Discussion of Short and Long, by Adao, Correia and Teles (BIP)

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Households

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[c(s^t) - V(n(s^t)) \right]$$

$$P(s^t)c(s^t) \leq M(s^t)$$

Technology

$$c(s^t) = A(s^t)n(s^t)$$

The intra-period marginal condition, once we replace for the equilibrium value of the wage is

$$\frac{1}{V'(n(s^t))} = \frac{R(s_t)}{A(s^t)}$$

Thus, pinning down $R(s_t)$ implies pinning down leisure and therefore consumption.

Then, the Fisher equation becomes

$$\frac{1}{P(s^t)} = \beta R(s_t) E_t \frac{1}{P(s^{t+1})}$$

and the prices of state contingent assets is

$$Q(s^{t+1}/s^t) = \beta \frac{P(s^t)}{P(s^{t+1})}$$

Assume $A(s^t) = 1$, $R(s_t) = R$ and search for deterministic equilibria.

There exists an equilibrium where

$$\frac{P_{t+1}}{P_t} = \beta R, \quad \frac{1}{V'(n)} = R, \quad \text{and } c = n$$

so the allocation and inflation rate are uniquely determined.

- But any pair $\{P_0, M_0 = cP_0\}$ is an equilibrium pair. S&W.

- Note however, that we obtain a unique inflation rate only if we restrict the search to deterministic outcomes.

- Can we have sunspots?

- Yes.

Let s_t be a finite support random variable.

The same allocation with a function $P(s_t)$ satisfying

$$\frac{1}{P(s_t)} = \beta R E_t \left[\frac{1}{P(s_{t+1})} \right]$$

is also an equilibrium, where $M(s^t)$ will be given by

$$P(s^t)c \leq M(s^t)$$

Thus, the degree of indeterminacy is of higher order than in SW.

Showing this higher - relative to my reading of SW - degree of indeterminacy is the first thing that BIP do.

Let us rule out sunspots and consider again the model with productivity shocks, but let

$$
\begin{aligned}
A_0 &= 1, \\
A_1 &= A > 1, \text{ with probability } \pi \\
A_1 &= B < 1, \text{ with probability } 1 - \pi \\
A_{t+1} &= A_t \text{ for } t \geq 1
\end{aligned}
$$

- In this case, the productivity shock could act as the sunspot.

- Note however, that while with the sunspot, the real quantity of money is constant across periods and states. Once you have productivity shocks that is not the case anymore, sin consumption will depend on the shock

Then, if $R_t = R$, the equilibrium conditions become

$$\begin{aligned}
\frac{1}{V'(n_0)} &= R, \text{ and } c_0 = n_0 \\
\frac{1}{V'(n^A)} &= \frac{R}{A}, \text{ and } c^A = n^A \text{ for } t \geq 1, \text{in the good state} \\
\frac{1}{V'(n^B)} &= \frac{R}{B}, \text{ and } c^B = n^B \text{ for } t \geq 1, \text{in the bad state}
\end{aligned}$$

as we assume sunspots away, there are 3 different price levels

$$P_0, P^A, P^B$$

and the following equilibrium conditions

$$\begin{aligned}
\frac{1}{P_0} &= \beta Q(A)\frac{1}{P^A} \\
\frac{1}{P_0} &= \beta Q(B)\frac{1}{P^B} \\
\frac{1}{P_0} &= \beta R\left[\frac{\pi}{P^A} + \frac{1-\pi}{P^B}\right]
\end{aligned}$$

- SW is about P_0 indeterminacy
- BIP is about P^A, P^B indeterminacy - although a linear combination of both is unique.

- I will eliminate SW indeterminacy by setting $P_0 = 1$.

- BIP argue they can do it with M_0.

Contribution of this paper

1. Make explicit this type of indeterminacy

2. Argue that, if we take this model seriously, monetary policy ought to be though of as setting

   $$Q(s^{t+1}/s^t) \text{ for all } s^t$$

   rather than

   $$R(s^t) \text{ for all } s^t$$

3. Provide a more natural alternative, using maturity structure.

4. The message is that the whole (maybe not even enough) maturity structure can - and should - be pinned down by the Central Bank.

- But isn't the Taylor principle a solution?

- BIP argue - convincingly to me - that it is not. See also Cochrane (2008).

One reaction

- BIP show that setting only the short term interest rates, leaves the price as a function of the state indeterminate.

- I view the conduct of Central Banks in the last 20 years as setting the short term interest rate only.

- According to the model, the price level could have been all over the place, with the resulting volatility of the allocation if prices were sticky.

- Should we convey to Central Bankers the idea that they should target the whole maturity structure?

Sticky prices

- With flexible prices, interest rate rules do not uniquely pinned down prices, but the allocation - and therefore welfare - is unique.

- BIP show that the same degree of indeterminacy arises with prices set in advance. What is special about that case?

- Consider the same model with a - permanent - productivity shock at time 1.

- Allow for a Dixit-Stiglitz aggregator for consumption.

- Technology

$$c(s^t) = \Delta\left[P(s^t), P(s^{t-1}), P(s^{t-2}), ...\right] A(s^t) n(s^t)$$

where $\Delta(s^t)$ measures the degree of price dispersion.

- The intra-period marginal condition, once we replace for the equilibrium value of the wage is

$$\frac{1}{V'(n(s^t))} = \kappa \frac{R(s_t)}{A(s^t)}$$

where κ is the mark-up.

- Thus, labor is pinned down by the interest rate, but consumption now depends on the interest rate and on the history of the price level.

- Depending on the type of price friction, there will be a pricing function

$$P(s^t) = F\left[s^t, P(s^{t-1}), P(s^{t-2}), ...\right]$$

- But the asset pricing equations are still given by

$$\begin{aligned}\frac{1}{P_0} &= 1 = \beta Q(1, A)\frac{1}{P_1^A} \\ \frac{1}{P_0} &= 1 = \beta Q(1, B)\frac{1}{P_1^B}\end{aligned}$$

so we have the same indeterminacy problem than with flexible prices for any form of the function

$$P(s^t) = F\left[P(s^{t-1}), P(s^{t-2}), ...\right]$$

- Does linearity on consumption matter? YES!

- With flexible prices it does not, since the allocations are uniquely pinned down by the interest rate rule...

- ..but with sticky prices it does, since consumption also depends on prices.

- Assume now a concave $U(c(s^t))$. Then, we have

$$\begin{aligned}\frac{U'(c_0)}{P_0} &= U'(c_0) = \beta Q(1, A)\frac{U'(c_1^A)}{P_1^A} \\ \frac{U'(c_0)}{P_0} &= U'(c_0) = \beta Q(1, B)\frac{U'(c_1^B)}{P_1^B}\end{aligned}$$

but

$$\begin{aligned} c_0 &= c_0(R, P_1^A, P_1^B) \\ c_1^A &= c_1^A(R, P_1^A, P_1^B) \\ c_1^B &= c_1^B(R, P_1^A, P_1^B) \end{aligned}$$

Then, we obtain

$$\begin{aligned} U'(c_0(R, P_1^A, P_1^B)) &= \beta Q(1, A) \frac{U'(c_1^A(R, P_1^A, P_1^B))}{P_1^A} \\ U'(c_0(R, P_1^A, P_1^B)) &= \beta Q(1, B) \frac{U'(c_1^B(R, P_1^A, P_1^B))}{P_1^B} \end{aligned}$$

- Given $Q(1, A)$ and $Q(1, B)$ - and therefore $R-$, this is a system of two equations in two unknowns (P_1^A, P_1^B), but it is not linear.

- With flexible prices, the "consumption functions" only depend on R.

- So, it is not clear that setting $Q(1, A)$ and $Q(1, B)$ is enough to uniquely pin down the price levels - and therefore the allocations.

- However, it would still appear to be very non-generic that by only setting the short term interest rate (only a linear combination of $Q(1, A)$ and $Q(1, B)$) one could pin down a unique equilibrium.

- Sticky prices imposes nonlinearities, so "counting" unknowns and variables is not a safe road.

- BIP show that when prices are set in advance, it is still a safe road.

- With sticky prices, the real interest rate depends on the prices - unless we have linear consumption. Then, setting the $Q's$ is not enough to pin down the price.

- It is clear, however, that by choosing the $Q(1, A)$ and $Q(1, B)$ that result in a stable price level under flexible prices, one possible equilibrium will be the flexible prices allocation. But there may be other.

- Thus, sticky prices can potentially generate additional degrees of multiplicity, since given the state contingent interest rates, there may be more than one solution.

- However, general properties of the "consumption functions" could be derived under some assumptions regarding the source of price frictions.

- Are there families of sticky price models such that one can prove the same results that with flexible prices?