A Macroeconomic Model with a Financial Sector

Discussion

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Summary

- Very Ambititous...brings together many important elements
  - macro
  - finance
- Very tractable...a nice framework
- Plan:
  - the paper through the lens of a simple 3-period model
  - a short comment on some normative aspects

Demographics and Preferences

- Build on Holmstrom and Tirole's LAPM
- Three periods: $t = 0, 1, 2$
- Two classes of agents:
  - entrepreneurs
  - consumers
- Risk neutrality, no discounting

Technology

- Two goods: capital, consumption
- p_t relative price of capital at t
- Between $t = 0$ and $t = 1$: $k_1 = \tilde{R} k_0$, with $\tilde{R}$ stochastic
- Between $t = 1$ and $t = 2$: $y = \frac{A}{\alpha^\alpha (1-\alpha)^{1-\alpha}} k^\alpha i^{1-\alpha}$

Frictions

- Two key frictions:
  - consumers cannot commit at $t = 0$ to payments at $t = 1$
  - only fraction ϕ of period 2 output y can be pledged to consumers
- Implications:
  - endogenous limit on available external finance at $t = 1$ (leverage constraint)
  - demand for liquidity between $t = 0$ and $t = 1$ (precautionary savings)
- A third friction to simplify and get some leverage at $t = 0$:
  - consumers can't directly invest...
  - ...they can only sign debt contracts with entrepreneurs

Solving Backwards: $t = 1$

- w net worth of representative entrepreneur at $t = 1$
- As long as financially constrained

$$\max_{k,i}(1-\phi)\frac{A}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}k^{\alpha}i^{1-\alpha}$$

s.t.

$$pk + i \leq w + \phi\frac{A}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}k^{\alpha}i^{1-\alpha}$$

- Scale and leverage

$$k = \frac{w}{p^{1-\alpha}}\frac{\alpha}{p^{\alpha}-\phi A}$$

- Return on net worth between 1 and 2

$$\frac{A(1-\phi)}{p^{\alpha}-\phi A}$$

Price and Net Worth to Capital

- Market clearing for capital at $t = 1$:

$$k = \frac{w}{p^{1-\alpha}} \frac{\alpha}{p^{\alpha} - \phi A}$$

- Result #1 price and net worth to capital $\frac{\partial p}{\partial (w/k)} > 0$

Volatility

- $1-\eta$ fraction of initial k_0 financed with debt
- η net worth to capital at $t=0$
- Net worth at $t=1$: $w=\left[p\tilde{R}-(1-\eta)p_0\right]k_0$
- Market clearing for capital at $t=1$:

$$\tilde{R}k_0=\frac{\left[p\tilde{R}-(1-\eta)p_0\right]k_0}{p^{1-\alpha}}\frac{\alpha}{p^{\alpha}-\phi A}$$

- Result #2 "excess volatility" $\frac{\partial p}{\partial \tilde{R}}>0$
  - leverage ($\eta<1$) $\Longrightarrow$ good shock increases net worth to capital
  - at given p, demand for capital increases more than supply
- Result #3 volatility and net worth to capital $\frac{\partial^2 p}{\partial \eta \partial \tilde{R}}<0$

Correlation

- Different types of capital (perfect substitutes in production at $t = 1$)
- Capital of type i: $k_1^i = \tilde{R}\tilde{R}^i k_0^i$ where $\tilde{R}_i$ and $\tilde{R}_j$ uncorrelated
- Result #4 correlation and net worth to capital

$$\frac{\partial Corr(\frac{p\tilde{R}}{p_0}\tilde{R}^i, \frac{p\tilde{R}}{p_0}\tilde{R}^j)}{\partial \eta} < 0$$

Expected Returns and Predictability

- SDF for entrepreneurs between 0 and 1

$$\frac{A(1-\phi)}{\left[p(\tilde{R};\eta)\right]^{\alpha}-\phi A}$$

- Apply it to capital to get expected return on capital

$$\pi \equiv \mathbb{E}\left[\frac{\tilde{R}p(\tilde{R};\eta)}{p_0}\right]$$

- Result #5 predictability $\frac{\partial \pi}{\partial \eta} < 0$
  - p reacts less to $\tilde{R}$ when η is high
  - price and quantity of risk
  - feedback between volatility and precautionary hoarding
  - potential liquidity premium on interest rate (not in paper)

Amplification

- Result #6 amplification (decreasing with η)

$$y = \frac{A}{\alpha} \tilde{R} k_0 \left[p(\tilde{R}; \eta) \right]^{1-\alpha}$$

A Comment on Some Normative Aspects

- Equilibrium constrained efficient...add externalities
- Fire sale externality $\Longrightarrow$ entrepreneurs overinvest at $t = 0$ (excessive leverage)
- Externalities on real economy a la Romer $Y_t = AK_t^{\alpha} k_t^{1-\alpha} I_t^{\alpha}$
  - proposed policy is delay in payout time...
  - ...but subsidy to capital restores constrained efficiency
  - targeting principle