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March 2002

Can a Baumol-Tobin model account for the short-run behavior of velocity?

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ABSTRACT

We exposit the link between money, velocity and prices in an inventory-theoretic model of the demand for money and explore the extent to which such a model can account for the short-run volatility of velocity, the negative correlation of velocity and the ratio of money to consumption, and the resulting "stickyness" of the aggregate price level as measured by the relative volatility of the ratio of money to consumption and the price level. We find that an inventory-theoretic model of the demand for money is a natural framework for understanding these aspects of the behavior of velocity in the short run.

The inventory-theoretic models of Baumol (1952) and Tobin (1956) provide a foundation for the transactions demand for money. In constructing his model, Tobin starts with the question of why people hold substantial amounts of money, a low yielding asset, rather than hold their transactions balances in assets with higher yields than money, shifting into money only at the time that they wish to purchase something. Tobin and Baumol base their models on the premise that people do not do so because there are pecuniary and non-pecuniary costs of making frequent transactions between money and other assets. This premise led to them to build models with the prediction that, *in the long run*, there should be a systematic relationship between the demand for money, as measured by its velocity, and the difference between the interest rate offered on transactions accounts and that offered on alternative accounts that are not convenient for transactions purposes.[1] To the best of our knowledge however, no one has explored the quantitative implications of an inventory-theoretic model of the demand for money for the behavior of money, velocity, and the price level that is observed *in the short run*.[2] That is the task that we take up here.

We explore the possibility that a Baumol-Tobin style model can account for two salient features of the data on the short-run behavior of money, velocity, and the aggregate price level. First, in the short run, the velocity of money is highly variable. Second, the short-run movements in the velocity of money are highly negatively correlated with the short-run movements in the stock of money relative to consumption. As a result of these comovements of money and velocity, prices look "sticky" in the data in the sense that the volatility of the price level in the short-run is smaller than the volatility of the stock of money relative to consumption.[3]

An inventory-theoretic model of the demand for money along the lines proposed by Baumol and Tobin provides a natural accounting of the variability of velocity in the short-run and its systematic negative correlation with the stock of money relative to consumption. In

[1] A number of authors — such as Jovanovic (1982), Romer (1986) and Chatterjee and Corbae (1992) — have studied the long-run properties of general equilibrium versions of Baumol-Tobin models.

[2] Akerloff (19??) discusses the idea that an inventory theoretic model of the demand for money might have interesting short run implications for the velocity of money. Manuelli and Sargent 19?? examine the short run behavior of the demand for money in a Townsend Turnpike model of money. This work is related in that it explores the distributional implications of money injections on the demand for money. See also Perez-Verdia (2000) for related work.

[3] We document these facts in Section 3.

such a model, different agents at different points in the cycle of depleting and replenishing their cash balances have different propensities to spend the cash that they have on hand, or, equivalently, different individual velocities of money. Those agents who have recently visited the asset market to replenish their cash balances will tend to spend their stock of cash slowly since they spread the spending of this cash over the interval of time that remains before they next visit the asset market. Hence, these agents will have a relatively low individual velocity of money. In contrast, those agents who have been away from the asset market for some time and anticipate visiting it again soon to replenish their cash balances will tend to spend the cash that they have at a relatively rapid rate, and thus have a relatively high individual velocity of money. Aggregate velocity at any point in time is determined by the weighted average of the individual velocities of money of all of the agents in the economy, with the weights determined by the distribution of cash holdings across agents.

Now consider the effects of a one-time increase in the money supply in a Baumol-Tobin model. When the central bank injects additional money via an open market operation, this money is purchased by those agents who are currently visiting the asset market to exchange money and interest bearing assets and the fraction of the money stock held by these agents rises. These agents have a lower propensity to spend this cash than would the average agent in this economy. As a result, an exogenous injection of cash via an open market operation leads to an endogenous reduction in the aggregate velocity of money and hence, a diminished response of the price level. In this way, an inventory-theoretic model of the demand for money provides a rich theory of the dynamic response of velocity and prices to even the simplest monetary experiment.

Our implementation of a Baumol-Tobin model is a standard cash-in-advance model with two modifications. The first modification concerns the frequency with which agents have the opportunity to trade money and interest bearing assets. In the standard cash-in-advance model all agents have the opportunity to trade money and other financial assets every period, and, hence, agents can choose to carry into goods markets only the money that they need to pay for consumption expenditures in that period. In contrast, in our model, each agent has the opportunity to trade money and other financial assets only once every N periods.[4] Hence,

[4] Grossman and Weiss (1983) and Rotemberg (1984) solve similar models with $N = 2$.

agents must carry into the goods market enough money to pay for consumption expenditures for several periods. We refer to the number of periods N as the *length of a shopping trip*. For simplicity, we take the length of a shopping trip as exogenous. As a result of this modification, and in contrast to the standard cash-in-advance model, the velocity of money is not determined by the length of a time period. Instead, aggregate velocity is determined by the length of a shopping trip.

Second, our model differs from the standard cash-in-advance model in that — following Lucas (1990), Alvarez and Atkeson (1997), and others — we organize agents into coalitions so that they may share the risk that arises because different households trade money and assets in different periods. Alvarez, Atkeson, and Kehoe (2002) discuss how this coalition problem can be decentralized with contingent claims to cash in the asset market. This device of organizing households into coalitions serves to make the model tractable since it eliminates the need to keep track of both the distribution of wealth across agents in addition to the distribution of money holdings across agents.

In terms of its implications, our model differs from a standard cash-in-advance model also because the asset market is segmented as a result of our assumption that agents cannot trade money and other assets every period. In the periods in which an agent has the opportunity to trade money and other financial assets, we refer to him as an *active agent*. In the other periods in which that agent does not have the opportunity to trade money and other financial assets, we refer to him as an *inactive agent*. Thus, in our paper, the asset market is segmented in the same way that it is in Grossman and Weiss (1983), Rotemberg (1984), and Alvarez and Atkeson (1997). As a result of this similarity with those models, our model will also have predictions for the effects of money injections on real interest rates and real exchange rates arising from the segmentation of the asset market related to the predictions in these papers and in Alvarez, Atkeson, and Kehoe (2002) and Alvarez, Lucas, and Weber (2002).

1. Economic Environment

Consider a cash-in-advance economy in which agents can transfer funds between an asset market and a goods market only every N periods. Time is discrete and denoted $t =$

$0, 1, 2, \ldots$. The exogenous shocks in this economy are money growth shocks with realizations denoted by μ_t. A finite history of length t is denoted $h^t = (h_0, h_1, ..., h_t)$ with $h_t = (\mu_t)$ and probability density $f_t(h^t)$ at h^t. The aggregate endowment is a constant y.

A coalition is comprised of workers and shoppers. To be precise, there is a measure 1 continuum of workers and N types of shoppers with each type having measure $1/N$. Each period, workers sell the coalition's endowment in the goods market for cash $P_t(h^t)y$. In the next period, a fraction $\gamma \in [0, 1]$ of this cash is distributed directly to each shopper in the goods market for use next period. We think of the fraction γ as the fraction of total income that agents receive regularly deposited as paychecks into their transactions accounts and we refer to γ as the "paycheck" parameter. The remaining fraction $1 - \gamma$ of this cash comes to the coalition as income in the asset market next period. This cash income that the coalition receives in the asset market can be distributed only to those shoppers who are active.

We denote by $s = 0, 1, ..., N-1$ the number of time periods since a shopper was active, i.e. since that shopper has had his most recent opportunity to trade money and other other securities in the asset market. Hence a shopper of type $s = 0$ is active in the asset market in this period, a shopper $s = 1$ was active last period, and a shopper $s = N - 1$ will be active next period. A shopper of type $s < N - 1$ in the current period will be type $s + 1$ in the next period. A shopper of type $s = N - 1$ in the current period will be type $s = 0$ in the next period.

The shoppers of type $s = 1, 2, \ldots, N-1$ are inactive in the asset market in the current period. These shoppers have cash held over from last period $Z_{t-1}(s, h^{t-1})$, receive the paycheck $\gamma P_{t-1}(h^{t-1})y$ from the earnings of the workers the period before, consume $P_t(h^t)c_t(s, h^t)$ and carry unspent cash into next period, $Z_t(s+1, h^t) \geq 0$. Thus, the cash flow constraint for these shoppers is:

$$P_t(h^t)c_t(s, h^t) + Z_t(s + 1, h^t) \leq Z_{t-1}(s, h^{t-1}) + \gamma P_{t-1}(h^{t-1})y \tag{1}$$

In the settings that we will be interested in, shoppers of type $s = N - 1$ will not hold over cash to bring into the asset market next period. That is, shoppers of type $s = N - 1$ will choose cash holdings of $Z_t(N, h^t) = 0$ all (t, h^t). This will be the case when the rate of return on money over N periods is always less than the nominal interest rate over N periods.

When a shopper is active in the asset market, and hence of type $s = 0$, his income is supplemented with a transfer of cash $P_t(h^t)x_t(h^t)$ from the asset market and his cash flow constraint becomes

$$P_t(h^t)c_t(0, h^t) + Z_t(1, h^t) \leq Z_{t-1}(N, h^{t-1}) + \gamma P_{t-1}(h^{t-1})y + P_t(h^t)x_t(h^t), \tag{2}$$

with $Z_t(1, h^t) \geq 0$.

This transfer of cash to active shoppers is decided by the coalition and is chosen subject to a budget constraint for the coalition in the asset market. Asset markets are complete in the sense that there is a complete set of contingent claims to cash delivered in the asset market at different dates and in different states of nature. Hence, for the coalition, we can write a single date-0 budget constraint restricting the present value of the transfers $P_t(h^t)x_t(h^t)$. Let $Q_t(h^t)$ denote the price in the asset market as of date-0 for a dollar delivered in the asset market at date t in state h^t. In per capita terms, this present value budget constraint is:

$$\begin{aligned} &\sum_{t=0}^{\infty} \int_{h^t} Q_t(h^t) \left[\frac{P_t(h^t)x_t(h^t)}{N} + A_t(h^t) \right] dh^t \\ &\leq B_0 + \sum_{t=1}^{\infty} \int_{h^t} Q_t(h^t) \left[(1-\gamma)P_{t-1}(h^{t-1})y - P_t(h^t)\tau_t(h^t) + A_{t-1}(h^{t-1}) \right] dh^t \end{aligned} \tag{3}$$

where B_0 denotes the coalition's nominal wealth in the initial state h_0 (this is a liability of the government), $\tau_t(h^t)$ denotes lump-sum taxes on the coalition and $A_t(h^t) \geq 0$ denotes cash held by the coalition in the asset market. Note that if the nominal interest rate from t to $t+1$ is positive in some state of nature h^t, then the coalition will not hold cash in the asset market in that state ($A_t(h^t) = 0$) since a one-period bond dominates cash in the asset market. Notice also that $P_t(h^t)x_t(h^t)$ is transferred to each of the $1/N$ shoppers of type $s = 0$ in the asset market while $(1-\gamma)P_{t-1}(h^{t-1})y$ is the money received from measure 1 workers.

Corresponding to the coalition's present value budget constraint is a government budget constraint which we write:

$$B_0 \leq \sum_{t=1}^{\infty} \int_{h^t} Q_t(h^t)[P_t(h^t)\tau_t(h^t) + M_t(h^t) - M_{t-1}(h^{t-1})]dh^t + M_0 - M_{-1}$$

where $M_0 - M_{-1}$ denotes the initial monetary injection.

Since this is an endowment economy, the goods market clearing condition is

$$\frac{1}{N}\sum_{s=0}^{N-1} c_t(s, h^t) = y \qquad \text{for all } (t, h^t),$$

while the money market clearing condition is:

$$\frac{1}{N}\sum_{s=0}^{N-1}[Z_t(s, h^t) + P_t(h^t)c_t(s, h^t)] + A_t(h^t) = M_t(h^t).$$

The total money supply at (t, h^t) is $M_t(h^t)$. At the end of the period, this money can be held by the coalition in either the asset market as $A_t(h^t)$, by the workers who sold the coalition's endowment $P_t(h^t)y$, or in different amounts by the N shoppers in the goods market $Z_t(s, h^t)$. By summing up the cash flow constraints (1)-(2) over s, dividing by N and then using the goods market clearing condition, we can derive an expression relating the size of the coalition's transfer in the asset market to the size of the monetary injection:

$$\frac{P_t(h^t)x_t(h^t)}{N} = [M_t(h^t) - M_{t-1}(h^{t-1})] - [A_t(h^t) - A_{t-1}(h^{t-1})] + (1-\gamma)P_{t-1}(h^{t-1})y. \quad (4)$$

A. Coalition problem

For each date and state and taking as given the prices and aggregate variables, the coalition chooses transfers $x_t(h^t)$, cash to hold over in the asset market, $A_t(h^t)$, consumption for each of the shoppers, $c_t(s, h^t)$, and money holdings for each of the shoppers, $Z_t(s+1, h^t)$, to maximize the equally weighted sum of the shoppers' expected utilities:

$$\frac{1}{N}\sum_{s=0}^{N-1}\sum_{t=0}^{\infty}\int_{h^t} u[c_t(s, h^t)]f_t(h^t)dh^t$$

subject to the set of cash flow constraints (1)-(2), the single present-value budget constraint (3) and non-negativity constraints. Alvarez, Atkeson, and Kehoe (2002) discuss how to decentralize problems of this form.

Let $\eta_t(s, h^t) \geq 0$ denote the Lagrange multiplier on the cash flow constraint of shopper s at (t, h^t), $\lambda \geq 0$ denote the single multiplier for the present value budget constraint, and let $\delta_t^A(h^t) \geq 0$ and $\delta_t^M(s, h^t) \geq 0$ denote respectively the multipliers on the non-negativity constraints for cash held in the asset market or cash held by shoppers of type s. The first order necessary conditions for this optimization problem include:

$$x_t(h^t): \qquad P_t(h^t)\eta_t(0, h^t) \leq \lambda Q_t(h^t) P_t(h^t) \frac{1}{N} \tag{5}$$

$$A_t(h^t): \qquad \delta_t^A(h^t) + \lambda \int_{h_{t+1}|h^t} Q_{t+1}(h^t, h_{t+1}) dh_{t+1} \leq \lambda Q_t(h^t) \tag{6}$$

$$c_t(s, h^t): \qquad \beta^t u'[c_t(s, h^t)] f_t(h^t) \frac{1}{N} \leq P_t(h^t)\eta_t(s, h^t) \tag{7}$$

$$Z_t(s+1, h^t): \qquad \delta_t^M(s, h^t) + \int_{h_{t+1}|h^t} \eta_{t+1}(s+1, h^t, h_{t+1}) dh_{t+1} \leq \eta_t(s, h^t) \tag{8}$$

Since u is strictly increasing, the consumption first order condition (7) will hold with equality and we can write out a familiar-enough expression for the marginal utility of a dollar of shopper s at (t, h^t):

$$\eta_t(s, h^t) = \beta^t \frac{u'[c_t(s, h^t)] f_t(h^t)}{P_t(h^t)} \frac{1}{N} > 0$$

From (5), this implies that asset prices are determined by the marginal utility of the active shoppers (of type $s = 0$) in the asset market:

$$\eta_t(0, h^t) = \lambda Q_t(h^t) \frac{1}{N} = \beta^t \frac{u'[c_t(0, h^t)] f_t(h^t)}{P_t(h^t)} \frac{1}{N} > 0 \tag{9}$$

Now define the nominal interest rate $i_t(h^t)$ and the price of a bond $q_t(h^t)$ that pays one dollar in the asset market for sure in any state h_{t+1} following h^t. These satisfy the relationship:

$$\frac{1}{1 + i_t(h^t)} = q_t(h^t) = \int_{h_{t+1}|h^t} \frac{Q_{t+1}(h^t, h_{t+1})}{Q_t(h^t)} dh_{t+1} \tag{10}$$

From (6), this implies $\delta_t^A(h^t) = 0$ if and only if $i_t(h^t) = 0$. If the nominal interest rate is positive, the coalition will not hold cash in the asset market.

For all the inactive shoppers who hold cash, $Z_t(s+1, h^t) > 0$, we have the familiar stochastic Euler equation for an agent who can save only with cash:

$$1 = \int_{h_{t+1}|h^t} \beta \frac{u'[c_{t+1}(s+1, h^t, h_{t+1})]}{u'[c_t(s, h^t)]} \frac{f_{t+1}(h^t, h_{t+1})}{f_t(h^t)} \frac{P_t(h^t)}{P_{t+1}(h^t, h_{t+1})} dh_{t+1}$$

to go along side our Euler equation for bonds.

$$q_t(h^t) = \int_{h_{t+1}|h^t} \beta \frac{u'[c_{t+1}(0, h^t, h_{t+1})]}{u'[c_t(0, h^t)]} \frac{f_{t+1}(h^t, h_{t+1})}{f_t(h^t)} \frac{P_t(h^t)}{P_{t+1}(h^t, h_{t+1})} dh_{t+1} \tag{11}$$

In the situations we will be interested in, only shoppers of types $s = 0, 1, ..., N-2$ will hold cash. The shopper of type $s = N-1$ who gets to transact in the asset market next will not hold over any cash and their first order condition for $Z_t(N, h^t)$ will be an inequality.[5]

B. Solving the model

We solve for the dynamics of money, prices, and velocity in this model by log-linearizing these equations around the deterministic steady-state. We solve the resultant system of stochastic difference equations using the method of undetermined coefficients as described in Uhlig (1997). When we specify an exogenous first-order autoregressive processes for the gross growth rate of the money supply, $\mu_t(h^t) = \mu_t(h^{t-1}, h_t) \equiv M_t(h^{t-1}, h_t)/M_{t-1}(h^{t-1})$ and solve for the corresponding equilibrium, the solution of the model is completely standard and can be accomplished easily with the MATLAB code that accompanies Uhlig (1997).[6] Later, we also use the method of undetermined coefficients to solve for a money supply process that results in nominal interest rates that follow an exogenous AR(1) process. It turns out, as explained in section 3c below and in the technical appendix to this paper that the solution in this case required some additional work.

[5] More generally, however, there may exist situations where shoppers cease to hold cash from one period to another even in the middle of their trip. The paycheck that shoppers receive makes it possible to consume while on their trip even if they are not storing cash; this may be optimal behavior if the return on cash is sufficiently low.

[6] These MATLAB programs are available at http://www.wiwi.hu-berlin.de/wpol/research.html.

2. How the model works

In this section we give some intuition as to how our model resembles a Baumol-Tobin model and how money injections affect prices and aggregate velocity. We then show impulse responses from the model following a one-time shock to the money supply to illustrate the dynamics of these effects.

We think of our model as resembling a Baumol-Tobin model in the following sense. In our model, in equilibrium, agents periodically withdraw cash from the asset market and carry that cash for some time in the goods market until they have the opportunity to make another withdrawal of cash from the asset market. As a result, agents' equilibrium paths for money holdings have a familiar *saw-toothed* shape that is characteristic of the Baumol-Tobin model, declining steadily over the length of a shopping trip before jumping up once the trip is over and the next cash transfer from the coalition is in hand.[7] In Figure 1, we illustrate the saw-tooth pattern in the steady-state path of real balances for an individual shopper. Note that, in this figure, the individual shopper runs his real balances down to zero at the end of every shopping trip. This is because, in equilibrium, his consumption jumps up when he starts the next shopping trip and, therefore, he has no incentive to carry money from the end of one shopping trip into the beginning of the next one.

Our model implies a relationship between the steady-state velocity of money and the length of a shopping trip similar to that implied by the Baumol-Tobin model. In particular, if the paycheck parameter $\gamma = 0$, so shoppers must carry enough cash on each shopping trip to pay for all of their consumption on that trip, then the reciprocal of steady-state velocity measured on an annual basis is roughly 1/2 the length of a shopping trip measured in years. (To match aggregate annual velocity of 1, shopping trips would have to last 2 years). The term 1/2 arises since, to compute velocity, one must compute the area under the triangle described by individual money holdings shown in Figure 1. This relationship between the length of a shopping trip and aggregate velocity changes with the paycheck parameter. If the paycheck parameter $\gamma > 0$, and if agents still choose to carry cash into the last period of each shopping trip, then the reciprocal of steady-state velocity on an annual basis is roughly

[7] A refinement of our model would allow shoppers to chose the length of their shopping trip subject to a fixed cost. At this point, we leave such a refinement of the model to future work.

$(1-\gamma)/2$ times the length of a shopping trip measured in years.

Our model's implications for the dynamics of money, velocity, and the price level outside of steady-state are determined in large part by the affect of a money injection on the distribution of cash across agents. The intuition for how money injections affect aggregate velocity in this model is as follows. Each shopper has an individual velocity that depends on the underlying state of nature and his type. These measures of individual velocity equal the flow of consumption obtained by that shopper relative to the his total cash holdings at the beginning of the period. We can immediately see that a shopper towards the end of his shopping trip is going to have a relatively high individual velocity. A shopper near the end of his trip will spend more as a fraction of its beginning-of-period cash holdings than a shopper early in its trip — simply because a shopper early in its trip will be away from the asset market for more future periods. In the first panel of Figure 2, we illustrate the pattern of these individual velocities for shoppers of type s. At any point in time, aggregate velocity is a weighted average of these individual velocities, where the weights are determined by the distribution of cash holdings among the shoppers of different types.

How does a one-time money injection affect velocity in this economy? In an equilibrium in which the nominal interest rate is positive, the coalition distributes the entire money injection to the active ($s = 0$) shoppers, and hence, a monetary injection shifts the distribution of cash holdings towards the active shoppers at the expense of the inactive shoppers. In the second panel of Figure 2, we illustrate this redistribution of cash holdings across shoppers of different types in response to a money injection. In particular, we plot the percentage deviation of the real balances held by shoppers of type s from their steady-state values in response to a one percent increase in the aggregate money stock. In this figure we see that the real balances held by active shoppers rise considerably while those of all the inactive shoppers fall, where this fall comes about because inflation devalues the money holdings of all inactive shoppers. By redistributing real balances towards the active shopper, a money injection tilts the distribution of cash holdings towards agents with low individual velocities and away from agents with high individual velocities, lowering aggregate velocity. Thus, aggregate velocity declines when there is a monetary injection. Put differently, aggregate velocity and the money supply are negatively correlated. The dynamics of aggregate velocity and prices that arise from this money injection are then determined by the response of the distribution of cash holdings and its evolution following the money injection back to the steady-state.

We have been somewhat loose in explaining this intuition in that we have discussed only the effects of a money injection on the distribution of cash across shoppers of different types and not the effects of a money injection on individual velocities, that is, the rate at which individual shoppers will spend the money that they do hold. It is possible to obtain an exact solution of this model when shoppers have log utility and the paycheck parameter $\gamma = 0$. In this parameterization of the model, the individual velocities do not vary with the date or the state of nature. Hence, it is necessarily the case that a money injection affects aggregate velocity in this parameterization of the model only to the extent to which this money injection changes the distribution of cash holdings across agents, and the intuition given above is complete. In alternative parameterizations of the model with $\gamma > 0$, however, individual velocities also vary with the state of the economy, and hence, the responses of velocity and the price level are more complex. We start, however, by displaying the impulse responses of money, prices, and velocity in the parameterization of the model with log utility and $\gamma = 0$ to illustrate the intuition given above.

In the two panels of Figure 3, we show the responses of $\log(M_t)$, $\log(P_t)$ and $\log(v_t)$ to a one time unit shock to money growth when agents have log utility and $\gamma = 0$. As shown in Figure 3, at time $t = 0$, the money supply blips up by one unit and stays at its new level thereafter. In response to this injection, aggregate velocity falls (is negatively correlated with the money supply) and the price level responds less than one for one with the money supply. Observe here that, at least in terms of the impact effect of a money injections, the movement in velocity is roughly the same magnitude as the movement in the price level and is negatively correlated with the movement in money. Hence, prices in this model are sticky in the sense that they move substantially less than would be predicted by the simplest quantity theory.

The model's implications for the dynamics of prices and velocity in response to a money injection can be understood as follows. Note that in this parameterization of the model, since shoppers spend a constant fraction of their money balances each period and carry the remaining constant fraction into the next period, the shoppers who were active at the time of the money injection carry an abnormally large stock of cash throughout their entire shopping trip. As shown in Figure 2, their individual velocities rise as they progress through this trip. Thus, aggregate velocity remains below its steady-state level for a time as these agents are at the beginning of their shopping trip and have a low individual velocity, rises past its steady-state level as they pass through the middle of their shopping trip, and then rises above its steady-state level as they come to the end of their shopping trip and have a high individual velocity. After N periods these agents have finished their shopping trip. The periodic structure of the model (the pattern of shopping trips) introduces a sequence of dampened oscillations in velocity as the changes in the cash distribution work their way through the system.

3. Quantitative implications of the model

The purpose of our paper is to exposit the links between money, velocity and prices in an inventory-theoretic model of the demand for money and to explore the extent to which it can account quantitatively for the short-run volatility of velocity, the negative correlation of velocity and the ratio of money to consumption, and the resulting stickiness of prices as measured by the relative volatility of the ratio of money to consumption and the price level when the parameters of the model are specified at reasonable values. We now turn to that quantitative exploration.

The features of the data to which motivate our study of this model are shown in Table 1 and Figures 4 and 5. The first feature of the data that we focus on is the fact that, in the short run, the velocity of money is highly variable. In the first panel of Table 1, we report several measures of the variability of the log of velocity in the short run in the United States using monthly data from 1959 to 2001. We compute three measures of velocity by taking the ratio of personal consumption expenditure to $M0$, $M1$, and $M2$ as well as a measure of households' holdings of assets similar to those included in $M2$ obtained from the Flow of Funds Accounts (2002). (This last data series is quarterly). We compute the short-run fluctuations in these series both by using a HP-filter to remove trends and by computing 12 month differences in the series.[8] We compare the standard deviation of these series to the

[8] For the monthly series we used an HP smoothing parameter of $3^4 * 1600$ and for quarterly series we used an HP smoothing parameter of 1600.

standard deviation of the corresponding HP filtered or 12th differenced log of the price level as measured by the personal consumption expenditure deflator. Figure 4 displays the HP filtered series for the log of $M2$ velocity and the log of the price level. As is evident in this table and figure, in the short-run, velocity is at least as variable as the price level.

The second feature of the data that we focus on is the fact that, in the short run, movements in the velocity of money are highly negatively correlated with the short-run movements in the stock of money relative to consumption. In the second panel of Table 1, we compute the correlation between the short-run movements in our three measures of the velocity of money (using $M0, M1$, $M2$, and the measure of $M2$ held by households obtained from the Flow of Funds Accounts) and the short-run movements in the corresponding stock of money relative to consumption as measured by real personal consumption expenditure in both HP filtered and 12 month differenced data. Figure 5 displays the HP filtered series for the log of $M2$ velocity and the log of the ratio of $M2$ to consumption. As is evident in this table and figure, in the short-run, the movements in velocity are highly negatively correlated with the movements in the stock of money relative to consumption.

As a result of this negative correlation of velocity and money, in the data prices look "sticky" in the sense that the volatility of the price level in the short-run is smaller than the volatility of the stock of money relative to consumption. In the third panel of Table 1, we report on the standard deviation of the log of the ratio of our four measures of money to consumption relative to the standard deviation of the log of the price level in both HP filtered and 12 month differenced data. As is evident in this table, in the short-run, the price level is less volatile than the stock of money relative to consumption.

A. What are reasonable values of N and γ?

The quantitative implications of this model depend critically on the values of the parameters N and γ. To specify these parameters, we must take a stand on the frequency with which agents trade money and other assets (to determine N) and the fraction of income that agents receive directly in the form of money (to determine γ). We are aware of only a limited number of sources of microeconomic data on these questions. Since the choice of N and γ determine the average level of velocity implied by this model, one can also use macro economic data on the velocity of money to guide the choice of these parameters. We choose these parameters so that the model produces an average level of velocity for a monetary aggregate that is relatively broad. In doing so, we experiment with specifications of the model with log utility and $\gamma = 0$ and alternative specifications with log utility and $\gamma > 0$. Recall that the specification of the model with log utility and $\gamma = 0$ is useful for developing intuition since individual velocities of money are constant and thus variation in aggregate velocity arises only from changes in the distribution of money holdings across agents. Specifications of the model with $\gamma > 0$ are more complex in that both individual velocities and the distribution of money holdings across agents change in response to a money injection.

We see the Baumol-Tobin model as designed to account for the observation that households hold assets that are *substantially* dominated in rate of return by some alternative, safe, but relatively illiquid asset such as short-term Treasury Securities. Accordingly, we choose a monetary aggregate based on the observed user cost of the types of assets included in that aggregate. This analysis leads us to choose the sum of currency, demand deposits, and savings and time deposits as our monetary aggregate. This aggregate is essentially the same as M2 less money-market mutual funds.

In choosing this monetary aggregate, we examine data on the rate of return paid on various types of bank deposits and other financial assets is available from the web site of the Federal Reserve Bank of St. Louis.[9] We summarize that data in Table 2. In the top panel of Table 2, we report on the average user cost of holding currency, demand deposits, time and savings deposits, and retail money market mutual funds over the full time period for which the data are available as well as over the decade 1990-2001. The user costs reported in this table are equal to the difference between the rate of return on short-term Treasury securities (as reported in the spreadsheet from which the data are taken) less the rate of return on the asset in question. We display data for both the full time period and the most recent decade to show that financial reform has not had a large impact on the relative opportunity cost of demand and time and savings deposits.

[9] See the file of input data msinputs.zip available at http://www.stls.frb.org/research/msi/index.html. This file contains a spreadsheet that reports the data on the user cost of various types of bank deposits that has been collected by the Research Department at the Federal Reserve Bank of St. Louis as part of their project to construct Divisia monetary aggregates.

As is clear from Table 2, the average opportunity cost of holding time and savings deposits is roughly similar to that of holding demand deposits, both over the period 1959-2001 and over the most recent decade 1990-2001. In contrast, the opportunity cost of holding retail money market mutual fund shares has been essentially zero on average. In panel b of Table 2, we show the average opportunity cost of $M1$, $M2$, and $M2$ less retail money market mutual fund shares. Here these average opportunity costs are measured as the weighted average of the opportunity cost of each type of deposit in the corresponding aggregate where the weights are given by the share of each type of deposit in the corresponding monetary aggregate. The opportunity cost of $M2$ less retail money market funds is on the order of 200 basis points (2 percentage points) and is not that substantially different than the opportunity cost of $M1$.

From the data in Table 2, we conclude that if one is to apply the Baumol-Tobin model to the study of the demand for a broader measure of money than simply currency, then one might reasonably aggregate currency, demand deposits, and savings and time deposits as a the measure of the monetary aggregate and regard retail money market funds and other assets as higher yielding alternatives that cannot be exchanged for money without incurring transactions costs. In Figure 6, we report on US households' holdings of currency and demand deposits, time and savings deposits, and retail money market mutual funds. These data are from the Flow of Funds Accounts (2002). Figure 6 is a stacked line chart of these holdings relative to personal consumption expenditure. The height of the lowest line indicates holdings of currency and demand deposits relative to Personal Consumption Expenditure. The gap between that line and the next highest line indicates holdings of time and savings deposits. The gap between that second line and the third line indicates holdings of retail money market mutual funds. Note that holdings of money market mutual funds were equal to zero before the middle of the 1970's. In Figure 6, we see that households' holdings of the aggregate of currency, demand, and savings and time deposits has been more stable relative to personal consumption expenditure than is the case for the narrower aggregate of currency and demand deposits, with perhaps some increase in the velocity of this broader aggregate in recent years as households have expanded their holdings of retail money market mutual funds. These data give a measure of the velocity of money relative to personal consumption expenditure (at least for the money held by households) averaging roughly 1.5 and rising more recently towards 2. We choose this average level of velocity of 1.5 to guide our choice of N and γ for the quantitative results that follow.

To parameterize our model to reproduce an average velocity of money of 1.5, we use two choices for the parameters N and γ. In one of these, we set $N = 15$ months and $\gamma = 0$. Since we assume log utility, with this parameterization individual velocities are constant and aggregate velocity changes only because of changes in the distribution of money across agents. In the other of these, we choose the paycheck parameter $\gamma = 0.6$ to match the fraction of personal income that is received as wage and salary disbursements observed in the data.[10] We then choose $N = 38$ so that with $\gamma = 0.6$, the model produces an average velocity of 1.5.

These values of $N = 15$ and $N = 38$ for the length of a shopping trip are the values that are required to account for the average level of low-yielding assets held by U.S. households. While we are not aware of extensive micro data on household transactions between their demand, savings, and time deposits and other higher yielding assets such as money-market mutual funds or stocks or bonds, these choices of N appear to be consistent with the data on household transactions that is available.

The Investment Company Institute (1999) conducted an extensive survey of households' holdings and trading of equity in 1998. They report on the frequency with which households traded stocks and stock mutual funds in 1998. They report that 48% of the households that held individual stocks outside of their retirement accounts neither bought nor sold any stock in 1998 and 63% of the households that held stock mutual funds outside of their retirement accounts neither bought nor sold funds in 1998. Since a household would have to buy or sell some of these assets to transfer funds between these higher yielding assets and a lower yielding transactions account, these data, interpreted in light of our model, would indicate choices of N ranging from roughly 24 (for roughly 1/2 of households trading these risky assets at least once within the year) to roughly 36 (for roughly 1/3 of households trading within the year). These data may also overstate the frequency with which households transfer funds between their equity accounts and their transactions accounts since some of

[10] From Table 2.1 of the National Income and Product Accounts, we observe that this fraction has been equal to 60% on average over the period from 1959-2001.

the instances of equity trading are simply a reallocation of the equity portfolio. The Investment Company Institute reports that more than 2/3 of those households that sold individual shares of stock in 1998 reinvested all of the proceeds, while 57% of those households that sold stock mutual funds reinvested all of the proceeds. In the context of our model, reallocation of the household portfolio in the asset market is costless and does not generate cash that can be used to purchase goods.

Vissing-Jorgensen (2002) reports on micro data on the frequency of household trading of stocks, bonds, mutual funds and other risky assets obtained from the Consumer Expenditure Survey. In figure 6 in her paper, she shows the fraction of households who bought or sold one of these assets over the course of one year as a function of their financial wealth at the beginning of the year. She finds that the fraction of agents who traded one of these assets ranges from roughly 1/3 to 1/2 of the households owning these assets at the beginning of the year. Again, these data would lead us to choose N between 24 and 36.

The estimates of the frequency of transfers between transactions accounts and other financial assets inferred from the data in Investment Company Institute (1999) and Vissing-Jorgensen (2002) may be on the high side since each of these papers examines the trading only of those households that hold positive amounts of risky assets.[11] She finds in panel data that individual households tend to switch between holding positive amounts of risky assets and zero amounts of those assets from one year to the next. Thus data on the frequency with which the average household trades risky assets may lead to a choice of N larger than 36.

Some researchers choose to apply the Baumol-Tobin framework to the study of relatively narrow monetary aggregates, such as currency, or currency and demand deposits. The work of Attanasio, Guiso, and Jappelli (1998) and Avery, Elliehausen, Kennickell, and Spindt (1986 and 1987) with micro data on household use of cash and checking accounts in Italy and

[11] Mulligan and Sala-i-Martin (2000) observe in data from the Survey of Consumer Finances that 59% of U.S. households do not hold any nonmonetary financial assets (i.e. financial assets other than currency, demand deposits, and savings accounts). Fernandez-Villaverde and Krueger (2002) use a life-cycle model with durable goods to study this pattern of asset holdings by households and argue that the observation that many households do not hold nonmonetary financial assets arises because young households are credit constrained and find it optimal to hold only monetary assets and durable goods. If we take the Fernandez-Villaverde-Krueger hypothesis as our explanation for the observed asset holdings of households, it would seem reasonable to assume that, for many households, the appropriate value of N would be quite large because of the fact that durable goods are not liquid substitutes for monetary assets.

the United States might lead one to choose the period length of the model to be one day and the length of a shopping trip to be something on the order of two weeks and the paycheck parameter to be something positive and perhaps substantial.[12] If one used macro data from the United States on households' holdings of a relatively narrow definition of money to guide the choice of N and γ, one would also be led to choose similar values of these parameters. In Figure 6, we show data on U.S. households' nominal holdings of currency and various types of deposits relative to nominal personal consumption expenditure for the period 1952-2001. The data are obtained from Flow of Funds Accounts (2002)[13]. These data differ from aggregate measures of the stock of currency and these various types of deposits in that they are constructed to exclude amounts of these assets held by firms, governments, and foreigners. As is clear in Figure 6, households in the United States hold very little currency and demand deposits relative to their personal consumption expenditure. Their holdings of these assets have been trending downward steadily since 1952 and are now represent less than one month worth of personal consumption expenditure. To date, we have not computed a specification of our model with, say, a period equal to one day and N on the order of 15 and γ in the range of 1/2. We suspect, however, that the variations in velocity that would occur in such a model would be at too high a frequency to be of interest relative to the data presented in Table 1.

B. Implications with exogenous money growth

We first study the quantitative implications of our model under the assumption that the log of the gross growth rate of the money supply follows a first-order autoregressive process with first order autocorrelation $\rho = 0.66$. This autocorrelation is the first-order autocorrelation of monthly data on the growth rate of M2.

Consider first the implications of our model with $N = 15$ months and $\gamma = 0$ for the

[12]The work of Attanasio, Guiso, and Jappelli (1998) would seem to be the most useful source of detailed micro data on the demand for money and the use by households of currency and various bank accounts as financial assets. They use a Baumol-Tobin style model to examine data from Italy. They focus on currency as a narrow definition of money and demand deposits (which bear interest in Italy) as the alternative interest bearing asset. They report that in Italy, roughly 45% of income is received in the form of currency, indicating a choice of γ in the range of .45. They report that consumers in their data make frequent visits to the bank to exchange currency and demand deposits, with a shopping trip of roughly two weeks being typical.

[13]Table B.100

response of velocity and the aggregate price level to such a money growth shock. In Figure 7, we show the impulse response of the logarithm of the money stock, the aggregate price level, and velocity to this shock. (Recall that aggregate output and consumption remain constant). This persistent increase in the money supply results in a large hump-shaped decline in velocity that lasts for roughly 12 months. Corresponding to this endogenous decline in velocity, the aggregate price level rises substantially less than the money stock in the first several months following the shock and does not catch up to the increase in the money stock until 12 months have passed. After 15 months, there are small oscillations in velocity as the model settles down to its steady state.

In Table 3, we report on the standard deviations and correlations of 12 month differences and HP filtered levels of the aggregate price level, the money stock relative to aggregate consumption, and velocity obtained from data simulated from this model. In this table, under the column labelled $N = 15$ and $\gamma = 0$, we see that this specification of the model produces velocity that is roughly 40% as volatile as the price level and a correlation between velocity and the ratio of money to consumption of roughly -0.5.

We regard these results as a promising first set of results from the model in that it has produced economically significant variation in velocity and a substantial negative correlation between the ratio of money to consumption and velocity. Recall that in this specification of the model, individual velocities are constant so all variation in aggregate velocity arises simply from changes in the distribution of money holdings across agents. In alternative specifications of the model, aggregate velocity can vary not only with changes in the distribution of money holdings across shoppers but also with changes in individual velocities as shoppers change their spending in response to changes in anticipated inflation.

While a promising first start, this specification of the model does not account for the variability of velocity observed in the data. In the last column of the table, we repeat these same statistics obtained from the data reported in Table 1 when $M2$ is used as the monetary aggregate. This specification of the model produces a variability of velocity that is smaller than that in the data and a negative correlation between the ratio of money to consumption and velocity that is smaller in magnitude than in the data. As a consequence of these facts, the model also produces a smaller gap between the standard deviation of the ratio of money to consumption and the aggregate price level.

Consider next the implications of our model with $N = 38$ months and $\gamma = 0.6$ in response to the same persistent shock to the growth rate of the money supply. In Figure 8, we show the impulse response of the log of the money stock, the aggregate price level, and velocity for this specification of the model. In this figure we see that, on impact, velocity *rises* in response to an increase in the money supply. This qualitative difference between the response of velocity in this case and the negative initial response of velocity in the specification with $N = 15$ and $\gamma = 0$ arises because here individual velocities respond to the change in anticipated inflation brought about by the money growth shock. Further comparison of this figure with Figure 7 shows that, here, the response of velocity is shallower but somewhat more persistent, so that now it takes roughly 30 months for the price level to catch up with the money supply.

Table 3 also reports on the quantitative results from this specification of the model. Our measures of the volatility of velocity relative to the price level in the short run are similar across the two specifications, but this specification yields a stronger negative correlation between the ratio of money to consumption and velocity. As a result, this specification produces a higher standard deviation of the ratio of money to consumption relative to that of the aggregate price level.

Because agents in this model trade money and interest bearing assets only infrequently, the asset market in this model is segmented in the sense of Grossman-Weiss (1983), Rotemberg (1984), Alvarez and Atkeson (1998), Alvarez, Lucas, and Weber (2002), and Alvarez, Atkeson, and Kehoe (2002). From equation (11), we see that it is the marginal utilities of active agents that are used to price assets and since money injections have disproportionate effects on these marginal utilities, money injections have a large, and, in this case, negative, effect on nominal interest rates. In Figure 9, we show the response of the nominal interest rate to the money growth shock studied in Figures 7 and 8 for the specification of the model with $N = 38$ and $\gamma = 0.6$. In this figure we see that this money injection has a strong liquidity effect — an *increase* in money growth is associated with a large *decrease* in the interest rate that is twice as large in magnitude as the movement in money growth rates. As a result of this strong liquidity effect, this specification of the model has the implication that money growth and nominal interest rates are strongly negatively correlated and that nominal interest rates are substantially more volatile than money growth. Neither of these observations is consistent with the observed standard deviations and correlations of M2 growth and the Federal Funds Rate.

Motivated by the excessively large movements in nominal interest rates shown in Figure 9, in the next two subsections, we consider an alternative specification of the model in which we solve endogenously for a stochastic process for money growth that produces, in equilibrium, a process for the nominal interest similar to that observed in the data. We first warn the reader about several technical issues that arise when one looks to solve this model with the process for the nominal interest rate exogenous. We then present results for this version of the model.

C. Technical issues regarding finding equilibria with exogenous interest rates

In the next subsection, we solve for a stochastic process for the gross growth rate of the money supply that results in an equilibrium in which the short-term nominal interest rate follows a first order autoregressive process similar to that estimated for the monthly Federal Funds Rate. Two technical issues arose when we solved this version of the model. We discuss these issues in greater detail in a technical appendix to this paper.

The first issue that arose had to do with the dynamics of equilibria in which the nominal interest rate follows an exogenously specified path. Under the assumption that the nominal interest rate follows a pre-specified path, one can show analytically that the matrix that describes the dynamics of the endogenous variables in this economy has eigenvalues that are all equal to zero. (This implies that, if the interest rate is set at its steady-state value but the initial distribution of money holdings is not, then the economy will reach steady-state in exactly N periods). Because these eigenvalues are repeated, this matrix is not diagonalizable, and hence, this variant of the model cannot be solved using standard methods such as those outlined by Blanchard and Kahn (1980) or Uhlig(1997). We also found that direct methods based on use of the generalized Schur form, as suggested by Klein (2000) and others, did not correctly identify that the matrix describing the equilibrium dynamics of the variables had eigenvalues all equal to zero. This appears to be a numerical issue since this methodology should in theory work in cases with repeated eigenvalues. We have not resolved why a direct attack on the problem using this procedure did not work. We developed a specific solution method for this model based on the use of the generalized Schur form that makes use of the information that the eigenvalues of the matrix that describes the equilibrium dynamics are all equal to zero.

The second technical issue that arose had to do with the invertibility of the equilibrium mapping between interest rates and money growth rates. In this model, there are many stochastic processes for money all consistent with the same exogenously specified path for nominal interest rates in equilibrium. In the experiments with the second variant of the model that we carry out below, we choose one of the many stochastic process for the gross growth rate of the money supply that result in an equilibrium in which the short-term nominal interest rate follows a first order autoregressive process similar to that estimated for the monthly Federal Funds Rate. The process for money growth that we choose has the property that a shock to the nominal interest rate, on impact, is associated with no movement in the current price level.

D. Implications of the model with exogenous interest rates

We now study the quantitative implications of our model having solved for a money growth process that results in equilibrium in which the log of the short-term gross interest rate first-order autoregressive process with first order autocorrelation $\rho = 0.97$. This autocorrelation is the first-order autocorrelation of monthly data on the Federal Funds Rate.

We report on the quantitative implications of our model for the specifications with $N = 15$ months, $\gamma = 0$ and $N = 38$ months, $\gamma = 0.6$ in Table 4. In this table, we see that, in contrast to the results for the version of the model above in which money growth follows an AR(1) (reported in Table 3), the results for these two specifications of the parameters of the model are quite different from each other. On the one hand, with $N = 15$ months and $\gamma = 0$, the model does not produce substantial short-term variability in velocity, nor are the short-term movements in velocity strongly negatively correlated with the ratio of money to consumption. On the other hand, with $N = 38$ months and $\gamma = 0.6$, the model produces large variations of velocity in the short-term relative to those of the price level and these variations in velocity are very strongly negatively correlated with the ratio of money to consumption. As a consequence of these results, as shown in the third panel of Table 4, the variability of the ratio of money to consumption in the model is high relative to that of the price level.

Figure 10 shows the impulse responses of the log of the money stock, velocity, and the aggregate price level in response to a shock to the short-term interest rate (also shown in the figure) for the specification of the model with $N = 38$ months and $\gamma = 0.6$. That this specification of the model generates large short-term movements in velocity that are strongly negatively correlated with the ratio of money to consumption can be seen clearly in these impulse responses. As a result of these negative comovements of money and velocity, the aggregate price level appears "sticky" in this impulse response in that it shows little or no response to the shock to interest rates for at least the first twelve months. It is only after two years have passed that the money stock and the price level begin to rise together in the manner that would be expected in a flexible price model following a persistent increase in the nominal interest rate.

Figure 11 shows the impulse responses of the log of the growth rate of the money stock, the growth rate of velocity, and the growth rate of the aggregate price level following the same shock to nominal interest rates shown in Figure 10. This figure shows that there are persistent liquidity effects in this model both in the sense that a movement in the nominal interest rate is associated with a movement in the money growth rate in the opposite direction and also in the sense that a movement in the nominal interest rate is associated, at least at first, with a movement in the real interest rate (the difference between the nominal interest rate and the growth of the price level). The aggregate price level again appears "sticky" in the sense that inflation does not respond much to the movement in the nominal interest rate.

4. Conclusion

We have analyzed the high frequency implications of a simple version of a Baumol-Tobin monetary model. We find that when the parameters of the model are selected so that average velocity corresponds to the one of a broad monetary aggregate similar to M2, the model reproduces salient features of the short-run behavior of velocity — its relatively high volatility and its strong negative correlation with the money supply.

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Table 1

Summary statistics from data

(a) *Standard deviations of velocity relative to prices*

	Velocity Measure			
Detrending method	M0	M1	M2	M2 (Flow Funds)
HP-filter	1.40	2.61	1.65	1.81
12th Differences	1.05	1.51	1.03	1.35

(b) *Correlations of velocity and money*

	Velocity Measure			
Detrending method	M0	M1	M2	M2 (Flow Funds)
HP-filter	-0.79	-0.92	-0.8	-0.84
12th Differences	-0.66	-0.82	-0.6	-0.76

(c) *Standard deviations of money relative to prices*

	Monetary Aggregate			
Detrending method	M0	M1	M2	M2 (Flow Funds)
HP-filter	1.63	2.51	1.45	1.45
12th Differences	1.28	1.72	1.18	1.51

Table 2

Opportunity Cost of Various Monetary Assets

(a) *Short-Term Treasury Rate less own rate*

Asset	average opportunity cost in basis points 1959-2001	1990-2001
Currency	522	432
Demand Deposits	198	133
Savings Deposits	150	171
Time Deposits	180	247
Retail Money Market Funds*	-33	-11

*1973-2001

(b) Short-Term Treasury Rate less own rate

Aggregate	average opportunity cost in basis points 1959-2001	1990-2001
M1	297	271
M2	180	184
M2 less Retail Money Market Funds	195	217

Opportunity cost data constructed from the spreadsheets TB1ASAM.WKS
and ADJSAM.WKS available on the website of the Federal Reserve Bank of St. Louis
http://www.stls.frb.org/research/msi/index.html
These data are collected as part of the St. Louis Fed's project to construct
Divisia monetary services indices

Table 3

Money growth is an AR(1) with monthly autocorrelation of 0.66

(a) *Standard deviations of velocity relative to prices*

Detrending method	N = 38 $\gamma = 0.6$	N = 15 $\gamma = 0.0$	Data (M2)
HP-filter	0.41	0.40	1.65
12th Differences	0.38	0.43	1.02

(b) *Correlations of velocity and money*

Detrending method	N = 38 $\gamma = 0.6$	N = 15 $\gamma = 0.0$	Data (M2)
HP-filter	-0.69	-0.47	-0.80
12th Differences	-0.64	-0.50	-0.60

(c) *Standard deviations of money relative to prices*

Detrending method	N = 38 $\gamma = 0.6$	N = 15 $\gamma = 0.0$	Data (M2)
HP-filter	1.24	1.13	1.45
12th Differences	1.21	1.15	1.18

Table 4

Nominal interest rates are an AR(1) with monthly autocorrelation of 0.97

(a) *Standard deviations of velocity relative to prices*

Detrending method	N = 38 $\gamma = 0.6$	N = 15 $\gamma = 0.0$	Data (M2)
HP-filter	0.98	0.20	1.65
12th Differences	0.62	0.15	1.02

(b) *Correlations of velocity and money*

Detrending method	N = 38 $\gamma = 0.6$	N = 15 $\gamma = 0.0$	Data (M2)
HP-filter	-0.90	-0.20	-0.80
12th Differences	-0.74	-0.15	-0.60

(c) *Standard deviations of money relative to prices*

Detrending method	N = 38 $\gamma = 0.6$	N = 15 $\gamma = 0.0$	Data (M2)
HP-filter	1.78	1.02	1.45
12th Differences	1.38	1.01	1.18

Figure 1: The shoppers have "sawtoothed" real balance holdings

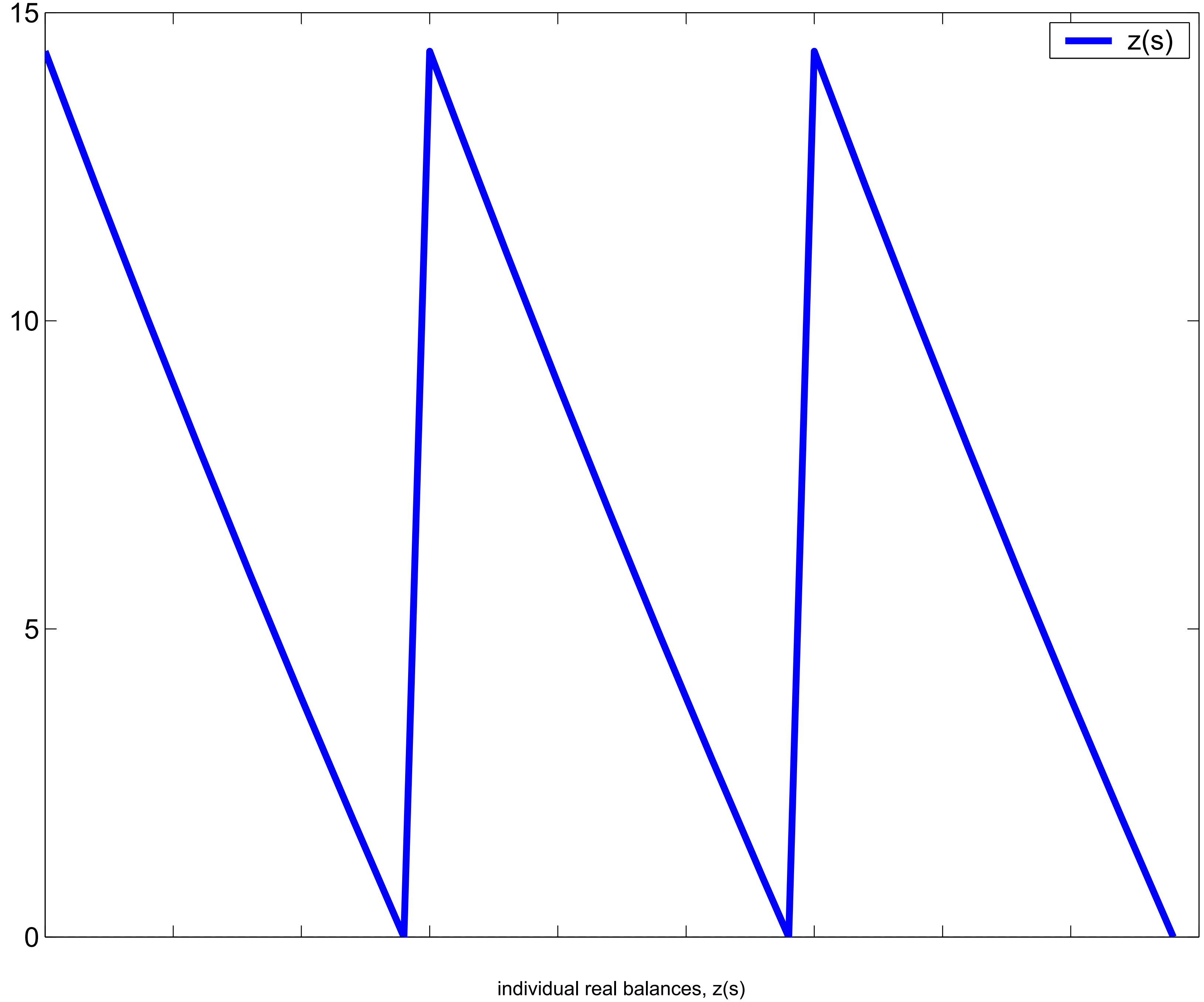

Figure 2: A one-time money growth shock redistributes to the shopper with the lowest velocity...

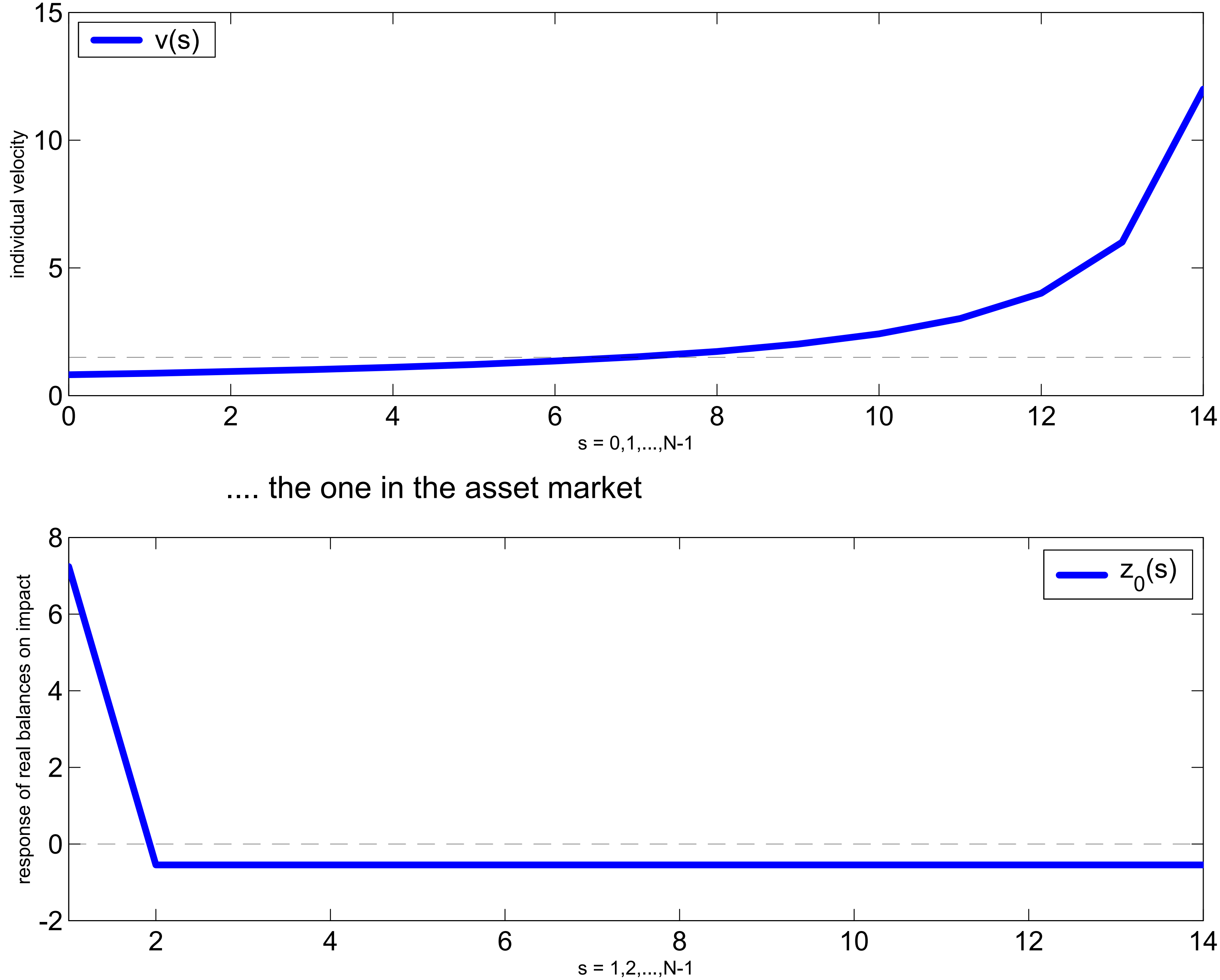

Figure 3: Impulse response to one-time unit shock to money growth (σ = 1, γ = 0)

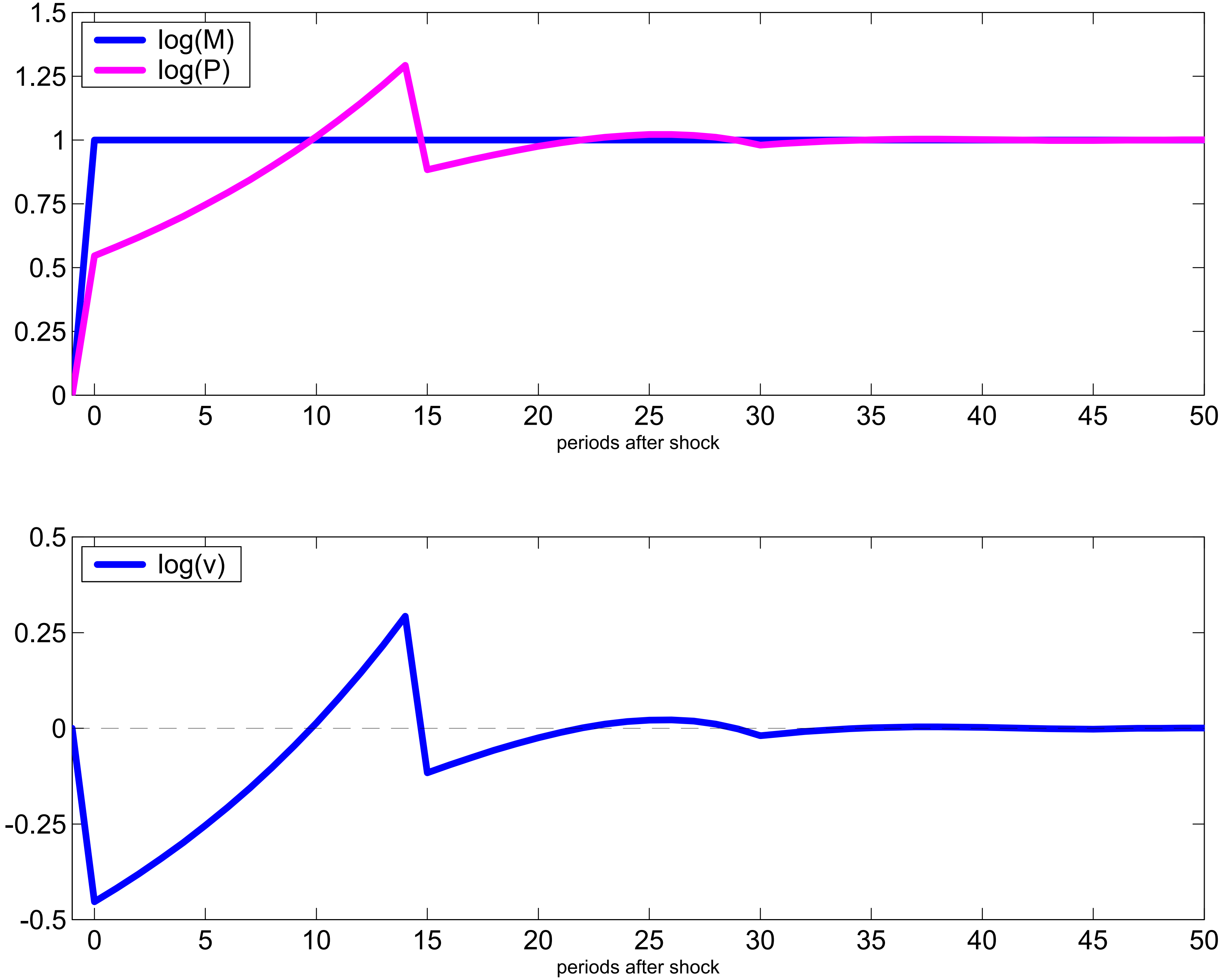

Figure 4: In the short-run, velocity is about 1.5 times as variable as the price level

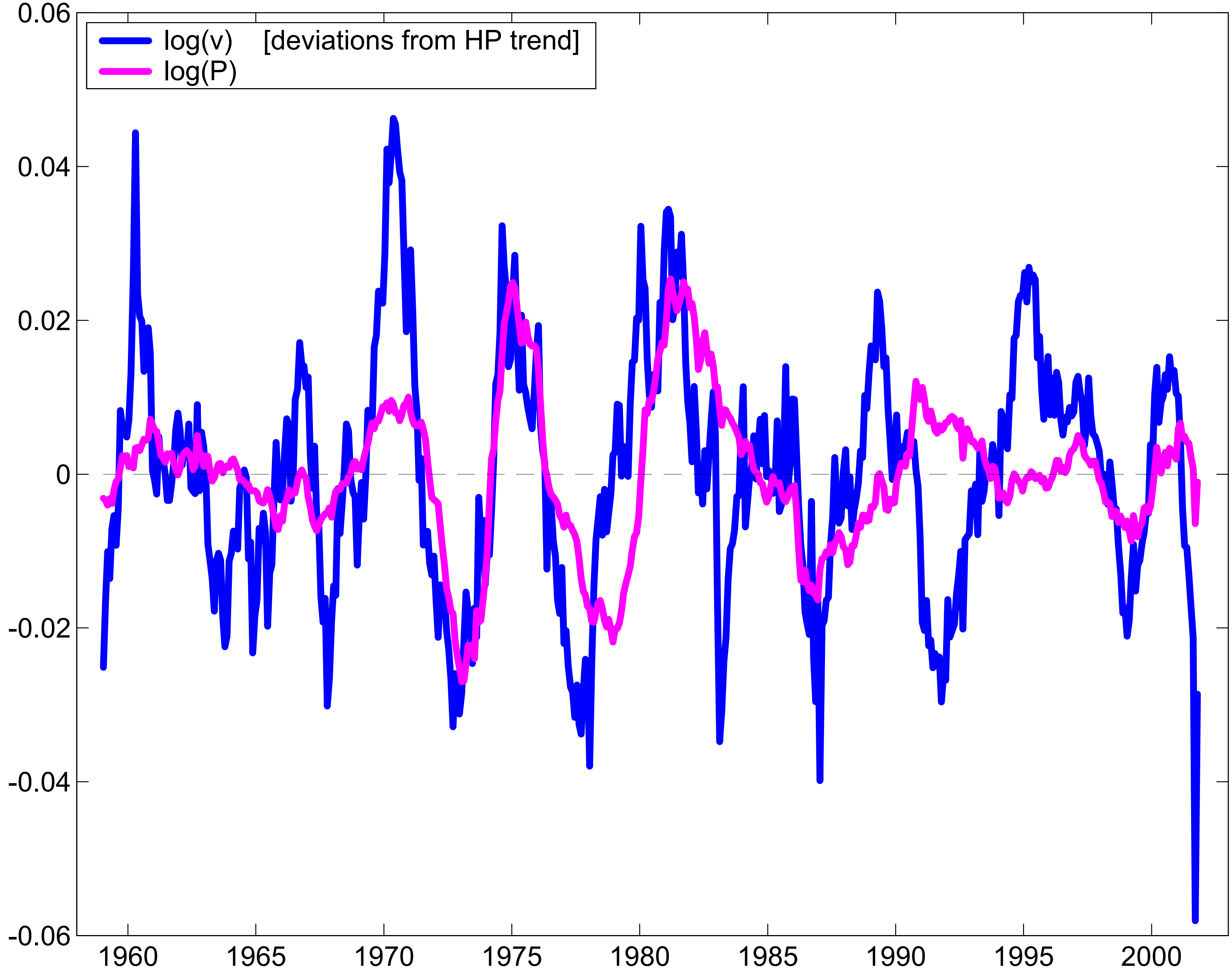

Figure 5: Movements in velocity are negatively correlated with money/consumption ratio

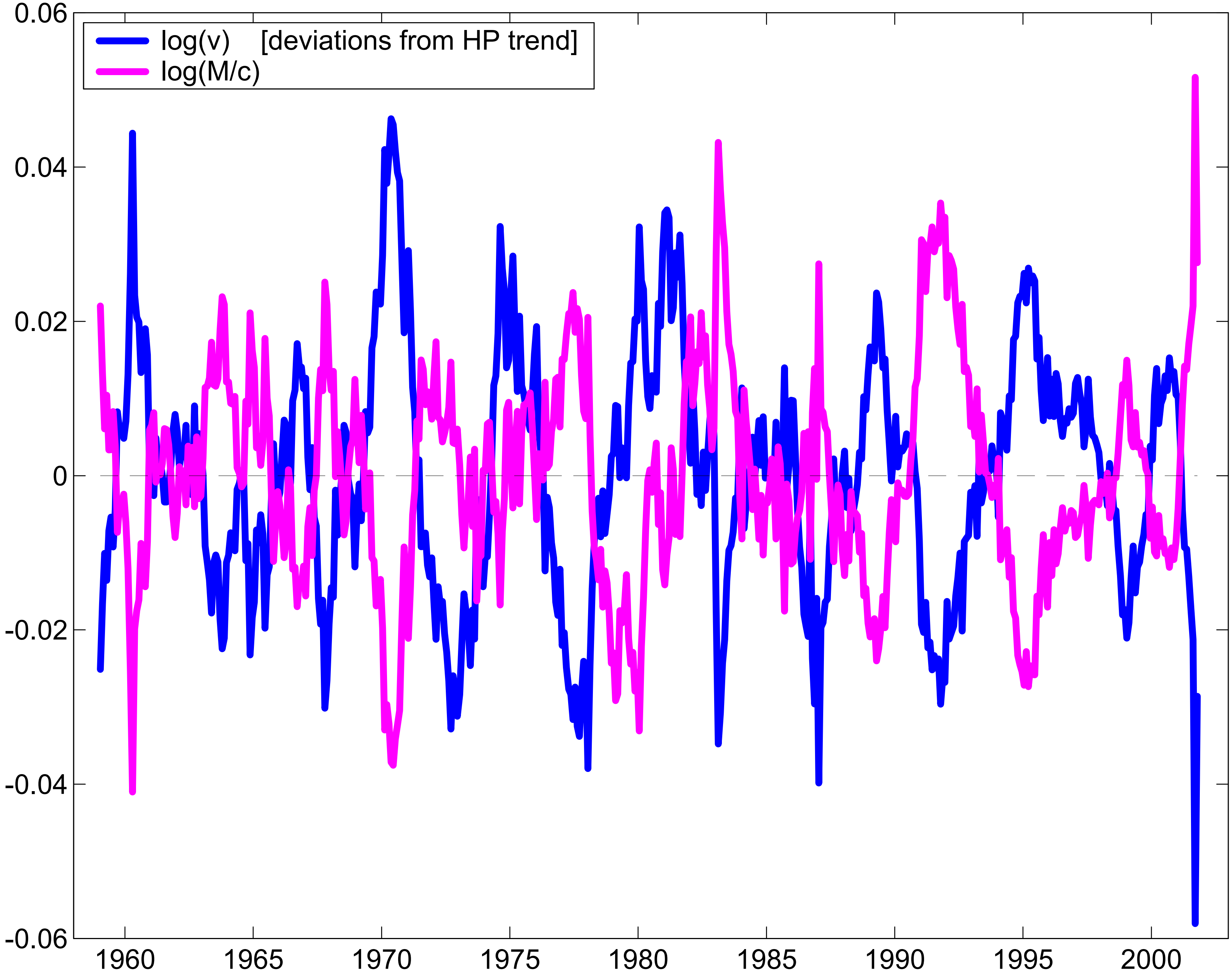

Figure 6:
Stacked Line Chart of Ratios of Monetary Assets to PCE

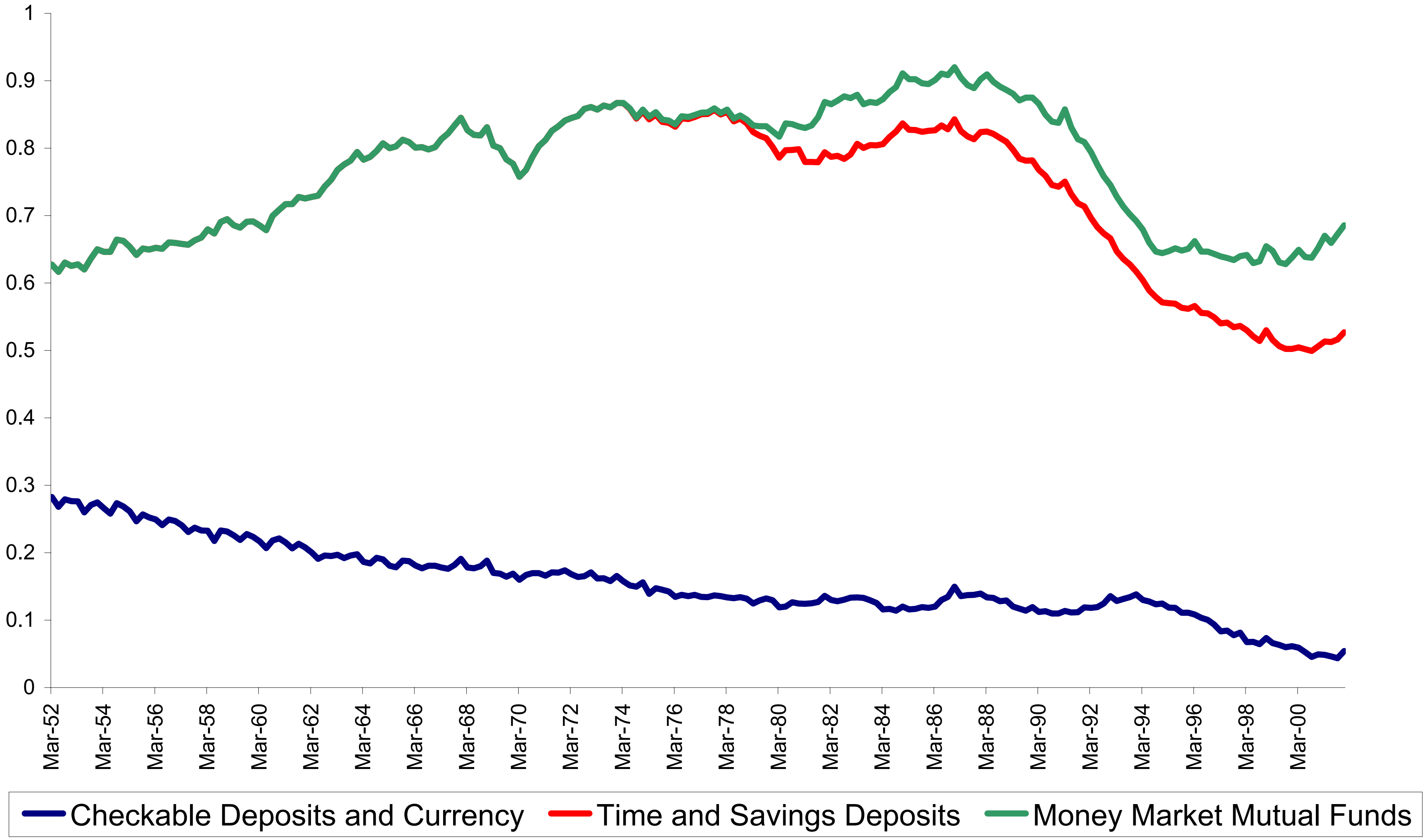

Figure 7: Money growth exogenous AR(1) with ρ=0.66 (N = 15, γ = 0.0)

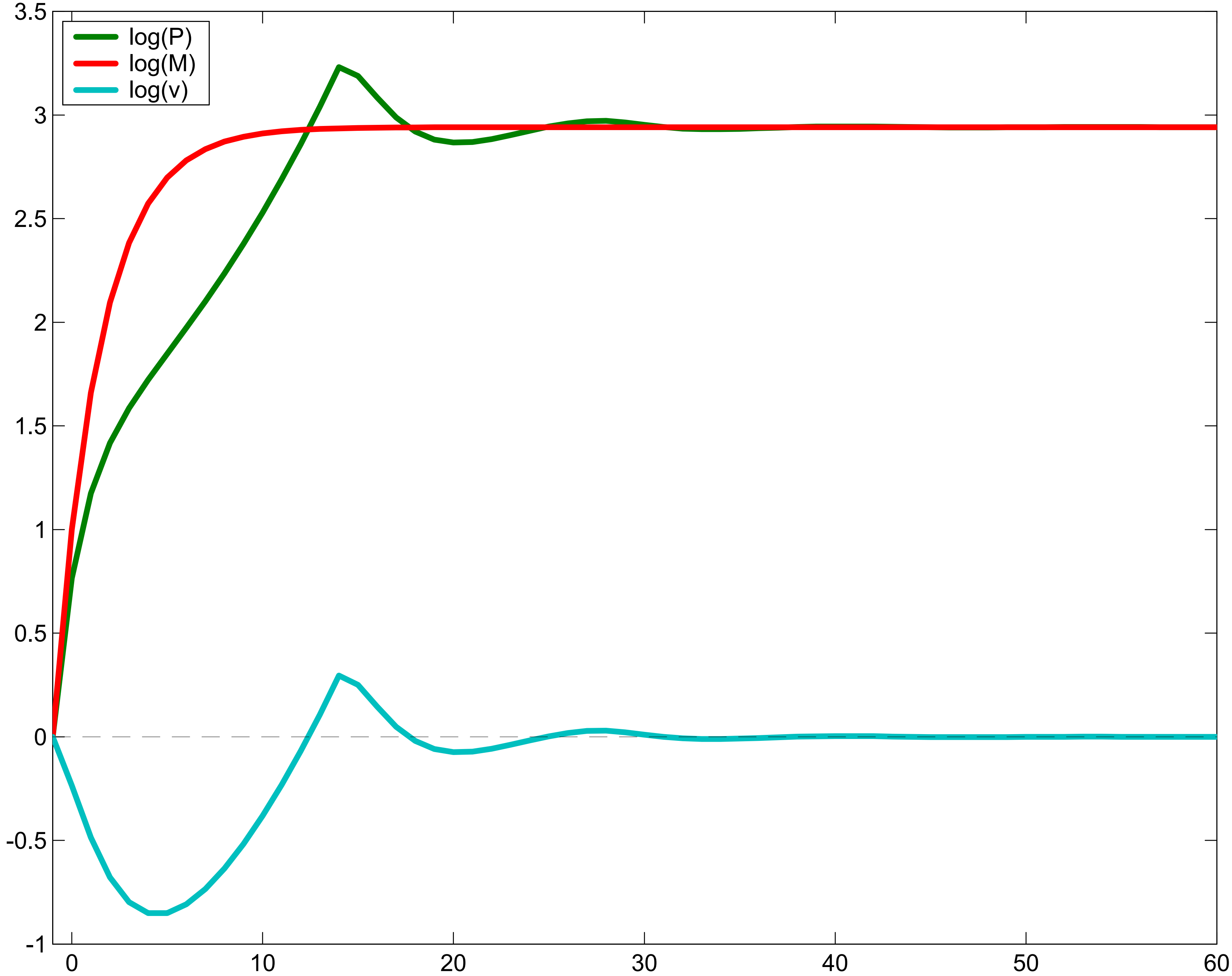

Figure 8: Money growth exogenous AR(1) with ρ=0.66 (N = 38, γ = 0.6)

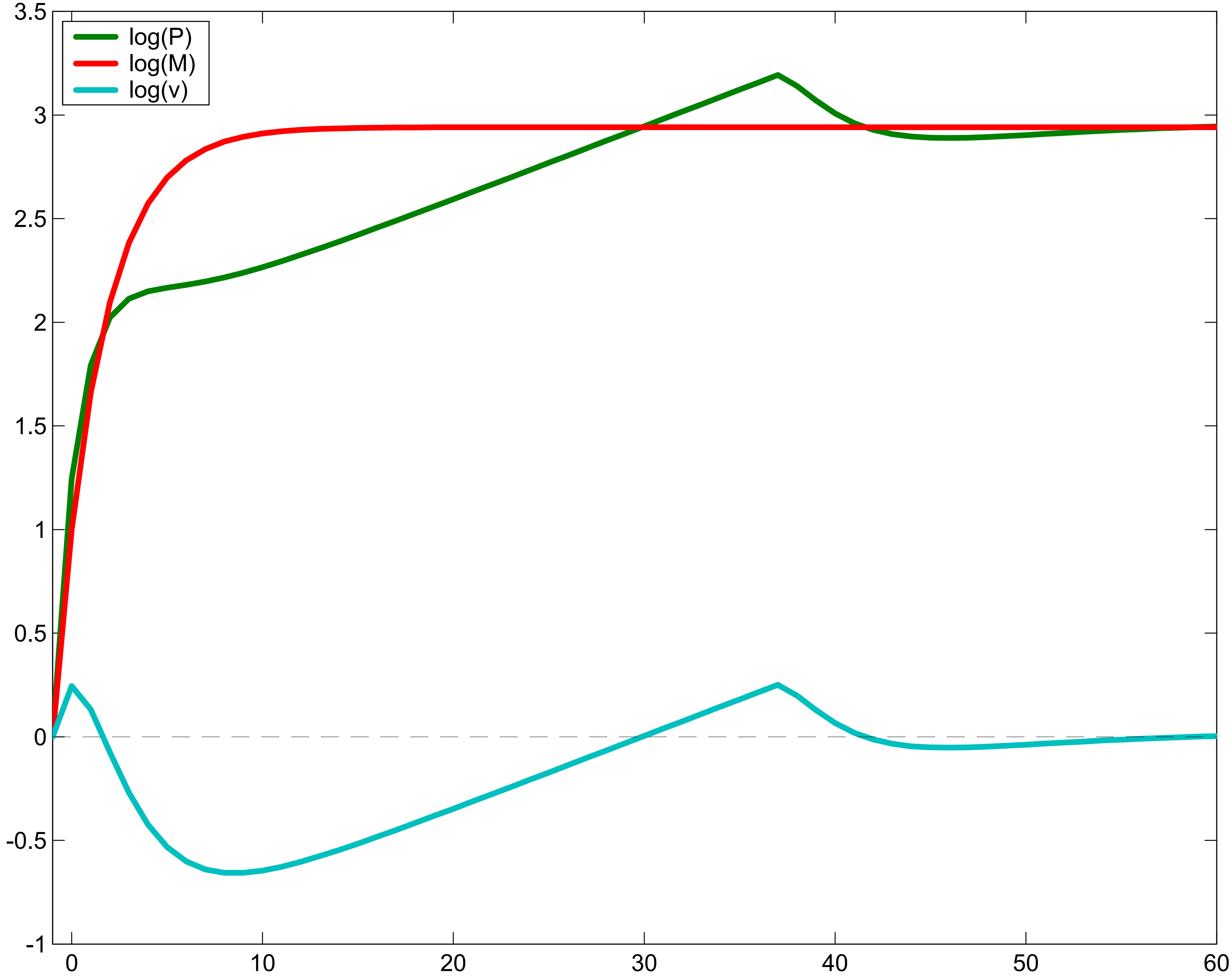

Figure 9: Nominal interest rates are too volatile with money exogenous

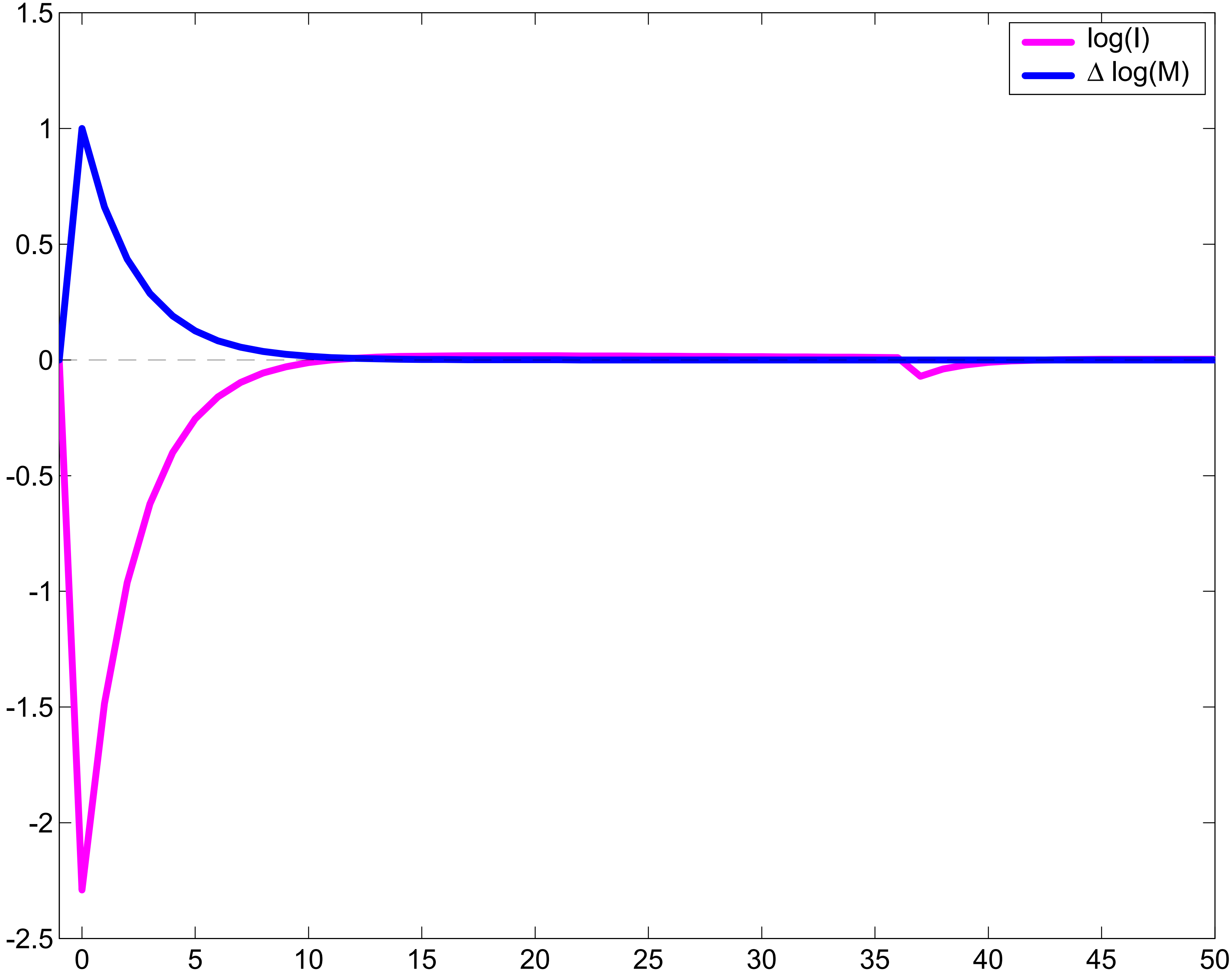

Figure 10: Nominal interest rates exogenous AR(1) with ρ = 0.97 (N = 38 γ =0.6)

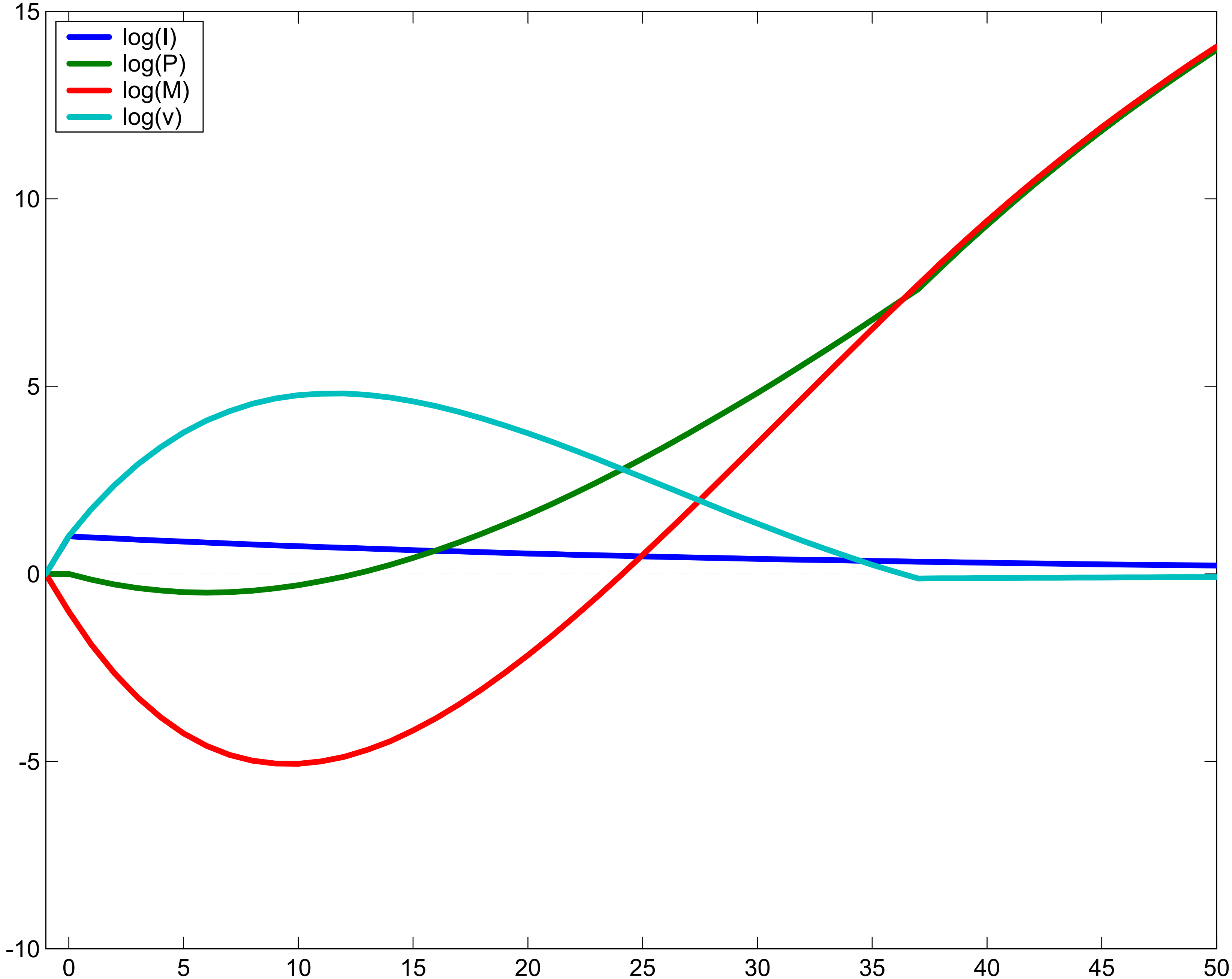

Figure 11: Nominal interest rates exogenous AR(1) with ρ = 0.97 (N=38 γ = 0.6)