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Exchange-rate-based versus money-based systems: The advantage of transparency

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ABSTRACT

In the literature on the choice of monetary systems it has been argued that an exchange-rate-based system has a natural advantage over a money-based system since the exchange rate provides a signal that is both clearer and easier to monitor than that provided by a money-based system. We formalize this argument in a simple model. In it the exhange-rate based system is more transparent then the money-based system in that it is easier to monitor policymakers actions in a system that uses exchange rate targets than it is to monitor them in a system that uses money growth targets. We find that the greater transparency of the exchange-rate-based regimes gives it an advantage of the money-based regime.

"True, the exchange rate has some special properties. In particular, it is easily observable, so the private sector can directly monitor any broken promises by the central bank. But we know of no convincing argument that turns these properties into an explanation for why it would be a more efficient method to achieve credibility to target the exchange rate rather than, say, the money growth rate." (Persson and Tabellini 1994 p17)

A classic question in international economics is whether it is better to set monetary policy according to some system based on targeting exchanges or one based on targeting money growth rates. A common argument is that an exchange-rate-based system has a natural advantage over a money-based system since the exchange rate provides a signal that is both clearer and easier to monitor than that provided by a money-based system [1]. This paper formalizes the argument that an exchange-rate-based system has an advantage over a money-based system because it is more transparent.

We formalize this argument using a simple model of sustainable monetary policy similar to that in Kydland and Prescott (1977) and Barro and Gordon (1983). In it each period the central bank chooses either an exchange-rate-based system or a money-based system for monetary policy. Under the exchange-rate-based system the central bank picks an exchange rate and realized inflation varies with shocks to foreign inflation. Under the money-based system the central bank has imperfect control of the money growth rate and can only set the mean of the realized growth rate of money as its money growth target. Hence, under this system, inflation varies with domestic shocks. In either system, the central bank has an incentive in the short run to surprise the public with higher than expected inflation. Our analysis focuses on the question of how the transparency of the system for monetary policy affects the ability of the central bank to sustain low inflation on average.

We say that a system for monetary policy is *transparent* if agents can observe the history of actions that the central bank has undertaken under that system. Throughout we assume that the exchange-rate-based system is transparent, in that private agents can directly observe the history of the exchange rates that the central banks has set. For money-based systems we consider two environments. In the first the money based system is transparent and agents see the history of money growth targets. In this environment we imagine there is a reliable agency that reports the money growth targets. In the second, the money based system is not transparent and agents only see the history of realized money growth rates and inflation rates. In this environment we imagine that there is no such reliable agency.

In the environment in which both systems are equally transparent our model reproduces the classic result: an exchange-rate-based system is preferred if and only if the volatility of foreign inflation shocks is smaller than that of domestic shocks. Here under either system the central bank can sustain a low mean inflation rate because agents can directly verify whether or not the central bank has deviated from its policy and thus any deviation leads to an immediate loss in credibility.

In the environment in which the exchange-rate-based system is transparent but the money-based system is not we find that the exchange-rate-based system has a natural advantage. Even when the volatility of foreign and domestic shocks are equal the exchange-rate-based system is strictly preferred. Here under the money-based system the central bank cannot sustain as low a mean inflation rate as it can under an exchange-rate-based system. This is because under the money-based system it is difficult for agents to detect such deviations in the money growth target while small deviations in the exchange rate target can be detected. Because of the inherent difficulty in monitoring the money growth target the best equilibrium under the money-based system necessarily involves periodic bouts of high inflation that are avoided in the best equilibrium under the exchange-rate-based system.

The equilibrium outcome under the best money-based system looks very different from the equilibrium outcome under the exchange-rate based system. This difference arises because the money-based system lacks transparency. Under a money-based system, agents cannot distinguish whether high realized inflation was the result of government's choice of a high target for money growth or simply the result of bad luck. As a result of this lack of transparency, under a money-based system, the best equilibrium outcome necessarily oscillates between two extremes. At first, government chooses a low target for money growth and low realizations of inflation are relatively likely. As long as low inflation is realized each period, play continues along this path. At some point, due to bad luck, high inflation is realized. At this point, the equilibrium outcome switches to a new regime in which government chooses a high target for money growth and high realizations of inflation are relatively likely. The utility associated with this high inflation regime is the utility obtained under the worst sustainable policy.

This high inflation regime is repeated unless a sufficiently high level of inflation is realized. Once a high enough inflation rate is realized, switches back to the low inflation regime. This switching between low and high inflation regimes has the effect of rewarding government for low inflation and punishing it for high inflation in the low inflation regime[2]. It is these rewards and punishments that induce the government to choose low inflation in the low inflation regime. Likewise, in the high inflation regime, the prospect of returning to the low inflation regime acts to reward government for high inflation and punish it for low inflation, giving government the incentive to choose a high money-growth target in the high inflation regime. This regime switching along the equilibrium path of the best money-based system is similar to that obtained by Green and Porter (1984) and Abreu, Pearce, and Stachetti (1986) in their analysis of equilibrium price wars among oligopolists.

In contrast, the best exchange-rate based regime is simple as a result of the transparency of the regime. In the best equilibrium, government picks a rate of depreciation for the exchange rate so as to achieve low average inflation. This outcome is repeated each period, regardless of the realization of inflation, as long as government sticks to the exchange rate. In the government ever were to deviate from the exchange rate, the outcome would switch to the worst sustainable outcome, but this path is never observed in equilibrium.

In terms of the literature on monetary policy our analysis is most related to the seminal contribution of Canzoneri (1985) who was the first to use the logic of Green and Porter (1984) to explain periodic bouts of high inflation. (See also Zarazaga 1993.) There is also some work in this literature on the issue of transparency in monetary policy. Cukierman and Meltzer (1986), and Faust and Svensson (1998, 1999) explore linear signalling outcomes in models with unobserved types.

In terms of international economics literature, the most related work is by Canavan and Tommasi (1997) and Herrendorf (1999) who use two-period signalling games to argue that governments that prefer low inflation signal their preference by choosing exchange-rate-based systems.

Here we have used simple reduced form model. Chang (1998) and Phelan and Stachetti (1999) have used the recursive methods of Abreu, Pearce, and Stachetti (1990) to analyze some general equilibrium macroeconomic models with perfect monitoring.

1. Exchange-rate-based vs. money-based policy

Here we present a model of monetary policy in which the government can either choose money growth or the exchange rate for its policy instrument. The model extends the work of Kydland and Prescott (1977) and Barro and Gordon (1983).

A. An economy

Time is discrete and denoted $t = 0, 1, 2, \ldots$. There is a government, who dislikes unemployment and inflation, and a continuum of agents who each choose their nominal wage. The timing of actions within each period is as follows. At the beginning of each period the government chooses to conduct monetary policy using either an exchange-rate based system or a money-based system. Under the exchange-rate based system, the government opens a trading desk at which it trades domestic and foreign currency with any agents who want to trade with it. Under the money-based system they do not open this desk. Agents then choose their nominal wages. Finally, the government chooses either the exchange rate or a target for money growth. We think of the presence or absence of the trading desk at the beginning of the period as being an observable indication of the system for monetary policy in the current period.

It is convenient to describe the economy for a given period t starting at the end of the period. The government takes as given the average rate of wage inflation x set by agents. Unemployment is equal to a constant U plus the gap between wage inflation and realized inflation π. The government's payoff for a given value of x and a realization of π is

$$(1) \qquad r(x, \pi) = -\frac{1}{2}\left[(U + x - \pi)^2 + \pi^2\right].$$

Realized inflation is a function of monetary policy as follows. Under the exchange-rate-based system, the government chooses a rate of change in the exchange rate denoted $e_t = s_t - s_{t-1}$. The choice of fixed exchange rate s_t is observed. Inflation in the home country is given by

$$(2) \qquad \pi = e + \pi^*$$

where π^* is inflation in the foreign country which is normal with mean 0 and variance $\sigma^2_{\pi^*}$. Foreign inflation π^* is observed only after the exchange rate e_t is chosen. We use $g(\pi, e)$ to denote the density of realized inflation at home given the choice of exchange rate e. The government's expected payoff from an exchange-rate based system is

$$S(x, e) = \int r(x, \pi) g(\pi, e) d\pi.$$

With our functional forms

$$(3) \qquad S(x, e) = -\frac{1}{2}\left[(U + x - e)^2 + e^2\right] - \sigma^2_{\pi^*}.$$

Under the money-based system, the government chooses a target for money growth μ. Given a target of μ, inflation π is distributed normally with mean μ and variance σ^2_π with density denoted $f(\pi, \mu)$. We can think of the imperfect connection between the target for money growth and inflation as arising from some combination of imperfect control over money growth and a noisy relation between money growth and inflation. We call σ^2_π the variance of domestic inflation shocks.

The government's expected payoff under a money-based system is given by

$$R(x, \mu) = \int r(x, \pi) f(\pi, \mu) d\pi.$$

With our assumed functional forms

$$(4) \qquad R(x, \mu) = -\frac{1}{2}\left[(U + x - \mu)^2 + \mu^2\right] - \sigma^2_\pi.$$

For technical reasons we assume that the policies e and μ are bounded above and below by some arbitrarily large constants. These bounds ensure that the government payoffs are bounded.

In the middle of the period, each agent chooses the change in his wage rate $z_t = w_t - w_{t-1}$. We let x_t denote the average change in the wage rate in period t. An agent's payoff for a given value of z and a realization of π is

$$(5) \qquad r^A(z, \pi) = -\frac{1}{2}\left[(z - \pi)^2 + \pi^2\right].$$

Each agent can choose z differently depending on whether the chosen system is an exchange-rate-based system or a money-based system. An agent's expected payoff under an exchange-rate-based system with exchange rate e is

$$(6) \qquad S^A(z_e, e) = \int r^A(z_e, \pi)g(\pi, e)d\pi = -\frac{1}{2}\left[(z_e - e)^2 + e^2\right] - \sigma^2_{\pi^*}$$

while his expected payoff under a money-based system with money growth target μ is

$$(7) \qquad R^A(z_\mu, \mu) = \int r^A(z_\mu, \pi)f(\pi, \mu)d\pi = -\frac{1}{2}\left[(z_\mu - \mu)^2 + \mu^2\right] - \sigma^2_{\pi}.$$

Notice that there the objective function of agents differs from that of the government. In our simple reduced form model this difference generates the conflict of interests between government and the agents that leads to a time consistency problem. We think of this setup as a reduced-form way of capturing the tension that occurs in a general equilibrium model in which agents and the governments share the same objectives but there are distortions in the economy. (See Chari, Kehoe, and Prescott 1989 for a more complete discussion.)

B. The one-shot game

Before we define strategies and equilibrium for the repeated game, it is useful to consider the one-shot game. In the one-shot game, the government strategy is a choice of either an exchange-rate based system or a money-based system and actions e and μ indicating what exchange rate or money growth target are chosen under the two systems. The agents choose wages z_e or z_μ depending on the government's choice of system.

An *equilibrium of the one shot-game* is a pair of strategies for government and agents such that (i) e and μ solve the problems

$$\max_e S(x_e, e),$$

and

$$\max_\mu R(x_\mu, \mu)$$

(ii) z_e and z_μ maximize (6) and (7), (iii) the exchange-rate-based system is chosen if $S(x_e, e) \geq R(x_\mu, \mu)$ and the money-based system is chosen otherwise, (iv) $z_e = x_e$ and $z_\mu = x_\mu$.

The solution of the one-shot game is straightforward. The government's best response to x, $B(x)$, under a exchange-rate-based system is

$$(8) \qquad e = B(x) = \frac{U + x}{2}.$$

while the government's best response to x under a money-based system is $\mu = B(x) = (U + x)/2$.

An agent's best response in the two systems are $z_e = e$ and $z_\mu = \mu$ so that agents simply set wage inflation equal to expected inflation. Thus, conditional on government's choice of an exchange-rate-based system, the equilibrium choices of x and e are $e = x = U$ and the expected payoff to the government is

$$S(U, U) = -U^2 - \sigma^2_{\pi^*}.$$

Conditional on government's choosing to operate a money-based system, the equilibrium choices of x and μ are $\mu = x = U$. and the expected payoff to the government is

$$R(U, U) = -U^2 - \sigma^2_{\pi}.$$

Thus, in the one-shot game, the equilibrium choice of exchange-rate based or money-based system depends only of the relative variance of foreign inflation $\sigma^2_{\pi*}$ versus the variance of domestic inflation σ^2_{π} around the money growth target. If $\sigma^2_{\pi*}$ is lower than σ^2_{π} then an exchange rate base system is chosen in equilibrium while if $\sigma^2_{\pi*}$ is greater than σ^2_{π} then a money-based system is chosen in equilibrium. In either case, mean inflation is equal to U.

For later it will be useful to consider the equilibria of a game in which the government can commit to its exchange rate and money policies at the beginning of the period. The timing in this game is that at the beginning of the period the government chooses both the system and the particular policies e or μ and then after that agents choose their wages. We refer to the equilibria of this game as the Ramsey equilibria. We solve this game in two steps. Conditional on the choice of an exchange rate-based system, the optimal exchange rate policy with commitment solves

$$\max_e S(e, e)$$

where we have imposed that agents correctly anticipate inflation $x = e$. The solution of this problem is $x = e = 0$ and the value attained is

$$(9) \qquad S(0,0) = -\frac{1}{2}U^2 - \sigma^2_{\pi^*}.$$

Likewise under a money-based system the solution is $x = \mu = 0$ and the value obtained is

$$(10) \qquad R(0,0) = -\frac{1}{2}U^2 - \sigma^2_{\pi}.$$

In a Ramsey equilibrium if $\sigma^2_{\pi^*}$ is lower than σ^2_{π} then an exchange rate base system is chosen in equilibrium while if $\sigma^2_{\pi^*}$ is greater than σ^2_{π} then a money-based system is chosen in equilibrium. In either case, mean inflation is equal to 0.

2. The repeated games

Here we consider two environments. In one environment monetary statistics are transparent in the sense that agents can observe the history of the government's monetary target μ in those periods in which it chose a money-based system. In the other, monetary statistics are opaque in the sense that agents cannot observe this history. We think of the environment with transparent monetary statistics as having an institution that is free from central bank manipulation and accurately reports monetary data at the end of each period. We think of the environment with opaque monetary statistics as lacking such an institution and hence all monetary data are subject to manipulation.

A. With transparent monetary statistics

To define strategies for the government and agents in this environment, we let i_s be an indicator variable that denotes the choice of regime in period s, where $i_s = 0$ if the exchange-rate-based system is chosen and $i_s = 1$ is the money-based system is chosen. We let h_t denote the history of the choice of systems and the policies for the government in periods $s = 0, \ldots, t-1$. In each period s the indicator variable i_s is observed and then either the policy e_s is observed or the policy μ_s is observed.so that $h_t = (i_0, e_0, \mu_0; \ldots; i_{t-1}, e_{t-1}, \mu_{t-1})$ with the understanding that when $i_s = 0$, $\mu_s = \varnothing$ and when $i_s = 1$, $e_s = \varnothing$.

A strategy for government is a sequence of functions $\sigma^G = \{i_t(h_t), e_t(h_t), \mu_t(h_t)\}$ which map histories into the choice of system i_t and corresponding money growth targets μ_t or exchange rates e_t. Here, $e_t(h_t)$ is only relevant if $i_t(h_t) = 0$ and $\mu_t(h_t)$ is only relevant if $i_t(h_t) = 1$. A strategy for agents is a sequence of functions $\sigma^A = \{z_{et}(h_t), z_{\mu t}(h_t)\}_{t=0}^{\infty}$ which map histories into actions z_t, where $z_{et}(h_t)$ is only relevant if $i_t(h_t) = 0$ and $z_{\mu t}(h_t)$ is only relevant if $i_t(h_t) = 1$. We also define a sequence of functions $\sigma^X = \{x_{et}(h_t), x_{\mu t}(h_t)\}_{t=0}^{\infty}$ which record the average wages chosen by agents after each history. Let $\sigma = (\sigma^G, \sigma^A, \sigma^X)$ denote the strategies of the government, agents, and the average wages. Notice that in the histories we need not record the past averages of the actions of agents since a deviation by any one agent cannot effect this average. (See, for example, Chari and Kehoe 1990 for details.)

The payoff to the government after a history h_t under the strategies σ is given by

$$V_t(\sigma, h_t) = (1-\beta)\sum_{s=t}^{\infty}\beta^{s-t}[(1-i_s(h_s))S(x_{es}(h_s), e_s(h_s)) + i_s(h_s)R(x_{\mu s}(h_s), \mu_s(h_s))]$$

where the future histories h_s are recursively induced by the strategy σ^G for the government and a given history h_t according to $h_{t+1} = (h_t, i_t(h_t), e_t(h_t), \mu_t(h_t))$. The payoff to a private agent after a history h_t under the strategies σ is given by

$$(1-\beta)\sum_{s=t}^{\infty}\beta^{s-t}[(1-i_s(h_s))S^A(z_{es}(h_s), e_s(h_s)) + i_s(h_s)R^A(z_{\mu s}(h_s), \mu_s(h_s))].$$

A *perfect equilibrium of the repeated game with transparent monetary statistics* is a collection of strategies σ that satisfy the following conditions: *i*) government optimality: for every history h_t

(11) $\quad V_t(\sigma, h_t) = \max[V_{et}(\sigma, h_t), V_{\mu t}(\sigma, h_t)]$

where

(12) $\quad V_{et}(\sigma, h_t) = \max_e (1-\beta)S(x_e(h_t), e) + \beta V_{t+1}(\sigma, (h_t, 0, e, \varnothing))$

(13) $\quad V_{\mu t}(\sigma, h_t) = \max_\mu (1-\beta)R(x_\mu(h_t), \mu) + \beta V_{t+1}(\sigma, (h_t, 1, \varnothing, \mu)).$

ii) agent optimality: $z_{et}(h_t)$ solves

(14) $\quad max_z S^A(z, e_t(h_t))$

and $z_{\mu t}(h_t)$ solves

(15) $\quad max_z R^A(z, \mu_t(h_t))$

iii) representativeness:

(16) $z_{et}(h_t) = x_{et}(h_t)$ and $z_{\mu t}(h_t) = x_{\mu t}(h_t)$.

Our recursive definition of government optimality exploits the well-known result that if there are no beneficial one-shot deviations from a strategy then there are no beneficial deviations of any kind. (It is here that we use the assumption that the government's payoffs are bounded. See, for example, Abreu 1988 for an exposition of this result.) Our definition of agent optimality exploits the fact that agents essentially face a one period problem similar to that they face in the one-shot game since nothing they do at t can effect their payoffs in future periods. Notice that given our functional forms (14), (15), and (16) imply that $x_{et}(h_t) = e_t(h_t)$ and $x_{\mu t}(h_t) = \mu_t(h_t)$ so that agents simply set wage inflation equal to expected inflation.

Let V denote the set of equilibrium payoffs. In what follows it will prove convenient to allow public randomization to guarantee that this set V is convex. This public randomization is accomplished by adding to the model a random variable θ_t that the agents and the government observe at the beginning of each period. We modify the histories h_t to include the realizations of this variable from period 0 through period t.

B. With opaque monetary statistics

In this environment, in any period t in which the government follows a money-based system agents do not see the target money growth rate μ_t but instead the inflation π_t is the best indicator of the money growth target. We let h_t denote the history of the choice of systems, the exchange rate policy of the government for the periods in which the exchange-rate-based system is chosen and the realized inflation rate in the periods in which the money-based system is chosen. Hence, $h_t = (i_0, e_0, \pi_0; \ldots; i_{t-1}, e_{t-1}, \pi_{t-1})$ with the understanding that when $i_s = 0$, $\pi_s = \varnothing$ and when $i_s = 1$, $e_s = \varnothing$. (Notice that in the game with transparent monetary statistics it is redundant to include realized inflation in the history because agents see the money growth target directly.)

Strategies and payoffs are defined in a similar fashion as before, except that now strategies and the exogenous uncertainty linking money targets to inflation induce probability distributions over future histories. Thus, for example, under a monetary regime the density of the conditional distribution of $h_{t+1} = (h_t, 1, \varnothing, \pi_t)$ induced by σ^G given h_t is $f(\pi_t, \mu_t(h_t))$.

A *perfect equilibrium of the repeated game with opaque monetary statistics* is a collection of strategies σ that satisfy (11), (12), (14), (15), (16) and

$$(17) \quad V_{\mu t}(\sigma, h_t) = \max_{\mu}(1-\beta)R(x_\mu(h_t), \mu) + \beta \int V_{t+1}(\sigma, (h_t, 1, \varnothing, \pi) f(\pi, \mu) d\pi.$$

We again let V denote the set of equilibrium payoffs. Here also agents simply set wage inflation equal to expected inflation.

3. Tradeoffs with transparent monetary statistics

In this section we characterize the best equilibrium in the environment with transparent monetary statistics. We show that the exchange-rate-based system is optimal if the variance of inflation under an exchange-rate-based system is lower than that of under a money-based system and the money-based system is optimal otherwise.

We apply the ideas of Abreu, Pearce and Stachetti (1986, 1990) to our environments. The basic idea we use is that one can think of strategies in as specifying current actions and continuation values based on those actions where each continuation value is the utility of the continuation of the strategy. In a perfect equilibrium these continuation values are also equilibrium payoffs for the repeated game starting from tomorrow on. This simple observation forms the basis for a recursive approach to finding the set of equilibrium payoffs.

We begin with some definitions. Given some closed interval of continuation values $W = [\underline{w}, \bar{w}]$ we say a value v is *sustainable under an exchange-rate-based system* if there are actions z_e, x_e, e and a continuation value function $w(e')$ such that for all e', $w(e') \in W$,

$$(18) \quad v = (1-\beta)S(x_e, e) + \beta w$$

e solves

$$(19) \quad \max_{e'}(1-\beta)S(x_e, e') + \beta w(e')$$

z_e solves

$$(20) \quad \max_{z} S^A(z, e)$$

and

$$(21) \quad x_e = z_e.$$

Likewise, we say a value v is *sustainable under a money-based system* if there are actions z_μ, x_μ, μ and a continuation value function $w(\mu')$ such that for all μ', $w(\mu') \in W$,

(22) $\quad v = (1-\beta)R(x_\mu, \mu) + \beta w$

μ solves

(23) $\quad \max_{\mu'} (1-\beta)R(x_\mu, \mu') + \beta w(\mu')$

z_μ solves

(24) $\quad \max_{z} R^A(z, \mu)$

and

(25) $\quad x_\mu = z_\mu.$

Conditions (18) and (22) require that the current actions and continuation values deliver the current value v. Conditions (19) and (23) are analogous to (12) and (13). Conditions (20) and (24) are analogous to (14) and (15), while conditions (21) and (25) are analogous to (16).

We define the operator T^h on the set $W = [\underline{w}, \bar{w}]$ which finds the highest value sustainable with respect to W by

(26) $\quad T^h(\underline{w}, \bar{w}) = \max\{T^h_e(\underline{w}, \bar{w}), T^h_\mu(\underline{w}, \bar{w})\},$

where $T^h_e(\underline{w}, \bar{w})$ and $T^h_\mu(\underline{w}, \bar{w})$ are the highest values of v that are sustainable with respect to W under an exchange-rate-based system and a money-based system respectively. Likewise, we define the operator T^l on the set $W = [\underline{w}, \bar{w}]$ which finds the lowest value sustainable with respect to W by

(27) $\quad T^l(\underline{w}, \bar{w}) = \max\{T^l_e(\underline{w}, \bar{w}), T^l_\mu(\underline{w}, \bar{w})\},$

where $T^l_e(\underline{w}, \bar{w})$ and $T^l_\mu(\underline{w}, \bar{w})$ are the lowest values of v that are sustainable with respect to W under an exchange-rate-based system and a money-based system respectively. The intuition for why the worst sustainable value is the maximum rather than the minimum of the worst sustainable values under the exchange-rate-based and the money-based systems is that the government gets to pick the system it prefers at the beginning of the period. We say that a value v is *sustainable with respect to* $[\underline{w}, \bar{w}]$, if $v \in [T^l(\underline{w}, \bar{w}),\ T^h(\underline{w}, \bar{w})]$.

We then use the following theorem in our characterization of the best equilibrium.

Proposition 1. The set of equilibrium payoffs $V = [\underline{v}, \bar{v}]$ is the largest interval such that $\bar{v} = T^h(\underline{v}, \bar{v})$ and $\underline{v} = T^l(\underline{v}, \bar{v})$.

Proof. See Appendix.

In what follows we use the following simplified versions of constraints (19)-(21) and (23)-(25):

(28) $\quad (1-\beta)S(x_e, e) + \beta w \geq (1-\beta)S(x_e, B(x_e)) + \beta \underline{w}$

(29) $\quad x_e = e.$

(30) $\quad (1-\beta)R(x_\mu, \mu) + \beta w \geq (1-\beta)R(x_\mu, B(x_\mu)) + \beta \underline{w}$

(31) $\quad x_\mu = \mu.$

Conditions (28) and (30) require that the government prefer to take the current actions and receive the continuation value to choosing the best one shot deviation in response to agent's nominal wages and receiving $(1-\beta)B(x_e)$ and $(1-\beta)B(x_\mu)$ in the current period followed by the lowest possible continuation payoff $\underline{w}$. Notice that these conditions do not require us to check all possible deviations and all possible continuation payoffs after deviations, but only a particular deviation, namely the best one shot deviation and a particular continuation value, namely the lowest one. Abreu (1988) showed that we need only check this simpler condition. In conditions (29) and (31) we have used the result that in the equilibrium of the repeated game, agents simply set wage inflation equal to expected inflation.

Our objective is to find conditions under with the exchange-rate-based system is preferred to the money-based system and vice-versa. It turns out that to make this comparison we need not solve for the equilibrium set of payoffs, rather we only need to solve for the forms of $T_e^h(\underline{w}, \bar{w})$ and $T_\mu^h(\underline{w}, \bar{w})$ for an arbitrary interval of continuation payoffs $W = [\underline{w}, \bar{w}]$. Our analysis gives the following proposition.

Proposition 2. The exchange-rate-based system is preferred to the money-based system if and only if $\sigma_{\pi*}^2 \leq \sigma_\pi^2$.

Proof. We prove this result by showing that for any $W = [\underline{w}, \bar{w}]$, $T_e^h(\underline{w}, \bar{w}) \geq T_\mu^h(\underline{w}, \bar{w})$ if and only if $\sigma_{\pi*}^2 \leq \sigma_\pi^2$. The value $T_e^h(\underline{w}, \bar{w})$ is the solution to choosing v, x_e^h, e^h and w^h to maximize v subject to (18) – (29). Consider the incentive constraint (28). Clearly, $w^h = \bar{w}$ since this value maximizes both the objective (18) and the left-side of the incentive constraint (28). If this constraint is slack at the Ramsey outcome $x_e = e = 0$ then this outcome is sustainable in the current period. If this constraint binds then we can use our functional forms to write

$$(32) \quad S(x, B(x)) - S(x, x) = -\frac{1}{2}\frac{(U+x)}{2}^2 - \frac{1}{2}\left[U^2 + x^2\right] = \frac{1}{4}[U-x]^2$$

and hence we can write the incentive constraint as

$$\frac{[U-x]^2}{4} = \frac{\beta}{(1-\beta)}(\bar{w} - \underline{w}).$$

The solution to our maximization problem is the smallest x that satisfies this equation. Combining the two cases we have

$$x_e^h = e^h = \max\left[0, U - 2\left[\frac{\beta}{1-\beta}(\bar{w} - \underline{w})\right]^{\frac{1}{2}}\right].$$

The value $T_\mu^h(\underline{w}, \bar{w})$ is the solution to choosing v, x_μ^h and μ^h to maximize v subject to (22) – (31). A similar analysis yields that the solution to this maximization problem is

$$x_\mu^h = \mu^h = \max\left[0, U - 2\left[\frac{\beta}{1-\beta}(\bar{w} - \underline{w})\right]^{\frac{1}{2}}\right].$$

Since the solution to these two problems is the same, given our functional forms (3) and (4), it follows that $T_e^h(\underline{w}, \bar{w}) - T_\mu^h(\underline{w}, \bar{w}) = \sigma_\pi^2 - \sigma_{\pi*}^2$ which proves the result. Q.E.D.

In order to prove our main result we do not need to characterize the solution to T^l. In our characterization of T^h, however, we have found that when the incentive constraint binds the best equilibrium is supported by the threat that the economy will revert to the worst payoff $\underline{w}$ in the event of a government deviation. By analyzing T^l we can describe the policies that sustain this worst payoff. We then have the following proposition.

Proposition 3. The worst sustainable policy is exchange-rate-based if $\sigma^2_{\pi^*} \leq \sigma^2_\pi$ and it is money-based otherwise. In either case, this policy is to have high inflation for one period and then to revert to the best sustainable policy thereafter.

Proof. We prove this result by showing that for any $W = [\underline{w}, \bar{w}]$, $T^l_e(\underline{w}, \bar{w}) \geq T^l_\mu(\underline{w}, \bar{w})$ if and only if $\sigma^2_{\pi^*} \leq \sigma^2_\pi$. The value $T^l_e(\underline{w}, \bar{w})$ is the solution to choosing v, x^l_e, e^l and w^l to minimize v subject to (18) – (29). Consider the incentive constraint evaluated at $x_e = e$

$$(33) \quad (1-\beta)S(x_e, x_e) + \beta w \geq (1-\beta)S(x_e, B(x_e)) + \beta \underline{w}.$$

Notice two points. First, the left-side of this constraint is the objective function that we are trying to minimize. Second the left-side of the constraint can be made arbitrarily small by increasing x_e. Hence, this constraint must bind.

Since the incentive constraint binds, we can find the solution by minimizing the right-side of the incentive constraint subject to (33) written as an equality. This problem is

$$(34) \quad \min_{x_e, w}(1-\beta)S(x_e, B(x_e)) + \beta \underline{w}$$

subject to

$$(35) \quad \beta w = (1-\beta)[S(x_e, B(x_e) - S(x_e, x_e)] + \beta \underline{w}$$

$$\underline{w} \leq w \leq \bar{w}.$$

Clearly, since $S(x_e, B(x_e))$ is decreasing in x_e, the solution involves finding the w that allows for the largest choice of x_e. From (32) $S(x_e, B(x_e) - S(x_e, x_e)$ is increasing in x_e for x_e greater than U. Hence, the solution involves setting $w = \bar{w}$ and choosing x_e to be the largest solution to (35) with $w = \bar{w}$. The resulting x_e is

$$x^l_e = U + 2\left[\frac{\beta}{1-\beta}(\bar{w} - \underline{w})\right]^{\frac{1}{2}}.$$

The analysis in the case of the money-based system is identical and the resulting $x^l_\mu = x^l_e$. Hence $T^l_e(\underline{w}, \bar{w})- T^l_\mu(\underline{w}, \bar{w}) = \sigma^2_\pi - \sigma^2_{\pi^*}$ which proves the first part of the proposition. Q.E.D.

The second part of the proposition follows from the result that optimal continuation value $w = \bar{w}$ for both systems.

This result is reminiscent of a result in Abreu (1986).

4. Tradeoffs with opaque monetary statistics

In this section we characterize the best equilibrium in the environment with opaque monetary statistics. Here the exchange-rate-based system is transparent because agents can directly observe the history of government policy while the money-based system is opaque because agents can only observe the history of inflation which is a noisy signal of policy. We show that relatively higher transparency of the exchange-rate-based system gives it an advantage over the money-based system and hence it preferred when the variance of foreign inflation shocks equals that of domestic inflation shocks. We establish this result by showing that the best money-based system necessarily involves random bouts of high inflation along the equilibrium path while the best exchange-rate-based system does not.

To characterize the best systems for monetary policy in an environment with opaque monetary statistics, we again use a recursive approach. For this environment, our definitions of enforcability are as follows. Given some arbitrary interval of continuation values $W = [\underline{w}, \bar{w}]$ we say a value v is *sustainable under an exchange-rate-based system* if there are actions x_e,e and a continuation value $w \in W$ that satisfy (18), (28), and (29) as before. We say a value v is *sustainable under a money-based system* if there are actions z_μ, x_μ, μ and a continuation value function $w(\pi)$ with $w(\pi) \in W$ for all values of π such that

$$(36) \quad v = (1-\beta)R(x_\mu, \mu) + \beta \int w(\pi) f(\pi, \mu) d\mu$$

μ solves

$$(37) \quad \max_{\mu'} (1-\beta)R(x_\mu, \mu') + \beta \int w(\pi) f(\pi, \mu') d\pi$$

and (24) and (25) where, of course, $\mu = \int \pi f(\pi, \mu) d\pi$. The key difference between this definition and our earlier one in the environment with transparent monetary statistics is that here continuation values depend on the realized value of inflation π, while in the earlier environment these values depend directly on the government policy μ. Thus, here when the government deviates from some target μ to some other μ' it shifts future outcomes only by affecting the probability distribution over inflation.

As before, we define the operator T^h on the set $W = [\underline{w}, \bar{w}]$ which finds the highest payoff sustainable with respect to W by (26) and we define the operator T^l on the set

$W = [\underline{w}, \bar{w}]$ which finds the lowest payoff sustainable with respect to W by (27). We say that a value v is *sustainable with respect to* $[\underline{w}, \bar{w}]$, if $v \in [T^l(\underline{w}, \bar{w}),\ T^h(\underline{w}, \bar{w})]$. Proposition 1 characterizing the best and worst equilibrium payoffs as the largest fixed point of the equations $\bar{v} = T^h(\underline{v}, \bar{v})$, $\underline{v} = T^h(\underline{v}, \bar{v})$ applies in this environment as well.

The problems of solving for the highest and lowest values that can be sustained under an exchange-rate-based system ($T_e^h(\underline{w}, \bar{w})$ and $T_e^l(\underline{w}, \bar{w})$) is the same as in the environment with transparent monetary statistics and thus have the solutions given in Propositions 2 and 3.

To solve for the highest and lowest values that can be sustained under a money-based system we proceed as follows. We begin by replacing the incentive constraint (37) by the first order condition associated with maximizing the left-side of this incentive constraint with respect to μ and evaluating it at the proposed government policy. The resulting constraint is

$$(38) \quad (1-\beta)R_\mu(x(0), \mu) + \beta \int w(\pi) f_\mu(\pi, \mu)\, d\pi = 0$$

where $R_\mu(x, \mu) = \frac{\partial}{\partial \mu} R(x, \mu)$ and $f_\mu(\pi, \mu) = \frac{\partial}{\partial \mu} f(\pi, \mu)$. This first-order condition is necessary and sufficent to ensure that (37) when the function defined by the left-side of (37) is concave in μ. We proceed as follows. In Proposition 4, we simply assume that this approach is valid and characterize the resulting $w(\pi)$ under this conjecture. In Proposition 5 we show that given the resulting form of $w(\pi)$ the left-side of (37) is concave in μ when σ_π^2 is sufficiently large.

Under the assumption that the first-order condition approach is valid we can write

$$(39) \quad T_\mu^h(\underline{w}, \bar{w}) = \max_{\mu, x, w(\pi)} (1-\beta)R(x, \mu) + \beta \int w(\pi) f(\pi, \mu)\, d\pi$$

subject to the constraints (36), (**??**), and (38). We show that a solution to this problem continuation values necessarily that have a bang-bang form: there is a cutoff inflation level π^h such that the continuation value $w^h(\pi)$ is set to its maximum value $\bar{w}$ if realized inflation is less than π^h and to its minimum value $\underline{w}$ if realized inflation is higher than π^h. This result is reminiscent of the equilibrium price wars in models of oligopoly discussed by Green and Porter (1984) and Abreu, Pearce, and Stachetti (1986).

Proposition 4. Under the assumption that the first-order condition approach is valid the optimal continuation value has the form

$$w^h(\pi) = \left\{ \begin{array}{c} \bar{w} \text{ if } \pi \leq \pi^h \\ \underline{w} \text{ if } \pi > \pi^h \end{array} \right\} \tag{40}$$

for some cutoff inflation level π^h.

Proof. Letting λ be the Lagrange multiplier on the government's incentive constraint (38), the first order conditions of our program with respect to $w(\pi)$ imply that

$$w^h(\pi) = \bar{w} \text{ if } (1 + \lambda \frac{f_\mu(\pi, \mu)}{f(\pi, \mu)}) > 0,$$

$$w^h(\pi) = \underline{w} \text{ if } (1 + \lambda \frac{f_\mu(\pi, \mu)}{f(\pi, \mu)}) < 0.$$

These first order conditions imply the optimal continuation value has a bang-bang form. The only issue is for values of π are the payoffs $\bar{w}$ and $\underline{w}$ assigned. The determine these values we start by observing that with our assumption of normality $f_\mu(\pi, \mu) = f(\pi, \mu)(\pi - \mu)/\sigma_\pi$ so that our densities satisfy the local monotone likelihood ratio property in that

$$\frac{f_\mu(\pi, \mu)}{f(\pi, \mu)} = (\pi - \mu)/\sigma_\pi$$

is increasing in π. Thus, $w^h(\pi)$ is increasing in π if $\lambda > 0$ and decreasing in π if $\lambda < 0$. We will show $\lambda < 0$ so that it is decreasing in π. This is intuitive, since a higher realized level of inflation is more likely the higher is the money growth rate and thus to induce the lowest sustainable current money growth target continuation values after high realizations of π must be low.

We demonstrate this formally as follows. First, note that at the optimum $R_\mu(x^h, \mu^h) \geq 0$, so that current period payoff for the government are increased when the government deviates to a higher money growth target. To see this note that a feasible point in the constraint set is to play the one shot sustainable plan today $x = \mu = U$ and give the highest reward, regardless of the π by setting $w(\pi) = \bar{w}$ for all π. The optimal solution must weakly improve upon this point and hence must have $x^h = \mu^h \leq U$. Since $R_\mu(x, \mu) = U + x - 2\mu$, then $\mu^h \leq B(x^h)$, and $R_\mu(x^h, \mu^h) \geq 0$.

Next, note that since $R_\mu(x^h, \mu^h) \geq 0$ the incentive constraint (38) implies that

$$\int w^h(\pi) f_\mu(\pi, \mu)\, d\pi \leq 0.$$

Since inflation is normally distributed with mean μ, increasing μ increases the distribution of inflation in the sense of first order stochastic dominance. Thus, increasing μ increases $\int w^h(\pi) f(\pi, \mu) d\pi$ when $w^h(\pi)$ is increasing and decreases this integral when $w^h(\pi)$ is decreasing. Thus, $w^h(\pi)$ must be decreasing. Q.E.D.

In the appendix we prove the following proposition justifying our use of the first order approach (38) to characterize the incentive constraint (37) is valid. We let ϕ and Φ denote the density and cumulative distribution function of a standard normal respectively.

Proposition 5. Given that $w^h(\pi)$ has the bang-bang form (40) and is decreasing, if $\sigma_\pi^2 > \frac{\beta}{1-\beta}(\bar{w} - \underline{w})\frac{\phi(1)}{2}$, then the incentive constraint (37) is satisfied if and only if the first order condition (38) holds.

We now present our main result comparing exchange-rate-based systems and money-based systems when monetary statistics are opaque.

Proposition 6. When monetary statistics are opaque and the variance of shocks to foreign inflation is equal to the variance of shocks to domestic inflation the exchange-rate-based system is strictly preferred to the money-based system.

Proof. We show that for any interval $W = [\underline{w}, \bar{w}]$, with $\bar{w} > \underline{w}$, $T_e^h(\underline{w}, \bar{w}) > T_\mu^h(\underline{w}, \bar{w})$. Recall that

$$T_e^h(\underline{w}, \bar{w}) = (1 - \beta) S(e^h, e^h) + \beta \bar{w}$$

$$T_\mu^h(\underline{w}, \bar{w}) = (1 - \beta) R(\mu^h, \mu^h) + \beta[\bar{w} F(\pi^h, \mu^h) + \underline{w}(1 - F(\pi^h, \mu^h))].$$

We prove our result in three steps.

First, the continuation value under the optimal exchange-rate-based system $\bar{w}$ is strictly greater than the expected continuation value under the optimal money-based system $\bar{w} F(\pi^h, \mu^h) + \underline{w}(1 - F(\pi^h, \mu^h)$, since this latter system puts strictly positive probability on the worst continuation value $\underline{w}$.

Second, since $\sigma_{\pi^*}^2 = \sigma_\pi^2$, (3) and (4) imply that the functions S and R are the same (in that sense that $S(a, b) = R(a, b)$ for any two values of a and b).

Finally, we show that $e^h \leq \mu^h$ and hence $S(e^h, e^h) \geq R(\mu^h, \mu^h)$. Recall, that the e^h is either 0 or the smallest e such that

$$(41) \quad S(e, B(e)) - S(e, e) = \frac{\beta}{1-\beta}(\bar{w} - \underline{w}).$$

Since the incentive constraint (37) holds for all deviations, in particular it holds for the best one shot deviation $B(\mu^h)$. Using the form of $w^h(\pi)$ we then have

$$(42) \quad R(\mu^h, B(\mu^h)) - R(\mu^h, \mu^h) \leq \frac{\beta}{1-\beta}(\bar{w} - \underline{w})[F(\pi^h, \mu^h) - F(\pi^h, B(\mu^h)].$$

Where $F(\pi, \mu)$ is the cumulative distribution function corresponding to density $f(\pi, \mu)$. Clearly, the left-side of (42) is strictly less than (41). Since the functions S and R are the same and the left-sides of (41) and (42) are decreasing in e and μ respectively, we have that $e^h \leq \mu^h$ (with a strict inequality if μ^h is greater than 0). Thus, $S(e^h, e^h) \geq R(\mu^h, \mu^h)$ and the result follows. Q.E.D.

For completeness, we now characterize the solution to the problems T_μ^h and T_μ^l.

Proposition 7. The μ^h that solves T_μ^h is the unique solution of

$$(43) \quad (1-\beta)(U - \mu^h) = \beta\frac{(\bar{w} - \underline{w})}{\sigma_\pi}\phi(\frac{\sigma_\pi}{\mu^h})$$

when $\mu^h \in [0, U]$, the corresponding $x^h = \mu^h$ and π^h is given by

$$(44) \quad \frac{\pi^h - \mu^h}{\sigma_\pi} = \frac{\sigma_\pi}{\mu^h}.$$

This proposition is proved in the appendix.

The worst money-based system (the solution to T_μ^l) is symmetric to the best money-based system (the solution to T_μ^h) in the following sense. In the best money-based system, continuation values $w^h(\pi)$ were assigned to give the government incentives to choose a lower money growth target than it would choose in the one-shot game. This entailed rewarding the government with high continuation values in the event that low inflation is realized and low continuation values in the event that high inflation is realized. In the worst money-based system, continuation values $w^l(\pi)$ are assigned to give the government the incentive to choose a higher money growth target than it would chose in the one-shot game. This entails rewarding the government with high continuation values in the event that high inflation is realized and low continuation values in the event that low inflation is realized. Thus, if the worst money-based system is realized as part of the path of equilibrium play, the government chooses a high money growth target and keeps choosing this high target unless a sufficiently high level of inflation is realized. In the event that a high level of inflation is realized, the path of play reverts to the best equilibrium path of play, whether that be an exchange-rate based system or a money-based system. In this sense, under the worst money-based system, extremely high inflation must be realized before it can fall. This result is proved in the next proposition.

As before, under the assumption that the first-order condition approach is valid we can write the problem of finding the worst sustainable value under a money-based system as

$$(45) \quad T^l_\mu(\underline{w}, \bar{w}) = \min_{\mu,x,w(\pi)} (1-\beta)R(x,\mu) + \beta \int w(\pi) f(\pi,\mu)\, d\pi$$

subject to the constraints (36), (**??**), and (38).

Proposition 8. The solution $\mu^l, x^l, w^l(\pi)$ to (45) has $w^l(\pi)$ that satisfies

$$(46) \quad w^l(\pi) = \left\{ \begin{array}{l} \underline{w} \text{ if } \pi \leq \pi^l \\ \bar{w} \text{ if } \pi > \pi^l \end{array} \right\}$$

for some cutoff inflation rate π^l, μ^l given by the unique solution to

$$(47) \quad (1-\beta)(U-\mu^l) = \beta \frac{(\underline{w}-\bar{w})}{\sigma_\pi} \phi(\frac{\sigma_\pi}{\mu^l}),$$

$x^l = \mu^l$, and π^l given by

$$(48) \quad \frac{\pi^l - \mu}{\sigma_\pi} = \frac{\sigma_\pi}{\mu^l}.$$

Proof. The proof is similar to that of propositions 5 and 6. Specifically, the first order conditions of the problem (45) with respect to $w(\pi)$ imply that $w(\pi)$ has a bang-bang form around some cutoff μ^l. To show that w^l must be increasing, note that at the optimum $R_\mu(x^l, \mu^l) \leq 0$ so that current period payoff for the government is decreased when the government deviates to a higher money growth target. To see this point note that a feasible point in the constraint set is to play the one shot sustainable plan today $x = \mu = U$ and give the lowest reward, regardless of the π by setting $w(\pi) = \underline{w}$ for all π. The minimizing solution must deliver a weakly lower value than under this plan and hence must have $x^h = \mu^h \geq U$.

Since $R_\mu(x,\mu) = U + x - 2\mu$, then $\mu^h \geq B(x^h)$, and $R_\mu(x^h, \mu^h) \leq 0$. Accordingly, the incentive constraint (38) implies that

$$\int w^h(\pi) f_\mu(\pi, \mu)\, d\pi \geq 0$$

which gives the result that $w^l(\pi)$ is increasing. The derivation of (47) and (48) is the same as that in the proof of Proposition 7.

5. Conclusion

Here we have considered the advantage of transparency in a model in which the exchange-rate-based policy is observable and the money-based policy is not. Our simple model abstracted from the fiscal side of either regime. In practice, fiscal policy is an important component of the sustainability of any given monetary policy. In an exchange-rate-based system it may be quite easy to monitor the exchange rate but quite difficult to monitor the associated fiscal policies. If one were to explicitly model the fiscal side of the economy the tradeoffs between the two types of monetary policies may be more complex than in our simple model.

Appendix

Proof of Proposition 1. The statement of Proposition 1 presumes that V is an interval. The opportunity for public randomization guarantees that if any two values v_1 and v_2 attained by strategies σ_1 and σ_2 then so is any $v = pv_1 + (1-p)v_2$ with $0 \leq p \leq 1$. The value v is obtained by randomizing in the initial period with probability p over strategy σ_1 and probability $1-p$ over strategy σ_2. This randomization is accomplished by the partitioning the set of realizations of the observable random variable θ_0.

Let $[\underline{v}, \bar{v}]$ be the smallest closed interval that contains V. We first show that show that $[\underline{v}, \bar{v}] \subseteq \left[T^l(\underline{v}, \bar{v}), T^h(\underline{v}, \bar{v})\right]$. We do so by showing that $V \subseteq \left[T^l(\underline{v}, \bar{v}), T^h(\underline{v}, \bar{v})\right]$. The result that $[\underline{v}, \bar{v}] \subseteq \left[T^l(\underline{v}, \bar{v}), T^h(\underline{v}, \bar{v})\right]$ then follows immediately since $\left[T^l(\underline{v}, \bar{v}), T^h(\underline{v}, \bar{v})\right]$ is a closed set that contains V and $[\underline{v}, \bar{v}]$ is, by definition, the smallest closed set that contains V.

That $V \subseteq \left[T^l(\underline{v}, \bar{v}), T^h(\underline{v}, \bar{v})\right]$ follows immediately from the definition of a perfect equilibrium. Specifically, let $v \in V$ and σ a strategy that attains v. Without loss of generality suppose that in period 0, σ specifies an exchange-rate-based system. Use this strategy to define the actions $e = e_0(h_0)$, $z_e = z_{e0}(h_0)$, $x_e = x_{e0}(h_0)$ together with the continuation value function $w(e') = V_1(\sigma, (h_0,\ 0,\ e',\ \varnothing))$. We claim that these actions and continuation values are sustainable with respect to $[\underline{v}, \bar{v}]$ under an exchange-rate-based system. This follows since the continuation of any perfect equilibrium strategy is also a perfect equilibrium strategy, $V_1(\sigma, (h_0,\ 0,\ e',\ \varnothing)) \in V$ and thus $w(e') \in [\underline{v}, \bar{v}]$ for all e' and since conditions (12), (14), and (16) imply conditions (19), (20), and (21). Thus, $v \leq T_e^h(\underline{v}, \bar{v}) \leq T^h(\underline{v}, \bar{v})$. Clearly, $v \geq T_e^l(\underline{v}, \bar{v})$. To finish the proof we need to show that $v \geq T_\mu^l(\underline{v}, \bar{v})$. This is because by definition of a perfect equilibrium, the actions $\mu = \mu_0(h_0)$, $z_\mu = z_{\mu 0}(h_0)$, $x_\mu = x_{\mu 0}(h_0)$ together with the continuation value function $w(\mu') = V_1(\sigma, (h_0,\ 1,\ \varnothing,\ \mu'))$ sustain some lower payoff under a money-based system.

We now show $\left[T^l(\underline{v}, \bar{v}), T^h(\underline{v}, \bar{v})\right] \subseteq [\underline{v}, \bar{v}]$. We do so by showing that for any set $W = [\underline{w}, \bar{w}]$, if $[\underline{w}, \bar{w}] \subseteq \left[T^l(\underline{w}, \bar{w}), T^h(\underline{w}, \bar{w})\right]$ then $\left[T^l(\underline{w}, \bar{w}), T^h(\underline{w}, \bar{w})\right] \subseteq [\underline{v}, \bar{v}]$. That is, we show that if an arbitrary interval of continuation values can be sustained by using only continuation values also in that interval then any value that can be sustained using those continuation values must be a perfect equilibrium value. To show this we construct the equilibrium strategies that attain these values. The basic idea is that is to pick a $w \in \left[T^l(\underline{w}, \bar{w}), T^h(\underline{w}, \bar{w})\right]$. Then there exist initial actions and a continuation value for any initial action that support this payoff. For any initial, either on the equilibrium path or for any deviation, the continuation values that are assigned are all in $[\underline{w}, \bar{w}]$, which by assumption are also in $\left[T^l(\underline{w}, \bar{w}), T^h(\underline{w}, \bar{w})\right]$. Thus, corresponding to any specified continuation value, there is a new action and a new continuation value that supports this specified value. Repeating this procedure we can recursively construct strategies for governments and agents. By construction these strategies satisfy the one shot incentive constraints in the definition of T^l and T^h and hence they satisfy the incentive constraints in the definition of a perfect equilibrium.

Proof of Proposition 5. Here we show that the solution to the problem with incentive constraint (37) is satisfied if and only if the first order condition (38) holds when $\sigma_\pi^2 > \frac{\beta}{1-\beta}(\bar{w} - \underline{w})\frac{\phi(1)}{2}$. Using (40), the constraint (37) can be written

$$(49) \quad \mu \in \arg\max_{\mu}(1-\beta)R(x,\mu) + \beta\left[\bar{w}\Phi(\frac{\pi^h - \mu}{\sigma_\pi}) + \underline{w}(1 - \Phi(\frac{\pi^h - \mu}{\sigma_\pi}))\right]$$

Since $F(\pi^h, \mu) = \Phi((\pi^h - \mu)/\sigma_\pi)$, we can write the first and second order conditions of the maximization problem (49) as

$$(50) \quad (1-\beta)R_\mu(x^h, \mu) - \beta\frac{(\bar{w} - \underline{w})}{\sigma_\pi}\phi(\frac{\pi^h - \mu}{\sigma_\pi}) = 0.$$

and for all μ

$$(51) \quad (1-\beta)R_{\mu\mu}(x^h, \mu) - \beta(\bar{w} - \underline{w})\frac{(\pi^h - \mu)}{\sigma_\pi^2}\phi(\frac{\pi^h - \mu}{\sigma_\pi}) \leq 0$$

which can be written

$$(52) \quad -2(1-\beta) - \beta\frac{(\bar{w} - \underline{w})}{\sigma_\pi^2}\phi(z)z \leq 0$$

for all $z \in [-\infty, \infty]$. The expression $\phi(z)z$ in (52) is maximized at $z = -1$. Since $\phi(-1) = \phi(1)$, the inequality $\sigma_\pi^2 > \frac{\beta}{1-\beta}(\bar{w} - \underline{w})\frac{\phi(1)}{2}$ guarantees that the second order condition holds globally and thus (38) is both necessary and sufficient for (37). Q.E.D.

Proof of Proposition 7. Using our functional form we have that T_μ^h can be written as the

$$T_\mu^h(\underline{w}, \bar{w}) = \max_{\mu, \pi^h}(1-\beta)\left[-\frac{1}{2}(U^2 + \mu^2) - \sigma_\pi^2\right] + \beta(\bar{w} - \underline{w})\Phi(\frac{\pi^h - \mu}{\sigma_\pi}) + \beta\underline{w}$$

subject to

$$(53) \quad (1-\beta)(U-\mu) - \beta\frac{(\bar{w}-\underline{w})}{\sigma_\pi}\phi(\frac{\pi^h-\mu}{\sigma_\pi}) = 0$$

(Note: For each μ there are at most two solutions π^h to (53). The higher one is the one that maximizes the objective function. The lower one is the one that minimizes the objective and is used to prove proposition 8.) Letting λ denote the Lagrange multiplier on (53), the first order conditions are for μ

$$(54) \quad -(1-\beta)\mu - \beta\frac{(\bar{w}-\underline{w})}{\sigma_\pi}\phi(\frac{\pi^h-\mu}{\sigma_\pi}) =$$

$$\lambda(1-\beta) + \lambda\beta\frac{(\bar{w}-\underline{w})}{\sigma_\pi}(\frac{\pi^h-\mu}{\sigma_\pi^2})\phi(\frac{\pi^h-\mu}{\sigma_\pi})$$

for π^h

$$(55) \quad \frac{\beta(\bar{w}-\underline{w})}{\sigma_\pi}\phi(\frac{\pi^h-\mu}{\sigma_\pi}) + \lambda\beta\frac{(\bar{w}-\underline{w})}{\sigma_\pi}(\frac{\pi^h-\mu}{\sigma_\pi^2})\phi(\frac{\pi^h-\mu}{\sigma_\pi}) = 0$$

Substituting (55) into (54) gives $\mu = -\lambda$. Simplifying (55), and using $\mu = -\lambda$ gives

$$(56) \quad 1 - \mu^h(\frac{\pi^h-\mu}{\sigma_\pi^2}) = 0$$

which implies (44). Substituting (44) into (53) gives (43). Q.E.D.

Notes

[1]For example, Calvo and Vegh (1999 p1589) argue that "the exchange rate provides a much clearer signal to the public of the government's intentions and actual actions than a money supply target. Thus, if the public's inflationary expectations are influenced to a large extent by the ability to easily track and continuously monitor the nominal anchor, the exchange rate has a natural advantage."

[2]In the model, of course, no one is punishing anyone else since agents simply choose their optimal strategies given their predictions of government behavior. We use this language here because it is a convenient, if somewhat loose, way to describe the outcomes in the two regimes.

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