Optimal Monetary Policy and Exchange Rate Volatility in a Small Open Economy.*

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Abstract

We lay out a tractable *small open economy* version of the canonical sticky price model, and use it as a framework to study the properties of three alternative monetary regimes: (a) optimal monetary policy, (b) a Taylor rule, and (c) an exchange rate peg. Several interesting results emerge from our analysis. First, the optimal policy is shown to entail a positive correlation between domestic and world interest rates. That doesn't prevent sizable fluctuations of nominal and real exchange rates from occurring, though the implied volatility of those variables is much smaller than the empirical one. Second, a Taylor rule generally leads to excess volatility of nominal variables, and excess smoothness of real variables, relative to the optimal policy. Finally, we show that a pure exchange rate peg seems to have better stabilization properties than a Taylor rule.

Keywords: small open economy, optimal monetary policy, sticky prices, Taylor rule.

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1 Introduction

Much recent work in macroeconomics has involved the development and evaluation of monetary models that bring imperfect competition and nominal rigidities into the dynamic stochastic general equilibrium structure that for a long time had been the hallmark of RBC theory. In the resulting models–often referred to as New Keynesian–changes in monetary settings generally have nontrivial effects on real variables. Monetary policy may thus become a potential stabilization tool, as well as an independent source of economic fluctuations. Not surprisingly, the study of the properties of alternative monetary policy rules (i.e. specifications of how the central bank changes the settings of its instrument in response to changes in macroeconomic conditions) has been a fruitful area of research in recent years and a natural application of the new generation of models.[1]

In the present paper we lay out a *small open economy* version of a model with staggered price-setting, and use it as a framework to analyze the properties and macroeconomic implications of three alternative monetary regimes: (a) the optimal monetary policy, (b) a policy based on a simple Taylor rule, and (c) an exchange rate peg. In contrast with most of the existing literature–where monetary policy is introduced by assuming that some monetary aggregate follows an exogenous stochastic process–we model monetary policy as endogenous, with a short-term interest rate being the instrument of that policy.[2] For this very reason our framework allows us to model alternative monetary regimes. Furthermore, we believe that our approach accords much better with the practice of modern central banks, and provides a more suitable framework for policy analysis than the standard one.

As mentioned above, we consider three monetary regimes. Our analysis of the optimal policy is meant to provide a useful benchmark, allowing us to address a number of interesting issues under a normative light: What is the exchange rate volatility associated with the optimal policy? How does it compare to the observed volatility ? What is the optimal policy response to changes in world interest rates, or in the level of economic activity abroad ? How does the degree of openness of the economy affect the nature of the optimal monetary policy ? Several interesting findings emerge from that analysis. Thus, we show that under a plausible parameter values the optimal policy entails a positive comovement between domestic and foreign

[1]The volume edited by Taylor (1999) contains several significant contributions to that literature. See, e.g., Clarida, Galí, and Gertler (1999) for a recent survey.

[2]A recent exception is given by Obstfeld and Rogoff (1999), which solve for the optimal money supply rule in the context of a model with one-period sticky wages. A more similar methodological approach can be found in Svensson (2000), in which optimal policy is derived from the minimization by the central bank of a quadratic loss function. His model, however, differs from the standard optimizing sticky price model analyzed here in that it assumes a predetermined output and inflation (which resulting from their dependence on lagged variables, with a somewhat arbitrary lag structure), the introduction of an ad-hoc cost-push shock in the inflation equation (which creates a trade off between the output gap and inflation), as well as some other features that might arguably make the model more realistic though at the cost of reduced tractability.

interest rates and, as a result, some exchange rate smoothing. In spite of that, the model implies fluctuations of nominal and real exchange rates that are substantial, though smaller than those observed empirically. Furthermore, we show that the optimal degree of interest rate synchronization will be higher–and, as a consequence, the exchange rate volatility will be lower–the more open is the economy. We also show that the volatility of output increases with openness, whereas that of consumption displays a non monotonic relationship.

After analyzing the optimal policy, we move on to study the properties of a monetary regime based on a simple Taylor rule, which links the domestic interest rate to measures of inflation and economic activity. A similar rule has been found to provide a good approximation to the actual monetary policy of several countries in recent years. Our analysis allows us to ascertain the extent to which a central bank of a small open economy would deviate from best practice by following a simple Taylor rule, and how such a policy may impinge on the behavior of inflation and other macroeconomic variables. We show that, indeed, the implied deviations from optimality are substantial, taking the form of excess volatility of nominal variables, and excess smoothness of real variables.[3]

Finally, we consider the consequences of a policy that pegs permanently the nominal exchange rate against a hypothetical world currency or, equivalently, adopts the latter as a domestic currency, thus joining a world monetary union. We show that such a regime has some desirable properties, largely coming from its anchoring of the level of nominal variables, and may approximate the optimal policy better than the Taylor rule.

Our modelling approach, based on the Calvo price-setting model, appears to us as a most natural extension to the open economy of the new generation of sticky price models.[4] It yields highly tractable (and intuitive) log-linear equilibrium conditions that can be easily related to their closed economy counterparts (and which have the latter as a limiting case).[5] Furthermore, by making use of a staggered price-setting structure, it allows for richer dynamic effects of monetary policy than those found in the models with one-period advanced price-setting that are common in the recent literature.[6] A second novelty of our approach lies in the modelling of the rest of the world as a limiting case of an economy whose degrees of openness is negligible. The resulting limiting economy corresponds to the canonical Calvo model of a closed

[3]See Ball (1999) and Svensson (2000) for an analysis of the properties of Taylor rules in the context of a more traditional Keynesian framework.

[4]See Lane (2000) for a useful survey of recent efforts to incorporate sticky prices into dynamic general equilibrium open economy models.

[5]See, e.g., King and Wolman (1996), Yun (1996), and Woodford (1999), for an analysis of the canonical closed economy Calvo model. The introduction of price staggering in an open economy model follows the lead of Kollman (1997) and Chari et al. (1998), though both papers specify monetary policy as exogenous, restricting their analysis to the effects of a monetary shock.

[6]See, e.g., Obstfeld and Rogoff (1996, 1999), Corsetti and Pesenti (1998), Betts and Devereux (1998), and Bachetta and Van Wincoop (1998)

economy.

The remainder of the paper is organized as follows. In section 2 we lay out the basic model. Section 3 derives the log-linearized equilibrium conditions. Section 4 discusses the optimal policy regime and how it is influenced by the degree of openness. Section 5 analyzes the properties of Taylor rules. Section 6 looks at the consequences of pegging the exchange rate. Section 7 concludes.

2 A Small Open Economy Model

2.1 Households

Our small open economy is inhabited by a representative household who seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t, N_t) \tag{1}$$

where N_t denotes hours of labor, and C_t is a composite consumption index defined by

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} \, C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \, C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

with $C_{H,t}$ and $C_{F,t}$ being indices of consumption of domestic and foreign goods. Such indices are in turn given by the following CES aggregators of the quantities consumed of each type of good:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad ; \qquad C_{F,t} = \left(\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Notice that under our specification η measures the elasticity of substitution between domestic and foreign goods. The elasticity of substitution among goods within each category is given by ε. We assume $\eta > 0$ and $\varepsilon > 1$.

The maximization of (1) is subject to a sequence of intertemporal budget constraints of the form:

$$\int_0^1 [P_{H,t}(i) C_{H,t}(i) + P_{F,t}(i) C_{F,t}(i)] \, di + E_t\{Q_{t,t+1} \, D_{t+1}\} \leq D_t + W_t N_t + T_t \tag{3}$$

for $t = 0, 1, 2, ...$where $Q_{t,t+1}$ is the stochastic discount factor, D_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t, W_t is the nominal wage, and T_t are lump-sum transfers/taxes (the three variables are expressed in domestic currency). We assume that households have access to a complete set of contingent claims. Notice that money does not appear in either the budget constraint or the utility function: throughout we specify monetary policy in terms of an interest rate rule; hence, we do not need to introduce money explicitly in the model.[7]

[7] We can think of money as playing the role of a unit of account only.

The optimal allocation of any given expenditure within each category of goods yields the demand functions:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \quad ; \quad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}$$

for all $i \in [0,1]$, where $P_{H,t} \equiv (\int_0^1 P_{H,t}(i)^{1-\varepsilon} di)^{\frac{1}{1-\varepsilon}}$ and $P_{F,t} \equiv (\int_0^1 P_{F,t}(i)^{1-\varepsilon} di)^{\frac{1}{1-\varepsilon}}$ are the price indexes for domestic and imported goods, both expressed in home currency. Throughout we assume that the law of one price holds, implying that $P_{F,t}(i) = e_t \; P^*_{F,t}(i)$, all $i \in [0,1]$, where e_t is the nominal exchange rate (the price of foreign currency in terms of home currency), and $P^*_{F,t}(i)$ is the price of foreign good i denominated in foreign currency.

The optimal allocation of expenditures between domestic and foreign goods implies:

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \quad ; \quad C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$

where $P_t \equiv [(1-\alpha) \; {P_{H,t}}^{1-\eta} + \alpha \; {P_{F,t}}^{1-\eta}]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI). Notice that, when the prices of domestic and foreign goods are equal (as in the steady state considered below), parameter α corresponds to the share of domestic consumption allocated to imported goods. It thus represents a natural index of openness.

Under the above optimality conditions, the intertemporal budget constraint can be rewritten as:

$$P_t C_t + E_t\{Q_{t,t+1} \; D_{t+1}\} \leq D_t + W_t N_t + T_t \tag{4}$$

In what follows we specialize the period utility to be of the form $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}$. Then we can rewrite the remaining optimality conditions for the household's problem as follows:

$$C_t^{\sigma} \; N_t^{\phi} = \frac{W_t}{P_t} \tag{5}$$

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1} \tag{6}$$

Taking conditional expectations on both sides of (6) and rearranging terms we obtain a conventional stochastic Euler equation:

$$\beta R_t \; E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right)\right\} = 1 \tag{7}$$

where $R_t^{-1} = E_t\{Q_{t,t+1}\}$ is the price of a riskless one-period bond (denominated in domestic currency) and, hence, R_t is its gross return.

2.1.1 International Risk Sharing

In the rest of the world (which, for convenience, we refer to as the world economy) a representative household faces a problem identical to the one outlined above. In its preferences, however, the weight of goods produced in the small economy, denoted by α^*, is assumed to be negligible.

Under the assumption of complete securities markets, a first order condition analogous to (6) must also hold for consumers in the foreign country:

$$\beta \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \left(\frac{P_t^*}{P_{t+1}^*}\right) \left(\frac{e_t}{e_{t+1}}\right) = Q_{t,t+1} \tag{8}$$

Let us define the real exchange rate as $q_t \equiv \frac{e_t P_t^*}{P_t}$. Combining (6) and (8) it follows, after iterating, that:

$$C_t = \kappa \, C_t^* \, q_t^{\frac{1}{\sigma}} \tag{9}$$

for all t, where κ is a constant that depends on initial conditions. Log-linearizing (9) around a steady state yields:

$$\widehat{C}_t = \widehat{C}_t^* + \frac{1}{\sigma} \, \widehat{q}_t \tag{10}$$

We thus see that the assumption of complete markets at the international level leads to a simple relationship linking consumption at home and abroad and the real exchange rate, all in percent deviations from the steady state. That relationship is independent of the relative sizes of the two economies involved, and of constant κ.

2.2 Firms

Each firm produces a differentiated good with a linear technology represented by the production function

$$Y_t(i) = Z_t \, N_t(i)$$

where $Z_t \equiv \exp\{z_t\}$, and $\{z_t\}$ follows an AR(1) process $z_t = \rho \, z_{t-1} + u_t$.

For reasons that will become clear below, we assume that the government fully offsets the distortion associated with the existence of market power by subsidizing employment at a constant rate $\frac{1}{\varepsilon}$.[8] Hence, the firm's nominal marginal cost will be given by

$$MC_t = (1 - \frac{1}{\varepsilon}) \, \frac{W_t}{Z_t}$$

Under the Calvo formalism, a measure θ of (randomly selected) firms sets new prices each period. As we show in Appendix 2, the optimal price-setting strategy for

[8] The subsidy is assumed to be financed through a lump-sum tax on households.

the typical firm choosing a price in period t can be approximated in a neighborhood of the zero-inflation steady state by the rule:

$$\widehat{p}_{H,t}^{new} = \sum_{k=1}^{\infty} (\beta\theta)^k \; E_t\{\pi_{t+k}\} + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{\widehat{mc}_{t+k}\} \tag{11}$$

where $\widehat{p}_{H,t}^{new} \equiv \log\left(\frac{P_{H,t}^{new}}{P_{H,t}}\right)$ is the percent deviation of newly set domestic prices–denoted by $P_{H,t}^{new}$–from the domestic price index $P_{H,t}$. $\widehat{mc}_t \equiv \log\left(\frac{mc_t}{mc}\right)$ is the percent deviation from its steady state value of the real marginal cost–the latter defined in terms of domestic goods, i.e., $mc_t \equiv \frac{MC_t}{P_{H,t}}$.[9]

2.3 Inflation, the Real Exchange Rate, and the Terms of Trade: Some Identities

We distinguish between *domestic inflation*–defined above as the rate of change in the index of domestic goods prices, i.e., $\pi_{H,t} \equiv \log\left(\frac{P_{H,t+1}}{P_{H,t}}\right)$–, and *CPI-inflation*–defined as the rate of change in the CPI, i.e. $\pi_t \equiv \log\left(\frac{P_{t+1}}{P_t}\right)$. There is a simple relationship linking the two, as we will see next.

Let $s_t \equiv \frac{P_{F,t}}{P_{H,t}}$ denote the *terms of trade*, i.e., the price of foreign goods in terms of home goods. Define the price ratio $p_{H,t} \equiv \frac{P_{H,t}}{P_t}$. Log-linearization of the CPI formula implies $\widehat{p}_{H,t} = -\alpha \; \widehat{s}_t$. Hence, after taking logs on the expression for $p_{H,t}$ and first differencing, we obtain

$$\pi_t = \pi_{H,t} + \alpha \; \Delta\widehat{s}_t \tag{12}$$

which makes the gap between our two measures of inflation proportional to the percent change in the terms of trade, with the coefficient of proportionality given by the index of openness α.

Next, let us derive a relationship between the terms of trade and the real exchange rate. From our assumption that the share of imports in the rest of the world's CPI is negligible it follows that $P_t^* = P_{F,t}^*$, and $\pi_t^* = \pi_{F,t}^*$, for all t. We can thus write the real exchange rate as $q_t \equiv \frac{e_t P_t^*}{P_t} = s_t \; p_{H,t}$, implying the log-linearized relationship $\widehat{q}_t = \widehat{s}_t + \widehat{p}_{H,t}$. Hence, and using the fact that $\widehat{p}_{H,t} = -\alpha \; \widehat{s}_t$, we obtain a simple relationship linking the real exchange rate and the terms of trade:

$$\widehat{q}_t = (1-\alpha) \; \widehat{s}_t, \tag{13}$$

Finally, combining (12) and (13), we can derive an alternative expression for the gap between domestic and CPI-inflation, now as a function of the change in the real exchange rate:

$$\pi_t = \pi_{H,t} + \left(\frac{\alpha}{1-\alpha}\right) \; \Delta\widehat{q}_t \tag{14}$$

[9] Notice that, in the steady state, the real marginal cost is given by $mc \equiv (1 - \frac{1}{\varepsilon})$, the inverse of the optimal markup under flexible prices.

3 Equilibrium

3.1 Aggregate Demand and Output Determination

3.1.1 World Consumption and Output

It is convenient to start by describing how consumption and output are determined in the world economy. As mentioned above, preferences of the representative household there are identical to those introduced above, but with a negligible weight on the goods imported from the small economy. The log-linearized Euler equation, combined with the market clearing condition $\widehat{Y}_t^* = \widehat{C}_t^*$, implies

$$\widehat{Y}_t^* = E_t\{\widehat{Y}_{t+1}^*\} - \frac{1}{\sigma}\ (r_t^* - E_t\{\pi_{t+1}^*\}) \tag{15}$$

which can be solved forward to obtain:

$$\widehat{Y}_t^* = -\frac{1}{\sigma}\ E_t\{\sum_{k=0}^{\infty}(r_{t+k}^* - \pi_{t+1+k}^*)\} \tag{16}$$

Hence, as in a standard closed economy model, world consumption is inversely related to current and anticipated world real interest rates.

3.1.2 Consumption and Output in the Small Open Economy

Market clearing in the small economy requires $Y_t(i) = C_{H,t}(i) + C_{H,t}^*(i)$, for all $i \in [0,1]$. Log-linearization around a steady state with balanced trade implies:

$$\widehat{Y}_t(i) = (1-\alpha)\ \widehat{C}_{H,t}(i) + \alpha\ \widehat{C}_{H,t}^*(i)$$

Let $Y_t \equiv \int_0^1 Y_t(i)\ di = Z_t \int_0^1 N_t(i)\ di = Z_t N_t$ denote aggregate domestic output. Log-linearizing around a symmetric steady-state we obtain $\widehat{Y}_t = \int_0^1 \widehat{Y}_t(i)\ di = z_t + \widehat{N}_t$. Similarly, log-linearization of $C_{H,t}$ yields $\widehat{C}_{H,t} = \int_0^1 \widehat{C}_{H,t}(i)\ di$ (and an analogous expression for $\widehat{C}_{H,t}^*$). Thus, it follows that

$$\widehat{Y}_t = (1-\alpha)\ \widehat{C}_{H,t} + \alpha\ \widehat{C}_{H,t}^*$$

which can be combined with the expression $\widehat{C}_t = (1-\alpha)\ \widehat{C}_{H,t} + \alpha\ \widehat{C}_{F,t}$ (obtained from the log-linearization of (2)) to yield

$$\begin{aligned}\widehat{Y}_t &= \widehat{C}_t + \alpha\ (\widehat{C}_{H,t}^* - \widehat{C}_{F,t}) \\ &= (1-\alpha)\ \widehat{C}_t + \alpha\ \widehat{C}_t^* + \alpha\ [(\widehat{C}_{H,t}^* - \widehat{C}_t^*) - (\widehat{C}_{F,t} - \widehat{C}_t)]\end{aligned}$$

But notice that $\widehat{C}_{F,t} - \widehat{C}_t = -\eta\ \widehat{p}_{F,t} = -\eta(1-\alpha)\ \widehat{s}_t$ and, by analogy, $\widehat{C}_{H,t}^* - \widehat{C}_t^* = -\eta\ \widehat{p}_{H,t}^* = \eta\ \widehat{s}_t$ (where we make use of the assumption $P_t^* = P_{F,t}^*$). Hence we can write:

$$\widehat{Y}_t = (1-\alpha)\ \widehat{C}_t + \alpha\ \widehat{C}_t^* + \alpha\eta\,(2-\alpha)\ \widehat{s}_t \tag{17}$$

which determines output as a weighted average of domestic and foreign expenditures, plus an "expenditure switching factor" which is proportional to the terms of trade (all in percent deviations from their steady state values). (In the next three subsections we show how each of those factors can be determined as a function of interest and inflation rates, both domestic and foreign.)

Furthermore, we can combine (10), (17), and () to obtain

$$\widehat{Y}_t = \widehat{Y}_t^* + \frac{\omega}{\sigma}\ \widehat{s}_t \tag{18}$$

where $\omega \equiv 1 + \alpha\ (\sigma\eta - 1)(2 - \alpha) > 0$.

3.1.3 Uncovered Interest Parity and Terms of Trade Determination

Under the assumption of complete international financial markets, the equilibrium price (in terms of home currency) of a riskless bond denominated in foreign currency is given by $e_t R_t^{*\ -1} = E_t\{Q_{t,t+1}\ e_{t+1}\}$, which can be combined with $R_t E_t\{Q_{t,t+1}\} = 1$ and (6) to obtain a version of the *uncovered interest parity* (UIP) condition:

$$E_t\{C_{t+1}^{-\sigma} P_{t+1}^{-1}\ [R_t - R_t^*\ (e_{t+1}/e_t)]\} = 0$$

Linearization around a perfect-foresight steady state yields the familiar expression:

$$r_t - r_t^* = E_t\{\Delta \log e_{t+1}\} \tag{19}$$

where $r_t \equiv \log\left(\frac{R_t}{R}\right)$ and $r_t^* \equiv \log\left(\frac{R_t^*}{R^*}\right)$. Taking logs on $s_t = \frac{e_t P_t^*}{P_{H,t}}$, first-differencing, and combining the resulting log-linear expression with (19) yields the following stochastic difference equation for the terms of trade:

$$\widehat{s}_t = (r_t^* - E_t\{\pi_{t+1}^*\}) - (r_t - E_t\{\pi_{H,t+1}\}) + E_t\{\widehat{s}_{t+1}\} \tag{20}$$

As we show in Appendix 1, the terms of trade are pinned down uniquely in the perfect foresight steady state. That fact, combined with our assumption of stationary driving forces, implies that $\lim_{T\to\infty} E_t\{\widehat{s}_T\} = 0$. Hence, we can solve (20) forward to obtain:

$$\widehat{s}_t = E_t\{\sum_{k=0}^{\infty}[(r_{t+k}^* - \pi_{t+k+1}^*) - (r_{t+k} - \pi_{H,t+k+1})]\} \tag{21}$$

i.e., variations in the terms of trade are a function of current and anticipated real interest rate differentials.

Notice that (), (), and (), determine the level of output in the small economy as a function of current and anticipated interest rates and inflation rates, both at home and abroad. In the next section, we show how inflation is determined in equilibrium.

3.2 The Supply Side: Marginal Cost and Inflation Dynamics

3.2.1 Marginal Cost and Inflation Dynamics in the Rest of the World

The dynamics of inflation in the world economy are described by the difference equation

$$\pi_t^* = \beta \ E_t\{\pi_{t+1}^*\} + \lambda \ \widehat{mc}_t^* \tag{22}$$

where, under our assumptions, marginal cost is given by

$$\widehat{mc}_t^* = (\phi + \sigma) \ \widehat{Y}_t^* - (1+\phi) \ z_t^* \tag{23}$$

Given a rule for the interest rate r_t^*, equations (15), (22), and (23) describe the equilibrium dynamics of the world economy around the steady state.

3.2.2 Marginal Cost and Inflation Dynamics in the Small Open Economy

The after some algebra, the inflation equation:

$$\pi_{H,t} = \beta \ E_t\{\pi_{H,t+1}\} + \lambda \ \widehat{mc}_t \tag{24}$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

Notice that the real marginal cost can be written as $mc_t = \frac{(1-\frac{1}{\varepsilon})}{p_{H,t}} \frac{W_t}{P_t Z_t}$. That expression can be log-linearized, and combined with labor supply schedule (5), market clearing condition (17), and the fact that $\widehat{p}_{H,t} = -\alpha \ \widehat{s}_t$, to yield:

$$\widehat{mc}_t = [\sigma + (1-\alpha)\phi] \ \widehat{C}_t + \alpha\phi \ \widehat{Y}_t^* + \alpha[1 + \phi\eta(2-\alpha)] \ \widehat{s}_t - (1+\phi) \ z_t$$

Hence, we see that the domestic marginal cost is increasing in domestic consumption (through its wealth and employment effects on real wages), foreign consumption (through the employment effect on the real wage), the terms of trade (through changes in the product wage, given the real wage, and through its effects on employment and hence the real wage), and technology (through its direct effect on labor productivity).

Combining the expression above with (10) yields

$$\widehat{mc}_t = \left(1 + \frac{\phi\omega}{\sigma}\right) \ \widehat{s}_t + (\sigma + \phi) \ \widehat{Y}_t^* - (1+\phi) \ z_t \tag{25}$$

Given a process for $\{\widehat{Y}_t^*, \ r_t^*, \pi_t^*\}$, which is exogenous with respect to domestic variables, equations (20), (24), and (25), combined with a *rule* for the domestic interest rate r_t, can be used to describe the local equilibrium dynamics of the small open economy.

Next we turn our attention to the analysis of the properties of such an equilibrium under alternative hypotheses regarding the way monetary policy is conducted.

4 Optimal Monetary Policy

In this section we derive and discuss the properties of the equilibrium dynamics when both the domestic and world monetary authorities pursue an optimal monetary policy. It is important to notice that, under our assumptions, the presence of nominal rigidities is the only source of suboptimality in the equilibrium allocation. First, the assumed subsidy to employment fully neutralizes the market power distortion, hence eliminating the policymaker's incentive to expand output beyond its natural rate. Second, by not assigning any explicit value to the holding of money balances we eliminate the monetary distortion that would pull the optimal policy towards the Friedman rule. Accordingly, *if* prices were fully flexible the equilibrium allocations would be efficient. Thus, it is optimal for the monetary authority to fully neutralize the effects of nominal rigidities and restore the allocation associated with the flexible price equilibrium.[10]

Under our assumptions, the equilibrium under flexible prices is characterized by a *constant* markup of size $\mu \equiv \frac{\varepsilon}{\varepsilon-1}$ for all firms. But that is precisely the markup that prevails in the zero-inflation steady state of the model with nominal rigidities, and around which we have linearized the equilibrium conditions. It follows that, in order to replicate the flexible price allocation, the central bank should seek to maintain real marginal costs constant at their steady state level, i.e., $\widehat{mc}_t = 0$, for all t. That policy implies, in turn, zero domestic inflation ($\pi_{H,t} = 0$, all t) and constant (and identical) prices and markups for each individual firm. The same would be true for the world economy, where the policymaker should seek to maintain $\widehat{mc}_t^* = \widehat{\pi}_t^* = 0$, for all t. The intuition behind that property of the optimal policy is straightforward: in that state of affairs no firm has an incentive to change its price even if it has the opportunity to do so, since the current price is always consistent with the markup that would be desired in the absence of constraints on price adjustment. The latter constraint, hence, becomes nonbinding under the optimal policy.

Let us next see how that optimal policy can be implemented.

4.1 Optimal Monetary Policy in the World Economy

To begin with, we assume that the monetary authority in the rest of the world pursues an optimal policy itself. As argued above, that policy seeks to stabilize the real marginal cost at its steady state level ($\widehat{mc}_t^* = 0$, all t), which in turn implies price stability ($\pi_t^* = 0$, all t).

Setting $\widehat{mc}_t^* = 0$ in equation (23) allows us to determine the equilibrium level of world output associated with that optimal policy:

$$\widehat{Y}_t^* = \Gamma \; z_t^* \tag{26}$$

[10] A similar approach to the modelling of optimal monetary policy, in the context of different models, can be found in Obstfeld and Rogoff (1999) and Woodford (1999).

where $\Gamma \equiv \frac{1+\phi}{\sigma+\phi}$.

Given (15), we can easily derive an expression for the interest rate that is consistent with the previous output process:[11]

$$r_t^* = -\sigma(1-\rho)\Gamma \; z_t^* \tag{27}$$

4.2 Optimal Monetary Policy in the Small Open Economy

Using (25), and conditional on the monetary authority in the world economy following an optimal policy, we can derive the equilibrium path for the terms of trade and the exchange rate that would be consistent with constant marginal cost and domestic inflation.

$$\widehat{e}_t = \widehat{s}_t = \sigma\Gamma\Theta \; (z_t - z_t^*) \tag{28}$$

where $\Theta \equiv \frac{\sigma+\phi}{\sigma+\phi\omega} > 0$, and Γ (defined above) is independent of α and η, the parameters that are specific to the open economy. The implied equilibrium process for the real exchange rate and the domestic CPI are given by

$$\widehat{q}_t = (1-\alpha) \; \sigma\Gamma\Theta \; (z_t - z_t^*) \tag{29}$$

$$\widehat{P}_t = \alpha \; \sigma\Gamma\Theta \; (z_t - z_t^*) \tag{30}$$

Furthermore, given (10) and (18), we see that the corresponding processes for output and consumption under the optimal policy are:

$$\widehat{C}_t = \Gamma \; \left[(1-\alpha)\Theta \; z_t + (1-(1-\alpha)\Theta) \; z_t^*\right]$$

and

$$\widehat{Y}_t = \Gamma \; \left[\omega\Theta \; z_t + (1-\omega\Theta) \; z_t^*\right]$$

Since $(1-\alpha)\Theta \in (0,1)$ domestic consumption will always increase in response to favorable technology shocks, both domestic and foreign, under the optimal policy. On the other hand, while output in the small open economy always increases in response to a positive technology shock at home, the sign of the response to an external shock is ambiguous. In particular, that effect will be negative if $\omega\Theta > 1$, which in turn requires that the expenditure-switching effect of a change in the world interest rate dominates the direct demand effect, which corresponds to the assumption $\sigma\eta > 1$, satisfied below by our benchmark calibration.

[11] In order for equation (27) to be interpretable as an optimal *rule*, we could add an extra term (e.g., $\varphi_\pi \pi_t^*$ with $\varphi_\pi > 1$). In that case we would eliminate the indeterminacy that would otherwise be associated with an interest rate that depends on exogenous variables only. Notice however that such a term will be zero in equilibrium. See, e.g., Bernanke and Woodford (1997), and Clarida, Galí and Gertler (1999) for a detailed discussion.

What interest rate rule will support the optimal allocation ? Substituting (28) and (27) into (20), we obtain the equilibrium process for the domestic interest rate associated with the optimal policy:[12]

$$r_t = -\sigma(1-\rho)\Gamma\ [\Theta\ z_t + (1-\Theta)\ z_t^*]$$

which can in turn be rewritten as:

$$r_t = \varphi_r\ r_t^* - \sigma(1-\rho)\Gamma\Theta\ z_t \tag{31}$$

where $\varphi_r \equiv \frac{\phi(\omega-1)}{\sigma+\phi\omega}$.

Hence, the optimal interest rate in the small open economy is a function of the world interest rate and domestic productivity. Notice that the sign of φ_r–the coefficient measuring the sensitivity to the world interest rate–is ambiguous. A necessary and sufficient condition for $\varphi_r > 0$ is given by $\sigma\eta > 1$, which is likely to be satisfied for empirically reasonable values of the degree of risk aversion σ, and the elasticity of substitution between foreign and domestic goods, η. To the extent that such a condition is satisfied two observations are in order. First, in that case we have $\varphi_r \in (0,1)$: the optimal policy involves a positive comovement of domestic and foreign interest rates and, thus, some degree of *exchange rate smoothing,* since fluctuations in the exchange rate are a function of interest rate differentials. That result is quite intuitive: under the assumption that $\sigma\eta > 1$, an increase in the world interest rate leads, by itself, to an expansion in economic activity and real marginal costs in the small economy (resulting from the expansionary effects of a real depreciation); in order to stabilize marginal costs, the domestic central bank has to respond by raising the domestic nominal rate,

The evidence of a positive correlation of domestic and foreign interest rates is often interpreted as suggestive of coordination among central banks, or the desire to stabilize nominal exchange rates. The novel aspect of our finding is that the positive interest rate comovement arises from optimizing behavior by independent central banks, in a context of flexible exchange rates. One can also show, in addition, that φ_r is increasing in the openness index α; hence, the more open the economy is, the smaller are the interest rate differentials allowed for by the optimal policy.

We can also derive expression for equilibrium real exports

$$\begin{aligned} \widehat{EXP}_t &= \widehat{C}_{H,t}^* \\ &= \eta\ \widehat{e}_t + \widehat{Y}_t^* \\ &= \sigma\eta\Gamma\Theta\ (z_t - z_t^*) + \Gamma\ z_t^* \\ &= \Gamma\ [\sigma\eta\Theta\ z_t + (1-\sigma\eta\Theta)\ z_t^*] \end{aligned}$$

[12] Once again, indeterminacy can be ruled out by having the nominal rate respond with sufficient strength to domestic inflation.

and imports (in terms of domestic goods):

$$\begin{aligned}
\widehat{IMP}_t &= \widehat{s}_t + \widehat{C}_{F,t} \\
&= [1-\eta(1-\alpha)]\ \widehat{e}_t + \widehat{C}_t \\
&= \Gamma\ \{\Theta(\sigma-(1-\alpha)(\sigma\eta-1))\ z_t + [1-\Theta(\sigma-(1-\alpha)(\sigma\eta-1))]\ z_t^*\}
\end{aligned}$$

Hence we see that, under the optimal policy, all the fluctuations in the variables of interest for the small economy are proportional to a weighted average of domestic and foreign productivity. Next we discuss some of the implied dynamics for a calibrated version of the model, with the help of some impulse response graphics.

4.2.1 Dynamic Effects of a Domestic Technology Shock

In this section we use the equilibrium conditions derived above to characterize and discuss the dynamic effects of a *domestic* technology shock (i.e., a unit shock to z_t). The results reported here are obtained using the following calibration. We set σ equal to unity, which corresponds to a log utility specification.[13] We assume $\phi = 1$, which implies a unit labor supply elasticity. Following much of the international RBC literature, we set the elasticity of substitution between domestic and foreign goods, η, equal to 1.5.[14] For the openness index α we assume a value of 0.4, which is not too far from the average share of imports in GDP for a "typical" small economy. Parameter θ is set equal to 0.75, a value consistent with an average period of one year between price adjustments. We assume $\beta = 0.99$, which implies a riskless return of about 4 percent in the steady state. Finally, we set $\rho = 0.9$, a value consistent with the fairly persistent technology variations assumed in the RBC literature. We assume identical values of σ, ϕ, β, and θ for the world economy. Notice that our calibration implies that $\sigma\eta > 1$, and hence $\omega > 1$ and $\Theta > 1$; as discussed above, that assumption implies that, ceteris paribus, a increase in the world real interest rate has an expansionary effect on the domestic economy.

The impulses responses for the different variables to a one percent innovation in domestic technology are displayed in Figure 1.[15] By design, domestic inflation and the real marginal cost remain unchanged. The optimal response requires an expansion of output that is absorbed by an increase in both consumption and net exports. We also see that a positive technology shock leads to a persistent reduction in the domestic interest rate under the optimal policy. Such a decline is largely mirrored by the real rate, and accounts for the increase in consumption on impact, as well as its gradual return to the initial level. Since the world interest rate remains unchanged, uncovered interest parity requires an initial depreciation of the domestic currency, followed by a gradual reversion to its initial level (i.e., an anticipated appreciation).

[13] That assumption makes the model consistent with sustained balanced growth, given our utility function.

[14] See, e.g., Backus, Kehoe, and Kydland (1994).

[15] By construction, the effect on world aggregates is negligible and is thus not displayed.

Notice also that the level of the CPI jumps up in the period of the shock (because of the exchange rate depreciation), and then reverts back to trend, thus generating the path of inflation seen in the figure. That response of CPI inflation accounts for the gap between the responses of the real exchange rate and that of the terms of trade (which corresponds to the nominal exchange rate).

The increase in consumption discussed above, combined with the increase in net exports, leads to an expansion of aggregate demand and output. Notice that about half of the increase in output is accounted for by the increase in the trade balance, which displays a persistent surplus. That surplus is the result of an increase in exports (resulting from the greater competitiveness associated with the real depreciation) above the increase experienced by imports that prevails under our parametrization.[16]

4.2.2 Dynamic Effects of a Technology Shock Abroad

Figure 2 displays the effects of a positive technology shock in the rest of the world. In order to stabilize marginal costs and inflation, the monetary authority overseas lowers the interest rate, which induces a persistent increase in both world output and consumption. The domestic central bank also lowers its interest rate, in order to counteract the decline in marginal cost resulting from the real appreciation caused by the lower world interest rate. Notice, however, that the domestic rate remains above the world rate all along, in a way consistent with the observed appreciation of the domestic currency on impact (and an induced fall in the CPI), followed by a gradual depreciation until both interest rates converge at their initial level. Hence, while the lower domestic interest rates account for the observed expansion in consumption, the exchange rate appreciation accounts for the negative contribution of the external sector to aggregate demand and output. As discussed above, the latter effect prevails under our calibration. That generates a negative correlation between outputs across countries, which can only be undone by allowing for a positive correlation between technology shocks.

4.2.3 Optimal Exchange Rate Volatility

Equation (**??**) represents the equilibrium behavior of the nominal exchange rate under the optimal policy. An interesting feature is worth emphasizing: under the optimal policy, the nominal exchange rate is stationary. This is consistent with full stability of prices (at home and abroad) combined with the stationarity of the terms of trade associated with the optimal allocation. It is in stark contrast, however, with the unit root that seems to characterize the empirical behavior of that variable.(reference?)

Of course, stationarity does not necessarily imply low volatility. The latter will be a function of the relative variances of domestic and world productivity, as well as the

[16]The literature, both theoretical and empirical, does not seem to agree on the importance of the expenditure switching effect in driving the short-run dynamics of the balance of trade. See Obstfeld-Rogoff (1999) for an empirical argument in favor of this effect.

correlation between the two. More precisely, the variance of the nominal exchange rate under the optimal policy is proportional to

$$(\sigma_z - \sigma_{z^*})^2 + 2\ \sigma_z \sigma_{z^*}\ (1 - \rho_{z,z^*}) \tag{32}$$

where σ_z and σ_{z^*} denote the standard deviation of domestic and world productivity, and ρ_{z,z^*} their correlation. Hence, we see that the "optimal volatility" of the nominal exchange rate is increasing with the extent of the asymmetry between the two shocks, both in terms of their magnitude (represented by the first term) and their comovement (measured by the second term). That volatility is likely to be related to the cost of pegging the exchange rate or joining a "large" monetary union, as it is typically pointed out in the literature on optimum currency areas.

In order to ascertain the order of magnitude of the optimal exchange rate volatility implied by our model, we compute second moments for our calibrated economy. They are reported in Table 1. We consider three scenarios: (a) domestic shocks only, (b) world shocks only, (c) both domestic and foreign shocks. Throughout, the variance of the technology shock is the same for the two economies and normalized so that the standard deviation of world output is equal to one under our calibration. The latter provides a useful benchmark since, under our calibration, it corresponds to the standard deviation of output for a closed economy (and, hence, for the world economy in our model). In addition we calibrate the covariance between the two shocks so that the correlation of domestic output with world output in the third scenario is 0.7, a value that we take as typical of many small open economies.[17]

Notice that when both shocks are operative the implied volatility of the real and nominal exchange rate is substantially lower than that of output, consumption and trade quantities and, hence, far lower than the relative volatility observed in the data.[18] We interpret this as the optimal exchange rate volatility implied by the model, conditional on our calibration. Hence, we see that nominal exchange rates are required to fluctuate substantially under the optimal policy, a result that could already be inferred from the impulse responses discussed above.

The previous result points to the perils of policies that may seek to stabilize the value of the currency (i.e., to reduce the volatility of the nominal exchange rate below that implied by the optimal policy). Those policies will either be a source of relative price distortions (i.e., real exchange rate "misalignment") or, even if they were to replicate the optimal real exchange rate path they could not be consistent with the optimal allocation (since in that case they would require variations in domestic prices that do not occur under flexible prices).

Notice, finally, that the volatility of the exchange rate decreases significantly in the two-shock scenario with respect to the one with one shock only. This is a consequence of the positive correlation between domestic and foreign shocks, since under our

[17]Thus, e.g., Backus et al. (1985) report a correlation of 0.76 between U.S. and Canadian GDP.

[18]The standard deviation of the real exchange rate relative to that of output reported in Backus et al. (1985) is 1.91 for the U.S., 2.0 for Canada, 1.76 for Germany, and 1.95 for the U.K..

assumption of equal variances of the shocks, the variance of the nominal exchange rate is proportional to $\sigma_z^2(1-\rho_{z,z^*})$.

4.2.4 Openness, Optimal Policy and Macroeconomic Volatility

How does the degree of openness of a small economy affect the characteristics of its optimal monetary policy and the resulting macroeconomic outcome?

In Figure 3 we first plot the relationship between our measure of openness α and the size of the coefficient φ_r, where the latter measures the size of the domestic interest rate response to a one percent change in the world rate under the optimal policy.[19] As mentioned above, and given that the condition $\sigma\eta > 1$ is satisfied under our calibration, φ_r lies between zero and one. Most interestingly, and as illustrated in the figure, it can also be shown to be strictly increasing in α.[20] Not surprisingly, in the closed economy limit (i.e., as $\alpha \rightarrow 0$), the domestic rate does not respond to changes in the world rate. As $\alpha \rightarrow 1$, φ_r does not exceed 0.2, in line with the idea that the optimal policy entails only a partial degree of exchange rate smoothing in the face of external productivity shocks. Notice also that the size of the optimal interest rate response to domestic shocks is decreasing in openness, since $\frac{\partial\Theta}{\partial\alpha} < 0$.

Are exchange rates intrinsically more volatile in more open economies? What are the consequences for the overall macroeconomic volatility ? Figure 4 displays the relationship between openness and the volatility of exchange rate, output, and consumption under our benchmark calibration. First, notice that the volatility of the exchange rate decreases as the economy becomes more open. This follows from the fact that the standard deviation of the interest rate differential is decreasing in α. By contrast, the volatility of output rises with α: the more open is the economy, the greater is the sensitivity of output to both domestic and foreign shocks.[21]In other words, the increasing stabilization of the nominal (and therefore real) exchange rate implies smaller room for real relative price adjustment as a stabilization tool, with a resulting higher instability of overall output. Notice that the previous results hold independently of the relative importance of domestic and foreign shocks.

Finally, the effect on consumption volatility is non monotonic. Under perfect international risk-sharing, the two extreme cases of a closed economy ($\alpha = 0$) and a fully open economy ($\alpha = 1$) are symmetric. In the former case, the volatility of consumption coincides with that of output in the closed economy, whereas in the latter it equals that of foreign consumption which, in turn, corresponds to that of output in the closed economy.[22] It is clear from equation (**??**) and the fact that $\frac{\partial\Theta}{\partial\alpha} < 0$ under our assumptions, that the volatility of consumption is inversely related to openness in the presence of domestic shocks only, but positively so if foreign shocks

[19] We keep constant the assumed settings for the remaining parameters.

[20] To see this, notice that ω is increasing in α.

[21] This can be simply shown by deriving the analytical expression for $var(y)$ implied by equation (**??**).

[22] In equation (**??**), in fact, $\widehat{C}_t = \Gamma\widehat{Z}_t = \widehat{Y}_t$ for $\alpha = 0$, and $\widehat{C}_t = \Gamma\widehat{Z}_t^* = \widehat{Y}_t^*$ for $\alpha = 1$.

are the dominant source of fluctuations. When both shocks are present the two effects combine to give the hump shaped relationship displayed in the bottom panel.

5 A Taylor Rule

How does a simple, empirically based, interest rate rule compare with the optimal policy? Since the work of Taylor (1994) a simple linear rule of the form:

$$r_t = b_\pi \; \pi_t + b_y \; \widehat{Y}_t \tag{33}$$

has been used frequently in monetary models of the macroeconomy. This popularity is mostly due to its ability to track well the behavior of interest rates in the US. and in other industrialized countries, especially after 1979.[23] In the present section we analyze the properties of the equilibrium of our small economy when the optimal rule (**??**) is replaced by (33), with $b_\pi = 1.5$ and $b_y = 0.5$ as in Taylor (1994).[24]

In Figures 5 and 6 we report impulse responses of different variables to domestic and world productivity shocks, respectively, under both the optimal policy (*solid line*) and the Taylor rule (*dashed*).

Several interesting differences emerge. Consider first the case of a favorable domestic productivity shock. Under a Taylor rule the real marginal cost falls below its optimal level and, as a result, so does domestic inflation. Such responses suggest that the Taylor rule implies too tight a policy in response to that shock. Yet, a look at the pattern of the nominal interest rate points to a much larger decline under the Taylor rule. What is going on? It turns out that, in contrast with the behavior of its nominal counterpart, the decline in the real interest rate is significantly smaller under the Taylor rule (this is true regardless of the inflation measure used to construct it). That has two consequences. First, the expansion of consumption is smaller. Second, the response of the terms of trade is also more muted, which leads to a more modest expansion of net exports. As a result, both output and employment remain below their optimal levels, which accounts for the observed decline in real marginal cost and domestic inflation. Therefore the key difference between the two rules lies in the short run dynamics of the real interest rate, with the latter not being sufficiently responsive under a Taylor rule. The previous observation suggests that the optimal path for the real rate could be approximated by making b_π arbitrarily large in equation (33).[25]

Two other differences are worth mentioning. First, under a Taylor rule the nominal exchange rate moves in the "wrong" direction and, furthermore, it becomes nonstationary. The latter property follows from the persistent response of inflation

[23] See also Taylor (1999), Judd and Rudebusch (1998), Clarida, Galí, and Gertler (1998, 2000), for an empirical analysis of Taylor-type rules.

[24] For comparison purposes we maintain the assumption of optimal policy in the rest of the world.

[25] In results not reported here we show that as $b_\pi \to \infty$ the dynamic adjustment of all the variables coincide under the two rules.

(which implies a permanent effect of the shock on the price level, and hence a unit root in the latter), combined with the stationarity of the real exchange rate (which is a more general property of the model). Second, CPI inflation falls on impact under the Taylor rule, as a result of both lower domestic inflation and the exchange rate appreciation.

Consider next the different implications of the two rules in response to a favorable productivity shock in the rest of the world. Looking at the response of domestic inflation and marginal cost suggests that the domestic central bank does not tighten policy sufficiently. That observation, however, seems at odds with the observed responses of the nominal rate under the two regimes. But it is the path of real interest rates that determines aggregate demand, and we see how both of its measures remain below their optimal paths. The difference in the CPI-based measures is almost negligible and, accordingly, so is the differential response of consumption. But this not true for the domestic inflation-based real rate which now remains closer to the world real rate along the adjustment path.[26] As a result, the size of the real appreciation is smaller under the Taylor rule, and so is the contraction in net exports. That accounts for the smaller decline in output, and the observed suboptimal pattern of marginal costs and inflation.

The second panel of Table 1 provides some statistics for the calibrated economy under a Taylor rule, which can be compared with their counterparts under the optimal policy. The results point to the following feature: with respect to the optimum, monetary policy under a Taylor rule delivers excess volatility of nominal variables, coexisting with excess smoothness of real variables. Independently of the distribution of the shocks, under the Taylor rule the volatility of output and consumption, and relative prices is lower than the optimal. Also, and as emphasized above, it is the insufficient responsiveness of the real interest rates that accounts for that lower volatility. On the other hand, a Taylor rule implies a volatility far above the optimal one for nominal interest rates, marginal costs, and domestic inflation. For CPI inflation, on the other hand, the ranking of volatilities under the two policies considered here is ambiguous and depends on the dominant source of fluctuations. When the two shock are present, CPI inflation is much more variable under a Taylor regime.

6 An Exchange Rate Peg

In the present section we consider a third monetary arrangement for the small open economy: a permanent (and credible) exchange rate peg. In the context of our model, this should be equivalent to the adoption by the small economy of the world currency, with the corresponding relinquishment of an autonomous monetary policy. For simplicity, we maintain the assumption of an optimal monetary policy in the

[26]Recall from Figure 2 that the world real rate remained well below the domestic rate under the optimal policy in reponse to the shock under consideration.

world economy. That policy is based on world aggregates, and is not affected by the joining of the world monetary union by our small economy, given the negligible size of the latter.[27]

In the absence of capital controls, an implication of monetary integration is the equalization of the domestic interest rate to the world interest rate. Under the assumption that the world monetary authority pursues an optimal policy it follows that:

$$r_t = -\sigma(1-\rho)\Gamma \ z_t^* \tag{34}$$

for all t.

Constancy of the nominal exchange rate and world prices (the latter resulting from the assumption of an optimal policy at the world level), imply that $\widehat{s}_t = -\widehat{P}_{H,t}$. Substituting the latter equality into (25) and combining the resulting expression for marginal cost with inflation equation (24), we can derive the following equilibrium process for domestic prices:

$$\gamma \ \widehat{P}_{H,t} = \widehat{P}_{H,t-1} + \beta \ E_t\{\widehat{P}_{H,t+1}\} + \lambda(1+\phi) \ (z_t^* - z_t)$$

where $\gamma \equiv 1+\beta+\lambda\left(1+\frac{\phi\omega}{\sigma}\right)$. The previous difference equation can be shown to have two positive, real eigenvalues, one stable and one unstable. Hence, $\widehat{P}_{H,t}$ is uniquely determined as a function of current and past realizations of productivity differentials $\{z_t^* - z_t\}$, and has a stationary ARMA(1,1) representation. The stationarity of the price level is a direct implication of the stationarity of the terms of trade, given the constancy of the nominal exchange rate and the world price level. Given the equilibrium process for $\{\widehat{P}_{H,t}\}$ it is straightforward to determine the corresponding processes for the terms of trade, the real exchange rate, output, consumption, etc. Furthermore, it is easy to check that the resulting equilibrium path for domestic consumption is consistent with the Euler equation, given the interest rate path in (34).

Notice that, in equilibrium, the variance of domestic inflation and the terms of trade will be proportional to the variance of the productivity differential, given by expression (32), and will thus be decreasing with the degree of comovement between domestic and world shocks. In the limiting case of perfect correlation, the optimal policy pursued by the world monetary authority will also be optimal for the small economy, leading to complete price and marginal cost stability in the latter.

Figures 7 and 8 display the responses of different variables to domestic and world productivity shocks, respectively, under both the optimal policy (*solid line*) and an exchange rate peg (*dashed*) implied by our calibrated model. One can then interpret the gap between the two as an indicator of the inefficiency associated with the relinquishment of monetary independence.

[27]We are very grateful to Chris Erceg for pointing out a critical mistake in the treatment of the exchange rate peg in an earlier version of the present paper.

As was the case with the Taylor rule, in response to a favorable domestic productivity shock the real marginal cost falls below its optimal level and, as a result, so does domestic inflation. This is a consequence of the impossibility of lowering the nominal rate and letting the currency depreciate, as would be needed in order to support an expansion in consumption and output as strong as that associated with the flexible price allocation.

Notice also that the stationarity of the price level implied by the peg requires that the initial, short-lived deflation is eventually followed by a period of persistent (albeit low) inflation. That pattern, combined with the constancy of the nominal rate, implies a decline in expected *long* real rates (not shown in the figure), which accounts for the expansion in consumption and the real depreciation (with the consequent rise in net exports). Those responses, in turn, explain the observed increase in aggregate demand and output.

The limited persistence of marginal costs and the eventual reversal of its sign lies behind the significant improvement observed in the short run trade-off between the output gap and inflation, relative to the Taylor regime. That result is reminiscent of the gains that the central bank derives from pursuing a price level targeting rule or those that result from its being able to commit to certain future actions (which are ex-post suboptimal), and which have been shown to arise in closed economies.[28]

The statistics in Table 1 reflect some of those results. Hence, we see that, in the presence of domestic shocks, the volatility of output, consumption, inflation, and the terms of trade under an exchange rate peg (shown in the third panel) is closer to the optimal than that implied by the Taylor rule.

Figure 8 illustrates the consequences of an exchange rate peg in the face of a favorable productivity shock abroad. In this case the need to match the persistent decline in the world interest rate causes a strong expansion of output at home, with short run inflationary consequences. The initial equilibrium is restored through the persistent real appreciation caused by higher domestic prices, with the resulting loss in competitiveness and decline in net exports. Notice that, under an exchange rate peg, the path of the real interest and the real exchange rate deviate persistently from their optimal paths; after a few quarters the size of that deviation, though small, is sufficient to generate a deflation that restores the initial price level. Interestingly, monetary integration leads to a positive correlation between domestic and world outputs, in contrast with the two other regimes considered. That synchronization of business cycles is a consequence of the identical interest rate responses to the world productivity shock. As illustrated in Table 1, and because of the inability to pursue its own stabilizing policy, output, consumption, and inflation are more volatile than under the Taylor rule, since the latter allows for some partial stabilization.

Notice finally that both the real exchange rate and the terms of trade are more stable under an exchange rate peg than under the optimal policy. That finding, which

[28] See, e.g., Clarida, Gali, and Gertler (1999), Woodford (1999), Rotemberg and Woodford (1999), Vestin (1999).

is consistent with the evidence of Mussa (1986), points to the existence of "excess smoothness" in real exchange rates under a peg, and arises from the inability of prices (which are sticky) to compensate for the constancy of the nominal exchange rate.[29]

7 Conclusions

In the present paper we have developed a *small open economy* version of a canonical sticky price model and used it to analyze the properties of three alternative monetary regimes: (a) optimal monetary policy, (b) a Taylor-rule based regime, and (c) an exchange rate peg. Two features of our model distinguish it from much of the existing literature. First, and because of staggered price setting, the model implies persistent deviations from the flexible price allocations. Second, we model monetary policy as endogenous, with a short-term interest rate being the instrument of that policy.

Our analysis yields a number of interesting insights, which we briefly summarize here.

1. The optimal monetary policy in our calibrated economy entails a positive correlation with the policy stance in the rest of the world. This arises in our analysis as a result of optimizing behavior by independent central banks, without no specific mandate to stabilize exchange rates.

2. The previous result notwithstanding, the optimal monetary policy requires sizeable volatility of nominal exchange rates. Yet, that volatility is significantly below that observed historically in flexible exchange rate periods. Furthermore, the optimal volatility of exchanges rates decreases with the degree of openness of the economy.

3. Simple interest rate rules of the Taylor type generally lead to excess volatility of nominal variables, and excess smoothness of real variables, relative to the optimal policy.

4. By anchoring the level of a nominal variable, an exchange rate peg appears to approximate the optimal policy (which stabilizes the price level) better than the Taylor rule (which seeks to stabilize inflation rate and output).

We are currently working on a two country version of the framework that we have developed here. We plan to use it to analyze a number of issues that cannot be addressed with the present model, including the importance of spillover effects in the design of optimal monetary policy, the potential benefits from monetary policy coordination, and the implications of exchange rate stabilization agreements.

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[29] See Monacelli (1999) for a detailed analysis of the implications of fixed exchange rates.

Appendix 1: The Steady State

Let us characterize the perfect foresight steady state of our small open economy model, taking Y^* as given and $Z = 1$. We use variables without time subscripts to refer to steady state values. Markups are constant in the steady state, implying a product wage $\frac{W}{P_H} = (1 - \frac{1}{\varepsilon})$. The latter fact can be combined with (5) and the identity $\frac{P}{P_H} = [(1-\alpha) + \alpha \ s^{1-\eta}]^{\frac{1}{1-\eta}} \equiv g(s)$ implies $C^\sigma Y^\phi = (1 - \frac{1}{\varepsilon}) \ \frac{1}{g(s)}$.

In steady state, risk sharing condition (9) takes the form $C^\sigma = (\kappa Y^*)^\sigma \ q(s)$ where $q(s) \equiv \frac{s}{g(s)}$ links real exchange rate and the terms of trade in the steady state. One can easily check that $g'(s) > 0$, and $q'(s) > 0$. Combining the previous results we obtain (up to a multiplicative constant):

$$Y = \left(\frac{1 - \frac{1}{\varepsilon}}{s \ (\kappa Y^*)^\sigma} \right)^{\frac{1}{\phi}} \equiv H(s, Y^*) \tag{35}$$

On the other hand, market clearing requires $Y = (1-\alpha) \ g(s)^\eta \ C + \alpha^* \ s^\eta \ Y^*$, which can be combined with (9) to yield:

$$Y = (1-\alpha) \ g(s)^\eta \ \kappa Y^* \ q(s)^{\frac{1}{\sigma}} + \alpha^* \ s^\eta \ Y^* \equiv J(s, Y^*) \tag{36}$$

Notice that $H_s < 0$, with $\lim_{s \to 0} H(s, Y^*) = +\infty$ and $\lim_{s \to \infty} H(s, Y^*) = 0$. On the other hand, $J_s > 0$, and $\lim_{s \to 0} J(s, Y^*) = 0$ and $\lim_{s \to \infty} J(s, Y^*) = +\infty$. Hence, and given a value of world output (and initial conditions implicit in κ), (35) and (36) jointly determine a unique, strictly positive, steady state value for s and $q(s)$.

For convenience, and without loss of generality, we can assume that initial conditions are such that $\frac{\alpha\kappa}{\alpha^*} = 1$. In that case, the steady state is characterized by $s = q(s) = 1$ and $C = Y = \kappa Y^*$, as well as balanced trade.

Appendix 2: Optimal Price Setting

Each firm may resets its price with probability $1 - \theta$ each period, independently of the time elapsed since the last adjustment. Thus, each period a measure $1 - \theta$ of firms reset their prices, while a fraction θ keep their prices unchanged. Let $P_t^{new}(j)$ denote the price set by firm j that adjusts its price in period t. Under the previous price-setting structure, $P_{t+k}(j) = P_t^{new}(j)$ with probability θ^k for $k = 0, 1, 2, ...$

When setting a new price in period t firm j will seek to maximize:

$$\min_{P_t^{new}} \sum_{k=0}^{\infty} (\beta\theta)^k \ E_t \left\{ Q_{t,t+k} \ [Y_{t+k}(j) \ (P_{H,t}^{new} - MC_{t+k})] \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k}(j) \leq \left(\frac{P_{H,t}^{new}}{P_{H,t+k}} \right)^{-\varepsilon} (C_{H,t+k} + C_{H,t+k}^*) \equiv Y_{t+k}^d(P_{H,t}^{new})$$

Thus, P_t^n must satisfy the first order condition

$$\sum_{k=0}^{\infty} \theta^k \; E_t \left\{ Q_{t,t+k} \; Y_{t+k}(j) \; (P_{H,t}^{new} - \mu \; MC_{t+k})] \right\} = 0 \tag{37}$$

where $\mu \equiv \frac{\epsilon}{\epsilon - 1}$ is the frictionless optimal markup. Using the fact that $Q_{t,t+k} = \beta^k \; (C_t/C_{t+k})^{\sigma} (P_t/P_{t+k})$, we can rewrite it as:

$$\sum_{k=0}^{\infty} (\beta\theta)^k \; E_t \left\{ C_{t+k}^{-\sigma} \; Y_{t+k}(j) \; (P_{H,t}^{new} - \mu \; MC_{t+k})] \right\} = 0$$

or, in terms of stationary variables,

$$\sum_{k=0}^{\infty} (\beta\theta)^k \; E_t \left\{ C_{t+k}^{-\sigma} \; Y_{t+k}(j) \; (p_{H,t}^{new} - \mu \; \Pi_{t,t+k} \; mc_{t+k})] \right\} = 0$$

where $p_{H,t}^{new} \equiv \frac{P_{H,t}^{new}}{P_{H,t}}$ and $\Pi_{t,t+k} \equiv \frac{P_{t+k}}{P_t}$ .Linearizing the previous condition around a perfect foresight, zero inflation, balanced trade steady state we obtain:

$$\widehat{p}_{H,t}^{new} = \sum_{k=1}^{\infty} \; (\beta\theta)^k \; E_t\{\pi_{t+k}\} + (1 - \beta\theta) \sum_{k=0}^{\infty} \; (\beta\theta)^k \; E_t\{\widehat{mc}_{t+k}\}$$

which corresponds to expression (11) in the text.

Under the assumed price-setting structure, the dynamics of the domestic price index are described by the equation

$$P_{H,t} \equiv \left[\theta \; P_{H,t-1}{}^{1-\varepsilon} + (1 - \theta) \; (P_{H,t}^{new})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{38}$$

which can be log-linearized to yield,

$$\pi_t = \left(\frac{1 - \theta}{\theta} \right) \; \widehat{p}_{H,t}^{new}$$

which can be combined with the expression for $\widehat{p}_{H,t}^{new}$ above to yield, after some algebra, (24) in the text. The inflation equation for the world economy can be derived in an analogous manner.

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Table 1
Macroeconomic Volatility under Alternative Monetary Policy Regimes

	Optimal Policy			*Taylor Rule*			*Exchange Rate Peg*		
	shocks			shocks			shocks		
	domestic	foreign	both	domestic	foreign	both	domestic	foreign	both
Output	1.14	0.13	1.03	0.67	0.10	0.60	0.87	0.35	0.98
Consumption	0.52	0.48	0.94	0.31	0.50	0.76	0.40	0.64	0.94
Interest Rate	0.08	0.01	0.09	0.63	0.03	0.60	0.00	0.10	0.10
Dom. Inflation	–	–	–	0.65	0.05	0.60	0.14	0.14	0.09
CPI Inflation	0.15	0.15	0.10	0.63	0.14	0.61	0.08	0.08	0.06
Exports	1.29	0.29	1.08	0.76	0.25	0.60	0.99	0.35	1.01
Imports	0.60	0.39	0.94	0.36	0.42	0.73	0.46	0.6	0.95
Nominal E. Rate	0.86	0.86	0.59	∞	∞	∞	0.00	0.00	0.00
Real E. Rate	0.52	0.52	0.36	0.31	0.50	0.33	0.40	0.40	0.27
Terms of Trade	0.86	0.86	0.59	0.51	0.83	0.55	0.66	0.66	0.45

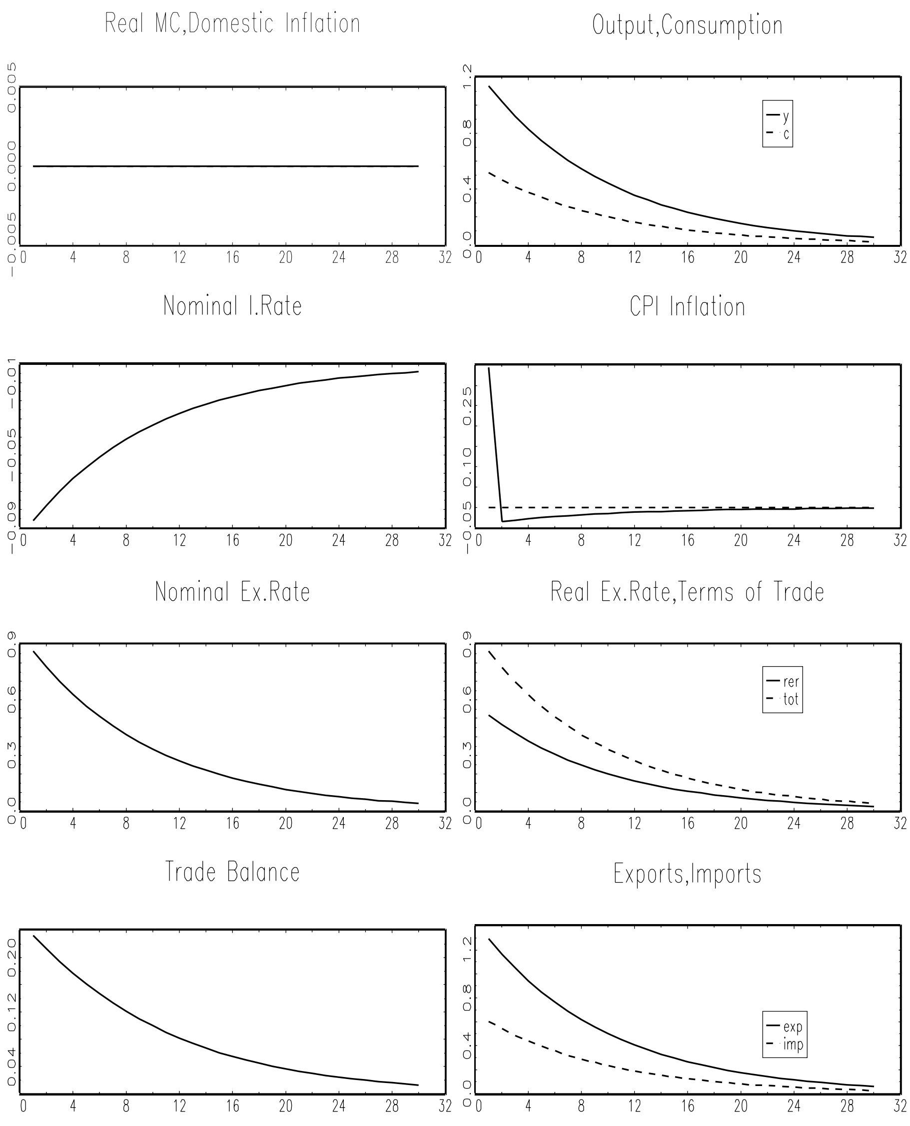

Figure 1: **Impulse Responses to a Domestic Productivity Shock**

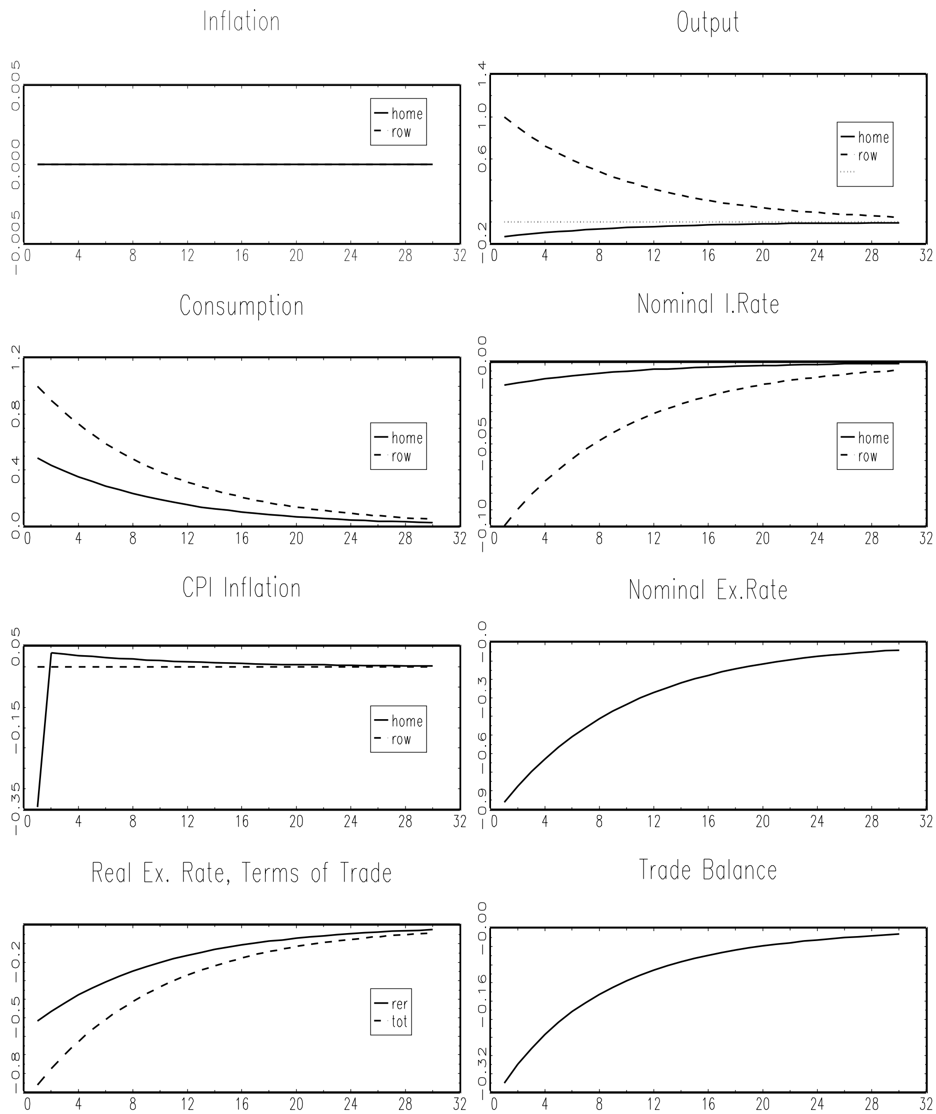

Figure 2: **Impulse Responses to a Productivity Shock in the Rest of the World.**

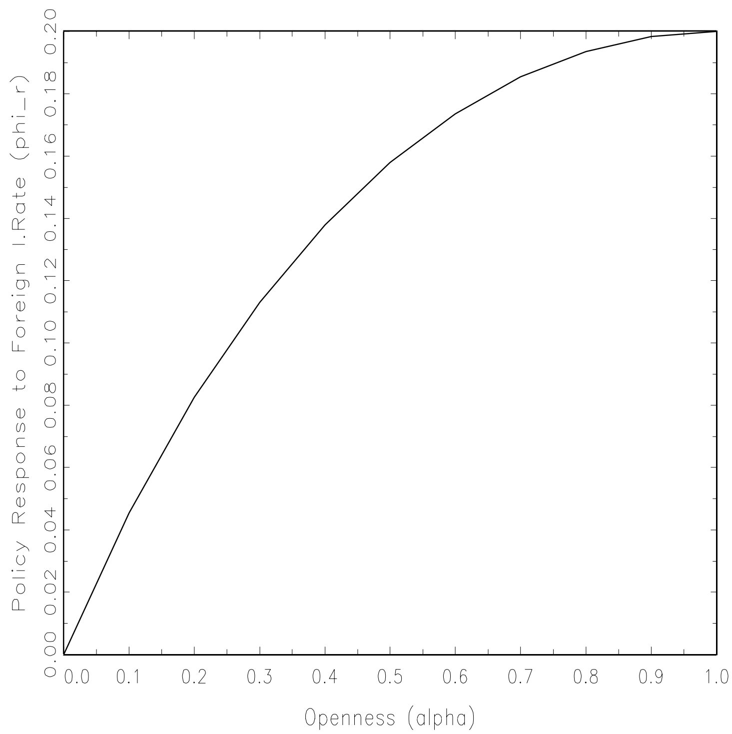

Figure 3: **Openness and Optimal Policy Response to the Foreign Interest Rate**

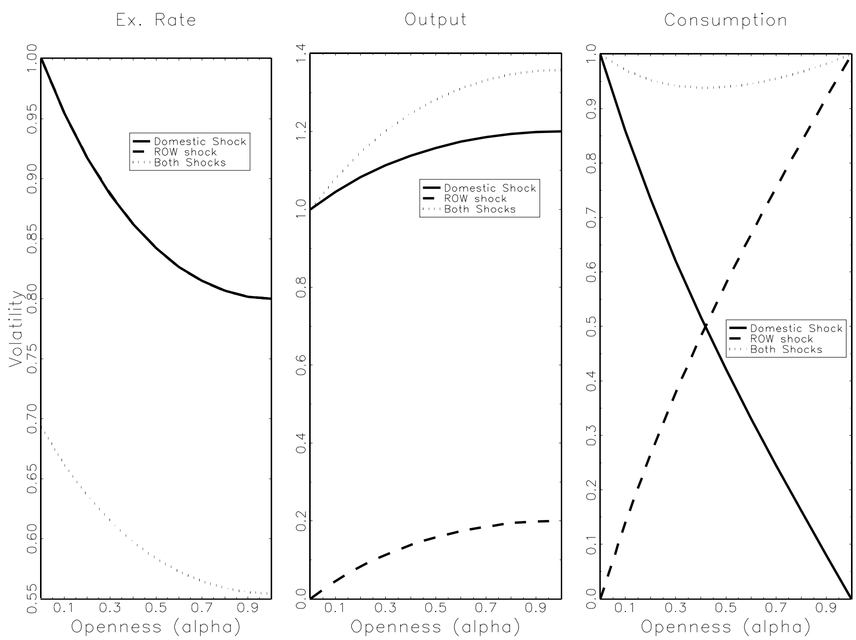

Figure 4: **Openness and Macroeconomic Volatility**

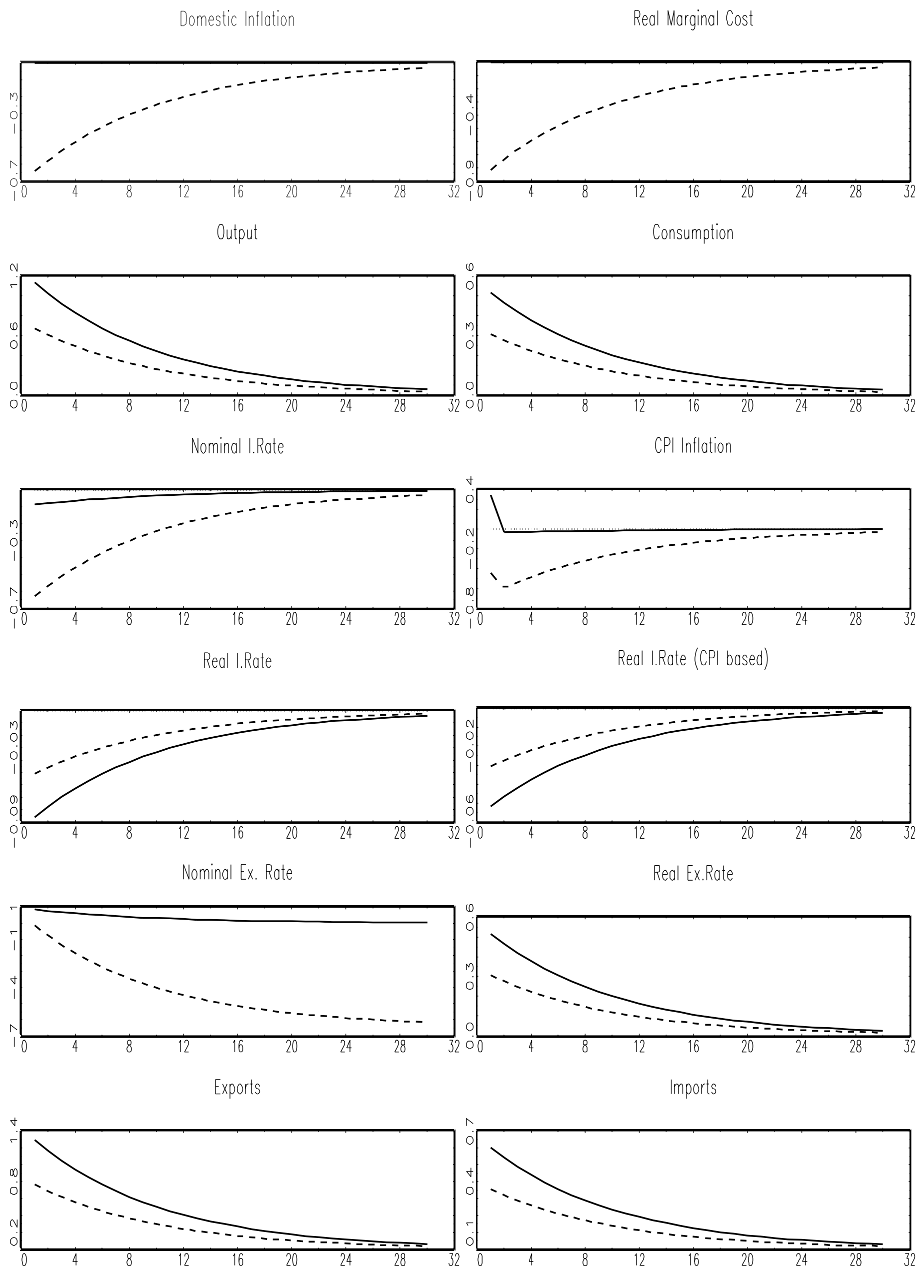

Figure 5: **Optimal (*solid*) vs. Taylor Rule (*dashed*): Domestic Productivity Shock.**

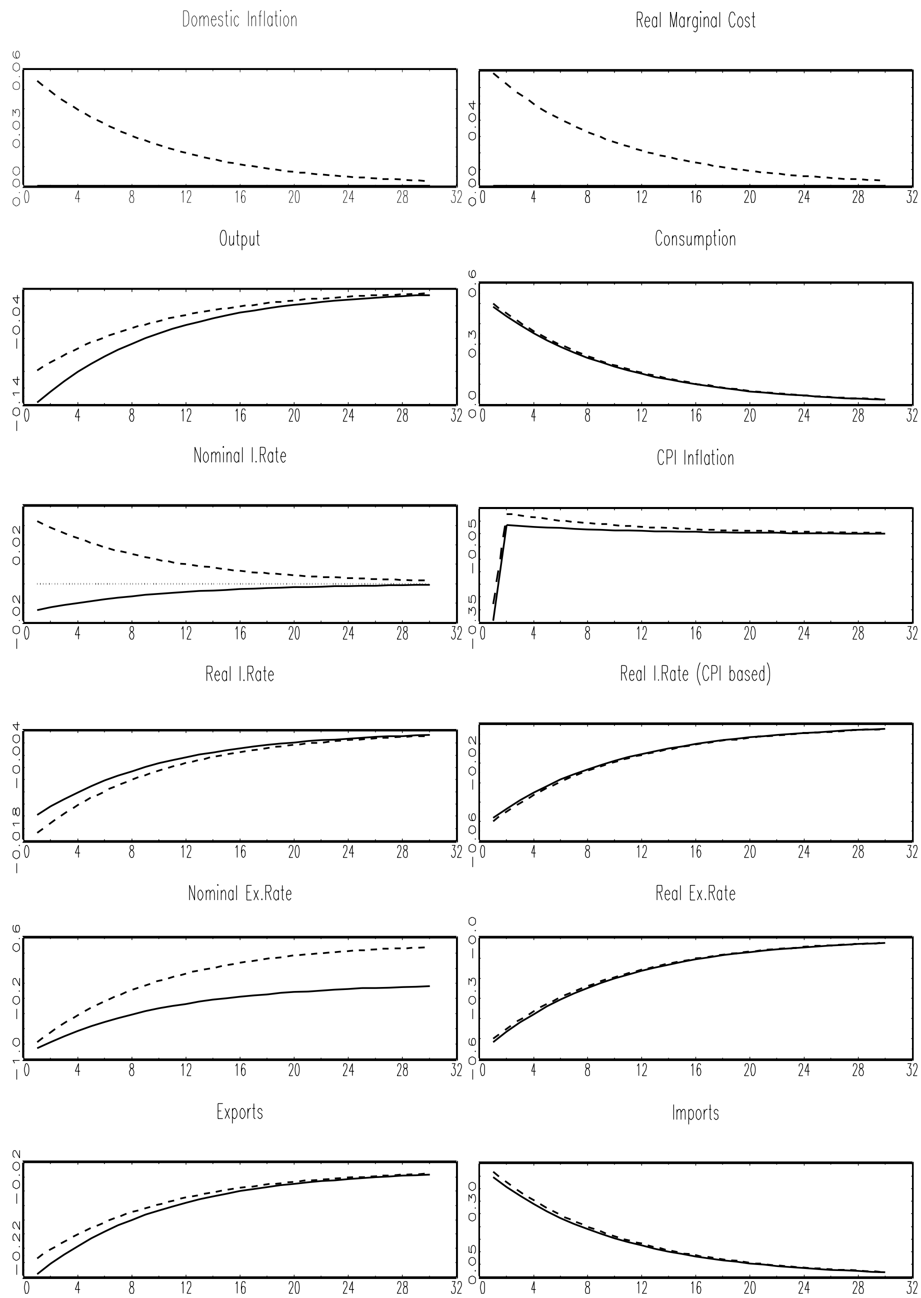

Figure 6: **Optimal (*solid*) vs. Taylor Rule (*dashed*): Productivity Shock in the Rest of the World.**

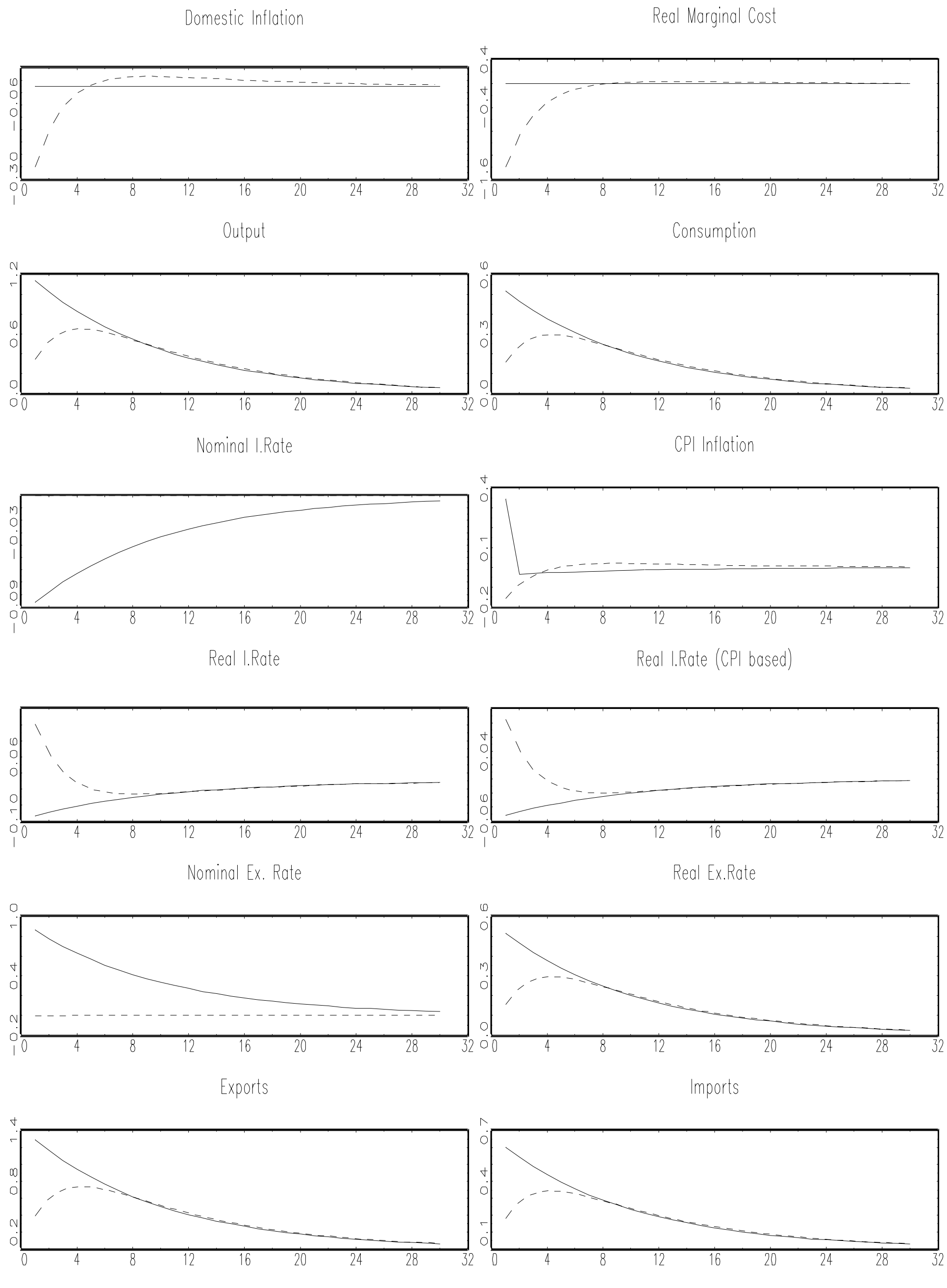

Figure 7: **Optimal (*solid*) vs. Monetary Integration (*dashed*): Domestic Productivity Shock.**

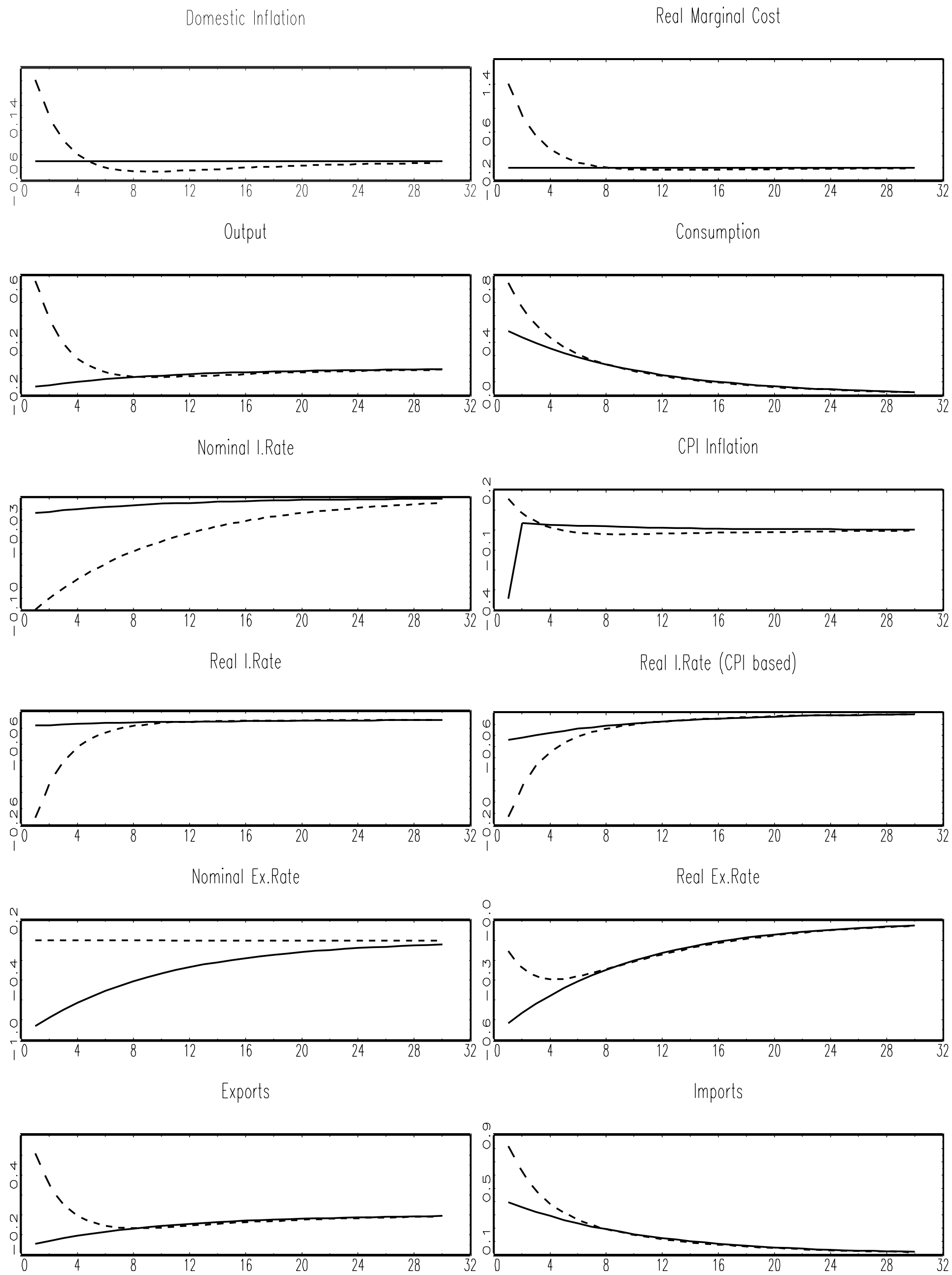

Figure 8: **Optimal (*solid*) vs. Monetary Integration (*dashed*): Productivity Shock in the Rest of the World.**