The Procyclical Effects of Basel II

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Abstract

We analyze the cyclical effects of moving from risk-insensitive (Basel I) to risk-sensitive (Basel II) capital requirements in the context of a dynamic equilibrium model of relationship lending in which banks are unable to access the equity markets every period. Banks anticipate that shocks to their earnings as well as the cyclical position of the economy (modeled as a two-state Markov switching process) can impair their capacity to lend in the future and, as a precautionary measure, may hold capital buffers. We find that the new regulation may change the behavior of these buffers from countercyclical to procyclical. Yet, the higher buffers maintained in expansions may be insufficient to prevent a significant contraction in the supply of credit at the arrival of a recession. This credit crunch can be reduced by smoothing the transition from low to high capital charges.

Keywords: Business cycles, Credit crunch, Loan defaults, Banking regulation, Capital requirements, Basel II, Relationship banking.

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1 Introduction

A widespread concern about the new risk-sensitive bank capital regulation, known as Basel II, is that it might force banks to restrict their lending when the economy goes into a recession. Under the internal-ratings based (IRB) approach of Basel II, capital requirements are an increasing function of the banks' estimates of the probabilities of default (PDs) and losses given default (LGDs) of their loans, and these inputs are likely to be higher in downturns.¹ The concern is that the increase in capital requirements during downturns might lead to a drastic contraction in the supply of credit. Clearly, such contraction can only occur if some banks find it difficult to respond to the higher regulatory requirements by issuing new equity and if some of their borrowers are unable to switch to other sources of finance.² However, when these conditions hold, banks anticipate that shocks to their earnings and changes in the cyclical position of the economy can impair their capacity to lend in the future and, as a precautionary measure, may hold capital in excess of the regulatory requirements. Hence, the assessment of the cyclical implications of the new regulation requires the explicit consideration of endogenous capital buffers. The critical question is whether the equilibrium buffers (if any) will be sufficient to neutralize the added procyclicality—an argument frequently made by the advocates of Basel II, but not formally checked so far.³

This paper analyzes the cyclical effects of Basel II in the context of a tractable dynamic equilibrium model of relationship banking in which the economic cycle is modeled as a two-state Markov switching process. We formalize banks' difficulties in accessing equity markets at will by assuming that they can only raise new equity every other period. With

¹Although Basel II stipulates that PD estimates "must be a long-run average of one-year default rates" (Basel Committee on Banking Supervision, 2004, paragraph 447), industry practices based on point-in-time rating systems, the dynamics of rating migrations, and composition effects make the effective capital charges on a representative loan portfolio very likely to be higher in recessions than in expansions. See, for example, Kashyap and Stein (2004), Catarineu-Rabell, Jackson, and Tsomocos (2005), and Gordy and Howells (2006).

²These conditions parallel the conditions stated by Kashyap, Stein, and Wilcox (1993) and Blum and Hellwig (1995) for the existence of a bank lending channel in the transmission of monetary policy and shocks to banks' earnings, respectively.

³To the best of our knowledge, no paper has formalized the holding of capital buffers under Basel II regulation. At an empirical level, some assessments have been made by extrapolating the observed behavior of buffers during the Basel I era, but obviously these exercises do not resist the Lucas' critique.

this assumption, we aim to capture in a convenient reduced-form manner the fact that, as recognized by the Basel Committee on Banking Supervision (2004, paragraph 757), "it may be costly for banks to raise additional capital, especially if this needs to be done quickly or at a time when market conditions are unfavorable." The standard argument is that, due to potential informational asymmetries, discretionary equity injections may involve prohibitive transaction and dilution costs.⁴

We do not assume that banks synchronize their access to equity markets. In every period there are banks that raise equity and, hence, have no binding limits to the their lending capacity. We assume, however, that borrowers become dependent on the banks with which they first start a lending relationship. Lock-in effects due to explicit switching costs or potential lemons problems faced by alternative banks when a borrower has previously borrowed from another bank are central to the literature on relationship banking—see Boot (2000) for a survey.⁵ Our modeling establishes a simple connection between the capital shortages of some banks in a given period and the credit rationing of some borrowers in that period.

We define equilibrium under the assumption of free entry into the banking sector, which implies that the net present value of the banks' excess returns at the equity issuance dates must be zero. We characterize the equilibrium loan rates and banks' capital decisions in each state of the economy, and derive a number of comparative statics results. We show that capital requirements increase equilibrium loan rates, but their effect on capital holdings is ambiguous. On the one hand, the higher prospects of ending up with insufficient capital call

⁴Consistent with this argument, Barakova and Carey (2001) show that banks need an average of 1.6 years to restore their capital positions after becoming undercapitalized. They study the time to recovery of US banks that became undercapitalized in the 1984-1999 period. They note that "banks are very opaque, making it difficult for outsiders to estimate franchise value and thus to price a seasoned equity issue. Especially as large credit losses are being experienced, the lemons problem may be so severe that equity issuance is impossible in the short run. To survive, a bank experiencing large credit losses may need equity sufficient to carry it through a period of many quarters or years during which it re-accumulates equity from retained earnings, rebalances the credit portfolio, and demonstrates viability to outside investors as a precursor to issuing equity."

⁵Several papers analyze the influence of explicit switching costs on the terms of the lending relationship (starting with Sharpe, 1990) as well as the trade-offs behind the possible use of multiple lenders (e.g., Detragiache *et al.*, 2000). By abstracting from these complexities, we are implicitly assuming that the costs of these alternative arrangements are prohibitively high.

for the holding a larger buffer; on the other hand, the higher capital requirement reduces the profitability of future lending and, thus, the bank's interest in preserving its future lending capacity. Our analytical expressions suggest that the shape of the distributions of loan losses in different states of the economy matter for determining which effect dominates. Since the impact of capital requirements on the supply of continuation loans is, therefore, analytically ambiguous, we assess it numerically.

For the numerical analysis, we describe the distributions of loan losses according to the single risk factor model of Vasicek (2002), which provides the foundation for the IRB capital requirements of Basel II. ⁶ We find that when the value of the on-going lending relationships is large enough and the cost of equity capital is not very large, banks optimally choose to keep capital buffers. Under realistic parameterizations, Basel II leads banks to hold buffers that range from 2% of assets in recessions to 6% in expansions. The procyclicality of these buffers reflect that banks are concerned about the upsurge in capital requirements that takes place when the economy goes into a recession. We find, however, that these equilibrium buffers are insufficient to neutralize the effects of the arrival of a recession, which may still cause a very significant reduction in the supply of credit.⁷ Under the flat capital requirements of the 1988 Basel Accord (Basel I), the same economies would exhibit countercyclical buffers and essentially no credit crunch effects.⁸

Our results also suggest that the probabilities of bank failure are likely to be substantially lower under Basel II than under Basel I. Moreover, due to the capital buffers and the net interest income earned on performing loans, the effective long-run average of the bank failure

⁶Under this model the IRB capital requirements have an exact value-at-risk interpretation: required capital is such that it can absorb the potential losses of a loan portfolio over a one-year horizon with probability (or confidence level) 99.9%. As shown by Gordy (2003), the single risk factor model also has the feature that the contribution of a given loan to value-at-risk is additive, that is, it depends on the loan's own characteristics and not on those of the portfolio in which it is included.

⁷Supervisors seem aware of this possibility. For instance, Greenspan (2002) claims that "The supervisory leg of Basel II is being structured to supplement market pressures in urging banks to build capital considerably over minimum levels in expansions as a buffer that can be drawn down in adversity and still maintain adequate capital."

⁸Some papers, starting with Bernanke and Lown (1991), point out that the introduction of Basel I caused a credit crunch in the US during the months preceding the cyclical peak of 1990. But no credit crunch episode has been detected after banks adjusted their capital holdings to the higher requirements.

rate under Basel II is barely one tenth of the 0.1% per year bound nominally targeted in the IRB approach. We argue that some state-contingent adjustments in the confidence level of the IRB approach (designed to smooth the cyclical transitions from low to high capital charges without compromising the long-run solvency target implied by Basel II) might substantially reduce the incidence of credit rationing over the business cycle.

The papers closest to ours are Estrella (2004) and Peura and Keppo (2006). Estrella (2004) considers the dynamic optimization problem of a single bank whose dividend policy and equity raising processes are subject to adjustment costs, and whose loan losses follow a second-order autoregressive process. According to an inventory model logic, the trade-off between the risk of bankruptcy and the costs of holding, raising and distributing capital determine the bank's optimal capitalization decisions. Peura and Keppo (2006) consider a single bank with an asset portfolio of exogenous size subject to minimum capital requirements whose compliance is checked by a supervisor at random intervals of time. Detected violations of the capital requirement lead to the closure of the bank, but raising equity takes time. The possibility that the bank's capital position remains undesirably impaired for some time may induce it to optimally hold some capital buffers. Relative to these papers, we simplify the details of the banks' dynamic optimization problem and plug such problem in the context of an equilibrium model of relationship banking, in which loan rates are endogenous and the dynamic behavior of aggregate bank lending comes to the forefront.

In our analysis, we abstract from demand-side cyclicality and feedback effects that might attenuate and exacerbate, respectively, the aggregate implications of the cyclicality in banks' lending capacity. We do so in order to isolate the potential cyclicality coming from the supply side of the bank loans' market. The model, however, could serve as a building block of a more comprehensive dynamic stochastic general equilibrium model in which part of the production comes from entrepreneurial projects that require relationship-banking finance.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we analyze the capital decision of a representative bank. Section 4 defines the equilibrium and provides the comparative statics of equilibrium loan rates and banks' capital holdings in each state of the economy. In Section 5 we summarize the numerical results concerning the size

and cyclical behavior of capital holdings, capital buffers, credit rationing, and probabilities of bank failure in a number of parameterizations of the model and under both Basel I and Basel II capital requirements. In Section 6 we comment on possible adjustments of the Basel II framework that might smooth the cyclical variability and expected incidence of credit rationing without compromising the long-term solvency targets set by regulators. Section 7 concludes. The Appendix contains the proofs of the analytical results.

2 The Model

Consider a discrete time economy in which time is indexed by t = 0, 1, 2, ... The economy is populated by three classes of risk-neutral agents: entrepreneurs, banks, and investors.

2.1 Entrepreneurs

Entrepreneurs belong to overlapping generations formed by a continuum of measure one of exante identical and penniless individuals who remain active for up to two periods (three dates). Entrepreneurs have the opportunity to undertake investment projects with the following characteristics. The initial project of an entrepreneur born at date t requires one unit of funds at that date. At date t + 1 the project yields 1 + a if it successful, and $1 - \lambda$ if it fails, with a > 0 and $0 < \lambda < 1$. Projects can be continued up to date t + 2 if some additional funding μ (which measures the scale of second-period operations) is provided at date t + 1. The return at date t + 2 of a continued project is $(1 + a)\mu$ if it is successful in its second period, and $(1 - \lambda)\mu$ if it fails.

All projects operating from date t to date t + 1 have an identical probability of failure denoted by p_t . The outcomes of contemporaneous projects exhibit positive but imperfect correlation, so their aggregate failure rate is a continuous random variable x_t with support [0, 1] and cumulative distribution function (cdf) $F_t(x_t)$ such that

$$p_t = E_t[x_t] = \int_0^1 x_t \, dF_t(x_t).$$

For simplicity, we consider the case in which the history of the economy up to date t only affects $F_t(x_t)$ (and, thus, p_t) through an observable state variable s_t that can take

two values, h and l, and follows a Markov chain with $q_h = \Pr[s_t = h \mid s_{t-1} = h]$ and $q_l = \Pr[s_t = h \mid s_{t-1} = l]$. Moreover, we assume that the cdfs in each of the two states, $F_h(\cdot)$ and $F_l(\cdot)$, are ranked in the sense of first-order stochastic dominance, so that the probabilities of business failure in each state satisfy $p_h > p_l$. Thus states h and l may be interpreted as recession (high business failure) and expansion (low business failure) states, respectively.

2.2 Banks

Banks are intermediaries specialized in channeling funds from investors to entrepreneurs. Following the literature on relationship banking, we assume that the financing of an entrepreneur in this economy relies on a sequence of one-period loans granted by the single bank from which the entrepreneur obtains his first loan. We also assume that setting up the relationship with the entrepreneur makes the bank incur some cost c, to be subtracted from first-period revenue.⁹ Finally, for simplicity, we abstract from the possibility that part of the required second-period investment μ is internally financed by the entrepreneur.¹⁰

Banks are funded with deposits and equity capital, both of which are raised from investors. To simplify the analysis we assume that deposits are fully insured (at a zero premium).¹¹ Their supply is perfectly elastic at an interest rate that we normalize to zero and reflects either the investors' rate of time preference or the return on some generally available risk-free technology.

Banks face two important imperfections concerning their equity financing. First, investors require an expected rate of return $\delta \geq 0$ on each unit of equity capital. Second, a bank can only raise new equity every other date. The cost of capital δ is intended to capture in a reduced-form manner distortions (such as agency costs of equity or debt tax shields) that introduce a comparative disadvantage of equity financing relative to deposit financing—in

⁹This cost might include personnel, equipment and other operating costs associated with the screening and monitoring functions emphasized in the literature on relationship banking.

¹⁰This simplification is standard in relationship-banking models—see, for example, Sharpe (1990, p. 1072) or von Thadden (2004, p. 14). Moreover, if entrepreneurs' first-period profits are small relative to the required second-period investment (as in our numerical analysis below), the quantitative effects of relaxing this assumption would be negligible.

¹¹In our numerical analysis the probability of bank failure is a small fraction of the 0.1% target of Basel II, so the required deposit insurance premium would be very small.

addition to deposit insurance.¹² The assumption that, for a given bank, recapitalization is impossible every other date is a simple way to capture the long delays or prohibitive dilution costs that the bank might face when organizing an urgent equity injection.

Banks are managed in the interest of their shareholders, who are protected by limited liability. Entry to the banking sector is free at all dates, but banks are subject to capital requirements that oblige them to hold a capital-to-loans ratio of at least γ_s on the loans made when the state of the economy is s. This formulation encompasses both state-invariant capital regulation ($\gamma_l = \gamma_h$), such as Basel I, and state-contingent regulation ($\gamma_h > \gamma_l$), such as Basel II.¹³

To guarantee that the funding of investment projects is attractive to banks at all dates, even when they are required to finance a minimum fraction of their loans with equity capital, we assume that

$$(1 - p_s)(1 + a) + p_s(1 - \lambda) - c > (1 - \gamma_s) + \gamma_s(1 + \delta),$$
(1)

for s = h, l. Thus, in all states of the economy, the expected return per unit of investment, net of the setup cost c, is greater than the cost of funding it with $1 - \gamma_s$ deposits and γ_s capital.

3 The Banks' Capital Decision

Suppose that entrepreneurs born at date t obtain their first-period loans from unrestricted banks that can raise capital at this date.¹⁴ Then the constraint on banks' recapitalization

 $^{^{12}}$ Further to the reasons for the extra cost of equity financing offered by the corporate finance literature, Holmström and Tirole (1997) and Diamond and Rajan (2000) provide agency-based explanations specifically related to banks' monitoring role.

¹³Under a perfectly acyclical rating system, the IRB approach of Basel II might also be state-invariant. Our state-contingent Basel II scenario is intended to capture the implications of the type of procyclical internal rating systems that are likely to be used in practice. See Kashyap and Stein (2004), Catarineu-Rabell, Jackson, and Tsomocos (2005), and Gordy and Howells (2006) for justifications of this point.

¹⁴The temporal coincidence of banks' access to the equity market with the renewal of their portfolio of lending relationships would be consistent with a microfoundation for the difficulties of raising equity at the interim dates based on asymmetric information. In a world in which banks learn about their borrowers after starting a lending relationship (like in Sharpe, 1990) and borrower quality is asymmetrically distributed across banks, the market for seasoned equity offerings might be affected by a "lemons problem" (like in

at date t + 1 is equivalent to assuming that the banking industry has an OLG structure in which banks operate for two periods, specialize in loans to contemporaneous entrepreneurs, and cannot issue equity at the interim date. This is because a bank that can raise capital at date t is entirely equivalent to a bank with identical equity created at that date.

In this economy, the supply of loans to the entrepreneurs who start up at date t might be affected by the recapitalization constraint faced by their banks at date t + 1. In fact, the banks will be aware of this and, in order to better accommodate the effect of negative shocks to their first-period income or possibly higher capital requirements in the case of risk-sensitive capital regulation, they may hold a buffer of equity capital on top of the first-period regulatory minimum. At a microeconomic level, the analysis of the model will help us understand the conditions under which banks' capital buffers are positive and the determinants of their size. At a macroeconomic level, the analysis will allow us to assess the extent to which the presence of (endogenous) buffers ameliorates the potential procyclicality in bank lending caused by minimum capital requirements.

To understand the financing problem faced by each generation of entrepreneurs in this economy, consider a representative bank that lends to the measure one continuum of entrepreneurs starting up at date t, possibly refinances them at date t + 1, and gets liquidated at date t + 2.¹⁵ Let s and s' denote the states of the economy at dates t and t + 1, respectively.

At date t the representative bank raises $1 - k_s$ deposits and $k_s \ge \gamma_s$ capital, and invests these funds in a unit portfolio of initial (or first-period) loans. The equilibrium interest rate on initial loans, r_s , will be determined endogenously, as explained below, but we can take it as given for the time being. Since the supply of deposits is perfectly elastic at a zero interest rate, r_s should be interpreted as the spread between initial loan rates and deposit rates.

At date t + 1 the bank gets $1 + r_s$ from the fraction $1 - x_t$ of the loans that do not default (that is, those extended to entrepreneurs whose projects are successful) and $1 - \lambda$ from the

Myers and Majluf, 1984). In essence, the banks with lending relationships of poorer quality would be the most interested in issuing equity at any given price, which might explain why the prices at which equity could be raised would be unattractive to higher quality banks.

¹⁵It will become obvious that banks that can raise equity face constant returns relative to the size of their loan portfolio.

fraction x_t of defaulted loans, and incurs the setup cost c, so its assets are $1+r_s-x_t(\lambda+r_s)-c$, while its deposit liabilities are $1-k_s$. Thus, its net worth (or available capital) at date t+1is

$$n_s(x_t) = k_s + r_s - x_t(\lambda + r_s) - c, \qquad (2)$$

where x_t is a random variable whose cdf conditional on the state of the economy at date t is $F_s(x_t)$. If $n_s(x_t) < 0$ the bank is closed, while if $n_s(x_t) \ge 0$ it can operate for a second period. Using the definition of $n_s(x_t)$, it is immediate to show that bank closure occurs when the default rate x_t exceeds the critical value

$$\widehat{x}_s = \frac{k_s + r_s - c}{\lambda + r_s}.$$
(3)

The entrepreneurs funded at date t demand an amount μ of continuation (or secondperiod) loans at date t+1.¹⁶ At this stage entrepreneurs are dependent on the bank, so their demand is inelastic insofar as the loan interest rate does not exceed the success return of the projects in the second investment period, a. Thus the continuation loan rate will be a.

Since the bank cannot issue new equity at date t + 1, its maximum lending capacity is $n_s(x_t)/\gamma_{s'}$, as determined by its available capital $n_s(x_t)$ and the capital requirement $\gamma_{s'}$, which depends on the state of the economy s' at date t + 1. Thus, whenever $n_s(x_t) \ge 0$ there are two cases to consider: the case with excess lending capacity, $n_s(x_t) \ge \gamma_{s'}\mu$, and the case with insufficient lending capacity, $n_s(x_t) < \gamma_{s'}\mu$. Using the definition of $n_s(x_t)$ in (2), it is immediate to show that the latter case arises when the default rate x_t exceeds the critical value

$$\widetilde{x}_{ss'} = \frac{k_s + r_s - c - \gamma_{s'}\mu}{\lambda + r_s},\tag{4}$$

which is obviously smaller than \hat{x}_s , defined in (3). Thus, whenever $0 < \tilde{x}_{ss'} < \hat{x}_s < 1$, one can find three different situations at date t + 1, depending on the realization of the default rate: for $x_t \in [0, \tilde{x}_{ss'}]$, the representative bank has excess lending capacity; for $x_t \in (\tilde{x}_{ss'}, \hat{x}_s]$, the bank has insufficient lending capacity; and for $x_t \in (\hat{x}_s, 1]$ the bank fails.

¹⁶Note that this includes entrepreneurs that defaulted on their initial loans. This is because under our assumptions such default does not reveal any information about the entrepreneurs' second-period projects.

Next we derive the expected continuation payoffs of the bank's shareholders in each of the two cases where the bank does not fail. When there is excess lending capacity at date t+1 the bank finances μ loans using $(1-\gamma_{s'})\mu$ deposits and $\gamma_{s'}\mu$ capital. Since $n_s(x_t) \geq \gamma_{s'}\mu$, the bank pays a dividend of $n_s(x_t) - \gamma_{s'}\mu$ to its shareholders at date t + 1.¹⁷ At date t + 2the bank gets 1+a from the fraction $1-x_{t+1}$ of the loans that do not default and $1-\lambda$ from the fraction x_{t+1} of defaulted loans, so its assets are $[1 + a - x_{t+1}(\lambda + a)]\mu$, while its deposit liabilities are $(1 - \gamma_{s'})\mu$. Thus shareholders' expected payoff, conditional on the state of the economy at date t + 1, can be expressed as $\mu\pi_{s'}$, where

$$\pi_{s'} = \int_0^1 \max\left\{\gamma_{s'} + a - x_{t+1}(\lambda + a), \ 0\right\} \ dF_{s'}(x_{t+1}) \tag{5}$$

measures the expected gross equity return on a per-unit-of-loans basis. Thus, the value of shareholders' stake in the bank at date t + 1, inclusive of the dividend $n_s(x_t) - \gamma_{s'}\mu$, can be written as

$$v_{ss'}(x_t) = (\beta \pi_{s'} - \gamma_{s'})\mu + n_s(x_t), \tag{6}$$

where $\beta \equiv 1/(1+\delta)$ is the shareholders' discount factor. The first term in (6) measures the net present value contribution of the capital that remains invested in the bank up to date t+2. Assumption (1) guarantees that $\beta \pi_{s'} > \gamma_{s'}$, so that such contribution is positive.¹⁸

When there is insufficient lending capacity at date t + 1 the bank finances $n_s(x_t)/\gamma_{s'}$ loans with $[n_s(x_t)/\gamma_{s'}] - n_s(x_t)$ deposits and $n_s(x_t)$ capital. At date t + 2 shareholders' expected payoff, conditional on the state of the economy at date t + 1, can be expressed as $[n_s(x_t)/\gamma_{s'}]\pi_{s'}$, where, as before, $\pi_{s'}$ is the expected gross equity return on a per-unit-of-loans basis given by (5). When there is insufficient lending capacity at date t + 1 the bank pays no dividends at that date and, hence, the value of shareholders' stake in the bank is just

$$v_{ss'}(x_t) = \frac{\beta \pi_{s'}}{\gamma_{s'}} n_s(x_t).$$
(7)

¹⁷Insofar as entrepreneurs born at date t + 1 borrow from banks that can raise equity at that date (which might be justified with the arguments in footnote 13), the best use of any excess capital held by the bank between dates t + 1 and t + 2 would be to substitute for an equivalent amount of deposit funding. However, under deposit insurance and $\delta \geq 0$, such a substitution is strictly suboptimal.

¹⁸To see this, notice that $\pi_{s'} > \int_0^1 [\gamma_{s'} + a - x_{t+1}(\lambda + a)] dF_{s'}(x_{t+1}) = \gamma_{s'} + a - p_{t+1}(\lambda + a)$, but (1) implies $a - p_{t+1}(\lambda + a) > \delta \gamma_{s'}$ and hence $\pi_{s'} > (1 + \delta) \gamma_{s'} = \gamma_{s'}/\beta$.

As before, assumption (1) implies that $\beta \pi_{s'} > \gamma_{s'}$, and hence shareholders strictly benefit from keeping $n_s(x_t)$ invested in the bank.

Putting together the two cases, as well as the case in which the bank is closed, we can express the value that shareholders obtain from the bank at date t+1, inclusive of dividends, as

$$v_{ss'}(x_t) = \begin{cases} (\beta \pi_{s'} - \gamma_{s'})\mu + n_s(x_t), & \text{if } x_t \leq \widetilde{x}_{ss'}, \\ \frac{\beta \pi_{s'}}{\gamma_{s'}} n_s(x_t), & \text{if } \widetilde{x}_{ss'} < x_t \leq \widehat{x}_s, \\ 0, & \text{if } x_t > \widehat{x}_s, \end{cases}$$
(8)

which is a continuous and piecewise linear function of x_t .¹⁹ Going one period backward, the net present value of the representative bank that in state s holds capital k_s and charges an interest rate r_s on its unit of initial loans is

$$v_s(k_s, r_s) = \beta E_t[v_{ss'}(x_t)] - k_s, \qquad (9)$$

where the operator $E_t[\cdot]$ takes care of the fact that, at date t, v_s is subject to the uncertainty about both the state of the economy at date t + 1, which affects $\pi_{s'}$ and $\gamma_{s'}$, and the default rate x_t of initial loans, which affects $n_s(x_t)$.

Taking as given the initial loan rate r_s , the representative bank that first lends to a generation of entrepreneurs in state s will choose its capital k_s so as to maximize $v_s(k_s, r_s)$ subject to the constraint $k_s \in [\gamma_s, 1]$. Since $v_s(k_s, r_s)$ is continuous in k_s , for any given interest rate r_s , the bank's capital decision always has a solution. In the Appendix we show that the function $v_s(k_s, r_s)$ is neither concave nor convex in k_s , and that we may have interior solutions or corner solutions with $k_s = \gamma_s$. When the solution is interior, there is a positive probability that the bank has insufficient lending capacity in state s' = h (and possibly also in state s' = l), and there is a positive probability that the bank has excess lending capacity in state s' = l (and possibly also in state s' = l). The intuition for this result is as follows: If in the two possible states at date t + 1 the bank had a zero probability of finding itself with insufficient lending capacity, then it would have an incentive to reduce the level of capital at date t in order to reduce its funding costs at that date. On the other hand, if in the two

¹⁹Recall from (2) that $n_s(x_t)$ is linear in x_t .

possible states at date t + 1 the bank had a zero probability of finding itself with excess lending capacity, then it would have an incentive either to increase the level of capital at date t and thereby relax the capital constraint at date t + 1, or to go to the corner $k_s = \gamma_s$.²⁰

4 Equilibrium

In the previous section we have characterized the banks' capital and lending decisions at the dates in which they can raise capital, as well as at the dates in which they cannot. This analysis has taken as given the interest rate r_s at the beginning of a lending relationship in state s, with the continuation loan rate being the success return a of the second-period investment projects. In order to define an equilibrium, it only remains to describe how the initial loan rate is determined.

For this, notice that free entry and the fact that banks that can issue equity face constant returns to scale make the situation in the market for initial loans equivalent to perfect (or Bertrand) competition. In equilibrium, the pricing of these loans must be such that the net present value of the bank is zero under the bank's optimal capital decision. Were it negative, no bank would extend loans. Were it positive, incumbent banks would have an incentive to expand, and new banks would profit from entering the market. Hence in each state of the economy s = h, l we must have

$$v_s(k_s^*, r_s^*) = 0, (10)$$

for

$$k_{s}^{*} = \arg \max_{k_{s} \in [\gamma_{s}, 1]} v_{s}(k_{s}, r_{s}^{*}).$$
(11)

Therefore we may define an *equilibrium* as a sequence of pairs $\{(k_t, r_t)\}$ describing the capital-to-loan ratio k_t of the banks that can issue equity at date t and the interest rate r_t charged on their initial loans, such that each pair (k_t, r_t) satisfies (10) and (11) for $s = s_t$, where s_t is the state of the economy at date t.

²⁰The possible preference for the corner $k_s = \gamma_s$ is due to the fact that, in this case, the function $v_s(k_s, r_s)$ is (locally) either decreasing or convex in k_s ; see the Appendix for the details.

The existence of an equilibrium is easy to establish. Differentiating (10) we have

$$\frac{dv_s}{dr_s} = \frac{\partial v_s}{\partial k_s^*} \frac{dk_s^*}{\partial r_s} + \frac{\partial v_s}{\partial r_s},$$

where the first term is zero, by the envelope theorem, and the second is positive (see the Appendix). So $v_s(k_s^*, r_s)$ is continuous and monotonically increasing in r_s . Moreover, for sufficiently low interest rates we have $v_s(k_s^*, r_s) < 0$, while for $r_s = a$ assumption (1) implies $v_s(k_s^*, r_s) > 0$. Hence we conclude that there is a unique r_s^* that satisfies $v_s(k_s^*, r_s^*) = 0.2^{21}$

4.1 Comparative statics

The structural parameters that describe the economy are the following: The profitability parameter a (which determines the interest rate of continuation loans), the loan demand expansion parameter μ , the loss given default parameter λ , the setup cost c, the cost of bank equity capital δ , the probabilities of transition from each state to the high default state q_h and q_l , and the possibly state-contingent capital requirements γ_h and γ_l . To complete the description, one must also specify the state-contingent cdfs of the default rate, $F_h(\cdot)$ and $F_l(\cdot)$.

Table 1 summarizes the comparative statics of the equilibrium interest rates on initial loans r_s^* , which can be obtained analytically (see the Appendix). The table shows the sign of the total derivatives of the form dr_s^*/dz obtained by totally differentiating (10) with respect to each exogenous parameter z (one at a time).

	z	=	a	μ	λ	С	δ	q_s	γ_h	γ_l
Sign of $\frac{dr_s^*}{dz}$			_	_	+	+	+	+	+	+

Table 1. Comparative statics of the initial loan rate r_s^*

Intuitively, the effects of the various parameters on r_s^* are inversely related to their effects on the profitability of banks' lending activity. Other things equal, a and μ impact positively on

²¹However, since the function $v_s(k_s, r_s)$ is neither concave nor convex in k_s , there may be multiple optimal values of k_s corresponding to r_s^* .

the overall profitability of continuation lending; λ affects negatively the profitability of both initial lending (directly) and continuation lending (directly and by reducing the availability of capital in the second period); c has a similar negative effect (with no direct effect on the profitability of continuation loans); δ increases the cost of equity funding in both periods; γ_h and γ_l increase the burden of capital regulation in the corresponding initial state, as well as in the corresponding continuation state (which will be h or l with probabilities q_s and $1 - q_s$, respectively); finally, q_s decreases the expected profitability of continuation lending because, in the high default state h, loan losses are higher and the corresponding capital requirement γ_h may also be higher.

Table 2 summarizes the comparative statics of the equilibrium initial capital k_s^* chosen by the banks in an interior solution—obviously, when the solution is at the corner $k_s^* = \gamma_s$, marginal changes in parameters other than γ_s do not change k_s^* . As further explained in the Appendix, the recursive nature of the comparative statics of the system given by (10) and (11) makes it convenient to decompose the effects of the change in any parameter z into a *direct effect* (for constant r_s^*) and a *loan rate effect* (due to the change in r_s^*):

$$\frac{dk_s^*}{dz} = \frac{\partial k_s^*}{\partial z} + \frac{\partial k_s^*}{\partial r_s^*} \frac{dr_s^*}{dz}$$

Loan rate effects can be easily determined. Differentiating the first-order condition that characterizes k_s^* in an interior solution gives

$$\frac{\partial^2 v_s}{\partial (k_s^*)^2} \frac{\partial k_s^*}{\partial r_s} + \frac{\partial^2 v_s}{\partial k_s^* \partial r_s} = 0,$$

where the coefficient of $\partial k_s^* / \partial r_s^*$ is negative, by the second-order condition, and the second term is negative (see the Appendix). Hence $\partial k_s^* / \partial r_s^*$ is negative and the signs of loan rate effects are the opposite to those in Table 1. Intuitively, the initial capital k_s and the initial loan rate r_s are substitutes in the role of providing the bank with sufficient capital $n_s(x)$ for its continuation lending (see (2)). In an interior solution, the marginal value of k_s is decreasing in k_s , and thus also in r_s , so a larger r_s reduces the bank's incentive to hold excess capital.

	z =	a	μ	λ	c	δ	q_s	γ_h	γ_l
Sign of $\frac{\partial k_s^*}{\partial z}$ (direct effect)		+	+	?	+	_	?	?	?
Sign of $\frac{\partial k_s^*}{\partial r_s^*} \frac{dr_s^*}{dz}$ (loan rate effect)		+	+	_	_	_	_	_	_
Sign of $\frac{dk_s^*}{dz}$ (total effect)		+	+	?	?	_	?	?	?

Table 2. Comparative statics of an interior equilibrium initial capital k_s^*

For the parameters a, μ , and δ , the direct and indirect effects point in the same direction, so the total effect can be analytically signed. In essence, higher profitability of continuation lending (captured by a and μ) and lower costs of capital (captured by δ) lead the bank to hold greater capital buffers. The greater net profitability of continuation lending encourages the bank to better self-insure against the default shocks that threaten its continuation lending capacity. For the setup cost c, the direct and loan rate effects have unambiguous but opposite signs so the total effect is ambiguous. The positive direct effect comes from the fact that, for constant r_s , the effects of rising c are the mirror image of the effects of lowering k_s —from (2), c subtracts to the bank's continuation lending capacity exactly like k_s adds to it, without affecting the profitability of such lending and hence the marginal gains from self-insuring against default shocks.

The direct effects of the parameters λ , q_s , γ_h , and γ_l have ambiguous signs. Increasing any of these parameters simultaneously reduces the profitability of continuation lending and impairs the expected capital position of the bank when such lending has to be made. The value of holding excess capital in the initial lending period falls but the prospects of ending up with insufficient capital increase. So the profitability of continuation lending and the need for self-insuring against default shocks move in opposite directions. The resulting ambiguity of the direct effects extends to the total effects. The details of the analytics suggest that the shape of the distributions of the default rates on initial and continuation loans matter for the determination of the total effect, which eventually becomes a question to be elucidated either empirically or by numerically solving the model under realistic parameterizations. Since the goal of the paper is to assess the potential impact of the yet not applied Basel II requirements, the resort to the second alternative is particularly in order.

5 Numerical Results

To further explore the forces that affect banks' initial capital buffers as well as to assess the implications for the dynamics of lending, we numerically solve the model in a number of realistic scenarios. Importantly, in all scenarios we assume that the state-contingent probability distributions of the default rate, described by the cdfs $F_h(\cdot)$ and $F_l(\cdot)$, conform to the single risk factor model that underlies the capital requirements associated with the IRB approach of Basel II.²² This means that we assess the implications of the new capital requirements under the assumption that the supervisor's model of reference is correct.²³

5.1 The single risk factor model

Suppose that an investment project *i* undertaken at date *t* is successful at date t + 1 if and only if $y_{it} \ge 0$, where y_{it} is a latent random variable defined by

$$y_{it} = \alpha_t + \sqrt{\rho_t} \ u_t + \sqrt{1 - \rho_t} \ \varepsilon_{it},$$

where α_t is a parameter determined by the state of the economy at date t, u_t is a single factor of systematic risk, ε_{it} is an idiosyncratic risk factor, and $\rho_t \in (0, 1)$ is a parameter determined by the state of the economy at date t that captures the exposure to the systematic risk factor; u_t and ε_{it} are N(0, 1) random variables, independently distributed from each other and over

 $^{^{22}}$ The single factor model is due to Vasicek (2002) and its use as a foundation for the capital requirements of Basel II is due to Gordy (2003).

²³Of course, the model could be similarly solved under alternative specifications of the relevant cdfs, but in that case the requirements set under the regulatory formula described below would not have the direct value-at-risk interpretation implied by our parameterization.

time, as well as, in the case of ε_{it} , across projects. Let $\Phi(\cdot)$ denote the cdf of a standard normal random variable. Clearly, conditional on the information available at date t, the probability of failure of project i is $p_t \equiv \Pr[y_{it} < 0] = \Phi(-\alpha_t)$, since $y_{it} \sim N(\alpha_t, 1)$, which implies $\alpha_t = -\Phi^{-1}(p_t)$.

When considering a continuum of projects, idiosyncratic risk is diversified away and the aggregate failure rate x_t is only a function of the realization of the single risk factor u_t . Specifically, by the law of large numbers, x_t coincides with the probability of failure of a (representative) project *i* conditional on the information available at *t* and the realization of u_t :

$$g_t(u_t) = \Pr\left[-\Phi^{-1}(p_t) + \sqrt{\rho_t} \ u_t + \sqrt{1 - \rho_t} \ \varepsilon_{it} < 0 \ | \ u_t\right] = \Phi\left(\frac{\Phi^{-1}(p_t) - \sqrt{\rho_t} \ u_t}{\sqrt{1 - \rho_t}}\right).$$

Hence, using the fact that $u_t \sim N(0, 1)$, the cumulative distribution function of the aggregate failure rate under the single risk factor model can be expressed as:

$$F_t(x_t) \equiv \Pr\left[g_t(u_t) \le x_t\right] = \Pr\left[u_t \le g_t^{-1}(x_t)\right] = \Phi\left(\frac{\sqrt{1-\rho_t} \ \Phi^{-1}(x_t) - \Phi^{-1}(p_t)}{\sqrt{\rho_t}}\right)$$

In Basel II the correlation parameter ρ_s is assumed to be a decreasing function of the state-contingent probability of default p_s . Hence, we postulate the following state-contingent probability distributions of the default rate

$$F_s(x) = \Phi\left(\frac{\sqrt{1-\rho_s} \, \Phi^{-1}(x) - \Phi^{-1}(p_s)}{\sqrt{\rho_s}}\right),\tag{12}$$

where, as stipulated by Basel II for corporate loans,

$$\rho_s = 0.12 \left(2 - \frac{1 - \exp(-50 \ p_s)}{1 - \exp(-50)} \right). \tag{13}$$

In the IRB approach of Basel II, capital must cover the losses due to loan defaults with a probability of 99.9%. Hence the capital requirement in state s is given by $\gamma_s = \lambda F_s^{-1}(0.999)$, where $F_s^{-1}(0.999)$ is the 99.9% quantile of the distribution of the default rate. Using (12), the Basel II capital requirement becomes:

$$\gamma_s = \lambda \Phi \left(\frac{\Phi^{-1}(p_s) + \sqrt{\rho_s} \Phi^{-1}(0.999)}{\sqrt{1 - \rho_s}} \right),$$
(14)

where ρ_s is given by (13). This is the formula for corporate, sovereign, and bank exposures that appears in Basel Committee on Banking Supervision (2004, paragraph 272), under the assumption of a one-year maturity.²⁴

5.2 Benchmark scenarios

Table 3 shows the set of parameter values that define the benchmark scenarios around which we focus the discussion below. Notice that, given our normalization of the risk-free rate to zero, all interest rates and rates of return in the parameterization below should be interpreted as spreads over the risk-free rate.

Panel A contains the parameters that are common to the three scenarios. The value of the profitability parameter a = 0.05 implies that the projects' (net) return-on-assets is at most 5% per period, and that the interest rate spread on continuation loans is 5%. The loan demand expansion parameter $\mu = 1$ provides a neutral starting point—fine tuning this parameter would require some empirical investigation so as to evaluate what is the growth rate of loan exposures along a typical corporate lending relationship or, alternatively, what is the growth rate of the assets in the representative business project financed by banks. The loss given default parameter $\lambda = 0.45$ is consistent with the "foundation IRB" formula for one-year corporate loans.²⁵ The setup cost c = 0.04 is chosen so as yield realistic initial loan rates. The cost of bank capital $\delta = 0.05$ can be seen as the result of assuming that half of the computable capital is Tier 1 equity capital, with an excess return of 10%, and the other half is Tier 2 subordinated debt, which empirically carries essentially no spread over the riskless rate.²⁶ The probabilities of transition from each state to the high default state, $q_h = 0.55$ and $q_l = 0.20$, are taken from Bruche and Gonzalez-Aguado (2006), who estimate a two-state Markov switching model for US bond defaults. These parameters imply an expected duration of 2.2 years for the high default state and 5 years for the low default

 $^{^{24}}$ The IRB formula incorporates a correction factor that is increasing in the maturity of the exposures and equals one for a one-year maturity.

²⁵In the "advanced IRB" approach, banks are allowed to use their own internal models to estimate λ . Since cyclical variation in λ would add cyclicality both to bank profits and to capital requirements, our results with constant λ provide a lower bound for the cyclical implications of the IRB approach.

²⁶Regulation requires at least 50% of regulatory capital to be Tier 1 capital.

state. 27

Panel B in Table 3 shows our choices for the state-contingent probabilities of default p_h and p_l (and, using (14), the state-contingent capital requirements γ_h and γ_l under Basel II). The cross-state variability in the probabilities of default are such that the long-run average capital requirement imposed by Basel II (given the underlying unconditional probabilities of visiting each state) is 8%, as under the risk-insensitive Basel I regulation. The three scenarios only differ in the importance of the cross-state variation—and the three of them are within a range that can be considered empirically plausible.

Table 3. Parameter values in the benchmark scenarios

	A. Common parameters										
	a	μ	λ	c	δ	q_h	q_l				
0	0.05	1.00	0.45	0.04	0.05	0.55	0.20				

D. I follability of default scenarios										
	Benchmark PDs	Basel II requirements								
	$p_s~(\%)$	$\gamma_s~(\%)$								
Low volatility scenario										
s = h	2.9	10.0								
s = l	1.3	7.1								
Medium volatility scenario										
s = h	3.7	11.2								
s = l	1.1	6.6								
High volatility scenario										
s = h	4.2	11.8								
s = l	1.0	6.3								

B. Probability of default scenarios

The three PD scenarios are defined so as to keep the expected capital charge under Basel II equal to 8%, which is the state-invariant capital requirement under Basel I.

²⁷In our Markov switching setup, the expected durations of states h and l can be computed as $1/(1 - q_h)$ and $1/q_l$, respectively. Although with a methodology different from Bruche and Gonzalez-Aguado (2006), Koopman et al. (2005) find a stochastic cycle in US business failure rates with a period between 8 and 11 years.

5.3 Capital buffers and procyclicality

Table 4 shows initial loan rates r_s^* , initial capital k_s^* , and the implied capital buffers $k_s^* - \gamma_s$, for s = h, l, in each of the scenarios described in Table 3 and under the two regulatory frameworks that we compare: Basel I, with flat requirements of 8%, and Basel II, with the requirements given by (14). The results show that initial loan rates are always higher in the high default state, reflecting the need to compensate the banks for both a higher PD and the lower prospective profitability of continuation lending (since the high default state h is more likely after state h than after state l). These rates are very similar across regulatory regimes, confirming previous results from static models predicting that the loan pricing implications of capital requirements are quantitatively small.²⁸ Interestingly, Basel II induces no significant rate reductions in the low default state, while it slightly increases initial loan rates in the high default state. This is because even for lending relationships originated in state l, banks anticipate that the transition to state h might imply a reduction in lending capacity and, hence, in the value of the relationship.

		Basel I		Basel II			
	r_s^*	k_s^*	Buffer	r_s^*	k_s^*	Buffer	
Low volatility scenario							
s = h	2.7	12.1	4.1	2.8	12.9	3.0	
s = l	1.6	11.8	3.8	1.6	12.5	5.4	
Medium volatility scenario							
s = h	3.3	12.1	4.1	3.4	13.5	2.3	
s = l	1.5	11.6	3.6	1.6	12.3	5.7	
High volatility scenario							
s = h	3.7	12.1	4.1	3.8	13.7	1.9	
s = l	1.5	11.6	3.6	1.5	11.1	4.8	

Table 4. Initial loan rates and capital buffers(all variables in %)

The parameters that define each of the scenarios (as well as the associated Basel II capital requirements) are described in Table 3. The Basel I capital requirement is always 8%.

In fact, in order to ameliorate the impact of capital shortages in the future, banks hold

 $^{^{28}\}mathrm{See}$ Repullo and Suarez (2004).

significant capital buffers. Under Basel I, the cyclical variation in PDs has a rather small impact on capital decisions, although excess capital tends to be larger in state h (where loan losses can be expected to cause a larger reduction in future lending capacity) than in state l^{29} Under Basel II the cross state variability in PDs visibly translates into greater variability of both total capital and capital buffers. Interestingly, the cyclical pattern of the buffers gets reversed, relative to Basel I. We can advance two complementary reasons for this reversal—consistent with the results on credit rationing shown in Table 5. First, in state lthe prospective profitability of continuation lending is higher, and so they are the marginal gains from preserving the bank's lending capacity through proper capitalization. Second, in state h the required capital γ_h already provides a significant buffer against potential losses and the implied reduction in lending capacity even if the economy continues in the high default state. But the same is not true for state l: the capital requirement γ_l is too low to guarantee continuation lending if the economy switches to the high default state (and the requirement jumps to γ_h). Precaution leads banks to hold larger buffers in good times, as shown in the last column of Table 4.³⁰ The long-run average increase in the buffers due to the move from Basel I to Basel II, in the medium volatility scenario, is of 0.9 percentage points.³¹

Table 5 compares the cyclical impact of Basel I and Basel II on bank lending. Lending in any given period is made up of initial loans—whose quantity is always one—and continuation loans—whose quantity varies with the lending capacity of the banks which are unable to issue equity in that period. We denote by *expected credit rationing* the expected percentage of continuation projects discontinued because of banks' insufficient lending capacity. Notice that in our simple model total expected investment and total expected gross output—the returns from the funded investment projects—are linearly related to total credit and the

²⁹This is consistent with the existing evidence about the behavior of capital buffers in the pre-Basel II era including Ayuso et al. (2004) with Spanish banks, Lindquist (2004) with Norwegian banks, and Bikker and Metzemakers (2004) with banks from 29 OECD countries—and calls for caution against the interpretation that such evidence necessarily reflects banks' myopia.

³⁰As shown in Table 5 below, these buffers contribute to dampen (but do not eliminate) the contractive effects of the arrival of a recession—the transition from state l to state h.

³¹To compute this, we weight the increase in each state by the unconditional expected frequency with which the state occurs over time: $q_l/(1-q_h+q_l)$ for state h and $(1-q_h)/(1-q_h+q_l)$ for state l.

state of the economy in every period. So we can use credit rationing as a summary statistic of aggregate economic activity.

In Basel I the expected credit rationing does not depend on whether state s' is a high or a low default state, since the capital requirement is constant. Rationing only depends on the profits realized during the previous period, which determines the capital available to the banks for continuation lending. The distribution of this random variable depends on the state of the economy in the previous period. This explains why the figures for Basel I on Table 5 only vary with s in each scenario, and are larger for s = h than for s = l.

	Expected credit rationing in s'				
	Basel I	Basel II			
Low volatility scenario					
$(s,s^{\prime})=(h,h)$	2.3	3.7			
(s,s')=(h,l)	2.3	0.8			
(s,s') = (l,h)	1.5	4.7			
$(s,s^{\prime})=(l,l)$	1.5	0.4			
Medium volatility scenario					
(s,s')=(h,h)	2.7	5.3			
(s,s')=(h,l)	2.7	0.6			
(s,s') = (l,h)	1.3	15.6			
(s,s')=(l,l)	1.3	0.3			
High volatility scenario					
(s,s')=(h,h)	3.0	6.5			
(s,s')=(h,l)	3.0	0.6			
$(s,s^{\prime})=(l,h)$	1.3	30.2			
$(s,s^{\prime})=(l,l)$	1.3	0.5			

Table 5. Procyclicality(all variables in %)

The parameters that define each of the scenarios (and the associated Basel II capital requirements) are described in Table 3. The Basel I capital requirement is always 8%. *Expected credit rationing* is the expected percentage of second-period projects discontinued because of banks' insufficient lending capacity. All the numbers correspond to periods where state s' is reached following the (s, s') sequence shown in the first column.

Under Basel II, the impact of bank profits is also present but, according to the reported results, the overall effects on expected credit rationing are dominated by the state-contingent

nature of the capital requirements, and its effect on initial capital holdings and continuation capital requirements. So the sequences (s, s') = (l, h), and then those with (s, s') = (h, h), systematically exhibit the largest expected credit rationing under Basel II. Intuitively, in the low default state l the transition to the high default state h is less likely than continuing in h after being in h. Additionally, in state l the required capital is lower than in state h. For both reasons, banks end up holding lower capital in s = l than in s = h (see Table 4). But if the economy ends up in s' = h, then the mixture of a lower capitalization in the previous period and a higher regulatory requirement in this period explain the very sizable contractions in lending capacity shown on Table 5 for the (l, h) sequences under Basel II.³²

Table 6 provides the signs of the comparative statics for the key variables of our analysis (initial capital, initial buffers, and expected credit rationing in s' over the sequences (s, s') =(h, h) and (s, s') = (l, h)) in the medium volatility scenario under Basel II. These results obtained from the numerical simulations complement the algebraic results in Section 4, signing effects that were ambiguous at an analytical level. Focusing on the most procyclical (l, h) sequence, the last column identifies that a, μ, q_l , and p_l mitigate the procyclicality problem, while λ , c, δ , and p_h exacerbate it.

³²Interestingly, for the sequences with s' = l (which entail the lowest expected credit rationing under Basel II), the effect of bank profits becomes visible again, producing lower rationing in the (l, l) sequence than in the (h, l) sequence.

	y										
	k_s^*		Buff	er	Rationing in $s' = h$						
z	s = h	1 1		s = l	s = h	s = l					
a	+	+	+	+	_	_					
μ	+	+	+	+	—	—					
λ	+	—	—	—	+	+					
c	+	+	+	+	+	—					
δ	—	_	—	_	+	+					
q_s	+	+	+	+	—	_					
p_h	+	_	—	—	+	+					
p_l	—	+	—	+	—	_					

Table 6. Comparative statics under Basel II (signs of dy/dz from the numerical results)

The signs describe the variations in the variables induced by discrete changes in the parameters in the neighbourhood of the medium volatility scenario of Table 3.

6 Policy analysis

To be written.

7 Concluding Remarks

In many supervisory and industry reports on the implications of Basel II, it is standard to first recognize the potential cyclical effects of the new risk-sensitive capital requirements and then qualify that, given than most banks hold capital in excess of the regulatory minima, the practical incidence of the procyclicality problem is likely to be small if not negligible. While some of these reports do not have the extension or the technical nature required to elaborate on the foundations of their claim, others unveil two related misconceptions at the heart of it. The first misconception is that the holding of capital buffers means that capital requirements are "not binding." Under a purely static perspective this would be tautologically true. In a convex optimization problem, it would also be true that small changes in the level of the requirements would not alter the optimal capital holdings. In a dynamic problem, however, this need not be the case: banks may hold capital buffers in a given period because they wish to reduce the risk of facing a statically "binding" requirement in the future. Perhaps these precautions make future requirements "not binding" when the time comes, but clearly their presence alters banks' capital decisions and the whole development of future events. So observing the holding of capital buffers among banks does not mean that capital requirements do not matter.

A second, related misconception is to accept that the cyclical behavior of capital buffers under Basel II can be somehow predicted from the empirical behavior of capital buffers in the Basel I era. If buffers are endogenously affected by the prevailing bank capital regulation (even if they appear not to "bind"), reduced-form extrapolations from the Basel I world to the Basel II world do not resist the Lucas' critique.

Our model provides a tractable framework in which it is possible to evaluate the cyclical effects of Basel II without incurring in the above mistakes. To keep the analysis as transparent as possible, we have simplified on a number of dimensions. For example, we abstract from demand side fluctuations and feedback effects that might mitigate and exacerbate, respectively, the supply-side effects that we identify. But one could take our model as a building block for a fuller dynamic stochastic general equilibrium model with a production sector partly composed of entrepreneurial firms that rely on relationship bank lending. One could also think about extensions that modify or generalize our modeling of the frictions related to banks' access to equity financing. It could be interesting to explore situations in which lending relationships extend over more periods and in which banks' ability to recapitalize follows a less deterministic pattern.³³ Our contribution, from this perspective, is to show that the interaction of relationship lending (which makes some borrowers dependent on the lending capacity of the specific bank with which they establish a relationship) with frictions in banks' access to equity markets (which makes some banks' lending capacity a function of their historically determined capital positions and the capital requirements imposed by regulation) has the potential to cause significant cyclical swings in the supply of

 $^{^{33}}$ For example, one could assume a structure similar to the one in the popular Calvo (1983) model of staggered price setting, i.e. that in each period a fraction of the banks can issue new equity.

bank lending.

Under realistic parameterizations, the model produces capital buffers and equilibrium loan rates whose levels and cyclicality in the Basel I regulatory environment are in line with those observed in the data. The same parameterizations when applied to the Basel II environment suggest that the new requirements might imply a substantial increase in the procyclicality induced by bank capital regulation. Specifically, despite banks taking precautions and holding larger buffers during expansions in order to have a reserve of capital for the time when a recession comes (and capital requirements rise), the arrival of recessions is normally associated with a sizeable credit crunch, as capital constrained banks are induced to ration credit to some of their dependent borrowers.

Having a model that explicitly accounts for the endogenous determination of capital buffers and equilibrium loan rates is also important for policy analysis. We have shown that some state-contingent adjustments in the confidence level of Basel II might substantially reduce the incidence of credit rationing over the business cycle without compromising the long-run solvency targets implied by the new regulation.

Appendix

Solutions to the representative bank's capital decision Using the definition of $v_{ss'}(x_t)$ in (8), the net present value $v_s(k_s, r_s)$ of the representative bank that in state s holds capital k_s and charges an interest rate r_s on its unit of initial loans can be written as

$$v_s(k_s, r_s) = q_s v_{sh}(k_s, r_s) + (1 - q_s) v_{sl}(k_s, r_s),$$
(15)

where

$$v_{ss'}(k_s, r_s) = \beta \left[\int_0^{\widetilde{x}_{ss'}} \left[(\beta \pi_{s'} - \gamma_{s'}) \mu + n_s(x) \right] dF_s(x) + \frac{\beta \pi_{s'}}{\gamma_{s'}} \int_{\widetilde{x}_{ss'}}^{\widehat{x}_s} n_s(x) \, dF_s(x) \right] - k_s.$$
(16)

By the definitions (4) and (3) of $\tilde{x}_{ss'}$ and \hat{x}_s , the function $v_{ss'}(k_s, r_s)$ has the following properties:

1. For $k_s \leq c - r_s$ we have $\widetilde{x}_{ss'} < \widehat{x}_s \leq 0$, so

$$\frac{\partial v_{ss'}}{\partial k_s} = -1 < 0$$

2. For $c - r_s < k_s \le c - r_s + \gamma_{s'} \mu$ we have $\widetilde{x}_{ss'} \le 0 < \widehat{x}_s$, so

$$\frac{\partial v_{ss'}}{\partial k_s} = \frac{\beta^2 \pi_{s'}}{\gamma_{s'}} F_s(\widehat{x}_s) - 1 \leq 0, \text{ and } \frac{\partial^2 v_{ss'}}{\partial k_s^2} = \frac{\beta^2 \pi_{s'} f_s(\widehat{x}_s)}{\gamma_{s'}(\lambda + r_s)} > 0.$$

3. For $c - r_s + \gamma_{s'} \mu < k_s < c + \lambda + \gamma_{s'} \mu$ we have $0 < \widetilde{x}_{ss'} < 1$, so

$$\frac{\partial v_{ss'}}{\partial k_s} = \frac{\beta}{\gamma_{s'}} \left[\beta \pi_{s'} F_s(\widehat{x}_s) - (\beta \pi_{s'} - \gamma_{s'}) F_s(\widetilde{x}_{ss'})\right] - 1 \leq 0,$$

and

$$\frac{\partial^2 v_{ss'}}{\partial k_s^2} = \frac{\beta}{\gamma_{s'}(\lambda + r_s)} [\beta \pi_{s'} f_s(\widehat{x}_s) - (\beta \pi_{s'} - \gamma_{s'}) f_s(\widetilde{x}_{ss'})] \leq 0.$$

4. For $c + \lambda + \gamma_{s'} \mu \leq k_s$ we have $1 \leq \widetilde{x}_{ss'} < \widehat{x}_s$, so

$$\frac{\partial v_{ss'}}{\partial k_s} = \beta - 1 < 0.$$

Hence the function $v_{ss'}(k_s, r_s)$ is linearly decreasing or strictly convex for $k_s \leq c - r_s + \gamma_{s'}\mu$, linearly decreasing for $k_s \geq c + \lambda + \gamma_{s'}\mu$, and may be increasing or decreasing and concave or convex for $c - r_s + \gamma_{s'}\mu < k_s < c + \lambda + \gamma_{s'}\mu$. Hence introducing the constraint $k_s \in [\gamma_s, 1]$ (and assuming that parameter values are such that $c + \lambda + \gamma_{s'}\mu < 1$) it follows that the problem $\max_{k_s \in [\gamma_s, 1]} v_{ss'}(k_s, r_s)$ has either a corner solution with $k_s = \gamma_s$, or an interior solution with $k_s \in (c - r_s + \gamma_{s'}\mu, c + \lambda + \gamma_{s'}\mu)$. In the latter case we have $0 < \tilde{x}_{ss'} < 1$, so there is a positive probability $F_s(\tilde{x}_{ss'})$ that the bank has excess lending capacity in state s', and a positive probability $1 - F_s(\tilde{x}_{ss'})$ that the bank has insufficient lending capacity in state s'. Since $\gamma_l \leq \gamma_h$ implies $c - r_s + \gamma_l \mu \leq c - r_s + \gamma_h \mu$ and $c + \lambda + \gamma_l \mu \leq c + \lambda + \gamma_h \mu$, we conclude that the problem $\max_{k_s \in [\gamma_s, 1]} q_s v_{sh}(k_s, r_s) + (1 - q_s) v_{sl}(k_s, r_s)$ has either a corner solution with $k_s = \gamma_s$, or an interior solution with $k_s \in (c + \gamma_l \mu - r_s, \lambda + c + \gamma_h \mu)$. In the latter case, there must be a positive probability that the bank has insufficient lending capacity in state s' = h(and possibly also in state s' = l), and a positive probability that the bank has excess lending capacity in state s' = l (and possibly also in state s' = h).

Comparative statics of the equilibrium interest rate on initial loans The sign of dr_s^*/dz for $z = a, \mu, \lambda, c, \delta, q_s, \gamma_h, \gamma_l$ can be obtained by total differentiation of (10):

$$\frac{\partial v_s}{\partial k_s}\frac{dk_s^*}{dz} + \frac{\partial v_s}{\partial r_s}\frac{dr_s^*}{dz} + \frac{\partial v_s}{\partial z} = 0.$$
(17)

When k_s^* is interior, the first-order condition for a maximum that follow from (11) gives $\partial v_s / \partial k_s |_{(k_s^*, r_s^*)} = 0$, so the first term in (17) vanishes. Moreover when k_s^* is interior it must be the case that $0 < \tilde{x}_{ss'} < 1$ for at least one state s', so differentiating (15) and (16) we have

$$\frac{\partial v_s}{\partial r_s} = q_s \frac{\partial v_{sh}}{\partial r_s} + (1 - q_s) \frac{\partial v_{sl}}{\partial r_s} > 0,$$

since

$$\frac{\partial v_{ss'}}{\partial r_s} = \beta \left[\int_0^{\widetilde{x}_{ss'}} (1-x) \ dF_s(x) + \frac{\beta \pi_{s'}}{\gamma_{s'}} \int_{\widetilde{x}_{ss'}}^{\widehat{x}_s} (1-x) \ dF_s(x) \right] \ge 0$$

with strict inequality for at least one state s'. Hence we are left with:

$$\frac{dr_s^*}{dz} = -\left(\frac{\partial v_s}{\partial r_s}\right)^{-1} \frac{\partial v_s}{\partial z}.$$
(18)

Similarly, in a corner solution with $k_s^* = \gamma_s$ we have $dk_s^*/dz = 0$ for all $z \neq \gamma_s$, in which case the first term in (17) also vanishes and (18) obtains again. Finally, for $z = \gamma_s$, we have

 $dk_s^*/d\gamma_s = 1$ and, thus,

$$\frac{dr_s^*}{d\gamma_s} = -\left(\frac{\partial v_s}{\partial r_s}\right)^{-1} \left(\frac{\partial v_s}{\partial \gamma_s} + \frac{\partial v_s}{\partial k_s}\right),\,$$

where $\partial v_s / \partial k_s |_{(k_s^*, r_s^*)} \leq 0$, since otherwise fixing $k_s^* = \gamma_s$ would not be optimal. With these expressions in mind, the results in Table 1 can be immediately related to the (selfexplanatory) signs of the partial derivatives of $v_s(k_s^*, r_s^*)$ that we summarize in Table A1 (and whose detailed expressions we omit, for brevity).

Table A1

	z =	r_s	a	μ	λ	c	δ	q_s	γ_h	γ_l
Sign of $\frac{\partial v_s}{\partial z}$		+	+	+	_	_	_	_	_	_

Comparative statics of the equilibrium initial capital When the optimal initial capital in state s is at the corner $k_s^* = \gamma_s$, with $\partial v_s / \partial k_s |_{(k_s^*, r_s^*)} < 0$, marginal changes in any parameter other than γ_s will have no impact on k_s^* , while obviously $dk_s^*/d\gamma_s = 1$. Thus, in what follows we focus on the more interesting interior solution case.³⁴

The sign of dk_s^*/dz for $z = a, \mu, \lambda, c, \delta, q_s, \gamma_h, \gamma_l$ can be obtained by total differentiation of the first-order condition $\partial v_s/\partial k_s = 0$ that characterizes an interior equilibrium:

$$\frac{\partial^2 v_s}{\partial k_s^2} \frac{dk_s^*}{dz} + \frac{\partial^2 v_s}{\partial k_s \partial r_s} \frac{dr_s^*}{dz} + \frac{\partial^2 v_s}{\partial k_s \partial z} = 0.$$
(19)

By the second-order condition we have $\partial^2 v_s / \partial k_s^2 < 0$, which gives

$$\frac{dk_s^*}{dz} = -\left(\frac{\partial^2 v_s}{\partial k_s^2}\right)^{-1} \left(\frac{\partial^2 v_s}{\partial k_s \partial z} + \frac{\partial^2 v_s}{\partial k_s \partial r_s} \frac{dr_s^*}{dz}\right)$$

Hence the sign of dk_s^*/dz coincides with the sign of the second term in brackets, which has two components: the direct effect of z on k_s^* (for constant r_s^*) and the loan rate effect (due

³⁴The case with $k_s^* = \gamma_s$ and $\partial v_s / \partial k_s |_{(k_s^*, r_s^*)} = 0$ is a mixture of both cases since, depending on the sign of the effect of the marginal variation in a parameter, the optimal decision might shift from being at the corner to being interior. A similar complexity may occur if the change in a parameter breaks some underlying indifference between an interior and a corner solution (or between two interior solutions). We will omit the discussion of these cases, for simplicity.

to the effect of z on r_s^*). The signs of the direct effects shown in the first row of Table 2 coincide with the signs of the cross derivatives $\partial^2 v_s / \partial k_s \partial z$ summarized in Table A2 (whose detailed expressions we omit, for brevity).

	z	=	r_s	a	μ	λ	c	δ	q_s	γ_h	γ_l
Sign of $\frac{\partial^2 v_{ss'}}{\partial k_s \partial z}$			_	+	+	?	_	_	?	?	?

The signs of the loan rate effects shown in the second row of Table 2 can be simply obtained from the results summarized on Table 1 and the fact that by differentiating (15) and (16) one can show that

$$\frac{\partial^2 v_s}{\partial k_s \partial r_s} = q_s \frac{\partial^2 v_{sh}}{\partial k_s \partial r_s} + (1 - q_s) \frac{\partial^2 v_{sl}}{\partial k_s \partial r_s} < 0,$$

where

$$\frac{\partial^2 v_{ss'}}{\partial k_s \partial r_s} = \frac{\beta}{\gamma_{s'}(\lambda + r_s)} [\beta \pi_{s'}(1 - \widehat{x}_s) f_s(\widehat{x}_s) - (\beta \pi_{s'} - \gamma_{s'})(1 - \widetilde{x}_{ss'}) f_s(\widetilde{x}_{ss'})].$$

To check this notice that the second-order condition $\partial^2 v_s / \partial k_s^2 < 0$ implies

$$\beta f_s(\widehat{x}_s) \left[q_s \frac{\pi_h}{\gamma_h} + (1 - q_s) \frac{\pi_l}{\gamma_l} \right] < q_s \frac{\beta \pi_h - \gamma_h}{\gamma_h} f_s(\widetilde{x}_{sh}) + (1 - q_s) \frac{\beta \pi_l - \gamma_l}{\gamma_l} f_s(\widetilde{x}_{sl})$$

Hence using the definitions (3) and (4) of \hat{x}_s and $\tilde{x}_{ss'}$, together with the fact that $\gamma_l \leq \gamma_h$, we have $1 - \hat{x}_s < 1 - \tilde{x}_{sl} \leq 1 - \tilde{x}_{sh}$, so we conclude that

$$\beta(1-\widehat{x}_s)f_s(\widehat{x}_s)\left[q_s\frac{\pi_h}{\gamma_h} + (1-q_s)\frac{\pi_l}{\gamma_l}\right] < q_s\frac{\beta\pi_h - \gamma_h}{\gamma_h}(1-\widetilde{x}_{sh})f_s(\widetilde{x}_{sh}) + (1-q_s)\frac{\beta\pi_l - \gamma_l}{\gamma_l}(1-\widetilde{x}_{sl})f_s(\widetilde{x}_{sl})$$

which after some reordering proves the result.

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