# Optimal Monetary Policy*

Aubhik Khan Robert G. King Alexander L. Wolman

May 2000

**Abstract**

Optimal monetary policy maximizes welfare, given frictions in the economic environment. Constructing a model with two sets of frictions – the Keynesian friction of costly price adjustment by imperfectly competitive firms and the Monetarist friction of costly exchange of wealth for goods – we find optimal monetary policy is governed by two familar principles.

First, the average level of the nominal interest rate should be sufficiently low, as suggested by Milton Friedman, that there should be deflation on average. Yet, the Keynesian frictions imply that the optimal nominal interest rate is positive.

Second, as various shocks occur to the real and monetary sectors, the price level should be largely stabilized, as suggested by Irving Fisher, albeit around a deflationary trend path. (In modern language, there is only small "base drift" for the price level path). Since expected inflation is roughly constant through time, the nominal interest rate must therefore vary with the Fisherian determinants of the real interest rate, i.e., as there is expected growth or contraction of real economic activity.

*The views expressed here are the authors' and not necessarily those of the Federal Reserve Banks of Philadelphia or Richmond or the Federal Reserve System. The authors thank Bernardino Adao, Isabel Correia, and Pedro Teles for helpful conversation, and Philip Jefferson for providing his data on currency holdings. Errors are our own.

# 1 Introduction

Three distinct intellectual traditions are relevant to the analysis of how optimal monetary policy should regulate the behavior of the nominal interest rate, output and the price level.

*The Fisherian view:* Early in this century, Irving Fisher [19xx, 19xx] argued that the business cycle was "largely a dance of the dollar" and called for stabilization of the price level, which he regarded as the central task of the monetary authority. Coupled with his analysis of the determination of the real interest rate [199x] and the nominal interest rate [199x], the Fisherian prescription implied that the nominal interest rate would fluctuate with those variations in real activity which occur when the price level is stabilized.

*The Keynesian view:* Stressing that the market-generated level of output could be inefficient, Keynes [1964 (1936)] called for stabilization of real economic activity by fiscal and monetary authorities. Within theoretical and quantitative models of macroeconomic activity constructed by his followers, stabilization policy typically mandated substantial variation in the nominal interest rate when shocks buffeted the economic system, particularly when there were shocks to aggregate demand. While most Keynesians viewed the price level as responding only gradually to these shocks, it typically changed over time as policy interventions focused on a real output target, with little importance attached to the path of the price level.

*The Monetarist view:* Evaluating monetary policy in a long-run context with fully flexible prices, Friedman [1969] found that an application of a standard microeconomic principle of policy analysis long used in public finance– that social and private cost should be equated – indicated that the nominal interest rate should be approximately zero. Using flexible price models of business fluctuations, later authors pointed out that the same reasoning also dictated that the nominal interest rate should not vary through time in response to real and nominal disturbances.

There are clear tensions between these three traditions if real forces produce expected changes in output growth that affect the real interest rate. If the price level is constant, then the nominal interest rate must mirror the real interest rate so that Friedman's rule must be violated. If the nominal interest rate is constant, as Friedman's rule suggests, then there must be expected inflation or deflation to accommodate the movement in the real rate so that Fisher's prescription cannot be maintained.

We construct a model economy that honors each of these intellectual traditions and study the nature of optimal monetary policy within this framework. There are Keynesian features to the economy: firms have market power, which means that output may be inefficiently low, and all prices cannot be frictionlessly adjusted. However, as in the New Keynesian research on price stickiness that begins with Taylor [1980], firms are forward-looking in their price setting, which has dramatic implications for the design of optimal monetary policy. In our economy, there are also costs of converting wealth into consumption. These costs can be mitigated by the use of money, so that there are social benefits to low nominal interest rates as in Friedman's analysis. The behavior of real and nominal interest rates in our economy is governed by Fisherian principles.

Following Ramsey [1927], Lucas and Stokey [1983] and Ireland [1996], we determine the allocation of resources which maximizes welfare (technically, it maximizes the expected, present discounted value of the utility of a representative agent) given the resource constraints of the economy and additional constraints that capture the fact that the resource allocation must be implemented in a decentralized private economy. We assume that there is full commitment on the part of a social planner for the purpose of determining these allocations. We find that two familiar principles govern monetary policy in our economy:

*The Friedman prescription for deflation:* The average level of the nominal interest rate should be sufficiently low, as suggested by Milton Friedman, that there should be deflation on average. Yet, the Keynesian frictions generally imply that there should be a positive nominal interest rate.

*The Fisherian prescription for eliminating price-level surprises:* As shocks occur to the real and monetary sectors, the price level should be largely stabilized, as suggested by Irving Fisher, albeit around a deflationary trend path. (In modern language, there is only a small "base drift" for the price level path). Since expected inflation is roughly constant through time, the nominal interest rate must therefore vary with the Fisherian determinants of the real interest rate, i.e., as there is expected growth or contraction of real economic activity.

The organization of the paper is as follows. In section 2 we outline the main features of our economic model. In section 3, we identify four distortions present in our economic model, which are summary statistics for how its behavior can differ from a fully competitive, nonmonetary business cycle model. In section 4, we describe the nature of the general optimal policy problem that we solve. In section 5, we discuss optimal monetary policy in two special cases, for which analytical results can be derived. First, suppressing price stickiness, we discuss how Friedman's analysis carries over to an economy with imperfect competition. Second, we discuss how optimal policy works in an economy where the distortions associated with money demand are arbitrarily small. An exact case for price stabilization along Fisherian lines emerged in a previous application of this kind of setup by King and Wolman [1999], who studied an environment with monopolistically competitive firms and sticky prices, but without the "monetary distortions" emphasized by Friedman. Our analysis reviews this reference sticky price case and interprets the prior results along the lines of Adao, Correia and Teles [2000]. In section 6, we discuss how we choose the parameters of our model economy.

In section 7, we discuss the results which lead to the first principle for monetary policy. The nominal interest rate should be set at an average level that implies deflation, but it should be positive. We show how this steady-state rate of deflation depends on various structural features of the economy, including the nature of the transactions cost function which gives rise to money demand and the degree of price-stickiness. At present, we have learned that a relatively small extent of transactions costs can generate a deflation of .25% per year with a substantial amount of monopoly power and price stickiness. Hence, the case for an average inflation rate of zero developed in King and Wolman [1999] is quite fragile. A smaller degree of market power, a smaller extent of price stickiness, a greater intensity of money (lower velocity) or a larger sensitivity of money demand to the interest rate all make for a larger extent of deflation. We defer a conclusion about the optimal rate of deflation until we have a model specification which is more carefully grounded in empirical work.

In section 8, we describe the near-steady state dynamics of the model under optimal policy. Looking across a battery of specifications, we find that these dynamics display only slight variation in the price level. Thus, we document that there is a robustness to Fisherian conclusion in King and Wolman [1999], which is that the price level should not vary in response to a range of shocks under optimal policy. In fact, the greatest price level variation that we find involves a .07% long-run change in the price level in response to a productivity shock which brings about a temporary deviation of output from trend, but which is large in the sense that the cumulative output deviation is 20%. Across the range of experiments, output under optimal policy closely resembles output which would occur if all prices were flexible and monetary distortions were absent: real activity resembles that in a core "real business cycle" model which underlies our framework. At the same time, we find that the real interest rate under the optimal policy does not always closely mimic that in the underlying RBC framework. To help interpret these results, we contrast some of them to benchmarks, including a real business cycle model, our model with simple money growth or interest rate rules, and a version of our model with the money demand distortions eliminated. Section 9 will conclude.

# 2 The model

The macroeconomic model we study is designed to be representative of two recent strands of macroeconomic research. First, we view money as a means of economizing on the use of costly credit.[1] Second, we use a new Keynesian approach to price dynamics, which views firms as imperfect competitors facing infrequent opportunities for price adjustment.[2] To facilitate the presentation of these mechanisms, we view the private sector as divided into three groups of agents. First, there are households which buy final consumption goods and supply factors of production. These households also trade in financial markets for assets, including a credit market, and acquire cash balances which can be exchanged for goods. Second, there are retailers, which sell final consumption goods to households and buy intermediate products from firms. Retailers can costlessly adjust prices.[3] Third, there are producers, who create the intermediate products that retailers use to produce final consumption goods. These firms have market power and face only infrequent opportunities to adjust prices.

[1] As in Prescott [1986], Dotsey and Ireland [1996] and Lacker and Schreft [1996].

[2] Taylor [1980], Calvo [1983]

[3] It is possible to eliminate the retail sector, but fleshing it out makes the presentation of the model easier.

## 2.1 Households

Households have preferences for consumption and leisure, which are represented by the time-separable expected utility function,

$$U_t = E_t\{\sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j})\} \tag{1}$$

The momentary utility function $u(c, l)$ is assumed to be increasing in consumption and leisure, strictly concave and differentiable as needed. Households divide their time allocation $\phi$ into leisure, market work $n$, and transactions time $h_t$ subject to

$$n_t + l_t + h_t = \phi \tag{2}$$

*Accumulation of wealth:* Households begin each period with a portfolio of claims on the intermediate product firms, holding a previously determined share $\gamma_{t-1}$ of the per capita value of these firms. This portfolio generates current nominal dividends of $\gamma_{t-1}Z_t$ and has nominal market value $\gamma_{t-1}V_t$. They also begin each period with a stock of nominal bonds $(B_{t-1})$ left over from last period which have matured and have market value $(1 + R_{t-1})B_{t-1}$. Finally, they begin each period with a nominal credit card balance arising from consumption purchases last period, in the amount $F_{t-1}$. So, their nominal wealth is:

$$\gamma_{t-1}V_t + \gamma_{t-1}Z_t + (1 + R_{t-1})B_{t-1} - F_{t-1} - T_t$$

where $T_t$ is the amount of a lump sum transfer to or from the government.

With this nominal wealth and current nominal wage income $W_t n_t$, they may purchase money $M_t$, buy current period bonds in amount $B_t$, or buy more claims on the intermediate product firms. Thus, they face the constraint

$$M_t + B_t + \gamma_t V_t \geq \left(\gamma_{t-1}V_t + \gamma_{t-1}Z_t + (1 + R_{t-1})B_{t-1} - F_{t-1} - T_t\right) + W_t n_t$$

We convert this nominal budget constraint into a real one, using a numeraire $P_t$. At present this is simply an abstract measure of nominal purchasing power but we are more specific later about its economic interpretation. The real flow budget constraint is

$$\begin{aligned} m_t + b_t + \gamma_t v_t \quad \geq \quad & \gamma_{t-1}v_t + \gamma_{t-1}z_t + (1 + R_{t-1})\frac{P_{t-1}}{P_t}b_{t-1} \\ & -\frac{F_{t-1}}{P_t} - \frac{T_t}{P_t} + w_t n_t \end{aligned}$$

with lower case letters representing real quantities when this does not produce notational confusion.

*Money and transactions:* Although households have been described as purchasing a single aggregate consumption good, we now reinterpret this involving many individual products – technically, a continuum of products on the unit interval – as in many studies following Lucas [19xx]. Each of these products is purchased from a separate retail outlet at a price $\overline{P}_t$. Each customer buys infinitesimally small amounts of each product, a fraction $\xi_t$ with credit and the remainder with cash. Hence, the households demand for nominal money satisfies

$$M_t = (1 - \xi_t)\overline{P}_t c_t \tag{3}$$

where $\overline{P}_t$ is the price which must be paid to a retailer for a unit of consumption. The customer's nominal credit card balance is

$$F_t = \xi_t \overline{P}_t c_t.$$

which must be paid next period. If credit is used, then there are time costs, which take the form

$$h_t = \int_0^{\xi_t} dh_t \tag{4}$$

with

$$dh_t(x) = \left\{ \begin{array}{ll} \nu_0 \exp(z_t)\, x^{1/\nu_1} - \nu_2 \text{ for } x > \left[\frac{\nu_2}{\nu_0 \exp(z_t)}\right]^{\nu_1} \\ 0 \qquad\qquad\qquad\qquad \text{for } x \leq \left[\frac{\nu_2}{\nu_0 \exp(z_t)}\right]^{\nu_1} \end{array} \right\}$$

where $z_t$ is a shock to the distribution.[4] As in Prescott [1987], Dotsey and Ireland [1996] and Lacker and Schreft [1996], we think of each final consumption goods purchase as having a random fixed cost – perhaps, the extent to

[4]Whenever $x > \left[\frac{\nu_2}{\nu_0 \exp(z_t)}\right]^{\nu_1}$, this function generates the following total time cost of credit.

$$\int_0^{\xi} dh(x; z_t)\, dx = \xi \left( \nu_0 \exp(z_t) \frac{\xi^{(1/\nu_1)}}{1 + (1/\nu_1)} - \nu_2 \right) + \left( \frac{\nu_2/\nu_1}{1 + (1/\nu_1)} \right) \left( \frac{\nu_2}{\nu_0 \exp(z_t)} \right)^{\frac{1}{\gamma}}$$

which small children are clamoring for candy in the checkout queue – that is known after the customer decides to purchase the product, but before the customer has decided on whether to use money or credit to finance the purchase. The household uses credit when the cost is below the critical level given by $dh(\xi)$ and uses money when the cost is higher.

*Consumption demand and labor supply:* Combining budget constraints, we can get a real flow budget constraint for the household,

$$\begin{aligned}(1-\xi_t)\frac{\overline{P}_t}{P_t}c_t + b_t + \gamma_t v_t \quad &\geq \quad \gamma_{t-1}v_t + \gamma_{t-1}z_t + (1+R_{t-1})\frac{P_{t-1}}{P_t}b_{t-1} \\ &-\xi_{t-1}c_{t-1}\frac{\overline{P}_{t-1}}{P_t} + w_t n_t - \frac{T_t}{P_t} \qquad (5)\end{aligned}$$

To characterize the solution to the household's problem, we consolidate the three constraints, (2), (4) and (5) into one:

$$(1-\xi_t)\frac{\overline{P}_t}{P_t}c_t + b_t + \gamma_t v_t - \Big(\gamma_{t-1}v_t + \gamma_{t-1}z_t + (1+R_{t-1})\frac{P_{t-1}}{P_t}b_{t-1}$$
$$-\xi_{t-1}c_{t-1}\frac{\overline{P}_{t-1}}{P_t} + w_t\Big(\phi - l_t - \int_0^{\xi_t} dh_t\Big) - \frac{T_t}{P_t}\Big) \geq 0$$

Let $\lambda_t$, which has the economic interpretation as the shadow value of wealth, represent the multiplier for this constraint at time $t$. The first-order conditions are given below.

$$c_t \; : \; \frac{\partial u(c_t, l_t)}{\partial c_t} = \lambda_t\left(1-\xi_t\right)\frac{\overline{P}_t}{P_t} + \beta E_t\left[\lambda_{t+1}\frac{\overline{P}_t}{P_{t+1}}\xi_t\right] \qquad (6)$$

$$\xi_t \; : \; \lambda_t\frac{\overline{P}_t}{P_t}c_t = \lambda_t w_t Dh\left(\xi_t\right) + \beta E_t\left[\lambda_{t+1}\frac{\overline{P}_t}{P_{t+1}}c_t\right] \qquad (7)$$

$$l_t \; : \; \frac{\partial u(c_t, l_t)}{\partial l_t} = w_t\lambda_t \qquad (8)$$

$$b_t \; : \; \lambda_t = \beta E_t\left[\lambda_{t+1}\left(1+R_t\right)\frac{P_t}{P_{t+1}}\right] \qquad (9)$$

$$\gamma_t \; : \; v_t\lambda_t = \beta E_t\left[\lambda_{t+1}\left(v_{t+1} + z_{t+1}\right)\right] \qquad (10)$$

The first efficiency condition states that the marginal utility of consumption must be equated to the full cost of consuming. The full cost of consuming involves a weighted average of the costs of purchasing goods with currency and credit. The second efficiency condition equates the marginal benefit of raising $\xi$ – decreasing current expenditure on consumption – to its marginal cost – the sum of current time cost and future repayment.

## 2.2 Retailers

We assume that retailers create units of the final good according to a constant elasticity of substitution aggregator of a continuum of intermediate products on the unit interval. In general, this will imply that $c$ units of final consumption are generated according to $c_t = [\int c_t(x)^{\frac{\varepsilon-1}{\varepsilon}} dx]^{\frac{\varepsilon}{\varepsilon-1}}$, where $\varepsilon$ is a parameter which controls the degree of substitutability. In our setup, however, there will be groups of firms which will all charge the same price for their good within a period, so that they can be aggregated easily. Let the $j$th group have fraction $\omega_j$ and charge price $P_{jt}$. Then the retailer allocates its demands for intermediates across the $J$ categories, minimizing $[\sum_{j=0}^{J-1} \omega_j P_{jt} c_{jt}]$ subject $c_t = [\sum_{j=0}^{J-1} \omega_j c_{jt}{}^{\frac{\varepsilon-1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}$. As is well known, this cost minimization problem leads to intermediate input demands of the form

$$c_{jt} = (\frac{P_{jt}}{P_t})^{-\varepsilon} c_t \tag{11}$$

where the unit cost of production – an intermediate goods price level of sorts – is given by

$$P_t = [\sum_{j=0}^{J-1} \omega_j P_{jt}^{(1-\varepsilon)}]^{\frac{1}{1-\varepsilon}}. \tag{12}$$

This is the price index which we use as numeraire in the analysis above.

Since the retail sector is competitive and all goods are produced according to the same technology, it follows that the final goods price must satisfy:

$$\overline{P}_t = (1 + R_t) P_t \tag{13}$$

For each unit of sales, the retail firm receives revenues in money or credit. Each of these are cash flows which are effectively in date t+1 dollars. If the firm receives money, then it must hold it "over night." If the firm takes a credit card, then it is paid only at date t+1 with no interest charges.

## 2.3 Producers

The producers of intermediate products are assumed to be monopolistic competitors and face irregularly timed opportunities for price adjustment. For this purpose, we use a generalized stochastic price adjustment model due to Levin [1991], as recently exposited in Dotsey, King and Wolman's [1999] analysis of state dependent pricing. In this setup, a firm which has held its price fixed for $j$ periods will be permitted to adjust with probability $\alpha_j$. The model is flexible in that it contains the Taylor [1980] staggered price adjustment model as one special case (a four quarter model would set $\alpha_1 = \alpha_2 = \alpha_3 = 0$ and $\alpha_4 = 1$), the Calvo [1983] stochastic adjustment model as another (this setup makes $\alpha_j = \alpha$ for all $j$), and can be used to match microeconomic data on price adjustment. In a steady state situation, an economy with a continuum of firms will have a distribution with fractions $\omega_j$ which are determined by the recursions $\omega_j = (1-\alpha_j)\omega_{j-1}$ for $j = 1, 2, ...J-1$ and $\omega_0 = 1 - \sum_{j=1}^{J-1} \omega_j$.

Each intermediate product $x$ on the unit interval is produced according to the production function

$$y_t(x) = a_t n_t(x)$$

with labor being paid a nominal wage rate of $W_t$ and being flexibly reallocated across sectors. Nominal marginal cost for all firms is accordingly $W_t/a_t$.

Firms are assumed to maximize the present discounted value of their real profits (*current* profits are $\frac{Z_t(x)}{P_t} = \frac{P_t(x)}{P_t} y_t(x) - \frac{W_t}{P_t} n_t(x)$) given the intermediate product demand described by (11) above and the stochastic structure of nominal price adjustment.

The model economy is one in which all firms that are adjusting at date $t$ will choose the same price, which we call $P_t^*$. This price is determined as part of the solution to the firm's dynamic programming problem

$$v_t^0 = \max_{P_t^*} \left\{ \frac{P_t^* y_{0t} - W_t n_{0t}}{P_t} + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \alpha_1 v_{t+1}^0 + (1-\alpha_1) v_{t+1}^1 \right) \right] \right\}$$

where the maximization takes place subject to the demand curve and the production function

$$\begin{aligned} y_{0t} &= (\frac{P_t^*}{P_t})^{-\varepsilon} d_t, \\ y_{0t} &= a_t n_{0t}, \end{aligned}$$

where $d_t$ is aggregate demand in period $t$. Aggregate demand will be made up of consumption ($c_t$) and exogenous, unproductive government spending ($g_t$):

$$d_t = c_t + g_t.$$

A few comments about the form of the dynamic program are in order. First, consistent with the discussion of the household, the dynamic program deflates the firm's nominal profits by $P_t$ and establishes asset values using the multiplier $\lambda_t$, which is the household's shadow value of wealth. Second, the firm is constrained by its production function and by its demand curve, which depends on aggregate consumption and government demand. Third, the firm knows that there are two possible situations at date t+1. With probability $\alpha_1$ it will adjust its price and the current pricing decision will be irrelevant to its market value ($v^0_{t+1}$). With probability $1-\alpha_1$ it will not adjust its price and the current price will be maintained, resulting in a market value ($v^1_{t+1}$), with the superscript $j$ in $v^j_t$ indicating the value of a firm which is maintaining its price fixed at the level set at date $t-j$, i.e., $P^*_{t-j}$. Thus, we have for $j = 1, \ldots, J-2$,

$$v^j_t = \frac{P^*_{t-j} y_{jt} - W_t n_{jt}}{P_t} + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \alpha_{j+1} v^0_{t+1} + (1-\alpha_{j+1}) v^{j+1}_{t+1} \right) \right]$$

and

$$v^{J-1}_t = \frac{P^*_{t-(J-1)} y_{J-1,t} - W_t n_{J-1,t}}{P_t} + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} v^0_{t+1} \right],$$

where

$$y_{jt} = (\frac{P^*_{t-j}}{P_t})^{-\varepsilon} (c_t + g_t) \quad (14)$$

$$y_{jt} = a_t n_{jt}. \quad (15)$$

An optimal pricing decision therefore requires that

$$0 = \frac{1}{P_t} \frac{\partial Z_{0t}}{\partial P^*_t} + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} (1-\alpha_1) \frac{\partial v_{1,t+1}}{\partial P^*_t} \right]$$

which is the requirement that, at the optimum, a small change in price have a zero effect on the present discounted value. It is straightforward to show that

$$\frac{\partial v^j_{t+j}}{\partial P^*_t} = \frac{1}{P_{t+j}} \frac{\partial Z_{jt+j}}{\partial P^*_t} + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} (1 - \alpha_{j+1}) \frac{\partial v_{j+1,t+j+1}}{\partial P^*_t} \right].$$

for $j = 1, \ldots, J-2$ and that

$$\frac{\partial v^{J-1}_{t+J-1}}{\partial P^*_t} = \frac{1}{P_{t+J-1}} \frac{\partial Z_{J-1,t+J-1}}{\partial P^*_t}.$$

Further,

$$\frac{1}{P_{t+j}} \frac{\partial Z_{jt+j}}{\partial P^*_t} = \frac{1}{P_{t+j}} (\frac{P^*_t}{P_{t+j}})^{-\varepsilon} (c_{t+j} + g_{t+j}) \left( (1-\varepsilon) + \varepsilon \frac{w_{t+j}}{a_{t+j}} \left( \frac{P^*_t}{P_{t+j}} \right)^{-1} \right)$$

where $w_{t+j} \equiv \frac{W_{t+j}}{P_{t+j}}$ is the real wage at time $t+j$, $j = 0, \ldots, J-1$. Repeated substitution of these results into the optimal pricing decision implies that price setting requires

$$0 = E_t \left[ \sum_{j=0}^{J-1} \beta^j \omega_j \lambda_{t+j} \left( \frac{1}{P_{t+j}} \frac{\partial Z_{j,t+j}}{\partial P^*_t} \right) \right] \tag{16}$$

i.e., that there is zero expected, discounted reward from a slightly higher or lower price. In this expression, the weights $\omega_j$ serve as indicators of the probability of the price being held fixed for at least $j$ periods, which is $\omega_j/\omega_0 = (1-\alpha_1)(1-\alpha_2)...(1-\alpha_j)$.

## 2.4 The government

The government enters our economy in two very different manners as we proceed through the analysis. First, the government enters as an actor in various markets of the economy. Second, the government enters as a social planner. We discuss each of these roles in turn.

### 2.4.1 The government as an actor in goods and other markets.

The government in our model economy takes fiscal actions which are very are limited in scope and monetary policy actions which may be richer in their nature.

*Fiscal actions:* The government demands final goods $g_t$ in a stochastically-varying manner that is exogenous to the economy. It levies lump sum taxes to pay for these goods (the lump sum taxes are a determinant of the term $T_t$ in the household's budget constraint above).[5] As in many macroeconomic analyses—including the optimal fiscal and monetary policy work of Lucas and Stokey [1983]—we assume that government purchases have no effect on either the utility of households or the productivity of firms, but simply involve a use of resources.

*Monetary policy actions:* We assume that the government can vary the money supply $M_t$ in response to the underlying disturbances in the economy. The changes in $M_t$ could be made through direct transfers to households so that $T_t = P_t g_t - (M_t - M_{t-1})$. Given the timing structure of our economy, in which asset markets are open prior to goods markets, a form of Ricardian equivalence should hold, so that such direct transfers should be equivalent to open market operations.

Notice that we do not permit our government to explicitly levy taxes on or make subsidies to the households or firms in our economy, despite the fact that there are good reasons that a government might wish to in our economy. For example, given the monopoly distortion in our economy, one use of fiscal policy might be to subsidize intermediate goods producers and levy lump-sum taxes on households to pay for these subsidies. A subsidy would be viewed as desirable by the residents of the economy because it could stimulate intermediate goods production, counteracting the effects of the monopoly power which intermediate goods firms have.

Notice also that we do not permit our government to pay interest on its money. Such a policy would typically be desirable as well.

[5] By requiring the government to buy final, rather than intermediate goods, we are assuming that there is a common demand elasticity for private and public consumption of the sticky price goods, if we dropped the intermediate good interpretation.

We are also assuming that the government buys from retailers without using money.

### 2.4.2 The government as planner

The major focus of the paper is on the government as a planner, in particular on (i) how it would calculate the constrained optimal policy in our economy; and (ii) on the characteristics of equilibria under the optimal policy. When we take this perspective, we assume that the government's objective is to maximize the expected utility of households, subject to the general equilibrium of the model. But we carry along the assumptions that the government has a limited set of instruments, ruling out the use of non-lump sum taxes or subsidies and the payment of interest on money. In this sense, our government is active as a traditional monetary policy decision-maker, varying the quantity of money to accomplish macroeconomic objectives. At the same time, it is passive as a fiscal decision-maker. We adopt this strategy because we are interested in the design of monetary policies which can be implemented without requiring simultaneous fiscal actions or changes in the nature of current monetary instruments.

# 3 Four distortions

Our macroeconomic model has the property that there are four, readily identifiable routes by which nominal factors can affect real economic activity. We discuss these four distortions in turn, using general ideas that carry over to a wider class of macroeconomic models.

*Relative price distortions:* In any model with asynchronized adjustment of nominal prices, there are distortions that arise when the price level is not constant. First, note that the definition of the perfect price index –which applies to both consumption and government spending–means that we can write nominal expenditure as

$$\sum_{j=0}^{J} \omega_j P_{jt}[c_{jt} + g_{jt}] = P_t[c_t + g].$$

Second, note that a simple sum aggregate output measure is given by

$$y_t = \int_0^1 y_t(x)dx = \sum_{j=0}^{J} \omega_j y_{jt}$$

and that this implicitly defines an implicit deflator $\mathcal{P}_t$ as

$$\mathcal{P}_t y_t = \sum_{j=0}^{J} \omega_j P_{jt} y_{jt}$$

Third, since the current model makes output linear in labor input, then we know that aggregate output $y_t$ is simply related to aggregate labor input $n_t = \sum_{j=0}^{J} \omega_j n_{jt}$ so that the above may be combined to yield

$$c_t + g_t = \delta_t a_t n_t$$

The factor $\delta_t \equiv \mathcal{P}_t / P_t$ works like a productivity shock in an aggregate production function for our basic economy. In fact, this result carries over to the entire class of setups suggested by Yun [1996], in which firms have technologies that imply constant marginal cost and factors can be flexibly reallocated across sectors.[6] Variations in $\delta$ can be described in another complementary way. Using $y_{jt} = (P_{jt}/P_t)^{-\varepsilon}(c_t + g_t)$, we find that $\delta_t$ is related to relative prices:

$$\delta_t = [\sum_{j=0}^{J} \omega_j (P_{jt}/P_t)^{-\varepsilon}].$$

If all relative prices are unity, then $\delta$ takes on a value of one. If relative prices deviate from unity, which is the unconstrained efficient level given the technology, then $\delta_t$ measures the extent of lost aggregate output which arises for this reason.

*The markup distortion:* If all firms have the same marginal cost functions, then we can write

$$W_t = \Psi_t a_t f'(n_t)$$

where $W$ is the nominal wage, $\Psi_t$ is nominal marginal cost and $a_t f'(n_t)$ is the common marginal product of labor. If we divide by the perfect (intermediate good) price index, then this expression can be stated in real terms as

$$w_t = \psi_t a_t f'(n_t) = \frac{1}{\mu_t} a_t f'(n_t)$$

[6] For example, if the production function had constant returns to scale labor and capital, then an analysis along the lines of Yun [1996] indicates that the text equation would be modified to $c_t + g_t = \delta_t a_t f(k_t, n_t)$.

so that real marginal cost $\psi_t$ acts like a sales tax shifter in any such setup. In our model, the marginal product of labor is constant.

Some authors such as Woodford [1995], King and Wolman [1996] and Goodfriend and King [1997], have described this second source of distortions as in the second equality in terms of the average markup $\mu_t \equiv P_t/\Psi_t$, which is the reciprocal of real marginal cost $\psi_t$. These authors have stressed that the monetary authority has temporary control over this markup tax because prices are sticky, enabling it to erode (or enhance) the market power of firms in response to various disturbances. According to this convention, which we follow here, a higher value of the markup lowers real marginal cost and works like a tax on productive activity.

Note that $\delta_t$ and $\mu_t$ (or $\psi_t$) are not necessarily related closely together, so thinking about these from the standpoint of fiscal analysis – in which there can be separate shocks to the level of the production function and its marginal products – is the relevant background to this analysis, rather than reasoning from the effects of productivity shocks which traditionally shift both in RBC analysis.

*Inefficient shopping time*: The next distortion is sometimes referred to as "shoe leather costs." But in our model, it is really "shopping time costs," as in McCallum and Goodfriend [19xx], since it is in time rather than goods units. Using the notation above, it is

$$h_t = \int_0^{\xi_t} dh\left(x_t, s_t\right) \tag{17}$$

From the standpoint of our economy, variations in $h_t$ work like a shock to the economy's time endowment. Pursuing the fiscal analogy discussed above, this is similar to a conscription (lump sum labor tax).

*The wedge of monetary inefficiency:* In transactions-based monetary models, there is also an effect of monetary policy on the full cost of consumption. Beginning with the efficiency condition (6), using the bond efficiency condition (9) to eliminate the expectations term and substituting out the pricing of the final product using (13), we arrive at a simple version of the efficiency condition for consumption:

$$D_1 u\left(c_t, l_t\right) = \lambda_t \left[1 + R_t \left(1 - \xi_t\right)\right]. \tag{18}$$

This equation expresses the wedge of monetary inefficiency as the product of the nominal interest rate and the extent of monetization of exchange $(1-\xi_t)$.

Pursuing the fiscal policy analogy discussed above, it is like a consumption tax relative to the non-monetary model.

# 4 Optimal policy

Our analysis of optimal policy is in the tradition of Ramsey [1928] and draws heavily on the modern literature on optimal policy in dynamic economies which follows from Lucas and Stokey [1993]. In general, the idea of optimal policy design is for the government to maximize expected utility subject to the conditions of dynamic equilibrium and the constraints on its instruments. Working in a dynamic competitive equilibrium setting, Lucas and Stokey showed the power of a multi-stage approach. First, one determines the conditions that circumscribe competitive equilibrium for arbitrary policies: in their initial analysis of a real economy subject to fiscal shocks, the relevant conditions which implicitly determined quantities and relative prices included the efficiency conditions of firms and households as well as the resource constraints of the economy plus private and public budget constraints. Second, these conditions were manipulated to eliminate all tax rates and relative prices, leaving only a group of constraints on real quantities. Third, the government maximized expected utility subject to the constraints on real quantities: this determined a unique path for real quantities. Fourth, relative prices and tax rates were determined which led these outcomes to be the result of a dynamic competitive equilibrium.

In this paper, as in King and Wolman [1999], we adopt a similar approach to an economy which has real and nominal frictions: monopolistic competition, price stickiness and the costly conversion of wealth into goods that can be altered by money holding. The outline of our multi-stage approach is as follows. First, we have previously determined the efficiency conditions of households and firms that restrict dynamic equilibria in our economy, as well as the various budget and resource constraints. Second, we manipulate these equations to determine a smaller subset of restrictions that governs real quantities. However, for various reasons, we find it convenient to leave in one relative price – the multiplier on the households budget constraint– and two nominal variables–the inflation rate and the nominal interest rate– in the subset of equations. Third, we maximize expected utility subject to these constraints. Fourth, we find the remaining absolute prices and monetary policy actions which lead these outcomes to be the result of dynamic equilibrium.[7]

### 4.1 Organizing the restrictions on dynamic equilibrium

We begin by combining the household's first-order conditions with retailers' zero profit conditions. Using (9), (8) and (13) we eliminate expectations from (6) and (7).

$$\frac{\partial u(c_t, l_t)}{\partial c_t} = \lambda_t \left(1 + R_t \left(1 - \xi_t\right)\right) \tag{19}$$

$$\lambda_t R_t c_t = \frac{\partial u(c_t, l_t)}{\partial l_t} Dh\left(\xi_t\right). \tag{20}$$

We rewrite the Euler equation (9) as

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \frac{1 + R_t}{\Pi_{t+1}}\right], \tag{21}$$

where $\Pi_t$ is the gross inflation rate:

$$\Pi_t = P_t / P_{t-1}.$$

Next, combining equations (2) and (4) we have a consolidated time constraint for the household in (22).

$$n_t + l_t + \int_0^{\xi_t} dh_t = \phi. \tag{22}$$

The constraints upon the policymaker that arise directly from the household or retailers are (19) - (22). Equation (10) is an asset-pricing equation that determines the real share price, $v_t$, to ensure that the household holds the market portfolio thereby ensuring that it receives all profits. As such, we may ignore it in the analysis of optimal policy. Equation (8) will be used to eliminate the real wage in the firm's asset-pricing equation to which we turn next.

[7] We do not consider the possibility that optimal policy might involve randomization, as suggested by Dupor [1999].

We eliminate nominal and wage terms from the optimal firm price-setting equation. Multiplying (16) by $P_t^*$ and manipulating the result using (8), (15) and (14) we have (23).

$$0 = E_t \sum_{j=0}^{J-1} \beta^j \omega_j X\left(\lambda_{t+j}, y_{j,t+j}, c_{t+j}, l_{t+j}, g_{t+j}, a_{t+j}\right) \tag{23}$$

where

$$X\left(\lambda, y_j, c, l, g, a\right) = \left[(1-\varepsilon)\left(\frac{y_j}{c+g}\right)^{\frac{\varepsilon-1}{\varepsilon}} \lambda + \varepsilon \frac{\partial u\left(c, l\right)}{\partial l} \frac{1}{a}\left(\frac{y_j}{c+g}\right)\right](c+g).$$

As discussed above, in any period $t$, $(1-\alpha_j)$ fraction of firms that set their prices $j$ periods ago, $j = 1, \ldots, J-2$, will be unable to reset their prices. As a result, their price in period $t$, $P_{t-j}^*$, is unchanged from their price in the previous period, $P_{t-(j-1)}^*$. Equation (14) then implies that the demand for the output of such firms will evolve over time according to

$$y_{j,t}\Pi_t^{\varepsilon} = y_{j-1,t-1}\frac{(c_t+g_t)}{c_{t-1}+g_{t-1}}. \tag{24}$$

That is: the past behavior of various real quantities is one way of summarizing the past nominal prices which are the natural state variables of sticky price models.

Next, noting $n_t = \sum_{j=0}^{J-1} \omega_j n_{jt}$ and using (15), we find that

$$a_t n_t = \sum_{j=0}^{J-1} y_{jt}. \tag{25}$$

Finally, the final goods production function is our last constraint.

$$(c_t + g_t) = [\sum_{j=0}^{J-1} \omega_j y_{jt}{}^{\frac{\varepsilon-1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}} \tag{26}$$

## 4.2 Posing the optimal policy problem

Two of the implementation conditions, the household's Euler equation (21) and the firm's price-setting condition (23) introduce expectations of future variables into the time $t$ constraint set. Thus the set of feasible policies for the monetary authority are constrained by the expectations of the private sector. The unusual nature of these constraints requires us to reformulate them prior to solving the optimal monetary policy problem. We begin by introducing $\Omega_t$ and $\Phi_t$ as the multipliers for (21) and (23), respectively. The optimal policy problem then solves

$$
\begin{aligned}
&\min_{\{\Phi_k,\Omega_k\}_{k=0}^{\infty}} \max_{\{\mathcal{X}_k\}_{k=0}^{\infty}} E_t\left\{\sum_{k=0}^{\infty}\beta^k\left(u\left(c_{t+k},l_{t+k}\right)\right.\right. \qquad (27)\\
&+\Phi_k\mathbf{E}_{t+k}\left[\sum_{j=0}^{J-1}\beta^j\omega_j\mathbf{X}\left(\lambda_{t+k+j},y_{j,t+k+j},c_{t+k+j},l_{t+k+j},g_{t+k+j},a_{t+k+j}\right)\right]\\
&\left.\left.+\Omega_k\mathbf{E}_{t+k}\left[\lambda_{t+k}-\left(1+R_{t+k}\right)\beta\frac{\lambda_{t+k+1}}{\Pi_{t+k+1}}\right]\right)\right\}
\end{aligned}
$$

where $\chi_k = \left\{c_{t+k},\xi_{t+k},l_{t+k},n_{t+k},\lambda_{t+k},\Pi_{t+k},R_{t+k},\left(y_{j,t+k}\right)_{j=0}^{J-1},\right\}$ and the solution is subject to the constraints (19) - (20), (22), (24) - (26) with $(y_{0,t-1},\ldots,y_{J-2,t-1},c_{t-1},g_{t-1},R_{t-1},a_t,g_t,z_t)$ given in each period $k=0,1,\ldots$.

This problem is inherently nonstationary. Kydland and Prescott [1980] began the analysis of how to describe such problems using recursive methods. Important recent work by Marcet and Marimon [1999] formally develops a recursive approach to such problems. Following their method, we convert our dynamic optimization problem into a recursive saddlepoint problem. We re-organise the terms in (27) involving expectations of future variables at time $t$. Grouping expectations of variables sharing the same date, we apply the law of iterated conditional expectation. Moreover, in re-organizing the constraints, we add terms involving variables dated before period $t$ to ensure that the constraints in the first $J-1$ periods are identical to those appearing in subsequent periods. This requires the introduction of lagged multipliers into the problem. As is well known, these lagged multipliers effectively convert a nonstationary problem into a stationary or recursive one. The nonstationarity that would otherwise arise originates through initial conditions – here predetermined nominal prices – that the monetary authority may exploit. Augmenting the optimal policy problem with lagged multipliers does not necessarily eliminate this initial period issue, this depends on their starting values. However, the introduction of lagged multipliers as state variables does allow us to analyse a recursive problem. We formulate the optimal monetary policy problem, under commitment, as solving

$$\min_{\{\Phi_k,\Omega_k\}_{k=0}^{\infty}} \max_{\{\mathcal{X}_k\}_{k=0}^{\infty}} E_t\Bigg\{\sum_{k=0}^{\infty}\beta^k\Bigg(u\left(c_{t+k},l_{t+k}\right) \\ +\sum_{j=0}^{J-1}\beta^j\Phi_{k-j}\omega_j\mathbf{X}\left(\lambda_{t+k},y_{j,t+k},c_{t+k},l_{t+k},g_{t+k},a_{t+k}\right) \\ +\left(\Omega_k-\Omega_{k-1}\frac{1+R_{t+k-1}}{\Pi_{t+k}}\right)\lambda_{t+k}\Bigg)\Bigg\} \tag{28}$$

where $\chi_k$ is unchanged from (27), the constraints are (19) - (20), (22), (24) - (26) as before. However, it is now the case that $\left(y_{0,t-1},\ldots,y_{J-2,t-1},c_{t-1},g_{t-1},R_{t-1},a_t,g_t,z_t,\Phi_{t-1},\ldots,\Phi_{t-(J-1)},\ \Phi_{t-1}\right)$ is given. Note that (28) does not contain constraints on the choice of current variables that involve the expectations of future quantities or prices.

## 4.3 Computation

We determine the efficiency conditions for the policy maker, given the problem posed in the previous subsection. We use these conditions in two ways. First, assuming certainty, we solve for a steady state. Second, we linearize around the steady state in order to determine the nature of response to various disturbances.

After we calculate the solution to the optimal policy problem, it is also direct to calculate the values of various nominal variables that are relevant to monetary policy. For example, we can study the behavior of the price level by exploiting two aspects of the economy. First, given that the optimal policy problem determines quantities, we can determine the relative price of an adjusting firm, $p_{0t}$, using the demand behavior of retailers ($p_{0t}=(y_{0t}/y_t)^{-1/\varepsilon}$). Second, we can determine the behavior of the price level using the predetermined nominal prices, the current relative price $p_{0t}$, and the price level (12) determined in our analysis of the retailer's problem. Given that (28) determines $\xi_t$ and $R_t$, we can also calculate the retail price level, the real quantity of money, and the nominal quantity of money that are associated with the constrained optimal policy.

# 5 Optimal policy in two special cases

In two special cases of the model, one can characterize optimal policy analytically. One of those cases is well known: if prices are flexible it is optimal to equate the private and social costs of money holding, which means keeping the net nominal interest rate equal to zero – the Friedman rule. In the second special case, the distortions associated with money demand are assumed to be arbitrarily small. There we can show that under a familiar elasticity condition on preferences it is optimal for the price level to be constant in response to productivity shocks.

## 5.1 Flexible Prices

To make prices flexible, set $\omega_0 = 1$. This immediately eliminates the relative price distortions, since every firm charges the same price. The markup distortion is still present, but it cannot be affected by the monetary authority: the markup is constant across time and across states at $\varepsilon / (\varepsilon - 1)$. The only distortions that the monetary authority can affect are shopping time and the wedge of monetary inefficiency. Zero nominal interest rates eliminate both of these distortions, hence zero nominal interest rates represent optimal policy.[8] The only novel feature here is that the presence of monopolistic competition makes the Friedman rule outcome second-best. In a sense, the monetary authority would like to make the nominal interest rate negative, to offset the monopoly inefficiency. Of course the nominal interest rate cannot be negative. However, this incentive implies that in the full model with sticky prices, it *may* still be optimal to pursue the Friedman rule.

## 5.2 Absence of Monetary Distortions

If the time costs of credit are such that the shopping time and monetary wedge distortions vanish regardless of the level of interest rates, the conditions describing optimal policy simplify dramatically. King and Wolman

[8] Cole and Kocherlakota [1998] discuss policies that actually *implement* the Friedman rule.

[1999] showed that price stability is optimal in the long run for a particular specification of preferences. In fact one can derive a sharper result, for the case where government spending is absent.

To derive the result, we proceed as in Adao, Correia and Teles [2000]. That is, we impose a constant price level on the equations describing optimal policy, and examine the conditions under which these equations are satisfied. The key implication of a constant price level across states is that the quantities produced by firms that set their prices at different times are identical. That is, $c_{j,t} = c_t$ and $n_{j,t} = n_t$. This implication allows us to derive the following condition, under which a constant price level is optimal:

$$\begin{aligned} & D_t \text{ is constant across time and states,} \\ & \text{where} \\ D_t \equiv & \left(\frac{c \cdot u_{cc}}{u_c} - \frac{c \cdot u_{cl}}{u_l}\right) - \frac{n}{l}\left(\frac{l \cdot u_{cl}}{u_c} - \frac{l \cdot u_{ll}}{u_l}\right). \end{aligned}$$

This condition is clearly satisfied if preferences are constant elasticity and separable between consumption and leisure.

# 6 Choice of parameters

Given the limited amount of existing research on optimal monetary policy using the approach of this paper and the starkness of our model economy, we have chosen the parameters with two objectives in mind. First, we want our economy to be as realistic as possible so that we calibrate certain parameters to match certain features of the U.S. economy as discussed below. Second, we want our economy to be familiar to economists who have worked with related models of business cycles, fiscal policy, money demand, and sticky prices. Our benchmark parametric model is as follows, with the time unit taken to be one quarter of a year:

## 6.1 Preferences

We assume the utility function is logarithmic, with a share parameter chosen so that a real economy would have individuals working one-fifth of the time. We assume also that the discount factor is such that there would be a one percent real interest rate per quarter (four percent annual real interest rate).

$$\begin{aligned} u(c,l) &= \ln c + 3.3 \ln(l) \\ \beta &= 0.99 \end{aligned}$$

We later explore some implications of a higher labor supply elasticity, assuming that $u(c,l) = \ln c + 3.3l$ which may be rationalized by indivisible labor as in Rogerson [1988] and Hansen [1985].

## 6.2 Monopoly power

There is a substantial amount of dispute about the extent to which firms have market power. In this analysis, we assume that the markup would be 10% over marginal cost in a real economy, as suggested by various recent studies. Since the gross markup is $\mu = \frac{\varepsilon}{\varepsilon - 1}$, this implies that

$$\varepsilon = 10$$

We later explore some implications of a lower elasticity of demand.

## 6.3 Distribution of price-setters

A key aspect of our economy is the extent of exogenously imposed price stickiness. We use a distribution suggested by Wolman [2000], which has the following features. First, it implies that firms expected a newly set price to remain in effect for five quarters. That is: the expected duration of a price chosen at $t$, which is $\alpha_1 1 + (1-\alpha_1)\alpha_2 2 + (1-\alpha_1)(1-\alpha_2)\alpha_3 3 + ...$ is equal to 5. Second, this estimate is consistent with the recent empirical work on aggregate price adjustment dynamics by Gali and Gertler [1999] and Sbordone [1998]. Third, rather than assuming a constant hazard $\alpha_i = \alpha$ as in the Calvo model, our weights involve an increasing hazard, which is consistent with available empirical evidence and recent work on calibrated models of state dependent pricing. The particular adjustment probabilities $\alpha_i$ and the associated distribution are:

**Table 1:**
**Price adjustment probabilities**
**and the associated distribution weights**

| $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\alpha_5$ | $\alpha_6$ | $\alpha_7$ | $\alpha_8$ | $\alpha_9$ |
|---|---|---|---|---|---|---|---|---|
| 0.014 | 0.056 | 0.126 | 0.224 | 0.350 | 0.504 | 0.686 | 0.897 | 1 |
| $\omega_0$ | $\omega_1$ | $\omega_2$ | $\omega_3$ | $\omega_4$ | $\omega_5$ | $\omega_6$ | $\omega_7$ | $\omega_8$ |
| 0.198 | 0.195 | 0.184 | 0.161 | 0.125 | 0.081 | 0.040 | 0.012 | 0.001 |

For a distribution of price adjustment, we can also calculate the average age of a price which is in place, $\sum_{j=0}^{9} j\omega_j$. This average is 2.3 for the benchmark parameterization.

We explore some implications of assuming much greater price flexibility below.

## 6.4 Credit costs and the demand for money:

In our economy, the distribution of credit costs implies the form of the demand for money. Since there has been much work on the demand for money, we use this aspect of the U.S. data to restrict the parameters of the credit cost function. That function had the form $\nu_0 x^{1/\nu_1} - \nu_2$,when the shock term is set to zero. Optimal credit use implies that $\xi = [\frac{\frac{Rc}{w}+\nu_2}{\nu_0}]^{\nu_1}$, with the demand for money taking the form $m = c(1-\xi)$. Our benchmark parameterization is that

$$\nu_0 = 0.131, \nu_1 = 1/9, \nu_2 = 0.0249.$$

These parameters were selected to match three facts.

First, at current interest rate levels, we wanted to match currency holdings by U.S. citizens (as estimated by Porter and Judson [1996] and studied by Jefferson [2000]) relative to aggregate consumption expenditure on nondurables and services. Our calculations indicated that domestically-held currency was about one-sixth of quarterly expenditure over the past ten years or so. Equivalently, we wanted our model to have a consumption velocity of a narrow monetary aggregate of 6.32, which is representative of recent of recent U.S. history. By using a narrow monetary aggregate (and only the domestically held portion of it), we have a velocity number which is much smaller than others which are sometimes employed. For example, since only about one-half of currency is held domestically, our target for $m/c$ would have been twice as high if we had used total currency. Further, if we had used a broader monetary aggregate, it is easy to make $m/c$ exceed one. Thus, one reaction to our parameters is that the "money intensity" of our model economy is too low. However, note that our economy will have a typical individual holding currency with value of more than two weeks of expenditure at current interest rate levels.

Second, many studies suggest that the demand for currency is not too sensitive to the nominal interest rate. We chose the parameters above to make the interest semielasticity of money demand roughly equal to $-1.0$.

Third, we wanted the money demand structure not to produce demonetization at rates of inflation below 10 percent per quarter. One can calculate that there is complete use of credit if $\frac{Rc}{w}+\nu_2=\nu_0$, so setting a specific level of the demonetization interest rate provides one restriction on the parameters.

Taking all of these considerations into account, our parameter choices imply that the fraction of goods purchased with credit is 0.84 at a six percent annual inflation rate. It also implies that the amount of transactions time used is a very small number (approximately one hundredth of a percent of work time). In some of the analysis below, we explore the implications of making (a) the money intensity higher ($m/c$ higher) while maintaining the semi-elasticity; (b) increasing the semi-elasticity while make the money intensity higher; and (c) raising both simultaneously. All of our experiments, however, imply that there is a relatively small amount of transactions time.[9]

# 7 Optimal policy in the long run

The preceding discussion established two reference points for thinking about optimal policy in the long run. The first reference point is Friedman's [1969] celebrated conclusion that the nominal interest rate should be sufficiently close to zero so that the private and social costs of money-holding coincide. At this point, the economy minimizes the costs of decentralized exchange.

[9] In experiments conducted after this conference draft was largely complete, we have concluded that this aspect of our model is a result of three basic factors: our requirement that we match $c/m \approx 6.32$ at a six percent nominal interest rate; our requirement that there be an elasticity of about -1 at that point; and the functional form that we have employed. In future research, we plan to alter the functional form so that we can maintain the first two factors, without requiring that transactions time be small. We also plan to estimate the parameters of this generalized functional form using U.S. time series data, building on the earlier analysis of Lucas [2000] and Wolman [1997] of broader monetary aggregates.

The second reference point is an average rate of inflation of zero, which minimizes relative price distortions in steady state: this is therefore the optimal long-run rate in the absence of the distortions which Friedman highlighted. In this section, we document the intuitive conclusion that the long-run inflation rate should be negative – but not as negative as suggested by Friedman's analysis – when both sticky price and exchange frictions are present.

In particular, when we solve the optimal policy problem for the benchmark model, we determine that the asymptotic rate of inflation – the steady state under the optimal policy – is negative twenty-five basis points ($-.25\%$ at an annual rate). Given that we assume a steady state real interest rate of 4 percent (as determined by time preference), the long-run rate of nominal interest is 3.75%. Hence, the long run more closely resembles the zero inflation case than it does the Friedman rule under the benchmark parameter values.

This result raises two sets of questions. First, how do the four distortions isolated earlier in the paper contribute to this finding? Second, how do variations away from the benchmark parameter values affect the optimal long-run inflation rate? Each of these questions is addressed in a subsection below.

## 7.1 Behind the benchmark long-run inflation rate

In order to look behind the benchmark zero inflation steady-state, we think it is useful to take three steps. First, we consider how the economy would work in the zero-inflation steady state, even if this is not optimal. Second, we consider reevaluating the optimal policy problem if one or more of our four distortions is eliminated as a consideration for the monetary authority. Third, having isolated relative price distortions as a key feature under this benchmark set of parameter values, we look further into how these distortions depend on the steady state rate of inflation.

### 7.1.1 The (suboptimal) zero inflation steady state

If there is zero inflation in the benchmark economy, then it is relatively easy to determine the levels of the four distortions.

Let us start by considering the effects of of sticky prices and imperfect competition: The *markup* is equal to that which prevails in the static monopoly problem, $\mu = \frac{\varepsilon}{\varepsilon - 1} = 1.10$ so that price is ten percent higher than real marginal cost in the steady-state. There are *no relative price distortions* – all firms are charging the same, unchanging price $P^*$– so that $\delta = 1$. In this situation, the nominal and real interest rates are each equal to 4% per annum. The parameters of the credit cost technology imply that 84 percent of transactions are financed with credit ($\xi = .84$) and that the ratio of real money to consumption is about 16.2 percent.

Let us next consider the effects of costly exchange of wealth for goods: *The wedge of monetary inefficiency* is positive, but relatively small in this steady state. It is calculated from the above discussion as

$$(1 + (1 - \xi) * R) = (1 + (1 - .84) * .01) = 1.0016$$

where the calculation of the wedge uses the quarterly nominal interest rate .01. From the discussion above, we know that the *time cost* $h$ is an extremely small number. At zero inflation, time costs associated with use of credit are less than one hundredth of a percent of labor time.

Even though the distortions associated with money demand are small at zero inflation, a monetary authority maximizing steady-state welfare would nonetheless choose a lower the rate of inflation, for the reasons stressed by Friedman [1969].

### 7.1.2 Optimal inflation with fewer distortions

We now imagine altering the monetary authority's problem – relative to the benchmark case – by selectively eliminating one or more distortions. For some of these modifications, there is an easy economic interpretation of our modified problem. For example, if we assume – as in King and Wolman [1999] – that there is interest on money at just below the market rate then there are no money demand distortions (no wedge and no resource costs). But to track down the origins of the benchmark inflation rate, it is sometimes necessary to consider other more abstract, modifications. Table 2 shows the effect of various modifications of the mix of distortions.

**Why is disinflation desirable?** Starting with the zero inflation steady state rate of inflation, the Table shows that both the wedge of monetary inefficiency and time costs play a role in reducing the inflation rate from zero to the benchmark level of -.25%.

*No variation in the wedge of monetary inefficiency:* Our discussion of the wedge of monetary inefficiency stressed that it captured the full price of converting wealth into final goods consumption, so that it was the product of the intensity of monetary exchange $(1-\xi)$ and the opportunity cost of holding money $R$. We now explore the implications of eliminating this wedge for the optimal rate of inflation. Mechanically, we fix the wedge at zero and resolve the monetary authority's optimal policy problem. One rationalization of this procedure is that there is a consumption subsidy, introduced into the household's problem and then varied in a manner that would neutralize the wedge of monetary inefficiency, i.e.,

$$(1+(1-\xi_t)*R_t)(1+\tau_t^c)=1$$

Table 2 shows that there is a significant influence of this distortion on the optimal long-run rate of inflation. If it is eliminated by itself, then the inflation rate rises from -.25% to -.10%, so that the wedge accounts for over one-half of the deviation from zero inflation.

*Resource costs of credit:* We can similarly eliminate the resource costs of credit usage from the optimal policy problem. Above, we used the idea that the wedge of monetary inefficiency is like a tax, so that it could be neutralized by a countervailing tax. In this case, we must envision a perturbation of the economy's resource constraint so that as the inflation rate is varied there are no effects on the economy's opportunities for work and leisure. That is: we must view the right-hand side of

$$l_t+n_t=\phi-h_t$$

as invariant the policymaker's choices. One possible interpretation is that a fiscal authority is adjusting the extent of a lump-sum confiscation of time to accomplish this elimination of resource costs of credit usage.

If we eliminate the resource costs by themselves, then the inflation rate rises from -.25% to -.15%, so that time costs account for somewhat less than one-half of the deviation from the zero inflation position.

**Why is there less deflation than at the Friedman Rule?** If prices are flexible, then the Friedman rule is optimal even though there is imperfect competition. In fact, Goodfriend [1997] notes that a positive markup makes the case stronger in a sense because the additional labor supply induced by declines in the wedge and time costs yield a social marginal product of labor which exceeds the real wage.

To evaluate why there is a benchmark rate of inflation of -.25% per annum – as opposed to a Friedman rule level of -4% per annum – it is necessary to eliminate either the relative price distortion or the markup distortion. We suppose that the policy maker cannot alter the average markup of firms, but can influence all of the other distortions. Why might this be the case? We have stressed that the markup acts like a sales tax, so one possibility is that an explicit sales tax is levied on intermediate goods producers and that it is varied so that

$$(1+\tau_t^i)\frac{1}{\mu_t} = \frac{\varepsilon}{\varepsilon - 1}$$

i.e., so that the markup always stays at its zero inflation level (the level also consistent with imperfect competition but no price stickiness).

With the markup distortion fixed, Table 2 shows that there is a slightly more negative rate of inflation. This finding is consistent with results of previous studies which documented that the average markup (i) is decreasing in the inflation rate near zero inflation; and (ii) does not respond importantly to variations in the inflation rate near zero inflation. The first finding of the previous studies explains why eliminating the distortion makes the optimal inflation rate more negative, since the monetary authority does not encounter an increasing markup in the modified problem as it lowers the inflation rate from a starting point of zero. The second finding explains why the effect is a small one quantitatively: since the price adjustment decisions of firms are forward-looking, the markup is not too affected by the trend rate of inflation.

### 7.1.3 Assessing Relative Price Distortions

Given that relative price distortions play a major role in the determination of the steady-state inflation rate, it is desirable to investigate more closely how these depend on the extent of price stickiness and other factors. There are three ingredients of the relationship between $\delta$ and the inflation rate. First, in an inflationary steady-state, relative prices are linked together by

$$\frac{P_{jt}}{P_t} = \frac{P_{t-j}^*}{P_{t-j}\Pi^j} = p_0\Pi^{-j}$$

where $\Pi$ is the gross inflation rate. Second, the definition of the price level implies that

$$1 = [\sum_{j=0}^{J} \omega_j (P_{jt}/P_t)^{1-\varepsilon} = p_0^{1-\varepsilon}[\sum_{j=0}^{J} \omega_j \Pi^{j(\varepsilon-1)}]$$

where the second line involves the use of the steady-state behavior of relative prices. This equation implicitly determines $p_0$ as a function of $\Pi$. Third, the $\delta$ measure may be written as

$$\delta_t = [\sum_{j=0}^{J} \omega_j (P_{jt}/P_t)^{-\varepsilon}] = p_0^{-\varepsilon}[\sum_{j=0}^{J} \omega_j \Pi^{j\varepsilon}]$$

where the second equality again follows from using the steady-state relative prices.

These expressions can be used to approximate the steady-state measure of relative price distortions as

$$\log(\delta) \approx -\frac{\varepsilon v}{2}(\Pi - 1)^2,$$

where $v = \sum_{j=0}^{J} j^2 \omega_j - [\sum_{j=0}^{J} j\omega_j]^2$ is a measure of the variance of the "age" of prices. The quality of this approximation can be evaluated since it is possible to calculate $\delta$ exactly and we have found that it is quite accurate for inflation rates between -1% and 10%, which would correspond to annual inflation rates of -4% to 40%.

This simple expression has a number of intuitive features. First, for small changes in the inflation rate near zero, there is no effect on the measure of relative price distortions.[10] For this reason, it is natural to conjecture that there will always be deflation in a setting which combines sticky prices and monopolistic competition with the costly conversion of wealth into consumption, although a markup which decreases with inflation can provide a disincentive for deflation. Second, distortions are larger if there is greater disparity between firms. Third, distortions are larger if there is a larger demand elasticity, which means that the inflation-induced changes in relative prices have a larger effect on the distribution of output across firms.

The relative price distortion implications of some commonly employed models of price adjustment due to Taylor [1980] and Calvo [1983] are easily

[10] $\frac{\partial \log(\delta_t)}{\partial \Pi} \approx -\varepsilon v(\Pi - 1)$ , which is zero at $\Pi$=1

evaluated using this formula. Each of these models has a single parameter which determines the distribution of prices by age. For the Taylor model, it is the length that every firm's price is held fixed, $J$, and the mean is $[\sum_{j=0}^{J} j\omega_j] = \frac{1}{J}[\sum_{j=0}^{J} j] = \frac{J-1}{2}$. For the Calvo model, it is the probability of price adjustment, $\alpha$, and the mean is $[\sum_{j=0}^{J} j\omega_j] = \alpha[\sum_{j=0}^{J} j(1-\alpha)^j] = \frac{1-\alpha}{\alpha}$. Further, letting the mean be $\overline{j}$ it can be shown that the variance measures take on the values $v = \frac{1}{3}(1+\overline{j})\overline{j}$ for the Taylor model and $v = (1+\overline{j})\overline{j}$ for the Calvo model. Accordingly, if the average age of prices is about 4 quarters, as suggested by the estimates of Gali and Gertler [1999] using the Calvo model, then $v = 20$. By contrast, a five quarter Taylor model – which implies that $[\sum_{j=0}^{J} j\omega_j] = 2$ – would have $v = 2$. These two common models suggest quite different costs of disinflation at the Friedman rule rate, if this is assumed, as in the balance of our analysis, to be -4% per annum (-1% per quarter) so that $\Pi - 1 = .01$. With a demand elasticity of $\varepsilon = 10$ then the formula implies that

$$\log(\delta) \approx -\frac{\varepsilon v}{2}(\Pi - 1)^2 \approx -\frac{10 * 20}{2}(.01)^2 = .01$$

or 1% of steady-state consumption for the Calvo specification. By contrast, with the five quarter Taylor structure, the welfare cost is only .1%.

Our benchmark parameterization implies that $\sum_{j=0}^{J} j\omega_j = 2.3$ and $v = \sum_{j=0}^{J} j^2\omega_j - [\sum_{j=0}^{J} j\omega_j]^2 = 3.3$, so that the $\delta$ lies between the example values given above: it is .165%. The relative price distortion at the Friedman rule is thus large (measured in terms of output) compared to the money demand distortions at zero inflation (discussed above). It is not surprising then that the solution to the optimal policy problem puts inflation much closer to zero than to the Friedman rule (this reasoning is informal, as the monetary authority balances *marginal* distortions).

## 7.2 Sensitivity Analysis

We now explore the sensitivity of the steady-state rate of inflation to various structural features of the model.

- Monopoly power: decreasing the demand elasticity ($\varepsilon$) to 4 leads to a larger deflation, 1.2% per year. This is to be expected, given the expression derived above for the relative price distortion: decreasing

$\varepsilon$ by a factor of 0.4 generates a corresponding decrease in the relative price distortion at any inflation rate. The money demand distortions become relatively more important, pushing the optimum closer to the Friedman rule.

- Labor supply elasticity: with infinite labor supply elasticity, policy moves just a bit closer to the Friedman rule; steady state has 32 basis points of deflation per year.

- Share of government spending: in our benchmark calibration, there is no government spending in the steady state. If the share of government spending rises to thirty percent, the steady state moves toward zero inflation – it involves deflation of 18 basis points per year. We conjecture that this occurs because an increase in government spending reduces the labor supply elasticity in response to a change in the real wage (our specification implies that the leisure demand elasticity is equal to negative one and the labor supply elasticity is $-l/n$ times this value; an increase in $g$ raises the level of $n$ and lowers the level of $l$, thus lowering this elasticity on both counts).

- Interest elasticity of money demand (loosely): we change the parameters of the function $dh(x)$ so that in a steady state with a six percent nominal interest rate the interest semi-elasticity of money demand is $-4$, with velocity remaining relatively high, at 5.6. This results in an optimal steady state with deflation of 70 basis points per year.

- Velocity of money (loosely): we change the parameters of the function $dh(x)$ so that in a steady state with a six percent nominal interest rate velocity is 1.15, with the interest semi-elasticity of money demand remaining low, at $-1.3$. This results in an optimal steady state with deflation of 1.5% per year.

- Interest elasticity *and* velocity: we change the parameters of the function $dh(x)$ so that in a steady state with a six percent nominal interest rate velocity is 1.52 and the interest semi-elasticity of money demand is $-2.5$. This results in an optimal steady state with deflation of 1.72% per year.

- Price stickiness: we change the distribution of prices ($\omega$) to $[0.4, 0.35, 0.25]$. With this distribution, the expected duration of a newly adjusted price is 2.5 quarters. The optimal deflation rate is 1.2%.

# 8 Dynamics under optimal policy

We now discuss the dynamics of the model under optimal policy, local to the benchmark steady state described above. Impulse response functions are presented for three shocks: to the level of productivity, to government spending (aggregate demand), and to the distribution of credit costs (money demand). We also compare the dynamics under optimal policy to what occurs under two simple policy rules, and investigate the robustness of the benchmark dynamics to a change in the parameters governing money demand.

## 8.1 Responses to three shocks

Figures 1, 2 and 3 illustrate how the economy behaves under optimal policy in response to persistent shocks to productivity, aggregate demand, and money demand. As a reference point in these figures, we plot the behavior of real variables that would occur in a real version of the model – with flexible prices and no money demand distortions. For each of these shocks, there is some movement in the price level, but it is negligible. With prices relatively stable, it follows that the real and nominal interest rates essentially move together.

With respect to the money demand shock, it may seem surprising that real variables move at all. Conventional wisdom (e.g., Poole [1970]) holds that monetary policy should smooth the nominal interest rate in response to money demand shocks and allow the quantity of money to adjust to the new demand. However, that conventional wisdom is associated with reduced form money demand shocks. Here, the shock impinges on the transactions cost, and thus under optimal policy it affects real quantities. Agents consume slightly less in response to the higher cost of transacting with credit. If prices were flexible, the transactions cost would always be irrelevant because nominal interest rates would be zero under optimal policy. Under flexible prices then, this type of shock would not affect real quantities. When prices are sticky however, the fact that optimal policy involves positive nominal interest rates means that shocks affecting the cost of credit do affect real quantities.

There is an interesting qualitative difference between optimal policy in response to productivity shocks and that in response to aggregate demand shocks. While there is little price level variation in either case, productivity shocks make the real interest rate behave approximately as in a real business cycle model, whereas aggregate demand shocks make the real interest rate vary significantly more in the initial periods than it would in a real business cycle model. To help explain this result, figure 4 plots the real interest rate for these two shocks, together with its behavior in the RBC model and in a version of our model where the money demand distortions have been eliminated. Figure 4 reveals that in response to a productivity shock, when money demand distortions are absent the real interest rate behaves as it would in an RBC model. This is consistent with King and Wolman's findings: when sticky price / monopolistic competition distortions are the only ones present, it is optimal to stabilize the price level in response to productivity shocks for certain forms of preferences – including the log-log preferences used here. However, in response to an aggregate demand shock, Figure 4.B shows that eliminating the money demand distortions does not restore RBC behavior of the real rate. This reveals a limitation of King and Wolman's price level stabilization result: when sticky price / monopolistic competition distortions are the only ones present, it is generally not optimal to exactly stabilize the price level (and restore the real business cycle) when there are aggregate demand shocks (the condition in (5.2) does not incorporate aggregate demand shocks).

## 8.2 Optimal policy versus simple alternatives

Figures 5 and 6 illustrate how the model's dynamics under optimal policy deviate from what they would be under two simple monetary policy rules, a constant money growth rule and interest rate rule which responds to deviations of inflation from target. The simple interest rate rule $(R_t = R^* + 1.5\,(\pi_t - \pi^*))$ comes much closer to replicating optimal policy than does constant money growth. However, even the interest rate rule generates substantial deviations of real variables from their optimal behavior. In particular, the interest rate rule stabilizes consumption behavior too much relative to the optimum.

## 8.3 Sensitivity to parameters governing money demand

Finally, figure 7 illustrates how the dynamics under optimal policy vary when we modify the distribution of credit costs $(dh\,())$ as in the combined higher interest elasticity and lower velocity case for which we presented steady state results in section 7.2. The behavior of real quantities does not vary significantly from the benchmark case. While the price level is much more variable than in the benchmark case, it still moves by less than one tenth of a percent in the long run, in response to a shock that has a cumulative impact on output of roughly twenty percent. Recall from section 7.2 that this case is one in which the steady state involves deflation of 1.72%.

# 9 Conclusions

(to be written)

# References

[1] Adao, Bernardino, Correia and Teles, 2000, "Gaps and Triangles," manuscript.

[2] Calvo, Guillermo A. (1983). "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics*, 12, 383-398.

[3] Cole, Harold L., and Narayana Kocherlakota (1998), "Zero Nominal Interest Rates: Why They're Good and How to Get Them," *Federal Reserve Bank of Minneapolis Quarterly Review,* 22(2), 2-10.

[4] Dotsey, Michael and Peter N. Ireland (1996), "The Welfare Cost of Inflation in General Equilibrium," *Journal of Monetary Economics*, 37(1), 29-47.

[5] Dotsey, Michael, Robert G. King, and Alexander L. Wolman (1999). "State Dependent Pricing and the General Equilibrium Dynamics of Money and Output", *Quarterly Journal of Economics.*

[6] Dupor, Bill (1999). "Optimal Monetary Policy with Nominal Rigidities," manuscript.

[7] Fisher, Irving

[8] Fisher, Irving

[9] Fisher, Irving

[10] Friedman, Milton (1969), "The Optimum Quantity of Money," in *The Optimum Quantity of Money, and Other Essays,* Aldine Publishing Company: Chicago.

[11] Gali, Jordi, and Mark Gertler (1999), "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics* 44(2), 195-222.

[12] Goodfriend, Marvin (1997), "A Framework for the Analysis of Moderate Inflations," *Journal of Monetary Economics* 39(1), 45-65.

[13] Goodfriend, Marvin and Robert G. King (1997), "The New Neoclassical Synthesis and the Role of Monetary Policy," in Bernanke, Ben S. and Rotemberg, Julio J., eds. *NBER macroeconomics annual 1997.* Cambridge and London: MIT Press, 1997, pages 231-83.

[14] Hansen, Gary D. (1985), "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, 16: 309-327.

[15] Ireland, Peter N. (1996), "The Role of Countercyclical Monetary Policy," *Journal of Political Economy,* 104,704-723.

[16] Jefferson, Philip, (2000) Jl of Economics and Business

[17] Keynes, John Maynard (1964), *The General Theory of Employment, Interest and Money*, Harbinger Edition, Harcourt, Brace and Jovanovich.

[18] King, Robert G., and Alexander L. Wolman (1999), "What Should the Monetary Authority Do When Prices are Sticky?" in Taylor, John B., ed., *Monetary Policy Rules*, Chicago: University of Chicago Press, 349-398.

[19] Kydland, Finn, and Edward C. Prescott (1980), "Dynamic Optimal Taxation, Rational Expectations and Optimal Control," *Journal of Economic Dynamics and Control* 2: 79-91.

[20] Lacker, Jeffrey M., and Stacey Schreft (1996), "Money and Credit as Means of Payment," *Journal of Monetary Economics*, 38(1), 3-23.

[21] Levin, Andrew (1991), "The Macroeconomic Significance of Nominal Wage Contract Duration," University of California, San Diego Discussion Paper 91-08.

[22] Lucas, Robert E., Jr. (2000), "Inflation and Welfare," *Econometrica* 68(2), 247-274.

[23] Lucas, Robert E., Jr., (1980), "Equilibrium in a Pure Currency Economy," *Economic Inquiry*, 18(2), 203-220.

[24] Lucas, Robert E. Jr., and Nancy L. Stokey (1983), "Optimal Fiscal and Monetary Policy in an Economy without Capital," *Journal of Monetary Economics 12*: 55-93.

[25] Marcet, Albert and Ramon Marimon (1999), "Recursive Contracts," manuscript.

[26] McCallum, Bennett T, and Marvin S. Goodfriend (1988), "Theoretical Analysis of the Demand for Money," *Federal Reserve Bank of Richmond Economic Review*, 74(1), 16-24.

[27] Porter, Richard D., and Ruth A. Judson (1996), "The Location of U.S. Currency: How Much Is Abroad?" *Federal Reserve Bulletin* 82(10), 883-903.

[28] Prescott, Edward C. (1987), "A Multiple Means-of-Payment Model," in Barnett, William A., and Singleton, Kenneth J. eds., *New approaches to monetary economics*, Cambridge; New York and Melbourne: Cambridge University Press, 42-51.

[29] Ramsey, Frank P. (1927), "A Contribution to the Theory of Taxation," *Economic Journal* 37: 47-61.

[30] Rogerson, Richard (1988), "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics,* vol. 21,: 3-16.

[31] Sbordone, Argia M. (1998), "Prices and Unit Labor Costs: A New Test of Price Stickiness," manuscript.

[32] Taylor, John B. (1980), "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy*, 88: 1-24.

[33] Wolman, Alexander L. (1999), "Sticky Prices, Marginal Cost and the Behavior of Inflation," forthcoming, *Federal Reserve Bank of Richmond Economic Quarterly.*

[34] Wolman, Alexander L. (1997), "Zero Inflation and the Friedman Rule: A Welfare Comparison," *Federal Reserve Bank of Richmond Economic Quarterly* 83(4), 1-22.

[35] Woodford, Michael, (1995), "Comment on, 'The Quantitative Analytics of the Basic Neomonetarist Model"', *Journal of Money, Credit, and Banking*, 27, 1278-1289.

[36] Yun, Tack (1996), "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics* 37: 345-370.

Table X:

## Effect of Eliminating Various Distortions on Optimal Long-Run Inflation Rate

(distortion eliminated is marked with x)

| | | | | Benchmark[1] | Lower Demand Elasticity[2] (ε=4) | Greater Money Intensity[3] | Greater Money Semi-Elasticity[4] | Greater Intensity and Semi-Elasticity[5] |
|---|---|---|---|---|---|---|---|---|
| Case | Markup | Time Cost | Wedge | | | | | |
| | | | | | | | | |
| 1 | | | | -0.25 | -1.20 | -1.52 | -0.70 | -1.72 |
| 2 | | | x | -0.10 | -.26 | -0.73 | -0.54 | -1.12 |
| 3 | | x | | -0.15 | -.98 | -0.89 | -0.15 | -0.63 |
| 4 | | x | x | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 5 | x | | | 0.27 | -1.47 | -1.64 | -0.76 | -1.86 |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

[1] The benchmark parameter values are described in the text.

[2] The benchmark demand elasticity (ε=10) is replaced with a lower elasticity (ε=4).

[3] The parameters of the cost function are modified, as described in the text, so as to produce a model with a higher ratio of real money to consumption (lower velocity) while retaining a similar semi-elasticity.

[4] The parameters cost function are modified, as described in the text, so as to produce a model with a higher semi-elasticity while maintaining a similar ratio of real money to consumption (similar velocity).

[5] The parameters of the cost function are modified, as described in the text, to increase both the ratio of real money to consumption and the semi-elasticity.

Figure 1

Response to a productivity shock under optimal policy

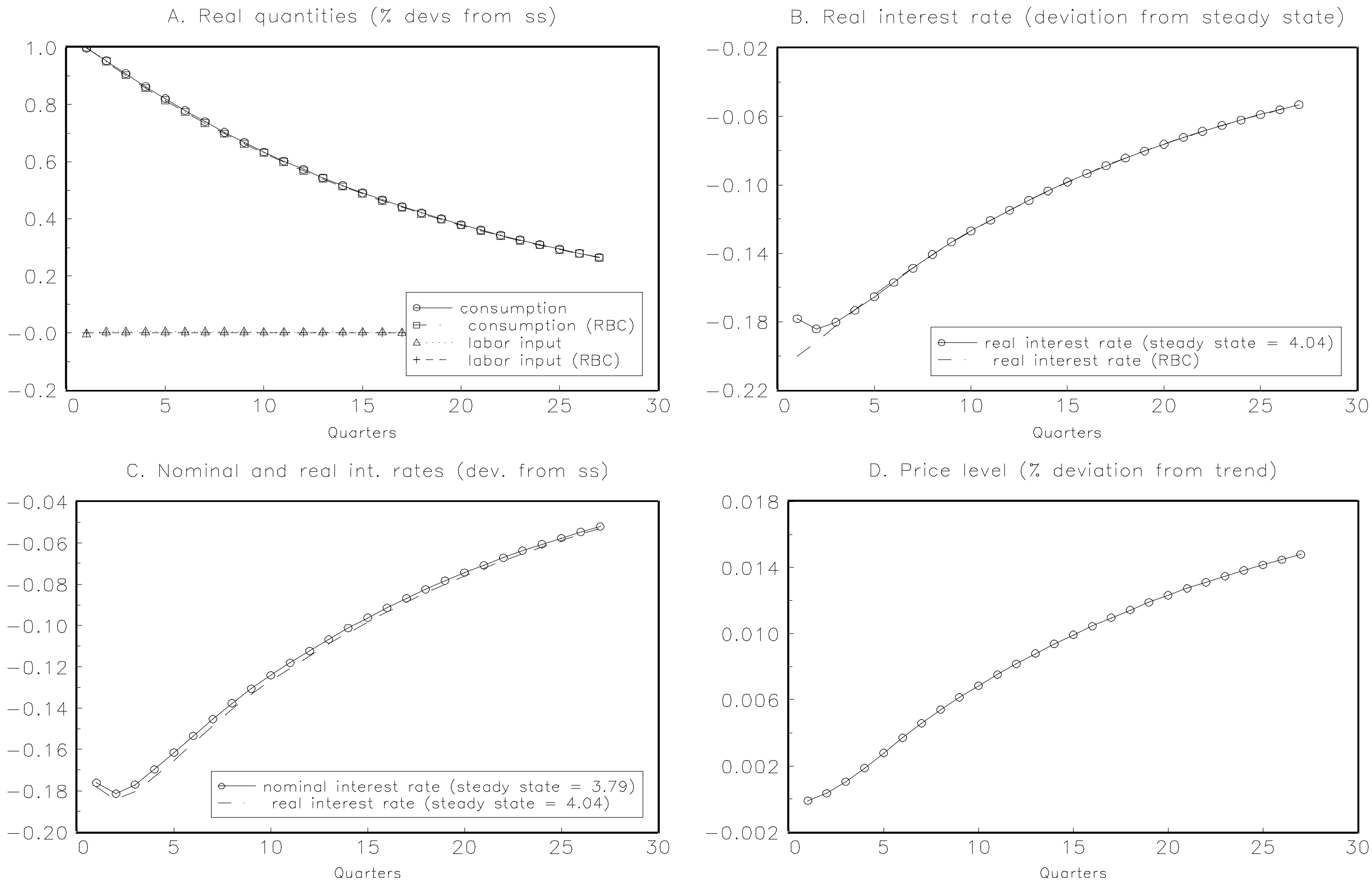


Figure 2

Response to a demand shock under optimal policy

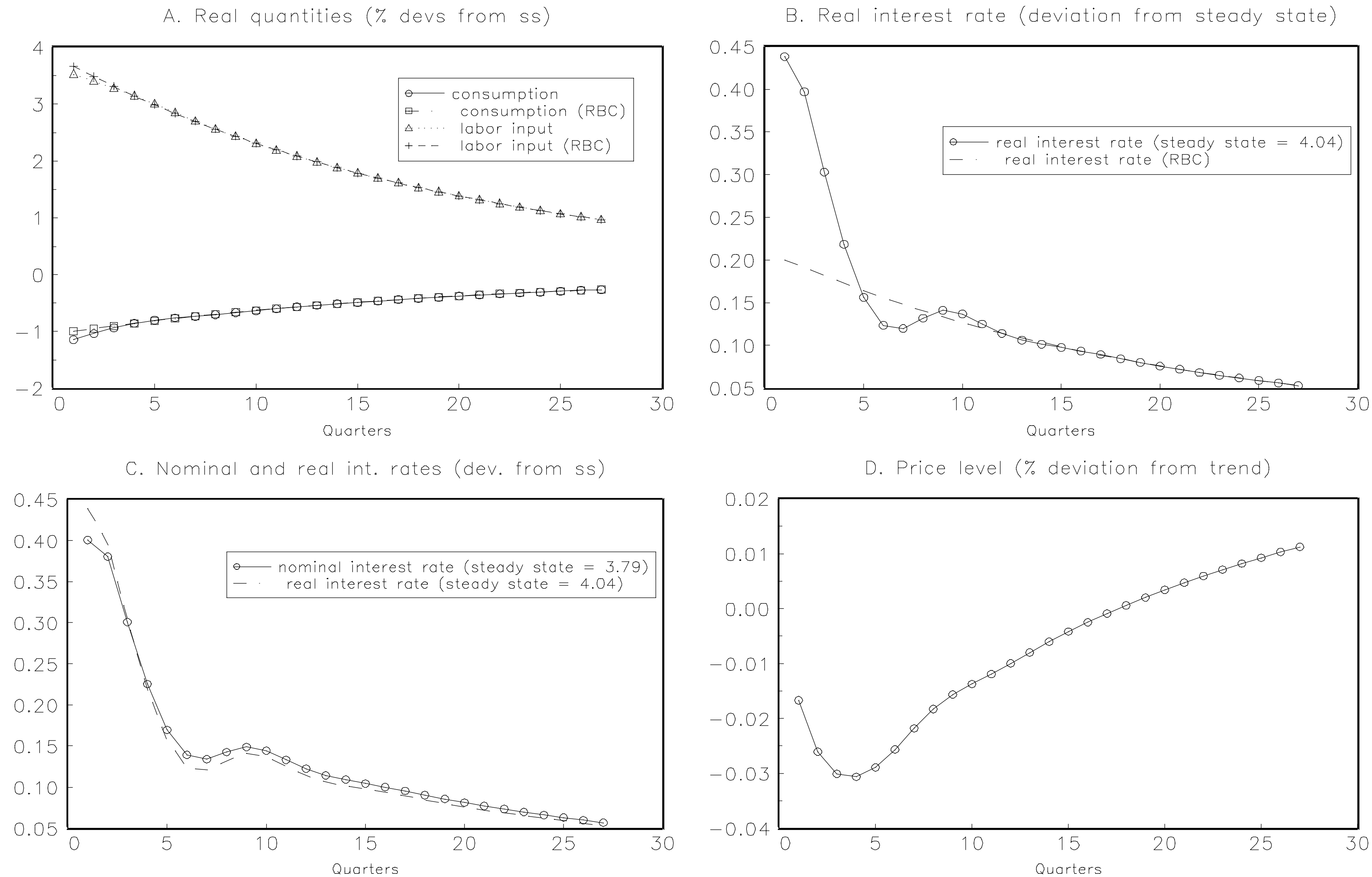


Figure 3

Response to a money demand shock under optimal policy

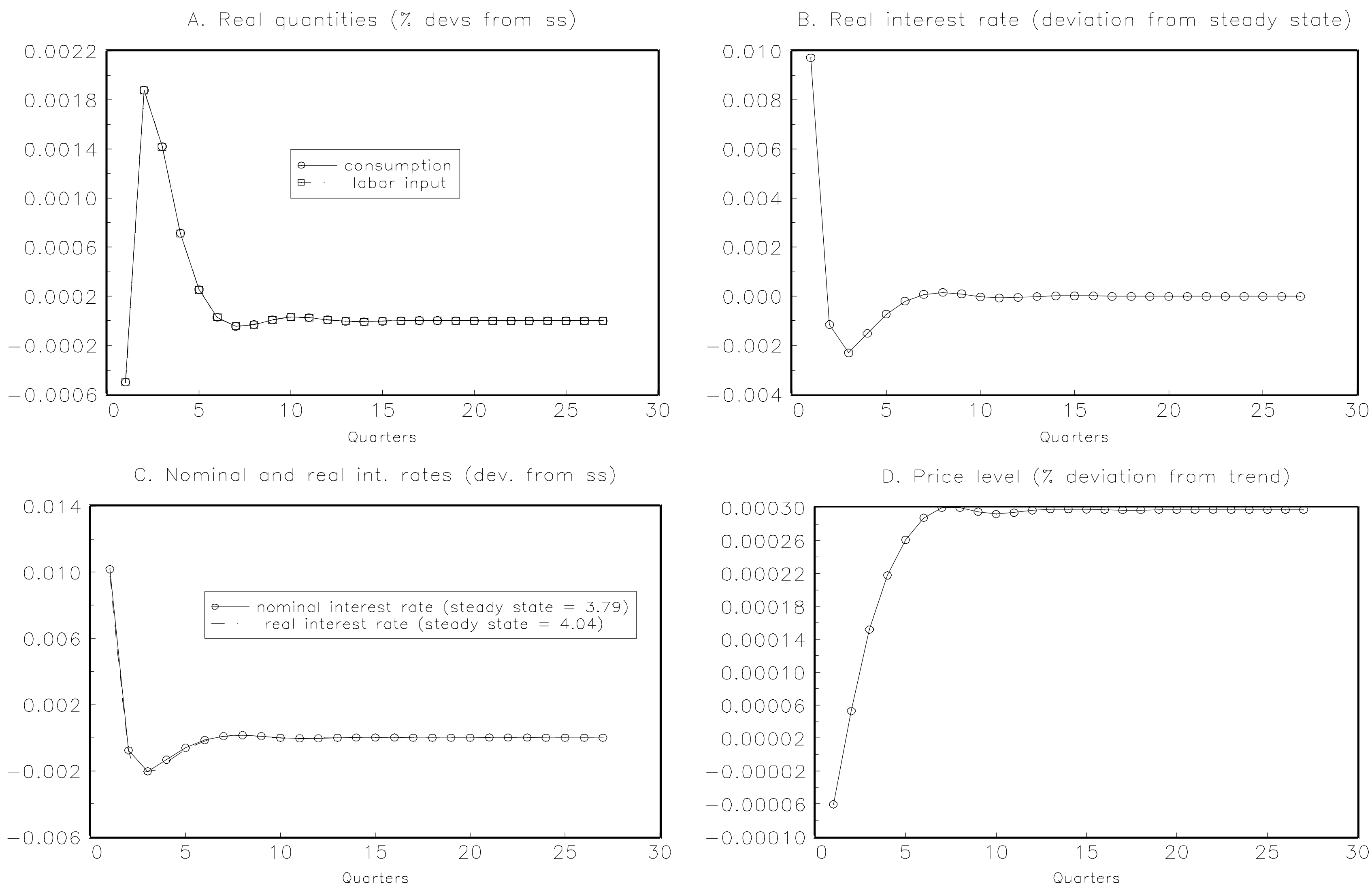
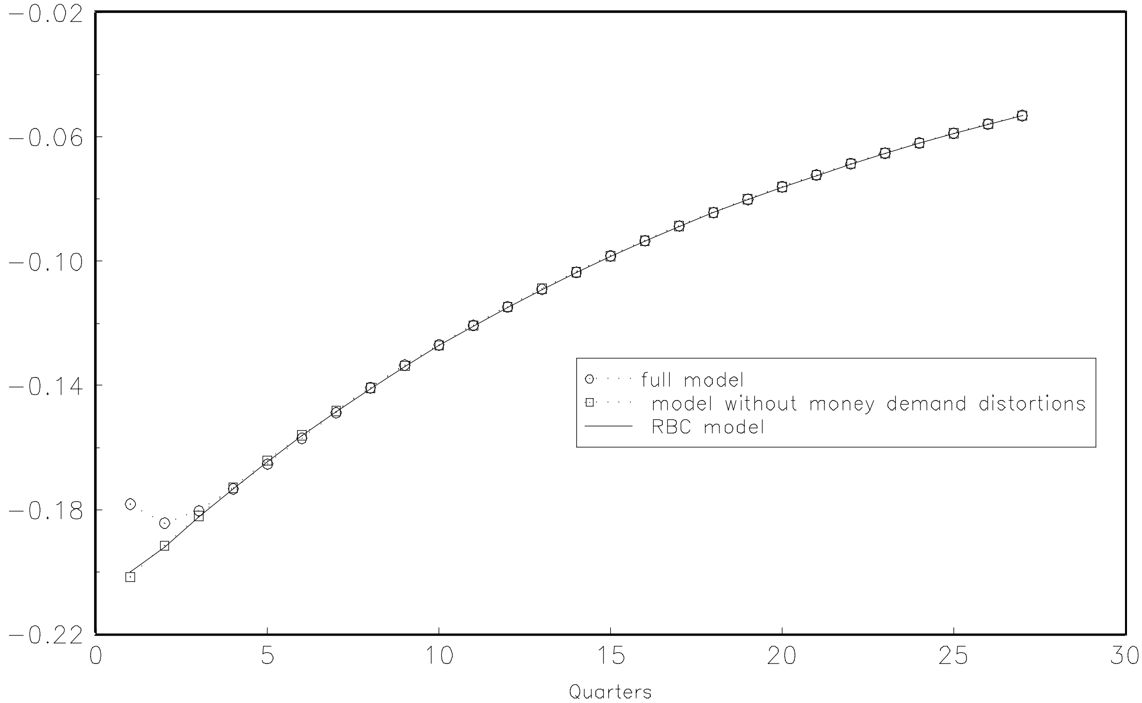


Figure 4
Productivity vs. aggregate demand shocks

A. Real interest rate following productivity shock




B. Real interest rate following demand shock

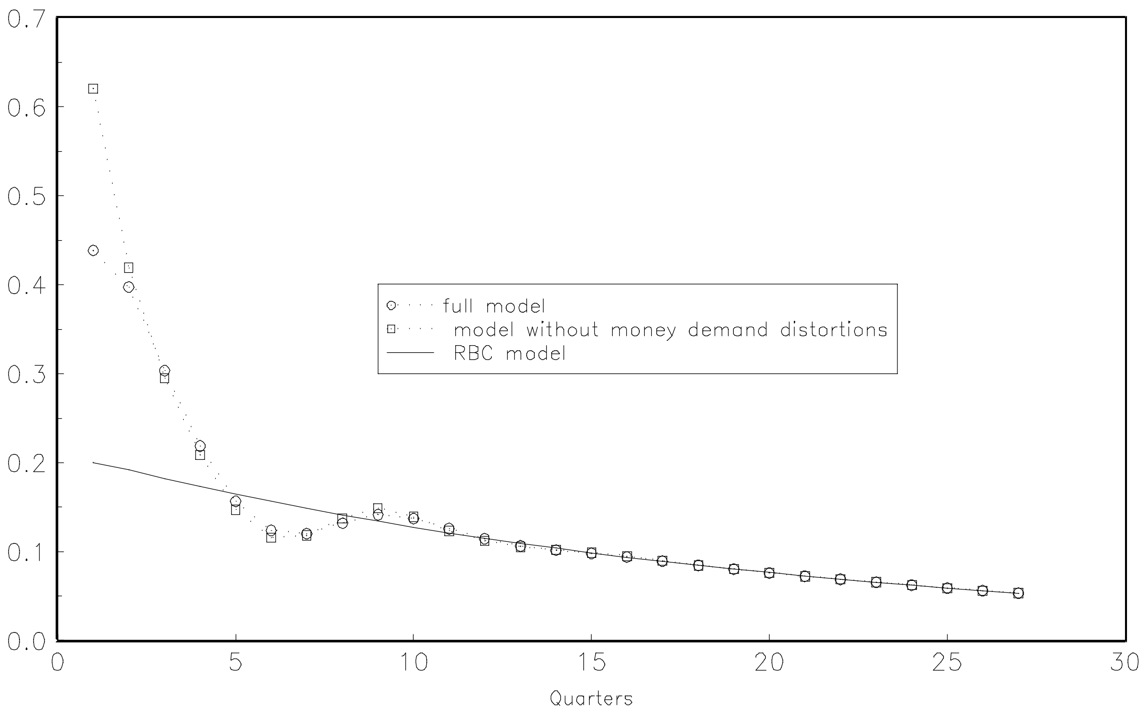
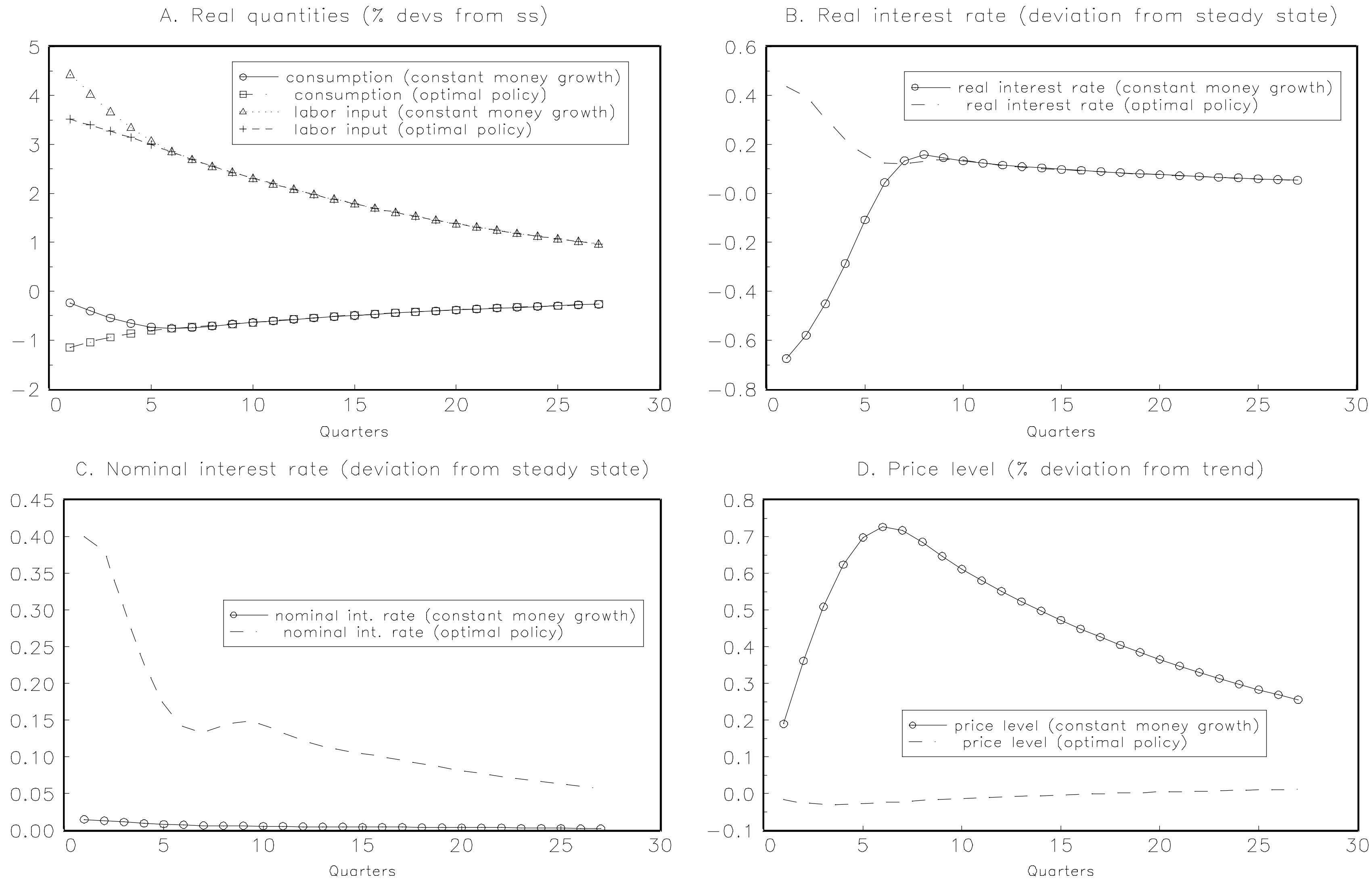


Figure 5: Aggregate Demand Shock
Comparison between constant money growth and optimal policy



# Figure 6: Aggregate Demand Shock
## Comparison between int. rate rule and optimal policy

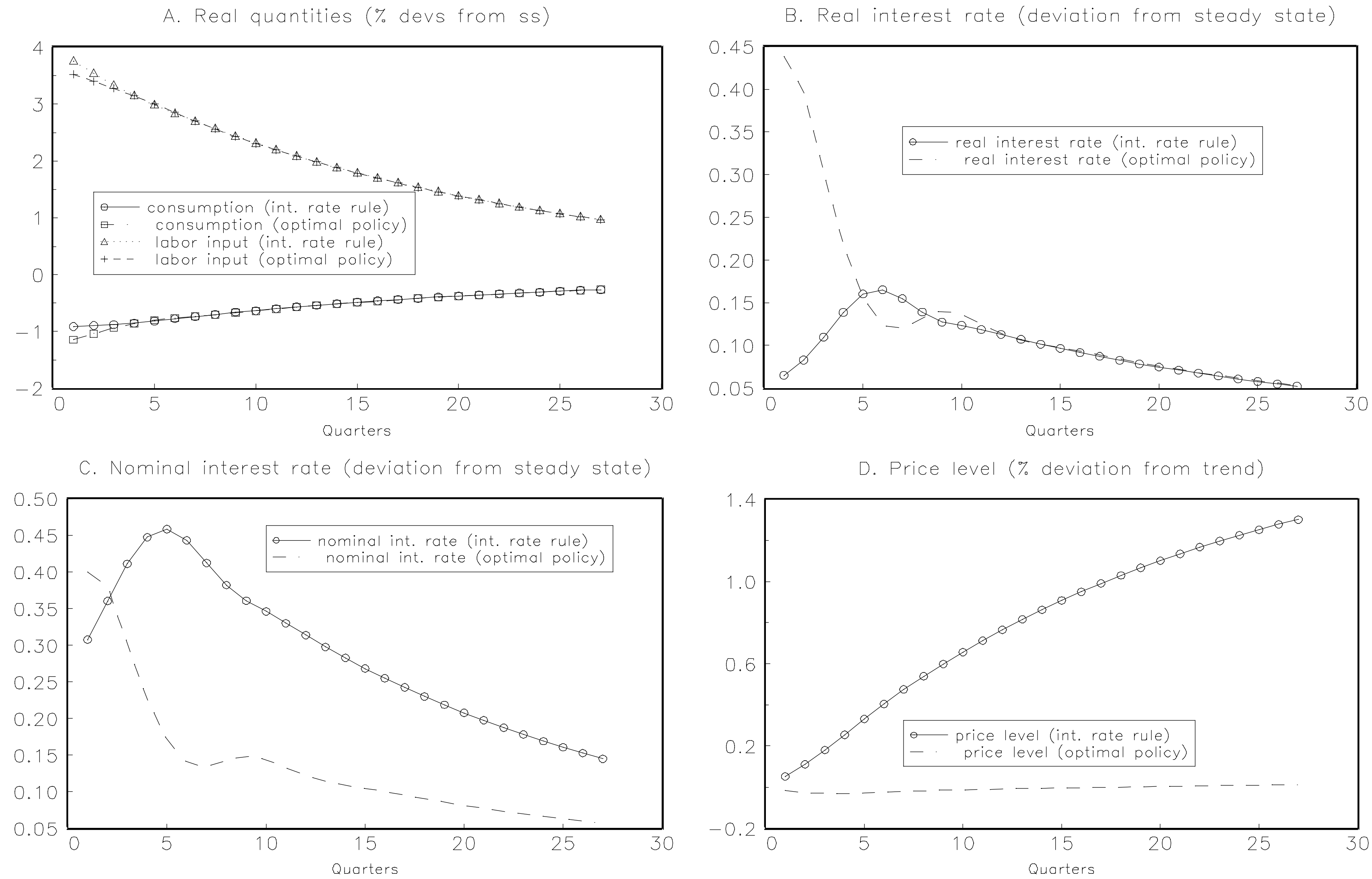


Figure 7: Dynamics Under Alternative Money Demand Specification
(high interest elasticity, low velocity) (productivity shock)