# Competition, Innovation and Growth with Limited Commitment<sup>\*</sup>

Ramon Marimon Universitat Pompeu Fabra, CREi and CREA

> Vincenzo Quadrini University of Southern California

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# Abstract

We study how *barriers to business start-up* affect the investment in knowledge capital when contracts are not enforceable. Barriers to business start-up lower the competition for knowledge capital and reduce the incentive to accumulate knowledge. We show in a dynamic general equilibrium model that this mechanism can account for significant cross-country income inequality. Other *barriers to knowledge capital mobility*, such as the strict enforcement of covenants or IPR, can have similar effects.

# 1 Introduction

Innovation and adoption of new technologies is essential for sustained levels of income and growth. In the process of innovation, the accumulation of human

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capital plays a central role. This is clearly visible in modern technologies such as information technologies, biotechnologies and nanotechnologies where *skilled* human capital or *knowledge* is a factor of production highly complementary to capital. Therefore, how capital is financed and innovation skills are rewarded is important for the rate of innovation and growth.

In many instances, the roles of 'investor' and 'innovator' are played by different parties and, to prevent opportunistic behaviors, contractual arrangements must be designed to provide the rights incentives. Several factors, however, limit the enforceability of these arrangements. First, knowledge capital can not be used as a collateral and innovators may quit the firm to pursue other innovative projects. Second, advance payments to the innovator are not incentive compatible if the accumulation of knowledge from the innovator requires effort. Third, once the innovation process has been completed, the investor may renege the payments promised to the innovator.

The severity of these contractual problems depend on the potential rewards that the parties can obtain outside the firm. In particular, the incentive of the innovator to accumulate knowledge depends on the value that the accumulated knowledge has outside the firm. This value may be curtailed by *barriers to mobility*. A very pervasive barrier is the difficulty to start up firms which, typically, have more incentive to innovate than incumbent firms. Another barrier can be a tight enforcement of covenants, precluding innovators to use, for a period of time, their acquired knowledge in a different firm. A similar barrier is a stringent system of Intellectual Property Rights, when the innovator does not have full control of the patent. As Boldrin & Levine (2006) suggest, a stringent IP system may deter innovation.

The evolution of the computer industry exemplifies these effects. As Bresnahan & Malerba (2002) emphasize, such industry has gone through different technological stages (from the main frames to the PCs and the Internet). Knowledge in this particular industry was geographically spread in many countries including Europe. Yet, the United States has been persistently the industry leader. According to them, such dominance can be explained by "...the existence of a large body of technical expertise in universities and the generally supportive environment for new firm formation in the United States", Bresnahan & Malerba (2002, page 69).

While lower barriers to business start up may have favored the computer leadership of United States, different enforcement of covenants—and, in general, of informational linkages across firms—may have determined the shift of regional leadership within the United States, from Massachusetts to California. As Gilson (1999) and others have argued,<sup>1</sup> Silicon Valley dominates over Route 128 due to a Californian legal and social tradition of not enforcing post-employment covenants resulting in high labor mobility and knowledge spillovers.

In this paper we develop a theory that formalizes these features and addresses the question of whether competition for innovators affects income levels through the accumulation of knowledge. We use a dynamic general equilibrium model where innovators can not be bound to the firm and, therefore, their limits to mobility are determined by other legal and social barriers. Our main result is that the degree of competition for knowledge capital is a determinant factor for innovation when neither the investor nor the innovator can commit to long-term contracts.

Competition for knowledge capital creates an outside value for the innovator which can be used as a threat against the investor's attempt to renegotiate the promised payments. Barriers to entry or mobility reduce the outside value of knowledge and, in absence of commitment from the investor, the innovator's incentive to accumulate knowledge. Without barriers, instead, innovators would maintain the incentive to accumulate knowledge because of its outside value. They may even have the incentive to over-accumulate knowledge because new firms have greater incentive to innovate. In principle, an incumbent firm could prevent the over-accumulation by making advance payments. However, advance payments are not incentive-compatible if contracts are not enforceable also for the innovator. It is in this sense that the double-sided limited commitment plays a crucial role. Although the higher accumulation of knowledge is inefficient at the firm level, it may still be efficient for the whole economy if there are 'externalities' or 'spillovers'.

Our results are first illustrated with a simple two-period model which is then extended to a dynamic infinite horizon set-up. The parametrization of the infinite horizon model allows us to quantify how much of the cross-country differences in income levels can be explained by cross-country differences in start-up costs. We then show how other barriers to mobility, such as covenants, can be incorporated in our model to account for regional differences. Finally, we show that the possibility of firms' or projects' failure exacerbate the effects of competition on the accumulation of knowledge.

This paper relates to three strands of literature. First, the labor literature that studies the the accumulation of skills within the firm (e.g., Acemoglu

<sup>&</sup>lt;sup>1</sup>See, for example, Hyde (2003) and Saxenian (1996).

(1997), Acemoglu & Pischke (1999), Acemoglu & Shimer (1999)). In this literature, higher outside value worsens the hold-up problem and leads to lower accumulation of skills. In our framework, instead, higher outside value increases human capital investment.

Second, the growth literature, starting with the pioneering work of Romer (1990, 1993), that studies the economics of ideas and the link between competition and growth (e.g., Greenwood & Jovanovic (1990), Aghion & Griffith (2005)). Whether free entry enhances innovation has been a major topic of research and debate since Schumpeter's claim that, while product market competition could deter innovation, competition in the innovation sector encourages innovation. Most of the subsequent literature has focused on market structure and product market competition. In particular, on the ability to gain market shares and appropriate the returns to R&D, as in Aghion, Bloom, Blundell, Griffith, & Howitt (2005). More closely related to our work is Aghion, Blundell, Griffith, Howitt, & Prantl (2004). They show—both, theoretically and empirically—that 'firm entry' spurs innovation in technological advanced sectors as firms try to 'escape competition' while Acemoglu, Aghion, & Zilibotti (2002) show that barriers to entry are especially costly for economies closer to the technology frontier. In contrast to these studies, we focus on the less studied—but we think, empirically relevant—dimension of 'human capital' competition. By emphasizing the role of barriers to mobility, our work also relates to the growth literature that, building on the work of economic historians (e.g., Mokyr (1990)), emphasizes the role of barriers to riches in slowing growth (Parente & Prescott (1990)).

The third branch of literature is on dynamic contracts with enforcement constraints as in Marcet & Marimon (1992). In this literature it is usually assumed that default or repudiation leads to market exclusion, while in our framework barriers to mobility matter precisely because there is no market exclusion (Kocherlachota (1996) and Cooley, Marimon, & Quadrini (2004) are exceptions). Many of the papers in this literature conclude that stronger commitment enhances income and growth. With our framework, instead, income and growth can be enhanced with specific forms of limited commitment.

# 2 Cross-country evidence on barriers to business start-up

A recent publication from the World Bank (2005) provides data on the quality of the business environment for a cross-section of countries. Especially important for this study are the variables that are proxy for the barriers to business start-up. There are three main variables. The first is the 'cost to start a new business'. This is the average pecuniary cost needed to set-up a corporation in the country, expressed in percentage of the country per-capita income.<sup>2</sup>

The second proxy for the barriers to business start-up is the 'number of bureaucratic procedures' that need to be filed before starting a new business. The third proxy is the average 'length of time' required to start a new business. Figure 1 plots the level of per-capita GDP in 2004 against these three indicators, where all variables are in log. All panels show a strong negative correlation indicating that the set-up of a new business is more costly and cumbersome in poor countries.

The cost of business start-up is also negatively correlated with economic growth. To show this, we regress the average growth in per-capita GDP from 2000 to 2004 (the five more recent years) to the cost of business startup. We also include the 1999 per-capita GDP to control for the initial level of development. The estimation results, with *t*-statistics in parenthesis, are reported in the top section of Table 1. As can be seen from the table, the cost of business start-up is negatively associated with growth even if we control for the level of economic development. Therefore, countries with lower barriers to entry tend to experience faster growth. This finding is robust to the choice of alternative years to compute the average growth rate. The other proxies for barriers to entry—specifically, the number of procedures and the time required to start a new business—are also negatively correlated with growth but they are not statistically significant at the conventional levels.

To show that these findings are not an artifact of normalizing the cost of business start-up by the level of per-capita income, the bottom section of Table 1 repeats the same regression estimation but using dollar values for the cost of business start-up (also in log). Again, the cost of business start-up is statistical significant with a negative sign.

To summarize, the general picture portrayed by the data is that the economic development and growth of a country is negatively associated with the cost of starting a business. The results presented above are simple correla-

<sup>&</sup>lt;sup>2</sup>The normalization of the cost of business start-up by the level of per-capita income better captures the importance of barriers to business start-up than the absolute dollar cost. What is relevant is the comparison between the cost of business start-up and the value of creating a business. Although the dollar cost is on average higher in advanced economies, the value of a new business is also likely to be higher.

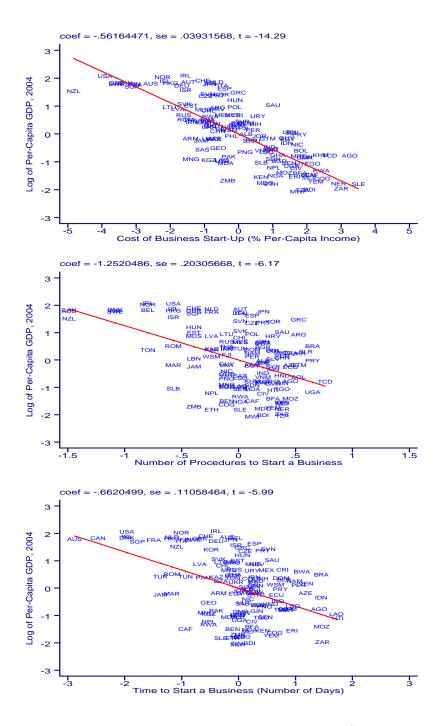


Figure 1: Barriers to business start-up and level of development.

		Constant	Initial Per-Capita GDP	Cost of Business Start-Up
(a)	$\begin{array}{c} \text{Coefficients} \\ t\text{-Statistics} \end{array}$	15.55 (5.01)	-1.16 (-3.81)	-1.04 (-4.92)
	R-square N. of countries	$\begin{array}{c} 0.150\\ 140 \end{array}$		
(b)	Coefficients $t$ -Statistics	6.02 (3.08)	$0.20 \\ (0.94)$	-0.83 (-3.75)
	R-square N. of countries	$\begin{array}{c} 0.093 \\ 140 \end{array}$		

Table 1: Cost of business start-up and growth.

NOTES: Dependent variable is the average annual growth rate in per-capita GDP for the five year period 2000-2004. Initial Per-Capita GDP is the log of per-capita GDP in 1999. In panel (a) the costs of business start-up is in percentage of the per-capital Gross National Income as reported in *Doing Business in 2005*. In panel (b) is the dollar value of this cost. Both measures of the cost of business start-up are in logs.

tions which, of course, do not imply causations. In the following sections we present a model where barriers to entry lead to lower income and growth. We will use again the cross country data presented here when we calibrate the model.

# 3 The model

There are two types of agents in the economy: a continuum of 'investors' of total mass m > 1 and a continuum of 'innovators' of total mass 1. Therefore, innovators are in short supply relatively to the number of investors. Investors own all the physical capital and innovators are the only holders of knowledge capital. The lifetime utilities for investors and innovators are, respectively,  $\sum_{t=0}^{\infty} \beta^t c_t$  and  $\sum_{t=0}^{\infty} \beta^t (c_t - e_t)$ , where  $c_t$  is consumption and  $e_t$  is the effort to accumulate human capital (knowledge) as specified below.

Innovators do not save. This assumption should be interpreted as an approximation to the case in which innovators discount more heavily than investors. The risk neutrality of investors implies that the equilibrium interest rate is equal to the intertemporal discount rate, that is,  $r = 1/\beta - 1$ .

Firms are owned by investors who need the management and innovation skills of innovators. The production function is:

$$y_t = z_t^{1-\alpha} k_t^{\alpha}$$

where  $z_t$  is the level of technology and  $k_t$  is the capital chosen at time t-1.

The variable  $z_t$  changes over time as the firm adopts new technologies. The key assumption is that the implementation of more advanced technologies requires higher knowledge. An innovator with knowledge  $h_t$  has the ability to install or implement any technology  $z_t \leq h_t$ .

Innovators are needed not only for the adoption and implementation of technologies but also to run the firm once the technology has been installed. In each period there are two subperiods. The technology is introduced in the first sub-period and run in the second. However, once the technology has been implemented, any innovator can run the firm, independently of his or her knowledge. The importance of this assumption will be clear later.

The investment in knowledge,  $h_{t+1} - h_t$ , requires effort from the innovator. The required effort depends on the economy-wide level of knowledge  $H_t$ , due to leakage or spillover effects. Formally,

$$e_t = \varphi(h_t, h_{t+1}; H_t)$$

The function  $\varphi$  is strictly decreasing in  $H_t$  and  $h_t$ , strictly increasing and convex in  $h_{t+1}$ , and satisfies  $\varphi(h_t, h_t; H_t) > 0$ . It is further assumed that the function is homogeneous of degree  $\rho > 1$ . The analysis can be easily extended to the case in which  $\rho = 1$ . In this case the model would generate long-term growth differences. With  $\rho > 1$ , which is the case considered here, the model generates long-term differences in income levels. Therefore, this is a semi-endogenous growth model as in Jones (1995).<sup>3</sup>

Physical capital is technology-specific. When the firm innovates, only part of the physical capital is usable with the new technology. Furthermore,

<sup>&</sup>lt;sup>3</sup>The model should be interpreted as a detrended version of an economy that grows at the exogenous rate dictated by the world-wide level of technology, external to an individual country. Let  $\overline{H}_t$  be the world-wide level of knowledge growing at the constant rate g. The cost to accumulate knowledge can be specified as  $\tilde{\varphi}(h_t, h_{t+1}; H_t, \overline{H}_t)$  where  $\tilde{\varphi}$  is strictly increasing in  $\overline{H}_t$  and homogeneous of degree 1. If we normalize all variables by  $\overline{H}_t$  we would have the stationary model studied here.

capital obsolescence increases with the degree of innovation. This is formalized by assuming that the depreciation rate increases with the size of the innovation, that is,

$$\delta_t = \delta \cdot \left(\frac{z_{t+1} - z_t}{z_t}\right)$$

Because of capital obsolescence, there is an asymmetry between *incumbent firms*—whose old capital depreciates with the adoption of more advanced technologies—and *new firms* that, without an old capital in place, have greater incentive to innovate (Arrow's 'replacement effect').

Firms remain productive with probability p. Whether a firm survives is revealed after the investment in knowledge. This assumption guarantees that, in the second stage of each period, the mass of innovators is larger than incumbent (surviving) firms. As we will see, this will avoid some technical issues. To facilitate the analysis we first assume that p is very close to 1 and, in the characterization of the individual problems, we will ignore it. Then, in Section 7, we discuss in more detail the general case with any p.

**Competitive structure and barriers to entry:** In each period there is a walrasian market for innovators who can move freely from one firm to the other. The market opens twice: before and after the accumulation of knowledge. Both incumbents and new firms participate. However, barriers to entry limit the effective competition created by potential new entrants.

There are different ways to model barriers to entry. Here we assume that new firms incur a deadweight loss proportional to the initial level of knowledge. Given  $h_{t+1}$  the initial knowledge, the entry cost is  $\tau \cdot h_{t+1}$ . We would like to emphasize that the key results of the paper are robust to alternative specifications of the entry cost. Our choice is only motivated by its analytical convenience.<sup>4</sup>

# 4 Two-period model

Before studying the general model with infinitely lived agents, we first consider a simplified version with only two periods. This allows us to gauge more

<sup>&</sup>lt;sup>4</sup>For example, we could assume that the cost is proportional to the initial capital  $k_{t+1}$  or to the initial output  $h_{t+1}^{1-\alpha}k_{t+1}^{\alpha}$  or to the discounted flows of outputs. The basic theory and results also apply when the entry cost is a fixed payment. The assumption of proportionality allows for a continuous impact of  $\tau$  while a fixed cost would have an impact only after it has reached the prohibitive level.

easily the intuitions underlying the key properties of the model. Although the intuitions can be shown with the simpler framework, the analysis of the infinite horizon model is still important. First, the infinite horizon structure allows us to derive the initial conditions endogenously as steady state values. In the two-period model, instead, the initial states—which are important in determining the level of income—are exogenous. Second, the infinite horizon model is better suited for the quantitative analysis of Section 6.

There are two periods: period zero and period one. The state variables of the firm at the beginning of period zero are  $h_0$  and  $k_0$ . After making the investment decisions,  $h_1$  and  $k_1$ , the firm generates output  $y_1 = z_1^{1-\alpha} k_1^{\alpha}$ in period one. Because  $z_1 = h_1$ , the output can also be written as  $y_1 = h_1^{1-\alpha}k_1^{\alpha}$ . In this simple version of the model we assume that physical capital fully depreciates after production. The innovator receives a payment from the firm (compensation) at the end of period zero, after the choice of  $h_1$ . Payments before the choice of  $h_1$  are not incentive-compatible because of the limited enforcement of contracts for the innovator, while allowing for additional payments in period 1 does not change the results. We also assume that there is no discounting and the effort cost does not depend on the economy-wide knowledge H. The leakage or spillover effect is not relevant when there are only two periods.

The timing of the model can be summarized as follows: The firm starts period zero with initial states  $h_0$  and  $k_0$ . At this stage the innovator decides whether to stay or quit the firm. If the innovator quits, she can be hired either by an incumbent firm or by a new firm (funded by a new investor). If the innovator decides to stay, she will choose the new level of knowledge  $h_1$  and implement the technology  $z_1 = h_1$ . The investor provides the funds to accumulate the new physical capital  $k_1$ . After the investment decision, the firm pays  $w_0$ . At this stage the innovator can still quit, but she cannot change the level of knowledge  $h_1$ . The investor is the residual claimant of the firm's output. In the analysis that follows we use the terms 'investor' and 'firm' interchangeably.

## 4.1 Equilibrium with investor's commitment

To show the importance of contract enforcement, we will first characterize the equilibrium under the assumption that the investor commits to the contract. In this case, all variables are chosen at the beginning of the first period to maximize the total surplus.

Let  $D(h_0)$  be the repudiation value for the innovator before choosing  $h_1$ and  $\widehat{D}(h_1)$  the repudiation value after choosing  $h_1$ . These functions are endogenous and will be derived below as the values that the innovator would get by quitting the firm. From now on we will use the *hat* sign to denote the functions that are defined *after* the investment in knowledge. The participation of the innovator requires that the value of staying is greater than the repudiation value before and after the knowledge investment, that is,

$$w_0 - \varphi(h_0, h_1) \geq D(h_0)$$
$$w_0 \geq \widehat{D}(h_1)$$

In our model the repudiation values are the values that the innovator would receive if she was to quit for a new firm. As we will show, once we impose this, the second constraint is always satisfied if the first constraint is satisfied. Therefore, in the derivation of the optimal policy, we can neglect the second constraint and write the optimization of the surplus as follows:

$$\max_{h_1, k_1, w_0} \left\{ -\varphi(h_0, h_1) - k_1 + \left[ 1 - \delta \cdot \left( \frac{h_1 - h_0}{h_0} \right) \right] k_0 + h_1^{1 - \alpha} k_1^{\alpha} \right\}$$
(1)  
s.t.

$$w_0 - \varphi(h_0, h_1) \ge D(h_0)$$
$$-w_0 - k_1 + \left[1 - \delta \cdot \left(\frac{h_1 - h_0}{h_0}\right)\right] k_0 + h_1^{1 - \alpha} k_1^{\alpha} \ge 0$$

where the second constraint is the participation condition for the investor.

From the maximization problem it can be verified that the investment choices (in knowledge and physical capital) are independent of the choice of the innovator's payment  $w_0$ . The value of  $w_0$  is determined by the division of the surplus, which we specify below.

To determine the repudiation value before the choice of  $h_1$ , we have to solve for the optimal investment when the innovator quits the current firm. The innovator could be hired by an incumbent or a new firm, whoever makes the best offer. Because an incumbent firm never offers more than a new firm, it becomes relevant to determine the offer that a new firm would make. This is derived by solving the contractual problem for a new firm, that is:

$$S(h_{0}) = \max_{h_{1},k_{1},w_{0}} \left\{ -\varphi(h_{0},h_{1}) - \tau h_{1} - k_{1} + h_{1}^{1-\alpha}k_{1}^{\alpha} \right\}$$
(2)  
s.t.  
$$w_{0} - \varphi(h_{0},h_{1}) \ge D(h_{0})$$

$$-w_0 - \tau h_1 - k_1 + h_1^{1-\alpha} k_1^{\alpha} \ge 0$$

This problem differs from the problem solved by an incumbent firm in two respects. First, a new firm does not have any initial physical capital, and therefore, it does not incur capital obsolescence. Second, a new firm has to pay the entry cost  $\tau h_1$ . It is still the case, however, that the choices of  $h_1$ and  $k_1$  are independent of  $w_0$ .

Because of competition among potential entrants and the limited supply of innovators, an innovator that decides to quit for a new firm will get the whole surplus generated by this firm, that is,  $S(h_0)$ . This implies that  $D(h_0) = S(h_0)$  and, if the innovator stays with the incumbent firm, the payment  $w_0$  must be at least  $\varphi(h_0, h_1) + S(h_0)$ . Formally, the participation constraint in problem (1) becomes  $w_0 - \varphi(h_0, h_1) \ge S(h_0)$ .

Problems (1) and (2) show the different incentive to invest for an incumbent versus a new firm. On the one hand, new firms do not have any physical capital and innovations do not generate capital obsolescence. On the other, they must pay the entry cost  $\tau h_1$ , which discourages knowledge and capital accumulation. This is clearly shown by the first order conditions with respect to  $h_1$ , in problems (1) and (2). These can be written as:

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} = \varphi_{h_1}(h_0, h_1) + \delta \cdot \left(\frac{k_0}{h_0}\right)$$
(3)

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} = \varphi_{h_1}(h_0, h_1) + \tau \tag{4}$$

where the subscripts denote derivatives. The left-hand-side terms are the marginal productivity of knowledge. The right-hand-side terms are the marginal costs. The marginal cost for an incumbent firm is the effort cost incurred

by the innovator plus the obsolescence of physical capital. For a new firm the obsolescence cost is replaced by the entry cost.

Let  $h_1^{Old}$  be the optimal knowledge investment of an incumbent firm and  $h_1^{New}$  the optimal investment of a new firm. The following proposition formalizes the relation between barriers to entry and knowledge investment.

**Proposition 1** The knowledge investment of a new firm  $h_1^{New}$  is strictly decreasing in the entry cost  $\tau$  and there exists  $\bar{\tau} > 0$  such that  $h_1^{New} = h_1^{Old}$ .

**Proof 1** The first order condition for the choice of  $k_1$  is  $\alpha(k_1/h_1)^{\alpha-1} = 1$ , for both incumbent and new firms. Using this condition, (3) and (4) become:

$$(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} = \varphi_{h_1}(h_0, h_1^{Old}) + \delta \cdot \left(\frac{k_0}{h_0}\right)$$
$$(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} = \varphi_{h_1}(h_0, h_1^{New}) + \tau$$

The proposition follows directly from these two conditions. Q.E.D.

In the equilibrium with investor's commitment, there will not be any entrance of new firms at the beginning of the period and the investment in knowledge is  $h_1 = h_1^{Old}$ . The potential entrance of new firms only affects the payment received by an innovator. In the second stage there will be the entrance of new firms because some incumbents exit (p < 1). However, the level of knowledge cannot be changed at this stage. The next step is to show that this is not an equilibrium when contracts are not enforceable for both sides, that is, there is also limited commitment from the investor.

# 4.2 Equilibrium without investor's commitment

When the investor can not commit to fulfill its promises, he will renegotiate the contract after the choice of  $h_1$ . To see this, we must derive the value that the innovator would get by quitting the firm when her knowledge has already been chosen to be  $h_1$ . This is the surplus generated by a new firm (that hires the innovator) defined as:

$$\widehat{S}(h_1) = \max_{k_1, w_0} \left\{ -\tau h_1 - k_1 + h_1^{1-\alpha} k_1^{\alpha} \right\}$$
(5)

$$w_0 \ge \widehat{D}(h_1)$$
  
 $-w_0 - \tau h_1 - k_1 + h_1^{1-\alpha} k_1^{\alpha} \ge 0$ 

Because of competition, the innovator gets the whole surplus generated by the new firm, that is,  $\widehat{D}(h_1) = \widehat{S}(h_1)$ . An incumbent firm will renegotiate the promised payment  $w_0$  if this is higher than  $\widehat{S}(h_1)$ . The renegotiation threat after the accumulation of knowledge is credible because the firm can always replace the current innovator with other innovators. This could be either an innovator still employed by an incumbent (surviving) firm, or an innovator who separated from an exiting firm. Because in the second stage there are only p firms that are still alive but the mass of innovators is still 1, innovators are in the long side of the market (relatively to the number of incumbent firms). This implies that they only get the reservation value.<sup>5</sup>

Based on the above discussion, we have that the innovator will receive  $w_0 = \hat{S}(h_1)$  and the total utility from staying with the firm is:

$$-\varphi(h_0, h_1) + w_0 = -\varphi(h_0, h_1) + \widehat{S}(h_1)$$
(6)

If instead the innovator quits at the beginning of the period, she will get the whole surplus  $S(h_0)$  generated by the new firm, that is,

$$S(h_0) = \max_{h} \left\{ -\varphi(h_0, h) + \widehat{S}(h) \right\} = -\varphi(h_0, h_1^{New}) + \widehat{S}(h_1^{New})$$
(7)

Equations (6) and (7) show that the value of quitting at the beginning of the period,  $S(h_0)$ , is greater than the value of staying, as long as  $h_1 \neq h_1^{New}$ . Therefore, the innovator will quit the firm at the beginning of the period unless the firm agrees to the same knowledge investment chosen by a new entrant firm, that is,  $h_1 = h_1^{New}$ . In this way the innovator keeps the repudiation value high and prevents the firm from renegotiating.<sup>6</sup>

s.t.

<sup>&</sup>lt;sup>5</sup>Notice that we have ignored this probability in the formulation of the contractual problem because we are looking at the limiting case in which p is very close to 1. The explicit consideration of p will be done in Section 7.

<sup>&</sup>lt;sup>6</sup>This proves that, if the enforcement constraint for the innovator is satisfied at the beginning of the period, it is also satisfied after the investment in knowledge.

**Proposition 2** Without commitment from the investor, the investment in knowledge is  $h_1 = h_1^{New}$ .

**Proof 2** This follows directly from the analysis above. Q.E.D.

Because  $h_1^{New}$  is decreasing in  $\tau$  (see Proposition 1), the accumulation of knowledge decreases with the cost of entry. Therefore, when the investor does not commit, there is a negative correlation between barriers to entry and the accumulation of knowledge.

To summarize, increasing competition (reducing barriers to business entry) leads to greater investment in knowledge. However, since the investment in knowledge capital is determined by entrant firms, such level is not necessarily efficient for incumbent firms. In particular, if  $\tau$  is small, incumbent firms accumulate too much knowledge. The presence of spillovers, however, may make the higher investment socially desirable. We will re-introduce the spillovers in the analysis of the infinite horizon model.

**Remarks:** There are three points that should be emphasized. First, the importance of p < 1. If p was equal to 1, we would have the same number of innovators as incumbent firms in the second stage. This may lead to multiple equilibria. Each firm would renegotiate if all other firms renegotiate. But each firm would not individually renegotiate if all other firms do not renegotiate because there are no innovators willing to move for a lower pay. The assumption of a positive probability of exit, although small, eliminates this multiplicity because there is at least one innovator who has lost her job and she is willing to accept a lower payment.

Second, the assumption that knowledge is only necessary for the implementation of technologies but not for managing a firm after the implementation (although the management still needs to be done by an innovator) eliminates the following problem. Suppose that we are in an equilibrium with  $h_1 = h_1^{New} \neq h_1^{Old}$ . Under this equilibrium, an individual firm may be able to make a credible promise to the innovator and convince her to accumulate  $h_1^{Old}$ . Because in the second stage this is the only innovator with knowledge  $h_1^{Old}$  (all others have accumulated  $h_1^{New}$ ), there is no innovator with the same knowledge that the firm can hire to replace the current innovator. Consequently, the firm will be unable to renegotiate.<sup>7</sup> This implies

<sup>&</sup>lt;sup>7</sup>This is true independently of whether  $h_1^{New}$  is greater or smaller than  $h_1^{Old}$ . If  $h_1^{New} < 1$ 

that  $h_1 = h_1^{New}$  cannot be an equilibrium. At the same time, if all firms chose  $h_1 = h_1^{Old}$ , firms will renegotiate. Thus,  $h_1 = h_1^{Old}$  cannot be an equilibrium either. This problem does not arise if the firm can use any innovator to run the firm, once the technology has been adopted.

Third, the sharing of the output, in this deterministic environment, is equivalent to the enforcement of promised payments. Therefore, the assumption that the investor is unable to commit to the contract also means that he is unable to promise a share of the output.

## 5 The infinite horizon model

In this section we generalize the model to an infinite horizon set-up. We first characterize the equilibrium when investors commit to the contract and then we turn to the case of limited commitment. The comparison between these two environments will clarify how contract enforcement is key for barriers to entry to affect the accumulation of knowledge. To present the results more compactly, we relegate most of the technical analysis and proofs to the appendix.

Before continuing, it will be convenient to define the gross output function, inclusive of undepreciated capital, as follows:

$$\pi(h_t, k_t, h_{t+1}) = \left[1 - \delta \cdot \left(\frac{h_{t+1} - h_t}{h_t}\right)\right] k_t + h_t^{1-\alpha} k_t^{\alpha} \tag{8}$$

In writing this expression, we assume that the firm uses the best technology implementable by the innovator, that is,  $z_t = h_t$ . It is easy to show that the choice of  $z_t < h_t$  is never optimal.

## 5.1 Equilibrium with investor's commitment

We concentrate the analysis to steady state equilibria where all aggregate states are constant. Therefore, in the analysis that follows we will ignore the aggregate states as an explicit argument in all value functions.

Although in equilibrium there is no entrance of firms (more precisely the number of firms entering is negligible), we still need to solve for the dynamics

 $h_1^{Old}$ , then all the replacements have lower knowledge and are unable to run the more advance technology. If  $h_1^{New} > h_1^{Old}$ , then the deviating firm could use innovators with higher knowledge. But these innovators must be paid more because their reservation value (when employed by a new firm) is higher.

of a potential new entrant in order to determine the outside or repudiation value for the innovator (which is the value of starting a new firm). Even though we are limiting the analysis to steady states, newly created firms do experience a transition to the long-term level of physical and knowledge capital.

Let  $V(h_t)$  be the repudiation value for the innovator at the beginning of the period, before investing in knowledge. This is the value that an innovator with knowledge  $h_t$  would receive from quitting for a new firm. Similarly, let  $\hat{V}(h_{t+1})$  be the value of quitting after choosing the knowledge investment, and therefore, after exercising effort. At this point the stock of knowledge is  $h_{t+1}$ .

Consider the optimization problem solved by a new firm (start-up investor) that hires an innovator with knowledge capital  $h_0$  at the beginning of period 0, before the innovator makes the new investment in knowledge. Because with competition the innovator gets the whole surplus, it will be convenient to characterize the optimal contract by maximizing the value for the innovator, subject to the enforceability and participation constraints. The optimization problem can be written as:

$$V(h_0) = \max_{\{w_t, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \Big[ w_t - \varphi(h_t, h_{t+1}; H) \Big]$$
(9)

subject to

$$\sum_{j=t}^{\infty} \beta^{j-t} \Big[ w_j - \varphi(h_j, h_{j+1}; H) \Big] \ge V(h_t), \quad \text{for } t > 0$$
$$w_t + \sum_{j=t+1}^{\infty} \beta^{j-t} \Big[ w_j - \varphi(h_j, h_{j+1}; H) \Big] \ge \widehat{V}(h_{t+1}), \quad \text{for } t \ge 0$$
$$-\tau h_1 - w_0 - k_1 + \sum_{t=1}^{\infty} \beta^t \Big[ \pi(h_t, k_t, h_{t+1}) - w_t - k_{t+1} \Big] \ge 0$$

The first two conditions are the intertemporal enforcement constraints for the innovator, before and after the investment in knowledge. The last constraint guarantees non-negative profits for the investor (participation). The problem is also subject to a non-negative constraint for  $w_t$ .

For an innovator hired by a new firm at time 0, after the investment in

knowledge, the value of the contract is:

$$\widehat{V}(h_1) = \max_{\{w_t, k_{t+1}, h_{t+2}\}_{t=0}^{\infty}} \left\{ w_0 + \sum_{t=1}^{\infty} \beta^t \Big[ w_t - \varphi(h_t, h_{t+1}; H) \Big] \right\}$$
(10)

subject to the same constraints of problem (9).

The key difference respect to the problem solved by a new firm entering at the beginning of the period, is that the effort to accumulate knowledge has already been exercised by the innovator and  $h_1$  is given at this point. Consequently, the current flow of utility for the innovator is only  $w_0$ . This also explains why the choice of knowledge starts at 2.

Appendix A derives the first order conditions for problem (9). Because of the entry cost and the obsolescence of physical capital, the optimality conditions in period 0, when the firm enters, is different from the optimality conditions in subsequent periods, that is, when the firm becomes an incumbent. For the first period optimization, the first order conditions are:

$$V(h_t) \le w_t - \varphi(h_t, h_{t+1}; H) + \beta V(h_{t+1})$$
 (11)

$$\beta \pi_2(h_{t+1}, k_{t+1}, h_{t+1}) = 1 \tag{12}$$

$$\tau + \varphi_2(h_t, h_{t+1}; H) = \beta \left[ \pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H) \right]$$
(13)

with t = 0.

Here we use subscripts to denote the derivative with respect to the particular argument. The first condition simply says that the value of quitting the current firm must be at least as big as the current flow of utility plus the discounted value of quitting next period. The second condition equalizes the gross marginal return of capital to its marginal cost, which is 1. The last condition equalizes the marginal cost to accumulate knowledge in a new firm to the discounted value of its return (greater production and lower cost of future accumulation of knowledge).

The first order conditions after entering (incumbent firm) are similar to the ones derived above with the exception of condition (13). For an incumbent firm this condition becomes:

$$-\pi_3(h_t, k_t, h_{t+1}) + \varphi_2(h_t, h_{t+1}; H) = \beta \Big[ \pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H) \Big]$$
(14)

for all t > 0.

Conditions (13) and (14) show the asymmetry between new and incumbent firms. While the marginal benefit from investing in knowledge (the right-hand-side) is the same, the marginal cost (the left-hand-side) differs. For new firms this includes the entry cost  $\tau$ . For incumbent firms the entry cost is replaced by the depreciation of physical capital,  $-\pi_3(h_t, k_t, h_{t+1})$ .

We can now characterize the steady state equilibrium. Because in equilibrium there is no entrance, in the steady state all firms will have the economywide level of knowledge H. The convergence to the economy-wide average is the result of the spillovers in the accumulation of knowledge. Because of this, firms with lower than average knowledge tend to invest more and firms with higher than average knowledge tend to invest less. Because of the complementarity between knowledge and physical capital, all firms will also have the economy-wide level of physical capital K. The steady state values of Hand K are determined by conditions (13) and (14), that is:

$$\beta \pi_2(H, K, H) = 1 \tag{15}$$

$$-\pi_3(H, K, H) + \varphi_2(H, H; H) = \beta \Big[ \pi_1(H, K, H) - \varphi_1(H, H; H) \Big]$$
(16)

Appendix B shows that there are unique values of H and K solving the above two conditions. Therefore, the steady state equilibrium is unique. After solving for H and K, we can solve for the payment w. However, in order to solve for w, we need to determine the value of a new firm V(H). This requires us to solve for the whole transition experienced by a 'new firm', as characterized by the first order conditions (11)-(14), even if in equilibrium there are neither quitting innovators nor entrance of new firms.

Conditions (15) and (16) show that the entry cost  $\tau$  does not affect the steady state values of K and H. We will see in the next section that this property no longer holds when the investor is unable to commit to the long-term contract.<sup>8</sup>

<sup>&</sup>lt;sup>8</sup>As we will show in Section 7, when p is not arbitrarily close to 1, the steady state with investor's commitment does depend on  $\tau$ . In this case the limited enforcement from the investor amplifies the negative effects of barriers to entry.

# 5.2 Equilibrium without investor's commitment

The intertemporal participation constraints, before and after the investment in knowledge, can be written as:

$$\sum_{j=t}^{\infty} \beta^{j-t} \Big[ w_j - \varphi(h_j, h_{j+1}; H) \Big] \ge V(h_t)$$
(17)

$$\sum_{j=t}^{\infty} \beta^{j-t} \Big[ w_j - \varphi(h_j, h_{j+1}; H) \Big] \ge -\varphi(h_t, h_{t+1}; H) + \widehat{V}(h_{t+1}) \quad (18)$$

As shown in Appendix A,  $V(h_t) > -\varphi(h_t, h_{t+1}; H) + \hat{V}(h_{t+1})$ . This implies that the investor has an ex-post incentive to renegotiate the promised payments. That is, the lack of credibility of the two-period economy is recurrent in the infinite horizon economy

Let  $h_{t+1}^{New} = g(h_t)$  be the investment in knowledge chosen by a new firm in the entry period, when the initial knowledge is  $h_t$  and the investor does not commit to the contract. The next proposition establishes that, without commitment from the investor, incumbent firms choose the same knowledge investment as new firms.

**Proposition 3** Without investor's commitment, the knowledge investment chosen by an incumbent firm is equal to the knowledge investment chosen by a new firm, that is,  $h_{t+1}^{Old} = h_{t+1}^{New} = g(h_t)$ .

**Proof 3** See Appendix C.

Since the firm can renegotiate the promised payments after the investment in knowledge, the innovator would not stay unless the firm agrees to the same knowledge chosen by a new firm. In this way, the innovator keeps the outside value high and prevents the firm from renegotiating.

Let  $J(h_t)$  be the repudiation value for the innovator when investors can not commit to the contract. Furthermore, let  $\hat{J}(h_{t+1})$  be the corresponding value after the investment in knowledge. Given the above proposition, the optimization problem for a new firm, started at time 0, can be written as:

$$J(h_0) = \max_{h_{t+1}, \{w_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \Big[ w_t - \varphi(h_t, h_{t+1}; H) \Big]$$
(19)

subject to

$$\sum_{j=t}^{\infty} \beta^{j-t} \Big[ w_j - \varphi(h_j, h_{j+1}; H) \Big] \ge J(h_t), \quad \text{for } t \ge 0$$
$$-\tau h_1 - w_0 - k_1 + \sum_{t=1}^{\infty} \beta^t \Big[ \pi(h_t, k_t, h_{t+1}) - w_t - k_{t+1} \Big] \ge 0$$
$$h_{t+1} = g(h_t), \quad \text{for } t > 0$$

Notice that only the initial knowledge  $h_1$  is chosen in this problem. Future values of h will be determined by the investment policy of future new firms, that is,  $h_{t+1} = g(h_t)$ . It is for this reason that we have not included the enforcement constraint after the investment in knowledge since it is already imbedded in  $g(h_t)$ .

The solution of this problem involves a non-trivial fixed point problem. First, as with the other optimization problems previously discussed, the enforcement constraints involve the outside value  $J(h_t)$ , which is the value function obtained by solving the optimization problem of a new entrant firm. Second, the policy function  $g(h_t)$ , which is taken as given by an incumbent firm, is also the policy function obtained as the solution of the same optimization problem. Solving for endogenous participation constraints is relatively new in the literature since they are often imposed exogenously by assuming autarky values. One exception is Cooley et al. (2004). Taking the policy function g(h) as given is specific to the problem considered here, although it is similar to the structure of recursive (or Markov perfect) equilibria with time inconsistency, as in Krusell, Martin, & Rios-Rull (2005) and Díaz-Giménez, Giovannetti, Marimon, & Teles (2006).

A detailed characterization of the solution to this problem is given in Appendix D. It should be noticed, however, that conditions (11) and (12) derived when the investor commits, are also valid in the case without commitment. The optimality condition for the accumulation of knowledge, however, is different. For new firms this is given by:

$$\tau + \varphi_2(h_t, h_{t+1}; H) = \beta \left\{ \pi_1(h_{t+1}, k_{t+1}, g(h_{t+1})) - \varphi_1(h_{t+1}, g(h_{t+1}); H) + \left[ \pi_3(h_{t+1}, k_{t+1}, g(h_{t+1})) + \tau \right] g_1(h_{t+1}) \right\}$$
(20)

For incumbent firms there is no optimality condition for the investment

in knowledge since they take as given the investment policy  $g(h_t)$ .

Imposing the steady state condition  $h_t = h_{t+1} = H$  and  $k_t = k_{t+1} = K$ , conditions (12) and (20) become:

$$\beta \pi_2(H, K, H) = 1$$

$$\tau + \varphi_2(H, H; H) = \beta \left\{ \pi_1 \left( H, K, g(H) \right) - \varphi_1 \left( H, g(H); H \right) + g_1(H) \left[ \pi_3 \left( H, K, g(H) \right) + \tau \right] \right\}$$
(22)

Differently from the case in which the investor commits to the contract, these two conditions are no longer sufficient to determine the steady state values of H and K. The unknown function g(H) also need to be determined. This requires us to solve for a fixed point problem. Denote by  $h' = \psi(h; g)$  the policy function that solves problem (19), for a given g. The policy function satisfies the first order condition (20). In equilibrium,  $g(H) = \psi(H; g)$ .

Because incumbent firms innovate at the same rate as new firms, condition (20) also determines the investment in knowledge of incumbent firms. Therefore, in order to determine whether the lack of commitment from the investor leads to higher or lower investment in knowledge, we have to compare this condition to (14), that is, the optimality condition for the investment in knowledge when the investor commits to the long-term contract.

Let  $H^C$  be the steady state knowledge in the economy in which the investor commits, and  $H^{NC}$  the steady state knowledge when the investor does not commit. We then have the following proposition:

**Proposition 4** Suppose that  $g_1(H) \leq 1$ . Then the steady state value of  $H^{NC}$  is strictly decreasing in  $\tau$  and there exists  $\bar{\tau} > 0$  such that  $H^{NC} > H^C$  for  $\tau < \bar{\tau}$  and  $H^{NC} < H^C$  for  $\tau > \bar{\tau}$ .

**Proof 4** See Appendix E.

Notice that the proof is based on the assumption that  $g_1(H) \leq 1$ , which we have checked numerically. Therefore, when contracts are not fully enforceable, neither for the innovator nor for the investor, the start-up cost is harmful for the accumulation of knowledge. With low barriers, the economy would experience a higher level of income than in the economy in which the investor commits. The higher investment can be welfare improving if there are spillovers in the accumulation of knowledge.

## 6 Quantitative application

In this section we investigate how much of the variation in cross-country incomes can be accounted by differences in the cost of business start-up. We first assign the parameter values and specify the functional forms.

The discount factor is  $\beta = 0.95$ , implying an annual interest rate of about 5 percent. The production function takes the form  $h^{1-\alpha}k^{\alpha}$ . The parameter  $\alpha$  represents the capital income share and it is set to 0.33. The depreciation of capital is specified as:  $\delta_t = \bar{\delta} + \delta \cdot (z_{t+1}/z_t - 1)$ . The parameter  $\delta$  is not important for the sensitivity of the steady state equilibrium to  $\tau$ . We set it to 0.1. The parameter  $\bar{\delta}$ , instead, determines the (physical) capital-income ratio independently of the start-up cost. We choose a value of 0.066 which implies a capital-income ratio of 2.8.

The effort cost function is derived from the accumulation equation for the stock of knowledge, which is assumed to take the form:

$$h_{t+1} = \phi h_t + \overline{H}^{\nu} \left( H_t^{\theta} e_t^{1-\theta} \right)^{1-\nu}$$

where  $\overline{H}$  is the worldwide knowledge external to the country,  $H_t$  is the economy-wide level of knowledge and  $e_t$  is the effort cost to accumulate knowledge. The parameter  $\nu < 1$  captures the importance of worldwide leakage or spillovers and  $\theta < 1$  captures the importance of domestic leakage or spillovers. Because  $\overline{H}$  is constant, without loss of generality we set it to 1. Then, by inverting, we get the effort cost function:

$$e_t = \varphi(h_t, h_{t+1}; H_t) = \frac{(h_{t+1} - \phi h_t)^{\frac{1}{(1-\theta)(1-\nu)}}}{H_t^{\frac{\theta}{1-\theta}}}$$

which is homogeneous of degree  $\rho = [1 - \theta(1 - \nu)]/[(1 - \theta)(1 - \nu)].$ 

Assuming that the economy grows at an exogenous rate of 3 percent, the stock of knowledge must also grow at this rate in the steady state. In the detrended economy this corresponds to a depreciation rate of 3 percent which requires  $\phi = 0.97$ . The parameter  $\theta$  is not important for the steady state. It determines only the optimality of the equilibrium. We set it to 0.1.

The important parameter is  $\nu$ , which determines the degree of homogeneity  $\rho$ . We start by choosing the value of  $\nu$  that optimizes the fit of the model with the data. More specifically,  $\nu$  is chosen to minimize the sum of square deviations between the outputs predicted by the model for each country (given the observed cost of business start up) and the actual per-capita GDP. We limit the sample to countries with a start-up cost smaller than 100 percent to eliminate outliers. This reduces the sample size to 104 countries. We also normalize the model so that it replicates the highest per-capita income with  $\tau = 0$ . In the 2004 sample the country with the highest per-capita GDP was Ireland with about 40,000 dollars.

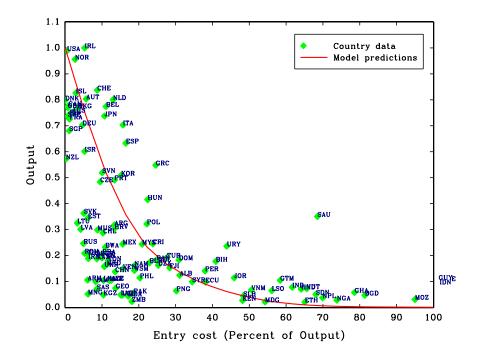


Figure 2: Steady state output for different entry costs.

Figure 2 plots the values of per-capita GDP and start-up costs for different countries, and the values predicted by the model when  $\nu = 0.014$  (and  $\rho = 1.0158$ ). This is the value that optimizes the fit of the model. As can be seen from the figure, the model is capable of capturing the main relation between the cost of business start-up and the level of per-capita income.

The next Figure 3 adds to the previous graph the predictions of the model for alternative values  $\nu$ . As we increase the degree of homogeneity of the cost function, the model is less successful in capturing large differences in per-capita income. A larger value of  $\nu$  implies that the economy depends more heavily on world-wide knowledge, which is external to an individual country. Therefore, it becomes more difficult to generate large

cross-country differences. On the other hand, when  $\rho$  is small, the economy depends only marginally on the world-wide knowledge. In the limit with  $\rho = 1$ , the model generates endogenous growth with long-term cross-country growth differences.

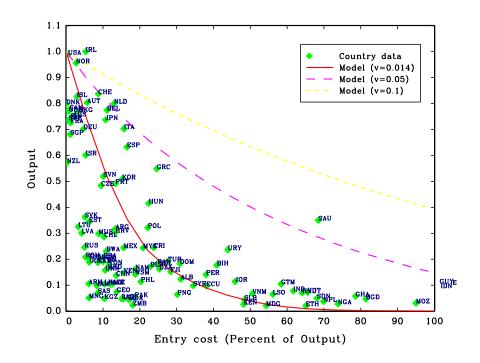


Figure 3: Steady state output for different entry costs.

We have also calculated the 'optimal' steady state level of output. This is the output produced if the investment in knowledge was chosen by a benevolent planner who takes into account the externality in the accumulation of knowledge. The steady state values of H and K in the planner solution are found by solving the first order conditions:

$$\beta \pi_2(H, K, H) = 1$$

$$\varphi_2(H, H; H) - \pi_3(H, K, H) = \beta \Big[ \pi_1(H, K, H) - \varphi_1(H, H; H) - \varphi_3(H, H; H) \Big]$$

These are similar to conditions (15) and (16) except for the additional term  $\varphi_3(H, H; H)$  in the second equation. This term captures the externality in

the accumulation of knowledge which is taken into account by the planner but ignored by the atomistic agents.

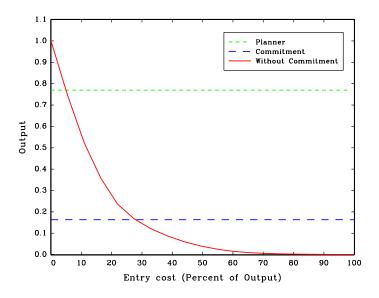


Figure 4: Steady state output for different entry costs.

Figure 4 plots the socially optimal level of output (planner's solution) for the baseline model where  $\theta = 0.1$ . The figure also plots the equilibrium output in the economies with and without investor's commitment. We would like to emphasize that the equilibrium output (with and without commitment) is not affected by  $\theta$ . It only affects the socially optimal output. For the particular parametrization, there is an optimal entry cost which is about 7 percent of output. As we increase  $\theta$ , the first line moves up while the other two lines do not change. When  $\theta = 0$ , the socially optimal output is equal to the equilibrium output when the investor commits.

# 7 The role of barriers when the probability of survival is p < 1

In this section we generalize the previous model by assuming that p < 1. We keep the assumption that the survival of the firm is observed after the investment in knowledge. Because of this, the level of  $h_{t+1}$  is predetermined for new firms. The initial physical capital, however, is still chosen as part of the optimal contract. If an incumbent firm survives, the innovators receives  $w_t$  and stay with the current employer. If the firm exits, the innovator is hired by a new firm and receives the lifetime utility  $\hat{V}(h_{t+1})$ . We can then define the pseudo utility flow for the innovator as follows:

$$U(h_t, h_{t+1}, w_t; H) \equiv -\varphi(h_t, h_{t+1}; H) + pw_t + (1-p)\widehat{D}(h_{t+1})$$

The contracting problem with investor's commitment can be written as in (9) after replacing the term  $w_t - \varphi(h_t, h_{t+1}; H)$  with  $U(h_t, h_{t+1}, w_t; H)$  and discounting future flows by  $p\beta$ . The first order conditions are derived in Appendix F. For a new entrant firm (first period optimization) they are:

$$V(h_t) \le -\varphi(h_t, h_{t+1}; H) + pw_t + (1-p)\hat{V}(h_{t+1}) + p\beta V(h_{t+1})$$
(23)

$$p\beta\pi_2(h_{t+1}, k_{t+1}, h_{t+2}) = 1 \tag{24}$$

$$(1-p)\tau + \tau + \varphi_2(h_t, h_{t+1}; H) = \beta \Big[ \pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H) \Big]$$
(25)

for t = 0. For an incumbent firm, the optimality conditions are (23), (24) and

$$(1-p)\tau - \pi_3(h_t, k_t, h_{t+1}) + \varphi_2(h_t, h_{t+1}; H) = \beta \Big[ \pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H) \Big]$$
(26)

The conditions for the accumulation of knowledge when the investor commits to the contract, are similar to the corresponding conditions derived with p = 1 (see (13) and (14)) with the exception of the constant term  $(1 - p)\tau$ . The most important difference with the case of p = 1 is that now the entry cost affects negatively the steady state value of H even if the investor commits to the contract.

Higher values of p (higher survival) increase the steady state value of knowledge for two reasons. First, it reduces the term  $(1 - p)\tau$  which corresponds to a reduction of the marginal cost of accumulating knowledge for both new and incumbent firms. Second, it increases the expected return from physical capital because the gross output  $\pi$  is generated only with probability p. This explains why in the optimality condition for the choice of physical capital, the gross marginal product is discounted by  $p\beta$ .

There is another point we should emphasize. With p < 1, there is a positive mass of firms entering in the second stage of the period. Because

they enter in the second stage, they survive with certainty to the next period. This implies that the optimality condition for the investment in physical capital is  $\beta \pi_2(h_{t+1}, k_{t+1}, h_{t+2}) = 1$ . This condition differs from the optimality condition of incumbent firms (see condition (24)) as new firms discount less the return from capital. As a result, they chose a higher  $k_{t+1}$ . This implies that new firms will experience some transition before converging to some long-run values  $\bar{K}$  and  $\bar{K}$ . Notice that the long-run value of knowledge  $\bar{H}$  is now different from the economy-wide average H, because there will be an invariant distribution of firms over h and k.

When the investor does not commit to the contract, the optimization problem can be written as in problem (19) once we replace  $w_t - \varphi(h_t, h_{t+1}; H)$ with  $U(h_t, h_{t+1}, w_t; H)$  and discount future flows by  $p\beta$ . The first order condition for the accumulation of knowledge of a new firm can be written as:

$$(1-p)\tau + \tau + \varphi_2(h_t, h_{t+1}; H) = \beta \Big\{ \pi_1 \Big( h_{t+1}, k_{t+1}, g(h_{t+1}) \Big) \\ - \varphi_1 \Big( h_{t+1}, g(h_{t+1}); H \Big) + g_1(h_{t+1}) \Big[ \pi_3 \Big( h_{t+1}, k_{t+1}, g(h_{t+1}) \Big) + \tau \Big] \Big\}$$
(27)

which differs from (20) only in the constant term  $(1-p)\tau$ .

Let  $H^C$  be the steady state level of knowledge when the investor commits and  $H^{NC}$  the steady state level when the investor does not commit. Thanking into account that the steady state equilibrium is now characterized by an invariant distribution of firms, Proposition 4 can be reformulated as follows:

**Proposition 5** Assume  $p \in (0, 1)$ . The steady state values of  $H^C$  and  $H^{NC}$  are both strictly decreasing in  $\tau$ . Moreover, there exists  $\bar{\tau} > 0$  such that  $H^{NC} > H^C$  for  $\tau < \bar{\tau}$  and  $H^{NC} < H^C$  for  $\tau > \bar{\tau}$ .

In general, barriers to entry affect the accumulation of knowledge even when the investor commits to the contract. However, their negative impact is stronger when the investor does not commit. The proof of the proposition uses the same logic of the proof of Proposition 4.

## 8 Covenants and other barriers to mobility

Other barriers to the mobility of innovators may have a similar effect in our model as the cost of business start up. As we have discussed in the Introduction, even within a similar legal and economic environment—resulting in similar costs for business start up—there may be differences in other barriers. Covenants is one of them. A covenant which is ex-post enforced will prevent the innovator from productively using her acquired knowledge for a period of time if she moves to another firm.

A natural way to model non-competitive covenants in our set-up is by assuming that a quitting innovator can use only a fraction  $\xi$  of the new investment in knowledge in the new firm. Notice, furthermore, that this formulation also captures the case that part of the knowledge acquired can not be used by the innovator due to the enforcement of IPR in the case in which the innovator does not have full control of the patent. To fix ideas, notice that a more stringent enforcement of covenants (or IPRs) is associated, in what follows, with a lower fraction  $\xi$ .

To keep the analysis brief, we limit the analysis to the two-period model. The extension to the infinite horizon will follow the same logic of the previous analysis with entry cost. The problem solved by a new firm, started at the beginning of the period can be written as follows:

$$S(h_{0}) = \max_{h_{1},k_{1},w_{0}} \left\{ -\varphi(h_{0},h_{1}) - k_{1} + (\xi h_{1})^{1-\alpha} k_{1}^{\alpha} \right\}$$
(28)  
s.t.  
$$w_{0} - \varphi(h_{0},h_{1}) \ge D(h_{0})$$
  
$$-w_{0} - k_{1} + (\xi h_{1})^{1-\alpha} k_{1}^{\alpha} \ge 0$$

The problem solved by an incumbent firm is as in problem (1). The first order conditions with respect to  $h_1$ , for incumbent and new firms, are:

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} = \varphi_{h_1}(h_0, h_1) + \delta \cdot \left(\frac{k_0}{h_0}\right)$$
(29)

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} = \varphi_{h_1}(h_0, h_1) \cdot \xi^{\alpha-1}$$
(30)

Because  $\xi < 1$  and  $\alpha < 1$ , the term  $\xi^{\alpha-1} > 1$ . Therefore, the noncompeting covenants have the effect of increasing the cost of accumulating knowledge and acts similarly to the entry cost  $\tau$ . Proposition 1 becomes:

**Proposition 6** The knowledge investment of a new firm  $h^{New}$  is strictly increasing in  $\xi$  and there exists  $\overline{\xi} > 0$  such that  $h^{New} = h^{Old}$ .

**Proof 5** Using the first order condition for the choice of physical capital, which is  $\alpha (k_1/h_1)^{\alpha-1} = 1$  for both incumbent and new firms, the above first order conditions can be rewritten as:

$$(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} = \varphi_{h_1}(h_0, h^{Old}) + \delta \cdot \left(\frac{k_0}{h_0}\right)$$
$$(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} = \varphi_{h_1}(h_0, h^{New})\xi^{-1}$$

The proposition follows directly from these two conditions. Q.E.D.

Given the symmetry with the case of barriers to business start up, it follows that all the results obtained in Section 4 also apply to the case of covenants and similar barriers to mobility.

# 9 Conclusion

We have developed a theory in which *barriers to knowledge mobility* affect the accumulation of knowledge, and therefore, the level of income and growth. It does not simply make the claim that "competition enhances income and growth". It also shows how different forms of contract enforcement affect the relation between competition, innovation and growth. In particular, when the investor can not commit to future promises of payments, the rate of innovation is determined by those firms that value innovation the most (start-up firms in our base model). As a result, high levels of innovation are associated with low barriers to knowledge mobility (low barriers to business start-up in our base model). At the firm level, this can results in overaccumulation of knowledge. However, if there are positive spillovers, such effect can enhance welfare.

In a semi-endogenous general equilibrium growth model, we have shown how *barriers to business start-up* can explain cross-country income differences. We take this to be the first step to bring our theory to the data. We also show how other *barriers to knowledge mobility*, such as strict enforcement of Covenants or Intellectual Property Rights, can have similar effects. This suggests a wide scope for the empirical application of the theory developed here.

### A First order conditions with investor's commitment

We first prove the following lemma:

**Lemma 1** The enforcement constraint 'after' the investment in knowledge is satisfied if the enforcement constraint is satisfied 'before' the investment in knowledge.

**Proof 1** The enforcement constraints can be rewritten as:

$$\sum_{j=t}^{\infty} \beta^{j-t} \Big[ w_j - \varphi(h_j, h_{j+1}; H) \Big] \geq V(h_t)$$
$$\sum_{j=t}^{\infty} \beta^{j-t} \Big[ w_j - \varphi(h_j, h_{j+1}; H) \Big] \geq -\varphi(h_t, h_{t+1}; H) + \widehat{V}(h_{t+1})$$

Therefore, to show that the second constraint is satisfied when the first constraint is satisfied, it is enough to show that  $V(h_t) \ge -\varphi(h_t, h_{t+1}; H) + \hat{V}_t(h_{t+1})$  for any value of  $h_{t+1}$ . Because  $V(h_t) = \max_{h} \{-\varphi(h_t, h; H) + \hat{V}(h)\}$ , we have that:

$$V(h_t) = \max_h \left\{ -\varphi(h_t, h; H) + \widehat{V}(h) \right\} \ge -\varphi(h_t, h_{t+1}; H) + \widehat{V}(h_{t+1})$$

for any  $h_{t+1}$ .

Let's consider now problem (9). Thanks to the above lemma we can ignore the enforcement constraint after the investment in knowledge. Let  $\gamma_t$  be the Lagrange multiplier associated with the enforcement constraint before the investment in knowledge and  $\lambda_0$  the Lagrange multiplier associated with the participation constraint for the investor. The Lagrangian can be written as:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^{t} \Big[ w_{t} - \varphi(h_{t}, h_{t+1}; H) \Big] \\ + \sum_{t=0}^{\infty} \beta^{t} \gamma_{t} \left\{ \sum_{j=t}^{\infty} \beta^{j-t} \Big[ w_{j} - \varphi(h_{j}, h_{j+1}; H) \Big] - V_{t}(h_{t}) \right\} \\ + \lambda_{0} \left\{ -w_{0} - \tau h_{1} - k_{1} + \sum_{t=1}^{\infty} \beta^{t} \Big[ \pi(h_{t}, k_{t}, h_{t+1}) - w_{t} - k_{t+1} \Big] \right\}$$

Define  $\mu_t$  recursively as follows:  $\mu_{t+1} = \mu_t + \gamma_t$ , with  $\mu_0 = 0$ . Using this variable and rearranging terms, the Lagrangian can be written as:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^{t} \left\{ \left( 1 + \mu_{t+1} \right) \left[ w_{t} - \varphi(h_{t}, h_{t+1}; H) \right] - \left( \mu_{t+1} - \mu_{t} \right) V(h_{t}) \right\} \\ + \lambda_{0} \left\{ -w_{0} - \tau h_{1} - k_{1} + \sum_{t=1}^{\infty} \beta^{t} \left[ \pi(h_{t}, k_{t}, h_{t+1}) - w_{t} - k_{t+1} \right] \right\}$$

This problem becomes recursive at any t > 0. Therefore, we can rewrite the problem as follows:

$$\mathcal{L} = \min_{\mu_1 \ge 0} \max_{\substack{w_0 \ge 0, \\ k_1, h_1}} \left\{ \lambda_0 \Big[ -w_0 - \tau h_1 - k_1 \Big] + (1 + \mu_1) \Big[ w_0 - \varphi(h_0, h_1; H) \Big] - \mu_1 V(h_0) + \beta W(\mu_1, h_1, k_1) \right\}$$
(31)

with the function W is defined recursively as follows:

$$W(\mu_{t}, h_{t}, k_{t}) = \min_{\mu_{t+1} \ge \mu_{t}} \max_{\substack{w_{t} \ge 0, \\ k_{t+1}, h_{t+1}}} \left\{ \lambda_{0} \Big[ \pi(h_{t}, k_{t}, h_{t+1}) - w_{t} - k_{t+1} \Big] + (1 + \mu_{t+1}) \Big[ w_{t} - \varphi(h_{t}, h_{t+1}; H) \Big] - (\mu_{t+1} - \mu_{t}) V(h_{t}) + \beta W(\mu_{t+1}, h_{t+1}, k_{t+1}) \right\}$$
(32)

for all t > 0.

The first optimization problem (equation (31)) is the problem solved by a new firm with initial state  $h_0$  and for a given  $\lambda_0$ . The lagrange multiplier  $\lambda_0$ is determined such that the participation constraint for the investor is satisfied. Tighter is this constraint and higher is the value of  $\lambda_0$ . The second optimization problem (equation (32)) is the one solved after entering. Therefore, this is the problem solved by an incumbent firm that starts with states  $\mu_t$ ,  $h_t$  and  $k_t$ .

Taking derivatives in problem (31) gives:

$$V(h_t) \le -\varphi(h_t, h_{t+1}; H) + w_t + \beta V(h_{t+1})$$
(33)

$$1 + \mu_{t+1} \le \lambda_0 \tag{34}$$

$$\beta \pi_2(h_{t+1}, h_{t+1}, k_{t+1}) = 1 \tag{35}$$

$$\lambda_0 \tau + (1 + \mu_{t+1})\varphi_2(h_t, h_{t+1}; H) = \beta W_2(\mu_{t+1}, h_{t+1}, k_{t+1})$$
(36)

for t = 0 and with the envelope term given by:

$$W_2(\mu_t, h_t, k_t) = \lambda_0 \pi_1(h_t, k_t, h_{t+1}) - (1 + \mu_{t+1})\varphi_1(h_t, h_{t+1}; H) - (\mu_{t+1} - \mu_t)V_1(h_t)$$

The first order conditions in problem (32) are (33)-(35) and

$$-\lambda_0 \pi_3(h_t, k_t, h_{t+1}) + (1 + \mu_{t+1})\varphi_2(h_t, h_{t+1}; H) = \beta W_2(\mu_{t+1}, h_{t+1}, k_{t+1})$$
(37)

As emphasized above, the value of  $\lambda_0$  depends on the tightness of the participation constraint for the investor. Assume that a new firm can choose  $h_1 < h_0$  without any cost. This is equivalent to assuming that the innovator choose to destroy part of the knowledge. Then we can prove that the investor is able to break even even if the contract chooses the unconstrained sequence of h. This implies that  $\lambda_0 = 1$  and, from condition (34),  $\mu_t = 0$  for all t. Using this and substituting the envelope term, conditions (33), (35), (36) and (37) become (11)-(14). Q.E.D.

#### **B** Steady state equilibrium when the investor commits

**Proposition 7** There is a unique steady state equilibrium in which all firms have the same knowledge H and physical capital K.

**Proof 6** Consider condition (16), which we rewrite here as follows:

$$\varphi_2(H, H; H) + \beta \varphi_1(H, H; H) = \pi_3(H, K, H) + \beta \pi_1(H, K, H)$$

The right-hand-side term remains constant for any value of H. In fact, taking into account the functional form of  $\pi$  (see equation (8), we have that  $\pi_3(H, K, H) = -\delta(K/H)$  and  $\pi_1(H, K, H) = \delta(K/H) + (1 - \alpha)(K/H)^{\alpha}$ . These two terms only depend on the ratio K/H. From condition (15) we have that  $\pi_2(H, K, H) = 1 + \alpha(K/H)^{\alpha-1} = 1$ , which uniquely determine the ratio K/H.

Let's look now at the left-hand-side term. Because  $\varphi$  is homogenous of degree  $\rho > 1$ , the derivatives  $\varphi_1$  and  $\varphi_2$  are homogeneous of degree  $\rho - 1$ . Therefore, the left-hand-side term can be written as

$$\varphi_2(H, H; H) + \beta \varphi_1(H, H; H) = \Big[\varphi_2(1, 1; 1) + \beta \varphi_1(1, 1; 1)\Big] H^{\rho - 1}$$

Because  $\rho > 1$ , this term is strictly increasing in H, converges to zero as  $H \to 0$ and to infinity as  $H \to \infty$ . Therefore, there exists a unique value of H that solves this condition. The uniqueness of H then implies the uniqueness of K. Q.E.D.

#### C Proof of Proposition 3

Suppose that the knowledge investment chosen by a new firm is different from the one chosen by an incumbent firm. Denote by  $h_{t+1}^{New}$  and  $h_{t+1}^{Old}$  the investment of new and incumbent firms, respectively. Because  $h_{t+1}^{New}$  solves the problem  $V_t(h_t) = \max_{h_{t+1}} \{-\varphi(h_t, h_{t+1}; H) + \hat{V}_t(h_{t+1})\}$ , we have that:

$$V_t(h_t) > -\varphi(h_t, h_{t+1}^{Old}; H) + \hat{V}_t(h_{t+1}^{Old})$$

if  $h_{t+1}^{Old} \neq h_{t+1}^{New}$ . But then constraints (17) and (18) cannot be both satisfied. Therefore, the only feasible solution is  $h_{t+1} = h_{t+1}^{New}$ . Q.E.D.

## D Derivation of the first order condition (20)

Following the same steps of Appendix A, we can show that in a steady state equilibrium, problem (19) can be reformulated as:

$$\mathcal{L} = \min_{\mu_1 \ge 0} \max_{\substack{w_0 \ge 0, \\ k_1, h_1}} \left\{ \lambda_0 \Big[ -w_0 - \tau h_1 - k_1 \Big] + (1 + \mu_1) \Big[ w_0 - \varphi(h_0, h_1; H) \Big] - \mu_1 J(h_0) + \beta W(\mu_1, h_1, k_1) \right\}$$
(38)

with the function W is defined recursively as follows:

$$W(\mu_{t}, h_{t}, k_{t}) = \min_{\mu_{t+1} \ge \mu_{t}} \max_{\substack{w_{t} \ge 0, \\ k_{t+1}}} \left\{ \lambda_{0} \Big[ \pi(h_{t}, k_{t}, g(h_{t})) - w_{t} - k_{t+1} \Big] + (1 + \mu_{t+1}) \Big[ w_{t} - \varphi(h_{t}, g(h_{t}); H) \Big] - (\mu_{t+1} - \mu_{t}) J(h_{t}) + \beta W(\mu_{t+1}, g(h_{t}), k_{t+1}) \Big\}$$
(39)

for all t > 0.

The first order condition with respect to  $h_1$  in problem (38) gives:

$$\lambda_0 \tau + (1 + \mu_1) \varphi_2(h_0, h_1; H) = \beta W_2(\mu_1, h_1, k_1)$$
(40)

with the envelope condition given by:

$$W_{2}(\mu_{t}, h_{t}, k_{t}) = \lambda_{0} \pi_{1} \Big( h_{t}, k_{t}, g(h_{t}) \Big) + \lambda_{0} \pi_{3} \Big( h_{t}, k_{t}, g(h_{t}) \Big) g_{1}(h_{t})$$

$$- (1 + \mu_{t+1}) \varphi_{1} \Big( h_{t}, g(h_{t}); H \Big) - \mu_{t+1} \varphi_{2} \Big( h_{t}, g(h_{t}); H \Big) g_{1}(h_{t})$$

$$- (\mu_{t+1} - \mu_{t}) J_{1}(h_{t}) + \beta W_{2}(\mu_{t+1}, h_{t+1}, k_{t+1}) g_{1}(h_{t})$$

$$(41)$$

With limited enforcement, condition (40) must be satisfied at any point in time. Substituting this condition in (41), we get:

$$W_{2}(\mu_{t}, h_{t}, k_{t}) = \lambda_{0} \pi_{1} \Big( h_{t}, k_{t}, g(h_{t}) \Big) - (1 + \mu_{t+1}) \varphi_{1} \Big( h_{t}, g_{t}(h_{t}); H \Big) \\ - (\mu_{t+1} - \mu_{t}) J_{1}(h_{t}) + \lambda_{0} \Big[ \pi_{3} \Big( h_{t}, k_{t}, g(h_{t}) \Big) + \tau \Big] g_{1}(h_{t})$$

Also in this case we can prove that the unconstrained investment in knowledge capital allows the investor to break-even. Therefore,  $\lambda_0 = 1$  and  $\mu_t = 0$ . Using this result and substituting the envelope in (40) we get condition (20). Q.E.D.

#### **E** Proof of Proposition 4

In the steady state without commitment, potential new firms start with the same knowledge H as incumbents firms. Because H = g(H), condition (22) can be written as:

$$\tau + \varphi_2(H, H; H) = \beta \Big[ \pi_1(H, K, H) - \varphi_1(H, H; H) \Big] + \beta g_1(H) \Big[ \pi_3(H, K, H) + \tau \Big]$$

which determines the steady state knowledge for incumbent and new firms when the investor does not commit (double-side limited enforcement).

This condition must be compared to the optimality condition that determines the steady state knowledge when the investor commits to the contract (one-side limited enforcement). This is given by equation (16), which we rewrite as:

$$\varphi_2(H, H; H) = \beta \left[ \pi_1(H, K, H) - \varphi_1(H, H; H) \right] + \pi_3(H, K, H)$$

The homogeneity of degree  $\rho$  of the cost function  $\varphi$  implies that the derivatives are homogeneous of degree  $\rho - 1$ . Therefore, the above two conditions can be rewritten as:

$$\left[ \varphi_2(1,1;1) + \beta \varphi_1(1,1;1) \right] H^{\rho-1} = \beta \pi_1(H,K,H) + \beta g_1(H) \pi_3(H,K,H) (42) - \tau \left[ 1 - \beta g_1(H) \right]$$

$$\left[\varphi_{2}(1,1;1) + \beta\varphi_{1}(1,1;1)\right]H^{\rho-1} = \beta\pi_{1}(H,K,H) + \pi_{3}(H,K,H)$$
(43)

Because  $\rho - 1 > 0$ , the left-hand-side terms are strictly increasing in H, converge to zero as  $H \to 0$  and to infinity as  $H \to \infty$ . We further observe that, as shown in the proof of Proposition 7, the terms  $\pi_1$  and  $\pi_3$  only depend on the ratio K/H. This term is uniquely pinned down by condition (12), which is the same for both economies. Therefore,  $\pi_1(H, K, H)$  and  $\pi_3(H, K, H)$  do not change as H changes.

Consider first the case in which the start-up cost is zero, that is,  $\tau = 0$ . If  $g_1(H) \leq 1$ , as postulated in the proposition, the term  $\beta g_1(H) < 1$ . Because  $\pi_3(H, K, H) < 0$  and  $\beta g_1(H) < 1$ , the right-hand-side of (42) is bigger than the right-hand-side of (43) for a given H. This implies that the value of H in the first equation must be bigger than in the second, that is,  $H^{NC} > H^C$ . Notice that, without capital obsolescence,  $\pi_3(H, K, H) = 0$ . Therefore, conditions (42) and (43) are indistinguishable if  $\tau = 0$ .

Let's consider now the case in which  $\tau > 0$ . This variable only affects condition (42). Because  $\beta g_1(H) < 1$ , then an increase in  $\tau$  reduces the right-hand-side of (42). The reduction in the left-hand-side term then requires a lower value of H. For a sufficiently large  $\tau$ , the steady state level of knowledge declines to the point in which  $H^{NC} < H^C$ . Q.E.D.

#### **F** First order conditions with p < 1

We repeat the steps used in Appendix A for the case p = 1 after replacing the term  $w_t - \varphi(h_t, h_{t+1}; H)$  with  $U(h_t, h_{t+1}, w_t; H)$  and discounting by  $p\beta$ . The first order conditions for an entrant firm is:

$$V(h_t) \le -\varphi(h_t, h_{t+1}; H) + pw_t + (1-p)\dot{V}(h_{t+1}) + p\beta V(h_{t+1})$$
(44)

$$1 + \mu_{t+1} \le \lambda_0 \tag{45}$$

$$p\beta\pi_2(h_{t+1}, k_{t+1}, h_{t+1}) = 1 \tag{46}$$

$$\lambda_0 \tau + (1 + \mu_{t+1})\varphi_2(h_t, h_{t+1}; H) = (1 + \mu_{t+1})(1 - p)\widehat{D}_1(h_{t+1}) + p\beta W_2(\mu_{t+1}, h_{t+1}, k_{t+1})$$
(47)

for t = 0 and with the envelope term given by:

$$W_2(\mu_t, h_t, k_t) = \lambda_0 \pi_1(h_t, k_t, h_{t+1}) - (1 + \mu_{t+1})\varphi_1(h_t, h_{t+1}; H) - (\mu_{t+1} - \mu_t)V_1(h_t)$$

The first order conditions for an incumbent firm are (44)-(46) and

$$-\lambda_0 \pi_3(h_t, k_t, h_{t+1}) + (1 + \mu_{t+1})\varphi_2(h_t, h_{t+1}; H) = (1 + \mu_{t+1})(1 - p)\widehat{V}_1(h_{t+1}) + p\beta W_2(\mu_{t+1}, h_{t+1}, k_{t+1})$$
(48)

Also in this case the investor breaks even when the contract chooses the unconstrained knowledge. Therefore,  $\lambda_0 = 1$  and  $\mu_t = 0$ . With all  $\mu$  set to 0, the function W is the surplus generated by an incumbent firm. Using this, the surplus generated by a new firm, after the investment in knowledge and after the realization of survival, can be written as:

$$\widehat{V}(h_t) = -\tau h_{t+1} - k_{t+1} - w_t + \beta W(1, h_{t+1}, k_{t+1})$$

from which we have  $\widehat{V}_1(h_{t+1}) = -\tau + \beta W_2(1, h_{t+1}, k_{t+1})$ . Therefore, conditions (47) and (48) can be rewritten as:

$$(1-p)\tau + \tau + \varphi_2(h_t, h_{t+1}; H) = \beta \Big[ \pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H) \Big]$$
  
(1-p)\tau - \pi\_3(h\_t, k\_t, h\_{t+1}) + \varphi\_2(h\_t, h\_{t+1}; H) = \beta \Big[ \pi\_1(h\_{t+1}, k\_{t+1}, h\_{t+2}) - \varphi\_1(h\_{t+1}, h\_{t+2}; H) \Big]  
$$- \varphi_1(h_{t+1}, h_{t+2}; H) \Big]$$
  
Q.E.D.

# References

- Acemoglu, D. (1997). Training and innovation in an imperfect labor market. *Review of Economic Studies*, 64(3), 445–64.
- Acemoglu, D., Aghion, P., & Zilibotti, F. (2002). Distance to frontier, selection, and economic growth. CEPR Discussion Paper #3467.
- Acemoglu, D. & Pischke, J. (1999). The structure of wages and investment in general training. *Journal of Political Economy*, 107(3), 539–72.
- Acemoglu, D. & Shimer, R. (1999). Holdups and efficiency with search frictions. International Economic Review, 40(4), 827–50.
- Aghion, P., Bloom, N., Blundell, R., Griffith, R., & Howitt, P. (2005). Competition and innovation: an inverted U-relationship. *Quarterly Journal* of Economics, 120(2), 701–28.
- Aghion, P., Blundell, R., Griffith, R., Howitt, P., & Prantl, S. (2004). Firm entry, innovation and growth: theory and micro evidence. Unpublished manuscript. Institute for Fiscal Studies & Harvard University.
- Aghion, P. & Griffith, R. (2005). Competition and Growth: Reconciling Theory and Evidence. MIT Press, Cambridge, Massachusetts.
- Anton, J. J. & Yao, D. A. (1994). Expropriation and inventions: appropriable rents in the absence of property rights. *American Economic Review*, 84(1), 190–209.
- Baccara, M. & Razin, R. (2004). Curb your innovation: corporate conservatism in the presence of imperfect intellectual property rights. Unpublished manuscript, New York University.
- Boldrin, M. & Levine, D. (2006). Against Intellectual Monopoly. Electronic version, http://www.econ.umn.edu/mboldrin/aim.html.
- Bresnahan, T. F. & Malerba, F. (2002). The value of competitive innovation and U.S. policy toward the computer industry. In Bai, C.-E. & Yuen, C.-W. (Eds.), *Technology and the New Economy*, chap. 2, pp. 49–93. MIT Press, Cambridge, Massachusetts.

- Cooley, T. F., Marimon, R., & Quadrini, V. (2004). Aggregate consequences of limited contracts enforceability. *Journal of Political Econ*omy, 111(4), 421–46.
- Díaz-Giménez, J., Giovannetti, G., Marimon, R., & Teles, P. (2006). Nominal debt as a burden on monetary policy. Unpublished manuscript, Universitat Pompeu Fabra.
- Gilson, R. J. (1999). The legal infrastructure of high technology industrial districts: silicon valley, route 128, and covenants not to compete. New York University Law Review, 74(3), 575–629.
- Greenwood, J. & Jovanovic, B. (1990). Financial development, growth, and the distribution of income. *Journal of Political Economy*, 98(5), 1076– 1107.
- Hyde, A. (2003). Working in Silicon Valley: Economic and Legal Analysis of a High-Velocity Labor Market. Sharpe, M.e., Inc., Armonk, New York.
- Jones, C. I. (1995). R&D Based models of economic growth. Journal of Political Economy, 103(4), 759–84.
- Kocherlachota, N. R. (1996). Implications of efficient risk sharing without commitment. *Review of Economic Studies*, 63(4), 595–609.
- Krusell, P., Martin, P. F., & Rios-Rull, J. V. (2005). On the determination of government debt. Unpublished manuscript, University of Pennsylvania.
- Marcet, A. & Marimon, R. (1992). Communication, commitment and growth. Journal of Economic Theory, 58(1), 219–249.
- Mokyr, J. (1990). The Lever of Riches: Technological Creativity and Economic Progress. Oxford University Press, New York.
- Parente, S. L. & Prescott, E. C. (1990). Barriers to Riches. MIT Press, Cambridge, Massachusetts.
- Romer, P. M. (1990). Endogenous technological change. Journal of Political Economy, 98(1), 71–102.
- Romer, P. M. (1993). Two strategies for economic development: using ideas and producing ideas. World Bank Economic Review, 7(1), 63–91.

- Saxenian, A. (1996). Regional Advantage: Culture and Competition in Silicon Valley and Route 128. Harvard University Press, Cambridge, Massachusetts.
- World Bank (2005). Doing Business in 2005: Removing Obstacles to Growth. World Bank, Washington.