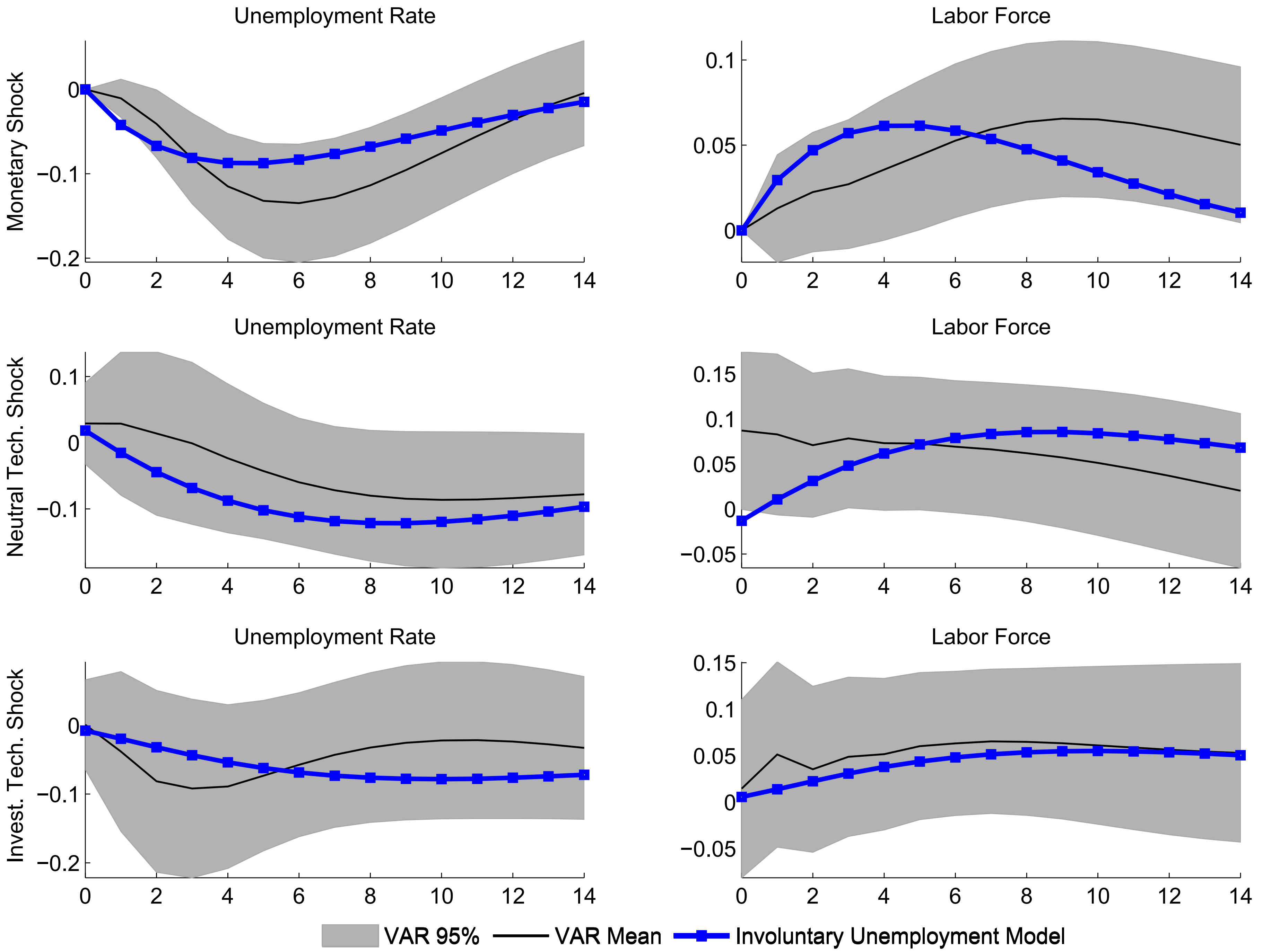
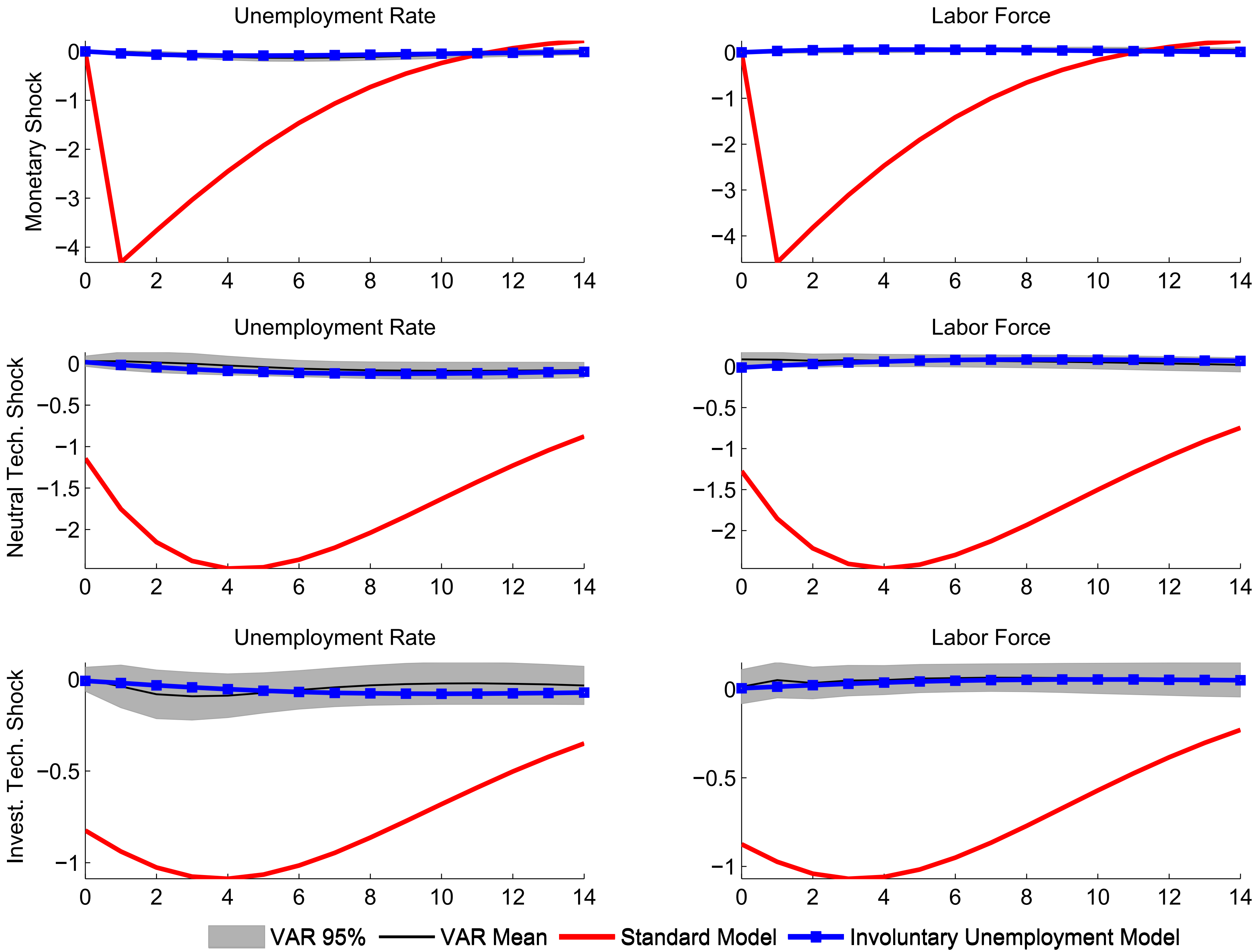


Figure 5: Dynamic Responses of Labor Market Variables to Three Shocks



Technical Appendix for "Involuntary Unemployment and the Business Cycle"

by

Lawrence J. Christiano, Mathias Trabandt, Karl Walentin

Apart from sections B and D below, the derivations in this appendix are standard and can be found in the technical appendices in ACEL and CEE, for example. We display them here so that the technical results are all available in one place and in a consistent notation. The material in sections B and D involve straightforward (but, sometimes tedious) extensions of the results in the manuscript. Equation numbers refer to equation numbers in the text of the paper.

A. Equilibrium Conditions in the Model Without Capital

We first derive the equilibrium conditions associated with price setting. We then derive the other conditions.

A.1. Price Setting Equilibrium Conditions

The discounted profits of the

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} \overbrace{\left[\overbrace{P_{i,t+j} Y_{i,t+j}}^{\text{revenues}} - \overbrace{P_{t+j} s_{t+j} Y_{i,t+j}}^{\text{total cost}} \right]}^{\text{period } t+j \text{ profits sent to household}},$$

where υ_{t+j} denotes the period $t+j$ Lagrange multiplier on household budget constraint. Let $\tilde{P}_t$ denote the price selected by each of the $1-\xi_p$ firms that have opportunity to reoptimize price in period t. Because firms have no state variables, they are only concerned about future histories in which they cannot reoptimize price. This leads to the following objective function for a firm that can reoptimize price in period t :

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \upsilon_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right].$$

Substitute out for intermediate good firm output using the demand curve:

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[\tilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon} \right],$$

where

$$\varepsilon \equiv \frac{\lambda_f}{\lambda_f - 1}$$

Differentiate with respect to $\tilde{P}_t$:

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[(1-\varepsilon) \left(\tilde{P}_t\right)^{-\varepsilon} + \varepsilon P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon-1} \right] = 0,$$

or,

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \lambda_f s_{t+j} \right] = 0.$$

Note that when $\xi_p = 0$, one obtains the standard result, that price is fixed markup over marginal cost.

Now, substitute out the multiplier:

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \overbrace{\frac{u'\left(C_{t+j}\right)}{P_{t+j}}}^{\text{marginal utility of currency } = \upsilon_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \lambda_f s_{t+j} \right] = 0,$$

or, given our assumption about log utility,

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} P_{t+j}^{\varepsilon} \left[\frac{\tilde{P}_t}{P_{t+j}} - \lambda_f s_{t+j} \right] = 0.$$

or,

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \lambda_f s_{t+j}\right] = 0,$$

where

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t}, X_{t,j} = \begin{cases} \frac{1}{\pi_{t+j}\pi_{t+j-1}\cdots\pi_{t+1}}, & j \geq 1 \\ 1, \ j = 0. \end{cases}, X_{t,j} = X_{t+1,j-1}\frac{1}{\pi_{t+1}}, j > 0$$

Solving for $\tilde{p}_t$:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \lambda_f s_{t+j}}{E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_{p,t}}{F_{p,t}},$$

We now obtain simple recursive expressions for $K_{p,t}$ and $F_{p,t}$.

Consider $K_{p,t}$ first. Accordingly,

$$\begin{aligned}
K_{p,t} &= E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f s_{t+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \sum_{j=1}^{\infty} \left(\beta\xi_p\right)^{j-1} \frac{Y_{t+j}}{C_{t+j}} \left(\frac{1}{\pi_{t+1}} X_{t+1,j-1}\right)^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f s_{t+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j X_{t+1,j}^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f \frac{Y_{t+1+j}}{C_{t+1+j}} s_{t+1+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p \overbrace{E_t E_{t+1}}^{=E_t} \left(\frac{1}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j X_{t+1,j}^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f \frac{Y_{t+1+j}}{C_{t+1+j}} s_{t+1+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} E_{t+1} \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j X_{t+1,j}^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f \frac{Y_{t+1+j}}{C_{t+1+j}} s_{t+1+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} K_{p,t+1}
\end{aligned}$$

so that

$$K_{p,t} = \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1}. \tag{A.1}$$

Similarly,

$$F_{p,t} \equiv E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{\frac{\lambda_f}{1-\lambda_f}} = \frac{Y_t}{C_t} + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} F_{p,t+1} \tag{A.2}$$

In (A.1), marginal cost is defined in (2.28) and (2.29), repeated here for convenience:

$$s_t = \frac{1}{\lambda_f} \frac{C_t z_h\left(h_t, \varsigma_t\right)}{A_t}.$$

Evaluating (2.27)

$$P_t = \left[\left(1-\xi_p\right) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p P_{t-1}^{\frac{1}{1-\lambda_f}}\right]^{1-\lambda_f}.$$

Dividing by P_t and rearranging, we obtain:

$$\tilde{p}_t = \left[\frac{1-\xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p}\right]^{1-\lambda_f} \tag{A.3}$$

We conclude that the equilibrium conditions associated with price setting are (A.1), (A.2) and:

$$\left[\frac{1-\xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p}\right]^{1-\lambda_f} = \frac{K_{p,t}}{F_{p,t}}. \tag{A.4}$$

In a zero inflation steady state without price distortions,

$$K_p = F_p \tag{A.5}$$

according to (A.4). It then follows from (A.1) and (A.2) that

$$s = 1/\lambda_f, \; F_p = \frac{\frac{1}{1-\eta_g}}{1-\beta\xi_p}, \tag{A.6}$$

where $1/\left(1-\eta_g\right)$ is the steady state output to consumption ratio, Y_t/C_t, and η_g is the steady state G_t/Y_t ratio. the object, η_g, is something we fix in the calculations. See below for further discussion.

In the analysis of the linearized equilibrium conditions in the manuscript, we set $g_t, \eta_g \equiv 0$. Differentiating (A.1), (A.2) and (A.4) in steady state:

$$\begin{aligned}
\hat{K}_{p,t} &= \left(1-\beta\xi_p\right)\hat{s}_t + \beta\xi_p\left[\frac{\lambda_f}{\lambda_f-1}\hat{\pi}_{t+1} + \hat{K}_{p,t+1}\right] \\
\hat{F}_{p,t} &= \beta\xi_p\left[\frac{1}{\lambda_f-1}\hat{\pi}_{t+1} + \hat{F}_{p,t+1}\right] \\
\hat{K}_{p,t} &= \frac{\xi_p}{1-\xi_p}\hat{\pi}_t + \hat{F}_{p,t}
\end{aligned}$$

Substitute the third equation into the first:

$$\frac{\xi_p}{1-\xi_p}\hat{\pi}_t + \hat{F}_{p,t} = \left(1-\beta\xi_p\right)\hat{s}_t + \beta\xi_p\left[\frac{\lambda_f}{\lambda_f-1}\hat{\pi}_{t+1} + \frac{\xi_p}{1-\xi_p}\hat{\pi}_{t+1} + \hat{F}_{p,t+1}\right]$$

Substitute the recursive expression for $\hat{F}_{p,t}$:

$$\frac{\xi_p}{1-\xi_p}\hat{\pi}_t + \beta\xi_p\left[\frac{1}{\lambda_f-1}\hat{\pi}_{t+1} + \hat{F}_{p,t+1}\right] = \left(1-\beta\xi_p\right)\hat{s}_t + \beta\xi_p\left[\frac{\lambda_f}{\lambda_f-1}\hat{\pi}_{t+1} + \frac{\xi_p}{1-\xi_p}\hat{\pi}_{t+1} + \hat{F}_{p,t+1}\right],$$

and rearrange, to obtain:

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\hat{s}_t.$$

This is the linearized Phillips curve used in the text.

A.2. Other Private Sector Equilibrium Conditions

We now derive the equilibrium relationship between aggregate consumption and aggregate inputs, (2.35), using the approach described in Yun (1996). Let Y_t^* denote the unweighted integral of intermediate inputs:

$$Y_t^* \equiv \int_0^1 Y_{i,t}di = \int_0^1 A_t h_{i,t}di = A_t h_t.$$

Using the demand curve, (2.26),

$$Y_t^* = \int_0^1 Y_{i,t} di = Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\frac{\lambda_f}{\lambda_f - 1}} di = Y_t P_t^{\frac{\lambda_f}{\lambda_f - 1}} \int_0^1 P_{i,t}^{-\frac{\lambda_f}{\lambda_f - 1}} di = Y_t P_t^{\frac{\lambda_f}{\lambda_f - 1}} \left(P_t^* \right)^{-\frac{\lambda_f}{\lambda_f - 1}},$$

say, where

$$P_t^* \equiv \left[\int_0^1 P_{i,t}^{-\frac{\lambda_f}{\lambda_f - 1}} di \right]^{\frac{-(\lambda_f - 1)}{\lambda_f}} = \left[(1 - \xi_p) \tilde{P}_t^{-\frac{\lambda_f}{\lambda_f - 1}} + \xi_p \left(P_{t-1}^* \right)^{-\frac{\lambda_f}{\lambda_f - 1}} \right]^{\frac{-(\lambda_f - 1)}{\lambda_f}}.$$

Combining the preceding three equations, we obtain (2.35), which we reproduce here for convenience:

$$\begin{aligned} \frac{C_t}{A_t} &= p_t^* h_t - g_t n_t \\ \log \frac{C_t}{C_{t-1}} &= \log(p_t^* h_t - g_t n_t) - \log\left(p_{t-1}^* h_{t-1} - g_{t-1} n_{t-1}\right) + g_{A,t} \end{aligned}$$

$$G_t + C_t = p_t^* A_t h_t. \tag{A.7}$$

In (A.7), include government consumption expenditures, which we model as follows:

$$G_t = g_t N_t,$$

where $\log g_t$ is potentially a stationary stochastic process independent of any other shocks in the system, such as A_t. Also,

$$N_t = A_t^{\gamma} N_{t-1}^{1-\gamma}, \; 0 < \gamma \leq 1. \tag{A.8}$$

In the extreme case, $\gamma = 1$, this reduces to the model of G_t studied in Christiano and Eichenbaum (1992). A problem with the latter model, however, is that it implies G_t moves immediately with shocks to A_t, an implication that seems implausible. With γ close to zero, the immediate impact of A_t on G_t is virtually nil. Yet, regardless of the value of γ, G_t/A_t converges to a constant in nonstochastic steady state. This is necessary if we are to have balanced growth in the case that A_t follows a growth path in the steady state. To see that G_t/A_t converges in steady state, note

$$n_t = \left(\frac{n_{t-1}}{g_{A,t}} \right)^{1-\gamma}, \; n_t \equiv \frac{N_t}{A_t}, \tag{A.9}$$

so that the steady state value of n_t is:

$$n = \left(\frac{1}{g_A} \right)^{\frac{1-\gamma}{\gamma}}. \tag{A.10}$$

From this we conclude that

$$\frac{G_t}{A_t} = g_t n_t, \tag{A.11}$$

is constant in a steady state too. In our steady state analysis, we are interested in fixing η_g, the steady state ratio of G_t to Y_t :

$$\eta_g = \frac{G}{Y} = \frac{G/A}{Y/A} = \frac{g\left(\frac{1}{g_A}\right)^{\frac{1-\gamma}{\gamma}}}{h}.$$

In practice, we fix η_g, g_A and h at their empirically relevant values and so the above equation can be thought of as determining a value for g.

In (A.7), we have used the goods clearing condition, (2.32), and p_t^* captures the distortions to output due to the price setting frictions:

$$p_t^* \equiv \left(\frac{P_t^*}{P_t}\right)^{\frac{\lambda_f}{\lambda_f - 1}}.$$

The law of motion of the distortions is, using (A.3) and (A.4):

$$p_t^* = \left[(1-\xi_p)\left(\frac{1-\xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1-\xi_p}\right)^{\lambda_f} + \frac{\xi_p \pi_t^{\frac{\lambda_f}{\lambda_f - 1}}}{p_{t-1}^*}\right]^{-1}. \tag{A.12}$$

By (A.1), we require an expression for $\lambda_f Y_t s_t / C_t$. After substituting out for C_t from (A.7) into the expression for marginal cost, (2.28), and using (2.29), we obtain:

$$\lambda_f \frac{Y_t}{C_t} s_t = \frac{1}{\lambda_f} \frac{C_t z_h\left(h_t, \varsigma_t\right)}{A_t} \lambda_f \frac{Y_t}{C_t} = \frac{Y_t z_h\left(h_t, \varsigma_t\right)}{A_t}.$$

Then,

$$\lambda_f \frac{Y_t}{C_t} s_t = p_t^* h_t z_h\left(h_t, \varsigma_t\right), \tag{A.13}$$

where z_h denotes the marginal disutility of labor and z is defined in (2.22). Equation (A.13) combines the marginal cost of intermediate good firms with the optimal employment decision by the family, (2.24). By (A.2) we require an expression for Y_t / C_t :

$$\frac{Y_t}{C_t} = \frac{p_t^* A_t h_t}{p_t^* A_t h_t - g_t n_t A_t} = \frac{p_t^* h_t}{p_t^* h_t - g_t n_t} \tag{A.14}$$

The family's intertemporal Euler equation is, using (2.23):

$$1 = \beta E_t \frac{p_t^* h_t - g_t n_t}{\left[p_{t+1}^* h_{t+1} - g_{t+1} n_{t+1}\right] g_{A,t+1}} \frac{R_t}{\pi_{t+1}}, \tag{A.15}$$

where we have used (A.7) to substitute out for C_t and C_{t+1} and

$$g_{A,t+1} \equiv \frac{A_{t+1}}{A_t}.$$

Writing the equilibrium conditions, (A.1) and (A.2), after using (A.13) and (A.14), we obtain:

$$K_{p,t} = p_t^* h_t z_h (h_t, \varsigma_t) + \beta \xi_p E_t \left(\frac{1}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} \tag{A.16}$$

$$F_{p,t} = \frac{p_t^* h_t}{p_t^* h_t - g_t n_t} + \beta \xi_p E_t \left(\frac{1}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} F_{p,t+1} \tag{A.17}$$

The 6 private sector equilibrium conditions are (A.16), (A.17), (A.4), (A.9), (A.12), and (A.15). There are 7 endogenous variables:

$$h_t, p_t^*, R_t, \pi_t, K_{p,t}, F_{p,t}, n_t.$$

When government spending is zero, then (A.9) and n_t drop from the system.

A.3. Closing the Model

The previous subsection enumerated 6 equilibrium conditions for determining 7 endogenous variables. One way to close the model is to consider the Ramsey-optimal allocations. Another is to add a Taylor rule equation.

A.3.1. Ramsey-optimal Allocations

We set government spending to zero here. Substituting out for C_t using (A.7), and for s_t using (A.13), the Lagrangian representation of the Ramsey problem is:

$$
\begin{aligned}
&\max_{p_t^*, h_t, R_t, \pi_t, F_{p,t}, K_{p,t}} E_0 \sum_{t=0}^{\infty} \beta^t \{ \log h_t + \log p_t^* - z(h_t, \varsigma_t) \\
&+\lambda_{1t} \left[\frac{1}{p_t^* h_t} - E_t \frac{\beta}{p_{t+1}^* h_{t+1} g_{A,t+1}} \frac{R_t}{\pi_{t+1}} \right] \\
&+\lambda_{2t} \left[\frac{1}{p_t^*} - \left((1-\xi_p) \left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1-\xi_p} \right)^{\lambda_f} + \frac{\xi_p \pi_t^{\frac{\lambda_f}{\lambda_f - 1}}}{p_{t-1}^*} \right) \right] \\
&+\lambda_{3t} \left[1 + \beta \xi_p E_t \left(\frac{1}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} F_{p,t+1} - F_{p,t} \right] \\
&+\lambda_{4t} p_t^* \left[h_t z_h(h_t, \varsigma_t) + \beta \xi_p E_t \left(\frac{1}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} \frac{p_{t+1}^*}{p_t^*} \frac{K_{p,t+1}}{p_{t+1}^*} - \frac{K_{p,t}}{p_t^*} \right] \\
&+\lambda_{5t} \left[F_{p,t} \left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1-\xi_p} \right)^{1-\lambda_f} - K_{p,t} \right] \}.
\end{aligned}
$$

In the fourth Lagrangian constraint, it is convenient to factor out p_t^*. We conjecture (and can later verify) that the first, third, fourth and fifth constraints are not binding on the problem. In particular, we can simply select R_t to satisfy the first constraint, $F_{p,t}$ to satisfy the third, $K_{p,t}$ to satisfy the fourth, and we then need to verify that the fifth constraint is satisfied, as well as $R_t \geq 1$.

Implementing the conjecture, the problem, with $g_t \equiv 0$, reduces to:

$$
\begin{aligned}
&\max_{p_t^*, h_t, \pi_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \log p_t^* + \log h_t - z(h_t, \varsigma_t) \\
&+\lambda_{2t} \left[\frac{1}{p_t^*} - \left((1-\xi_p) \left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1-\xi_p} \right)^{\lambda_f} + \frac{\xi_p \pi_t^{\frac{\lambda_f}{\lambda_f - 1}}}{p_{t-1}^*} \right) \right] \},
\end{aligned}
$$

where $\log A_t$ in the utility function is ignored because it cannot be controled. This leads to the efficiency condition for hours and p_t^*:

$$
h_t z_h(h_t, \varsigma_t) = 1. \tag{A.18}
$$

Interestingly, this coincides with the first-best setting for h_t. The efficiency conditions for p_t^*

and π_t are, respectively,

$$\begin{aligned} p_t^* &= \lambda_{2t} - \beta\lambda_{2t+1}\xi_p \pi_{t+1}^{\frac{\lambda_f}{\lambda_f-1}} = 0 \\ \left(\frac{1-\xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p}\right)^{\lambda_f-1} &= \frac{\pi_t}{p_{t-1}^*}. \end{aligned} \tag{A.19}$$

Rearranging, and substituting into (A.12), we obtain:

$$p_t^* = \left[\left(p_{t-1}^*\right)^{\frac{1}{\lambda_f-1}} \xi_p + \left(1-\xi_p\right)\right]^{\lambda_f-1} \tag{A.20}$$

$$\pi_t = \frac{p_{t-1}^*}{p_t^*}. \tag{A.21}$$

We now verify that all the constraints on the Ramsey problem assumed to be non-binding are in fact satisfied. Consider the fourth Lagrangian constraint, after substituting out (A.18), (A.20), and (A.21):

$$1 + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}} \frac{K_{p,t+1}}{p_{t+1}^*} = \frac{K_{p,t}}{p_t^*}.$$

We let this equation define $K_{p,t}$, so that the fourth Lagrangian is satisfied. The requirement that the third Lagrangian constraint also be satisfied implies

$$\frac{K_{p,t}}{p_t^*} = F_{p,t}. \tag{A.22}$$

Letting (A.22) define $F_{p,t}$ we have that the third Lagrangian is satisfied. From (A.19) and (A.21),

$$\left(\frac{1-\xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p}\right)^{\lambda_f-1} = p_t^*. \tag{A.23}$$

It follows from (A.22) and (A.23) that the fifth Lagrangian constraint is satisfied. The first Lagrangian constraint is trivially satisfy if we use it to define the nominal rate of interest, R_t. As long as the shocks are not too big, $R_t \geq 1$.

We have established that allocations in which hours worked, the price distortions and inflation satisfy (A.18), (A.20), and (A.21), respectively, solve the Ramsey problem. Because h_t is independent of A_t according to (A.18), it follows that the welfare cost of business cycles in the Ramsey problem is the same, regardless of whether the underlying economy is the standard DSGE model or our model of involuntary unemployment.

A.3.2. Taylor Rule Equilibrium

Consider the following policy rule:

$$R_t = R^{1-\rho_R} R_{t-1}^{\rho_R} \pi_t^{(1-\rho_R)r_\pi} x_t^{(1-\rho_R)r_y}, \tag{A.24}$$

where

$$x_t = \frac{Y_t}{\tilde{Y}_t},$$

and $\tilde{Y}_t$ is the first-best level of consumption. We can write

$$x_t = \frac{p_t^* A_t h_t}{A_t \tilde{h}_t} = \frac{p_t^* h_t}{\tilde{h}_t},$$

where h_t is equilibrium hours worked and $\tilde{h}_t$ is the value of hours worked in the efficient equilibrium, which is define by:

$$\frac{A_t}{\tilde{C}_t} = z_h\left(\tilde{h}_t, \varsigma_t\right).$$

Here, $\tilde{C}_t$ denotes consumption in the efficient equilibrium. Using the resource constraint with no price distortions, (A.7) with $p_t^* = 1$, and rewriting:

$$\frac{A_t}{A_t \tilde{h}_t - G_t} = \frac{1}{\tilde{h}_t - g_t n_t} = z_h\left(\tilde{h}_t, \varsigma_t\right),$$

or,

$$\left(\tilde{h}_t - g_t n_t\right) z_h\left(\tilde{h}_t, \varsigma_t\right) = 1.$$

Equilibrium is characterized by the requirement that (A.24) as well as the above equation be satisfied, and also (A.16), (A.17), (A.4), (A.9), (A.12), and (A.15). Summarizing:

$$0 = E_t\left[p_t^* h_t z_h\left(h_t, \varsigma_t\right) + \beta\xi_p\left(\frac{1}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} - K_{p,t}\right] \tag{A.25}$$

$$0 = E_t\left[\frac{p_t^* h_t}{p_t^* h_t - g_t n_t} + \beta\xi_p E_t\left(\frac{1}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}} F_{p,t+1} - F_{p,t}\right] \tag{A.26}$$

$$K_{p,t} = \left[\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p}\right]^{1-\lambda_f} F_{p,t} \tag{A.27}$$

$$p_t^* = \left[\left(1 - \xi_p\right)\left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p}\right)^{\lambda_f} + \frac{\xi_p \pi_t^{\frac{\lambda_f}{\lambda_f - 1}}}{p_{t-1}^*}\right]^{-1} \tag{A.28}$$

$$1 = \beta E_t \frac{p_t^* h_t - g_t n_t}{\left(p_{t+1}^* h_{t+1} - g_{t+1} n_{t+1}\right) g_{A,t+1}} \frac{R_t}{\pi_{t+1}} \tag{A.29}$$

$$R_t = R^{1-\rho_R} R_{t-1}^{\rho_R} \pi_t^{(1-\rho_R) r_\pi} \left(\frac{p_t^* h_t}{\tilde{h}_t}\right)^{(1-\rho_R) r_y} \tag{A.30}$$

$$1 = \left(\tilde{h}_t - g_t n_t\right) z_h\left(\tilde{h}_t, \varsigma_t\right) \tag{A.31}$$

$$n_t = \left(\frac{n_{t-1}}{g_{A,t}}\right)^{1-\gamma} \tag{A.32}$$

The 8 variables to be determined by these 8 equations are:

$$K_{p,t}, F_{p,t}, h_t, \tilde{h}_t, p_t^*, \pi_t, R_t, n_t \tag{A.33}$$

The present discounted value of utility is:

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - z\left(h_t, \varsigma_t\right)\right] && \text{(A.34)} \\ = \; & E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(p_t^* h_t - g_t n_t\right) - z\left(h_t, \varsigma_t\right)\right] + \frac{\log A_0}{1-\beta} + \frac{1}{1-\beta} E_0 \sum_{t=1}^{\infty} \beta^t \log g_{A,t} \end{aligned}$$

The piece that is exogenous and additive is not interesting. Define

$$\begin{aligned} w\left(g_{A,0}, p_{-1}^*, \varsigma_0\right) &= E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(p_t^* h_t - g_t n_t\right) - z\left(h_t, \varsigma_t\right)\right] \\ &= \log\left(p_0^* h_0 - g_0 n_0\right) - z\left(h_0, \varsigma_0\right) + \beta E_0 w\left(g_{A,1}, p_0^*, \varsigma_1\right). \end{aligned}$$

We can add this equation to the list, (A.25)-(A.30), above,

$$E_t \left[\log\left(p_t^* h_t - g_t n_t\right) - z\left(h_t, \varsigma_t\right) + \beta w_{t+1} - w_t\right] = 0, \tag{A.35}$$

giving us one additional variable, w_t, and one additional equation.

This system can be solved in Dynare. Simply type in equations (A.25)-(A.32) and (A.35). We have three stochastic processes, g_t, $g_{A,t}$ and ς_t. Let x_t denote one of these. Then, the law of motion of x_t is:

$$\log x_t = (1-\rho_x) \log x + \rho_x \log x_{t-1} + \varepsilon_t^x,$$

for $x_t = g_t,\ g_{A,t},\ \varsigma_t$.

A.4. Steady State

From (A.5) and (A.6),

$$K_p = F_p,\ \ s = 1/\lambda_f,\ \ F_p = \frac{\frac{1}{1-\eta_g}}{1-\beta\xi_p}.$$

From (A.12), in a zero inflation steady state,

$$p^* = 1.$$

We want to impose that government spending is a given proportion, η_g, of total output in the steady state:

$$\eta_g = \frac{G}{C+G} = \frac{gnA}{Ah} = \frac{gn}{h} = \frac{g\left(\frac{1}{g_A}\right)^{\frac{1-\gamma}{\gamma}}}{h},$$

by (A.10). Thus, the steady state value of g is:

$$g = \eta_g h g_A^{\frac{1-\gamma}{\gamma}},\ gn = \eta_g h \tag{A.36}$$

Combining (A.13), the result for the steady state value of s and (A.7), (A.11) to obtain:

$$\frac{h}{h - gn} = h z_h (h, \varsigma),$$

or, using (A.36),

$$1 = \left[1 - \eta_g\right] h z_h (h, \varsigma). \tag{A.37}$$

We now proceed to develop the formulas necessary to compute z_h. Recall, the disutility of labor function, $z(h_t)$ in (2.22). Write this in terms of m_t :

$$Z(m_t) = \log\left[Q(m_t)\left(e^{F+\varsigma_t(1+\sigma_L)m_t^{\sigma_L}} - 1\right) + 1\right] - \frac{a^2\varsigma_t^2(1+\sigma_L)\sigma_L^2}{2\sigma_L+1} m_t^{2\sigma_L+1} - \eta\varsigma_t\sigma_L m_t^{\sigma_L+1}, \tag{A.38}$$

where

$$\begin{aligned} h_t &= m_t\eta + a^2\varsigma_t\sigma_L m_t^{\sigma_L+1} \equiv Q(m_t), \\ m_t &= Q^{-1}(h_t), \end{aligned} \tag{A.39}$$

where Q^{-1} is inverse function of Q, defined by:

$$h_t = Q\left(Q^{-1}(h_t)\right). \tag{A.40}$$

Then,

$$z(h_t) = Z\left(Q^{-1}(h_t)\right). \tag{A.41}$$

We require the first and second derivatives of z. Thus,

$$z_h(h_t) = Z_m\left(Q^{-1}(h_t)\right)\left[Q^{-1}\right]'(h_t), \tag{A.42}$$

where $[Q^{-1}]'$ denotes the derivative of the function, Q^{-1}. To obtain an expression for $[Q^{-1}]'(h_t)$, differentiate (A.40) with respect to h_t :

$$1 = Q'\left(Q^{-1}(h_t)\right)\left[Q^{-1}\right]'(h_t),$$

so that:

$$\left[Q^{-1}\right]'(h_t) = \frac{1}{Q'\left(Q^{-1}(h_t)\right)}. \tag{A.43}$$

We also require the second derivative of Q^{-1}. Differentiating (A.40) a second time with respect to h_t :

$$0 = Q''\left(Q^{-1}(h_t)\right)\left(\left[Q^{-1}\right]'(h_t)\right)^2 + Q'\left(Q^{-1}(h_t)\right)\left[Q^{-1}\right]''(h_t).$$

Substituting from the first derivative:

$$0 = Q''\left(Q^{-1}(h_t)\right)\left(\frac{1}{Q'\left(Q^{-1}(h_t)\right)}\right)^2 + Q'\left(Q^{-1}(h_t)\right)\left[Q^{-1}\right]''(h_t),$$

so that

$$\left[Q^{-1}\right]''(h_t) = -Q''\left(Q^{-1}(h_t)\right)\left(\frac{1}{Q'\left(Q^{-1}(h_t)\right)}\right)^3. \qquad \text{(A.44)}$$

Substituting (A.43) into (A.42),

$$z_h(h_t) = \frac{Z_m\left(Q^{-1}(h_t)\right)}{Q'\left(Q^{-1}(h_t)\right)}.$$

Differentiating z in (A.41) a second time,

$$z_{hh}(h_t) = Z_{mm}\left(Q^{-1}(h_t)\right)\left(\left[Q^{-1}\right]'(h_t)\right)^2 + Z_m\left(Q^{-1}(h_t)\right)\left[Q^{-1}\right]''(h_t),$$

which, after substituting from (A.43) and (A.44), is:

$$\begin{aligned}
z_{hh}(h_t) &= Z_{mm}\left(Q^{-1}(h_t)\right)\left(\frac{1}{Q'\left(Q^{-1}(h_t)\right)}\right)^2 - Z_m\left(Q^{-1}(h_t)\right)Q''\left(Q^{-1}(h_t)\right)\left(\frac{1}{Q'\left(Q^{-1}(h_t)\right)}\right)^3 \\
&= \left(\frac{1}{Q'\left(Q^{-1}(h_t)\right)}\right)^2 Z_m\left(Q^{-1}(h_t)\right)\left[\frac{Z_{mm}\left(Q^{-1}(h_t)\right)}{Z_m\left(Q^{-1}(h_t)\right)} - \frac{Q''\left(Q^{-1}(h_t)\right)}{Q'\left(Q^{-1}(h_t)\right)}\right] \\
&= \left(\frac{1}{Q'(m_t)}\right)^2 Z_m(m_t)\left[\frac{Z_{mm}(m_t)}{Z_m(m_t)} - \frac{Q''(m_t)}{Q'(m_t)}\right],
\end{aligned}$$

where the expressions for Q', Q'', Z_m, Z_{mm} can be obtained by symbolic differentiation of the underlying functions, (A.38) and (A.39).

The endogenous variables are

$$h, m, \sigma_z, u, \bar{p},$$

and the equations are:

$$\begin{aligned}
(1) h &= m\eta + a^2 \varsigma \sigma_L m^{\sigma_L + 1} \\
(2) \bar{p} &= \eta + \varsigma a^2 (1 + \sigma_L) m^{\sigma_L} \\
(3) 1 &= \left[1 - \eta_g\right] h z_h(h, \varsigma) \\
(4) \sigma_z &= \frac{z_{hh} h}{z_h} \\
(5) \kappa^{okun} &= \frac{a^2 \varsigma \sigma_L^2 m^{\sigma_L}(1-u)}{1 - u + a^2 \varsigma \sigma_L^2 m^{\sigma_L}} \\
u &= \frac{m - h}{m}.
\end{aligned} \qquad \text{(A.45)}$$

The parameters are:

$$F, \varsigma, a, \eta, \sigma_L.$$

We solve the steady state using two different strategies, depending on our purposes. The first strategy, the straightforward one, takes the parameters as given and computes the steady values of the endogenous variables. We implement this strategy by placing a grid of values of m on the unit interval. For each value of m on the grid we compute h using (1) and evaluate (3). We count the number of places on the grid where there is a switch in the sign of (3), *i.e.*, in $1 - \left[1 - \eta_g\right] hz_h\left(h, \varsigma\right).$ For each sign switch, we then narrowed down the corresponding value of m that implements (3) exactly using a nonlinear equation solver. After this, we computed $\bar{p}$ using (2) as well as steady state unemployment, u. In practice, we found two sets of solutions to the steady state equations, but one was inadmissible because it implied $u < 0$. We applied this 'first strategy' to compare the steady states of the involuntary unemployment model studied in this section with the steady state of the full information model in section B holding model parameters fixed. Section B shows that the steady state of the full information model is also characterized by (A.45). The difference between the two models lies in the details of the function, z_h. With this approach to computing the steady state we are able to evaluate the impact on welfare and other variables of the assumption of limited information. We compute the value of information in consumption units as follows. According to (A.34), steady state utility is, apart from an additive constant, as follows:

$$U = \frac{1}{1-\beta}\left[\log\left(h\right) - z\right].$$

Here, h denotes steady state employment and z denotes the steady state disutility of labor (the fact, $h - gn = \left(1 - \eta_g\right) h$, in steady state has been used here). Let utility in the steady state of the full information model be denoted:

$$U^{fi} = \frac{1}{1-\beta}\left[\log\left(h^{fi}\right) - z^{fi}\right]$$

Let λ denote the percent increase in consumption in the involuntary unemployment model that makes households in that model indifferent between staying in that environment, or converting to the environment with full information. Let $U\left(\lambda\right)$ denote the level of utility in the involuntary unemployment equilibrium when consumption is raised from h to $h\left(1 + \lambda/100\right).$ We seek λ such that

$$U\left(\lambda\right) = U^{fi}.$$

Note:

$$\begin{aligned} U\left(\lambda\right) &= \frac{1}{1-\beta}\left[\log\left(h\right) + \log\left(1 + \lambda/100\right) - z\right] \\ &= U + \frac{1}{1-\beta}\log\left(1 + \lambda/100\right), \end{aligned}$$

so that

$$\lambda = 100\left[e^{(1-\beta)\left(U^{fi} - U\right)} - 1\right].$$

Our second strategy for computing the steady state takes the values of $\bar{p}$, m, u and σ_z as given. Thus, the 'parameters' become:

$$\bar{p}, m, u, \sigma_z, \sigma_L \tag{A.46}$$

and the 'endogenous' variables become:

$$F, \varsigma, a, \eta, h.$$

We wish to solve the system for the endogenous variables, for a given set of values for the parameters. To do so, note first that h is determined by

$$h = m(1-u).$$

We then proceed to solve equations (1)-(4) with respect to F, ς, a, η using a nonlinear multipler equation solver. This solution strategy is useful when we want to compute observationally equivalent parameterizations for the models considered in this paper. When we do this, we take observed data as given. Variables like m and u are measured from time averages of unemployment and the labor force, and σ_z can be estimated from applying time series techniques to the reduced form of the model using macroeconomic data that do not include unemployment and the labor force.

In our analysis, we find it convenient to compare the model with involuntary unemployment with two other models. This includes the CGG model and the model in which the family problem is based on full information. The lattter is treated in the section devoted to that model. Here, we develop the steady state equations for the CGG model.

The only change required for computing the steady state lies in the specification of z_h in (A.37). We define the disutility of labor in the CGG model as the one implicit in the 'standard model', (3.21):

$$z(h_t, \varsigma_t) = \varsigma h_t^{1+\sigma_L}.$$

Then, (A.37) reduces to:

$$1 = \left[1-\eta_g\right] h z_h(h, \varsigma) = \left[1-\eta_g\right] \varsigma (1+\sigma_L) h^{\sigma_L+1}.$$

Note that as η_g increases, h does too. In the CGG model $m = h$ because there is no job search.

We follow the approach taken in the involuntary unemployment model in calibrating m, and thus h too (see (A.46)). We thus define the preference shock in this model as

$$\varsigma = \frac{1}{\left[1-\eta_g\right](1+\sigma_L) h^{1+\sigma_L}}.$$

Of course the steady state unemployment rate is zero (as it is outside of steady state too) because there is no monopoly power in the labor market. In addition, we parameterizer the curvature, σ_z, of the disutility of labor. This is simply

$$\sigma_L = \sigma_z$$

in the CGG model. Then, labor supply is simply:

$$z_h \left(h_t, \varsigma_t\right) = \varsigma \left(1 + \sigma_L\right) h_t^{\sigma_L},$$

B. The Family Utility Function Under Full Information

Our model of involuntary unemployment makes two sorts of assumptions: (i) households have different aversions to work and must make an effort to find work and (ii) their type and effort levels are private information. The fact that there is unemployment in the BLS sense follows from (i) only and (ii) is required for unemployment to be 'involuntary'. This section allows us to determine the impact on the analysis of (ii), by deriving the family utility function that applies when (ii) is not satisfied.

The first subsection derives the family utility function. The second subsection discusses the steady state when this model of unemployment is introduced into CGG.

B.1. Family Utility Function

Thus, we assume that the family observes everything about the individual household: the effort it exerts to find a job, if any, and its aversion to work. The family selects a consumption allocation and level of search effort, conditional on a household's type. It does so by optimizing the ex ante utility of an arbitrary household or, equivalently, by optimizing the average utility of all households ex post. Households with $0 \leq l \leq m$ participate in the labor force and those with $1 \geq l \geq m$ do not, where m is a choice variable. We drop subscripts to simplify the notation. The family optimization problem is:

$$\max_{m,\{e_l\},c^w,c^{nw}} \int_0^m \left(p\left(e_l\right)\left[\log\left(c^w\right) - F - \varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right] + \left(1 - p\left(e_l\right)\right)\log\left(c^{nw}\right) - \frac{1}{2}e_l^2 \right) dl$$
$$+ \left(1 - m\right)\log\left(c^{nw}\right)$$

subject to the resource constraint, (2.18), and the link between the number employed, h, and the labor force, m, (2.11). We reproduce these constraints here for convenience:

$$\begin{aligned} C &= hc^w + \left(1 - h\right)c^{nw} \\ h &= \int_0^m p\left(e_l\right) dl = m\eta + a\int_0^m e_l dl. \end{aligned}$$

Rewrite the objective:

$$\max_{m,\{e_l\},c^w,c^{nw}} \int_0^m \left(p(e_l) \left[\log\left(\frac{c^w}{c^{nw}}\right) - F - \varsigma (1+\sigma_L) l^{\sigma_L} \right] - \frac{1}{2} e_l^2 \right) dl + \log(c^{nw}).$$

From the resource constraint:

$$\frac{c^w}{c^{nw}} = \frac{\frac{C}{c^{nw}} - (1-h)}{h}.$$

So that, after substituting out the resource constraint, we get:

$$\max_{m,\{e_l\},c^{nw}} \int_0^m \left(p(e_l) \left[\log\left(\frac{\frac{C}{c^{nw}} - (1-h)}{h}\right) - F - \varsigma (1+\sigma_L) l^{\sigma_L} \right] - \frac{1}{2} e_l^2 \right) dl + \log(c^{nw})$$
$$+\lambda \left[\int_0^m p(e_l)\, dl - h \right],$$

where λ is the multiplier on the restriction linking m and h. The first order condition for c^{nw} is:

$$c^{nw} = C,$$

so that the objective reduces to:

$$\max_{m,\{e_l\}} \int_0^m \left(-p(e_l) \left[F + \varsigma (1+\sigma_L) l^{\sigma_L} \right] - \frac{1}{2} e_l^2 \right) dl + \log(C) \tag{B.1}$$
$$+\lambda \left[\int_0^m p(e_l)\, dl - h \right].$$

Optimization with respect to e_l implies:

$$e_l = \lambda a - a \left[F + \varsigma (1+\sigma_L) l^{\sigma_L} \right],$$

so that, using (2.5),

$$p(e_l) = \eta + \lambda a^2 - a^2 \left[F + \varsigma (1+\sigma_L) l^{\sigma_L} \right].$$

Also,

$$h = \int_0^m p(e_l)\, dl = m \left(\eta + \lambda a^2 - a^2 \left[F + \varsigma m^{\sigma_L} \right] \right), \tag{B.2}$$

or,

$$\lambda = \frac{\frac{h}{m} - \eta}{a^2} + F + \varsigma m^{\sigma_L}. \tag{B.3}$$

Optimality of the choice of m in (B.1) implies, by Leibniz's rule:

$$-p(e_m) \left[F + \varsigma (1+\sigma_L) m^{\sigma_L} \right] - \frac{1}{2} e_m^2 + \lambda p(e_m) = 0,$$

or,

$$-\left[\eta + \lambda a^2 - a^2 x \right] x - \frac{1}{2} (\lambda a - a x)^2 + \lambda \left(\eta + \lambda a^2 - a^2 x \right) = 0,$$

where

$$x \equiv F + \varsigma\left(1 + \sigma_L\right) m^{\sigma_L}. \tag{B.4}$$

Simplifying,

$$-\left(\eta + \lambda a^2\right) x + a^2 x^2 - \frac{1}{2}\left(\lambda a\right)^2 + \lambda a^2 x - \frac{1}{2} a^2 x^2 + \lambda\left(\eta + \lambda a^2\right) - \lambda a^2 x = 0,$$

or,

$$f\left(x\right) \equiv x^2 - 2\left(\frac{\eta + \lambda a^2}{a^2}\right) x + \lambda\left[\lambda + 2\frac{\eta}{a^2}\right] = 0. \tag{B.5}$$

It is easy to verify that there are two values of x with the property, $f\left(x\right) = 0$:

$$\frac{1}{a^2}\left(\lambda a^2 + 2\eta\right),\ \lambda.$$

Substituting out for λ from (B.3) these solutions reduce to:

$$\frac{\frac{h}{m} + \eta}{a^2} + F + \varsigma m^{\sigma_L},\ \frac{\frac{h}{m} - \eta}{a^2} + F + \varsigma m^{\sigma_L}.$$

Consider the first solution:

$$F + \varsigma\left(1 + \sigma_L\right) m^{\sigma_L} = \frac{\frac{h}{m} + \eta}{a^2} + F + \varsigma m^{\sigma_L}$$

or,

$$m^{\sigma_L} = \frac{\frac{h}{m} + \eta}{a^2 \varsigma \sigma_L} \tag{B.6}$$

There is a unique value of m, $m \geq 0$, that satisfies (B.6). To see this, note that the left side of (B.6) starts at zero and increases without bound as m increases. The right side starts at plus infinity (thus, greater than the left size) with $m = 0$ and declines monotonically to a finite number as m increases (thus, the right side is eventually below the left side). By continuity and monotonicity, there is a unique value of m that satisfies the equality in (B.6). Of course, the only admissible solution satisfies $m \in \left(h, 1\right)$. The second solution to $f\left(x\right) = 0$ implies

$$F + \varsigma\left(1 + \sigma_L\right) m^{\sigma_L} = \frac{\frac{h}{m} - \eta}{a^2} + F + \varsigma m^{\sigma_L},$$

or

$$m^{\sigma_L} = \frac{\frac{h}{m} - \eta}{a^2 \varsigma \sigma_L}. \tag{B.7}$$

Interestingly, this also reduces to the expression in (A.39):

$$h = \eta m + a^2 \varsigma \sigma_L m^{\sigma_L + 1} \equiv Q\left(m\right). \tag{B.8}$$

There is a unique value of m, $m \geq 0$, that satisfies (B.7) for any $h \geq 0$. This is because m is monotone increasing for $m \geq 0$ and $Q\left(0\right) = 0$. Whether one or both of (B.6) and (B.7) correspond to local maxima requires examining the relevant second order condition. We do so now, graphically. Of course, we can anticipate that the smaller of the two solutions, the one associated with (B.7), is likely to correspond to the maximum sought in (B.1).

The existence of more than one solution to (B.5) implies that we must investigate second order conditions. Differentiate (B.5) with respect to m :

$$f'(x)\frac{dx}{dm} = \left[x - \frac{\eta + \lambda a^2}{a^2}\right] 2\frac{dx}{dm}.$$

Since $2dx/dm > 0$ the sign of the above expression corresponds to the sign of the object in square brackets, which is, after substituting out for x from (B.4) and for λ from (B.3):

$$\varsigma \sigma_L m^{\sigma_L} - \frac{\frac{h}{m}}{a^2}$$

It is easy to verify that (B.6) implies the above expression is positive, while (B.7) implies the above expression is negative. Thus, (B.7) satisfies the first and second order conditions necessary for an optimum. Thus, (B.7) is a local optimum, while (B.6) is a local minimum.

It is interesting to investigate whether (B.9) satisfies the usual bounds for a probability, given (B.3) and (B.7). Using (B.3) to substitute out for λ in (B.9) and rearranging,

$$p(e_l) = \frac{h}{m} + a^2 \varsigma \left[m^{\sigma_L} - (1 + \sigma_L) l^{\sigma_L}\right].$$

Using (B.7) to substitute out for h/m in the previous expression, we obtain:

$$p(e_l) = \eta + (1 + \sigma_L) a^2 \varsigma \left[m^{\sigma_L} - l^{\sigma_L}\right]. \tag{B.9}$$

Interestingly, this is the same function obtained for the involuntary unemployment model (to see this, substitute the incentive constraint, (2.9), into the probability function for that model, (2.6)).

We would like to have an expression for the family utility function, the function that attains the optimum in (B.1) for given C and h. Consider the object under the integral in the objective function, (B.1):

$$
\begin{aligned}
& -p\left(e_{l}\right)\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]-\frac{1}{2} e_{l}^{2} \\
= & -\left[\eta+\lambda a^{2}-a^{2}\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]\right]\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]-\frac{1}{2}\left[\lambda a-a\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]\right]^{2} \\
= & -\left(\eta+\lambda a^{2}\right)\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]+a^{2}\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]^{2}-\frac{1}{2}\left[\lambda a-a\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]\right]^{2} \\
= & -\left(\eta+\lambda a^{2}\right)\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]+a^{2}\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]^{2} \\
& -\frac{1}{2}\left[(\lambda a)^{2}-2 \lambda a^{2}\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]+a^{2}\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]^{2}\right] \\
= & -\left(\eta+\lambda a^{2}\right)\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]+a^{2}\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]^{2} \\
& -\frac{1}{2}(\lambda a)^{2}+\lambda a^{2}\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]-\frac{1}{2} a^{2}\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]^{2} \\
= & -\eta\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]+\frac{1}{2} a^{2}\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]^{2}-\frac{1}{2}(\lambda a)^{2}
\end{aligned}
$$

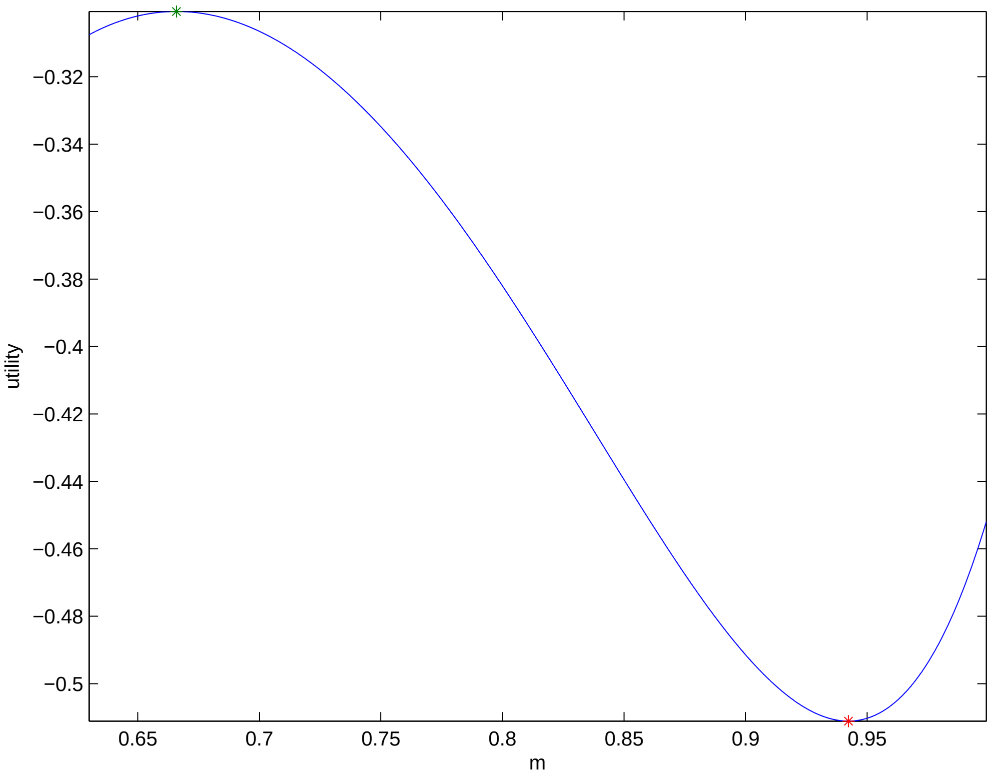
Note,

$$
\begin{aligned}
\int_{0}^{m}\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right] d l &= F m+\varsigma m^{\sigma_{L}+1} \\
\int_{0}^{m}\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]^{2} d l &= \int_{0}^{m}\left[F^{2}+2 \varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}} F+\varsigma^{2}\left(1+\sigma_{L}\right)^{2} l^{2 \sigma_{L}}\right] d l \\
&= m F^{2}+2 \varsigma m^{\sigma_{L}+1} F+\varsigma^{2}\left(1+\sigma_{L}\right)^{2} \frac{m^{2 \sigma_{L}+1}}{2 \sigma_{L}+1}
\end{aligned}
$$

Then, the integral in (B.1) is

$$
\begin{aligned}
& \int_{0}^{m}\left\{-\eta\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]+\frac{1}{2} a^{2}\left[F+\varsigma\left(1+\sigma_{L}\right) l^{\sigma_{L}}\right]^{2}-\frac{1}{2}(\lambda a)^{2}\right\} d l \\
= & -\eta m\left[F+\varsigma m^{\sigma_{L}}\right]+\frac{1}{2} a^{2} m\left[F^{2}+2 \varsigma m^{\sigma_{L}} F+\varsigma^{2}\left(1+\sigma_{L}\right)^{2} \frac{m^{2 \sigma_{L}}}{2 \sigma_{L}+1}\right]-\frac{1}{2}(\lambda a)^{2} m,
\end{aligned}
$$

with λ given by (B.3) and m given by (B.6). The graph of this object, for $m \in (h, 1)$ is as follows:



Note that there are two values of m where this function is flat. They correspond to (B.6) and (B.7). Consistent with the algebra above, the second flat point is a local minimum, while the first is a local maximum.

To construct the utility function of the representative agent, write the following function of m :

$$Z(m,h) = \eta m\left[F + \varsigma m^{\sigma_L}\right] - \frac{1}{2}a^2 m\left[F^2 + 2\varsigma m^{\sigma_L}F + \varsigma^2(1+\sigma_L)^2\frac{m^{2\sigma_L}}{2\sigma_L+1}\right] + \frac{1}{2}(\lambda(m,h)a)^2 m, \tag{B.10}$$

where $\lambda(m,h)$ is the particular function of m and h given by (B.3). Express (B.8) as

$$m = Q^{-1}(h). \tag{B.11}$$

Then, the utility function of the representative agent is:

$$u(C,h) = \log(C) - z(h), \tag{B.12}$$

where

$$z(h) = Z\left(Q^{-1}(h), h\right), \tag{B.13}$$

where Q^{-1} is defined as the inverser function of Q (see (A.40)).

Our calculations require the derivative of z, $z_h(h)$. According to (B.13),

$$z_h(h) = Z_1\left(Q^{-1}(h), h\right)\left[Q^{-1}\right]'(h) + Z_2\left(Q^{-1}(h), h\right),$$

where $\left[Q^{-1}\right]'(h)$ is the derivative of the inverse function of Q, defined in terms of Q' in (A.43). Using (A.43):

$$z_h(h) = \frac{Z_1\left(Q^{-1}(h), h\right)}{Q'\left(Q^{-1}(h_t)\right)} + Z_2\left(Q^{-1}(h), h\right). \tag{B.14}$$

We also require the second derivative of z, z_{hh}:

$$\begin{aligned} z_{hh}(h) &= Z_{11}\left(Q^{-1}(h), h\right)\left(\left[Q^{-1}\right]'(h)\right)^2 + Z_{12}\left(Q^{-1}(h), h\right)\left[Q^{-1}\right]'(h) \\ &\quad + Z_1\left(Q^{-1}(h), h\right)\left[Q^{-1}\right]''(h) + Z_{21}\left(Q^{-1}(h), h\right)\left[Q^{-1}\right]'(h) \\ &\quad + Z_{22}\left(Q^{-1}(h), h\right), \end{aligned}$$

where the second derivative of the inverse function, $\left[Q^{-1}\right]''(h)$, can be computed using (A.44). Using (A.43) and (A.44):

$$\begin{aligned} z_{hh}(h) &= Z_{11}\left(Q^{-1}(h), h\right)\left(\frac{1}{Q'\left(Q^{-1}(h_t)\right)}\right)^2 + \frac{Z_{12}\left(Q^{-1}(h), h\right)}{Q'\left(Q^{-1}(h_t)\right)} \\ &\quad - Z_1\left(Q^{-1}(h), h\right)Q''\left(Q^{-1}(h_t)\right)\left(\frac{1}{Q'\left(Q^{-1}(h_t)\right)}\right)^3 \\ &\quad + \frac{Z_{21}\left(Q^{-1}(h), h\right)}{Q'\left(Q^{-1}(h_t)\right)} + Z_{22}\left(Q^{-1}(h), h\right), \end{aligned} \tag{B.15}$$

Given (B.14) and (B.14), we can compute the curvature of utility:

$$\sigma_z = \frac{z_{hh}h}{z_h}.$$

All these formulas can be computed symbolically given the definitions of the Z and Q functions in (B.10) and (B.8), respectively.

B.2. Steady State

We discuss two ways of computing the steady state of the model. The first is the 'natural' one. This takes the parameters of the model as given and computes the implied exogenous variables. This is useful for doing one type of comparison of the full information version of the model with our involuntary unemployment model. For example, we can ask how the level of unemployment is affected by the absence of full information. The second approach allows us to impose certain endogenous variables exogenously, and back out parameters. This approach is useful when we want to ask what an econometrician observing data (and, hence given endogenous variables) would infer about the welfare cost of business cycles.

B.2.1. A First Approach to Computing the Steady State

The structural parameters of the model are

$$F, \varsigma, a, \eta, \sigma_L.$$

The endogenous variables of interest are

$$\bar{p}, h, m, u, \sigma_z.$$

The relevant equations are (A.45):

$$\begin{aligned}
(1)h &= m\eta + a^2\varsigma\sigma_L m^{\sigma_L+1} \\
(2)\bar{p} &= \eta + \varsigma a^2 (1+\sigma_L) m^{\sigma_L} \\
(3)1 &= \left[1-\eta_g\right] h z_h (h, \varsigma) \\
(4)\sigma_z &= \frac{z_{hh} h}{z_h} \\
u &= \frac{m-h}{m}.
\end{aligned}$$

Notably, equations (1) and (2) in (A.45) are literally unchanged from what they are in the full information model because the expressions for $p(e_l)$ and the mapping from m to h are unchanged. In addition, equations (3) and (4) are formally the same, although the details of z_h and z_{hh} are different for the partial information model because z is different. The steady state is found by solving (3) for h. Then, (1) is solved for m and (2) is solved for $\bar{p}$. Finally, the last equation is solved for u.

B.2.2. A Second Approach to Solving the Steady State

We proceed here in a way that is parallel to the approach taken in the involuntary unemployment model, in section A.4. In particular, the exogenous 'parameters' are (A.46):

$$\bar{p}, m, u, \sigma_z, \sigma_L.$$

treated asand the 'endogenous' variables are:

$$F, \varsigma, a, \eta, h.$$

As in A.4, h is derived trivially from the parameters:

$$h = (1-u)m.$$

There now remain four endogenous variables to be solved for, F, ς, a, η, using (1)-(4). As before, we select F, ς, a, η to set these equations to zero for given values of the exogenous parameters.

C. Quantitative Properties of the Model Without Capital

This section explores various quantitative properties of our simple model without capital. The assumption of limited information lies at the heart of our model of unemployment, and we quantity the impact of this assumption by comparing the properties of the model with and without limited information. We also address a question raised by Lucas', analysis of the welfare cost of business cycles. Lucas used aggregate data and the assumption that insurance markets are perfect to argue that the welfare cost of business cycles is at most very small. In reality insurance markets are of course not perfect, but Lucas conjectured that this failure of his assumption does not have a first-order impact on his conclusions. We explore the validity of Lucas' conjecture in our model. We generate data from our involuntary unemployment model and investigate the impact on the estimate of the welfare cost of business cycles of wrongly assuming the existence of perfect insurance. The experiments we perform to reach this conclusion are preliminary and we indicate what it would take to do a full analysis. For example, a more reliable calculation would be one done on the model that we fit to the US data. However, we ran into a technical difficulty with this model. The first order condition for wage setting in that model takes the form of an infinite-ordered present value. To work with it, the expression must be written in recursive form. We were not able to express the actual non-linear first order condition in recursive form, though we can do so after it is linearized. The problem is that the welfare calculations require second order approximations. This is why we do not report the results of welfare calculations in our medium-sized DSGE model. We do, however, report calculations based on the model without capital, and our provisional finding is that Lucas' assumption of perfect insurance does not substantially distort his conclusions.

C.1. Parameter Values

To do the numerical calculations, we must assign values to our involuntary unemployment model parameters. The laws of motion of the exogenous shocks are as follows. Let x_t denote the shock. Then,

$$\log x_t = (1-\rho_x)\log x + \rho_x \log x_{t-1} + \varepsilon_t^x, \; E\left(\varepsilon_t^x\right)^2 = \left(\sigma_x\right)^2,$$

for $x = g, g_A, \varsigma$. The monetary policy shock, ε_t, is assumed to be an iid process with standard deviation denoted σ_R. Table A1 reports the values of a subset of parameters that are relatively uncontroversial. For example, we set values for the Taylor rule that imply the Taylor principle is satisfied (i.e., $r_\pi > 1$), we assume substantial interest rate smoothing (ρ_R is large) and the feedback on the output gap is modest (r_y is small). The steady state share, η_g, of government consumption to gross output is 20 percent. Government consumption is a long weighted average of current and past technology (i.e., γ is close to zero). The autocorrelation of the three shocks in the model is large. Finally, the innovation standard deviation of the three shocks was chosen so that each shock, when operative in isolation, causes the standard deviation of quarterly output growth in the model to be around 1 percent, the corresponding post WWII average in US data. The exception is the standard deviation of the preference shock which is set to zero throughout the analysis in this section.

We treated the structural parameters associated with households' job finding function, $p(e)$, and their disutility of labor differently. These parameters are:

$$F,\ \varsigma,\ a,\ \eta,\ \sigma_L. \tag{C.1}$$

We do not have direct observations on these parameters, nor are we aware of any estimates of these parameters in the literature. So, we chose values for them so that, conditional on the value of η_g given in Table A1, the steady state of the involuntary unemployment model implies the values of

$$m,\ u,\ 1/\sigma_z,\ 1/\kappa^{okun},\ \bar{p} \tag{C.2}$$

that are reported in the top left panel of Table A2. The indicated steady state value of the labor force participation rate, $m = 2/3$, corresponds to the average value of this variable in recent years. The steady state value of unemployment, $u = 0.056$, corresponds to the average value of unemployment in the post war US data. The compensated family labor supply elasticity, $1/\sigma_z = 2$, is roughly the corresponding object used in the real business cycle literature.[41] This is where the distinction between the compensated family labor supply elasticity and the Frisch elasticity is important. If $1/\sigma_z$ were interpretable as a Frisch elasticity, then the labor literature implies a value of $1/\sigma_z$ well below unity. However, as noted above, $1/\sigma_z$ has no connection to any individual agent's willingness to vary hours worked in response to a wage change in our model. The value of κ^{okun} was selected so that the model is consistent with a standard estimate of Okun's law. Finally, we chose a value for $\bar{p}$ to ensure that in the stochastic version of the model, the likelihood of violating the upper bound constraint on the job finding probability is small. The mapping from (C.2) to (C.1) using the steady state equilibrium conditions is described in detail in section A.4 of this technical appendix.

[41] A standard real business cycle model (see, e.g., the 'divisible labor' model in Christiano and Eichenbaum, 1992) uses preferences, $\sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \psi(1 - h_t)\right]$, where h_t denotes time worked of the representative agent, as a fraction of available time. The labor first order condition associated with the agent's optimal labor choice is $\psi C_t/(1 - h_t) = w_t$, where w_t denotes the real wage. The consumption compensated (actually 'consumption constant', with these preferences) labor supply function is (apart from a constant), $log(1 - h_t) = \log w_t$. This implies the following steady state elasticity of employment with respect to the wage:

$$\frac{d \log h_t}{d \log w_t} = -\frac{1 - h}{h} = -2.$$

Here, we assume that in steady state, 1/3 of available time is devoted to market work, i.e., $h = 1/3$.

C.2. Welfare

Following is a brief discussion of the welfare calculations that we report below. The approach is standard, and we discuss it here only for completeness. Let

$$W\left(g_{A,t}, p_{t-1}^{*}, \varsigma_{t}, g_{t}, \varepsilon_{t}, \sigma\right)$$

denote the welfare of the representative family in our model, conditional on the period t state, $g_{A,t}$, p_{t-1}^{*}, ς_{t}, g_{t}, ε_{t}. Here, $g_{A,t}$ denotes the period t realization of the growth rate of technology, the previous period's measure of price distortions is p_{t-1}^{*} and the current realization of the aggregate utility shock is ς_{t}. Finally, ε_{t} denotes the period t realization of the monetary policy shock. The parameter, σ, is a scalar that multiplies the 3×1 vector composed of ε_{t} and the innovations to $g_{A,t}$ and ς_{t}. Thus, $\sigma = 0$ corresponds to the nonstochastic version of the model and $\sigma = 1$ corresponds to the stochastic version. We think of σ as a continuous variable, $0 \leq \sigma \leq 1$. In the numerical examples considered, we allow only one shock to be stochastic at a time. We always set the preference shock, ς_{t}, to a constant.

We define the welfare cost of business cycles as

$$\Delta = W\left(g_{A,t}, p_{t-1}^{*}, g_{t}, \varepsilon_{t}, 0\right) - W\left(g_{A,t}, p_{t-1}^{*}, g_{t}, \varepsilon_{t}, 1\right).$$

We compute Δ for three models: the 'standard model' (i.e., the CGG model), our involuntary unemployment model, and our model with full information as described in section B of this appendix. We measure the impact on the welfare cost of business cycles of the assumption of imperfect insurance markets by comparing these three measures of welfare.

Our numerical strategy for approximating Δ is as follows. Perturbation methods can be used to solve the system by a second order approximation about nonstochastic steady state:

$$\varepsilon_{t} = 0, g_{A,t} = g_{A},\ p_{t-1}^{*} = 1,\ g_{t} = g,\ \sigma = 0. \tag{C.3}$$

Let x_{t} denote one of the variables, (A.33), in the system, $x = K_{p}, F_{p}, h, p^{*}, \pi, R, w$. The solution to x_{t} is a function of the state, $g_{A,t}$, $p_{t-1}^{*}, g_{t}, \varepsilon_{t}$. In addition, it is a function of σ. The second order approximate solution is witten

$$\begin{aligned}
x\left(g_{A,t}, p_{t-1}^{*}, \varsigma_{t}, \sigma\right) &\simeq \hat{x}\left(g_{A,t}, p_{t-1}^{*}, g_{t}, \varepsilon_{t}, \sigma\right) \\
&\equiv x^{ss} + x_{g_{A}}\left(g_{A,t} - g_{A}\right) + x_{p^{*}}\left(p_{t-1}^{*} - 1\right) + x_{g}\left(g_{t} - g\right) + x_{\varepsilon}\varepsilon_{t} + x_{\sigma}\sigma \\
&\quad + \frac{1}{2}\left[x_{g_{A},g_{A}}\left(g_{A,t} - g_{A}\right)^{2} + x_{p^{*},p^{*}}\left(p_{t-1}^{*} - 1\right)^{2} + x_{g,g}\left(g_{t} - g\right)^{2} + x_{\varepsilon\varepsilon}\varepsilon_{t}^{2} + x_{\sigma,\sigma}\sigma^{2}\right] \\
&\quad + x_{g_{A},p^{*}}\left(g_{A,t} - g_{A}\right)\left(p_{t-1}^{*} - 1\right) + x_{g_{A},g}\left(g_{A,t} - g_{A}\right)\left(g_{t} - g\right) \\
&\quad + x_{g_{A},\varepsilon}\left(g_{A,t} - g_{A}\right)\left(\varepsilon_{t} - \varepsilon\right) + x_{g_{A},\sigma}\left(g_{A,t} - g_{A}\right)\sigma \\
&\quad + x_{p^{*},g}\left(p_{t-1}^{*} - 1\right)\left(g_{t} - g\right) + x_{p^{*},\varepsilon}\left(p_{t-1}^{*} - 1\right)\varepsilon_{t} + x_{p^{*},\sigma}\left(p_{t-1}^{*} - 1\right)\sigma \\
&\quad + x_{g,\varepsilon}\left(g_{t} - g\right)\varepsilon_{t} + x_{g,\sigma}\left(g_{t} - g\right)\sigma.
\end{aligned}$$

Here, the subscripts on x denote differentiation with respect to the indicated arguments, evaluated at (C.3). Also, x^{ss} denotes the value of x in steady state, (C.3). It can be shown that $x_\sigma = x_{g_A,\sigma} = x_{p^*,\sigma} = x_{g,\sigma} = 0$. That is, uncertainty only affects the intercept terms in the second order approximation of policy rules. Slope terms are not affected. The objects in the second order approximation can be computed using the programming package, Dynare.

We define the cost of technology shocks by doing the above computations with all shock variances set to zero, except the innovation to $g_{A,t}$. Then, the welfare cost of the shocks, g_{At}, is

$$\hat{w}\left(g_{A,t}, p_{t-1}^*, \varsigma, 0\right) - \hat{w}\left(g_{A,t}, p_{t-1}^*, \varsigma, 1\right) = -\frac{w_{\sigma,\sigma}}{2}.$$

Interestingly, it is not necessary in the models we work with, for the above cost to be positive. Still, in the numerical examples we considered, $w_{\sigma,\sigma} < 0$. The above second derivative is reported by Dynare in the printout of the results when a second order approximation is requested.[42]

We need to translate a utility impact into consumption units. The present discounted value of utility in the stochastic equilibrium is:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - z\left(h_t, \varsigma\right)\right] = \hat{w}\left(g_{A,t}, p_{t-1}^*, g_t, \varepsilon_t, 1\right),$$

plus a constant term that is not dependent on the variance of the technology shock. When we shut down the variance of technology, utility is:

$$\bar{U}_0 = \sum_{t=0}^{\infty} \beta^t \left[\log \bar{C}_t - z\left(\bar{h}_t, \varsigma\right)\right] = \hat{w}\left(g_{A,t}, p_{t-1}^*, g_t, \varepsilon_t, 0\right),$$

plus a constant. The change in utility from shutting down the technology shock (say) is, approximately,

$$\Delta U = \bar{U}_0 - U_0 = \hat{w}\left(g_{A,t}, p_{t-1}^*, g_t, \varepsilon_t, 0\right) - \hat{w}\left(g_{A,t}, p_{t-1}^*, g_t, \varepsilon_t, 1\right) = -\frac{w_{\sigma,\sigma}}{2}.$$

We can convert ΔU into consumption units by asking the following. Suppose we increase consumption in the stochastic equilibrium from C_t to $(1+\lambda/100)\, C_t$, for all t for some constant λ. What would the value of λ have to be to make utility in the stochastic equilibrium equal to utility in the nonstochastic equilibrium? In the stochastic equilibrium with adjusted consumption, utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - z\left(h_t, \varsigma\right)\right] + \frac{\log\left(1+\lambda/100\right)}{1-\beta} = U_0 + \frac{\log\left(1+\lambda/100\right)}{1-\beta},$$

[42] We interpret the term, '(correction)', in the Dynare printout as corresponding to $w_{\sigma,\sigma}/2$.

using the fact that logs convert multiplication into addition. We want to find λ such that

$$U_0 + \frac{\log(1+\lambda/100)}{1-\beta} = \bar{U}_0,$$

or,

$$\lambda = 100\left[e^{-(1-\beta)\frac{w_{\sigma,\sigma}}{2}} - 1\right].$$

C.3. Numerical Experiments

The results are reported in Table A2. The first column displays the model parameters in the bottom panel. Note how low the replacement ratio is, 20 percent. This is much lower than what is reported in data. Our medium sized DSGE model displays a much higher replacement ratio because the preferences in that model have habit persistence. In effect, this creates a stronger desire for insurance, and this is manifest in a much higher replacement ratio.

The second column of the table indicates the impact of information per se. The structural parameters of our model are all the same in the second column. Notice from the first panel how more people join the labor force, and the employment rate in the population, h, is higher in that model. This is not surprising. Getting people to work is costlier in the involuntary unemployment model because a premium has to be offered to people that are lucky to find work, and this in effect reduces insurance. In the full information model, no insurance sacrifice is required to get people to work. In that model, the family issues instructions to each household that are contingent on that household's aversion to work. No incentives are required to enforce those instructions, and so the family can allocate consumption equally across families. We calculate the price of information by asking what constant percent increase in consumption (analogous to λ in the previous section) would make households in the involuntary unemployment model indifferent between remaining in that world and going to the full information world. The price is 0.2 percent of consumption.

The third and fourth columns represents our attempt to address the Lucas conjecture. Those columns address the question, "what would an econometrician living in our involuntary unemployment world, who incorrectly assumes insurance markets are perfect, conclude about the welfare cost of business cycles?". The correct approach to this question, in our view, is to make a plausible assumption about the data available to the econometrician, e.g., output, consumption, government consumption, inflation, the interest rate generated by the model in column 1 of Table A2. A reasonable first step would avoid sampling uncertainty by supposing that the econometrician has a large data set. Then, one identifies the parameter values that the econometrician who does Gaussian maximum likelihood would estimate (the Gaussian likelihood function is easy to characterize when there is a large sample of data by expressing it in the frequency domain). We did not do this computation. We did an alternative computation instead, and we describe this when we describe our experiments in detail below.

In addition to deciding how the econometrician comes up with parameter values, one has to determine exactly what model the econometrician works with. Our first approach assumes that the econometrician uses the full information model described in section B. That model, relative to the model in the first column has precisely one specification error. It makes the mistake of supposing the family has all information on households, whereas in fact the family only knows the household's employment status. A second approach is to imagine the econometrician uses what we call the standard model, the CGG model. This in effect makes two specification errors. It assumes effort is not required to find a job and although households still face risk in terms of their aversion to work, they have full consumption insurance against this risk. The results in column three ('Full information model, observational equivalent') refer to the first approach and the results in the next column refer to the second. The first approach has the appealing feature that it isolates clearly the nature of the specification error being commited. The second approach has the appealing feature that it works with the standard, simple utility function used in practice in the analysis of DSGE models.

C.3.1. The Full Information Model

For our computation, we assumed that the data are generated by the model in column 1 of Table A2. The model used by the econometrician is the full information model. We imagine that the econometrician is provided with m and h, the steady state values of the labor force and hours worked, respectively. Of course, a trivial identify then permits him to compute the steady state unemployment rate. The econometrician is also provided with the parameter values reported in Table A1. Regarding the Taylor rule, we may perhaps suppose that he has data on the variables in that equation (including the output gap!) and he runs an instrumental variables regression on the policy rule, using the lagged gap, inflation and interest rates as instruments (recall, the policy shock is in fact iid, and the econometrician knows it). Regarding the price stickiness parameter, we may interpret the econometrician's knowing the value of this parameter as reflecting that he read a micro study about price stickiness. In the case of λ_f, he may have gotten this parameter from an Industrial Organization colleague. The steady state of technology growth can be inferred from the average growth rate of output in the economy. The steady state government consumption to GDP ratio could have been computed from a time series average of that variable.

The econometrician is also provided with the values of κ^{Okun} and σ_z. We may suppose that ordinary least squares using data on the inflation rate and the output gap is used to estimate the slope of the Phillips curve, (2.45). Then, σ_z can be unravelled from the slope given the known values of ξ_p, β and η_g. Similarly, κ^{Okun} may be found by computing the ratio of the output gap and the unemployment gap in any date. It turns out that this is not enough for the econometrician to solve the model. For this, he needs to determine values of the five parameters in (C.1). We need to provide him with one additional piece of information. We provide the econometrician with $\bar{p}$, the steady state probability of finding a job by the household who has the least aversoin to work, $l = 0$. The fact that we must provide $\bar{p}$ indicates that, of the data we have already provided, there is not enough to determine the values of the parameters in (C.1).

Given this strategy attributed to the econometrician for parameterizing his misspecified model, the results are given in the third column of table A2. Note how the results in the first panel are the same for the two models (except, of course, for the replacement ratio). The structual parameters, (C.1), estimated by the econometrician (see the middle panel) are of course different from their correct values, the ones reported in the first column. The inconsistency of the econometrician's estimator reflects the specification error he commits when he assumes that the family has full information. The inconsistency in two parameters, η and σ_L, is so small it is not evident given the number of significant digits reported in the table. The inconsistency of the econometrician's estimator of the three other parameters, however, is substantial.

The bottom panel reports the welfare cost of business cycles computed by the econometrician using his estimated model. This is to be compared with the true welfare cost of inflation, reported in the first column. The notable result here is that the welfare costs are all identical to many significant digits, regarless of the shocks considered. This result was a surprise to us. It is true that the econometrician estimates, relative to the data provided to him, a model that is observationaly equivalent to the true model. But, this estimation is presumed to be done using the first order properties of the model. Because labor supply is part of the system (and this involves the first derivative of the utility function), this means that the econometrician using first order expansions chooses parameters to match the second derivative of the utility function. But, welfare is computed by taking a second order expansion of the system, and so one presumes that it involves the third derivative of the utility function. That is, our welfare computation focuses on a feature of the utility function (i.e., its third derivative) that the econometrician has no reason to match to the corresponding true value in the process of estimation. This is why we expected the welfare cost of business cycles to be different between the models. The fact that the numbers are numerically so extremely close suggests to us that there exists a theorem that declares the numbers are mathematically the same. We have not developed a proof for that theorem. From this perspective, it is interesting to note that when we perturbed the computations in particular ways, the actual magnitude of the welfare losses did not change. In particular, we perturbed the parameters of the data generating mechanism, i.e., the model in the first column, so as to change the values of the endogenous variables provided to the econometrician. In these experiments, we only considered the case of neutral technology shocks (we suppose that the results would have been had we worked with the other shocks). We redid the computations holding $\bar{p}$, h, m, σ_z fixed at their values in Table A2 and changed the value of σ_L (thereby implicitly changing κ^{Okun}) to 4. We then changed the value of $\bar{p}$ to 0.97, holding h, m, σ_z, σ_L fixed at their values reported in the table. We changed the value of u to 0.07, holding $\bar{p}$, m, σ_z, σ_L to their values in the table. In all these cases, the magnitude of the welfare cost of business cycles remained unchanged. Only when we changed σ_z (to unity) did the welfare cost of business cycles change. However, even in this case, the econometrician's estimate of the welfare cost of business cycles is the same, to 10 digits, as the true cost.[43]

The computations in the third column of the table are consistent with the proposition that abstracting from incomplete insurance does not distort assessments about the welfare cost of business cycles.

C.3.2. The Standard Model

Consider now the fourth column. This column presents results under the assumption that the econometrician's model corresponds to the standard model, the one in which the family utility function is given by (3.21) with $b = 0$. Again, we must specify what the econometrician does to parameterize his misspecified model. As before, we assume that the econometrician knows the values of the parameters in Table A1. Now, however, the econometrician has only to estimate two parameters of the utility function, the curvature and slope parameters, σ_L, and ς, respectively.

We suppose that the econometrician knows the slope of the Phillips curve and can therefore recover σ_z. In the econometician's model the value of σ_L coincides with that of σ_z. The econometrician then estimates ς by requiring that steady state hours worked in his model coincide with the actual steady state of hours worked. We suppose that the econometrician simply ignores unemployment and labor force data, as in standard monetary DSGE models.

The results are given in the fourth column of table A2. The key finding in Table A2 is that the welfare cost of business cycles estimated by the econometrician coincides with the true welfare cost, up to 12 significant digits!

[43] In this case, the welfare cost of business cycles is 0.553167318878733 for column 1 and 0.553167318879688 for column 3.

D. Non-Separability in Utility

In the manuscript, we work with a household utility function in which consumption and leisure are additively separable. In this appendix, we show that the analysis can also easily be done with two non-separable utility functions that have been used extensively in the literature.

D.1. King-Plosser-Rebelo Preferences

We replace the preferences in (2.4) with:

$$p(e_t)\frac{(c_t^w)^{1-\gamma}}{1-\gamma}v(l)+(1-p(e_t))\frac{(c_t^{nw})^{1-\gamma}}{1-\gamma}v(0)-\frac{1}{2}\frac{e_t^2}{1-\gamma},\ \gamma\ >\ 1$$
$$v(l)=F+\varsigma_t(1+\sigma_L)l^{\sigma_L}.$$

With these preferences, utility is decreasing in l, and the marginal utility of consumption is increasing in l. In this case, the household with aversion to work, l, sets

$$e_{l,t}=\max\left\{a\left[(c_t^w)^{1-\gamma}v(l)-(c_t^{nw})^{1-\gamma}v(0)\right],0\right\}.$$

In this case, households that participate in the labor market receive utility

$$\frac{1}{2}a^2\left[\frac{(c_t^w)^{1-\gamma}}{1-\gamma}v(l)-\frac{(c_t^{nw})^{1-\gamma}}{1-\gamma}v(0)\right]^2+\frac{(c_t^{nw})^{1-\gamma}}{1-\gamma}v(0),$$

while households that do not participate receive

$$\frac{(c_t^{nw})^{1-\gamma}}{1-\gamma}v(0).$$

The incentive constraint (i.e., the analog of (2.9)) requires:

$$(c_t^w)^{1-\gamma}v(m)=(c_t^{nw})^{1-\gamma}v(0) \tag{D.1}$$

The mapping from the labor force, m_t, to the number of people working, h_t, is given by:

$$h_t=\int_0^{m_t}p(e_{l,t})\,dl=a^2\frac{(c_t^w)^{1-\gamma}}{\gamma-1}\varsigma_t(1+\sigma_L)\int_0^{m_t}\left[m_t^{\sigma_L}-l^{\sigma_L}\right]dl=a^2\frac{(c_t^w)^{1-\gamma}}{\gamma-1}\varsigma_t\sigma_L m_t^{\sigma_L+1}. \tag{D.2}$$

Combining the resource constraint, (2.18), with the incentive constraint, (D.1), we obtain:

$$c_t^w = \frac{C_t}{h_t+(1-h_t)\left[\frac{v(m_t)}{v(0)}\right]^{\frac{1}{1-\gamma}}} \tag{D.3}$$

$$c_t^{nw} = \frac{C_t\left[\frac{v(m_t)}{v(0)}\right]^{\frac{1}{1-\gamma}}}{h_t+(1-h_t)\left[\frac{v(m_t)}{v(0)}\right]^{\frac{1}{1-\gamma}}}. \tag{D.4}$$

Integrating utility over all the households in the family, the analog of (2.20) is:

$$u\left(c_t^w, c_t^{nw}, m_t\right) = a^2 \frac{\left(c_t^w\right)^{2(1-\gamma)}}{(1-\gamma)^2}\left(1+\sigma_L\right)\frac{\sigma_L^2 m_t^{2\sigma_L+1}}{(2\sigma_L+1)} + \frac{\left(c_t^{nw}\right)^{1-\gamma}}{1-\gamma} v(0). \tag{D.5}$$

Equations (D.2), (D.3) and (D.4) provide a mapping from C_t and h_t to c_t^w, c_t^{nw} and m_t. Utility is then given by (D.5). Thus, we have family utility in terms of C_t and h_t only.

D.2. Greenwood-Hercowitz-Huffman Preferences

We now replace the preferences in (2.4) with:

$$p(e_t)\frac{\left(c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right)^{1-\gamma} - 1}{1-\gamma} + (1 - p(e_t))\frac{\left(c_t^{nw}+F\right)^{1-\gamma} - 1}{1-\gamma} - \frac{1}{2}e_t^2,\ \gamma > 0.$$

In this case,

$$e_{l,t} = \max\left\{a\left[\frac{\left(c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right)^{1-\gamma} - \left(c_t^{nw}+F\right)^{1-\gamma}}{1-\gamma}\right], 0\right\},$$

so that the utility of a household that participates in the labor force is:

$$\frac{1}{2}a^2\left[\frac{\left(c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right)^{1-\gamma} - \left(c_t^{nw}+F\right)^{1-\gamma}}{1-\gamma}\right]^2 + \frac{\left(c_t^{nw}+F\right)^{1-\gamma} - 1}{1-\gamma}.$$

Comparing this with the utility of households that choose to be out of the labor force, we obtain the incentive constraint:

$$c_t^w - (1+\sigma_L) m_t^{1+\sigma_L} = c_t^{nw} \tag{D.6}$$

The mapping between the labor force and the number of people working is provided by:

$$h_t = \int_0^{m_t} p(e_{l,t})\, dl = \frac{a^2}{1-\gamma}\int_0^{m_t}\left[\begin{array}{c}\left(c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right)^{1-\gamma} \\ -\left(c_t^w + F - (1+\sigma_L) m_t^{1+\sigma_L}\right)^{1-\gamma}\end{array}\right] dl. \tag{D.7}$$

Now,

$$\begin{aligned}
& \int_0^{m_t}\left[c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right]^{1-\gamma} dl \\
= \quad & m_t\left[c_t^w + F - (1+\sigma_L) m_t^{1+\sigma_L}\right]^{1-\gamma}\left(\frac{c_t^w + F - (1+\sigma_L) m_t^{(1+\sigma_L)}}{c_t^w + F}\right)^{-(1-\gamma)} \\
& \times\mathcal{F}\left(\left[\begin{array}{cc} -(1-\gamma) & \frac{1}{1+\sigma_L}\end{array}\right]; 1 + \frac{1}{1+\sigma_L}; \frac{(1+\sigma_L) m_t^{1+\sigma_L}}{c_t^w + F}\right),
\end{aligned}$$

where

$$\mathcal{F}(x; a; b)$$

denotes the hypergeometric function where x is a 1×2 row vector and a and b are scalars.[44] In this way, (D.7) defines a mapping from c_t^w, c_t^{nw} and m_t to h_t :

$$h_t = f\left(c_t^w, c_t^{nw}, m_t\right). \tag{D.8}$$

Combining the resource constraint, (2.18), with the incentive constraint, (D.6), we obtain:

$$\begin{aligned} c_t^w &= C_t + (1 - h_t)(1 + \sigma_L) m_t^{1+\sigma_L} && \text{(D.9)} \\ c_t^{nw} &= C_t - h_t (1 + \sigma_L) m_t^{1+\sigma_L} && \text{(D.10)} \end{aligned}$$

Ex ante utility of all the households in the family, the analog of (2.20), is (after using the incentive constraint):

$$\begin{aligned} u\left(c_t^w, c_t^{nw}, m_t\right) &= \frac{1}{2}\frac{a^2}{(1-\gamma)^2}\int_0^{m_t}\left[\begin{array}{c}\left(c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right)^{1-\gamma} \\ -\left(c_t^{nw} + F\right)^{1-\gamma}\end{array}\right]^2 dl \\ &\quad + \frac{\left(c_t^{nw} + F\right)^{1-\gamma} - 1}{1-\gamma} \end{aligned} \tag{D.11}$$

Expanding the square term:

$$\begin{aligned} & \int_0^{m_t}\left[\left(c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right)^{1-\gamma} - \left(c_t^{nw} + F\right)^{1-\gamma}\right]^2 dl \\ = & \int_0^{m_t}\left(c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right)^{2(1-\gamma)} dl \\ & -2\left(c_t^{nw} + F\right)^{1-\gamma}\int_0^{m_t}\left(c_t^w + F - (1+\sigma_L) l^{1+\sigma_L}\right)^{2(1-\gamma)} dl \\ & + m_t\left(c_t^{nw} + F\right)^{2(1-\gamma)}, \end{aligned}$$

which can be evaluated using formulas analogous to the one after (D.7). Equations (D.8), (D.9) and (D.10) provide a mapping from C_t and h_t to c_t^w, c_t^{nw} and m_t. Utility is then given by (D.11). Thus, we have family utility in terms of C_t and h_t only.

E. Solving the Model Used in the Empirical Analysis

We first derive the equilibrium conditions associated with optimal wage setting. We then indicate the remaining equilibrium conditions of the model. Finally, we describe a strategy for solving the model's steady state.

[44] This formula may be found at http://integrals.wolfram.com/index.jsp. In MATLAB, the hypergeometric function is evaluated using hypergeom(x,a,b).

E.1. Scaling of Variables

We adopt the following scaling of variables. The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable, Ψ_t, is an embodied shock to technology and it is convenient to define the following combination of embodied and neutral technology:

$$\begin{aligned} z_t^+ &\equiv \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &\equiv \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{E.1}$$

Capital, $\bar{K}_t$, and investment, I_t, are scaled by $z_t^+\Psi_t$. Consumption goods C_t, government consumption G_t and the real wage, W_t/P_t are scaled by z_t^+. Also, υ_t is the multiplier on the nominal household budget constraint in the Lagrangian version of the household problem. That is, υ_t is the marginal utility of one unit of currency. The marginal utility of a unit of consumption is $\upsilon_t P_t$. The latter must be multiplied by z_t^+ to induce stationarity. Thus, our scaled variables are:

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{z_t^+\Psi_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+\Psi_t},\ i_t = \frac{I_t}{z_t^+\Psi_t},\ c_t = \frac{C_t}{z_t^+},\ g_t = \frac{G_t}{z_t^+},\ \bar{w}_t = \frac{W_t}{z_t^+P_t} \\ \psi_{z^+,t} &= \upsilon_t P_t z_t^+,\ \tilde{y}_t = \frac{Y_t}{z_t^+},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t},\ w_t = \frac{\tilde{W}_t}{W_t},\ \dddot{w}_t \equiv \frac{\dddot{W}_t}{W_t},\ \overset{\infty}{w}_t = \frac{\overset{\infty}{W}_t}{W_t}. \end{aligned} \tag{E.2}$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k',t} = \Psi_t P_{k',t},\ \bar{r}_t^k = \Psi_t r_t^k.$$

where $P_{k',t}$ is in units of the homogeneous good. We define the following inflation rates:

$$\pi_t = \frac{P_t}{P_{t-1}},\ \pi_t^i = \frac{P_t^i}{P_{t-1}^i}.$$

Here, P_t is the price of the homogeneous output good and P_t^i is the price of the domestic final investment good.

E.2. Wage Setting by the Family

We consider the problem of a monopolist who represents households that supply the type j labor service. That monopolist optimizes the utility function of $j-$type households, (2.21), subject to Calvo frictions. With probability $1-\xi_w$ the monopolist reoptimizes the wage and with probability ξ_w the monopolist sets the current wage rate according to (3.10). In each period, type j households supply the quantity of labor dictated by demand, (3.6). Because the $j-$type family has perfect consumption insurance, the monopolist can take the $j-$type family's consumption as given. However, the monopolist does assign a weight to the revenues from $j-$type labor that corresponds to the value, υ_t, assigned to income by the family. Ignoring terms beyond the control of the monopolist the monopolist seeks to maximize:

$$E_t^j \sum_{i=0}^{\infty} \beta^i \left[-z\left(h_{t+i,j}, \zeta_{t+i}\right) + \upsilon_{t+i} W_{t+i,j} h_{t+i,j} \right].$$

Here, υ_t denotes the Lagrange multiplier on the type j family's time t flow budget constraint, (2.2). The function, z, is defined in (2.21), but we reproduce it here for convenience:

$$z\left(h_{j,t}, \zeta_t\right) = \log\left[h_{j,t}\left(e^{F+\varsigma(1+\sigma_L)f(h_t)^{\sigma_L}} - 1\right) + 1\right] - \frac{a^2\varsigma^2\left(1+\sigma_L\right)\sigma_L^2}{2\sigma_L+1} f\left(h_{j,t}\right)^{2\sigma_L+1} - \eta\varsigma\sigma_L f\left(h_{j,t}\right)^{\sigma_L+1}.$$

Here, $f\left(h_{j,t}, \zeta_t\right)$ is the unique value of m_t that satisfies

$$h_{j,t} = Q\left(m_{j,t}, \zeta_t\right) \equiv m_{j,t}\eta + a^2\varsigma_t\sigma_L m_{j,t}^{\sigma_L+1}, \tag{E.3}$$

for a given value of m_t. That is,

$$m_{j,t} = f\left(h_{j,t}, \zeta_t\right) \equiv Q^{-1}\left(h_{j,t}, \zeta_t\right),$$

where Q^{-1} is the inverse function of Q. In the case of the standard model, z is implicitly defined in (3.21).

E.2.1. Differentiating the Family Disutility of Labor

In the calculations that follow, we require the derivatives of z and f, evaluated in steady state. In the case of the standard model, these calculations are trivial. We compute the derivatives for our model with involuntary unemployment here. We drop the j subscript for convenience, as well as the stochastic shock. From the definition of the inverse function,

$$m_t = f\left(Q\left(m_t\right)\right).$$

We find the derivatives of f by differentiating this expression twice with respect to m_t :

$$\begin{aligned} 1 &= f'\left(Q\left(m_t\right)\right) Q'\left(m_t\right) \\ 0 &= f''\left(Q\left(m_t\right)\right)\left[Q'\left(m_t\right)\right]^2 + f'\left(Q\left(m_t\right)\right) Q''\left(m_t\right) \end{aligned}$$

From the first expression,

$$f'\left(Q\left(m_t\right)\right) = \frac{1}{Q'\left(m_t\right)}.$$

Substituting this into the second expression and solving:

$$0 = f''\left(Q\left(m_t\right)\right)\left[Q'\left(m_t\right)\right]^2 + \frac{Q''\left(m_t\right)}{Q'\left(m_t\right)},$$

so that

$$f''\left(Q\left(m_t\right)\right) = -\frac{Q''\left(m_t\right)}{\left[Q'\left(m_t\right)\right]^3},$$

From (E.3),

$$\begin{aligned} Q'\left(m_t\right) &= \eta + \left(\sigma_L + 1\right) a^2 \varsigma \sigma_L m_t^{\sigma_L} \\ Q''\left(m_t\right) &= \left(\sigma_L + 1\right) a^2 \varsigma \sigma_L^2 m_t^{\sigma_L - 1}, \end{aligned}$$

so that, in steady state,

$$\begin{aligned} f_h &= \frac{1}{\eta + \left(\sigma_L + 1\right) a^2 \varsigma \sigma_L m^{\sigma_L}} \\ f_{hh} &= -\frac{\left(\sigma_L + 1\right) a^2 \varsigma \sigma_L^2 m^{\sigma_L - 1}}{\left[\eta + \left(\sigma_L + 1\right) a^2 \varsigma \sigma_L m^{\sigma_L}\right]^3}, \end{aligned} \tag{E.4}$$

where m denotes the steady state value of m, computed below.

Let,

$$\begin{aligned} Z\left(m_t\right) &= \log\left[Q\left(m_t\right)\left(e^{F + \varsigma\left(1 + \sigma_L\right) m_t^{\sigma_L}} - 1\right) + 1\right] - \frac{a^2 \varsigma^2 \left(1 + \sigma_L\right) \sigma_L^2}{2\sigma_L + 1} m_t^{2\sigma_L + 1} - \eta \varsigma \sigma_L m_t^{\sigma_L + 1} \\ h_t &= Q\left(m_t\right) \equiv m_t \eta + a^2 \varsigma \sigma_L m_t^{\sigma_L + 1}, \end{aligned}$$

so that

$$z\left(h_t\right) \equiv Z\left(Q^{-1}\left(h_t\right)\right) = Z\left(f\left(h_t\right)\right),$$

and

$$\begin{aligned} z_h\left(h_t\right) &= Z'\left(f\left(h_t\right)\right) f_h\left(h_t\right) \\ z_{hh}\left(h_t\right) &= Z''\left(f\left(h_t\right)\right)\left[f_h\left(h_t\right)\right]^2 + Z'\left(f\left(h_t\right)\right) f_{hh}\left(h_t\right). \end{aligned}$$

Evaluating this in steady state,

$$z_h = Z' f_h, \;\; z_{hh} = Z'' f_h^2 + Z' f_{hh}.$$

In this case,

$$\sigma_z \equiv \frac{z_{hh} h}{z_h} = \frac{\left[Z'' f_h^2 + Z' f_{hh}\right] h}{Z' f_h} = \frac{Z'' f_h h}{Z'} + \frac{f_{hh} h}{f_h}$$

From (E.4),

$$f_h = \frac{1}{Q'}, \;\; f_{hh} = -\frac{Q''}{\left[Q'\right]^3},$$

so that

$$\sigma_z = \frac{Z'' f_h h}{Z'} + \frac{f_{hh} h}{f_h}$$
$$= \frac{Z'' h}{Z' Q'} - \frac{Q'' h}{[Q']^2}$$
$$= \frac{Q}{Q'}\left[\frac{Z''}{Z'} - \frac{Q''}{Q'}\right]$$

E.2.2. First Order Condition Associated with Family Wage Setting

Consider the monopoly wage setter, j, that has an opportunity to reoptimize the wage rate. The objective function with $h_{t+i,j}$ substituted out using labor demand, (3.6), and ignoring terms beyond the control of the monopolist, is as follows:

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i [-z\left(\zeta_{t+i}, \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)$$
$$+\upsilon_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}],$$

where

$$\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}$$

is the nominal wage rate of the monopolist which sets wage $\tilde{W}_t$ in period t and cannot reoptimize again afterward. Also, z is the function described in the case of our model with unemployment, and it is (3.21) in the case of the standard model. We adopt the following scaling convention:

$$w_t = \frac{\tilde{W}_t}{W_t},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t},\ \psi_{z^+,t} = \upsilon_t P_t z_t^+.$$

With this notation, the objective can be written,

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i [-z\left(\zeta_{t+i}, \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right) + \psi_{z^+,t+i} w_t^{\frac{1}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}],$$

where:

$$X_{t,i} = \frac{\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+i}\pi_{t+i-1} \cdots \pi_{t+1} \mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}.$$

Differentiating with respect to w_t,

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i [-z_{h,t+i}^t \frac{\lambda_w}{1-\lambda_w} w_t^{\frac{\lambda_w}{1-\lambda_w}-1} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}$$
$$+\frac{1}{1-\lambda_w} \psi_{z^+,t+i} w_t^{\frac{1}{1-\lambda_w}-1} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}],$$

where

$$z_{h,t+i}^{t} \equiv z_h \left(\zeta_{t+i}, \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right).$$

Here, $h_{2,t+i}^{t}$ denotes the marginal utility of labor in period $t+i$, for a monopolist who last reoptimized the wage rate in period t. Dividing and rearranging,

$$E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} [\psi_{z^+,t+i} w_t \bar{w}_t X_{t,i} - \lambda_w z_{h,t+i}^{t}] = 0. \tag{E.5}$$

The first object in square brackets is the marginal utility real wage in period $t+i$ and the second is a markup, λ_w, over the marginal utility cost of working. According to (E.5) the monopolist attempts to set a weighted average of the term in square brackets to zero. The structure of $z_{z,t+i}^{t}$ makes it difficult to express (E.5) in recursive form. This is because we have not found a way to express $z_{h,t+1}^{t} = Z_t z_{h,t+1}^{t+1}$, for some variable, Z_t. The expression, (E.5), is recursive after linearizing it about steady state. Thus,

$$\hat{z}_{h,t+i}^{t} \equiv \frac{dz_h \left(\zeta_{t+i}, \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)}{z_h \left(\zeta, w^{\frac{\lambda_w}{1-\lambda_w}} H \right)},$$

where a variable without a time subscript denotes non-stochastic steady state. Expanding this expression:

$$\hat{z}_{h,t+i}^{t} = \sigma_\varsigma \hat{\zeta}_{t+i} + \alpha_{h,1} \left(\hat{w}_t + \hat{\bar{w}}_t - \hat{\bar{w}}_{t+i} + \hat{X}_{t,i} \right) + \sigma_z \hat{H}_{t+i},$$

where,

$$\sigma_\varsigma \equiv \frac{z_{h\zeta} \zeta}{z_h}, \ \sigma_z \equiv \frac{z_{hh} H}{z_h}, \ \alpha_{h,1} \equiv \frac{\lambda_w}{1-\lambda_w} \sigma_z.$$

Also,

$$\hat{X}_{t,i} = \hat{\tilde{\pi}}_{w,t+i} + \cdots + \hat{\tilde{\pi}}_{w,t+1} - \hat{\pi}_{t+i} - \hat{\pi}_{t+i-1} - \cdots - \hat{\pi}_{t+1} - \hat{\mu}_{z^+,t+i} - \cdots - \hat{\mu}_{z^+,t+1}.$$

However, note:

$$\hat{\tilde{\pi}}_{w,t+1} = \kappa_w \hat{\pi}_t.$$

Then,

$$\hat{X}_{t,i} = -\Delta_{\kappa_w} \hat{\pi}_{t+i} - \Delta_{\kappa_w} \hat{\pi}_{t+i-1} - \cdots - \Delta_{\kappa_w} \hat{\pi}_{t+1} - \hat{\mu}_{z^+,t+i} - \cdots - \hat{\mu}_{z^+,t+1},$$

where

$$\Delta_{\kappa_w} \equiv 1 - \kappa_w L,$$

where L denotes the lag operator.

Write out (E.5) in detail:

$$
\begin{aligned}
& H_t[\psi_{z^+,t} w_t \bar{w}_t - \lambda_w z_{h,t}^t] \\
& +\beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} X_{t,1}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+1}[\psi_{z^+,t+1} w_t \bar{w}_t X_{t,1} - \lambda_w z_{h,t+1}^t] \\
& +(\beta\xi_w)^2 \left(\frac{\bar{w}_t}{\bar{w}_{t+2}} X_{t,2}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2}[\psi_{z^+,t+2} w_t \bar{w}_t X_{t,2} - \lambda_w z_{h,t+2}^t] + \ldots = 0
\end{aligned}
$$

In expanding this expression, we can simply set the terms outside the square brackets to their steady state values. The reason is that the term inside the brackets are equal to zero in steady state. Thus, the expansion of the previous expression about steady state:

$$
\begin{aligned}
& H[d\left(\psi_{z^+,t} w_t \bar{w}_t\right) - \lambda_w d\left(z_{h,t}^t\right)] \\
& +\beta\xi_w H[d\left(\psi_{z^+,t+1} w_t \bar{w}_t X_{t,1}\right) - \lambda_w d\left(z_{h,t+1}^t\right)] \\
& +(\beta\xi_w)^2 H[d\left(\psi_{z^+,t+2} w_t \bar{w}_t X_{t,2}\right) - \lambda_w d\left(z_{h,t+2}^t\right)] + \ldots = 0
\end{aligned}
$$

or,

$$
\begin{aligned}
& H[\psi_{z^+}\bar{w}\left(\hat{\psi}_{z^+,t} + \hat{w}_t + \widehat{\bar{w}}_t\right) - \lambda_w z_h \hat{z}_{h,t}^t] \\
& +\beta\xi_w H[\psi_{z^+}\bar{w}\left(\hat{\psi}_{z^+,t+1} + \hat{w}_t + \widehat{\bar{w}}_t + \hat{X}_{t,1}\right) - \lambda_w z_h \hat{z}_{h,t+1}^t] \\
& +(\beta\xi_w)^2 H[\psi_{z^+}\bar{w}\left(\hat{\psi}_{z^+,t+2} + \hat{w}_t + \widehat{\bar{w}}_t + \hat{X}_{t,2}\right) - \lambda_w z_h \hat{z}_{h,t+2}^t] + \ldots = 0
\end{aligned}
$$

Note that in steady state, $\psi_{z^+}\bar{w} = \lambda_w z_h$, so that, after multiplying by $1/\left(H\psi_{z^+}\bar{w}\right)$, we obtain:

$$
\begin{aligned}
& \hat{\psi}_{z^+,t} + \hat{w}_t + \widehat{\bar{w}}_t - \hat{z}_{h,t}^t \\
& +\beta\xi_w[\hat{\psi}_{z^+,t+1} + \hat{w}_t + \widehat{\bar{w}}_t + \hat{X}_{t,1} - \hat{z}_{h,t+1}^t] \\
& +(\beta\xi_w)^2[\hat{\psi}_{z^+,t+2} + \hat{w}_t + \widehat{\bar{w}}_t + \hat{X}_{t,2} - \hat{z}_{h,t+2}^t] + \ldots = 0
\end{aligned}
$$

Substitute out for $\hat{z}_{h,t+i}^t$ and $\hat{X}_{t,i}$:

$$
\begin{aligned}
0 \;=\; & \hat{\psi}_{z^+,t} + \hat{w}_t + \widehat{\bar{w}}_t - \left[\sigma_\varsigma\hat{\zeta}_t + \alpha_{h,1}\hat{w}_t + \sigma_z\hat{H}_t\right] \\
& \\
& +\beta\xi_w[\hat{\psi}_{z^+,t+1} + \hat{w}_t + \widehat{\bar{w}}_t - \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right) \\
& -\left(\sigma_\varsigma\hat{\zeta}_{t+1} + \alpha_{h,1}\left(\hat{w}_t + \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t+1} - \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right)\right) + \sigma_z\hat{H}_{t+1}\right)] \\
& \\
& +(\beta\xi_w)^2[\hat{\psi}_{z^+,t+2} + \hat{w}_t + \widehat{\bar{w}}_t - \left(\Delta_{\kappa_w}\hat{\pi}_{t+2} + \hat{\mu}_{z^+,t+2}\right) - \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right) \\
& -\left(\sigma_\varsigma\hat{\zeta}_{t+2} + \alpha_{h,1}\left(\begin{array}{c} \hat{w}_t + \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t+2} \\ -\left(\Delta_{\kappa_w}\hat{\pi}_{t+2} + \hat{\mu}_{z^+,t+2}\right) - \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right) \end{array}\right) + \sigma_z\hat{H}_{t+2}\right)] + \ldots
\end{aligned}
$$

Collecting terms:

$$\begin{aligned}
0 &= \sum_{j=0}^{\infty} (\beta\xi_w)^j \left[\hat{\psi}_{z^+,t+j} - \left(\sigma_\varsigma \hat{\zeta}_{t+j} + \sigma_z \hat{H}_{t+j}\right)\right] + \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\hat{w}_t \\
&\quad + \frac{1-\alpha_{h,1}\beta\xi_w}{1-\beta\xi_w}\widehat{\bar{w}}_t + \alpha_{h,1}\sum_{j=1}^{\infty}(\beta\xi_w)^j\,\widehat{\bar{w}}_{t+j} \\
&\quad - (1-\alpha_{h,1})\,\beta\xi_w\left[\left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right)\right] \\
&\quad - (1-\alpha_{h,1})\,(\beta\xi_w)^2\left[\left(\Delta_{\kappa_w}\hat{\pi}_{t+2} + \hat{\mu}_{z^+,t+2}\right) + \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right)\right] \\
&\quad - ...
\end{aligned}$$

or,

$$\begin{aligned}
0 &= \sum_{j=0}^{\infty} (\beta\xi_w)^j \left[\hat{\psi}_{z^+,t+j} - \left(\sigma_\varsigma \hat{\zeta}_{t+j} + \sigma_z \hat{H}_{t+j}\right)\right] + \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\hat{w}_t \\
&+ \frac{1-\alpha_{h,1}\beta\xi_w}{1-\beta\xi_w}\widehat{\bar{w}}_t + \sum_{j=1}^{\infty}(\beta\xi_w)^j\left[\alpha_{h,1}\widehat{\bar{w}}_{t+j} - \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+j} + \hat{\mu}_{z^+,t+j}\right)\right].
\end{aligned}$$

Note

$$\begin{aligned}
S_t &= X_t + \beta\xi_w X_{t+1} + (\beta\xi_w)^2 X_{t+2} + ... \\
&= X_t + \beta\xi_w \overbrace{[X_{t+1} + \beta\xi_w X_{t+2} + ...]}^{S_{t+1}},
\end{aligned}$$

so that the log-linearized first order condition can be written:

$$0 = F_{,t} + \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\hat{w}_t + \frac{1-\alpha_{h,1}\beta\xi_w}{1-\beta\xi_w}\widehat{\bar{w}} + G_t, \tag{E.6}$$

where

$$\begin{aligned}
F_t &= \sum_{j=0}^{\infty} (\beta\xi_w)^j \left[\hat{\psi}_{z^+,t+j} - \left(\sigma_\varsigma \hat{\zeta}_{t+j} + \sigma_z \hat{H}_{t+j}\right)\right] \\
&= \hat{\psi}_{z^+,t} - \left(\sigma_\varsigma \hat{\zeta}_t + \sigma_z \hat{H}_t\right) + \beta\xi_w F_{t+1} \\
G_t &= \sum_{j=1}^{\infty}(\beta\xi_w)^j\left[\alpha_{h,1}\widehat{\bar{w}}_{t+j} - \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+j} + \hat{\mu}_{z^+,t+j}\right)\right] \\
&= \beta\xi_w\alpha_{h,1}\widehat{\bar{w}}_{t+1} - \frac{(1-\alpha_{h,1})\,\beta\xi_w}{1-\beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right) + \beta\xi_w G_{t+1}
\end{aligned}$$

Note:

$$\left(1-\beta\xi_w L^{-1}\right)F_t \equiv F_t - \beta\xi_w F_{t+1} = \hat{\psi}_{z^+,t} - \left(\sigma_\varsigma \hat{\zeta}_t + \sigma_z \hat{H}_t\right) \tag{E.7}$$

$$\left(1-\beta\xi_w L^{-1}\right)G_t \equiv G_t - \beta\xi_w G_{t+1} = \beta\xi_w\alpha_{h,1}\widehat{\bar{w}}_{t+1} - \frac{(1-\alpha_{h,1})\,\beta\xi_w}{1-\beta\xi_w}\left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right)$$

We now obtain a second restriction on $\hat{w}_t$ using the relation between the aggregate wage rate and the wage rates of individual households:

$$W_t = \left[(1-\xi_w) \left(\tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} W_{t-1} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w}.$$

Dividing both sides by W_t :

$$1 = (1-\xi_w) (w_t)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t} W_{t-1}}{W_t} \right)^{\frac{1}{1-\lambda_w}}.$$

Note,

$$\pi_{w,t} \equiv \frac{W_t}{W_{t-1}} = \frac{\bar{w}_t z_t^+ P_t}{\bar{w}_{t-1} z_{t-1}^+ P_{t-1}} = \frac{\bar{w}_t \mu_{z^+,t} \pi_t}{\bar{w}_{t-1}},$$

so that

$$1 = (1-\xi_w) (w_t)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\frac{\bar{w}_{t-1} \tilde{\pi}_{w,t}}{\bar{w}_t \mu_{z^+,t} \pi_t} \right)^{\frac{1}{1-\lambda_w}}.$$

Differentiate and make use of $w = 1$, $\tilde{\pi}_w = \mu_{z^+} \pi$:

$$0 = (1-\xi_w) \frac{1}{1-\lambda_w} \hat{w}_t + \xi_w \frac{1}{1-\lambda_w} \left[\widehat{\bar{w}}_{t-1} + \widehat{\tilde{\pi}}_{w,t} - \widehat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \hat{\pi}_t \right],$$

or,

$$\hat{w}_t = -\frac{\xi_w}{1-\xi_w} \left[\widehat{\bar{w}}_{t-1} - \widehat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \Delta_{\kappa_w} \hat{\pi}_t \right].$$

Use this expression to substitute out for $\hat{w}_t$ in (E.6):

$$\frac{1-\alpha_{h,1}}{1-\beta\xi_w} \frac{\xi_w}{1-\xi_w} \left[\widehat{\bar{w}}_{t-1} - \widehat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \Delta_{\kappa_w} \hat{\pi}_t \right] = F_t + \frac{1-\beta\xi_w \alpha_{h,1}}{1-\beta\xi_w} \widehat{\bar{w}}_t + G_t.$$

Multiply by $(1-\beta\xi_w L^{-1})$ and use (E.7):

$$\begin{aligned} & \frac{1-\alpha_{h,1}}{1-\beta\xi_w} \frac{\xi_w}{1-\xi_w} \left(1-\beta\xi_w L^{-1} \right) \left[\widehat{\bar{w}}_{t-1} - \widehat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \Delta_{\kappa_w} \hat{\pi}_t \right] \\ = \; & \hat{\psi}_{z^+,t} - \left(\sigma_\varsigma \hat{\zeta}_t + \sigma_z \hat{H}_t \right) + \left(1-\beta\xi_w L^{-1} \right) \frac{1-\beta\xi_w \alpha_{h,1}}{1-\beta\xi_w} \widehat{\bar{w}}_t \\ & + \beta\xi_w \alpha_{h,1} \widehat{\bar{w}}_{t+1} - \frac{\left(1-\alpha_{h,1}\right)\beta\xi_w}{1-\beta\xi_w} \left(\Delta_{\kappa_w} \hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1} \right), \end{aligned}$$

or,

$$\begin{aligned} & \frac{1-\alpha_{h,1}}{1-\beta\xi_w} \frac{\xi_w}{1-\xi_w} \left[\begin{array}{c} \widehat{\bar{w}}_{t-1} - \beta\xi_w \widehat{\bar{w}}_t - \widehat{\bar{w}}_t + \beta\xi_w \widehat{\bar{w}}_{t+1} - \hat{\mu}_{z^+,t} \\ + \beta\xi_w \hat{\mu}_{z^+,t+1} - \Delta_{\kappa_w} \hat{\pi}_t + \beta\xi_w \Delta_{\kappa_w} \hat{\pi}_{t+1} \end{array} \right] \\ = \; & \hat{\psi}_{z^+,t} - \left(\sigma_\varsigma \hat{\zeta}_t + \sigma_z \hat{H}_t \right) + \frac{1-\beta\xi_w \alpha_{h,1}}{1-\beta\xi_w} \left[\widehat{\bar{w}}_t - \beta\xi_w \widehat{\bar{w}}_{t+1} \right] \\ & + \beta\xi_w \alpha_{h,1} \widehat{\bar{w}}_{t+1} - \frac{\left(1-\alpha_{h,1}\right)\beta\xi_w}{1-\beta\xi_w} \left(\Delta_{\kappa_w} \hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1} \right). \end{aligned}$$

Note that the wage does not simply enter via nominal wage inflation. To see this, note

$$\widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1} = \hat{\pi}_{w,t} - \hat{\mu}_{z^+,t} - \hat{\pi}_t,$$

where $\hat{\pi}_{w,t}$ denotes nominal wage inflation. But, it is not simply $\widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1}$ that enters in this expression. That is, if we tried to express the above expression in terms of nominal wage inflation, we would simply add another variable to it, $\hat{\pi}_{w,t}$, without subtracting any, such as the real wage, $\widehat{\bar{w}}_t$. Collecting terms:

$$\begin{aligned} 0 &= E_t[\eta_0 \widehat{\bar{w}}_{t-1} + \eta_1 \widehat{\bar{w}}_t + \eta_2 \widehat{\bar{w}}_{t+1} + \eta_3 \hat{\pi}_{t-1} + \eta_4 \hat{\pi}_t + \eta_5 \hat{\pi}_{t+1} + \eta_6 \hat{\mu}_{z^+,t} + \eta_7 \hat{\mu}_{z^+,t+1} \\ &\quad + \eta_8 \hat{\psi}_{z^+,t} + \eta_9 \hat{H}_t + \eta_{10} \hat{\zeta}_t], \end{aligned} \tag{E.8}$$

where

$$\begin{aligned} \eta_0 &= \frac{1-\alpha_{h,1}}{1-\beta\xi_w} \frac{\xi_w}{1-\xi_w},\ \eta_1 = -\eta_0 \left(1+\beta\xi_w\right) - \frac{\left(1-\beta\xi_w\alpha_{h,1}\right)}{1-\beta\xi_w}, \\ \eta_2 &= \beta\xi_w \left(\eta_0 + \frac{\left(1-\beta\xi_w\alpha_{h,1}\right)}{1-\beta\xi_w} - \alpha_{h,1}\right),\ \eta_3 = \eta_0\kappa_w, \\ \eta_4 &= -\eta_0 \left(1+\kappa_w\beta\xi_w\right) - \frac{\left(1-\alpha_{h,1}\right)\beta\xi_w}{1-\beta\xi_w}\kappa_w, \\ \eta_5 &= \eta_0\beta\xi_w + \frac{\left(1-\alpha_{h,1}\right)\beta\xi_w}{1-\beta\xi_w}, \\ \eta_6 &= -\eta_0,\ \eta_7 = \eta_5,\ \eta_8 = -1,\ \eta_9 = \sigma_z,\ \eta_{10} = \sigma_\varsigma. \end{aligned}$$

Note that (E.8) is the same for the standard model and for our model with involuntary unemployment. The difference between the two models has only to do with the construction of σ_ς and σ_z.

The wage equation can be thought of, for computational purposes, as a nonlinear equation, if we treat

$$\widehat{\bar{w}}_t = \frac{\bar{w}_t - \bar{w}}{\bar{w}},$$

and the other hatted variables in the same way.

E.3. Other Equilibrium Conditions

E.3.1. Firms

We let s_t denote the firm's marginal cost, divided by the price of the homogeneous good. The standard formula, expressing this as a function of the factor inputs, is as follows:

$$s_t = \frac{\left(\frac{r_t^k P_t}{\alpha}\right)^\alpha \left(\frac{W_t R_t^f}{1-\alpha}\right)^{1-\alpha}}{P_t z_t^{1-\alpha}}.$$

When expressed in terms of scaled variables, this reduces to:

$$s_t = \left(\frac{\bar{r}_t^k}{\alpha}\right)^{\alpha}\left(\frac{\bar{w}_t R_t^f}{1-\alpha}\right)^{1-\alpha}. \tag{E.9}$$

Productive efficiency dictates that s_t is also equal to the ratio of the real cost of labor to the marginal product of labor:

$$s_t = \frac{\left(\mu_{\Psi,t}\right)^{\alpha}\bar{w}_t R_t^f}{(1-\alpha)\left(\frac{k_{i,t}}{\mu_{z^+,t}}/H_{i,t}\right)^{\alpha}}. \tag{E.10}$$

The only real decision taken by intermediate good firms is to optimize price when it is selected to do so under the Calvo frictions. According The first order necessary conditions associated with price optimization are, after scaling:[45]

$$E_t\left[\psi_{z^+,t}y_t + \left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}}\beta\xi_p F_{t+1}^f - F_t^f\right] = 0 \tag{E.11}$$

$$E_t\left[\lambda_f\psi_{z^+,t}y_t s_t + \beta\xi_p\left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}}K_{t+1}^f - K_t^f\right] = 0, \tag{E.12}$$

$$\mathring{p}_t = \left[(1-\xi_p)\left(\frac{1-\xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right)^{\lambda_f} + \xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_f}{1-\lambda_f}}\right]^{\frac{1-\lambda_f}{\lambda_f}}, \tag{E.13}$$

$$\left[\frac{1-\xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right]^{(1-\lambda_f)} = \frac{K_t^f}{F_t^f}, \tag{E.14}$$

$$\tilde{\pi}_{f,t} \equiv \left(\pi_{t-1}\right)^{\kappa_d}\left(\pi\right)^{1-\kappa_d}. \tag{E.15}$$

In terms of scaled variables, the law of motion for the capital stock is as follows:

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t + \Upsilon_t\left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)i_t. \tag{E.16}$$

The aggregate production relation is:

$$y_t = \left(\mathring{p}_t\right)^{\frac{\lambda_f}{\lambda_f-1}}\left[\epsilon_t\left(\frac{1}{\mu_{\Psi,t}}\frac{1}{\mu_{z^+,t}}\bar{k}_t u_t\right)^{\alpha}H_t^{1-\alpha} - \phi\right].$$

[45] When we linearize about the steady state and set $\varkappa_d = 0$, we obtain,

$$\hat{\pi}_t - \hat{\bar{\pi}}_t = \frac{\beta}{1+\kappa_d\beta}E_t\left(\hat{\pi}_{t+1} - \hat{\bar{\pi}}_{t+1}\right) + \frac{\kappa_d}{1+\kappa_d\beta}\left(\hat{\pi}_{t-1} - \hat{\bar{\pi}}_t\right) - \frac{\kappa_d\beta\left(1-\rho_\pi\right)}{1+\kappa_d\beta}\hat{\bar{\pi}}_t + \frac{1}{1+\kappa_d\beta}\frac{\left(1-\beta\xi_d\right)\left(1-\xi_d\right)}{\xi_d}\widehat{mc}_t,$$

where a hat indicates log-deviation from steady state.

Finally, the resource constraint is:

$$y_t = g_t + c_t + i_t + a\left(u_t\right)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}.$$

E.3.2. Family

We now derive the equilibrium conditions associated with the household, apart from the wage condition, which was derived in a previous subsection. The Lagrangian representation of the household's problem is:

$$\begin{aligned}
&E_0^j \sum_{t=0}^{\infty} \beta^t \{[\ln\left(C_t - bC_{t-1}\right) - z\left(h_{t,j}, \zeta_t\right)] \\
&\upsilon_t \left[\begin{array}{c} W_{t,j}h_{t,j} + X_t^k \bar{K}_t + \left(R_{t-1} - \tau^R\left(R_{t-1}-1\right)\right)B_t \\ +a_{t,j} - P_t\left(C_t + \frac{1}{\Psi_t}I_t\right) - B_{t+1} - P_t P_{k',t}\Delta_t \end{array}\right] \\
&+\omega_t \left[\Delta_t + \left(1-\delta\right)\bar{K}_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right)I_t - \bar{K}_{t+1}\right]\}
\end{aligned}$$

The first order condition with respect to C_t is:

$$\frac{1}{C_t - bC_{t-1}} - E_t\frac{b\beta}{C_{t+1} - bC_t} = \upsilon_t P_t,$$

or, after expressing this in scaled terms and multiplying by z_t^+:

$$\psi_{z^+,t} = \frac{1}{c_t - b\frac{c_{t-1}}{\mu_{z^+,t}}} - \beta b E_t \frac{1}{c_{t+1}\mu_{z^+,t+1} - bc_t}. \tag{E.17}$$

The first order condition with respect to Δ_t is, after rearranging:

$$P_t P_{k',t} = \frac{\omega_t}{\upsilon_t}. \tag{E.18}$$

The first order condition with respect to I_t is:

$$\omega_t\left[1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right) - \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + E_t\beta\omega_{t+1}\tilde{S}'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 = \frac{P_t\upsilon_t}{\Psi_t}.$$

Making use of (E.18), multiplying by $\Psi_t z_t^+$, rearranging and using the scaled variables,

$$\begin{aligned}
&\psi_{z^+,t}p_{k',t}\left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right] \\
&+\beta\psi_{z^+,t+1}p_{k',t+1}\tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1}i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\mu_{z^+,t+1}\mu_{\Psi,t+1} \quad = \quad \psi_{z^+,t},
\end{aligned} \tag{E.19}$$

Optimality of the choice of $\bar{K}_{t+1}$ implies the following first order condition:

$$\omega_t = \beta E_t\upsilon_{t+1}X_{t+1}^k + \beta E_t\omega_{t+1}\left(1-\delta\right) = \beta E_t\upsilon_{t+1}\left[X_{t+1}^k + P_{t+1}P_{k',t+1}\left(1-\delta\right)\right],$$

using (E.18). Using (E.18) again,

$$v_t = E_t \beta v_{t+1} \left[\frac{X_{t+1}^k + P_{t+1} P_{k',t+1} (1-\delta)}{P_t P_{k',t}} \right] = E_t \beta v_{t+1} R_{t+1}^k, \tag{E.20}$$

where R_{t+1}^k denotes the rate of return on capital:

$$R_{t+1}^k \equiv \frac{X_{t+1}^k + P_{t+1} P_{k',t+1} (1-\delta)}{P_t P_{k',t}}$$

Multiply (E.20) by $P_t z_t^+$ and express the results in scaled terms:

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1} \frac{R_{t+1}^k}{\pi_{t+1} \mu_{z^+,t+1}}. \tag{E.21}$$

Expressing the rate of return on capital, (3.14), in terms of scaled variables:

$$R_{t+1}^k = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{u_{t+1} \bar{r}_{t+1}^k - a(u_{t+1}) + (1-\delta) p_{k',t+1}}{p_{k',t}}. \tag{E.22}$$

The first order condition associated with capital utilization is:

$$\Psi_t r_t^k = a'(u_t),$$

or, in scaled terms,

$$\bar{r}_t^k = a'(u_t). \tag{E.23}$$

The first order condition with respect to B_{t+1} is:

$$v_t = \beta v_{t+1} R_t.$$

Multiply by $z_t^+ P_t$:

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1} \pi_{t+1}} R_t. \tag{E.24}$$

E.4. Steady State

We describe two strategies for computing the steady state. In each case, the strategy is applied to our model with involuntary unemployment, and we indicate what changes are required for the standard model. The first steady state strategy takes all the model parameters as given and computes the endogenous variables. The second imposes values for several endogenous variables and solves for an equal number of parameters.

E.4.1. First Algorithm

Consider the equilibrium conditions associated with price setting. In steady state, these reduce to (we have used that $\tilde{\pi}^f = \pi$ using (3.4)):

$$
\begin{aligned}
F^f &= \frac{\psi_{z+}y}{1-\beta\xi_p} \\
K^f &= \frac{\lambda_f \psi_{z+} ys}{1-\beta\xi_p} \\
\mathring{p} &= 1 \\
\frac{K_t^f}{F_t^f} &= 1
\end{aligned}
$$

These equations imply

$$(1)s = \frac{1}{\lambda_f}.$$

Thus,

$$
\begin{aligned}
(2)s &= \left(\frac{\bar{r}^k}{\alpha}\right)^{\alpha}\left(\frac{\bar{w}R^f}{1-\alpha}\right)^{1-\alpha} = \frac{1}{\lambda_f} \rightarrow \bar{w} = \frac{(1-\alpha)}{R^f}\left[\frac{1}{\lambda_f}\left(\frac{\alpha}{\bar{r}^k}\right)^{\alpha}\right]^{\frac{1}{1-\alpha}}, \\
(3)s &= \frac{(\mu_\Psi)^{\alpha}\bar{w}R^f}{\epsilon(1-\alpha)\left(\frac{\bar{k}}{\mu_{z+}}/H\right)^{\alpha}} = \frac{1}{\lambda_f}, \rightarrow \frac{\bar{k}}{H} = \mu_{z+}\left[\frac{\lambda_f(\mu_\Psi)^{\alpha}\bar{w}R^f}{\epsilon(1-\alpha)}\right]^{\frac{1}{\alpha}}
\end{aligned}
$$

where

$$(4)R^f = \nu^f R + 1 - \nu^f$$

The equilibrium conditions associated with wage setting is:

$$(5)\bar{w} = \lambda_w \frac{z_h}{\psi_{z+}},$$

which is the usual wage markup equation. Also,

$$(6)h = H.$$

The consumption-compensated elasticity of labor supply is

$$(7)\sigma_z = \frac{z_{hh}h}{z_h}.$$

This reduces to σ_L in the case of the standard model, and is derived above in the case of the model with involuntary unemployment.

Also,

$$(8)\left[1 - \frac{1-\delta}{\mu_{z+}\mu_\Psi}\right]\bar{k} = \Upsilon i,$$

and

$$\begin{aligned}(9)\psi_{z^+} &= \frac{1}{c}\frac{\mu_{z^+}-\beta b}{\mu_{z^+}-b}, \\ p_{k'} &= 1\end{aligned}$$

Consider the utilization adjustment cost function. We specify that as follows, with $\sigma_b = \bar{r}^k$:

$$\begin{aligned}a(u) &= 0.5\sigma_b\sigma_a u^2 + \sigma_b\left(1-\sigma_a\right)u + \sigma_b\left(\left(\sigma_a/2\right)-1\right) \\ a(1) &= \sigma_b\sigma_a + \sigma_b\left(1-\sigma_a\right) - \sigma_b\end{aligned}$$

Then, $a'(u) = \bar{r}^k$ implies

$$u = 1,\ a(1) = 0.$$

We use the latter two results in what follows. The household intertemporal Euler equation for capital implies:

$$(10) 1 = \beta\frac{R^k}{\pi\mu_{z^+}},$$

with

$$(11) R^k = \frac{\pi}{\mu_\Psi}\left[\bar{r}^k + 1 - \delta\right].$$

The intertemporal Euler equation for nominal bonds:

$$(12) 1 = \frac{\beta R}{\mu_{z^+}\pi}.$$

The production function and resource constraint:

$$\begin{aligned}(13)y &= \epsilon\left(\frac{1}{\mu_\Psi}\frac{1}{\mu_{z^+}}\bar{k}\right)^{\alpha}H^{1-\alpha} - \phi, \\ (14)y &= g + c + i\end{aligned}$$

The 17 variables whose steady state values are to be determined are:

$$y, c, i, R, h, H, \bar{k}, \bar{r}^k, R^k, \bar{w}, \psi_{z^+}, s, R^f, u, \bar{p}, m, \sigma_z.$$

Here is a strategy for solving these equations. Equation (1) produces s; R from (12); R^f from (4); R^k from (10); $\bar{r}^k$ from (11); $\bar{w}$ from (2); $\bar{k}/H$ from (3).

The remaining variables can be found using a one dimensional search. Fix a value for h. By (6), we have H; from $\bar{k}/H$ we have $\bar{k}$; from (13) we have y; from (8) we have i; from (14) and

$$\eta_g = \frac{g}{y},$$

we have c; from (9) we have ψ_{z^+}; in the case of the model with involuntary unemployment, compute m from the steady state version of (2.12):

$$(15) h = m\eta + a^2 \varsigma \sigma_L m^{\sigma_L + 1};$$

compute σ_z using (7) or $\sigma_z = \sigma_L$ in the case of the standard model; we are now in a position to evaluate (5):

$$f(h) = \bar{w} - \lambda_w \frac{z_h}{\psi_{z^+}}.$$

Adjust h until $f(h) = 0$.

In the case of our model with involuntary unemployment, we proceed as follows. Compute u from

$$(16) u = \frac{m-h}{m}.$$

Let $\bar{p}$ denote the steady state probability that the marginal household with $l = 0$ finds a job. According to (2.15):

$$(17)\, \bar{p} = \eta + \varsigma a^2 (1 + \sigma_L) m^{\sigma_L}.$$

This algorithm solves for 17 endogenous variables using 17 equations.

In the case of the standard model, m solves the steady state version of (3.23):

$$\psi_{z^+} \bar{w} = \varsigma (m)^{\sigma_L},$$

and then the unemployment rate is computed using m and h.

E.4.2. A Second Algorithm

We find it convenient to shift three endogenous labor market variables to the list of exogenous variables:

$$u,\ m,\ \bar{p},\ \sigma_z.$$

Corresponding to this, we shift four parameters to the list of endogenous variables:

$$F, \varsigma, a, \eta.$$

Thus, we must solve for the following 17 variables:

$$y, c, i, R, h, H, \bar{k}, \bar{r}^k, R^k, \bar{w}, \psi_{z^+}, s, R^f, F, \varsigma, a, \eta.$$

We compute the steady state as follows. As before, equation (1) produces s; R from (12); R^f from (4); R^k from (10); $\bar{r}^k$ from (11); $\bar{w}$ from (2); $\bar{k}/H$ from (3). Compute h from (16); H is found from (6); $\bar{k}$ from $\bar{k}/H$; y from (13); i from (8); c from (14) and

$$\eta_g = \frac{g}{y};$$

ψ_{z^+} from (9).

Equations (5), (7), (15), and (17) remain to be solved for F, ς, a, η. Fix values of F and a. We compute η as follows. Rewrite (15):

$$m^{\sigma_L+1} = \frac{h - m\eta}{a^2 \varsigma \sigma_L}.$$

Multiply both sides of (17) by m and substitute out for m^{σ_L+1} from the previous expression:

$$\frac{\sigma_L}{1+\sigma_L}\left(\bar{p} + \frac{\eta}{\sigma_L}\right) m = h$$

Substitute this into (16):

$$u = \frac{m-h}{m} = 1 - \frac{\sigma_L}{1+\sigma_L}\left(\bar{p} + \frac{\eta}{\sigma_L}\right) = \frac{1+\sigma_L(1-\bar{p}) - \eta}{1+\sigma_L},$$

or,

$$\eta = 1 + \sigma_L(1-\bar{p}) - (1+\sigma_L)u,$$

so that we now have η (this must be non-negative). Rewriting (15), we have ς :

$$(15')\ \varsigma = \frac{h - m\eta}{a^2 m^{\sigma_L+1} \sigma_L}.$$

Next, adjust F and a so that (5) and (7) are satisfied.

E.4.3. Steady State Replacement Ratio

Here, we compute the steady state consumption of employed and non-employed households. By the analog of (2.9), the incentive constraint for the j^{th} family is:

$$c_{t,j}^{w} = bC_{t-1} + \left(c_{t,j}^{nw} - bC_{t-1}\right) e^{F+\zeta_t(1+\sigma_L)m_{t,j}^{\sigma_L}}.$$

The j^{th} family's resource constraint is:

$$h_{t,j} c_{t,j}^{w} + (1-h_{t,j})\, c_{t,j}^{nw} = C_t.$$

Substituting out for $c_{t,j}^{w}$ from the incentive constraint, we obtain:

$$bC_{t-1} + \left(c_{t,j}^{nw} - bC_{t-1}\right) e^{F+\zeta_t(1+\sigma_L)m_{t,j}^{\sigma_L}} + \frac{(1-h_{t,j})}{h_{t,j}} c_{t,j}^{nw} = \frac{C_t}{h_{t,j}}.$$

Rearrange,

$$c_{t,j}^{nw} = \frac{C_t + \left(e^{F+\zeta_t(1+\sigma_L)m_{t,j}^{\sigma_L}} - 1\right) h_{t,j} bC_{t-1}}{h_{t,j}\left(e^{F+\zeta_t(1+\sigma_L)m_{t,j}^{\sigma_L}} - 1\right) + 1}.$$

Scaling,

$$\tilde{c}_{t,j}^{nw} = \frac{c_t + \left(e^{F+\zeta_t(1+\sigma_L)m_{t,j}^{\sigma_L}} - 1\right) h_{t,j} \frac{b}{\mu_{z+}} c_{t-1}}{h_{t,j}\left(e^{F+\zeta_t(1+\sigma_L)m_{t,j}^{\sigma_L}} - 1\right) + 1}.$$

In steady state:

$$\tilde{c}^{nw} = c\frac{1 + \left(e^{F+\zeta(1+\sigma_L)m^{\sigma_L}} - 1\right) h \frac{b}{\mu_{z+}}}{h\left(e^{F+\zeta(1+\sigma_L)m^{\sigma_L}} - 1\right) + 1},$$

and

$$\tilde{c}^{w} = \frac{b}{\mu_{z+}}c + \left(c^{nw} - \frac{b}{\mu_{z+}}c\right) e^{F+\zeta(1+\sigma_L)m^{\sigma_L}}$$

F. Relationship of Our Work to Gali (2010)

Our paper emphasizes labor supply in its explanation of the dynamics of unemployment and the labor force. Another recent paper that adopts this perspective is Gali (2009). To better explain our model, it is useful to compare its properties with those of Gali's model. Gali demonstrates that with a modest reinterpretation of variables, the standard DSGE model already contains a theory of unemployment. In particular, one can define the unemployed as the difference between the number of people actually working and the number of people that would be working if the marginal cost of work were equated to the wage rate. This difference is positive and fluctuating in the standard DSGE model because of the presence of wage-setting frictions and monopoly power. In effect, unemployment is a symptom of social inefficiency. People inflict unemployment upon themselves in the quest for monopoly profits. By contrast, in our model unemployment reflects frictions that are necessary for people to find jobs. The existence of unemployment does not require monopoly power. This point is dramatized by the fact that we introduce our model in the CGG framework, in which wages are set in competitive labor markets. At the same time, the logic of our model does create a positive relationship between monopoly power and unemployment. In our model, the employment contraction resulting from an increase in the monopoly power of unions produces a reduction in the incentives for households to work. Households' response to the reduced incentives is to allocate less effort to search, implying higher unemployment. So, our model shares the prediction of Gali's model that unemployment should be higher in economies with more union monopoly power. However, our model has additional implications that could differentiate it from Gali's. Ours implies that in economies with more union power both the labor force and the consumption premium for employed workers over non-employed workers are reduced. Gali's model predicts that with more union monopoly power, the labor force will be larger. The exact amount by which the labor force increases depends on the strength of wealth effects on leisure.

Other important differences between our model of unemployment and Gali's is that the latter fails to satisfy characteristics (i) and (iii) above. The model assumes that the available jobs can be found without effort. Because the model does not satisfy (i), unemployment does not meet the official US definition of unemployment. In addition, the presence of perfect insurance in Gali's model implies that the employed have lower utility than the non-employed, violating (iii).

There are more differences between ours and Gali's theory unemployment. In standard DSGE models, labor supply plays little role in the dynamics of standard macro variables like consumption, output, investment, inflation and the interest rate. The reason is that the presence of wage setting frictions reduces the importance of labor supply. This is why the New Keynesian literature has been relatively unconcerned about all the old puzzles about income effects on labor and labor supply elasticities that were a central concern in the real business cycle literature. However, CTW show that these problems are back in full force if one adopts Gali's theory of unemployment. This is because labor supply corresponds to the labor force in that theory. To see how this brings back the old problems, we study the standard DSGE model's predictions for unemployment and the labor force in the wake of an expansionary monetary policy shock. Because that model predicts a rise in consumption, the model also predicts a decline in labor supply, as the income effect associated with increased consumption produces a fall in the value of work. The drop in labor supply is counterfactual, according to our VAR-based evidence. In addition, the large drop in the labor force leads to an counterfactually large drop in unemployment in the wake of an expansionary monetary policy shock.

Gali (2010), Gali, Smets and Wouters (2010) and CTW show, in different ways, that changes to the household utility function that offset wealth effects reduce the counterfactual implications of the standard model for the labor force. In effect, our paper proposes a different strategy. We preserve the additively separable utility function that is standard in monetary DSGE models, and our model nevertheless does not display the labor force problems in the standard DSGE model. This is because in our model the labor force and employment have a strong tendency to comove. In our model, the rise in employment in the wake of an expansionary monetary policy shock is accomplished by increasing people's incentives to work. The additional incentives not only encourage already active households to intensify their job search, but also to shift into the labor force. More generally, the analysis highlights the fact that modeling unemployment requires thinking carefully about the determinants of the labor force.[46]

[46] Our argument complements the argument in Krusell, Mukoyama, Rogerson, and Sahin (2009), who also

G. Aggregate Hours Worked

Given our linear approximation, and the assumptions that imply that steady state is undistorted by wage frictions, we have

$$\hat{h}_t = \hat{H}_t.$$

Although this is a well known result (see, e.g., Yun (1996)), we derive it here for completeness. Recall,

$$h_t \equiv \int_0^1 h_{j,t} dj.$$

Invert the demand for labor, (3.6), to obtain an expression in terms of $h_{j,t}$. Substitute this into the expression for h_t to obtain:

$$h_t = H_t \int_0^1 \mathring{w}_{j,t}^{\frac{\lambda_w}{1-\lambda_w}} dj, \tag{G.1}$$

where

$$\mathring{w}_{j,t} \equiv \frac{W_{j,t}}{W_t}.$$

Here, W_t denotes the aggregate wage rate, which one obtains by substituting (3.5) into (3.6):

$$W_t = \left[\int_0^1 W_{j,t}^{\frac{1}{1-\lambda_w}} dj \right]^{1-\lambda_w}.$$

Because all families are identical in steady state (see the discussion after (3.10)), $\mathring{w}_j = 1$ for all j. Totally differentiating (G.1),

$$\hat{h}_t = \hat{H}_t + \int_0^1 \widehat{\mathring{w}}_{j,t} dj.$$

Thus, to determine the percent deviation of aggregate employment from steady state, we require the integral of the percent deviations of type j wages from the aggregate wage, over all j. We now show that this integral is, to first order, equal to zero.

Express the integral in (G.1) as follows:

$$h_t = \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t,$$

say, where

$$\mathring{w}_t \equiv \left[\int_0^1 \mathring{w}_{j,t}^{\frac{\lambda_w}{1-\lambda_w}} dj \right]^{\frac{1-\lambda_w}{\lambda_w}}. \tag{G.2}$$

---

stress the importance of understanding employment, unemployment and the labor force.

Pursuing logic that is standard in the Calvo price/wage setting literature we obtain:

$$\begin{aligned} W_t &= \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} W_{t-1}\right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w} & \text{(G.3)} \\ \mathring{w}_t &= \left[(1-\xi_w)\, w_t^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mathring{w}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}, & \text{(G.4)} \end{aligned}$$

where:

$$w_t \equiv \frac{\tilde{W}_t}{W_t},\ \pi_{w,t} \equiv \frac{W_t}{W_{t-1}},$$

and $\tilde{W}_t$ denotes the wage set by the $1-\xi_w$ families that have the opportunity to reoptimize in the current period. Because all families are identical in steady state

$$w = \mathring{w} = \frac{\tilde{\pi}_w}{\pi_w} = 1, \tag{G.5}$$

where $\tilde{\pi}_{w,t}$ is defined in (3.10) and $\pi_{w,t}$ denotes wage inflation:

$$\pi_{w,t} \equiv \frac{W_t}{W_{t-1}}.$$

Dividing (G.3) by W_t and solving,

$$w_t = \left[\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right]^{1-\lambda_w}. \tag{G.6}$$

Differentiating (G.4) and (G.6) in steady state:

$$\begin{aligned} \widehat{\mathring{w}}_t &= (1-\xi_w)\,\hat{w}_t + \xi_w \left(\widehat{\tilde{\pi}}_{w,t} - \hat{\pi}_{w,t} + \widehat{\mathring{w}}_{t-1} \right) & \text{(G.7)} \\ \hat{w}_t &= -\frac{\xi_w}{1-\xi_w} \left(\widehat{\tilde{\pi}}_{w,t} - \hat{\pi}_{w,t} \right) \end{aligned}$$

Using the latter to substitute out for $\hat{w}_t$ in (G.7):

$$\widehat{\mathring{w}}_t = \xi_w \widehat{\mathring{w}}_{t-1}.$$

Thus, to first order the wage distortions evolve according to a stable first order difference equation, unperturbed by shocks. For this reason, we set

$$\widehat{\mathring{w}}_t = 0, \tag{G.8}$$

for all t.

Totally differentiating (G.2) and using (G.5), (G.8):

$$\int_0^1 \widehat{\mathring{w}}_{j,t} dj = 0.$$

That is, to first order, the integral of the percent deviations of individual wages from the aggregate is zero.

H. Functional Forms of Cost Functions

We adopt the following functional form for the capacity utilization cost function a :

$$a(u) = 0.5\sigma_b\sigma_a u^2 + \sigma_b (1 - \sigma_a) u + \sigma_b ((\sigma_a/2) - 1), \tag{H.1}$$

where σ_a and σ_b are the parameters of this function. The parameter, σ_b is equated to the steady state value of the rental rate of capital. $a' = \sigma_b\sigma_a u + \sigma_b (1 - \sigma_a)$.

We assume that the investment adjustment cost function takes the following form:

$$S(x) = \frac{1}{2}\left\{\exp\left[\sqrt{S''}(x - \mu_{z^+}\mu_\Psi)\right] + \exp\left[-\sqrt{S''}(x - \mu_{z^+}\mu_\Psi)\right] - 2\right\}, \tag{H.2}$$

where S'' is a parameter.

Table A1: Structural Parameters of Small Model Held Fixed Across Numerical Experiments

Parameter	Value	Description
β	$1.03^{-.25}$	Discount factor
g_A	1.0047	Technology growth
ξ_p	0.75	Price stickiness
λ_f	1.2	Price markup
ρ_R	0.8	Taylor rule: interest smoothing
r_π	1.5	Taylor rule: inflation
r_y	0.2	Taylor rule: output gap
η_g	0.2	Government consumption share on GDP
γ	0.001	Diffusion speed of technology into government consumption
ρ_g	0.8	AR(1) government consumption
ρ_{g_A}	0.8	AR(1) technology
ρ_ς	0.8	AR(1) disutility of labor
σ_g	0.05489[a]	Standard deviation government consumption shock
σ_a	0.00377[a]	Standard deviation technology shock
σ_R	0.00449[a]	Standard deviation monetary policy shock
σ_ς	0	Standard deviation disutiliy shock

[a] Set such that implied standard deviation of output growth equals 1% given no other shocks.

Table A2: The Impact of Imperfect Information in the Small Model

Variable	Involuntary Unemp. Model (Imperfect Information)	Full Information Model		Standard Model	Description
		Fixed Structural Params[b]	Observational Equivalent[c]	Observational Equivalent[d]	
			Steady State Properties		
m	0.67	0.69	0.67	0.63	Labor force
h	0.63	0.68	0.63	0.63	Employment
u	0.056	0.015	0.056	n.a.	Unemployment rate
$\bar{p}$	0.95	0.99	0.95	n.a.	Max $p(e)$
$1/\sigma_z$	2.0	0.80	2.0	2.0	Family labor supply elasticity
$1/\kappa^{Okun}$	2.0	1.64	2.0	n.a.	Okun's law coefficient
c^{nw}/c^{w}	0.18	1.0	1.0	1.0	Replacement ratio
	0.189		n.a.	n.a.	Price (% of C) of info.[a]
			Structural Parameters[e]		
a	0.53	0.53	0.74	n.a.	Slope, $p(e)$
η	0.86	0.86	0.86	n.a.	Intercept, $p(e)$
ς	4.64	4.64	2.45	1.67	Slope, labor disutility
F	1.39	1.39	1.83	n.a.	Intercept, labor disutility
σ_L	13.31	13.31	13.31	0.5	Power, labor disutility
			Welfare Cost of Business Cycles		
		Technology shock only			
λ	0.520684131141	0.566191290230	0.520684131141	0.520684131142	% of consumption
		Government consumption shock only			
λ	0.112215458271	0.125326644511	0.112215458271	0.112215458271	% of consumption
		Monetary policy shock only			
λ	0.071331553871	0.100111000086	0.071331553871	0.071331553871	% of consumption

[a] Percent increase in consumption in steady state of involuntary unemployment model that makes steady state utility in that model equal to steady state utility of model with full information.

[b] Full information model with same structural parameters as involuntary unemployment model.

[c] Full information model with parameter values in Table A1, plus parameter values in bottom panel of this table, chosen so that full information model steady state properties in the top panel (except c^{nw}/c^{w}) coincide with those in involuntary unemployment model.

[d] Standard model with parameter values in Table A1, plus parameter values for ς and σ_L so that h and $1/\sigma_z$ coincide with those of the involuntary unemployment model.

[e] Model structural parameter values are those listed in Table A1 plus the ones indicated in the bottom panel of this table.

Table A3: Benchmark Parameter Values and Steady State Properties of the Small Model for the Sensitivity Analysis of the Costs of Business Cycles

Variable Name	Value	Description
β	$1.03^{-.25}$	Discount factor
g_A	1.0047	Technology growth
ξ_p	0.75	Price stickiness
λ_f	1.2	Price markup
ρ_R	0.8	Taylor rule: interest smoothing
r_π	1.5	Taylor rule: inflation
r_y	0.2	Taylor rule: output gap
$1/\sigma_z$	1	Family labor supply elasticity
η_g	0.2	Government consumption share on GDP
u	0.055	Unemployment rate
m	2/3	Labor force participation
$\overline{p}$	0.97	max p(e)
γ	0.001	Diffusion speed of technology into gov. spending
F	0.31	Intercept, labor disutility
σ_L	4	Power, labor disutility
ς	0.99	Slope, labor disutility
a	0.36	Slope, $p(e)$
η	0.85	Intercept, $p(e)$
ρ_g	0.8	AR(1) government consumption
ρ_{g_A}	0.8	AR(1) technology
ρ_ς	0.8	AR(1) disutility of labor
σ_g	0.06045[a]	Standard deviation government consumption shock
σ_a	0.00373[a]	Standard deviation technology shock
σ_R	0.00449[a]	Standard deviation monetary policy shock
σ_ς	0	Standard deviation disutiliy shock

[a] Set such that implied standard deviation of output growth equals 1% given no other shocks.

Table A4a: Welfare Costs of Business Cycles in the Small Model: Consumption Equivalents in %

Model	Benchmark	ξ_p=0	$r_\pi = 100$	$\eta_g = 0$	$\gamma = 1$
		Technology Shocks			
Standard Model	0.54145973949766	0.48843255026769	0.48856383934627	0.03602282953172	0.04299383343424
Involuntary Unemp.	0.54145973949775	0.48843255026769	0.48856383934645	0.03602282953172	0.04299383343424
Full Information	0.54145973949761	0.48843255026775	0.48856383934459	0.03602282953172	0.04299383343424
		Government Spending Shocks			
Standard Model	0.14695322840805	0.13822951084114	0.13825099674533	n.a.	0.14695322840805
Involuntary Unemp.	0.14695322840805	0.13822951084114	0.13825099674533	n.a.	0.14695322840805
Full Information	0.14695322840803	0.13822951084114	0.13825099674531	n.a.	0.14695322840803
		Monetary Policy Shocks			
Standard Model	0.09736866565077	n.a.	0.00023970621749	0.08157775324686	0.09736866565077
Involuntary Unemp.	0.09736866565077	n.a.	0.00023970621749	0.08157775324686	0.09736866565077
Full Information	0.09736866565075	n.a.	0.00023970621749	0.08157775324686	0.09736866565075

Table A4b: Welfare Costs of Business Cycles in the Small Model:
Consumption Equivalents in %

Model	Benchmark	$\sigma_L = 20, \sigma_z = 1$	$\sigma_L = 1.5, \sigma_z = 1$	$\sigma_L = 4, \sigma_z = 3$	$\sigma_L = 4, \sigma_z = 0.1$
Technology Shocks					
Standard Model	0.54145973949766	0.54145973949766	0.54145973949766	0.62045916723039	0.47453492071094
Involuntary Unemp.	0.54145973949775	0.54145973949786	0.54145973949753	0.62045916723059	0.47453492071103
Full Information	0.54145973949761	0.54145973949788	0.54145973949699	0.62045916723036	0.47453492071116
Government Spending Shocks					
Standard Model	0.14695322840805	0.14695322840805	0.14695322840805	0.18461277975589	0.12684875171187
Involuntary Unemp.	0.14695322840805	0.14695322840807	0.14695322840805	0.18461277975589	0.12684875171187
Full Information	0.14695322840803	0.14695322840807	0.14695322840805	0.18461277975589	0.12684875171187
Monetary Policy Shocks					
Standard Model	0.09736866565077	0.09736866565077	0.09736866565077	0.16925151157873	0.05852588699248
Involuntary Unemp.	0.09736866565077	0.09736866565088	0.09736866565075	0.16925151157873	0.05852588699248
Full Information	0.09736866565075	0.09736866565086	0.09736866565077	0.16925151157873	0.05852588699248