# Economic and Regulatory Capital What is the Difference?*

Abel Elizalde
CEMFI and UPNA

Rafael Repullo
CEMFI and CEPR

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**Abstract**

This paper analyzes the determinants of regulatory capital (the minimum required by regulation), economic capital (that chosen by shareholders in the absence of capital regulation), and actual capital (that chosen with such regulation) in the context of the single risk factor model of Basel II. The results show that economic and regulatory capital do not depend on the same variables and do not react in the same way to changes in their common determinants. For plausible parameter values, they are both increasing in the loans' probability of default and loss given default, but variables that affect economic but not regulatory capital, such as the intermediation margin and the cost of capital, can move them significantly apart. Actual capital follows more closely the behavior of regulatory capital. The results also show that market discipline, proxied by the coverage of deposit insurance, increases economic and actual capital, although the effects are generally small.

*We would like to thank Jaime Caruana, Charles Goodhart, Rosa Lastra, Julio Segura, and Javier Suarez for their comments. Address for correspondence: CEMFI, Casado del Alisal 5, 28014 Madrid. Tel: +34-914290551. Fax:+34-914291056. E-mail: elizalde@cemfi.es, repullo@cemfi.es.

# 1 Introduction

Economic and regulatory capital are two terms frequently used in the analysis of the new framework for bank capital regulation recently finalized by the Basel Committee on Banking Supervision (2004), known as Basel II. In particular, many discussions preceding the publication of the new regulation have highlighted the objective of bringing regulatory capital closer to economic capital. For example, Gordy and Howells (2004, p.1) state that "the primary objective under Pillar 1 (of Basel II) is better alignment of regulatory capital requirements with 'economic capital' demanded by investors and counterparties."

To compare economic and regulatory capital we first must clarify the meaning of each term. The definition of *regulatory capital* is clear: it is the minimum capital required by the regulator, which in this paper we identify with the capital charges in the Internal Ratings-Based (IRB) approach of Basel II. Economic capital is usually defined as the capital level that is required to cover the bank's losses with a given probability; see, for example, Jones and Mingo (1998) or Carey (2001). It is implicitly assumed that shareholders are the ones choosing such probability in order to maximize the market value of the bank. Therefore, *economic capital* may be understood, and this is the definition that we will use hereafter, as the capital level that bank shareholders would choose in absence of capital regulation.

The main purpose of this paper is to analyze the differences between economic and regulatory capital in the context of the single risk factor model that underlies the IRB capital requirements of Basel II. To compute economic capital we use a dynamic model in which shareholders choose, at the beginning of each period, the level of capital in order to maximize the value of the bank, taking into account the possibility that the bank be closed if the losses during the period exceed the initial level of capital.[1] Thus economic capital trades-off the costs of funding the bank with costly equity against the benefits of reducing the probability of losing its franchise value, which appears as a key endogenous value in the bank's maximization problem. It is important to stress that in our model bank shareholders only choose the level of capital, not the risk characteristics of its loan portfolio which are taken as given.

[1] This could be justified by assuming that either the bank supervisor withdraws its license or that a bank run takes place before the shareholders can raise new equity to cover the losses.

We show that economic and regulatory capital do not depend on the same variables: the former (but not the latter) depends on the intermediation margin and the cost of bank capital, while the latter (but not the former) depends on the confidence level set by the regulator. Moreover, economic and regulatory capital do not respond in the same manner to changes in the common variables that affect them, such as the loans' probability of default, loss given default, and exposure to the systematic risk factor.

Due to the difficulty of obtaining analytical results for economic capital, we use a numerical procedure to compute it. The results show that Basel II regulatory capital only approaches economic capital for a limited, although reasonable, range of parameter values. The results also show that the relative position of economic and regulatory capital is mainly determined by the cost of bank capital: economic capital is higher (lower) than regulatory capital for values of the cost of bank capital lower (higher) than a given critical value. Another key variable in the shareholders' economic capital decision is the intermediation margin, which has two opposite effects. On the one hand, a higher margin increases the bank's franchise value and consequently shareholders' incentives to contribute capital. On the other hand, a higher margin increases bank revenues and therefore reduces the role of capital as a buffer to absorb future losses, acting as a substitute of economic capital. We show that the net effect of the intermediation margin on economic capital is positive in very competitive banking markets and negative otherwise.

The numerical results also show that increases in the loans' probability of default and loss given default increase regulatory capital, while they only increase economic capital for a range of (reasonable) values of these variables. At any rate, the correlation between economic and regulatory capital in Basel II is clearly higher than in the 1988 Basel Accord (Basel I), where regulatory capital was practically independent of risk.[2]

The paper also addresses the determinants of *actual capital*, which is defined as the capital chosen by bank shareholders taking into account regulatory constraints. In

[2]Basel I required a minimum capital equal to 8% of the bank's risk weighted assets. Two basic criteria were used to compute these weights: the institutional nature of the borrower and the collateral provided. In particular, the weights were 0% for sovereign risks with OECD countries, 20% for interbank risks, 50% for mortgages, and 100% for all other risks.

particular, two regulations are considered. First, banks must have, at the beginning of each period, a capital level no lower than regulatory capital. Second, banks whose capital level at the end of a period falls below a minimum (positive) threshold are considered critically undercapitalized and closed. The first regulation guarantees that for operating banks, i.e. banks whose franchise value under these regulations is positive, actual capital is no lower than regulatory capital. The threat of closing undercapitalized banks, borrowed from US banking regulation, and in particular the Prompt Corrective Action provisions of the Federal Deposit Insurance Corporation Improvement Act (FDICIA), induces them to choose an actual capital level well above regulatory capital for a wide range of parameter values. Therefore, such measures provide an explanation for why banks typically hold a buffer of capital above the minimum required by regulation.

The model proposed in the paper allows us to analyze the effect of market discipline, proxied by the coverage of deposit insurance, on economic and actual capital. We consider two alternative scenarios: one in which depositors are fully insured and where the deposit interest rate is equal to the risk free rate, and another one in which depositors are uninsured. In this scenario, depositors require an interest rate such that the expected return of their investment is equal to the risk free rate. The results suggest that measures aimed at increasing market discipline, such as those contemplated in Pillar 3 of Basel II, have a positive effect on economic capital, though its magnitude is generally small, except in very competitive markets for high risk loans. The impact of market discipline on actual capital is even lower and almost negligible.

It is important to emphasize some of the limitations of our analysis, such as the assumption that bank risk level is exogenous or the use of the single risk factor model. The inclusion of the bank's level of risk as an endogenous variable, together with capital, in the shareholders' maximization problem, as well as the analysis of more complex models of bank risk are left for future research.

The academic literature on this topic is very small, and in no case economic and regulatory capital are compared. From a theoretical perspective, the most interesting paper (which is the basis of our analysis of economic capital) is Suarez (1994), who constructs a dynamic model of bank behavior in which shareholders choose not only the capital level as in our model but also the asset risk.

Although analyzing risk-taking incentives rather than capital choice, Calem and Rob (1999) present a dynamic model similar to ours where the bank's charter value is an endogenous variable and shareholders take their decisions subject to minimum capital requirements. The model is calibrated with empirical data from the banking industry for 1984-1993, focusing on the impact of risk-based versus flat-rate capital requirements on banks' risk taking, which is shown to be ambiguous across banks depending on their capital levels.

Repullo (2004) also analyzes capital and risk taking decisions in a dynamic model of imperfect competition where the franchise value of the bank is an endogenous variable. Banks compete in the deposit market and the cases with flat-rate and risk-based minimum capital requirements are analyzed. He shows that capital requirements reduce the banks' incentives to take risk, and that risk-based requirements are more efficient regulatory tools. In contrast with our model, actual capital is always equal to the minimum required by regulation.

From an empirical perspective, Flannery and Rangan (2002) analyze the relationship between regulatory and actual bank capital between 1986 and 2000 for a sample of US banks. The authors conclude that the increase in regulatory capital during the first part of the 1990s could explain the increase in the capital levels of the banking industry during those years, but that the additional increase in capital in the second part of the 1990s is mainly driven by market discipline. These two empirical results support two predictions of our model: actual capital is an increasing function of regulatory capital and of the level of market discipline. However, our results suggest that the threat of closing critically undercapitalized banks could have played a more important role in boosting bank capital levels than market discipline.

This paper is organized as follows. Section 2 presents the model and characterizes the determinants of regulatory, economic and actual capital. Section 3 derives the numerical results, and Section 4 concludes. Appendix A discusses the comparative statics of economic capital, and Appendix B contains a proof of the negative relationship between bank capital and the interest rate on uninsured deposits.

## 2 The Model

Consider a bank that at the beginning of each period $t = 0, 1, 2, ...$ in which it is open has an asset size that is normalized to 1. The bank is funded with *deposits*, $1 - k_t$, that have an interest rate $c$, and *capital*, $k_t$, that requires an expected return $\delta$. We assume that the *deposit rate* $c$ is smaller than the *cost of capital* $\delta$. The bank is owned by risk-neutral shareholders who enjoy limited liability and, in the absence of minimum capital regulation, choose the capital level $k_t \in [0, 1]$. When $k_t = 0$ the bank is fully funded with deposits, while when $k_t = 1$ the bank is fully funded with equity capital. To simplify the presentation, we assume that there are no intermediation costs.

In each period $t$ in which the bank is open, its funds are invested in a portfolio of loans paying an exogenously fixed interest rate $r$. The return of this investment is stochastic: a random fraction $p_t \in [0, 1]$ of these loans default, in which case the bank loses the interest $r$ as well as a fraction $\lambda \in [0, 1]$ of the principal. Therefore, the bank gets $1 + r$ from the fraction $1 - p_t$ of the loans that do not default, and it recovers $1 - \lambda$ from the fraction $p_t$ of defaulted loans, so the value of its portfolio at the end of period $t$ is given by

$$a_t = (1 - p_t)(1 + r) + p_t(1 - \lambda). \tag{1}$$

Since the bank has to pay depositors an amount $(1 - k_t)(1 + c)$, its capital level at the end of period $t$ is

$$k_t' = a_t - (1 - k_t)(1 + c) = k_t + r - (1 - k_t)c - (\lambda + r)p_t. \tag{2}$$

There exists a supervisor that, at the end of each period $t$, verifies the bank's capital $k_t'$ and withdraws its license whenever $k_t' < 0$, that is whenever the losses during the period exceed the initial capital level $k_t$. In that case, the assumption of limited liability implies that bank shareholders do not make any payment to the depositors. By the definition (2) of $k_t'$ the bank will fail if

$$p_t > p(k_t) = \min\left\{\frac{k_t + r - (1 - k_t)c}{\lambda + r}, 1\right\}, \tag{3}$$

that is, if the *default rate* $p_t$ exceeds the critical level $p(k_t)$. Notice that $p(k_t)$ is increasing in the initial capital level $k_t$, with $p(k_t) \geq 1$ when

$$k_t \geq k_{\max} = \frac{\lambda + c}{1 + c}, \tag{4}$$

in which case the probability of bank failure is zero.

Let $I_{t+1} \in \{0,1\}$ be a random variable that indicates whether the bank is closed ($I_{t+1}=0$) or open ($I_{t+1}=1$) at the beginning of period $t+1$. The closure rule can be formalized as

$$I_{t+1} = I_t h(k_t'), \tag{5}$$

where

$$h(k_t') = \begin{cases} 0, & \text{if} \quad k_t' < 0, \\ 1, & \text{otherwise.} \end{cases} \tag{6}$$

Thus when $k_t' < 0$ the bank's license is permanently withdrawn by the supervisor.

We assume that the probability distribution of the default rate $p_t$ is the one derived from the *single risk factor* model of Vasicek (2002) that is used for the computation of the capital charges in the Internal Ratings-Based (IRB) approach of Basel II.[3] Its distribution function is given by

$$F(p_t) = N\left(\frac{\sqrt{1-\rho}\; N^{-1}(p_t) - N^{-1}(\overline{p})}{\sqrt{\rho}}\right), \tag{7}$$

where $N(\cdot)$ denotes the distribution function of a standard normal random variable, $\overline{p} \in [0,1]$ is the loans' (unconditional) probability of default, and $\rho \in [0,1]$ is their exposure to the systematic risk factor: when $\rho = 0$ defaults are statistically independent, so $p_t = \overline{p}$ with probability 1, while when $\rho = 1$ defaults are perfectly correlated, so $p_t = 0$ with probability $1-\overline{p}$, and $p_t = 1$ with probability $\overline{p}$. We also assume that $p_t$ is independent over time.

The distribution function $F(p_t)$ is increasing, with $F(0) = N(-\infty) = 0$ and $F(1) = N(\infty) = 1$. Moreover, it can be shown that

$$E(p_t) = \int_0^1 p_t \; dF(p_t) = \overline{p}$$

and

$$Var(p_t) = \int_0^1 (p_t - \overline{p})^2 \; dF(p_t) = N_2(N^{-1}(\overline{p}), N^{-1}(\overline{p}); \rho) - \overline{p}^2,$$

where $N_2(\cdot,\cdot;\rho)$ denotes the distribution function of a zero mean bivariate normal random variable with standard deviation equal to one and correlation coefficient $\rho$; see

[3]As shown by Gordy (2003), this is the only model for which the contribution of a given asset to value-at-risk (and hence the corresponding capital charge) is portfolio-invariant, that is, depends on the asset's own characteristics and not on those of the portfolio in which it is included.

Vasicek (2002, p.161). Therefore, the expected value of the default rate is precisely the probability of default $\overline{p}$, while its variance is increasing in the correlation parameter $\rho$, with $Var(p_t) = 0$ for $\rho = 0$, and $Var(p_t) = \overline{p}(1-\overline{p})$ for $\rho = 1$.

## 2.1 Regulatory capital

According to the IRB approach of Basel II bank capital must cover losses due to loan defaults with a given probability (or confidence level) $\alpha = 99,9\%$.[4] Specifically, let $\widehat{p}$ denote the $\alpha$-quantile of the distribution of the default rate $p_t$, that is the critical value such that

$$\Pr(p_t \leq \widehat{p}) = F(\widehat{p}) = \alpha.$$

Hence we have $\widehat{p} = F^{-1}(\alpha)$, so making use of (7) we get the capital requirement

$$\widehat{k} = \lambda \widehat{p} = \lambda N\left(\frac{N^{-1}(\overline{p}) + \sqrt{\rho}\, N^{-1}(\alpha)}{\sqrt{1-\rho}}\right). \tag{8}$$

This is the formula that appears in BCBS (2004, paragraph 272), except for the fact that we are assuming a one year maturity (which implies a maturity adjustment factor equal to 1) and that the correlation parameter $\rho$ is in Basel II a decreasing function of the default probability $\overline{p}$.

From this expression we can immediately identify the *determinants of regulatory capital* $\widehat{k}$, which are the loans' probability of default $\overline{p}$, loss given default $\lambda$, and exposure to systematic risk $\rho$, and the confidence level $\alpha$ set by the regulator.

To analyze the effects on the level of regulatory capital $\widehat{k}$ of changes in its determinants, we differentiate the function (8) which gives

$$\frac{\partial \widehat{k}}{\partial \overline{p}} > 0, \quad \frac{\partial \widehat{k}}{\partial \lambda} > 0 \quad \text{and} \quad \frac{\partial \widehat{k}}{\partial \alpha} > 0.$$

Moreover, we also get

$$\frac{\partial \widehat{k}}{\partial \rho} > 0 \quad \text{if and only if} \quad N^{-1}(\alpha) + \sqrt{\rho}\, N^{-1}(\overline{p}) > 0,$$

[4] In principle, regulatory capital should be derived from the maximization of a social welfare function that takes into account both the costs (e.g. the increase in the cost of credit) and the benefits (e.g. the reduction in the probability of bank failure) of capital regulation; see Repullo and Suarez (2004) for a discussion of this issue.

which for $\alpha = 99.9\%$ and $\rho \leq 0.24$ (the maximum value in Basel II for corporate, sovereign, and bank exposures) holds for all $\overline{p} \geq 0.03\%$ (the minimum value in Basel II). Therefore, we conclude that regulatory capital $\widehat{k}$ is an increasing function of its four determinants.

## 2.2 Economic capital

To derive the level of capital chosen by the bank shareholders in the absence of minimum capital regulation we solve a dynamic programming problem in which the state variable $I_t \in \{0,1\}$ indicates whether the bank is closed ($I_t = 0$) or open ($I_t = 1$) at the beginning of period $t$.

The value function of this problem, $V(I_t)$, is implicitly defined by the Bellman equation

$$V(I_t) = \max_{k_t \in [0,1]} I_t \left[ -k_t + \frac{1}{1+\delta} E\left[\max\{k_t', 0\} + V(I_{t+1})\right] \right]. \qquad (9)$$

According to this expression, $V(0) = 0$ is the value of a bank that is closed, and $V(1)$ is the *franchise value* of a bank that is open. This value, which henceforth will simply be denoted by $V$, results from maximizing with respect to capital $k_t$ an objective function that has three terms: the first one, with negative sign, is the capital contribution of shareholders at the beginning of period $t$; the second one is the discounted expected payoff at the end of period $t$, which under limited liability is equal to $\max\{k_t', 0\}$; and the third one is the discounted expected value of remaining open in period $t+1$, and therefore of having the possibility of receiving a stream of future payoffs. Notice that the discount rate used in the last two terms is the return required by bank shareholders or cost of capital $\delta$.

Therefore, assuming that $I_t = 1$, there are two possible scenarios at the end of period $t$: if $k_t' < 0$ the bank fails and the shareholders get a final payoff of zero; and if $k_t' \geq 0$ the bank remains open in period $t+1$ and the shareholders receive a dividend payment (or make a capital contribution, depending on the sign) of $k_t' - k_{t+1}$, that is the difference between the capital at the end of period $t$ and the capital that they would like to keep in the bank for period $t+1$.

As noted above, when $k_t \geq k_{\max}$ the probability of bank failure is zero, so

$$E\left[\max\{k_t', 0\} + V(I_{t+1})\right] = E(k_t') + V = k_t + r - (1-k_t)c - (\lambda + r)\overline{p} + V.$$

In this case the derivative with respect to $k_t$ of the expression in the right hand side of the Bellman equation (9) equals $(c-\delta)/(1+c)$, which is negative by the assumption that the deposit rate $c$ is smaller than the cost of capital $\delta$. Hence bank shareholders will never choose a capital level $k_t$ higher than $k_{\max}$.

This result is easy to explain. Bank shareholders might be willing to contribute capital, instead of funding the bank with cheaper deposits, as long as capital provides a buffer that reduces the probability of failure, and consequently increases the probability of receiving a stream of future dividends. However, if $k_t \geq k_{\max}$ capital covers the bank loses at the end of period $t$ even when 100% of the loans in its portfolio default, which means that any additional capital will only increase the bank's funding costs without reducing its probability of failure (which is zero). It follows then that bank shareholders will never want to provide $k_t > k_{\max}$, so we can limit the range of values for $k_t$ in the Bellman equation (9) to the interval $[0, k_{\max}]$.

Substituting the definition (2) of $k_t'$ into $E\left[\max\{k_t', 0\}\right]$, and integrating by parts taking into account the restriction $k_t \leq k_{\max}$ yields

$$E\left[\max\{k_t', 0\}\right] = (\lambda + r) \int_0^{p(k_t)} F(p_t)\ dp_t.$$

Moreover, by the definition (3) of $p(k_t)$ we also have

$$E\left[V(I_{t+1})\right] = \Pr(k_t' \geq 0)V = F(p(k_t))V.$$

Since the shareholders' maximization problem is identical in all periods, we can leave out the temporal subindex $t$ and rewrite the Bellman equation (9) as

$$V = \max_{k \in [0, k_{\max}]} G(k, V), \tag{10}$$

where

$$G(k, V) = -k + \frac{1}{1+\delta}\left[(\lambda + r)\int_0^{p(k)} F(p)\ dp + F(p(k))V\right]. \tag{11}$$

The solution of this equation gives the level of economic capital $k^*$ that bank shareholders would like to hold in the absence of minimum capital regulation, as well as the bank's franchise value $V$. In addition, this equation allows us to identify the *determinants of economic capital* $k^*$, which are the loans' probability of default $\overline{p}$, loss given default $\lambda$, exposure to systematic risk $\rho$, loan rate $r$, deposit rate $c$, and the cost of bank capital $\delta$. Notice that these last three variables do not affect regulatory capital $\widehat{k}$, while the confidence level $\alpha$ set by the regulator does not affect economic capital $k^*$.

Appendix A shows that economic capital can be at the corner $k^* = 0$, and that if there is an interior solution comparative static results cannot be derived analytically, except for the cost of capital $\delta$, for which we obtain

$$\frac{\partial k^*}{\partial \delta} < 0.$$

Thus the higher the bank's equity funding costs the lower the capital provided by its shareholders.

To conclude, it is important to highlight the different determinants of economic and regulatory capital. Both of them depend on the loans' probability of default $\overline{p}$, loss given default $\lambda$, and exposure to systematic risk $\rho$. However, while an increase in any of these variables increases regulatory capital, its effect on economic capital is, in general, ambiguous. Moreover, economic capital depends on the loan rate $r$, the deposit rate $c$, and the cost of bank capital $\delta$, whereas regulatory capital depends on the confidence level $\alpha$ set by the regulator.

## 2.3 Actual capital

We next derive the capital level chosen by the bank shareholders when their choice is restricted by two regulatory constraints. First, we assume that the regulator audits the bank at the beginning of each period, and requires the bank to hold at least the regulatory capital $\widehat{k}$ in order to operate. Second, in line with US regulation, and in particular the Prompt Corrective Action provisions of the Federal Deposit Insurance Corporation Improvement Act (FDICIA), we assume that banks whose capital at the end of a period falls below a certain critical level $\widehat{k}_{\min}$ are closed by the supervisor.[5] Such capital level will be called *actual capital* and denoted by $k^a$.

In this setup the Bellman equation that characterizes the solution to the share-

[5] According to FDICIA, banks whose tangible equity ratio falls below 2% are considered *critically undercapitalized* and, among other things, placed in receivership or conservationship; see Comptroller of the Currency (1993). Tangible equity ratio is defined as Tier 1 capital plus cumulative preferred stock and related surplus less intangibles except qualifying purchased mortgage servicing rights (PMSR) divided by total assets less intangibles except qualifying PMSR.

holders' maximization problem is

$$V = \max\left\{ \max_{k_t \in [\widehat{k},1]} \left[ -k_t + \frac{1}{1+\delta} E\left[ \max\{k_t',0\} + \Pr(k_t' \geq \widehat{k}_{\min})V \right]\right], 0 \right\}. \tag{12}$$

There are two differences between this equation and the one for economic capital. First, the bank is not allowed to operate when $k_t < \widehat{k}$, so the choice of $k_t$ is restricted to the interval $[\widehat{k}, 1]$. But with this constraint the shareholders may find it optimal not to operate the bank, in which case $V = \max\{\cdot, 0\} = 0$. Second, equation (12) takes into account that the bank is closed by the supervisor when $k_t' < \widehat{k}_{\min}$, so the discounted value of remaining open in period $t+1$ is multiplied by $\Pr(k_t' \geq \widehat{k}_{\min})$.

Substituting the definition (2) of $k_t'$ into $E\left[\max\{k_t', 0\}\right]$, integrating by parts, and using the definition (3) of $p(k_t)$ and the definition (4) of $k_{\max}$ yields

$$\begin{aligned} E\left[\max\{k_t',0\}\right] &= \left[(k_t + r - (1-k_t)c - (\lambda + r)p_t)F(p_t)\right]_0^{p(k_t)} + (\lambda + r)\int_0^{p(k_t)} F(p_t)\ dp_t \\ &= \max\{(1+c)(k_t - k_{\max}), 0\} + (\lambda + r)\int_0^{p(k_t)} F(p_t)\ dp_t. \end{aligned}$$

Moreover, we also have

$$\Pr(k_t' \geq \widehat{k}_{\min})V = F(\widehat{p}(k_t))V$$

where

$$\widehat{p}(k_t) = \min\left\{ \frac{k_t + r - (1-k_t)c - \widehat{k}_{\min}}{\lambda + r}, 1 \right\}.$$

Therefore, leaving out the temporal subindex $t$, the Bellman equation (12) can be rewritten as

$$V = \max\left\{ \max_{k_t \in [\widehat{k},1]} \widehat{G}(k, V), 0 \right\}, \tag{13}$$

where

$$\widehat{G}(k,V) = -k + \frac{1}{1+\delta}\left[ \max\{(1+c)(k_t - k_{\max}), 0\} + (\lambda + r)\int_0^{p(k)} F(p)\ dp + F(\widehat{p}(k))V \right].$$

The solution of this equation gives the actual level of capital $k^a$ that bank shareholders would like to hold given the assumed regulation, as well as the bank's franchise value $V$. This equation also identifies the *determinants of actual capital* $k^a$, which are the same six variables that determine economic capital, plus the minimum capital requirement $\widehat{k}$ and the critical level $\widehat{k}_{\min}$.

As in the case of economic capital, actual capital can be at the corner $k^a = \widehat{k}$, in which case none of the other variables matters for actual capital. And if there is an interior solution, comparative static results cannot be derived analytically, except for the cost of capital $\delta$ and the minimum capital requirement $\widehat{k}$, for which we obtain

$$\frac{\partial k^a}{\partial \delta} < 0 \quad \text{and} \quad \frac{\partial k^a}{\partial \widehat{k}} = 0.$$

Thus, when the shareholders choose an interior solution for actual capital, an increase in the bank's equity funding costs reduces the level of capital, while an increase in the minimum capital requirement does not have any effect on their choice.

An important difference between economic and actual capital is that in choosing the former bank shareholders have the option of providing no capital, which implies that the bank will always have a positive franchise value, while in choosing the latter they have to provide at least the minimum capital required by regulation, which in some cases may lead them to prefer not to operate the bank ($V = 0$). Whenever shareholders choose to operate the bank ($V > 0$), actual capital will, by construction, be greater than or equal to regulatory capital. In contrast, economic capital may be below regulatory capital. Finally, the bank's franchise value $V$ will always be higher for economic than for actual capital, because the constraints imposed by the regulator reduce the value of the bank.

Since comparative static results cannot in general be derived analytically, in the following section we resort to numerical solutions to discuss the relationship between regulatory, economic, and actual capital for plausible values of the parameters that determine them.

# 3 Numerical results

This section compares the values of regulatory $\widehat{k}$, economic $k^*$, and actual capital $k^a$ obtained by, respectively, computing the IRB formula (8), and solving the Bellman equations (10) and (13), for different values of the parameters of the model.[6]

[6] The Bellman equations (10) and (13) are solved by an iterative procedure. For example, in the case of (10), given an initial franchise value $V_0$ we compute $V_1 = \max_k G(k, V_0)$, and iterate the process until convergence to a value $V$. Economic capital is then given by $k^* = \arg\max_k G(k, V)$.

For the benchmark case, we assume a probability of default $\overline{p}$ of 2%, and a loss given default $\lambda$ of 45% (the value specified in the IRB foundation approach for senior claims on corporates, sovereigns and banks not secured by recognized collateral). For computing regulatory capital, we use the confidence level $\alpha = 99,9\%$ set in Basel II.

The exposure to systematic risk parameter $\rho$ will be assumed to be a decreasing function of the probability of default $\overline{p}$, according to functional form specified in Basel II for corporate, sovereign and bank exposures, which is

$$\rho(\overline{p}) = 0.24 - 0.12\,\frac{1 - e^{-50\overline{p}}}{1 - e^{-50}}.$$

Thus the maximum value of the exposure to systematic risk is $\rho(0) = 0.24$, the minimum value is $\rho(1) = 0.12$, and for the benchmark probability of default we have $\rho(0.02) = 0.16$. The effect of this assumption is to flatten (relative to the case with a constant $\rho$) the function that relates regulatory capital $\widehat{k}$ to the probability of default $\overline{p}$. However, it is important to stress that our conclusions do not vary qualitatively when $\rho$ is constant.

With regard to the loan rate $r$, instead of taking it as exogenous, we assume that it is determined according to the equation

$$(1 - \overline{p})r - \overline{p}\lambda = \mu, \tag{14}$$

that equals the expected return of a loan, $(1 - \overline{p})r - \overline{p}\lambda$, to a margin $\mu$ over the risk free rate, which is normalized to zero.[7] Rearranging (14) we obtain

$$r = \frac{\mu + \overline{p}\lambda}{(1 - \overline{p})},$$

so the loan rate $r$ is an increasing function of the probability of default $\overline{p}$, the loss given default $\lambda$, and the intemediation margin $\mu$. In the benchmark case we take a value for $\mu$ of 1%.

For the deposit rate $c$, we assume that the return required by depositors is equal to the risk free rate, which has been normalized to zero, and we consider two alternative scenarios. In the first one depositors are fully insured by a deposit insurance agency, and therefore (ignoring the deposit insurance premium) the deposit rate $c$ is equal to

[7] Equation (14) is an approximation to the loan rate equation derived by Repullo and Suarez (2004) for a perfect competition model.

the risk free rate, i.e. $c = 0$. In the second one depositors are uninsured, so under the assumption of risk neutrality the deposit rate $c$ has to verify the participation constraint

$$E\left[\min\left\{a, (1-k)(1+c)\right\}\right] = 1-k. \tag{15}$$

To understand this equation notice that when the value of the bank's assets $a$ is greater than or equal to the deposits' principal and interest, that is when $k' = a - (1-k)(1+c) \geq 0$, depositors receive $(1-k)(1+c)$, whereas when $k' < 0$ the bank is closed by the supervisor and depositors receive the liquidation value of the bank, which (ignoring bankruptcy costs) is equal to $a$. Thus the left hand side of (15) is the expected value of the depositors' claim at the end of each period, while the right hand side is the gross return that they require on their investment.[8]

The last parameter that has to be specified is the return $\delta$ required by bank shareholders, which in the benchmark case will be set equal to 6%.[9] Since we have normalized the risk free rate to zero, this value should be interpreted as a spread over the risk free rate.

Table 1 summarizes the parameter values in the benchmark case, as well as the range of values for which regulatory, economic, and actual capital will be computed (keeping the rest of the parameters at their benchmark levels).

| Parameter | Benchmark case | Range of values |
|---|---|---|
| Probability of default $\bar{p}$ | 2% | $0-20\%$ |
| Intermediation margin $\mu$ | 1% | $0-5\%$ |
| Cost of bank capital $\delta$ | 6% | $0-10\%$ |
| Loss given default $\lambda$ | 45% | $0-100\%$ |

Table 1. Parameter values used in the numerical exercise.

Our model only considers deposits and equity capital as the sources of bank funding, but one should bear in mind that in reality the are many instruments in between.

[8] Although this argument is based on our model of economic capital, the participation constraint (15) also applies to our model of actual capital, because when the bank is closed for $0 \leq k' < \widehat{k}_{\min}$ the uninsured depositors get $(1-k)(1+c)$.

[9] We take it from the evidence on the equity premium; see, for example, Kocherlakota (1996).

For regulatory purposes, Basel II distinguishes between Tier 1 and Tier 2 capital.[10] Tier 1 comprises equity capital and reserves from retained earnings, while Tier 2 represents "supplementary capital" such as undisclosed reserves, revaluation reserves, general loan-loss reserves, hybrid (debt/equity) capital instruments, and subordinated debt. Ignoring the special treatment of loan-loss provisions, Basel II involves two constraints: Tier 1 plus Tier 2 capital should be greater than the minimum capital requirement, and Tier 1 capital should be greater than 50% of the minimum requirement. Obviously, the latter would be the only relevant constraint for a bank with plenty of Tier 2 capital. Given that we analyze the case of uninsured deposits (which may be similar to subordinated debt), and that as we will see below deposit insurance is not an important determinant of the bank's capital choice, in what follows we will restrict attention to the Tier 1 capital requirement $\widehat{k}_1 = \widehat{k}/2$, where $\widehat{k}$ is computed from the IRB formula (8).

Finally, to compute actual capital we follow FDICIA and set the threshold for critically undercapitalized banks at $\widehat{k}_{\min} = 2\%$.

When discussing the effects of the different parameters on economic $k^*$ and actual $k^a$ capital we will distinguish between the cases of insured and uninsured deposits. To understand the differences between these two cases it is important to analyze the effect of bank capital on the interest rate required by uninsured depositors derived from the participation constraint (15). Appendix B shows that this equation has a unique solution $c(k) \geq 0$ for all $k$ and that $c'(k) < 0$, except for $k \geq \lambda$ in which case $c'(k) = c(k) = 0$.

Figure 1 represents the cost of uninsured deposits $c$ as a function of the capital level $k$, that is the function $c(k)$, for the benchmark case parameters, $\overline{p} = 2\%$ and $\mu = 1\%$, as well as the effects of an increase in $\overline{p}$ and in $\mu$. The negative effect of $k$ on the uninsured deposit rate $c$ is significant for small values of $k$, for which the probability of bank failure is relatively high. An increase in the intermediation margin $\mu$ from 1% to 2% reduces this probability and consequently the deposit rate $c$, whereas an increase in the probability of default $\overline{p}$ from 2% to 5% has the opposite effect.

The left panel of Figure 2 plots regulatory capital $\widehat{k}_1$ and economic capital with insured and uninsured deposits, $k_i^*$ and $k_u^*$, as functions of the loan's probability of

[10] The definition of eligible regulatory capital has not changed from Basel I; see BCBS (1988, paragraph 14) and BCBS (2004, paragraphs 40 and 41).

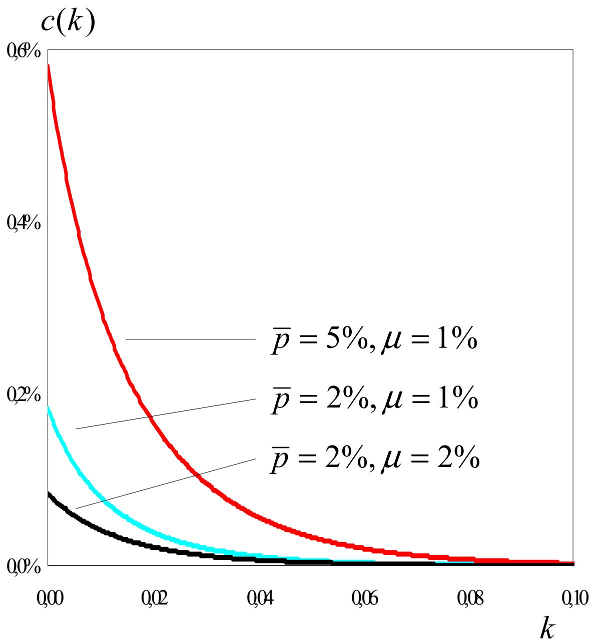


Figure 1: Effect of bank capital on the uninsured deposits' interest rate.

default $\overline{p}$, and the right panel plots regulatory capital $\widehat{k}_1$ and actual capital with insured and uninsured deposits, $k_i^a$ and $k_u^a$, as functions of $\overline{p}$.

As discussed in Section 2, an increase in the probability of default $\overline{p}$ increases regulatory capital, but has an ambiguous effect on economic capital. In particular, the Figure shows that economic capital with insured deposits $k_i^*$ is increasing in the probability of default for values of $\overline{p}$ smaller than 12%, it is decreasing for values of $\overline{p}$ between 12% and 18%, and it jumps to the corner solution $k_i^* = 0$ for higher values of $\overline{p}$. Economic capital with uninsured deposits $k_u^*$ is also first increasing and then decreasing in the probability of default $\overline{p}$, although for much higher levels of $\overline{p}$.

The reason why the relationship between the probability of default $\overline{p}$ and economic capital is nonmonotonic is that, for high values of $\overline{p}$, the bank probability of failure is so high that bank shareholders prefer to reduce (even to zero) their capital contribution. This result holds for relative higher levels of the probability of default $\overline{p}$ when deposits are uninsured because, as illustrated in Figure 1, the uninsured deposits' interest rate is a decreasing function of the level of capital, which implies that shareholders have an additional incentive to provide capital. It should be noted, however, that these

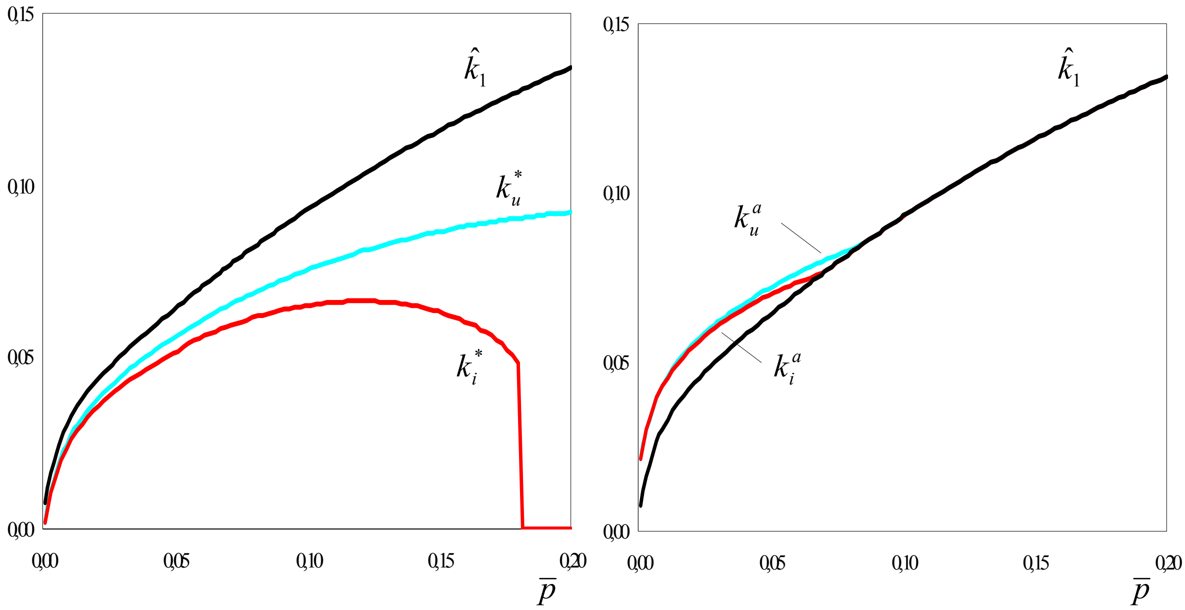


Figure 2: Effect of the probability of default on regulatory, economic, and actual capital.

somewhat surprising results only hold for implausibly high values of the probability of default $\overline{p}$, so their practical relevance may be limited.

Economic capital with insured deposits $k_i^*$ is always below economic capital with uninsured deposits $k_u^*$, because in the latter case shareholders have an additional incentive to provide capital in order to reduce the cost of uninsured deposits. Hence we conclude that the market discipline introduced by the need to provide the required expected return to the uninsured depositors implies higher bank capital. Figure 2 shows that this effect is more important when the loans' probability of default $\overline{p}$ is higher because of the higher impact of the capital level $k$ on the uninsured deposits' interest rate $c$ noted above. As we shall see later, the difference between economic capital with insured and uninsured deposits, $k_i^*$ and $k_u^*$, is also decreasing in the intermediation margin $\mu$, while it is increasing in the cost of bank capital $\delta$ and in the loans' loss given default $\lambda$.

With respect to actual capital, the right panel of Figure 2 shows that actual capital with insured and uninsured deposits, $k_i^a$ and $k_u^a$, are strictly greater than

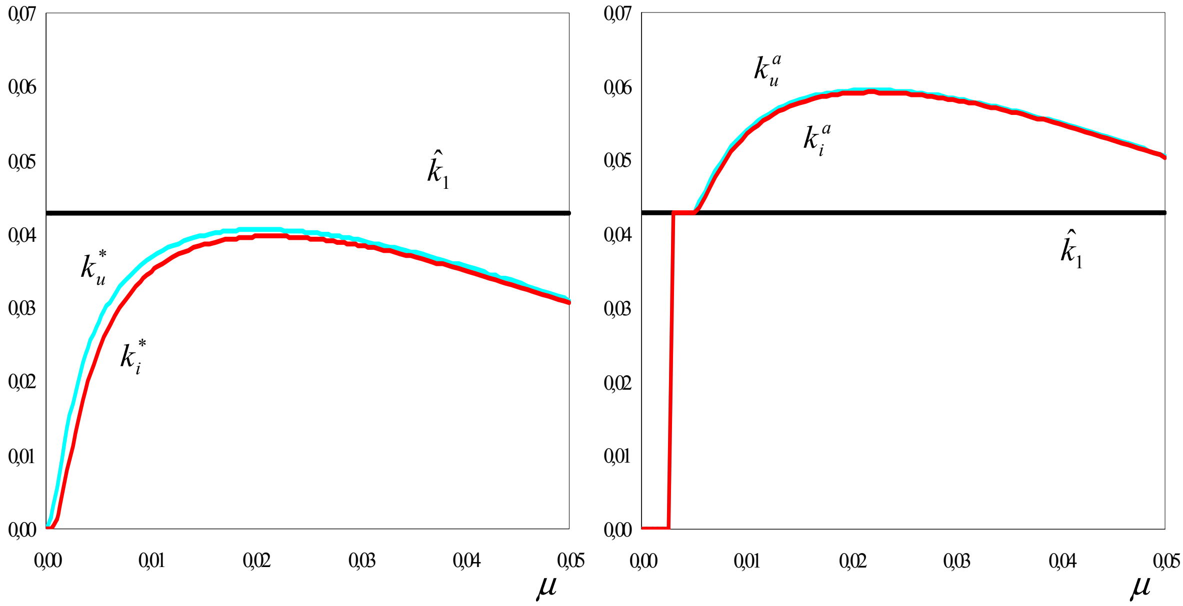


Figure 3: Effect of the intermediation margin on regulatory, economic, and actual capital.

regulatory capital $\widehat{k}_1$ for default probabilities $\overline{p}$ lower than 7.7 and 8.5%, respectively. It also shows that, for those cases where an interior solution exists, actual capital with insured deposits $k_i^a$ is always below actual capital with uninsured deposits $k_u^a$. For higher default probabilities those capital levels are equal to $\widehat{k}_1$, except when $\overline{p}$ is greater than 32%, in which case shareholders do not open the bank when the deposits are not insured. Finally, the gap between actual capital with uninsured and insured deposits, $k_u^a - k_i^a$, is (whenever the bank operates) smaller than that gap for economic capital, $k_u^* - k_i^*$, because since actual capital is greater than economic capital shareholders have less incentives to provide capital in order to reduce the cost of uninsured deposits. As we shall see in the Figures below, this is a general result.

The left panel of Figure 3 shows regulatory capital $\widehat{k}_1$ and economic capital with insured and uninsured deposits, $k_i^*$ and $k_u^*$, as functions of the intermediation margin $\mu$, and the right panel shows regulatory capital $\widehat{k}_1$ and actual capital with insured and uninsured deposits, $k_i^a$ and $k_u^a$, as functions of $\mu$.

Starting with economic capital, an increase in the intermediation margin $\mu$ increases both $k_i^*$ and $k_u^*$, and reduces the distance between them. The first result stems from the fact that a higher margin $\mu$ implies a higher franchise value $V$, so shareholders have greater incentives to provide capital in order to preserve it. The second result holds because a higher margin $\mu$ increases bank solvency and therefore reduces the interest rate on uninsured deposits, bringing it closer to the interest rate on insured deposits.

It is important to note that the intermediation margin $\mu$ has two opposite effects on economic capital. On the one hand, as we have already mentioned, a higher margin $\mu$ increases the bank's franchise value $V$, and therefore shareholders' incentives to provide capital. On the other hand, by assumption (14), a higher margin $\mu$ increases the loan rate $r$, which increases value $a$ of the bank's portfolio and reduces the need to hold capital in order to protect the franchise value $V$. From this perspective, economic capital $k^*$ and the intermediation margin $\mu$ are substitutes, which may account for a possible negative relationship between them.

The left panel of Figure 3 shows that, for values of the intermediation margin $\mu$ below 2.1%, increases in the margin increase both levels of economic capital, bringing them closer to regulatory capital (which does not vary with $\mu$), but the relationship becomes negative for higher values of the margin $\mu$. Thus, for relatively competitive banking markets, the positive effect of the intermediation margin $\mu$ on economic capital $k^*$, via an increase in the bank's franchise value $V$, outweighs its negative effect, via the substitution between economic capital $k^*$ and the margin $\mu$, while for oligopolistic markets the negative effect dominates.

With respect to actual capital, the right panel of Figure 3 shows that when the intermediation margin is below 0.25%, the shareholders prefer to close the bank rather than provide the minimum capital $\widehat{k}_1$. Beyond this point, and for those values of the margin for which the restriction $k^a \geq \widehat{k}_1$ is not binding, actual capital has a similar shape than economic capital, although the maximum actual capital is achieved for a higher intermediation margin $\mu$ than in the economic capital case. Again, whenever the bank operates, actual capital is higher than economic capital, which implies that the increase in actual capital for uninsured deposits is lower than for economic capital and almost negligible.

In all cases analyzed so far, we have found economic capital below regulatory

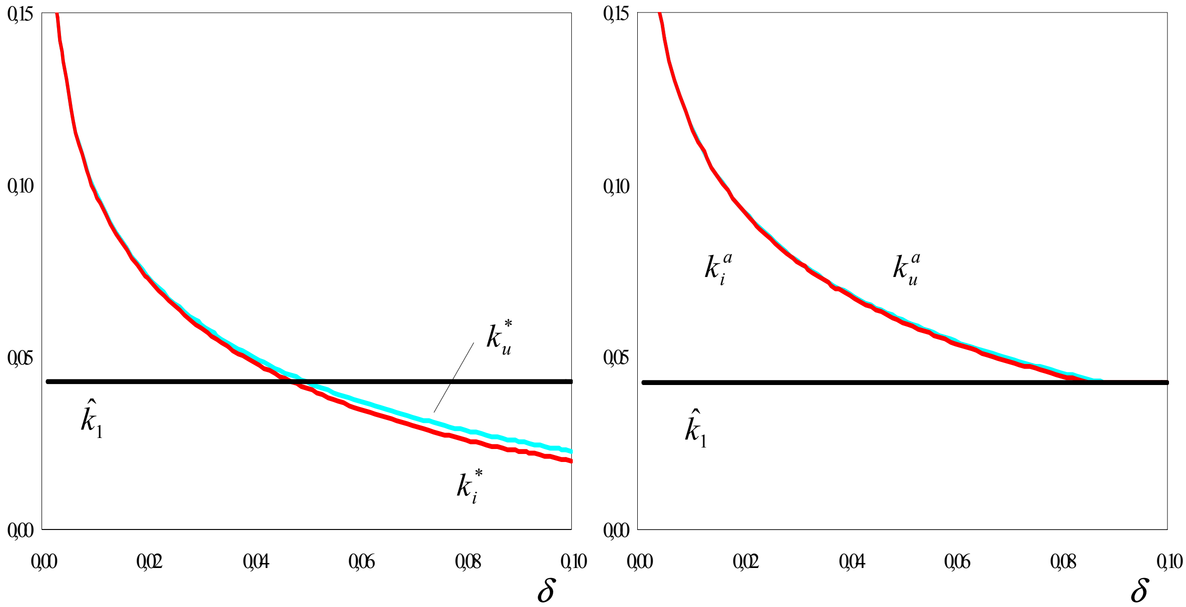


Figure 4: Effect of the cost of bank capital on regulatory, economic, and actual capital.

capital. This is mainly due to our benchmark parameter value for the cost of bank capital $\delta$. The left panel of Figure 4 plots regulatory capital $\widehat{k}_1$ and economic capital with insured and uninsured deposits, $k_i^*$ and $k_u^*$, as functions of the cost of capital $\delta$, and the right panel plots regulatory capital $\widehat{k}_1$ and actual capital with insured and uninsured deposits, $k_i^a$ and $k_u^a$, as functions of $\delta$. As shown in Appendix A, economic capital is a decreasing function of the cost of capital ($\partial k^*/\partial\delta < 0$). Moreover, for values of the cost of capital $\delta$ below 5% (approximately), both levels of economic capital, with and without insured deposits, are above regulatory capital. The reason is obvious: the lower the cost of capital $\delta$, the higher the incentives of bank shareholders to contribute capital. In fact, for values of $\delta$ sufficiently close to zero, shareholders choose capital levels that effectively guarantee the bank's survival regardless of the fraction of the loans in its portfolio that default.

The relative position of actual capital with respect to economic and regulatory capital follows the same pattern than in previous cases. Actual capital is higher than regulatory capital for values of $\delta$ below than 8.4 and 8.8%, respectively for the insured and uninsured deposits cases, and from those levels onwards they are equal to $\widehat{k}_1$. The

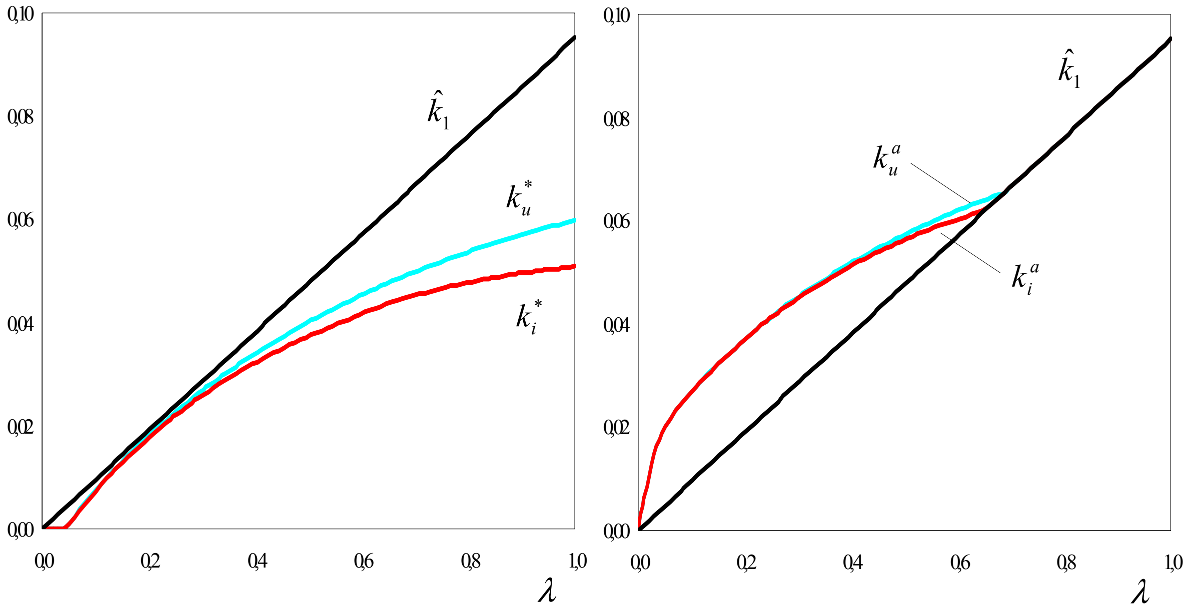


Figure 5: Effect of the loss given default on regulatory, economic, and actual capital.

shareholders do not operate the bank for unreasonably high values for $\delta$ (23.5%).

Finally, the left panel of Figure 5 shows regulatory capital $\widehat{k}_1$ and economic capital with insured and uninsured deposits, $k_i^*$ and $k_u^*$, as functions of the bank loans' loss given default $\lambda$, and the right panel shows regulatory capital $\widehat{k}_1$ and actual capital with insured and uninsured deposits, $k_i^a$ and $k_u^a$, as functions of $\lambda$. According to the IRB formula (8), regulatory capital $\widehat{k}_1$ is a linear function of the loss given default $\lambda$. On the other hand, while the effect of $\lambda$ on economic capital is positive in Figure 5, as noted in Section 2 this is not true in general. For example, if the probability of default $\overline{p}$ equals 7%, $k_i^*$ and $k_u^*$ start decreasing for values of $\lambda$ of 57% and 94%, respectively.

To sum up, we have found that both regulatory and economic capital depend positively on the loans' probability of default and loss given default risk for reasonable values of these variables. However, variables that only affect economic capital, such as the intermediation margin and the cost of capital, may significantly move it away from regulatory capital. Actual capital, which by definition is higher than regulatory capital, always lies above economic capital. We have also found that market discipline, proxied by the coverage of deposit insurance, has a positive impact on economic capital, but the effect is in general small and very sensitive to the values of the rest of the determinants of economic capital. Since actual capital is higher than economic capital, it is even less affected by market discipline.

# 4 Conclusion

Basel II establishes a new way of computing minimum capital requirements that is intended to "contribute to reducing the gap between regulatory and economic capital." (European Central Bank, 2001). Our analysis of the determinants of economic and regulatory capital for a bank whose loan default rates are described by the single risk factor model that underlies the capital charges in the IRB approach shows that there does not exist a direct relationship between both capital levels.

First, economic and regulatory capital do not depend on the same variables: regulatory capital (but not economic capital) depends on the confidence level set by the regulator, while economic (but not regulatory capital) depend on the intermediation margin and the cost of bank capital. These last two variables play a key role in determining the differences between them. Economic capital is only higher than regulatory capital for low values of the cost of capital, and when this cost increases, the former quickly falls below the latter. The effect of the intermediation margin on economic capital is only positive in fairly competitive credit markets, which is explained by the existence of two opposite effects: on the one hand, a higher margin increases the bank's franchise value, and hence shareholders' incentives to contribute capital in order to preserve it, but on the other hand a higher margin provides a source of income that reduces the need to hold capital as a buffer against losses. The first (positive) effect outweights the second (negative) in sufficiently competitive credit markets. Therefore, changes in the market power of banks, due for example to public policies, may have very different effects on economic capital depending on the initial level of competition.

Second, variables that affect both economic and regulatory capital, such as the loans' probability of default and loss given default, have a positive impact on both capital levels for reasonable values of these variables. However, when they reach certain critical values their effect on economic capital becomes negative, increasing the gap with regulatory capital.

However, the debate around whether Basel II closes the gap between regulatory and economic capital is somewhat artificial, because in practice banks choose their capital structure considering the regulations in place, i.e. they choose actual, rather than economic, capital. We define actual capital as the equity capital chosen by the bank shareholders when their choice is restricted by two regulations: an initial capital higher than the minimum required by regulation, and a closure rule for critically undercapitalized banks.

The first regulation alone makes actual capital equal to the maximum of economic and regulatory capital, which according to our results coincides almost always with the latter (except for small values of the cost of capital). Therefore, whenever actual capital is higher than regulatory capital this is due to the second regulation. Our results indicate that the threat of closing critically undercapitalized banks (banks with Tier 1 capital below 2%) significantly increases actual bank capital for reasonable ranges of parameter values. This regulation was introduced in the US by the Federal Deposit Insurance Corporation Improvement Act (FDICIA), but it is not explicitly contemplated in Basel II. However, under Pillar 2 (supervisory review process) of the New Accord national supervisors have discretion to introduce it. According to our results, this seems an effective way to induce banks to hold a buffer above regulatory capital.

The comparison of economic capital with insured and uninsured deposits reveals that, even though the latter is never below the former, their differences are, in general, small and very sensitive to the values of the rest of the determinants of economic capital. In the case of actual capital those differences are even smaller and almost negligible. Therefore, the effects of policies aimed at increasing market discipline, such as those contemplated in Pillar 3 of Basel II may be very limited.

To conclude we want to stress that there are other reasons why regulatory capital need not approach economic capital, in particular those that relate to the different objectives of shareholders and regulators. As noted by Crockett (1998), "It is entirely natural that the level of capital required by regulators to protect systemic stability should be higher than banks might choose voluntarily. There are many reasons for this, not least the differences between the private and social costs of failure, and the distortion in incentives associated with safety net arrangements. Even if they start from the same basis of risk assessment, regulators would, in general, want a higher capital cushion than the industry."

# Appendix

## A Comparative statics of economic capital

This Appendix discusses the effects on economic capital $k^*$ of changes in its determinants, namely the loans' probability of default $\overline{p}$, loss given default $\lambda$, and exposure to systematic risk $\rho$, the loan rate $r$, the deposit rate $c$, and the cost of bank capital $\delta$. It is shown that only for the last variable one can derive analytically the sign of the effect on economic capital.

To this end we first analyze the properties of the function $G(k, V)$ in the Bellman equation (11). Its derivatives with respect to $k$ are given by

$$\frac{\partial G}{\partial k} = -1 + \frac{1+c}{1+\delta}\left[F(p(k)) + \frac{f(p(k))V}{\lambda + r}\right], \tag{16}$$

$$\frac{\partial^2 G}{\partial k^2} = \frac{(1+c)^2}{(1+\delta)(\lambda + r)}\left[f(p(k)) + \frac{f'(p(k))V}{\lambda + r}\right], \tag{17}$$

where $f(p) = F'(p)$ is the density function of the default rate and $f'(p)$ is its derivative. Whilst the first term of (17) is nonnegative (since $f(p(k))$ is a density), the second term can either be positive (if $f'(p(k)) > 0$) or negative (if $f'(p(k)) < 0$). Thus $G(k, V)$ is not, in general, a convex or a concave function of $k$, which implies that we may have both corner and interior solutions. However, since $F(p(k_{\max})) = F(1) = 1$ and $f(p(k_{\max})) = f(1) = 0$, our assumption $\delta > c$ implies that the derivative of $G(k, V)$ with respect to $k$ evaluated at $k_{\max}$ is always negative, so a corner solution with $k = k_{\max}$ can be ruled out. Therefore, the only possible corner solution is $k^* = 0$.

If an interior solution exists, it would be characterized by the first-order condition $\partial G/\partial k = 0$ and the second-order condition $\partial^2 G/\partial k^2 < 0$. Differentiating the first-order condition and taking into account the definition (10) of the franchise value $V$ gives

$$\frac{\partial k^*}{\partial x} = -\left(\frac{\partial^2 G}{\partial k^2}\right)^{-1}\left(\frac{\partial^2 G}{\partial k \partial x} + \frac{\partial^2 G}{\partial k \partial V}\frac{\partial V}{\partial x}\right),$$

where $x$ is any of the six variables that determine economic capital $k^*$. Since

$$\frac{\partial^2 G}{\partial k \partial V} = \frac{(1+c)f(p(k))}{(1+\delta)(\lambda + r)} > 0$$

and by the second order-condition we have $\partial^2 G/\partial k^2 < 0$, we need to find the signs of $\partial^2 G/\partial k \partial x$ and $\partial V/\partial \delta$. For $x = \delta$ it is easy to check that

$$\frac{\partial^2 G}{\partial k \partial \delta} = -\frac{1+c}{(1+\delta)^2}\left[F(p(k)) + \frac{f(p(k))V}{\lambda + r}\right] < 0,$$

and by the envelope theorem we have

$$\frac{\partial V}{\partial \delta} = -\frac{1+c}{(1+\delta)^2}\left[1 - \frac{1+c}{1+\delta}F(p(k))\right]^{-1}\left[(\lambda + r)\int_0^{p(k)} F(p)\ dp + F(p(k))V\right] < 0,$$

which implies $\partial k^*/\partial \delta < 0$. However for $x = \overline{p}$, $\lambda$, and $\rho$ the sign of $\partial^2 G/\partial k \partial x$ is ambiguous, for $x = r$ we have $\partial^2 G/\partial k \partial r < 0$ and $\partial V/\partial r > 0$, and for $x = c$ we have $\partial^2 G/\partial k \partial c > 0$ and $\partial V/\partial c < 0$.

# B Uninsured deposits' interest rate

The uninsured deposits' interest rate $c$ is obtained by solving the participation constraint (15) that equates the expected value of the depositors' claim at the end of each period, $E\left[\min\left\{a, (1-k)(1+c)\right\}\right]$, to the gross return that they require on their investment, $1-k$. This Appendix shows that the equation

$$U(c,k) = E\left[\min\left\{a, (1-k)(1+c)\right\}\right] - (1-k) = 0 \tag{18}$$

has a unique solution $c(k) \geq 0$ for all $k$ and that $c'(k) < 0$, except for $k \geq \lambda$ in which case $c'(k) = c(k) = 0$.

For $k \geq \lambda$ it is immediate to check that $U(c,k) \geq U(0,k) = 0$, with strict inequality for $c > 0$, so $c = 0$ is the unique solution.

For $k < \lambda$, given that $0 \leq p(k) < 1$ for all $0 < c \leq (k+r)/(1-k)$, substituting the definition (1) of $a$ into (18), integrating by parts, and making use of the definition (3) of $p(k)$ gives

$$U(c,k) = k - \lambda + (\lambda + r)\int_{p(k)}^{1} F(p)\ dp. \tag{19}$$

To prove that (18) has a unique solution $c(k) > 0$ it suffices to show that $U(0,k) < 0 < \max_c U(c,k)$, and that $\partial U/\partial c > 0$. First, since $F(p(k)) < 1$, using (19) and the definition (3) of $p(k)$ implies

$$U(0,k) < k - \lambda + (\lambda + r)(1 - p(k)) = 0.$$

Second, using the fact $\int_0^1 F(p)\, dp = 1 - \overline{p}$ together with the fact that $(1-\overline{p})r - \overline{p}\lambda = \mu > 0$, (19) implies

$$\max_c U(c,k) = k - \lambda + (\lambda + r)\int_0^1 F(p)\, dp = k + (1-\overline{p})r - \overline{p}\lambda = k + \mu > 0.$$

And third, differentiating (19) with respect to $c$ gives

$$\frac{\partial U}{\partial c} = (1-k)\, F(p(k)) > 0. \tag{20}$$

Finally, totally differentiating $U(c,k) = 0$ and using (20) we have that $c'(k) < 0$ if

$$\frac{\partial U}{\partial k} = 1 - (1+c)\, F(p(k)) > 0.$$

But for $c = c(k)$ we have

$$U(c,k) = (1-k)(1+c)F(p(k)) + \int_{p(k)}^1 a\; dF(p) - (1-k) = 0,$$

which implies

$$1 - (1+c)\, F(p(k)) = (1-k)^{-1}\int_{p(k)}^1 a\; dF(p) > 0.$$

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