# Using Asset Prices to Measure the Cost of Business Cycles

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May 26, 2000

## Abstract

We propose a method to measure the welfare cost of economic fluctuations without full specification of consumer preferences, and instead uses asset prices. The method is based on *the marginal cost of consumption fluctuations,* the per unit benefit of a marginal reduction in consumption fluctuations expressed as a fraction consumption. We show that this measure is an upper bound to the benefit of reducing consumption fluctuations. We also clarify the link between the cost of consumption uncertainty, the equity premium and the slope of the real term structure. To measure the marginal cost of fluctuations we fit a variety of pricing kernels that reproduce key asset pricing statistics. We show that the pricing kernel has to be non-stationary in order to explain simultanously the slope of the real term structure and the equity premium. We find that consumers would be willing to pay a very high price for eliminationg all the uncertainty in aggregate consumption. However, for consumption fluctuations corresponding to the business cycle frequencies, we estimate the marginal cost to be about 0.55% of lifetime consumption based on the period 1889-1997 and about 0.30% based on 1954-1997.

# 1. Introduction

In a seminal contribution, Lucas (1987) proposes a measure of the welfare cost of economic fluctuations. His measure is defined as the compensation required to make the representative agent indifferent between consumption plans with business cycle fluctuations and without. He finds a very small cost of business cycles. Subsequently, several studies have proposed estimates of this cost of business cycles under alternative assumptions on preferences and the consumption processes. Primarily as a function of the specification and parameterization of preferences, these estimates vary widely across studies.[1] In this paper, we propose to measure the welfare cost of business cycles without fully specifying consumer preferences, instead we directly use financial market data. We believe that by directly measuring the premia for aggregate risk we can circumvent some of the difficulties related to specifying a utility function.

We start by generalizing Lucas' cost of business cycles along two dimensions. First, we derive *the marginal cost of consumption fluctuations*. Lucas' cost of business cycles measures the welfare gain from removing all the business cycle risk, it can be thought of as a total cost. We define a cost that measures the welfare benefits from reduced fluctuations *at the margin*. We find two useful features to this definition. First, because it is a marginal cost we can use asset prices to estimate the cost of business cycles for a representative agent. Second, given that most economic policies would not intend to eliminate business cycle fluctuations entirely, knowing the potential benefits at the margin may be useful in itself. Our second dimension of generalization concerns the type of consumption fluctuations that one associates with business cycles. Lucas' consumption was subject to only temporary deviations from a deterministic trend. Several later studies specified consumption as a nonstationary process (for instance, Obstfeld (1994), Campbell and Cochrane (1995)), while the cost of business cycles continued to be measured as the gain from eliminating consumption uncertainty entirely, including the risks associated with the permanent stochastic components. Our definition is general enough to encompass this case, but also allows for business cycle fluctuations to be

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[1] Lucas' estimates were well below one percent of consumption. Obstfeld (1994) finds slightly higher costs than Lucas when allowing for stochastic growth trends and preferences that distinguish between risk aversion and intertemporal elasticity of substitution. Campbell and Cochrane (1998) find larger costs of business cycles by using habit-formation preferences that are also able to explain the equity premium. Atkeson and Phelan (1994) present an example of an incomplete markets economy that can generate the equity premium, but nevertheless has only small costs of business cycles. Other examples include: Dolmas (1998), Hansen, Sargent and Tallarini (1998), Krusell and Smith (1999), Otrok (1998), Tallarini (1998); and for the related literature about the welfare gains from international integration: Lewis (1996) and Van Wincoop (1999).

defined differently, for instance, as the cyclical consumption volatility excluding a possibly stochastic trend. We find this distinction useful because *a priori* we think that many policies affect primarily economic behavior at business cycle frequencies without affecting the low frequency behavior of the economy. Even if one disagrees with this distinction about the effect of many policies, it may still be interesting to know the costs of the relatively short lived fluctuations versus the costs of all consumption uncertainty.

Before presenting our estimates, we derive some analytical results. Under general specification we show that the marginal cost of reducing fluctuations is higher than the total cost of these fluctuations, hence our estimates are upper bounds. For more specific cases, such as expected discounted utility, we obtain a sharper bound where the marginal cost of consumption uncertainty is twice the total cost. The marginal cost is also an upper bound with consumption externalities of a general class we label as *envy*.

Several of the above cited studies have highlighted the relationship between Lucas' low cost of business cycles and the equity premium puzzle. In particular, studies with preference specifications that can solve the equity premium puzzle report usually costs of business cycles larger than Lucas' original estimates. We show that the marginal cost of consumption uncertainty is not equal to the (consumption) equity premium in general. The equity premium is the excess return of equity over a risk-less rate of return for a given holding period. In contrast, the marginal cost of consumption uncertainty is equal to the excess in the price of a perpetual bond with growing coupons over the price of a consumption equity claim. Two of the factors that make the marginal cost of consumption uncertainty different from the equity premium are the steepness of the term structure and the persistence of the shocks. We clarify the link between the two and provide examples where the quantitative differences are of first-order importance.

The marginal cost of all consumption uncertainty is given by the ratio of the prices of two assets, the price of a claim to the expected value of consumption divided by the price of a claim to consumption. Equivalently, it can be measured as the ratio of the dividend yield of the claim to consumption to the long-term yield on real debt minus the expected growth rate of consumption. If consumption were equal to the dividends of a portfolio of common stocks, this formula could be used to measure the cost of all uncertainty using the dividend-price ratio for such portfolio. Measuring the cost of economic uncertainty in this way, we find the costs to be about 619% (173%) of the level of consumption for period covering 1889-1997 (1954-97 respectively).

To relax the assumption of our simplified expression for the marginal cost, and to better measure the relevant prices we estimate a pricing kernel. We estimate a

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pricing kernel as a way to interpolate from the available asset prices the prices for the assets that measure the cost of consumption fluctuations. Our asset pricing kernel is consistent with U.S. historical returns on equity, the term structure and the price/dividend ratios, all features that we show, on theoretical grounds, are important for measuring the cost of business cycles. In the state space for the pricing kernel we include variables that determine the market price of risk in different popular models. Since consumption is an element of the state space, our pricing kernel allows to differentiate consumption from dividends and to model its risk premia. We model consumption and the pricing kernel as non-stationary. This choice has quantitative implications for some of our measures. Our choice is motivated by a bound, in the spirit of the Hansen-Jagannathan bound. It implies that the pricing kernel cannot be stationary and explain simultanously the historical real term structure and the equity premium. As we show, this is a very general result that should be of interest independent of the main question of this paper.

Our definition of business cycles is based on a frequency domain approach with business cycles corresponding to cycles of length 8 years and less. Our preferred filter is derived through an ad hoc adjustment to a one-sided band-pass filter so as to match the volatility of the deviations from trend obtained with a more accurate two-sided filter.[2] Based on this, we estimate the marginal cost to be about 0.55% of lifetime consumption for the period 1889-1997 and about 0.30% for the period 1954-1997. We also estimate the cost of all consumption uncertainty, that is, including cycles of any length. We find very large costs. With cointegrated consumption and dividends, in many cases costs are around 30% of lifetime consumption, without cointegration they amount in general to several hundred percents. Finally, we link these estimates with the benefit of increasing growth—a small modification of our framework allows us to do this in a consistent way. The benefit from stabilization in terms of additional long-term growth is shown to be equal to the marginal cost expressed in percents of consumption divided by the duration of an equity claim with consumption as payouts. We find that eliminating business cycle fluctuations is worth less than 1 basis point in additional growth. Eliminating all consumption uncertainty is valued at roughly 1 percent in additional growth.

As a way to check our quantitative results, and to provide intuition, we also examine the special case where the logarithms of consumption and the pricing kernel follow random walks. We derive a simple expression for the marginal cost of consumption fluctuations. The marginal cost is approximately equal to the excess return of an equity claim to consumption multiplied by a factor that is a

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[2]As becomes clear below, our approach requires the use of a one-sided filter.

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simple function of the moving average coefficients used to define the business cycle components. Estimates of this simplified expression, where the pricing kernel is only required to fit the excess returns of different CRSP portfolios, yield findings that are similar to the ones obtained with the more general specification.

The paper is organized as follows. Section 2 defines the marginal cost of consumption fluctuations. Section 3 compares marginal and total cost. Section 4 studies the relationship between the marginal cost of consumption uncertainty and the equity premium. Section 5 presents our empirical estimates of the marginal cost of business cycles. Section 6 compares the cost of fluctuations with the effects of growth. Section 7 concludes.

## 2. Defining the Marginal Cost of Consumption Fluctuations

We start by defining our measures of the cost of business cycles. We generalize Lucas' definition along two dimensions. First, allowing for partial reduction in consumption fluctuations we define a total cost function, and, as its derivative evaluated at the point where the consumers bear all the consumption fluctuations, we define the *marginal cost of economic fluctuations*. As a second dimension of generalization, we leave it open for the moment what type of consumption fluctuations are eliminated. As we discuss more in detail in the following sections, this could be, for instance, all consumption uncertainty or only cyclical consumption fluctuations for some frequencies.

The analysis is carried out under the assumptions of a representative agent economy. In each period $t$, the economy experiences one of finitely many events $z_t \in Z$. We denote by $z^t = (z_0, z_1...z_t)$ the history of events up through and including period $t$. We index consumption by histories, so we write $C : \mathbf{Z} \to R_+$, where $\mathbf{Z} \equiv \prod_{t \geq 1} Z^t$, or simply $\{C\} = \left\{ C_t\left(z^t\right) : \forall t \geq 1, z^t \in Z^t \right\}.$

**Definition 2.1.** *We define the* **total cost of consumption fluctuations function** $\Omega(\alpha)$ *as the solution of*

$$U\left(\left(1 + \Omega(\alpha)\right)\{C\}\right) = U\left(\left(1 - \alpha\right)\{C\} + \alpha\left\{\mathfrak{C}\right\}\right), \tag{2.1}$$

*where* $\alpha \in [0,1]$, $C : \mathbf{Z} \to R_+$ *denotes the process of consumption and* $\mathfrak{C} : \mathbf{Z} \to R_+$ *denotes the process for the trend starting in period 1.* $U\left(.\right)$ *is a utility function, mapping consumption processes into* $R$.

The scalar $\alpha$ measures the fraction of risky consumption $C$ that has been replaced by the less risky trend consumption $\mathfrak{C}$.[3] The total cost function gives the

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[3] Our use of the word *trend* does not imply any statistical statement. It is simply a label for a consumption process different from actual consumption and that is presumably less risky.

total benefit from reducing consumption fluctuations as a function of the fraction of the reduction in fluctuations. It is straightforward to see that $\Omega(0) = 0$, so that no reduction in fluctuations generates no benefit.

The next definition serves mainly notational convenience. We define the **total cost of consumption fluctuations** $\omega$, as $\omega \equiv \Omega(1)$, or equivalently $U\left((1+\omega)\{C\}\right) = U\left(\{\mathfrak{C}\}\right)$. As a particular case, define the trend consumption to be $\{\mathfrak{C}\} = \{E_0(C)\}$, i.e. where $\mathfrak{C}\left(z^t\right) = E_0(C_t)$ for all $t$, and $z^t$. In this case, we have

$$U\left(\,(1+\omega)\{C\}\,\right) = U\left(\,\{E_0(C)\}\,\right), \tag{2.2}$$

which is Lucas' definition of the cost of business cycles. Thus, Lucas' definition of the cost of business cycles can be seen as the total benefit associated with eliminating all the consumption fluctuations, that is $\alpha = 1$, and where consumption fluctuations are defined as consumption uncertainty, that is, resulting in the exchange of consumption for its expected path. When we use this specification for $\mathfrak{C}$ we refer to $\Omega$ as the cost of consumption *uncertainty*, since $\mathfrak{C}$ is deterministic, to distinguish it from the more general case, where we refer to $\Omega$ as the cost of consumption *fluctuations*.

Note, the specification in equation (2.2) differs slightly from Lucas' and the literature's standard specification because we choose to begin compensation as of $t = 1$, the standard has been to start compensation at $t = 0$. We choose this departure because our definition is more consistent with the idea of ex-dividend security prices, some of our qualitative results present themselves more tractably, and finally, the quantitative difference between the two will be insignificant.

For the next definition we assume that $U$ is differentiable with respect to each $C_t\left(z^t\right)$ for all $t$ and $z^t$. We denote the partial derivatives by $U_{z^t}\left(\{C\}\right) \equiv \partial U\left(\{C\}\right)/\partial C_t\left(z^t\right)$.

**Definition 2.2.** *We define the* **marginal cost of consumption fluctuations** $\omega^m$ *as the derivative of the total cost function* $\Omega$ *evaluated at* $\alpha = 0$, *i.e.*

$$\omega^m \equiv \Omega'(0) = \frac{\sum_{t=1}^{\infty}\sum_{z^t} U_{z^t}\left(\{C\}\right)\cdot\left(\mathfrak{C}\left(z^t\right) - C_t\left(z^t\right)\right)}{\sum_{t=1}^{\infty}\sum_{z^t} U_{z^t}\left(\{C\}\right)\cdot C_t\left(z^t\right)}. \tag{2.3}$$

Thus, $\omega^m$ measures the per unit benefit of a marginal reduction in consumption fluctuations expressed in percents of consumption. It can be considered as the market price of consumption fluctuations.[4]

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[4] For a discussion about the thought experiment underlying this definition we refer the reader to section 3.3. about the costs of consumption fluctuations with consumption externalities.

For any process $X : \mathbf{Z} \to R$, define $V_0\left[\{X\}\right]$ as follows,

$$V_0\left[\{X\}\right] \equiv \sum_{t=1}^{\infty} \sum_{z^t} U_{z^t}\left(\{C\}\right) \cdot X_t\left(z^t\right).$$

This is the shadow price, for the representative agent, of an asset with payouts given by $\{X\}$. Under this convention, it is immediate to see that

$$\omega^m = \frac{V_0\left[\{\mathfrak{C}\}\right]}{V_0\left[\{C\}\right]} - 1. \tag{2.4}$$

Thus, one can interpret the marginal cost of consumption fluctuations as a ratio of the values of two securities: a claim to the consumption trend, $V_0\left[\{\mathfrak{C}\}\right]$, and a consumption equity claim, $V_0\left[\{C\}\right]$.

We think that these generalizations of Lucas' cost of business cycles have some attractive features. First, focusing on a *marginal* cost, we can hope to measure it by using information on the representative agent's marginal evaluation contained in security prices. Second, we can now think of the benefits of partial reduction in consumption fluctuations. Moreover, as our analysis in the next section shows, we can use our measure of the marginal cost to bound the standard total cost of business cycles for a large class of preference specifications, without the need to fully parameterize these preferences.

## 3. Comparing Marginal Cost and Total Cost of Consumption Fluctuations

The marginal cost defined in the previous section can be measured using asset prices, while the total cost requires a fully specified utility function. In this section we show that under some assumptions it is possible to gain information about the total cost once we know the marginal cost. Alternatively, the marginal cost may be interesting in itself, since some policies may only move partially in the direction of eliminating fluctuations. For instance, if most of the benefits from stabilization occur already for low values of $\alpha$, then implementing a policy that moves in this direction–even if it does not make consumption equal to the trend value–will be worthwhile.

The main results of this section are the following. First, concavity of $U$ implies that $\Omega$ is concave for small $\alpha$, and hence the marginal cost $\omega^m$ is an upper bound for $\Omega\left(\alpha\right)/\alpha$ for small $\alpha$. Second, if $U$ is concave and homothetic, so that the total cost does not depend on the scale of the economy, then the total cost $\Omega\left(\alpha\right)$ is concave, so that the marginal cost is an upper bound for the total cost, i.e.

$\omega^m = \Omega'(0) \geq \omega = \Omega(1)$. Many preference specifications used in quantitative asset pricing studies satisfy this assumption, since many researchers impose the same form of scale invariance. Third, we consider the case when $U$ is given by the expected value of a concave utility and the trend is given by the expected value of consumption. In this case, $\Omega$ is not only concave for $\alpha$ close to zero, but also for $\alpha$ close to one. If in addition, we have time separable utility, we show that for small variance $\omega = \frac{1}{2}\omega^m$ and the total cost is given by the insurance risk premium.

We make the following initial assumptions: $U(\{C\})$ is increasing and concave in $\{C\}$. We also assume that the process $\{\mathfrak{C}\}$ is preferred to $\{C\}$, that is, $U(\{\mathfrak{C}\}) > U(\{C\})$. Under these conditions, it is straightforward to see that $\Omega(\alpha) \geq 0$. Our first result is that $\Omega$ is concave for small $\alpha$.

**Proposition 3.1.** *Let $U$ be increasing, concave and twice differentiable. Then $\Omega''(0) \leq 0$.*

This proposition implies that $\omega^m \equiv \Omega'(0) \geq \frac{\Omega(\alpha)}{a}$, that is, the marginal cost, $\omega^m \equiv \Omega'(0)$, is larger than the average cost, $\frac{\Omega(\alpha)}{a}$.

### 3.1. Homothetic preferences and scale-free cost functions

If we require that the cost of fluctuations $\Omega(\alpha)$ be the same for the processes $\{C\}$ and $\{\mathfrak{C}\}$ as for the processes $\{\lambda C\}$ and $\{\lambda \mathfrak{C}\}$, where $\lambda$ is any positive scalar, then we must impose some additional restrictions on the utility function $U$. This requirement implies that the cost of consumption fluctuations will not differ merely because economies are rich and poor. Specifically, we require $U$ to be homothetic; that is, $U$ is homogeneous of degree $1 - \gamma$, *i.e.*, for any positive scalar $\lambda > 0$, and for $\lambda \{C\}$ defined as $\lambda \{C\}_t (z^t) = \lambda C_t (z^t)$ for each $z^t$ we have

$$U(\lambda \{C\}) = \lambda^{1-\gamma} U(\{C\}).$$

Under this assumption, we obtain that the marginal cost is higher than the total cost.

**Proposition 3.2.** *Assume that $U$ is increasing, concave and homothetic. Also assume that $\{\mathfrak{C}\}$ is preferred to $\{C\}$, i.e. $U(\{\mathfrak{C}\}) > U(\{C\})$ Then $\Omega(\alpha)$ is concave, and thus*

$$\omega^m \equiv \Omega'(0) \geq \Omega(1) \equiv \omega.$$

Examples from the literature that satisfy this homogeneity property are the preferences used in Abel (1999), Epstein and Zin (1991), Mehra and Prescott

(1985), and Tallarini (1998). Some utility functions with additive habit such as Campbell and Cochrane (1998), Constantinides (1991) and Jermann (1998) do not satisfy this property for a predetermined habit level.[5]

### 3.2. The cost of all uncertainty with expected utility

In this sub-section we present some implications for the total and marginal cost $\Omega$ and $\Omega'$ when the utility $U$ is given by expected utility. We also assume that the trend $\{\mathfrak{C}\}$ is given by the expected value of consumption, so we evaluate the elimination of all uncertainty.

Let $U$ be given by the expected value of a function $u : R_+^\infty \to R$. This specification allows time non-separabilities, including models where consumption display habit formation or durability. Notice that if $u$ is concave, then $U$ is concave and hence from our previous proposition $\Omega$ is concave for $\alpha$ close to zero. Now we obtain a complementary result, by showing that $\Omega$ is concave for values of $\alpha$ close to 1.

**Proposition 3.3.** *Let $U$ be given by the expected value of a concave and differentiable function $u$, and let $\mathfrak{C}_t\left(z^t\right) = E_0\left(C_t\right)$ for all $z^t$ and $t$. Then $\Omega'(1) = 0$ and $\Omega(\alpha)$ is concave for $\alpha$ close to one.*

In the rest of this sub-section, we further specialize $u$ by eliminating time non-separabilities and by considering the case where consumption fluctuations are small. We show that, for an approximation up to order of the variance of consumption, the total cost of uncertainty is half of the marginal cost, that is $\Omega(1) = 1/2\ \Omega'(0)$. Moreover, we show that in this case the marginal cost is given by a weighted average of the product of risk aversion and the variance of consumption for different periods. We also consider a higher order approximation to examine the role of skewness in consumption fluctuations. We show that if the period utility function $u$ displays "prudence", *i.e.* $u''' > 0$, and if consumption fluctuations have negative skewness, then we obtain a stronger inequality, *i.e.* $\Omega(1) < \frac{1}{2}\Omega'(0)$. This combination of assumptions has been used in the literature to obtain high equity premium with low risk aversion. In particular, Rietz (1988) assumes that there is a small probability of a large drop in consumption, motivated by the great depression, and he is able to substantially increase the excess returns on equity. Our result shows that skewness impacts the marginal cost and the total cost differently.

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[5]If the habit level is a function of only a few lags and when there is some separability across time, such as for instance in Jermann (1998), it can be shown that a result similar to proposition 3.2. holds after a suitiable restatement of the proposition.

Let's consider first the one period case, where consumption is given by

$$C = \bar{C}(1 + \sigma\varepsilon)$$

for a zero mean random variable $\varepsilon$. The parameter $\sigma$ indexes the amount of risk. The "trend" is given by the expected value, i.e. $\mathfrak{C} = \bar{C} \equiv E[C]$. Notice that the variance of $C$ is proportional to $\sigma^2$, $i.e.$ $Var(C/\bar{C}) = \sigma^2 E\varepsilon^2$, and that its third moment is proportional to $\sigma^3$. We include $\sigma$ as an argument of the total and the marginal cost, which are given by

$$E\left[u\left(C\left(1 + \Omega\left(1,\sigma\right)\right)\right)\right] \equiv E\left[u\left(\bar{C}\left(1 + \sigma\varepsilon\right)\left(1 + \Omega\left(1,\sigma\right)\right)\right)\right] = u\left(\bar{C}\right), \qquad (3.1)$$

and

$$\Omega'\left(0,\sigma^2\right) = \frac{E\left[u'\left(C\right)\left(\bar{C} - C\right)\right]}{E\left[u'\left(C\right)C\right]} \equiv \frac{E\left[u'\left(\bar{C}\left(1 + \sigma\varepsilon\right)\right)\left(\bar{C} - C\right)\right]}{E\left[u'\left(\bar{C} + \sigma\varepsilon\right)\left(\bar{C} + \sigma\varepsilon\right)\right]}. \qquad (3.2)$$

**Proposition 3.4.** If $E\left[u''''\left(\bar{C}\left(1 + \varepsilon\right)\right)\varepsilon^4\right]$ is finite, then

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\begin{aligned}
\Omega\left(1,\sigma\right) &= \frac{1}{2}\left[-\frac{\bar{C}u''\left(\bar{C}\right)}{u'\left(\bar{C}\right)}\sigma^2 E\varepsilon^2 - \frac{1}{3}\frac{\bar{C}^2 u'''\left(\bar{C}\right)}{u'\left(\bar{C}\right)}\sigma^3 E\varepsilon^3\right] + o\left(\sigma^3\right), \\
\Omega'\left(0,\sigma\right) &= \left[-\frac{\bar{C}u''\left(\bar{C}\right)}{u'\left(\bar{C}\right)}\sigma^2 E\varepsilon^2 - \frac{1}{2}\frac{\bar{C}^2 u'''\left(\bar{C}\right)}{u'\left(\bar{C}\right)}\sigma^3 E\varepsilon^3\right] + o\left(\sigma^3\right)
\end{aligned}
$$

where $h\left(\sigma\right) = f\left(\sigma\right) + o\left(\sigma^p\right)$ means that $\lim_{\sigma \to 0}\left[h\left(\sigma\right) - f\left(\sigma\right)\right]/\sigma^p = 0$.

Notice that these expressions imply that

$$\Omega'\left(0,\sigma\right) = 2\Omega\left(1,\sigma\right) - \frac{\sigma^3}{6}\frac{\bar{C}^2 u'''\left(\bar{C}\right)}{u'\left(\bar{C}\right)}E\varepsilon^3 + o\left(\sigma^3\right).$$

Thus, ignoring all the terms smaller than $\sigma^2$, the total marginal cost is twice the total cost, $\Omega'\left(0,\sigma\right) = 2\Omega\left(1,\sigma\right)$. Furthermore, considering higher moments, and under the assumptions that $u'''\left(\bar{C}\right) > 0$ and $E\varepsilon^3 < 0$, we obtain $\Omega'\left(0,\sigma\right) > 2\Omega\left(1,\sigma\right)$.

The next example illustrates these approximations for a distribution with approximately zero third moment.

**Example 3.5.** *Let $u$ be given a utility with constant relative risk aversion $\gamma$, i.e. $u(C) = \frac{C^{1-\gamma}}{1-\gamma}$, and $\log C$ be distributed as $N(\mu, \hat{\sigma}^2)$. Recall that if $C$ is for log-normal,*

$$Var\left(C/\bar{C}\right) \equiv \frac{Var(C)}{[E(C)]^2} = \left(e^{\hat{\sigma}^2} - 1\right) = \hat{\sigma}^2 + o\left(\hat{\sigma}^3\right)$$

*Note that for small $\sigma$, $\sigma\varepsilon \cong \log C$, which is normal distributed, and hence it has zero third moment. Simple computations show that,*

$$\Omega(1, \sigma) = e^{\frac{1}{2}\gamma\hat{\sigma}^2} - 1 = \frac{1}{2}\gamma\hat{\sigma}^2 + o\left(\hat{\sigma}^3\right),$$

$$\Omega'(0, \sigma) = e^{\gamma\hat{\sigma}^2} - 1 = \gamma\hat{\sigma}^2 + o\left(\hat{\sigma}^3\right).$$

*As the previous proposition states, since the centered third moments are zero, the marginal cost is twice the total cost, and both costs are proportional to the variance of consumption relative to its expected value.*

Now we consider the multiperiod case, where

$$U(\{C\}) = \sum_{t=1}^{T} \beta^t E_0\left[u\left(C_t\right)\right]$$

where $C_t = \bar{C}_t(1 + \sigma\varepsilon_t)$, with $\varepsilon_t$ a random variable satisfying $E_0[\varepsilon_t] = 0$. We keep the assumption that $\mathfrak{C}_t = \bar{C}_t \equiv E_0[C_t]$. The next proposition says that expressions for the cost in the multiperiod case are equal to a weighted average of the ones for the one period case.

**Proposition 3.6.** *If*

$$\beta^t E_0\left[\frac{\bar{C}_t^2 u''''\left(\bar{C}_t(1 + \varepsilon_t)\right)}{u'\left(\bar{C}_t\right)}\varepsilon_t^4\right]$$

*is finite, then*

$$\Omega(1, \sigma) = \sum_{t=1}^{T} \bar{w}_t\left\{-\frac{1}{2}\frac{\bar{C}_t u''\left(\bar{C}_t\right)}{u'\left(\bar{C}_t\right)}\sigma^2 E_0\varepsilon_t^2 - \frac{1}{6}\frac{\bar{C}_t^2 u'''\left(\bar{C}_t\right)}{u'\left(\bar{C}_t\right)}\sigma^3 E_0\varepsilon_t^3\right\} + o\left(\sigma^3\right),$$

$$\Omega'(0, \sigma) = \sum_{t=1}^{T} \bar{w}_t\left\{-\frac{\bar{C}_t u''\left(\bar{C}_t\right)}{u'\left(\bar{C}_t\right)}\sigma^2 E_0\varepsilon_t^2 - \frac{1}{2}\frac{\bar{C}_t^2 u'''\left(\bar{C}_t\right)}{u'\left(\bar{C}_t\right)}\sigma^3 E_0\varepsilon_t^3\right\} + o\left(\sigma^3\right)$$

*where the weights $\bar{w}_t$ are defined as*

$$\bar{w}_t = \frac{\beta^t u'\left(\bar{C}_t\right)\bar{C}_t}{\sum_{j=1}^{\infty}\beta^j u'\left(\bar{C}_j\right)\bar{C}_j}.$$

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Notice that this proposition implies that

$$\Omega'(0,\sigma) = 2\Omega(1,\sigma) + \sum_{t=1}^{T} \bar{w}_t \left\{ -\frac{\sigma^3}{6} \frac{\bar{C}_t^2 u'''(\bar{C}_t)}{u'(\bar{C}_t)} E_0 \varepsilon_t^3 \right\} + o(\sigma^3),$$

which neglecting the terms of order higher than $\sigma^2$, can be stated as $\Omega'(0,\sigma) = 2\Omega(1,\sigma)$. The following example uses this approximation for a CRRA utility function.

**Example 3.7.** *Let $u$ be a utility function with constant relative risk aversion $\gamma$, so that $u(C) = C^{1-\gamma}/(1-\gamma)$. In this case, neglecting the terms of order higher than $\sigma^3$,*

$$\Omega'(0,\sigma) = \sum_t \bar{w}_t \left\{ \gamma \sigma^2 E_0 \varepsilon_t^2 - \frac{1}{2}\gamma(1+\gamma)\sigma^3 E_0 \varepsilon_t^3 \right\},$$

$$\Omega(1,\sigma) = \sum_t \bar{w}_t \left\{ \frac{1}{2}\gamma \sigma^2 E_0 \varepsilon_t^2 - \frac{1}{6}\gamma(1+\gamma)\sigma^3 E_0 \varepsilon_t^3 \right\}.$$

We end this section with a numerical example that illustrates the accuracy of the approximation for small $\sigma^2$. In each panel of Figure 1 we plot the total costs, $\Omega(\alpha)$, the marginal cost, $\Omega'(\alpha)$, as a function of the fraction of risk removed, $\alpha$. The distribution of consumption has the same mean and variance in the three panels, but it differs in its skewness. In particular, the standard deviation is 3.6%, the mean of 1, and $u$ is given by a CRRA function with risk aversion $\gamma = 10$. We introduce skewness by making down-moves 4 times as large a up-moves for negative skewness. Notice that, as our proposition says, total cost are very close half of the value of the marginal cost –indicated by doted lines – i.e. $\frac{1}{2}\omega^m = \frac{1}{2}\Omega'(0) \approx \Omega(1) = \omega$. In particular, for the first panel where skewness is zero, the total cost is about 0.6%, almost exactly half of the marginal cost, which is about 0.6%. For the second panel, where skewness is negative, the total cost is smaller than half of the marginal cost, which is the previous proposition says for utility functions, such as CRRA, for which $u''' > 0$. The third panel shows that for positive skewness the opposite holds. In a related comparison, Hansen, Sargent and Tallarini (1999) compute local and global mean-risk trade-off for consumers with risk-sensitive preferences, they find their local measure to be between 2 and 4 times higher than the global one.

### 3.3. Cost of fluctuations with consumption externalities

Up to now, we have considered the case of a representative agent economy without consumption externalities. We define aggregate consumption externalities by

including aggregate consumption $\{\bar{C}\}$ in the utility function as a separate argument, together with the agent's consumption $\{C\}$, so that utility is given by $U\left(\{C\},\{\bar{C}\}\right)$.

With externalities, the interpretation of the compensation $\Omega$ depends on the exact nature of the experiment. In particular, there are two ways of thinking about reducing fluctuations in consumption. One experiment is to stabilize consumption of only one agent, keeping aggregate consumption unchanged. Another experiment is to stabilize consumption of all the agents. In the first experiment, the total cost function is given by

$$U\left(\left(1+\Omega\left(\alpha\right)\right)\{C\},\{C\}\right)=U\left(\alpha\{C\}+\left(1-\alpha\right)\{\mathfrak{C}\},\{C\}\right). \qquad (3.3)$$

Then, the benefit of a small reduction in fluctuations can be measured by the corresponding marginal cost. As explained above, the marginal cost $\Omega'\left(0\right)$ equals the ratio of the prices of two securities, as in (2.4) where $U_{z^t}\left(\{C\}\right)$ is replaced by $U_{z^t}\left(\{C\},\{C\}\right)$ defined as

$$U_{z^t}\left(\{C\},\{\bar{C}\}\right)\equiv\frac{\partial U\left(\{C\},\{\bar{C}\}\right)}{\partial C_t(z^t)}.$$

In the second experiment, the total cost function $\tilde{\Omega}$ is defined as

$$U\left(\left(1+\tilde{\Omega}\left(\alpha\right)\right)\{C\},\{C\}\right)=U\left(\alpha\{C\}+\left(1-\alpha\right)\{\mathfrak{C}\},\alpha\{C\}+\left(1-\alpha\right)\{\mathfrak{C}\}\right). \tag{3.4}$$

The function $\tilde{\Omega}$ differs from $\Omega$, since it includes the effect of the externality. However, because market prices do not internalize the effect of aggregate consumption, they do not equal $\tilde{\Omega}'\left(0\right)$.

While consumption externalities can be of various forms, the consumption externalities used to account for the determinants of aggregate risk are of a particular type. We call "envy" a general type of externality that is related to the ones used in the literature, such as the "catching up with the Joneses" in Abel (1999), and the "external habit" in Campbell and Cochrane (1995).

**Definition 3.8.** *We say that the representative agent exhibits envy if for any $C$ and $\hat{C}$ such*

$$U\left(\left\{\hat{C}\right\},\{C\}\right)\geq U\left(\{C\},\{C\}\right),$$

*then*

$$U\left(\left\{\hat{C}\right\},\{C\}\right)\geq U\left(\left\{\hat{C}\right\},\left\{\hat{C}\right\}\right).$$

The interpretation of this definition is that each agent uses the consumption of the representative agent as a benchmark for his own consumption. The next proposition shows that if the representative agent exhibits envy, the total benefit of stabilizing everybody's consumption are even smaller than the benefits from stabilizing one person's consumption.

**Proposition 3.9.** *Let's assume that the trend consumption $\{\mathfrak{C}\}$ is such that*

$$U\left(\{\mathfrak{C}\},\{C\}\right) > U\left(\{C\},\{C\}\right).$$

*If $U$ is increasing in its first argument and concave, and if Definition 3.8 holds, then*

$$\Omega\left(\alpha\right) \geq \tilde{\Omega}\left(\alpha\right)$$

*for $\alpha \in [0,1]$.*

As a corollary of the previous proposition, if $U\left\{\cdot, C\right\}$ has the properties of $U\left(\cdot\right)$ in Proposition 3.2, then

$$\omega^m \equiv \Omega'\left(0\right) \geq \Omega\left(1\right) \geq \tilde{\Omega}\left(1\right).$$

Thus, the ratio of the market prices of the securities in (2.4) is an upper bound for the total cost of fluctuations in both experiments, that is, whether consumption is stabilized for one agent or for all agents.

## 4. Comparing the Marginal Cost of Consumption Uncertainty with the Equity Premium

It seems widely recognized that there is a relationship between the equity premium and the cost of consumption uncertainty. Indeed, both are measures of the compensation required to bear aggregate risk. To our knowledge however no detailed comparison has been made in the literature. In this section, we compare the marginal cost of consumption uncertainty with the equity premium. To facilitate the comparison, we select an equity whose dividends are identical to consumption, which we call consumption-equity. Our analysis clearly exposes the differences, conceptually and quantitatively, between the marginal cost of consumption uncertainty and the equity premium. Our analysis also provides insights into how to extract information contained in the equity premium to learn about the cost of business cycles.

We approach the comparison from two complementary angles. First, in Section (4.1), we use a decomposition into the fundamental risk components inspired

by Campbell (1986) and Jermann (1998). This characterization has two important implications for estimating the marginal cost using asset price data. The first implication is that the slope of the term structure is an important determinant for the equity premium, but it is not a determinant for the marginal cost. The steeper the term structure is, the larger the equity premium can be, relative to the marginal cost of uncertainty. The second implication is that the degree of persistence of the pricing kernel and of the consumption processes have different impacts on the marginal cost and in the equity premium. The more persistent these processes are, the larger the marginal cost can be, relative to the equity premium. Second, in Section (4.2), we derive simple expressions for the equity premium and the marginal cost as functions of three elements: the dividend/price ratio, the real yield and the expected growth rate of the economy. We also present a first estimate of the benefits of eliminating all uncertainty in U.S. corporate dividends where we find very high costs.

## 4.1. Decomposition into the fundamental risk components

Recall that in (2.4) we have defined $V_0\left[\{X\}\right]$ as the time zero implicit price of an asset that pays dividends $\{X\}$. Define $V_0\left[X_t\right]$ as the time zero price of an asset that pays a single dividend $X_t$ at time $t$. In this subsection, we also consider these prices at times different from zero, for instance $V_1\left[X_t\right]$ denotes the price at time $t = 1$ of a security that pays $X_t$ at time $t$. By a no-arbitrage argument the price of a consumption equity claim equals the value of a portfolio of claims to a single dividend equal to the consumption of each period, which we call, by analogy to the terms used for bonds, equity *strips*. Thus,

$$V_0\left[\{C\}_{t=1}^{\infty}\right] = \sum_{t=1}^{\infty} V_0\left[C_t\right].$$

To use the same notation for the prices of bonds, we denote by $1_t$ a dividend that is equal to one at $t$ for all $z^t \in Z^t$ and zero otherwise. Define as $R_{0,t}\left[X_t\right]$ the time zero return until maturity of an asset with a single payment $X_t$ at $t$, i.e.

$$R_{0,t}\left[X_t\right] \equiv \frac{V_t\left[X_t\right]}{V_0\left[X_t\right]} = \frac{X_t}{V_0\left[X_t\right]}.$$

Specializing the previous definition, denote the one-period holding return of an asset with single payment $X_t$ at $t$ as

$$R_{0,1}\left[X_t\right] \equiv \frac{V_1\left[X_t\right]}{V_0\left[X_t\right]}.$$

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Using these definitions, denote by $\nu_0$ the multiplicative excess expected equity premium,

$$1 + \nu_0 \equiv \frac{E_0 \left( R_{0,1} \left[ \{C\}_{t=1}^\infty \right] \right)}{R_{0,1} \left[ 1_1 \right]}.$$

**Proposition 4.1.** *Define the weights $w's$ as follows*

$$w_0 \left[ C_t \right] \equiv \frac{V_0 \left[ C_t \right]}{V_0 \left[ \{C\}_{t=1}^\infty \right]}, \tag{4.1}$$

*the multiplicative consumption-equity premium can be written as*

$$1 + \nu_0 = \sum_{t=1}^\infty w_0 \left[ C_t \right] \left( \frac{E_0 \left( R_{0,1} \left[ C_t \right] \right)}{R_{0,1} \left[ 1_1 \right]} \right), \tag{4.2}$$

*and the marginal cost of consumption uncertainty can be written as*

$$1 + \omega_0^m = \sum_{t=1}^\infty w_0 \left[ C_t \right] \left( \frac{E_0 \left( R_{0,t} \left[ C_t \right] \right)}{R_{0,t} \left[ 1_t \right]} \right). \tag{4.3}$$

Equation (4.3) says that the marginal cost of consumption uncertainty equals **a weighted sum of dividend strip return premia with holding period until maturity**. In contrast, the multiplicative equity premium to a share that pays aggregate consumption, is equal to a **weighted sum of dividend strip premia for one-period holding returns.** Note, the weights, $w_0 \left[ C_t \right]$, are indeed the same for the marginal cost and for the equity premium!

When comparing the two expressions, (4.2) and (4.3), for the equity premium and the marginal cost, respectively, another fundamental difference becomes apparent. If the payouts are not random, then the holding returns until maturity, that determine the marginal cost, are not random either, that is,

$$R_{0,t} \left[ X_t \right] = R_{0,t} \left[ 1_t \right] \equiv \frac{1_t}{V_0 \left[ 1_t \right]},$$

and thus, the multiplicative strip premium is zero,

$$E_0 \left( R_{0,t} \left[ 1_t \right] \right) = \frac{E_0 \left( R_{0,t} \left[ 1_t \right] \right)}{R_{0,t} \left[ 1_t \right]} = 1.$$

Therefore, the marginal cost is pure compensation for payout risk. This is in contrast to the one period holding returns that make up the equity premium. Realized returns depend on the valuation at time 1,

$$R_{0,1} \left[ X_t \right] = R_{0,1} \left[ 1_t \right] \equiv \frac{V_1 \left[ 1_t \right]}{V_0 \left[ 1_t \right]},$$

so that premia are in general non-zero,

$$E_0\left(R_{0,1}\left[1_t\right]\right) = \frac{E_0\left(R_{0,1}\left[1_t\right]\right)}{R_{0,1}\left[1_1\right]} \neq 1.$$

This comparison shows that the difference between equity premium and marginal cost will be a function of the slope of the term structure, as we discuss in more detail for the loglinear case in the next subsection.

### 4.1.1. Log-linear environment

We introduce a log-linear environment for two reasons. First, it allows us to sharpen the comparison made in the previous section between the equity premium and marginal cost. In particular, it helps to explain the different impact of the term structure risk and the payout uncertainty risk on the marginal cost and the equity premium. While the distinction between payout uncertainty risk and term structure risk is a general concept, in the log-linear environment these risk premia can be expressed in a separable way.[6] Additionally, we will use the log-linear environment later in the paper for estimating the marginal cost of business cycles.

As it is well known, if there are no arbitrage opportunities, under technical regularity conditions, there must exists a non-negative process $\beta^t M_t$ such that all prices satisfy

$$V_0\left[\{X\}\right] = \sum_{t=1}^{\infty} \beta^t E_0 \left[\frac{M_t}{M_0} X_t\right].$$

We call the process $\beta^t M_t$ a stochastic pricing kernel. We assume that the logarithm of the kernel and the logarithm of the dividends of the assets of interest can be represented as linear functions of a linear VAR model. Specifically, let $s_t$ be a state vector following a multivariate, homoscedastic VAR

$$s_t = A s_{t-1} + \varepsilon_t, \tag{4.4}$$

where $\varepsilon_t$ is a multivariate normal vector, i.i.d. through time. The square matrix $A$ determines the dynamics of the system. We require the roots of $A$ to be all smaller or equal to one, thus allowing for I(1) nonstationarity. Assume that

$$\ln(C_t) = \left(\ln\left(1+g\right)^t\right) + l_c \cdot s_t,$$
$$\ln(M_t) = l_m \cdot s_t,$$

---

[6]See Campbell (1986), Jermann (1998) and Abel (1999) for studies that focus on the distinctions between these two types of risk.

where the loadings, $l_c$ and $l_m$, are row vectors and $g$ is the trend growth rate.

Following the calculations in Jermann (1998), the multiplicative premium for one-period holding returns of a risky strip can be separated in two parts. One part is the term premium, i.e. the excess return of a long bond over the short rate, and the other is a compensation for the riskiness of the payout. Thus, the equity premium can be written as

$$1 + \nu_0 (s_0) = \sum_{t=1}^{\infty} w [C_t] (s_0) \left\{ \frac{E_0 (R_{0,1} [1_t])}{R_{0,1} [1_1]} \exp (-cov_0 (\log M_1, E_1 \log C_t)) \right\} \quad (4.5)$$

where we use the notation $\nu_0 (s_0)$, and $w [C_t] (s_0)$ to indicate that the equity premium and the weights depend on the current state vector $s_0$. Notice that the equity premium is a weighted average of two terms. The first term is the expected return of a $t$ period real zero coupon bond over the short rate. This term is higher, when the real term structure is steeper. The second term, $-cov_0 (\log M_1, E_1 \log C_t)$, measures how undiversifiable the risk about news on period $t$ consumption is.[7]

By similar calculations the marginal cost equals

$$1 + \omega_0^m (s_0) = \sum_{t=1}^{\infty} w_0 [C_t] (s_0) \exp (- cov_0(\log M_t, \log C_t)) \quad (4.6)$$

where $- cov_0(\log M_t, \log C_t)$ measures how undiversifiable consumption risk at horizon $t$ is. Notice that for the marginal cost of consumption uncertainty, there is no term involving the real term structure.

In the case where $\log M_t$ and $\log C_t$ are very persistent, this covariance can be much larger in absolute value than the covariance in the expression for the equity premium.

Let us compare the covariances in the expressions for the marginal cost (4.5) and the equity premium (4.6). By definition of a the coefficient of correlation,

$$
\begin{aligned}
cov_0(\log M_t, \log C_t) &= \rho_0 (\log M_t, \log C_t) \sigma_0 (\log M_t) \sigma_0 (\log C_t) \\
cov_0 (\log M_1, E_1 \log C_t) &= \rho_0 (\log M_1, E_1 \log C_t) \sigma_0 (\log M_1) \sigma_0 (E_1 \log C_t)
\end{aligned}
$$

The more persistent the processes for $\log M_t$, $\log C_t$ are the bigger the differences $\sigma_0 (\log M_t) - \sigma_0 (\log M_1)$ and $\sigma_0 (\log C_t) - \sigma_0 (E_1 \log C_t)$. Thus, the more persistent these processes are, the larger the marginal cost is, relative to the equity premium. For instance, if both $\log M_t$ and $\log C_t$ are $I (1)$, then the standard deviations in the marginal cost, $\sigma_0 (\log M_t)$ and $\sigma_0 (\log C_t)$, grow without bound, while the standard deviations for the equity premium, $\sigma_0 (\log M_1)$ and $\sigma_0 (E_1 \log C_t)$, are bounded.

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[7] Given the assumptions of the log-linear, homoskedastic system, the covariance is independent of the state $s_0$ and depends only on the maturity of the strip.

**Example: Separable CRRA utility.** We illustrate the difference between the marginal cost of uncertainty and the equity premium using three specifications of a well understood economy. With time separable expected discounted CRRA utility, the kernel satisfies $\ln(M_t/M_0) = -\gamma \ln(C_t/C_0)$. The following results can be shown by direct calculations.

1. First, assume that consumption follows a random walk. In this case, interest rates are constant and thus there are no term premia. Payout uncertainty premia for $k$-period holding are related to the one-period return premia by

$$-cov_0 \left(\log M_t, \log C_t\right) = -t \; cov_0 \left(\log M_1, E_1 \log C_t\right) = t \; \gamma \sigma_\varepsilon^2$$

   Clearly, in this case, the marginal cost, $t \; \gamma \sigma_\varepsilon^2$, is bigger, likely substantially so, than the consumption-equity premium, $\gamma \sigma_\varepsilon^2$.

2. Second, assume that consumption growth rates follow an AR(1) with positive serial correlation. It can then easily be shown that the term structure is downward sloping, so that

$$\frac{E_0 \left(R_{0,1} \left[1_t\right]\right)}{R_{0,1} \left[1_1\right]} < 1,$$

   and
$$-cov_0 \left(\log M_t, \log C_t\right) > -t \; cov_0 \left(\log M_1, E_1 \log C_t\right) > 0,$$

   so that there are even more reasons for which the marginal cost is bigger than the consumption-equity premium.

3. Third, the one case where the consumption-equity premium might actually be bigger than the cost of consumption uncertainty is with an AR(1) in consumption levels, with positive serial correlation, *i.e.* the growth rate of consumption is negatively autocorrelated . In this case, the term structure is upward sloping,
$$\frac{E_0 \left(R_{0,1} \left[1_t\right]\right)}{R_{0,1} \left(1_1\right)} > 1,$$

   and
$$0 < -cov_0 \left(\log M_t, \log C_t\right) < -t \; cov_0 \left(\log M_1, E_1 \log C_t\right).$$

**4.2. Decomposition into yields and growth rates**

As shown in equation (2.4), the marginal cost of consumption uncertainty, can, by definition, be viewed as the ratio of the prices of two securities. For the particular case of comparing actual consumption to its expected growth path these two securities have intuitive interpretation and close real-world counterparts. Indeed, the appropriate version of equation (2.4) is

$$1 + \omega_0^m = \frac{V_0\left(\{E_0\left(C\right)\}\right)}{V_0\left(\{C\}\right)}.$$

In the numerator, $V_0\left(\{E_0\left(C\right)\}\right)$ is a perpetual bond with coupons that are growing at the expected consumption growth rate. In the denominator, $V_0\left(\{C\}\right)$ is a consumption equity claim. Dividing both sides of the fraction by $C_0$ we have

$$1 + \omega_0^m = \frac{V_0\left(\{E_0\left(C\right)\}\right)/C_0}{V_0\left(\{C\}\right)/C_0},$$

suggesting that the marginal cost is a function of two "price/dividend ratios," or, reciprocally, two yields. If we assume that aggregate consumption has a constant expected trend growth rate, $g_0$, this can be written as

$$1 + \omega_0^m = \frac{\sum_{i=1}^{\infty}\left(\frac{1+g_0}{1+r_0}\right)^i}{\frac{1}{dp_0}} = (1 + g_0)\left[\frac{dp_0}{r_0 - g_0}\right], \qquad (4.7)$$

where $r_0$ is the real yield of a bond with the duration of the growing perpetuity, and where $dp_0$ is the dividend yield of the consumption-equity.

In general, the expected excess return of equity cannot be written as a function of $r_0$, $dp_0$ and $g_0$. Nevertheless if we make some simplifying assumption we obtain an expression that depends on similar factors. If we assume that dividend yields are constant, we can write the expected return of equity as $(1 + g_0) \cdot (1 + dp_0)$. By definition, the equity premium is the expected excess return of equity over a risk-less investment, yielding a known return of $y_0$. Thus,

$$1 + \nu_0 = \frac{(1 + g_0) \cdot (1 + dp_0)}{(1 + y_0)}. \qquad (4.8)$$

In general $y_0$ will be smaller than the yield of the long term bond $r_0$ used in equation (4.7). They will be equal only in special cases, for instance, if the term structure is flat. Clearly the formulae for the equity premium in (4.8) is different from the one for the marginal cost in (4.7). Notice that the equity premium $\nu_0$, in (4.8) is approximately $\nu_0 \simeq g_0 + dp_0 - y_0$, and hence it cannot exceed a few

percentage points. This is quite different for the marginal cost, $\omega_0^m$, in equation (4.7), which is approximately, $\omega_0 \simeq dp_0 / (r_0 - g_0)$, and hence it could be much larger. In the next subsection, we provide quantitative content to equation (4.7) by measuring the marginal cost.

### 4.2.1. A measure of the marginal cost of dividend uncertainty

For a first quantitative cut, we plug historic averages into the formula for the marginal cost given by equation (4.7). Indeed, we can consider the dividend-price ratio, $dp$, to be the historical dividend-price ratio of the value weighted U.S. stock market. Of course, we are thus looking at how *dividend* uncertainty reduces the price of equity relative to a perpetual bond paying as coupons the expected dividends, we will focus on *consumption* fluctuations in the next section.[8]

Combining the average historical dividend-price ratio with a historical long-term real yield and the dividend growth rate, we report in Table 1 a marginal cost for dividend uncertainty of 619% and 174% for the periods 1889-1997 and 1954-1997 respectively. That is, for the long sample period, the buyer of an equity claim would have to be given a more than six times higher dividend to make him value equity as much as a perpetual bond paying coupons growing at the 1.31% trend growth rate of dividends. Notice that the lower cost of dividend uncertainty for the post-war period corresponds indeed to a period of lower volatility as measured by the standard deviations of the growth rates of consumption and dividends. If dividends were equal to aggregate consumption, we would have had here our estimate of the marginal cost of consumption uncertainty. The additional statistics in Table 1 document some aspects of dividend and consumption behavior.

## 5. Measuring the Marginal Cost of Business Cycles

The approach used to measure the cost of business cycles in the previous section has the advantage of being simple and intuitive, but it has several shortcomings. First, it measures the elimination of all the uncertainty, as opposed to the elimination of business cycle fluctuations only. Second, the theory requires the use of an asset whose dividends are perfectly correlated with consumption. The measurement in the previous section uses a diversified equity portfolio, whose dividends display positive correlation with consumption, but that it is much smaller

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[8]To be precise, the expected marginal cost should be estimated by the average of the corresponding marginal costs. Here we compute the marginal cost for the average yields, average dividend price ratios and average growth rates, since we only observe ex-post real yields and growth rates.

than one. Third, the calculation of the marginal cost can be very sensitive to small inconsistencies in the measurement of the different yields and growth rates. This is due to the non-linearities in (4.7), where $dp_0$ is divided by $r_0 - g_0$. For instance, the calculation of the duration adjusted yield, $r_0$, is not straightforward without specifying jointly the term structure and the expected growth rate of consumption.

In this section we address these three shortcomings. First, we will isolate the cost of consumption fluctuations corresponding to business cycle frequencies and compare it to the cost of all consumption uncertainty that is usually computed in the literature. Specifically, we isolate cyclical consumption movements corresponding to cycles of at most 8 years by using a band-pass filter approach. Second, we will price an asset that pays aggregate consumption, as opposed to aggregate dividends. Third, by explicitly modeling a pricing kernel, our estimates of dividend yields and growth rates will be consistent with each other. As a way to check our quantitative results, and to provide intuition, we also examine the special case where the logarithms of consumption and the pricing kernel follow random walks. We derive a simple expression for the marginal cost of consumption fluctuations, and we provide an estimate with a pricing kernel that is required to fit the excess returns of different CRSP portfolios.

Let us start here by overviewing the different steps involved in our estimation procedure, a more detailed step-by-step discussion follows in the subsequent subsections. Based on the log-linear environment defined in Section (4.1.1):

1. We select state variables that determine marginal utility of consumption based on popular models, and we estimate a linear autoregressive law of motion for the logarithm of these state variables, $s_t$.

2. We estimate the loading vector for the pricing kernel, $l_m$, and the effective time discount rate, $\beta$, from a set of asset pricing moment conditions determined by the theoretical considerations discussed above.

3. We specify the trend $\{\mathfrak{C}_\tau\}_{t=\tau}^{\infty}$, compute $V_t\left(\{\mathfrak{C}_\tau\}_{\tau=1}^{\infty}\right)(s_t)$ and $V_t\left(\{C_\tau\}_{\tau=1}^{\infty}\right)(s_t)$ for every $s_t$ of our data-set, and report the sample mean of the marginal costs.

## 5.1. The state variables

We choose a multidimensional state vector to capture the dependence of marginal utility on non-contemporaneous consumption. These dependence has been emphasized in the literature in models with habit formation and in models with non-expected utility.

The state variables include (1) aggregate consumption, (2) aggregate dividends, (3) the dividend-price ratio, (4) the long-short government bond yield spread and (5) the value-weighted realized stock return. This choice is motivated by empirical work on intertemporal asset pricing. For instance, Campbell (1996) has used dividend-price ratios, yield spreads, realized stock returns, and aggregate output as his state. We include consumption in the state space since we need to estimate the price of an asset with payouts equal to consumption.

We use annual time series covering 1889-1997, data sources and definitions are described in detail in the Data Appendix. We start by removing a linear deterministic growth trend from consumption and dividends. Specifically, we extract a drift that is computed as the mean of the (log) growth rates. We also demean all variables. For instance, starting with raw consumption $C_t$, we get $c_t = \log(C_t) - c_0 - \mu t$, where the drift and the mean are defined respectively as $\mu \equiv mean(\log \Delta c_t)$ and $mean(c_t; c_0) = 0$. We then estimate by OLS a VAR for growth rates for consumption and dividends and for levels for the remaining variables. We estimate systems with and without an error-correction term that forces cointegration between consumption and dividends. A system in levels, as in equation (4.4), is then recovered. The variance-covariance matrix of the innovations is obtained from the residuals of the estimated VAR.

By estimating the VAR in growth rates for consumption and dividends, we have introduced one or two unit roots into the matrix A, depending on whether we introduce an error-correction term. This decision to model consumption and the pricing kernel as non-stationary has first-order quantitative implications for the measurement of the cost of consumption uncertainty. In the next subsection, we explain the reasons why we choose to model them as non-stationary.

### 5.1.1. Stationary versus nonstationary kernels

It is certainly possible to argue on statistical grounds that consumption and/or dividends can be modelled as (trend) stationary processes. However, we will select a nonstationary representation for consumption due to the fact that a stationary pricing kernel cannot *possibly* explain simultaneously the relatively flat historical term structure of interest rates and the relatively large historical equity premium, as we show below. Intuitively, a stationary pricing kernel that is volatile enough to generate high equity risk premia has too much predictable movements to be consistent with the relatively low term spread. In other words, long term interest rates contain valuable information about expected growth rates of the pricing kernel at long horizons. This result applies not only to the particular specification considered here, but to *any* stationary pricing kernel. Given the requirement

of a nonstationary pricing kernel, we choose consumption and dividends to be nonstationary. We think that specifying consumption to be stationary while having a non-stationary pricing kernel would be an a priori unappealing property of any representation.

The rest of this subsection makes the claims in the previous paragraph precise. First we set up some definitions for the term structure. We define **the continuously compounded term premium for a $k-$period discount bond** as

$$h_t^{cc}(k) = E_t \left\{ \log \left( \frac{R_{t,t+1} \left[ 1_{t+k} \right]}{R_{t,t+1} \left[ 1_{t+1} \right]} \right) \right\}$$

where $R_{t,t+1} \left[ 1_{t+k} \right]$ measure the gross return from holding from time $t$ to $t+1$ a claim to one unit of the consumption good to be delivered at time $t + k$. Define the **continuously compounded yield differential** between a $k$-period discount bond and a one-period, risk-less bond as

$$y_t^{cc}(k) = \log \left( \frac{V_t \left[ 1_{t+1} \right]}{\left( V_t \left[ 1_{t+k} \right] \right)^{1/k}} \right).$$

We say that a kernel is stationary if $\{M_{t+1}\}$ is stationary and ergodic. We show here that with a stationary pricing kernel the excess returns for a discount bond that matures very far in the future takes a very simple form.

**Proposition 5.1.** *If the pricing kernel is stationary then*

$$h_t^{cc}(\infty) = \log E_t \left[ M_{t+1} \right] - E_t \log M_{t+1},$$
$$y_t^{cc}(\infty) = \log E_t \left[ M_{t+1} \right] - \log M_t.$$

Now we relate our expression for the long-run term premia with some other observable quantities. We follow a similar argument as in Cochrane (1992) or Bansal and Lehmann (1997) to relate the size of the term premia with the return on the *growth optimal* portfolio.

**Proposition 5.2.** *If the pricing kernel is stationary, then*

$$E \left[ h_t^{cc}(\infty) \right] = E \left[ y_t^{cc}(\infty) \right] > E \left[ \log \left( R_{t,t+1}^j \right) \right] - E \left[ \log \left( R_{t,t+1} \left[ 1_{t+1} \right] \right) \right]. \qquad (5.1)$$

*where $R_{t,t+1}^j$ is the holding return on any asset.*

Equation (5.1) implies that, if the pricing kernel is stationary, the average long-short term spread is superior to the continuously compounded expected excess return of any asset, for instance the equity premium. This inequality can easily be rejected by historical averages. Consider the return on the aggregate U.S. stock market as the asset $R_{t,t+1}^{j}$. In this case, the bound is clearly violated as shown in the table below:

|  | 1989-1997 | 1926-1997 |
|---|---|---|
| (1) $E\left[\log\left(R_{t,t+1}^{Eq}\right)\right]$ | 0.0666 | 0.0710 |
| (2) $E\left[\log\left(R_{t,t+1}\left(1_{t+1}\right)\right)\right]$ | 0.0118 | 0.0058 |
| (1)-(2) | 0.0548 | 0.0652 |
| $E\left[y_{t}^{cc}\left(\infty\right)\right]$ | 0.0059 | 0.0140 |
| $E\left[h_{t}^{cc}\left(\infty\right)\right]$ | 0.0045 | 0.0136. |

## 5.2. Estimating the loading vector $l_m$ and $\beta$

Once the law of motion of the state vector is estimated, we need to estimate the various loading coefficients on the states, $l_m$, plus the time discount factor, $\beta$. We will choose $(l_m, \beta)$ in order to minimize $\Im = [\theta - f(l_m, \beta)]' \cdot W \cdot [\theta - f(l_m, \beta)]$, where $\theta$ is a vector of moments to match, $f(l_m, \beta)$ contains the corresponding moments generated by our asset pricing model and $W$ is a weighting matrix.

Our analysis in Section 4 of the fundamental components of the equity premium and of the marginal cost of consumption uncertainty suggests we focus on a pricing kernel that is good at explaining historical aggregate stock price behavior and the real term structure. Specifically, we choose the asset pricing kernel to replicate the U.S. average dividend-price ratio and the equity premium. We consider two ways of replicating the equity premium. First, by applying the pricing kernel to the estimated process of dividends, our model generates a series of conditional equity premia for which we compute the sample mean

$$f_j(l_m, \beta) = \frac{1}{T} \sum_{t=1}^{T} \left[1 + \nu_t^d\left(s_t; l_m, \beta\right)\right],$$

where $\nu_t^d(.)$ is defined as $\nu(.)$ in equation (4.5) except that the payout process represents aggregate dividends instead of aggregate consumption. Our second moment condition relative to the equity premium applies the pricing kernel to realized U.S. excess stock returns, $VWR_{t,t+1} - R_{t,t+1}^{f}$, so that

$$f_i(l_m, \beta) = \frac{1}{T} \sum_{t=1}^{T} \beta \exp\left(l_m\left(s_{t+1} - s_t\right)\right) \cdot \left(VWR_{t,t+1} - R_{t,t+1}^{f}\right),$$

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with $\theta_i = 0$. In Table 2, we refer to these two conditions as $E\left(R^d/R^f\right)$ and $E\left(VWR - R^f\right)$ respectively. We also make the kernel fit the average real return on a risk-less short term bond and the average real yield on a 20 year bond.[9]

## 5.3. Defining business cycles

We now describe how we specify the process corresponding to the consumption trend $\{\mathfrak{C}\}$. We choose two approaches. As our first approach, we specify the logarithm of the consumption trend to be a one-sided moving average of current and past consumption, $\ln\left(\mathfrak{c}_\tau\right) = \sum_{k=0}^{K} a_k \cdot c_{\tau-k}$, for appropriate moving average coefficients.[10] As our second approach, we set trend consumption equal to its conditional expectation, $\mathfrak{C}_\tau = E_t\left(C_\tau\right)$.

We define business cycles to be the cyclical movements that last 8 years and less, so that our trend $\{\mathfrak{C}\}$ contains movements of 8 years and more. Assuming business cycles to last up to 8 years is consistent with the definition of Burns and Mitchel (1946) and with many recent studies describing business cycle properties. This also corresponds approximately to the definition of business cycles implied by the widely used Hodrick-Prescott filter for quarterly data with a smoothing parameter of 1600. For comparison we also report results for trends with frequency cutoffs at 12, 16 and 20 years.

We choose the moving-average coefficients, $\{a_k\}$, so as to represent a band-pass filter that lets pass frequencies that correspond to cycles of $Y$ years and more. Band-pass filters are represented in the time domain by infinite order two-sided moving averages. However, a requirement of our analysis is to have trend consumption at time $t$ to be function of information available at time $t$, thus our choice of a one-sided moving average.

We chose the moving average coefficients of our trend consumption using the procedures presented by Baxter and King (1995, 8). Let $\beta\left(\omega\right)$ be the frequency response function of the desired low-pass filter, which in our case is equal to one for frequencies lower than $Y$ years and zero otherwise. Let $\alpha_K\left(\omega\right)$ be the frequency response function associated with a set of moving average coefficients $\{a_k\}_{k=1}^{K}$. We select the moving average coefficients $\{a_k\}$ so that $\alpha_K$ approximates $\beta$. In particular, our choice of $\{a_k\}$ minimizes

$$\int_{\pi}^{-\pi} \left|\beta\left(\omega\right) - \alpha_K\left(\omega\right)\right|^2 f\left(\omega\right) d\omega,$$

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[9] Practically, we make the model fit the average yield of a zero coupon bond of 13 years maturity, given that this corresponds approximately to the duration of the 20 year government coupon bond in our historical data set.

[10] With this definition of the trend $\mathfrak{C}$, $E_0\left(C_t\right) \neq E_0\left(\mathfrak{C}_t\right)$, because $\mathfrak{C}$ is a nonlinear function of $C$. Quantitatively, however, this difference is negligible as we show in the appendix.

where $f(\omega)$ is a weighting function representing (an approximation to) the spectral density of the series to be filtered. In this minimization, we impose the condition $\alpha_K(0) = 1$, which implies that $\sum_{k=0}^{K} a_k = 1$.

We fix the number of lagging moving average coefficients to be fifteen ( *i.e.*, $K = 15$), and we use the spectral density of an AR(1) with autocorrelation 0.99 for the weighting function $f$, because this matches approximately the spectral density of consumption.

Unfortunately, one-sided filters are far from perfect low-pass filters. Being one-sided, these filters cannot avoid to introduce a phase shift, *i.e.* the trend and its associated deviations will be lagging the series. The objective function displayed above is closely related to the variance of the difference of the desired series and the implied filtered series. Thus, this objective function trades-off the phase shift of the filtered series with the desired shape of the spectral density.[11] This can be seen by plotting the transfer function of the filter as in Figure 2 in the left panel. The transfer function should be one in-between the desired frequencies and zero for higher frequencies. Instead, it tends to let pass up to 30% of the variance at higher frequencies, so that the computed trend contains a substantial amount of cyclical variability. As shown in the right panel of Figure 2, the corresponding two-sided band-pass filter fits the ideal filter's step function much closer—remember that a symmetric two-sided filter does not introduce a phase shift. The corresponding time-domain representation is in Figure 3A in the first panel. Clearly, the one-sided filter generates cyclical movements that are less volatile than those from the corresponding two-sided filter shown by the thin line. The second panel in Figure 3A and Figure 3B present consumption deviations from trends corresponding to cutoffs of 12, 16 and 20 years, allowing for comparisons with the deviations obtained with the two-sided filter for 8 year

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[11] This can be seen by rewriting the objective function as follows:

$$\int_{-\pi}^{\pi} |\beta(\omega) - \alpha_K(\omega)|^2 f(\omega)\, d\omega$$

$$= \int_{-\pi}^{\pi} (|\beta(\omega)| - |\alpha_K(\omega)|)^2 f(\omega)\, d\omega$$

$$+ \int_{-\pi}^{\pi} 2\beta(\omega) |\alpha_K(\omega)| [1 - \cos(\psi_K(\omega))] f(\omega)\, d\omega$$

where $\psi_K(\omega)$ denotes the phase shift of the filter. Thus, the objective function can be written as the square of the differences of the gains of the filters, $(|\beta(\omega)| - |\alpha_K(\omega)|)^2$, plus a term that depends on the phase shift. This second term is zero, if the filter has no phase shift. For instance, a symmetric two-sided filter has no phase shift, so it also fits the gain of the approximate filter (and thus the transfer function). Instead, a one-sided filter has to find the trade-off between these two forces.

cycles.

Based on this comparisons we decide to make an ad hoc adjustment to the one-sided filter so as to replicate the amount of business cycle volatility obtained from the more accurate two-sided filter. As shown in Figure 2A, the series generated by the one-sided filter is strongly correlated with the one from the two-sided filter, but it is less volatile. Thus we choose to scale up the volatility by multiplying the cyclical deviations by a constant $\theta > 1$. This is achieved by defining the adjusted trend, $\mathfrak{c}_t^*$, as

$$\mathfrak{c}_t^* = (1 - \theta) c_t + \theta \mathfrak{c}_t.$$

With $\theta = 1.4$ the standard deviation of the scaled one-sided filter is about equal to the one from the two-sided filter. The implied transfer function of the adjusted low-pass filter defining the trend is presented in Figure 4: the deviations from the ideal filter are somewhat larger around the cutoff frequencies, for the business cycle frequencies the adjusted filter represents a significant improvement. As we show below, the approximate marginal cost implied by the adjusted filter is obtained by the same scaling so that $\omega^{m*} \cong \theta \cdot \omega^m$.

We also report results based on some other popular moving-average filters. A geometric filter is specified so that $a_k = \delta_g^{k+1}$ and $\sum_{k=0}^{K} a_k = 1$, for $K = 5$; thus $\delta_g = 0.5041$. A linear filter is specified so that $a_k = \delta_l \left(1 - \frac{k}{K+1}\right)$ and $\sum_{k=0}^{K} a_k = 1$, which gives a slope so that the next potential weight $a_{K+1} = 0$; thus $\delta_l = 0.2857$ with $K = 5$. We choose a lag length of $K = 5$ because the one-sided frequency domain filters with 15 lags that we use have moving average coefficients that are not too different from zero for lags larger than 5. Figure 3C shows the cyclical consumption components in these cases.

## 5.4. Findings and discussion

In table 2A to 2D we report estimates for various state-space systems, various loading states and various moments to fit for the complete sample and for the postwar period. We have chosen the following algorithm for minimizing the sum of squares of the discrepancies between model generated moments and the data, $\Im$. We start by estimating systems with as many coefficients as there are moments, using a diagonal weighting matrix that assigns a weight of 1/5 to the moments involving the equity premium, and 1 for the remaining moments. As shown in tables 2A to 2D in the last column entitled "Fit" , for many systems we are able to drive $\Im$ to 0. As a minimum requirement for fit we impose a maximum discrepancy of 1.5% for the moments involving the equity premium and 0.3% for the others. In some cases we are able to obtain a satisfactory fit only after

removing one of the forecasting variables or changing slightly the sample period. If this exactly identified approach fails to provide a satisfactory fit, we move to an overidentified system marked by 'OI', by adding one moment condition, and we report the result with the lowest $\Im$.

Overall, for fluctuations defined by the 8-year filter we find on average a marginal cost of about 0.40% of lifetime consumption for the period 1889-1997 and of about 0.20% for the period 1954-97. With our adjustment to the one-sided filter, as explained in the previous section with $\theta = 1.4$, we estimate the marginal cost of business cycles to be about 0.55% based on the entire sample and about 0.30% for the post-war period. Imposing cointegration between consumption and dividends yields an estimate of a few basis points higher than without cointegration.[12] For comparison, the linear and the geometric filters imply marginal costs of about 0.75% and 1.75% of consumption for the entire sample and the post-war period, respectively.

The column labelled 'inf' reports the cost of all consumption uncertainty, that is, including cycles of any length. Specifically, $\mathfrak{C}_\tau = E_t(C_\tau)$. We find very large costs. With cointegrated consumption and dividends, many systems yield costs at around 30% of lifetime consumption, without cointegration they amount in general to several hundred percents. These numbers are consistent with the orders of magnitudes we found for the cost of dividend uncertainty in section (4.2.1). These numbers are also consistent with the intuition provided by the log-linear random walk case in section (4.1.1). Fundamentally, consumers very much dislike the possibility that consumption can wander off very far from its expected path—consistent with this interpretation, the additional structure provided by the cointegration restriction yields lower costs.[13]

Looking across the different specifications, our measures appear robust for the different moving average filters, even though the loading coefficients on some states differ sometimes substantially across cases. There is more variability for measures of consumption uncertainty. One reason for the variability in measuring the costs of consumption uncertainty is that in some cases prices for the perpetual bonds with growing coupons become very large, *i.e.* long term yields are low relative to the trend growth rate.

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[12] Taking an average over all reported estimates corresponding to the 8-year filter in Table 2 we have 0.35% and 0.42% for the period 1889-1997 without and with cointegration between consumption and dividends, 0.17% and 0.22% for the period 1854-1997 without and with cointegration between consumption and dividends.

[13] As explained in Section (5.2) we model consumption as non-stationary because a stationary consumption process is not consistent with having a kernel that can simultaneously explain a large equity risk premium and a relatively flat real term structure.

## 5.5. The marginal cost of fluctuations in a simpler case

In this section we specialize the kernel and the payoffs by assuming that they are given by random walks. Under the random walk assumption, most of the dynamics are eliminated: interest rates, dividend-price ratios, and expected excess returns are all constant. For this reason, we can present the marginal cost in a simple and intuitive form. The marginal cost of fluctuations is approximately equal to the consumption equity premium multiplied by a simple expression of $\{a_i\}$, the moving average coefficients that define $\{\mathfrak{C}_t\}$ as a function of $\{C_t\}$. For instance, for the 8, 12, 16 and 20 year filters used in the previous section the marginal cost of fluctuations equals approximately 0.38, 0.72, 077 and 0.88 times the consumption equity premium. We show that the consumption equity premium equals the covariance between the pricing kernel and the consumption growth rate, and we estimate this covariance by fitting a pricing kernel to average excess returns. We find that the results are similar to the ones obtained fitting a more general version of the pricing kernel in the previous section. Additionally, we explore the consequences of introducing more assets, namely CRSP decile portfolios, in our analysis.

We start by specializing the log-linear framework to the following:

$$s_{t+1} = s_t + \varepsilon_{t+1}$$

where $\varepsilon_{t+1}$ is multivariate normal i.i.d through time with mean zero. In this case, the pricing kernel can be written as:

$$\Delta m_{t+1} \equiv \log \frac{M_{t+1}}{M_t} = l_m \cdot \varepsilon_{t+1}.$$

Notice that implicit in this specification of the state space is the assumption that consumption and dividends of any asset included in the state space follow random walks, possibly with correlated innovations. Let $D_t$ be the dividend of any such asset and let $\Delta d_{t+1} \equiv \log \frac{D_{t+1}}{D_t}$. Under these assumption we have the following well known results.

**Proposition 5.3.** *If the log pricing kernel and the log dividends are random walks with homoskedastic innovations then interest rates, dividend-price ratios and expected returns of equities are all constant; moreover, expected returns of strips are all equal to the expected returns of the corresponding equity claims. Furthermore, assuming that the innovations are normal, the ratio of the expected return of a risky strip paying $D_{t+s}$ at date $t + s$ to the one period interest rate $R_f$ and the excess return for equity paying dividends equal to the process $\{D\}$*

are given by

$$\frac{E_t\left(R_{t+1}\left[\{D\}\right]\right)}{R_f} = \frac{E_t\left(R_{t+1}\left[D_{t+s}\right]\right)}{R_f} = \exp\left(-cov\left(m_{t+1}, d_{t+1}\right)\right) \quad \text{for all } s \geq 1.$$
(5.2a)

If $E\left(R_{t+1}^e\right)$ is a vector of excess expected returns then the loading vector is given by

$$\log E\left(R_{t+1}\right) = l_m \cdot \Sigma$$
(5.3)

where $\Sigma = \left[cov\left(\Delta s_{t+1,i}, \Delta d_{t+1,j}\right)\right]_{i,j}$ is the covariance matrix of the innovations in the growth rate of state $i$ with the innovations in the growth rate of the dividends of stock $j$.

Under the assumptions used in the previous proposition, dividend growth rates and total returns are perfectly correlated, since dividend-price ratios are constant. In our implementation, we use consumption and/or returns of broad indices as states. With consumption as the dividend, the previous proposition says that the multiplicative excess return of consumption equity equals minus the covariance of the kernel with consumption growth:

$$\frac{E\left(R_{t+1}\left[\{C_{t+s}\}_{s=1}^{\infty}\right]\right)}{R_f} = \exp\left(-cov\left(\Delta m_{t+1}, \Delta c_{t+1}\right)\right).$$

Now we present a proposition that links the multiplicative excess return of consumption equity to the moving average coefficients of the trend $\{\mathfrak{C}_t\}$.

**Proposition 5.4.** *Let the trend $\mathfrak{C}_t$ be defined as*

$$\log \mathfrak{C}_t = \tau\left(t\right) + \sum_{i=0}^{K} a_i \log C_{t-i}$$

*where $\tau\left(t\right)$ are constant chosen so that*

$$E_0\left(\mathfrak{C}_t\right) = E_0\left(C_t\right).$$

*Under the assumption of the log-linear system the ratio between the price of a risky strip paying $\mathfrak{C}_t$ and one paying $C_t$ is given by*

$$\frac{V_0\left[\mathfrak{C}_t\right]}{V_0\left[C_t\right]} = \exp\left[cov_0\left(m_t, \sum_{j=0}^{J} a_j c_{t-j}\right) - cov_0\left(m_t, c_t\right)\right].$$

*Furthermore, if the pricing kernel and consumption are random walks then for the strips for which $t > K$,*

$$\frac{V_0\left[\mathfrak{C}_t\right]}{V_0\left[C_t\right]} = \left(\frac{E\left(R_1\left[C_t\right]\right)}{R_f}\right)^{\left[1-a_0+1-(a_0+a_1)+\cdots 1-(a_0+a_1..+a_K)\right]}. \tag{5.4}$$

The role of the constants $\tau\left(t\right)$ is to ensure that $\mathfrak{C}_t$ has the same conditional expectation as $C_t$. This adjustment involves a Jensen's inequality correction, due to our assumption that the logarithm of $\mathfrak{C}_t$, as opposed to just its level, is a moving average of the logarithms of $C_t$. We show in the appendix that this adjustment is quantitatively negligible.

These propositions complement our earlier results. The first proposition explains how we identify the consumption equity premium by using excess return on securities. The second proposition explains how we use that information and the values of the moving average coefficients for measuring the marginal cost of fluctuations. In particular, recall that the marginal cost of economic fluctuations is given by the ratio of the prices of a claim paying trend consumption relative to a claim with paying consumption. The ratio of the prices of these long lived securities can be written as

$$\frac{V_0\left[\{\mathfrak{C}\}\right]}{V_0\left[\{C\}\right]} = \sum_{t=1}^{\infty} w_t\left[C_t\right]\frac{V_0\left[\mathfrak{C}_t\right]}{V_0\left[C_t\right]}$$

where $w_t\left[C_t\right] = \frac{V_0\left[C_t\right]}{V_0\left[\{C\}\right]}$. Thus, the marginal cost of economic fluctuations is given by a weighted average of the ratio of strip prices. The ratio of these strip prices, with the exception of the strips for the first $K$ securities, are independent of the date where they pay, and equal to the expression in (5.4). Therefore this expression is also approximately equal to the marginal cost

$$\omega^m \cong \log\left(1 + \omega^m\right) \cong \left(r_{ce} - r_f\right)\left[1 - a_0 + 1 - (a_0 + a_1) + \cdots 1 - (a_0 + a_1.. + a_K)\right], \tag{5.5}$$

where $\left(r_{ce} - r_f\right)$ stands for the difference between the net expected return of consumption equity and the net risk free interest rate. The second approximation is due to the first few strips for which $j \leq K$. As we discuss in the appendix, the approximation error is negligible for our applications.

Equation (5.5) has the following interpretation: $\left(r_{ce} - r_f\right)$ captures the risk premium associated with consumption; the factor $\left[1 - a_0 + \cdots\right]$ captures the variability of the deviations from trend. To understand how this factor works, consider the case of moving average coefficients $\{a_i\}$ that are positive and decreasing in $i$ and consider the following change: increase the values of $a_i's$ corresponding

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to long lags (large $i$) and decrease the ones corresponding to earlier lags (small $i$). This operation increases the factor and thus the marginal cost. This operation also makes the trend smoother, since it distributes the weights more evenly. Consequently, the deviations from trend are more volatile and thus a higher marginal cost makes sense.

In Table 3, we present computations for the expression (5.5)) representing the marginal cost. As states we use consumption and/or returns to various CRSP portfolios. The loading vector is fitted to the mean log excess returns of various CRSP return portfolios. The table also contains summary statistics, such as standard deviations and correlations of consumption growth rates and the market returns that show intuitively what factors are driving the estimates.

We would like to highlight three types of findings. First, estimates of the consumption equity premium lie between 0.22% and 1.66%, which is well below the premium for the market return which is above 7%. This is due to the relatively low consumption volatility and the correlation properties of consumption, returns and the states. Second, results for the marginal cost of consumption fluctuations are similar to our estimates obtained from the more general specification in the previous section. And third, adding additional equity portfolios such as the CRSP size decile portfolios, does not change much the results. This is due to the fact that once a portfolio has been included, additional portfolios have small extra power explaining the variance of the consumption growth rate.

In section (5.3), we have proposed an ad hoc adjustment for the one-sided filter that consists in scaling up the volatility of the cyclical component by the constant $\theta$. The next proposition shows how this adjustment affects the implied marginal cost.

**Proposition 5.5.** *Let the trend $\mathfrak{C}_t^*$ be defined as*

$$\log \mathfrak{C}_t^* = \tau^* (t) + (1 - \theta) \log C_t + \theta \log \mathfrak{C}_t$$
$$= \tau^* (t) + (1 - \theta) \log C_t + \theta \sum_{i=0}^{K} a_i \log C_{t-i}$$

*where the $\tau^* (t)$ are constant chosen so that*

$$E_0 \left( \mathfrak{C}_t^* \right) = E_0 \left( C_t \right).$$

*Under the assumption of the log-linear system the ratio between the price of a risky strip paying $\mathfrak{C}_t$ and one paying $C_t$ is given by*

$$\frac{V_0 \left[ \mathfrak{C}_t \right]}{V_0 \left[ C_t \right]} = \theta \exp \left[ cov_0 \left( m_t, \sum_{j=0}^{J} a_j c_{t-j} \right) - cov_0 \left( m_t, c_t \right) \right].$$

*Furthermore, if the pricing kernel and consumption are random walks then for the strips for which $t > K$,*

$$\frac{V_0\left[\mathfrak{C}_t\right]}{V_0\left[C_t\right]} = \left(\frac{E\left(R_1\left[C_t\right]\right)}{R_f}\right)^{\theta\left[1-a_0+1-(a_0+a_1)+\cdots 1-(a_0+a_1..+a_K)\right]}. \tag{5.6}$$

Given that the Jensen's inequality terms determining $\tau^*(t)$ are of second-order importance, the marginal cost with the adjusted trend is therefore approximately equal to original marginal cost scaled by $\theta$, that is,

$$\omega^{*m} \cong \theta \cdot \omega^m. \tag{5.7}$$

We end this section by illustrating the effect of a "phase shift" for the filter that defines the trend consumption. Given that we have defined the trend as a one-sided moving average, we have introduced a phase shift. The effect of such a phase shift is to make the marginal cost larger. The intuition is that a phase shift makes trend consumption more desirable since it becomes closer to the expected value. To be precise, we show that if a trend—defined by moving average coefficients $\{a_i\}$—is lagged by one period, then the cost of fluctuations goes up by the value of the consumption equity premium.

**Proposition 5.6.** *Let $a^K = \{a_i\}_{i=0}^K$ be the moving average coefficients that define the consumption trend, and let $m\left(a^K\right)$ be the factor that multiplies the consumption equity premium giving the corresponding marginal cost of fluctuations. Consider the moving average coefficients $a^{K+1}$ satisfying*

$$a_0^{K+1} = 0, \text{ and } a_{i+1}^{K+1} = a_i^K \text{ for } i = 1, 2, ..., K.$$

*Then $m\left(a^{K+1}\right) = m\left(a^K\right) + 1$.*

Denoting by $\omega^m(a)$ the marginal cost for a trend corresponding to moving average coefficients $a$, the two previous propositions imply that

$$\omega^m\left(a^{K+1}\right) - \omega^m\left(a^K\right) \cong r_{ce} - r_f.$$

This means that with a phase shift of a year, the cost of business cycles increase by an amount equal to the consumption equity premium.

## 6. Growth and Economic Fluctuations

In this section we analyze the relationship between growth and economic fluctuatations.. We consider this issue along two dimensions. First, we use our

framework to estimate the benefit from increasing growth. We find the benefit from an increase in the annual growth rate of 1% to be substantial. It is about 50% of lifetime consumption with cointegrated dividends and consumption, and even larger without cointegration. Second, we present a modified version of our framework that allows us to directly estimate the trade-off between long term growth and fluctuations. We find that reducing business cycle fluctuations is valued at less than 1 basis point increase in the per annum long term growth rate of consumption.

### 6.0.1. The benefits from increasing growth

We can use our definition of $\Omega$ to analyze the benefits of a permanent increase in the growth rate of the economy, as in Lucas (1987). For that we simply let $\{\mathfrak{C}\}$ be defined as

$$\mathfrak{C}_t \left( z^t \right) = [1 + x]^t C_t \left( z^t \right).$$

Assuming $x > 0$, this is a special case of our framework and all the results derived in section 3 for the general case directly apply.

The marginal cost can be written as

$$1 + \Omega' \left( 0 \right) = \frac{\sum_{t=1}^{\infty} V_0 \left[ (1+x)^t C_t \right]}{\sum_{t=1}^{\infty} V_0 \left[ C_t \right]} = \frac{\sum_{t=1}^{\infty} (1+x)^t V_0 \left[ C_t \right]}{\sum_{t=1}^{\infty} V_0 \left[ C_t \right]} = \sum_{t=1}^{\infty} (1+x)^t w_0 \left[ C_t \right],$$

where the weights $w_0$,

$$w_0 \left[ C_t \right] = \frac{V_0 \left[ C_t \right]}{\sum_{j=1}^{\infty} V_0 \left[ C_j \right]},$$

are the same as in the equity premium and the cost of consumption uncertainty. Based on our estimated pricing kernel Table 3 reports the (marginal) benefits of increasing the growth rate by 1% that is for $x = 0.01$. The estimates are very high, about 50% of lifetime consumption with cointegrated dividends and consumption, and higher without cointegration. These numbers are much larger than the one computed by Lucas, which is 17%.

To gain some intuition about these results we consider again the random walks specification, where

$$\Omega' \left( 0 \right) = \sum_{t=1}^{\infty} (1+x)^t w_0 \left[ C_t \right] - 1 = \frac{\gamma \left( 1 + r_{ce} \right)}{1 + r_{ce} - (1 + g_c) \left( 1 + x \right)} \cong \frac{x}{r_{ce} - g_c - x}$$

with $g_c = E \left[ \frac{C_{t+1}}{C_t} \right] - 1$ is the net rate of consumption growth.[14] In the previous section, we estimate $r_{ce} - r_f$ to be less than 1.5%. Thus if the risk free rate is $r_f =$

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[14] Note that lognormality is not required for this derivation.

2.5% and the growth rate of consumption $g_c = 2\%$ then $\Omega'(0) = 0.01/(0.015 + 0.025 - 0.02 - 0.01)$ equals 100%; with a smaller consumption equity premium the marginal cost further increases.[15]

## 6.1. Trade-off between fluctuations and growth

In order to directly measure the trade-off between fluctuations and growth we modify here our framework. Instead of measuring the cost of fluctuations by the uniform compensation $\Omega$, the compensation for fluctuations is expressed in terms of additional long term growth. We find that eliminating business cycle fluctuations is worth less than 1 basis point in additional growth.[16] Eliminating all consumption uncertainty is valued at 1 percent in additional growth or more; or equivalently, the cost of all consumption uncertainty represents about half the value of long-term historical growth.

Define $\lambda(\alpha)$ as the solution to

$$U\left(\left\{[1 + \lambda(\alpha)]^t C_t\right\}\right) = U\left((1 - \alpha)\{C\} + \alpha\{\mathfrak{C}\}\right) \tag{6.1}$$

where $\left\{[1 + \lambda(\alpha)]^t C_t\right\}$ is the process which has value $[1 + \lambda(\alpha)]^t C_t\left(z^t\right)$ at time $t$, event $z^t$. Notice that by definition $\lambda(0) = 0$. Assuming that $U(\cdot)$ is differentiable, differentiating both sides of (6.1) and evaluating them at $\alpha = 0$ we obtain

$$\lambda'(0) \sum_{t \geq 1} \sum_{z^t \in Z^t} U_{z^t}(\{C\})\ C_t\left(z^t\right)\ t = \sum_{t \geq 1} \sum_{z^t \in Z^t} U_{z^t}(\{C\})\ \left[\mathfrak{C}_t\left(z^t\right) - C_t\left(z^t\right)\right].$$

Rearranging terms, we can write this expression as

$$\lambda'(0) = \frac{\omega^m}{\sum_{t=1}^\infty t\ w_0\ [C_t]}, \tag{6.2}$$

where $w_0$ are the strip weights as in 4.1. We call $\sum_{t=1}^\infty t\ w_0\ [C_t]$ the duration of the consumption equity, by analogy with the definition of duration in the context of coupon bonds. Thus the trade-off between growth and fluctuations is given by the ratio of the marginal cost of fluctuations to the duration of the consumption equity. Before presenting estimates of $\lambda'(0)$, we notice that $\lambda$ is concave for small $\alpha$. In this case $\lambda'(0) \geq \lambda(\alpha)/\alpha$ for small $\alpha$, that is, the marginal cost is larger than the average cost.

---

[15] We select this high value for the risk free rate $r_f$ corresponding to the long term yield since in the random walk case the term structure is flat.

[16] We thank John Cochrane for suggesting this extension.

**Proposition 6.1.** *Let $U$ be increasing, concave and twice differentiable. Then $\lambda''(0) \leq 0$.*

Using our estimated pricing kernels in Table 2A to 2D we can see the duration of consumption equity to be about 50 years; with cointegration these numbers are usually smaller, without cointegration there are larger in many cases. These durations can be combined with the estimates of the marginal cost of fluctuations to arrive at the trade-off between growth and cyclical variations. Based on this, eliminating business cycle fluctuations is worth less than 1 basis point in additional growth. Eliminating all consumption uncertainty is valued at 1 percent in additional growth or more; or equivalently, the cost of all consumption uncertainty represents about half the value of long-term historical growth.

Under the random walk assumption for consumption and the pricing kernel the duration of the consumption equity also has a particularly simple form

$$\sum_{t=1}^{\infty} t \, w_0 \, [C_t] = \frac{1 + r_{ce}}{r_{ce} - g_c}.$$

To get some quantitative feel, we plug in again the same estimates as in the previous subsection. We consider $r_{ce} - r_f$ at 1.5%, the risk free rate $r_f = 2.5\%$ and the growth rate of consumption $g_c = 2\%$. In this case, the duration of the consumption equity, $\sum_{t=1}^{\infty} t \, w_0 \, [C_t]$, is about 50, that is, $\frac{1+r_{ce}}{r_{ce}-g_c} = 1.04/(0.04 - 0.02) = 52$. Thus, with the marginal cost of business cycle fluctuations for the 8-year filter usually below 50 basis points in terms of the uniform compensation $\Omega'(0)$, eliminating business cycles is worth less than 1 basis point in terms of additional long-term growth $\lambda'(0)$.

## 7. Summary and Conclusions

In this paper we have measured the cost of business cycle fluctuations using asset prices. We use a new approach that focuses on the *marginal* cost of consumption fluctuations, because asset prices correspond to marginal valuations of market participants. We have shown that the marginal cost of consumption fluctuations corresponds to the ratio of two asset prices. We establish that the marginal cost of consumption fluctuations provides an upper bound to the benefits of reducing fluctuations completely. Our analysis shows that the equity premium and the cost of consumption uncertainty are related, but clearly distinct, conceptually and quantitatively. The steepness of the term structure and the persistence of the shocks are two of the features that make the equity premium differ from the marginal cost of consumption uncertainty.

We have estimated a pricing kernel, as a way to interpolate from existing asset prices the prices of the assets that measure the marginal cost of consumption fluctuations. We use a non-stationary pricing kernel. This is motivated by a bound that we have developed, similar in spirit to the Hansen-Jagannathan bound. This bound shows that, to be consistent with the term structure, the pricing kernel cannot be stationary. Our quantitative analysis highlights that addressing more specifically the issue of what business cycle fluctuations correspond to is of first-order importance. Specifically, we estimated the marginal cost of *all* consumption uncertainty to be very large. Nevertheless, when we define business cycles as fluctuations with frequencies less than or equal to 8 years we estimate the marginal cost of business cycles fluctuations to about one half of a percent of lifetime consumption or less. Alternatively, the benefits of eliminating this type of fluctuations are smaller than the benefits of a permanent increase of the growth rate of consumption of 1 basis points per year.

In this paper we measure the cost of fluctuations for a representative agent, i.e., an agent whose consumption is proportional to aggregate consumption and that participates in frictionless asset markets. Tractable general equilibrium models that incorporate heterogeneous agents and asset market frictions have been introduced to study, among other things, the determinants of equity premia. We leave the investigation of the effects of heterogeneity and asset market frictions in the measurement of the cost of fluctuations for future research.

## Data Appendix

The data used in this paper is annual, covering the period 1889-1997 and comes mainly from two sources: Shiller (1998) and Ibbotson Associates. The breakdown by time series is given hereafter. For additional details we refer the reader to the original sources.

Consumption is real per capita consumption for nondurables and services from Shiller. We updated this series for the period after 1985 with NIPA data and population data from the US. Bureau of the Census.

Stock prices and corresponding dividends are the Standard and Poor Composite Stock Price Indexes from Shiller.

Short term rates are one-year returns based on 6-month commercial paper returns from Shiller, adjusted for a default premium for the period before 1926; after 1926, they are based on monthly holding periods for T-Bills from Ibbotson Associates.

The long-short yield spreads for government bonds are from Campbell (1996) for the period before 1926; after 1926, from Ibbotson Associates.

All series that have not been originally deflated were deflated by the producer price index from Shiller for the period before 1926; after 1926 by the CPI from Ibbotson Associates.

## Appendix

**Proof of Proposition 3.1.** Regarding $C = \{C\}$ as a vector on $R^\infty$ let $\frac{\partial U(C)}{\partial C}$ and $\frac{\partial^2 U(C)}{\partial C \partial C'}$ be the first and second derivatives evaluated at $C$. Define $C_\alpha = (1-\alpha)\,\mathfrak{C} + \alpha C$. Then twice differentiating with respect to $\alpha$ the equation defining $\Omega$,

$$\frac{\partial^2 \Omega(\alpha)}{\partial \alpha \partial \alpha} \frac{\partial U([1 + \Omega(\alpha)]C)}{\partial C} C + \left(\frac{\partial \Omega(\alpha)}{\partial \alpha} C\right)' \frac{\partial^2 U([1 + \Omega(\alpha)]C)}{\partial C \partial C'} \left(\frac{\partial \Omega(\alpha)}{\partial \alpha} C\right)$$

$$= (\mathfrak{C} - C)' \frac{\partial^2 U(C_\alpha)}{\partial C \partial C'} (\mathfrak{C} - C)$$

Taking $\alpha \downarrow 0$ since $C_0 = [1 + \Omega(0)]C = C$,

$$\frac{\partial^2 \Omega(0)}{\partial \alpha \partial \alpha} \frac{\partial U(C)}{\partial C} C = \left(\mathfrak{C} - [1 + \frac{\partial \Omega(0)}{\partial \alpha}]C\right)' \frac{\partial^2 U(C)}{\partial C \partial C'} \left(\mathfrak{C} - [1 + \frac{\partial \Omega(0)}{\partial \alpha}]C\right) \leq 0$$

where the inequality follows from the concavity of $U$. Since $U$ is increasing $\frac{\partial^2 \Omega(0)}{\partial \alpha \partial \alpha} \leq 0$. By concavity $\Omega(\alpha) \leq \Omega(0) + \Omega'(0)\alpha$ for small $\alpha$. $\blacksquare$

**Proof of Proposition 3.2.** If $U$ is increasing and concave in $\{C\}$ there must exist a homogenous of degree one, positive and quasi-concave utility $v$ that satisfies

$$U(\{C\}) = \frac{[v(\{C\})]^{1-\gamma}}{1 - \gamma}.$$

First, we show that $\Omega(\alpha)$ is concave in $\alpha$. By homogeneity of $U$

$$(1 + \Omega(\alpha))^{1-\gamma} \frac{[v(\{C\})]^{1-\gamma}}{1 - \gamma} = \frac{[v((1-\alpha)\{C\} + \alpha\{\mathfrak{C}\})]^{1-\gamma}}{1 - \gamma}$$

thus, after multiplying by $(1 - \gamma)$, taking the $1/(1 - \gamma)$ power and dividing by $v(\{C\})$ in both sides, we obtain

$$1 + \Omega(\alpha) = \frac{v((1-\alpha)\{C\} + \alpha\{\mathfrak{C}\})}{v(\{C\})}.$$

Since $v(\cdot)$ is positive, quasi-concave and homogenous of degree one, it is concave. With $(1-\alpha)\{C\} + \alpha\{\mathfrak{C}\}$ linear in $\alpha$, $v(.)$ it is also concave in $\alpha$, thus $\Omega(\alpha)$ is concave. Now we use the concavity to obtain the desired inequalities,

$$\Omega(1) = \Omega(0) + \int_0^1 \Omega'(\alpha)d\alpha \leq \Omega'(0),$$

where the inequality uses $\Omega(0) = 0$, the concavity of $\Omega$ and that $\alpha \leq 1$. $\blacksquare$

**Proof of Proposition 3.3.** From the concavity of $u$ and the definition of $\mathfrak{C}$

$$
\begin{aligned}
& U\left((1-\alpha)\{C\} + \alpha\{\mathfrak{C}\}\right) \\
={} & E\left[u\left((1-\alpha)C_1 + \alpha\mathfrak{C}_1, (1-\alpha)C_2 + \alpha\mathfrak{C}_2, ..., (1-\alpha)C_t + \alpha\mathfrak{C}_t, ...\right)\right] \\
\leq{} & u\left(\mathfrak{C}_1, \mathfrak{C}_2, ..., \mathfrak{C}_t, ...\right) = U\left(\{\mathfrak{C}\}\right)
\end{aligned}
$$

for any $\alpha$. By the definition of $\Omega$ and the monotonicity of $U$, $\Omega$ has to reach a maximum on $\alpha = 1$ and thus $\Omega'(1) = 0$ and $\Omega(\alpha)$ is concave for $\alpha$ close to 1. $\blacksquare$

**Proof of Proposition 3.4.** To simplify the notation denote $\Omega'(0, \sigma)$ by $\Omega'(\sigma)$ and $\Omega(0, \sigma)$ by $\Omega(\sigma)$. Let's first consider the total cost $\Omega(\sigma)$. We use Taylor expansions for the right and left hand side of the definition of the total cost to find and expression for $\Omega(\sigma)$. The utility $u(\bar{C})$ can be approximated around $\bar{C}(1 + \Omega(\sigma))$ as

$$
u\left(\bar{C}\right) = u\left(\bar{C}(1 + \Omega(\sigma))\right) - u'\left(\bar{C}_\sigma\right)\bar{C}\Omega(\sigma)
$$

where $\bar{C}_\sigma \in \left[\bar{C}, \bar{C}(1 + \Omega(\sigma))\right]$ and $\lim_{\sigma \to 0} \bar{C}_\sigma = \bar{C}$. The utility $u\left(\bar{C}(1 + \sigma\varepsilon)(1 + \Omega(\sigma))\right)$ can be approximated around $\bar{C}(1 + \sigma\varepsilon)$ for each $\varepsilon$ as

$$
\begin{aligned}
& u\left(\bar{C}(1 + \sigma\varepsilon)(1 + \Omega(\sigma))\right) \\
={} & u\left(\bar{C}(1 + \Omega(\sigma))\right) + u'\left(\bar{C}(1 + \Omega(\sigma))\right)\ (1 + \Omega(\sigma))\bar{C}\sigma\varepsilon \\
& + \frac{1}{2}u''\left(\bar{C}(1 + \Omega(\sigma))\right)\ \left((1 + \Omega(\sigma))\bar{C}\sigma\varepsilon\right)^2 \\
& + \frac{1}{6}u'''\left(\bar{C}(1 + \Omega(\sigma))\right)\ \left((1 + \Omega(\sigma))\bar{C}\sigma\varepsilon\right)^3 + \frac{1}{24}u''''\left(\bar{C}_\sigma(\varepsilon)\right)\ \left(\bar{C}\sigma\varepsilon\right)^4
\end{aligned}
$$

where $\bar{C}_\sigma(\varepsilon) \in \left[\bar{C}(1 + \sigma\varepsilon), \ \bar{C}(1 + \sigma\varepsilon)(1 + \Omega(\sigma))\right]$ for positive $\varepsilon$ and in $\left[\bar{C}(1 + \sigma\varepsilon)(1 + \Omega(\sigma)), \ \bar{C}(1 + \sigma\varepsilon)\right]$ for negative one, and $\lim_{\sigma \to 0} \bar{C}_\sigma(\varepsilon) = \bar{C}$. Then taking expected values,

$$
\begin{aligned}
& E\left[u\left(\bar{C}(1 + \sigma\varepsilon)(1 + \Omega(\sigma))\right)\right] \\
={} & u\left(\bar{C}(1 + \Omega(\sigma))\right) + \frac{1}{2}u''\left(\bar{C}(1 + \Omega(\sigma))\right)\ E\left[(1 + \Omega(\sigma))\bar{C}\sigma\varepsilon\right]^2 \\
& + \frac{1}{6}u'''\left(\bar{C}(1 + \Omega(\sigma))\right)\ E\left[(1 + \Omega(\sigma))\bar{C}\sigma\varepsilon\right]^3 + o\left(\sigma^3\right).
\end{aligned}
$$

Using the definition of $\Omega(\sigma)$,

$$
\begin{aligned}
& -u'\left(\bar{C}_\sigma\right)\bar{C}\Omega(\sigma) && (7.1) \\
={} & \frac{1}{2}u''\left(\bar{C}(1 + \Omega(\sigma))\right)\ E\left[(1 + \Omega(\sigma))\bar{C}\sigma\varepsilon\right]^2 \\
& + \frac{1}{6}u'''\left(\bar{C}(1 + \Omega(\sigma))\right)\ E\left[(1 + \Omega(\sigma))\bar{C}\sigma\varepsilon\right]^3 + o\left(\sigma^3\right)
\end{aligned}
$$

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or

$$\Omega\left(\sigma\right) = -\frac{\sigma^2}{2}\frac{\bar{C}u''\left(\bar{C}\left(1+\Omega\left(\sigma\right)\right)\right)\left(1+\Omega\left(\sigma\right)\right)^2}{u'\left(\bar{C}_\sigma\right)}E\varepsilon^2$$
$$-\frac{\sigma^3}{6}\frac{\bar{C}^2u'''\left(\bar{C}\left(1+\Omega\left(\sigma\right)\right)\right)\left(1+\Omega\left(\sigma\right)\right)^3}{u'\left(\bar{C}_\sigma\right)}E\varepsilon^3 + o\left(\sigma^3\right)$$

As an intermediate step we first show that

$$\Omega\left(\sigma\right) = \frac{\sigma^2}{2}\frac{\bar{C}u''\left(\bar{C}\right)}{u'\left(\bar{C}\right)}E\varepsilon^2 + o\left(\sigma^2\right) \equiv k\sigma^2 + o\left(\sigma^2\right)$$

which follows since $\Omega\left(\sigma\right) \to 0$,and $\bar{C}_\sigma \to \bar{C}$ as $\sigma \to 0$. Now we show that

$$u'\left(\bar{C}_\sigma\right)\bar{C}\Omega\left(\sigma\right) = u'\left(\bar{C}\right)\bar{C}\Omega\left(\sigma\right) + o_1\left(\sigma^3\right)$$

which follows from

$$u'\left(\bar{C}_\sigma\right)\bar{C}\Omega\left(\sigma\right) = u'\left(\bar{C}\right)\bar{C}\Omega\left(\sigma\right) + u''\left(C\left(\sigma\right)\right)\bar{C}\Omega\left(\sigma\right)\left(\bar{C}_\sigma - \bar{C}\right)$$

for some $C\left(\sigma\right) \in \left[\bar{C},\bar{C}_\sigma\right]$. Recall that by definition of $\bar{C}_\sigma$

$$0 \leq \left(\bar{C}_\sigma - \bar{C}\right) \leq \bar{C}\Omega\left(\sigma\right)$$

so

$$\bar{C}\Omega\left(\sigma\right)\left(\bar{C}_\sigma - \bar{C}\right) \leq \left(\bar{C}\Omega\left(\sigma\right)\right)^2 = \sigma^4 k + o\left(\sigma^3\right)$$

for some constant $k$. Also we show that that

$$u''\left(\bar{C}\left(1+\Omega\left(\sigma\right)\right)\right)\left(1+\Omega\left(\sigma\right)\right)^2$$
$$= u''\left(\bar{C}\right) + o\left(\sigma^1\right),$$

which follows from

$$u''\left(\bar{C}\left(1+\Omega\left(\sigma\right)\right)\right)\left(1+\Omega\left(\sigma\right)\right)^2$$
$$= u''\left(\bar{C}\left(1+\Omega\left(\sigma\right)\right)\right)\left(1 + 2k\sigma^2 + o\left(\sigma^2\right) + o\left(\sigma^4\right) + k^2\sigma^4\right)$$

and from

$$u''\left(\bar{C}\left(1+\Omega\left(\sigma\right)\right)\right) = u''\left(\bar{C}\right) + u'''\left(\bar{C}\left(\sigma\right)\right)\bar{C}\Omega\left(\sigma\right)$$
$$= u''\left(\bar{C}\right) + u'''\left(\bar{C}\left(\sigma\right)\right)\left(k\sigma^2 + o\left(\sigma^2\right)\right).$$

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Using these last two results we can write (7.1) as

$$-u'\left(\bar{C}\right)\bar{C}\Omega\left(\sigma\right)+o\left(\sigma^{3}\right)$$

$$=\quad\frac{1}{2}u''\left(\bar{C}\right)\ E\left[\bar{C}\sigma\varepsilon\right]^{2}+\frac{\sigma^{2}o\left(\sigma\right)}{2}E\left[\bar{C}\varepsilon\right]^{2}+\frac{1}{6}u'''\left(\bar{C}\left(1+\Omega\left(\sigma\right)\right)\right)\ E\left[\bar{C}\sigma\varepsilon\right]^{3}+o\left(\sigma^{3}\right),$$

dividing by $u'\left(\bar{C}\right)$ in both sides we obtain

$$\Omega\left(\sigma\right)=-\frac{\sigma^{2}}{2}\frac{\bar{C}u''\left(\bar{C}\right)}{u'\left(\bar{C}\right)}E\varepsilon^{2}-\frac{\sigma^{3}}{6}\frac{\bar{C}^{2}u'''\left(\bar{C}\right)}{u'\left(\bar{C}_{\sigma}\right)}E\varepsilon^{3}+o\left(\sigma^{3}\right).$$

Now let's consider the marginal cost $\Omega'\left(\sigma\right)$

$$\Omega'\left(\sigma\right)=-\frac{E\left[u'\left(\bar{C}\left(1+\sigma\varepsilon\right)\right)\ \bar{C}\sigma\varepsilon\right]}{E\left[u'\left(\bar{C}\left(1+\sigma\varepsilon\right)\right)\bar{C}\left(1+\sigma\varepsilon\right)\right]}.$$

Let's approximate $u'\left(\bar{C}\left(1+\sigma\varepsilon\right)\right)$ around $\bar{C}$ for each $\varepsilon$ to obtain

$$\begin{aligned}u'\left(\bar{C}\left(1+\sigma\varepsilon\right)\right)&=&u'\left(\bar{C}\right)+u''\left(\bar{C}\right)\ \bar{C}\sigma\varepsilon\\&&+\frac{1}{2}u'''\left(\bar{C}\right)\ \left(\bar{C}\sigma\varepsilon\right)^{2}+\frac{1}{6}u''''\left(\bar{C}_{\sigma}\left(\varepsilon\right)\right)\ \left(\bar{C}\sigma\varepsilon\right)^{3}\end{aligned}$$

where $\bar{C}_{\sigma}\left(\varepsilon\right)\in\left[\bar{C},\bar{C}\left(1+\sigma\varepsilon\right)\right]$ for positive $\varepsilon$ and $\left[\bar{C}\left(1+\sigma\varepsilon\right),\bar{C}\right]$ for negative $\varepsilon$ and where $\lim_{\sigma\to0}\bar{C}_{\sigma}\left(\varepsilon\right)=\bar{C}$. Then

$$\begin{aligned}&&E\left[u'\left(\bar{C}\left(1+\sigma\varepsilon\right)\right)\ \bar{C}\sigma\varepsilon\right]\\&=&u''\left(\bar{C}\right)\ \left(\bar{C}\sigma\right)^{2}E\varepsilon^{2}+\frac{1}{2}u'''\left(\bar{C}\right)\ \left(\bar{C}\sigma\right)^{3}E\varepsilon^{3}+o\left(\sigma^{3}\right)\end{aligned}$$

Then the marginal cost can be expressed as

$$\Omega'\left(\sigma\right)=-\frac{u''\left(\bar{C}\right)\ \left(\bar{C}\sigma\right)^{2}E\varepsilon^{2}+\frac{1}{2}u'''\left(\bar{C}\right)\ \left(\bar{C}\sigma\right)^{3}E\varepsilon^{3}+o\left(\sigma^{3}\right)}{E\left[u'\left(\bar{C}\left(1+\sigma\varepsilon\right)\right)\bar{C}\left(1+\sigma\varepsilon\right)\right]}.\qquad(7.2)$$

As an intermediate step we show that

$$\Omega'\left(\sigma\right)=-\frac{\bar{C}u''\left(\bar{C}\right)\ \sigma^{2}E\varepsilon^{2}}{u'\left(\bar{C}\right)}+o\left(\sigma^{2}\right)=q\sigma^{2}+o\left(\sigma^{2}\right)$$

for a constant $q$, which follows since

$$E\left[u'\left(\bar{C}\left(1+\sigma\varepsilon\right)\right)\bar{C}\left(1+\sigma\varepsilon\right)\right]=u'\left(\bar{C}\right)\bar{C}+o\left(1\right)$$

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and thus

$$\Omega'(\sigma) = -\frac{u''(\bar{C})\ (\bar{C}\sigma)^2 E\varepsilon^2 + \frac{1}{2}u'''(\bar{C})\ (\bar{C}\sigma)^3 E\varepsilon^3 + o(\sigma^3)}{u'(\bar{C})\ \bar{C} + o(1)}.$$

Now we argue that

$$E\left[u'\left(\bar{C}(1+\sigma\varepsilon)\right)\bar{C}(1+\sigma\varepsilon)\right] = u'(\bar{C})\ \bar{C} + p\sigma^2 + o(\sigma^2)$$

for some constant $p$, which follows by using a first order approximation on $u'\left(\bar{C}(1+\sigma\varepsilon)\right)$ around $\sigma = 0$

$$u'\left(\bar{C}(1+\sigma\varepsilon)\right) = u'(\bar{C}) + u''(\bar{C})\ \sigma\varepsilon + o(\sigma)$$

and taking expected values,

$$\begin{aligned}
E\left[u'\left(\bar{C}(1+\sigma\varepsilon)\right)\bar{C}(1+\sigma\varepsilon)\right] &= u'(\bar{C})\ \bar{C} + u''(\bar{C})\ \sigma\bar{C}E\varepsilon(1+\sigma\varepsilon) + o(\sigma^2) \\
&= u'(\bar{C})\ \bar{C} + u''(\bar{C})\ \sigma^2\bar{C}^2 E\varepsilon^2 + o(\sigma^2) \\
&= u'(\bar{C})\ \bar{C} + p\sigma^2 + o(\sigma^2).
\end{aligned}$$

Now consider the expression

$$\begin{aligned}
& \frac{E\left[u'\left(\bar{C}(1+\sigma\varepsilon)\right)\bar{C}(1+\sigma\varepsilon)\right]}{u'(\bar{C})\ \bar{C}}\Omega'(\sigma) \\
=\ & \frac{\left[u'(\bar{C})\ \bar{C} + p\sigma^2 + o(\sigma^2)\right]}{u'(\bar{C})\ \bar{C}}\Omega'(\sigma) \\
=\ & \Omega'(\sigma) + \frac{pq\sigma^4}{u'(\bar{C})\ \bar{C}} + \frac{o(\sigma^2)\,o(\sigma^2)}{u'(\bar{C})\ \bar{C}} \\
=\ & \Omega'(\sigma) + o(\sigma^3).
\end{aligned}$$

Then, replacing these expressions in (7.2), we get

$$\Omega'(\sigma) + o(\sigma^3) = -\frac{u''(\bar{C})\ (\bar{C}\sigma)^2 E\varepsilon^2 + \frac{1}{2}u'''(\bar{C})\ (\bar{C}\sigma)^3 E\varepsilon^3}{u'(\bar{C})\ \bar{C}} + o(\sigma^3)$$

which finishes the proof.

**Proof of Proposition 3.6.** The core of this proof is very similar to the one period case, so we only sketch this case. To simplify the notation denote $\Omega'(0,\sigma)$ by $\Omega'(\sigma)$ and $\Omega(1,\sigma)$ by $\Omega(\sigma)$. Let's first consider the total cost $\Omega(\sigma)$ defined as

$$\sum_{t=1}^{T}\beta^t E_0\left[u\left(\bar{C}_t(1+\sigma\varepsilon_t)(1+\Omega(\sigma))\right)\right] = \sum_{t=1}^{T}\beta^t u(\bar{C}_t)$$

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Using exactly the same arguments than the one period case for each period $t$ we can write $u\left(\bar{C}_t\right)$ as

$$u\left(\bar{C}_t\right) = u\left(\bar{C}_t\left(1 + \Omega\left(\sigma\right)\right)\right) - u'\left(\bar{C}_{t,\sigma}\right)\bar{C}\Omega\left(\sigma\right)$$

where $\lim_{\sigma \to 0} \bar{C}_{t,\sigma} = \bar{C}_t$ for all $t$. Also for each $t$ we can write $E_0\left[u\left(C_t\left(1 + \Omega\left(\sigma\right)\right)\right)\right]$ as

$$
\begin{aligned}
&E_0\left[u\left(\bar{C}_t\left(1 + \sigma\varepsilon_t\right)\left(1 + \Omega\left(\sigma\right)\right)\right)\right] \\
={}& u\left(\bar{C}_t\left(1 + \Omega\left(\sigma\right)\right)\right) + \frac{1}{2}u''\left(\bar{C}_t\left(1 + \Omega\left(\sigma\right)\right)\right)\ E_0\left[\bar{C}_t\sigma\varepsilon_t\right]^2 \\
&+\frac{1}{6}u'''\left(\bar{C}_t\left(1 + \Omega\left(\sigma\right)\right)\right)\ E_0\left[\bar{C}_t\sigma\varepsilon_t\right]^3 + o_t\left(\sigma^3\right)
\end{aligned}
$$

Replacing the expansions into the definition of $\Omega\left(\sigma\right)$,

$$
\begin{aligned}
&\sum_{t=1}^{T}\left\{u\left(\bar{C}_t\left(1 + \Omega\left(\sigma\right)\right)\right) - \Omega\left(\sigma\right)u'\left(\bar{C}_{t,\sigma}\right)\bar{C}_t\right\} \\
={}& \sum_{t=1}^{T}\beta^t\left\{\begin{array}{c} u\left(\bar{C}_t\left(1 + \Omega\left(\sigma\right)\right)\right) + \frac{1}{2}u''\left(\bar{C}_t\left(1 + \Omega\left(\sigma\right)\right)\right)\ E_0\left[\bar{C}_t\sigma\varepsilon_t\right]^2 \\ +\frac{1}{6}u'''\left(\bar{C}_t\left(1 + \Omega\left(\sigma\right)\right)\right)\ E_0\left[\bar{C}_t\sigma\varepsilon_t\right]^3 + o_t\left(\sigma^3\right) \end{array}\right\}
\end{aligned}
$$

and simplifying

$$
\begin{aligned}
&\Omega\left(\sigma\right)\sum_{t=1}^{T}\beta^t u'\left(\bar{C}_{t,\sigma}\right)\bar{C}_t \\
={}& \sum_{t=1}^{T}\beta^t\left\{\begin{array}{c} -\frac{1}{2}u''\left(\bar{C}_t\left(1 + \Omega\left(\sigma\right)\right)\right)\ E_0\left[\bar{C}_t\sigma\varepsilon_t\right]^2 \\ -\frac{1}{6}u'''\left(\bar{C}_t\left(1 + \Omega\left(\sigma\right)\right)\right)\ E_0\left[\bar{C}_t\sigma\varepsilon_t\right]^3 \end{array}\right\} + o\left(\sigma^3\right)
\end{aligned}
$$

Defining

$$w_t\left(\sigma\right) = \frac{\beta^t u'\left(\bar{C}_t\right)}{\sum_{j=1}^{T}\beta^t u'\left(\bar{C}_{j,\sigma}\right)\bar{C}_j}$$

we can write where $w_t\left(\sigma\right) \to \bar{w}_t$. By following a two step procedure similar to the one in the proof for the one period case we can write $\Omega\left(\sigma\right)$ as

$$\Omega\left(\sigma\right) = \sum_{t=1}^{T}w_t\left(\sigma\right)\left\{\begin{array}{c} -\frac{1}{2}\frac{\bar{C}_t u''\left(\bar{C}_t\left(1 + \Omega\left(\sigma\right)\right)\right)}{u'\left(\bar{C}_{t,\sigma}\right)}\sigma^2 E_0\left[\varepsilon_t\right]^2 \\ -\frac{1}{6}\frac{\bar{C}_t^2 u'''\left(\bar{C}_t\left(1 + \Omega\left(\sigma\right)\right)\right)}{u'\left(\bar{C}_{t,\sigma}\right)}\sigma^3 E_0\left[\varepsilon_t\right]^3 \end{array}\right\} + o\left(\sigma^3\right)$$

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and using that as $\sigma \to 0$, $\Omega(\sigma) \to 0$, $w_t(\sigma) \to \bar{w}_t$ and $\bar{C}_{t,\sigma} \to \bar{C}_t$ then

$$\Omega(\sigma) \sum_{t=1}^{T} \beta^t u'(\bar{C}_t)$$

$$= \sum_{t=1}^{T} \left\{ -\frac{1}{2}\bar{C}_t u''(\bar{C}_t) \sigma^2 E_0 [\varepsilon_t]^2 - \frac{1}{6}\bar{C}_t^2 u'''(\bar{C}_t) \sigma^3 E_0 [\varepsilon_t]^3 \right\} + o(\sigma^3)$$

or

$$\Omega(\sigma) = \sum_{t=1}^{T} \bar{w}_t \left\{ \begin{array}{c} -\frac{1}{2}\frac{\bar{C}_t u''(\bar{C}_t)}{u'(\bar{C}_t)}\sigma^2 E_0 [\varepsilon_t]^2 \\ -\frac{1}{6}\frac{\bar{C}_t^2 u'''(\bar{C}_t)}{u'(\bar{C}_t)}\sigma^3 E_0 [\varepsilon_t]^3 \end{array} \right\} + o(\sigma^3).$$

Now let's consider the marginal cost. Notice that we can write

$$\Omega'(\sigma) = \frac{\sum_{t=1}^{T} \beta^t E\left[u'(C_t)\left(\bar{C}_t - C_t\right)\right]}{\sum_{t=1}^{T} \beta^t E\left[u'(C_t) C_t\right]} = \sum_{t=1}^{T} a_t(\sigma) \frac{E_0\left[u'(C_t)\left(C_t - \bar{C}\right)\right]}{u'(\bar{C}_t) \bar{C}_t}$$

where $a_t$ are defined as

$$a_t(\sigma) = \frac{\beta^t E_0\left[u'(C_t) C_t\right]}{\sum_{j=1}^{T} \beta^j E_0\left[u'(C_j) C_j\right]}$$

For future reference note that $\lim_{\sigma \to 0} a_t(\sigma) \to \bar{w}_t$. For each $t$ using the the same expansions than in the one period case we can write $E_0\left[u'(C_t)\left(\bar{C}_t - C_t\right)\right]$ as

$$-E_0\left[u'\left(\bar{C}_t(1+\sigma\varepsilon_t)\right) \bar{C}\sigma\varepsilon_t\right]$$

$$= -u''(\bar{C}_t) \left(\bar{C}_t\sigma\right)^2 E_0\varepsilon_t^2 - \frac{1}{2}u'''(\bar{C}_t) \left(\bar{C}_t\sigma\right)^3 E_0\varepsilon_t^3 + o_t(\sigma^3)$$

Then

$$\Omega'(\sigma) = \sum_{t=1}^{T} a_t(\sigma) \left\{ \frac{-u''(\bar{C}_t) \left(\bar{C}_t\sigma\right)^2 E_0\varepsilon_t^2 - \frac{1}{2}u'''(\bar{C}_t) \left(\bar{C}_t\sigma\right)^3 E_0\varepsilon_t^3 + o_t(\sigma^3)}{u'(\bar{C}_t) \bar{C}_t} \right\}.$$

Using a two step procedure analogous to the one used in the proof for the one period case we show that

$$\Omega'(\sigma) = \sum_{t=1}^{T} \bar{w}_t \left\{ -\frac{\bar{C}_t u''(\bar{C}_t)}{u'(\bar{C}_t)}\sigma^2 E_0\varepsilon_t^2 - \frac{1}{2}\frac{\bar{C}_t^2 u'''(\bar{C}_t)}{u'(\bar{C}_t)}\sigma^3 E_0\varepsilon_t^3 + o_t(\sigma^3) \right\}.$$

**Proof of Proposition 3.9.** Notice that the left hand side of (3.3) and (3.4) are the same. Consider any arbitrary $\alpha$. By definition of envy, making $\left\{\hat{C}\right\} = (1 - \alpha)\left\{C\right\} + \alpha\left\{\mathfrak{C}\right\}$, we obtain the desired result. ∎

**Proof of Proposition 4.1.** Using the definition of the $w's$ in equation (4.1), multiplying and dividing by $E_0\left(C_t\right)$ and using the definitions of $R_{0,t}\left[C_t\right]$ and $R_{0,t}\left[1_t\right]$ we have,

$$
\begin{aligned}
1 + \omega_0^m &= \frac{\sum_{t=1}^{\infty} V_0\left[E_0\left(C_t\right)\right]}{\sum_{t=1}^{\infty} V_0\left[C_t\right]} \\
&= \sum_{t=1}^{\infty} w_0\left[C_t\right]\left(\frac{V_0\left[E_0\left(C_t\right)\right]}{V_0\left[C_t\right]}\right) \\
&= \sum_{t=1}^{\infty} w_0\left[C_t\right]\left(\frac{E_0\left(C_t\right)}{V_0\left[C_t\right]} \Big/ \frac{E_0\left(C_t\right)}{V_0\left[E_0\left(C_t\right)\right]}\right) \\
&= \sum_{t=1}^{\infty} w_0\left[C_t\right]\left(R_{0,t}\left[C_t\right] / R_{0,t}\left[1_t\right]\right).
\end{aligned}
$$

Using the definition of the equity premium, the decomposition of the price of equity as the sum of the strips, the definitions of $w_0\left[C_t\right]$, and the definition of excess one-period holding returns, we have:

$$
\begin{aligned}
1 + \nu_0 &= \frac{E_0\left(V_1\left[\{C_t\}\right]\right)}{V_0\left[\{C_t\}\right]} \Big/ \frac{1}{V_0\left(1_1\right)} \\
&= \frac{E_0\left(\sum_{t=1}^{\infty} V_1\left[C_t\right]\right)}{V_0\left[\{C_t\}\right]} \Big/ \frac{1}{V_0\left(1_1\right)} \\
&= \frac{V_0\left[\{C_t\}\right] E_0\left(\sum_{t=1}^{\infty} \frac{V_0\left[C_t\right]}{V_0\left[\{C_t\}\right]} \frac{V_1\left[C_t\right]}{V_0\left[C_t\right]}\right)}{V_0\left[C_t\right]} \Big/ \frac{1}{V_0\left(1_1\right)} \\
&= E_0\left(\sum_{t=1}^{\infty} w_0\left[C_t\right] \frac{V_1\left[C_t\right]}{V_0\left[C_t\right]}\right) \Big/ \frac{1}{V_0\left(1_1\right)} \\
&= \sum_{t=1}^{\infty} w_0\left[C_t\right] \frac{E_0\left(V_1\left[C_t\right]\right)}{V_0\left[C_t\right]} \Big/ \frac{1}{V_0\left(1_1\right)} \\
&= \sum_{t=1}^{\infty} w_0\left[C_t\right]\left(\frac{E_0\left(R_{0,1}\left[C_t\right]\right)}{R_{0,1}\left[1_1\right]}\right). \blacksquare
\end{aligned}
$$

**Proof of Proposition 5.1.** By definition

$$
h_t^{cc}\left(\infty\right) = \lim_{k \to \infty}\left\{E_t\left(\log\left[\frac{R_{t,t+1}\left[1_{t+k}\right]}{R_{t,t+1}\left[1_{t+1}\right]}\right]\right)\right\}
$$

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$$
\begin{aligned}
&= \lim_{k\to\infty}\left\{E_t\log\left[\left(\frac{\frac{\beta^{k-1}\cdot E_{t+1}[M_{t+k}]}{M_{t+1}}}{\frac{\beta^k E_t[M_{t+k}]}{M_t}}\frac{\beta E_t[M_{t+1}]}{M_t}\right)\right]\right\}\\
&= \lim_{k\to\infty}\left\{E_t\left(\log\left[\frac{E_{t+1}[M_{t+k}]}{M_{t+1}}\frac{E_t[M_{t+1}]}{E_t[M_{t+k}]}\right]\right)\right\}\\
&= \lim_{k\to\infty}\left\{E_t\left(\log E_{t+1}[M_{t+k}]-\log M_{t+1}+\log E_t[M_{t+1}]-\log E_t[M_{t+k}]\right)\right\}\\
&= \lim_{k\to\infty}\left\{E_t\log E_{t+1}[M_{t+k}]-E_t\log M_{t+1}+\log E_t[M_{t+1}]-\log E_t[M_{t+k}]\right\}\\
&= \log E_t[M_{t+1}]-E_t\log M_{t+1}+\lim_{k\to\infty}\left\{E_t\log E_{t+1}[M_{t+k}]\right\}-\lim_{k\to\infty}\left\{\log E_t[M_{t+k}]\right\}\\
&= \log E_t[M_{t+1}]-E_t\log M_{t+1},
\end{aligned}
$$

where the last step passes first the limit through the expectations operator—we assume enough regularity for this to be admissible (Lebesgue dominated convergence). Second, assuming that stationary is defined as requiring the existence of a finite unconditional mean, which implies that $\lim_{k\to\infty}\{\log E_{t+1}[M_{t+k}]\}=\lim_{k\to\infty}\{\log E_t[M_{t+k}]\}=\log E[M]$. This last equality shows how the price of a discount bond maturing in the distant future is deterministic in units of utility.

With yield differentials

$$
\begin{aligned}
y_t^{cc}(\infty) &= \lim_{k\to\infty}\left\{\log\left[\frac{V_t[1_{t+1}]}{(V_t[1_{t+k}])^{1/k}}\right]\right\}\\
&= \lim_{k\to\infty}\left\{\log\left[\frac{\beta E_t[M_{t+1}]}{M_t}\left(\frac{\beta^k\cdot E_t[M_{t+k}]}{M_t}\right)^{-1/k}\right]\right\}\\
&= \lim_{k\to\infty}\left\{\log E_t[M_{t+1}]-\log M_t+(1/k)\log E_t[M_{t+k}]-(1/k)\log M_t\right\}\\
&= \log E_t[M_{t+1}]-\log M_t.
\end{aligned}
$$

This concludes the proof. ∎

**Proof of Proposition 5.2.** First, that $E[h_t^{cc}(\infty)]=E[y_t^{cc}(\infty)]$ can be shown by taking unconditional expectations on the expression obtained in Proposition (5.1). The continuously compounded holding premium can be written as

$$
\begin{aligned}
h_t^{cc}(\infty) &= \log E_t[M_{t+1}]-E_t\log M_{t+1}, &\text{(7.3)}\\
&= \log\beta E_t\left[\frac{M_{t+1}}{M_t}\right]-\log\beta-E_t\log\frac{M_{t+1}}{M_t} &\text{(7.4)}\\
&= -\log R_{t,t+1}(1_{t+1})-\log\beta-E_t\log\frac{M_{t+1}}{M_t}.
\end{aligned}
$$

For any risky gross asset return $R_{t,t+1}^j$:

$$1 = \beta E_t \left[ R_{t,t+1}^j \frac{M_{t+1}}{M_t} \right]$$

so that,

$$
\begin{aligned}
0 &= \log(1) = \log \left( \beta E_t \left[ R_{t,t+1}^j \frac{M_{t+1}}{M_t} \right] \right) \\
&> E_t \left[ \log \beta \left( R_{t,t+1}^j \frac{M_{t+1}}{M_t} \right) \right] = E_t \left[ \log \left( \frac{M_{t+1}}{M_t} \right) \right] + \log \beta + E_t \left[ \log \left( R_{t,t+1}^j \right) \right]
\end{aligned}
$$

where the strict inequality follows from the strict concavity of $\log$. Then

$$-E_t \left[ \log \left( \frac{M_{t+1}}{M_t} \right) \right] - \log \beta > E_t \left[ \log \left( R_{t,t+1}^j \right) \right]. \qquad (7.5)$$

Combining the two expressions we obtain

$$h_t^{cc}(\infty) > E_t \left[ \log \left( R_{t,t+1}^j \right) \right] - \log R_{t,t+1}(1_t)$$

for any asset return $R_{t,t+1}^j$. Taking unconditional expectations we conclude the proof. ∎

**Proof of Proposition 5.3.** First notice that the random walk assumption implies that interest rate are constant, since

$$E_t \left( \frac{M_{t+1}}{M_t} \right) = E \left( \frac{M_{t+1}}{M_t} \right)$$

and hence the term premium is zero. Then

$$
\begin{aligned}
\frac{E_t(R_{t+1}[D_{t+s}])}{E_t R_{t+1}(1_{t+1})} &= \frac{E_t(R_{t+1}[D_{t+s}/D_t])}{E_t R_{t+1}(1_{t+1})} = E_t \left( \beta \frac{M_{t+1}}{M_t} \right) \frac{E_t \left( \frac{D_{t+1}}{D_t} E_{t+1} \left[ \beta^{s-1} \frac{M_{t+s}}{M_{t+1}} \frac{D_{t+s}}{D_{t+1}} \right] \right)}{E_t \left[ \beta^s \frac{M_{t+s}}{M_t} \frac{D_{t+1}}{D_t} \frac{D_{t+s}}{D_{t+1}} \right]} \\
&= \exp \left( -cov_t \left( \log \frac{M_{t+1}}{M_t}, E_t \left( \log \frac{D_{t+s}}{D_t} \right) \right) \right) \\
&= \exp \left( -cov_t(m_{t+1}, d_{t+1}) \right) \quad \text{for all } s \geq 1
\end{aligned}
$$

where the first inequality follows since $D_t$ is known at $t$, the second from the definition of returns and pricing kernel, the third from the log-normality of $D's$ and $M's$ as in (4.5) since the term premium is zero, and the forth follows by the

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random walk assumptions. From this we obtain that the excess expected return of a stock paying $\{D_{t+s}\}$ is

$$\frac{E_t\left(R_{t+1}\left[\{D_{t+s}\}_{s=1}^{\infty}\right]\right)}{E_t R_{t+1}\left(1_{t+1}\right)} = \sum_{s=1}^{\infty} w\left(\left[D_{t+s}\right]\right) \frac{E_t\left(R_{t+1}\left[D_{t+s}\right]\right)}{E_t R_{t+1}\left(1_{t+1}\right)} = \frac{E_t\left(R_{t+1}\left[D_{t+s}\right]\right)}{E_t R_{t+1}\left(1_{t+1}\right)}$$

where the weights $w\left[D_{t+s}\right] = V_t\left[\{D_{t+s}\}\right]/V\left[\{D_{t+j}\}_{j=1}^{\infty}\right]$ add up to one. For this stock, the price-dividend ratio is given by

$$
\begin{aligned}
V_t\left[\{D_{t+s}\}_{s=1}^{\infty}\right]/D_t &= \sum_{s=1}^{\infty}\beta^s E_t\left[\frac{M_{t+s}}{M_t}\frac{D_{t+s}}{D_t}\right] = \sum_{s=1}^{\infty}\prod_{i=1}^{s} E_t\left[\beta\frac{M_{t+i}}{M_{t+i-1}}\frac{D_{t+i}}{D_{t+i-1}}\right] \\
&= \sum_{s=1}^{\infty}\left(E\left[\beta\frac{M_{t+1}}{M_t}\frac{D_{t+1}}{D_t}\right]\right)^s = \frac{E\left[\beta\frac{M_{t+1}}{M_t}\frac{D_{t+1}}{D_t}\right]}{1 - E\left[\beta\frac{M_{t+1}}{M_t}\frac{D_{t+1}}{D_t}\right]}
\end{aligned}
$$

where the third inequality uses the random walk assumption. Finally, notice for each asset $i$ we have

$$\log E_t\left(R_{t+1,i}\right) = -cov_t\left(\Delta m_{t+1}, \Delta d_{t+1}^i\right) = -cov\left(l \cdot \varepsilon_{t+1}, \Delta d_{t+1}^i\right).$$

∎

**Proof of Proposition 5.4.** First we find an expression for $\tau\left(t\right)$. Notice that using lognormality we get

$$E_0\left(\mathfrak{C}_t\right) = \exp\left(\tau\right)E_0\exp\left(\sum_{i=1}^{K} a_i c_{t-j}\right) = \exp\left(\tau\left(t\right)\right)\exp\left(\sum_{i=1}^{K} a_i E_0\left(c_{t-j}\right) + \frac{1}{2}var_0\left(\sum_{i=1}^{K} a_i c_{t-j}\right)\right)$$

and

$$E_0\left(C_t\right) = E_0\exp\left(c_t\right) = \exp\left(E_0 c_t + \frac{1}{2}var_0\left(c_t\right)\right)$$

thus $\tau\left(t\right)$ solves $E_0\left(\mathfrak{C}_t\right) = E_0\left(C_t\right)$ so

$$\tau\left(t\right) = \exp\left(E_0 c_t - \sum_{i=1}^{K} a_i E_0\left(c_{t-j}\right) + \frac{1}{2}var_0\left(c_t\right) - \frac{1}{2}var_0\left(\sum_{i=1}^{K} a_i c_{t-j}\right)\right).$$

Using the definition of the pricing kernel and lognormality

$$\frac{V_0\left[\mathfrak{C}_t\right]}{V_0\left[C_t\right]} =$$

$$= \frac{E_0 \exp\left[\tau(t) + E_0 m_t - m_0 + \sum_{j=0}^{K} a_j c_{t-j}\right]}{E_0 \exp\left[m_t - m_0 + c_t\right]}$$

$$= \frac{\exp\left[\tau(t) + E_0\left(m_t - m_0 + \sum_{j=0}^{K} a_j c_{t-j}\right) + \frac{1}{2}var_0(m_t)\right]}{\exp\left[E_0\left(m_t - m_0 + c_t\right) + \frac{1}{2}var_0(m_t) + \frac{1}{2}var_0(c_t) + cov_0(m_t, c_t)\right]}$$

$$\times \exp\left[\frac{1}{2}var_0\left(\sum_{j=0}^{K} a_j c_{t-j}\right) + cov_0\left(m_t, \sum_{j=0}^{K} a_j c_{t-j}\right)\right]$$

$$= \exp\left[\begin{array}{c} E_0\left(\sum_{j=0}^{K} a_j c_{t-j}\right) - E_0(c_t) + \\ \tau(t) + \frac{1}{2}var_0\left(\sum_{j=0}^{K} a_j c_{t-j}\right) - \frac{1}{2}var_0(c_t) \\ + cov_0\left(m_t, \sum_{j=0}^{K} a_j c_{t-j}\right) - cov_0(m_t, c_t) \end{array}\right].$$

and by definition of $\tau(t)$ we get

$$\frac{V_0[\mathfrak{C}_t]}{V_0[C_t]} = \exp\left[cov_0\left(m_t, \sum_{j=0}^{K} a_j c_{t-j}\right) - cov_0(m_t, c_t)\right].$$

By direct computation in the random walk case:

$$= \exp\left[cov_0\left(m_t, \sum_{j=0}^{K} a_j c_{t-j}\right) - cov_0(m_t, c_t)\right]$$

$$= \exp\left[cov_0\left(\sum_{k=(t-K)}^{t} \varepsilon_k^m, \sum_{k=(t-K)}^{t}\sum_{j=0}^{K} a_j \varepsilon_{k-j}^c\right) - cov_0\left(\sum_{k=(t-K)}^{t} \varepsilon_k^m, \sum_{k=(t-K)}^{t} \varepsilon_{k-j}^c\right)\right]$$

$$= \exp\left[\begin{array}{c} cov_0\left(\varepsilon_t^m, a_0 \varepsilon_k^c\right) - cov_0\left(\varepsilon_t^m, \varepsilon_t^c\right) \\ cov_0\left(\varepsilon_{t-1}^m, (a_0 + a_1)\varepsilon_{t-1}^c\right) - cov_0\left(\varepsilon_{t-K-1}^m, \varepsilon_{t-1}^c\right) + \cdots \\ cov_0\left(\varepsilon_{t-K}^m, (a_0 + a_1 + \cdots + a_K)\varepsilon_{t-K-1}^c\right) - cov_0\left(\varepsilon_{t-K}^m, \varepsilon_{t-K}^c\right) \end{array}\right]$$

$$= \exp\left[\sigma_{\Delta c \Delta m}\left[a_0 - 1 + (a_0 + a_1) - 1 + \cdots + (a_0 + a_1 + \cdots + a_K) - 1\right]\right]$$

where $\varepsilon^m$ and $\varepsilon^c$ are the innovations of the log of the pricing kernel and the log of consumption. Where the first step uses the fact that the first $t - K$ shocks are common to both covariances; the second separates the covariances, and the third reorders and uses the result that for last term the sum of all the alphas equals 1. Finally, by the previous proposition we have that

$$\frac{E(R_1[C_t])}{R_f} = \exp\left(-cov(\Delta m_t, \Delta c_t)\right) \equiv \exp\left(-\sigma_{\Delta c, \Delta m}\right)$$

which combined with the previous equality finishes the proof. ∎

**Proof of Proposition 5.5.** The proof is a straightforward variation of the proof of proposition 5.4.

**Proof of Proposition 5.6.** Direct computation shows that

$$
\begin{aligned}
m\left(a^{k+1}\right) &= \left(a_0^{K+1} - 1\right) + \left(a_0^{K+1} + a_1^{K+1} - 1\right) + \left(a_0^{K+1} + a_1^{K+1} + a_2^{K} - 1\right) + \\
&\quad + \cdots + \left(a_0^{K+1} + a_1^{K+1} + \cdots + a_{K+1}^{K} - 1\right) \\
&= -1 + \left(a_0^{K} - 1\right) + \left(a_0^{K} + a_1^{K} - 1\right) + \cdots + \left(a_0^{K+1} + a_1^{K+1} + \cdots + a_K^{K} - 1\right) \\
&= m\left(a^K\right) + 1.
\end{aligned}
$$

∎

**Proof of Proposition 6.1.** Regarding $C = \{C\}$ as a vector on $R^\infty$ let $\frac{\partial U(C)}{\partial C}$ and $\frac{\partial^2 U(C)}{\partial C \partial C'}$ be its first and second derivatives evaluated at $C$. Define $C_\alpha = (1 - \alpha)\mathfrak{C} + \alpha C$ and $\left(C_t\left[1 + \lambda(\alpha)\right]^t\right)$ be the vector where each component consists of $C_t\left(z^t\right)\left[1 + \lambda(\alpha)\right]^t$. Then twice differentiating with respect to $\alpha$ the equation defining $\lambda$,

$$
\frac{\partial \lambda(\alpha)}{\partial \alpha} \frac{\partial U\left(\left(\left[1 + \lambda(\alpha)\right]^t C_t\right)\right)}{\partial C}\left(C_t\left[1 + \lambda(\alpha)\right]^{t-1} t\right) = \frac{\partial U\left(C_\alpha\right)}{\partial C}\left(\mathfrak{C} - C\right)
$$

and

$$
\begin{aligned}
& \frac{\partial^2 \lambda(\alpha)}{\partial \alpha \partial \alpha} \frac{\partial U\left(\left(\left[1 + \lambda(\alpha)\right]^t C_t\right)\right)}{\partial C}\left(C_t\left[1 + \lambda(\alpha)\right]^{t-1} t\right) + \\
& \left[\frac{\partial \lambda(\alpha)}{\partial \alpha}\right]^2 \frac{\partial U\left(\left(\left[1 + \lambda(\alpha)\right]^t C_t\right)\right)}{\partial C}\left(C_t\left[1 + \lambda(\alpha)\right]^{t-2} t(t-1)\right) \\
& + \left(\frac{\partial \lambda(\alpha)}{\partial \alpha}\left(C_t\left[1 + \lambda(\alpha)\right]^{t-1} t\right)\right)' \frac{\partial^2 U\left(\left(\left[1 + \lambda(\alpha)\right]^t C_t\right)\right)}{\partial C \partial C'}\left(\frac{\partial \lambda(\alpha)}{\partial \alpha}\left(C_t\left[1 + \lambda(\alpha)\right]^{t-1} t\right)\right) \\
&= \left(\mathfrak{C} - C\right)' \frac{\partial^2 U\left(C_\alpha\right)}{\partial C \partial C'}\left(\mathfrak{C} - C\right)
\end{aligned}
$$

Taking $\alpha \downarrow 0$ since $C_\alpha = C$ and $\lambda(0) = 0$ then,

$$
\begin{aligned}
& \frac{\partial^2 \lambda(0)}{\partial \alpha \partial \alpha} \frac{\partial U(C)}{\partial C}\left(C_t t\right) = -\left[\frac{\partial \lambda(\alpha)}{\partial \alpha}\right]^2 \frac{\partial U(C)}{\partial C}\left(C_t t(t-1)\right) \\
& + \left(\mathfrak{C} - C - \frac{\partial \lambda(\alpha)}{\partial \alpha}\left(C_t\, t\right)\right)' \frac{\partial^2 U(C)}{\partial C \partial C'}\left(\mathfrak{C} - C - \frac{\partial \lambda(\alpha)}{\partial \alpha}\left(C\, t\right)\right) \leq 0
\end{aligned}
$$

where the inequality follows from the concavity of $U$ and the monotonicity of $U$. Since $U$ is increasing , then $\frac{\partial^2 \lambda(0)}{\partial \alpha \partial \alpha} \leq 0$ ∎

**Details about approximations made with the log-linear system**

**A. Bias in the trend**

In section (5.3) in the paper we define the trend as a one-sided moving average of current and past consumption, $\ln(\mathfrak{c}_t) = \sum_{k=0}^{K} a_k \cdot c_{t-k}$. With this definition of the trend $\mathfrak{C}$ , $E_0(C_t) \neq E_0(\mathfrak{C}_t)$, because $\mathfrak{C}$ is a nonlinear function of $C$. We show here that this bias is quantitatively negligible for our applications.

Assuming we want to impose as a constraint that the conditional mean of the trend and of consumption are equal, then the following could be done. Based on our definition of the marginal cost we have

$$
\begin{aligned}
1 + \omega_0^m &= \frac{\sum_{t=1}^{\infty} V_0[X_t]}{\sum_{t=1}^{\infty} V_0[C_t]}, \\
&= \sum_{t=1}^{\infty} w_t \frac{V_0[X_t]}{V_0[C_t]},
\end{aligned}
$$

where $w_t \equiv V_0[C_t] / \sum_{t=1} V_0[C_t]$. Assume we want to adjust every strip of the trend so that

$$
1 + \omega_0^m = \sum_{t=1}^{\infty} w_t \frac{V_0[X_t]\,\phi(t)}{V_0[C_t]},
$$

the issue is how to define the adjustment $\phi(t)$. One possibility is to make the conditional mean of the two equal, so that

$$
\phi(t) = \frac{E_0[C_t]}{E_0[X_t]},
$$

in this case, $\phi(t)$ is a state dependent adjustment.

The random walk case can provide an idea of the order of magnitude of the adjustment, it is easy to compute and close to the cases analyzed in the paper. In this case, simple algebra gives that for $t > K$,

$$
\phi(t) = \frac{E_0 C_t}{E_0 X_t} = e^{\left[1-(\alpha_0)^2+1-(\alpha_0+\alpha_1)^2+1-(\alpha_0+\alpha_1+\alpha_2)^2+\dots+1-(\alpha_0+\dots+\alpha_J)^2\right]\frac{\sigma_c^2}{2}} \geq 1.
$$

In words, expected consumption is larger because consumption is more volatile than the trend, given that the trend depends partially on earlier realizations that have a lower variance. For the first few periods where $t \leq K$, the terms are a bit different. They will coincide if we take the expectations conditional on time $-K$, so that the lags of the trend are not known; otherwise, they will depend on recent realizations of consumption.

For the coefficient values implied by the 8 year filter in the paper, based on $\sigma_c^2 = 0.0328^2$ and $\sigma_c^2 = 0.0115^2$ corresponding to the variance of the consumption growth rate for the period 1889-1997 and 1954-1997 respectively, we have

$$\exp\left(0.5752\frac{0.0328^2}{2}\right) = 1.000309, \text{ and}$$

$$\exp\left(0.5752\frac{0.0115^2}{2}\right) = 1.000038.$$

That is, the bias is barely 3 basis points with consumption for 1889-1997, the bias is just about one third of a basis point with consumption for 1954-1997.

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Table 1

## Marginal Cost of Dividend Uncertainty

|  | 1889-1997 | 1954-1997 |
|---|---|---|
| Dividend-Price ratio | 4.60% | 3.63% |
| Longterm yield | 1.96% | 2.51% |
| Consumption trend growth | 1.68% | 1.85% |
| Dividend trend growth | 1.31% | 1.17% |
| | | |
| ((1+g)*dp) / (r - g) | 7.1937 | 2.7385 |
| *Marginal Cost of Dividend Uncertainty* | 619.37% | 173.85% |
| | | |
| Std ( $\Delta$ log C ) | 3.28% | 1.15% |
| Std ( $\Delta$ log D ) | 12.01% | 3.78% |
| Corr( $\Delta$ log C,$\Delta$ log D) | 0.32 | 0.42 |

*See the Appendix for data sources*

Table 2A

## Marginal Cost of Consumption Fluctuations (1889-1997)

| Moments the kernel fits: | Coefficients on states: | | | | | | Marginal cost: Low pass filters | | | | Other Filters | | | Benefit of 1% growth | Duration Consumption Equity | Fit |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | c | d | trm | d/p | vwr | β | 8 years | 12years | 16 years | 20 years | infinite | geometric | linear | | | |
| 1. E(D/P),E(VWR-Rf),Y(1),Y(13), | -9.31 | -1.56 | 8.05 | | | 0.90 | 0.43% | 0.79% | 0.89% | 1.06% | 353.26% | 1.01% | 1.85% | 227.60% | 70.19 | 0 |
| 2. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), | -9.21 | -1.58 | 8.19 | | | 0.90 | 0.43% | 0.79% | 0.89% | 1.05% | 347.07% | 0.94% | 1.73% | 227.26% | 70.16 | 0 |
| 3. E(D/P),E(Rd/Rf),Y(1),Y(13) | -9.08 | -1.59 | 8.31 | | | 0.90 | 0.42% | 0.78% | 0.88% | 1.05% | 341.72% | 0.99% | 1.83% | 230.56% | 70.47 | 0 |
| 4. E(D/P),E(VWR-Rf),Y(1),Y(13), | -4.04 | -2.37 | | 9.64 | | 0.91 | 0.32% | 0.60% | 0.67% | 0.69% | 543.28% | 0.72% | 1.41% | 1560.88% | 98.90 | 0 |
| 5. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), | -0.19 | -3.31 | | 10.09 | | 0.90 | 0.24% | 0.48% | 0.54% | 0.64% | 604.27% | 0.61% | 1.15% | 780.47% | 115.55 | 0 |
| 6. E(D/P),E(Rd/Rf),Y(1),Y(13), | -4.31 | -2.57 | | 10.22 | | 0.90 | 0.35% | 0.65% | 0.73% | 0.86% | 1956.35% | 0.82% | 1.52% | 4141.52% | 109.09 | 1.43E-05 |
| 7. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf) | -9.60 | -1.53 | 8.19 | -0.49 | | 0.90 | 0.43% | 0.80% | 0.90% | 1.07% | 362.85% | 1.02% | 1.88% | 221.43% | 69.59 | 0 |
| 8. E(D/P),E(VWR-Rf),Y(1),Y(13), | -6.53 | -1.93 | | | -0.53 | 0.90 | 0.40% | 0.73% | 0.80% | 0.93% | 2862.13% | 0.93% | 1.68% | 1344.64% | 97.20 | 0 |
| 9. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), | 0.06 | -3.30 | | | -0.70 | 0.89 | 0.26% | 0.49% | 0.54% | 0.63% | 1263.46% | 0.63% | 1.16% | 16561.54% | 123.50 | 0 |
| 10. E(D/P),E(Rd/Rf),Y(1),Y(13), | -1.00 | -2.88 | | | -0.69 | 0.91 | 0.27% | 0.51% | 0.55% | 0.64% | 6151.07% | 0.64% | 1.19% | 154985.17% | 151.48 | 2.62E-05 |
| 11. E(D/P),E(VWR-Rf),Y(1),Y(13), | -7.18 | -1.98 | | | -0.40 | 0.90 | 0.40% | 0.71% | 0.79% | 0.93% | 502.50% | 0.90% | 1.65% | 571.11% | 86.50 | 0 |
| 12. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), | -0.69 | -3.24 | | | -0.63 | 0.89 | 0.26% | 0.49% | 0.55% | 0.65% | 480.22% | 0.63% | 1.16% | 8664.40% | 116.65 | 0 |
| 13. E(D/P),E(Rd/Rf),Y(1),Y(13), [OI+E(VWR-Rf)] | -5.36 | -2.25 | | | -0.49 | 0.91 | 0.36% | 0.64% | 0.72% | 0.84% | 605.66% | 0.82% | 1.49% | 1498.99% | 98.47 | 2.72E-06 |
| 14. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf) | 0.06 | -2.16 | 44.74 | 1.66 | | 0.93 | 0.20% | 0.43% | 0.54% | 0.68% | 39.70% | 0.49% | 0.95% | 109.37% | 52.82 | 0 |
| 15. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf) | -13.44 | -1.04 | 8.61 | | 0.41 | 0.88 | 0.50% | 0.92% | 1.06% | 1.27% | 206.03% | 1.18% | 2.17% | 110.82% | 53.17 | 5.78E-08 |
| Standard deviation of consumption deviations from trend | | | | | | | 1.35% | 1.68% | 2.02% | 2.36% | | 1.80% | 2.92% | | | |

Moments  Dividend/price ratio: E(D/P) = 0.046
Equity premium: E(VWR-Rf) = 0.0677
Multiplicative equity premium: E(Rd/Rf-1) = 0.0698
Riskless rate: E(logY(1))= 0.0108
Yield spread: E(logY(13)-logY(1))= 0.0059

States  c = consumption
d = dividends
trm=long-short term spread
d/p = dividend-price ratio
vwr = realized value weighted stock return

Forecasting  1.-7.   c,d,trm,d/p
variables  8.-10.,14.  c,d,dp,vwr
11.-13,15.  c,d,trm,vwr

Fit  stands for the sum of squared residuals from data moments

Table 2B

## Marginal Cost of Consumption Fluctuations (1954-1997)

| Moments the kernel fits: | Coefficients on states: | | | | | | Marginal cost: Low pass filters | | | | Other filters | | | Benefit of 1% growth | Duration Consumption Equity | Fit |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | c | d | trm | d/p | vwr | β | 8 years | 12 years | 16 years | 20 years | inf | geometric | linear | | | |
| 1. E(D/P),E(VWR-Rf),Y(1),Y(13), | -28.64 | -2.82 | 23.95 | | | 0.89 | 0.22% | 0.42% | 0.51% | 0.62% | 22.33% | 0.51% | 0.96% | 6341.00% | 39.28 | 0 |
| 2. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), | -19.30 | 6.49 | 41.62 | | | 0.85 | 0.15% | 0.31% | 0.35% | 0.41% | 302.10% | 0.37% | 0.70% | 312531307.00% | 211.65 | 0 |
| 3. E(D/P),E(Rd/Rf),Y(1),Y(13), [OI + E(VWR-Rf)] | -12.61 | -2.40 | 20.16 | | | 0.94 | 0.11% | 0.22% | 0.27% | 0.33% | 11.54% | 0.26% | 0.48% | 103.93% | 51.5078 | 6.38E-05 |
| 4. E(D/P),E(VWR-Rf),Y(1),Y(13), | -10.94 | -4.40 | | 53.35 | | 0.92 | 0.14% | 0.22% | 0.24% | 0.26% | 17.86% | 0.24% | 0.41% | 203949.00% | 102.06 | 0 |
| 5. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), [no trm, OI + E(VWR-Rf)] | -15.96 | -2.93 | | 41.14 | | 0.91 | 0.15% | 0.27% | 0.32% | 0.39% | 32.77% | ,0036 | 0.66% | 144.60% | 59.76 | 6.63E-06 |
| 6. E(D/P),E(Rd/Rf),Y(1),Y(13), [no trm state] | -7.19 | -3.22 | | 41.53 | | 0.93 | 0.09% | 0.16% | 0.18% | 0.23% | 18.72% | 0.21% | 0.38% | 217.64% | 69.32 | 1.02E-06 |
| 7. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf) | no fit | | | | | | | | | | | | | | | |
| 8. E(D/P),E(VWR-Rf),Y(1),Y(13), | -11.85 | -4.28 | | | -2.02 | 0.89 | 0.18% | 0.34% | 0.36% | 0.41% | 115570.55% | 0.44% | 0.81% | 290802426.00% | 508.09 | 0 |
| 9. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), | -36.53 | -5.93 | | | -2.13 | 0.75 | 0.32% | 0.57% | 0.60% | 0.69% | 1156.26% | 0.72% | 1.29% | 2530.00% | 103.28 | 0 |
| 10. E(D/P),E(Rd/Rf),Y(1),Y(13),   (1947-1997) | -23.70 | -5.20 | | | -1.88 | 0.84 | 0.21% | 0.34% | 0.37% | 0.42% | 54.54% | 0.43% | 0.75% | 23430.00% | 70.94 | 7.66E-07 |
| 11. E(D/P),E(VWR-Rf),Y(1),Y(13),   (1947-1997) | -8.62 | -4.99 | | | -2.04 | 0.92 | 0.14% | 0.21% | 0.24% | 0.28% | 18.32% | 0.25% | 0.43% | 2965341.00% | 129.94 | 0 |
| 12. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), [no trm, OI + E(D/P)] | -0.95 | -4.50 | | | -1.90 | 0.92 | 0.07% | 0.12% | 0.12% | 0.14% | 248.02% | 0.16% | 0.27% | 141029900.00% | 419.44 | 8.78E-06 |
| 13. E(D/P),E(Rd/Rf),Y(1),Y(13), [no trm state] | -0.88 | -3.84 | | | -1.72 | 0.93 | 0.07% | 0.10% | 0.11% | 0.13% | 38.50% | 0.14% | 0.23% | 335717.05% | 163.47 | 7.95E-06 |
| 14. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf) | -40.57 | 5.40 | | 94.47 | 1.16 | 0.71 | 0.42% | 0.73% | 0.90% | 1.01% | 35.57% | 0.97% | 1.79% | 3149.00% | 24.44 | 0 |
| 15. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf) | no fit | | | | | | | | | | | | | | | |
| Standard deviation of consumption deviations from trend | | | | | | | 0.55% | 0.69% | 0.67% | 0.70% | | 0.73% | 1.21% | | | |

Moments   Dividend/price ratio: E(D/P) =  0.0363
Equity premium: E(VWR-Rf) =  0.0628
Multiplicative equity premium: E(Rd/Rf-1) =0.0613
Riskless rate: E(logY(1))=  0.0121
Yield spread: E(logY(13)-logY(1))=0.0123

States   c = consumption
d = dividends
trm=long-short term spread
d/p = dividend-price ratio
vwr = realized value weighted stock return

Forecasting   1.-7.   c,d,trm,d/p
variables   8.-10.,14.   c,d,dp,vwr
11.-13.,15.   c,d,trm,vwr

Fit  stands for the sum of squared residuals from data moments

Table 2C

## Marginal Cost of Consumption Fluctuations: Dividends and Consumption Cointegrated (1889-1997)

| Moments the kernel fits: | Coefficients on states: | | | | | | Low pass filters | Marginal cost: | | | Other filters | | | Benefit of 1% growth | Duration Consumption Equity | Fit |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | c | d | trm | d/p | vwr | β | 8 years | 12years | 16 years | 20 years | infinite | geometric | linear | | | |
| 1. E(D/P),E(VWR-Rf),Y(1),Y(13), [OI + E(Rd/Rf) | -11.55 | -1.40 | -8.19 | | | 0.90 | 0.36% | 0.66% | 0.78% | 0.96% | 47.68% | 0.88% | 1.60% | 45.98% | 31.74 | 2.19E-04 |
| 2. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), [OI + E(D/P)] | -11.55 | -1.40 | -8.19 | | | 0.90 | 0.36% | 0.66% | 0.78% | 0.96% | 47.68% | 0.88% | 1.60% | 45.98% | 31.74 | 2.19E-04 |
| 3. E(D/P),E(Rd/Rf),Y(1),Y(13), [OI + E(WVR-Rf)] | -11.55 | -1.40 | -8.19 | | | 0.90 | 0.36% | 0.66% | 0.78% | 0.96% | 47.68% | 0.88% | 1.60% | 45.98% | 31.74 | 2.19E-04 |
| 4. E(D/P),E(VWR-Rf),Y(1),Y(13), | -17.93 | -3.47 | | -32.32 | | 0.81 | 0.37% | 0.81% | 0.98% | 1.25% | 196.30% | 1.20% | 2.29% | 64.22% | 39.12 | 0.00E+00 |
| 5. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13),  [OI + E(D/P)] | -16.40 | -1.18 | | -10.29 | | 0.86 | 0.49% | 0.90% | 1.07% | 1.31% | 88.99% | 1.19% | 2.20% | 51.31% | 34.09 | 1.91E-04 |
| 6. E(D/P),E(Rd/Rf),Y(1),Y(13), [OI + E(WVR-Rf)] | -16.40 | -1.18 | | -10.29 | | 0.86 | 0.49% | 0.90% | 1.07% | 1.31% | 88.99% | 1.19% | 2.20% | 51.31% | 34.09 | 1.91E-04 |
| 7. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf) | no fit | | | | | | | | | | | | | | | |
| 8. E(D/P),E(VWR-Rf),Y(1),Y(13), | -10.28 | -3.54 | | | 1.05 | 0.88 | 0.23% | 0.56% | 0.71% | 0.93% | 55.17% | 0.84% | 1.65% | 40.02% | 28.94 | 0 |
| 9. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), | -20.06 | 0.23 | | | 0.41 | 0.83 | 0.67% | 1.22% | 1.37% | 1.63% | 367250.00% | 1.59% | 2.90% | 231.55% | 70.40 | 0 |
| 10. E(D/P),E(Rd/Rf),Y(1),Y(13), [OI + E(WVR-Rf)] | -8.80 | -2.44 | | | 0.40 | 0.91 | 0.27% | 0.55% | 0.68% | 0.86% | 45.20% | 0.75% | 1.43% | 44.36% | 31.06 | 1.66E-04 |
| 11. E(D/P),E(VWR-Rf),Y(1),Y(13), [no trm state] | -21.79 | -3.54 | | | 3.17 | 0.75 | 0.43% | 1.05% | 1.23% | 1.53% | 560455113.38% | 1.42% | 0.0278 | 214.25% | 66.59 | 0 |
| 12. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13),[no trm state] | -21.95 | 0.03 | | | 1.03 | 0.81 | 0.66% | 1.27% | 1.39% | 1.63% | 1.17E+12 | 1.64% | 3.02% | 5.05E+04 | 224.70 | 5.37E-05 |
| 13. E(D/P),E(Rd/Rf),Y(1),Y(13), [OI + E(VWR-Rf)] | -10.94 | -2.20 | | | 0.95 | 0.90 | 0.28% | 0.58% | 0.70% | 0.87% | 61.68% | 0.77% | 1.46% | 55.90% | 35.95 | 2.77E-04 |
| 14. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf) | -10.50 | 1.79 | | 75.51 | 3.09 | 0.90 | 0.51% | 0.86% | 1.08% | 1.32% | 19.94% | 1.07% | 1.96% | 22.74% | 18.86 | 0 |
| 15. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf) | no fit | | | | | | | | | | | | | | | |
| Standard deviation of consumption deviations from trend | | | | | | | 1.35% | 1.68% | 2.02% | 2.36% | | 1.80% | 2.92% | | | |

| Moments | | States | | Forecasting | | |
|---|---|---|---|---|---|---|
| | Dividend/price ratio: E(D/P) = 0.046 | | c = consumption | variables | 1.-7. | c,d,trm,d/p |
| | Equity premium: E(VWR-Rf) = 0.0677 | | d = dividends | | 8.-10.,14. | c,d,dp,vwr |
| | Multiplicative equity premium: E(Rd/Rf-1) = 0.0698 | | trm=long-short term spread | | 11.-13.,15. | c,d,trm,vwr |
| | Riskless rate: E(logY(1))= 0.0108 | | d/p = dividend-price ratio | | | |
| | Yield spread: E(logY(13)-logY(1))= 0.0059 | | vwr = realized value weighted stock return | | Fit stands for the sum of squared residuals from data moments | |

Table 2D

## Marginal Cost of Consumption Fluctuations: Dividends and Consumption Cointegrated (1954-1997)

| Moments the kernel fits: | Coefficients on states: | | | | | | Marginal cost: Low pass filters | | | | Other filters | | | Benefit of 1% growth | Duration Consumption Equity | Fit |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | c | d | trm | d/p | vwr | β | 8 years | 12 years | 16 years | 20 years | inf | geometric | linear | | | |
| 1. E(D/P),E(VWR-Rf),Y(1),Y(13), | -17.62 | -1.68 | 31.10 | | | 0.91 | 0.22% | 0.42% | 0.52% | 0.66% | 25.55% | 0.53% | 1.00% | 56.39% | 36.66 | 0 |
| 2. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), [no d/p, OI +E(D/P)] | 6.54 | -4.95 | 22.05 | | | 0.95 | 0.11% | 0.19% | 0.30% | 0.40% | 0.26% | 0.22% | 0.42% | 52.12% | 34.70 | 3.28E-05 |
| 3. E(D/P),E(Rd/Rf),Y(1),Y(13),[no d/p, OI + E(Rd/Rf)] | 6.54 | -4.95 | 22.05 | | | 0.95 | 0.11% | 0.19% | 0.30% | 0.40% | 0.26% | 0.22% | 0.42% | 52.12% | 34.70 | 3.28E-05 |
| 4. E(D/P),E(VWR-Rf),Y(1),Y(13), | -5.92 | -2.39 | | 64.75 | | 0.94 | 0.19% | 0.28% | 0.34% | 0.42% | 8.07% | 0.33% | 0.58% | 45.07% | 31.68 | 0 |
| 5. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), | -9.89 | -0.52 | | 60.82 | | 0.95 | 0.17% | 0.25% | 0.28% | 0.33% | 21.06% | 0.31% | 0.53% | 137.86% | 58.81 | 2.27E-06 |
| 6. E(D/P),E(Rd/Rf),Y(1),Y(13) | 3.47 | -3.07 | | 57.59 | | 0.95 | 0.12% | 0.16% | 0.21% | 0.27% | 0.16% | 0.17% | 0.29% | 46.35% | 32.23 | 5.97E-07 |
| 7. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf), | 0.83 | -2.97 | 1.72 | 56.73 | | 0.95 | 0.140% | 0.190% | 0.260% | 0.320% | 2.41% | 0.23% | 0.38% | 45.34% | 31.77 | 3.54E-06 |
| 8. E(D/P),E(VWR-Rf),Y(1),Y(13), | -13.95 | -4.61 | | | -1.89 | 0.88 | 0.33% | 0.59% | 0.71% | 0.87% | 48.16% | 0.77% | 1.40% | 57.17% | 36.97 | 0 |
| 9. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), | -18.46 | -2.94 | | | -1.97 | 0.88 | 0.35% | 0.67% | 0.71% | 0.82% | 462789900.00% | 0.86% | 0.16% | 89197900.00% | 375.47 | 0 |
| 10. E(D/P),E(Rd/Rf),Y(1),Y(13), | -6.54 | -4.50 | | | -1.46 | 0.93 | 0.24% | 0.41% | 0.51% | 0.65% | 20.54% | 0.53% | 0.96% | 46.22% | 32.20 | 0 |
| 11. E(D/P),E(VWR-Rf),Y(1),Y(13), | -13.34 | -4.58 | | | -1.92 | 0.91 | 0.29% | 0.48% | 0.58% | 0.70% | 20.53% | 0.59% | 1.05% | 49.25% | 33.60 | 0 |
| 12. E(Rd/Rf),E(VWR-Rf),Y(1),Y(13), [OI + E(D/P)] | -10.18 | -4.45 | | | -1.64 | 0.91 | 0.28% | 0.49% | 0.60% | 0.75% | 31.13% | 0.64% | 1.15% | 50.85% | 34.31 | 4.55E-06 |
| 13. E(D/P),E(Rd/Rf),Y(1),Y(13), [OI + E(VWR-Rf)] | -10.18 | -4.45 | | | -1.64 | 0.91 | 0.28% | 0.49% | 0.60% | 0.75% | 31.13% | 0.64% | 1.15% | 50.85% | 34.31 | 4.55E-06 |
| 14. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf), | -4.73 | -4.84 | -1.61 | | -2.78 | 0.94 | 0.22% | 0.34% | 0.43% | 0.53% | 9.80% | 0.41% | 0.73% | 45.30% | 31.72 | 1.33E-05 |
| 15. E(D/P),E(Rd/Rf),Y(1),Y(13), E(VWR-Rf), | -12.87 | -3.42 | 31.18 | | -0.70 | 0.90 | 0.28% | 0.50% | 0.60% | 0.75% | 38.05% | 0.65% | 1.19% | 54.36% | 35.90 | 5.53E-06 |
| Standard deviations of consumption deviations from trend | | | | | | | 0.55% | 0.69% | 0.67% | 0.70% | | 0.73% | 1.21% | | | |

| Moments | | States | | Forecasting | 1.-7. | c,d,trm,d/p |
|---|---|---|---|---|---|---|
| Dividend/price ratio: E(D/P) = 0.0363 | | c = consumption | | variables | 8.-10.,14. | c,d,dp,vwr |
| Equity premium: E(VWR-Rf) = 0.0628 | | d = dividends | | | 11.-13.,15. | c,d,trm,vwr |
| Multiplicative equity premium: E(Rd/Rf-1) =0.0613 | | trm=long-short term spread | | | | |
| Riskless rate: E(logY(1))= 0.0121 | | d/p = dividend-price ratio | | | | |
| Yield spread: E(logY(13)-logY(1))=0.0123 | | vwr = realized value weighted stock return | | Fit stands for sum of squared residuals from data moments | | |

Table 3

# Random Walk Case: Consumption Equity Premium and Marginal Cost

| | E(re-rf) | State | Loading | Consumption Equity Premium | Marginal Cost Frequency domain filters | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 8 years | 12 years | 16 years | 20 years | | | |
| **1954-1997** | | | | | | | | | | std($\Delta$c) | 0.011 |
| | Mkt | $\Delta$c | -83.9 | 1.09% | 0.42% | 0.79% | 0.84% | 0.96% | | std(Mkt) | 0.169 |
| | Mkt | Mkt | -2.6 | 0.22% | 0.08% | 0.16% | 0.17% | 0.19% | | corr($\Delta$c,Mkt) | 0.453 |
| | 10 CRSP size | | | 0.37% | 0.14% | 0.27% | 0.29% | 0.33% | | E(Mkt-rf) | 7.30% |
| | Dec.: 1,10 | $\Delta$c, Mkt | | 0.31% | 0.12% | 0.22% | 0.24% | 0.27% | | | |
| **1926-1997** | | | | | | | | | | std($\Delta$c) | 0.025 |
| | Mkt | $\Delta$c | -26.3 | 1.59% | 0.61% | 1.15% | 1.23% | 1.40% | | std(Mkt) | 0.197 |
| | Mkt | Mkt | -2.1 | 0.63% | 0.24% | 0.45% | 0.49% | 0.56% | | corr($\Delta$c,Mkt) | 0.628 |
| | 10 CRSP size | | | 0.64% | 0.24% | 0.46% | 0.49% | 0.56% | | E(Mkt-rf) | 8.01% |
| | Dec.: 1,10 | $\Delta$c, Mkt | | 1.66% | 0.63% | 1.20% | 1.28% | 1.47% | | | |
| Factor multiplying consumption equity premium [1-$a_0$ + 1 - ($a_0$+$a_1$) …1-($a_0$+$a_1$..$a_K$)] | | | | | 0.38 | 0.72 | 0.77 | 0.88 | | | |

Figure 1: Marginal and Total Cost Functions
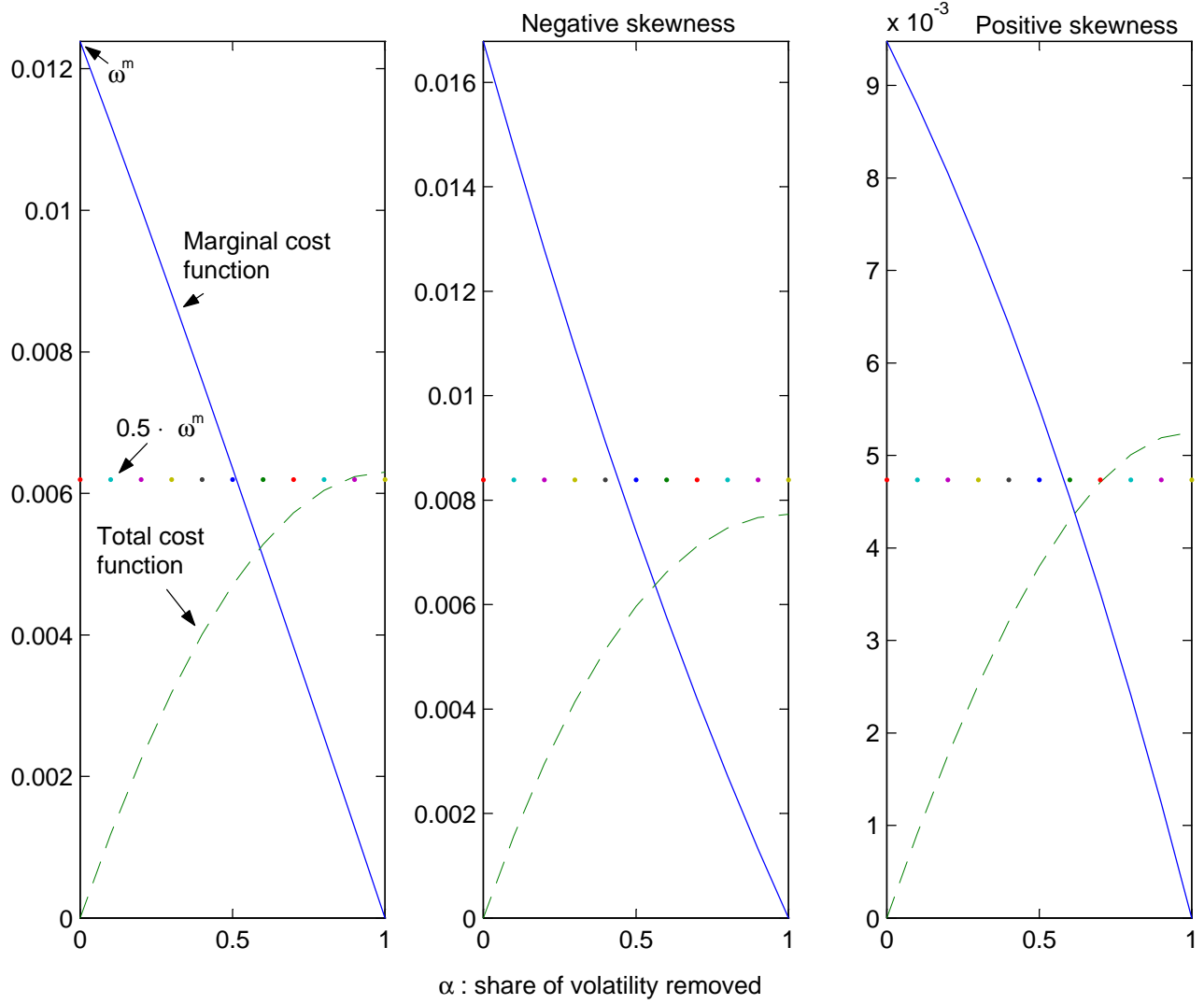
Figure 2

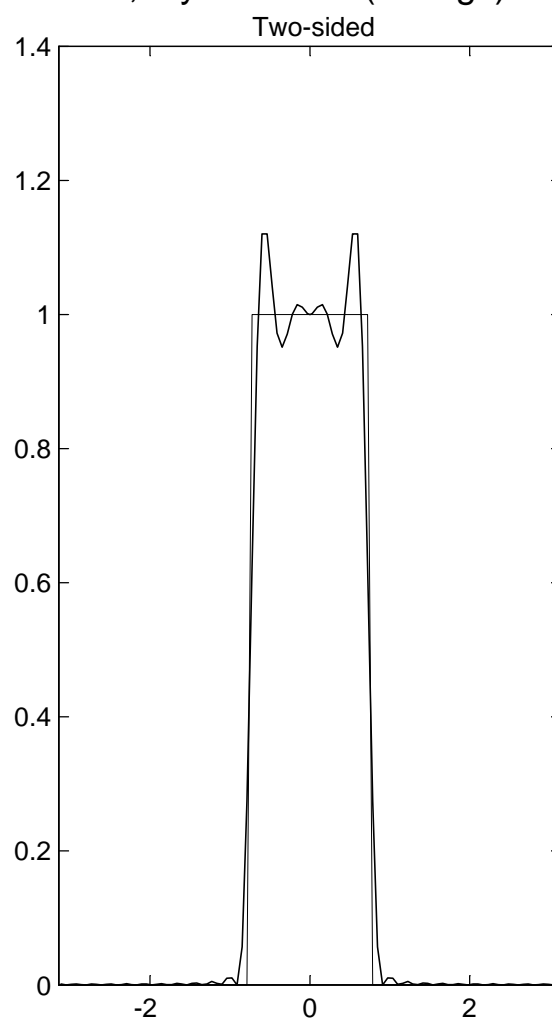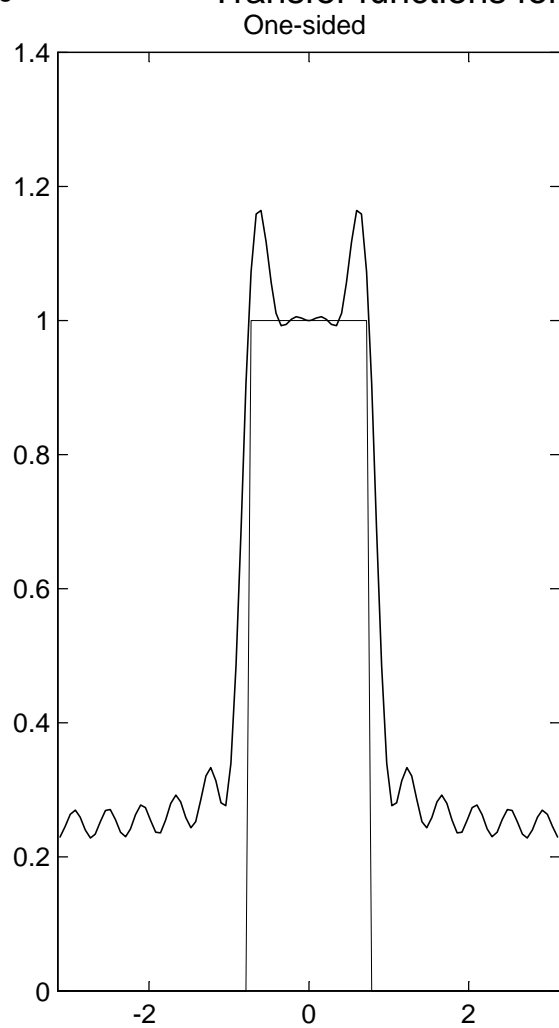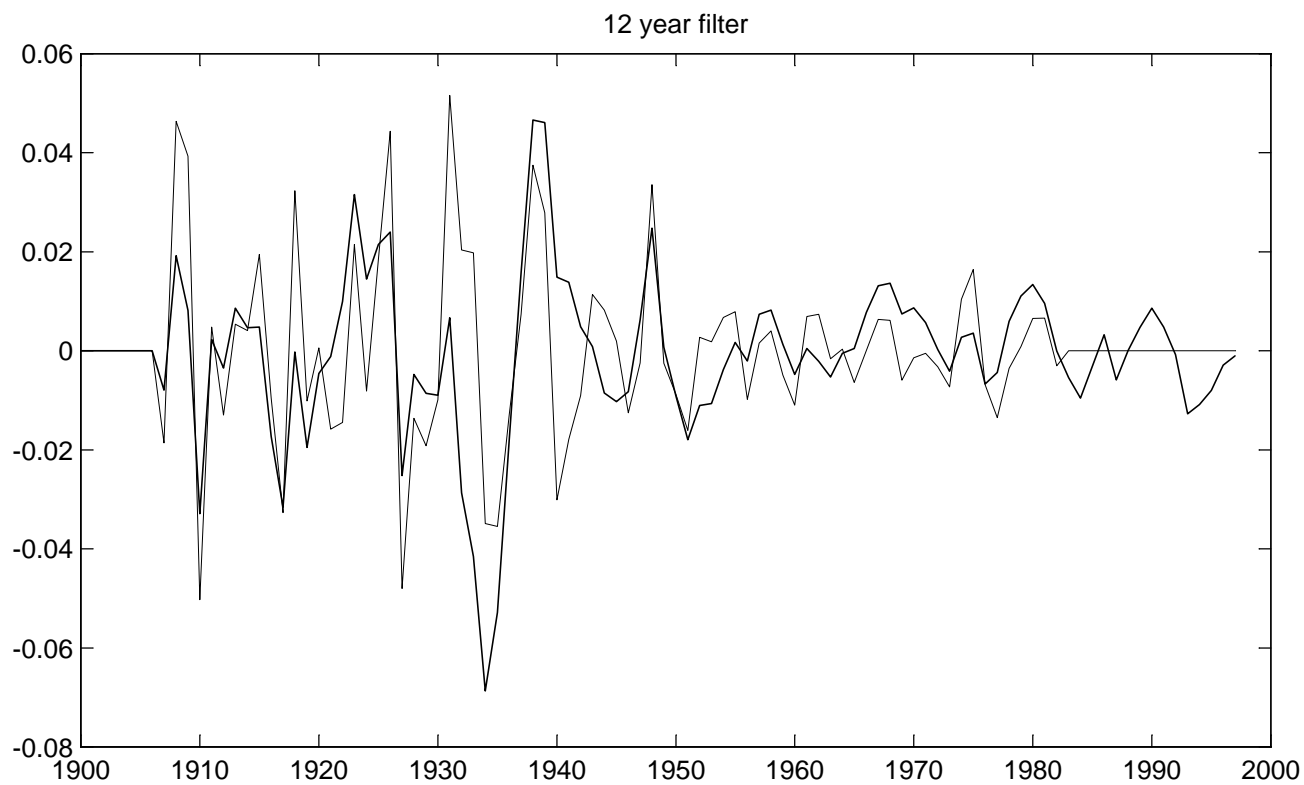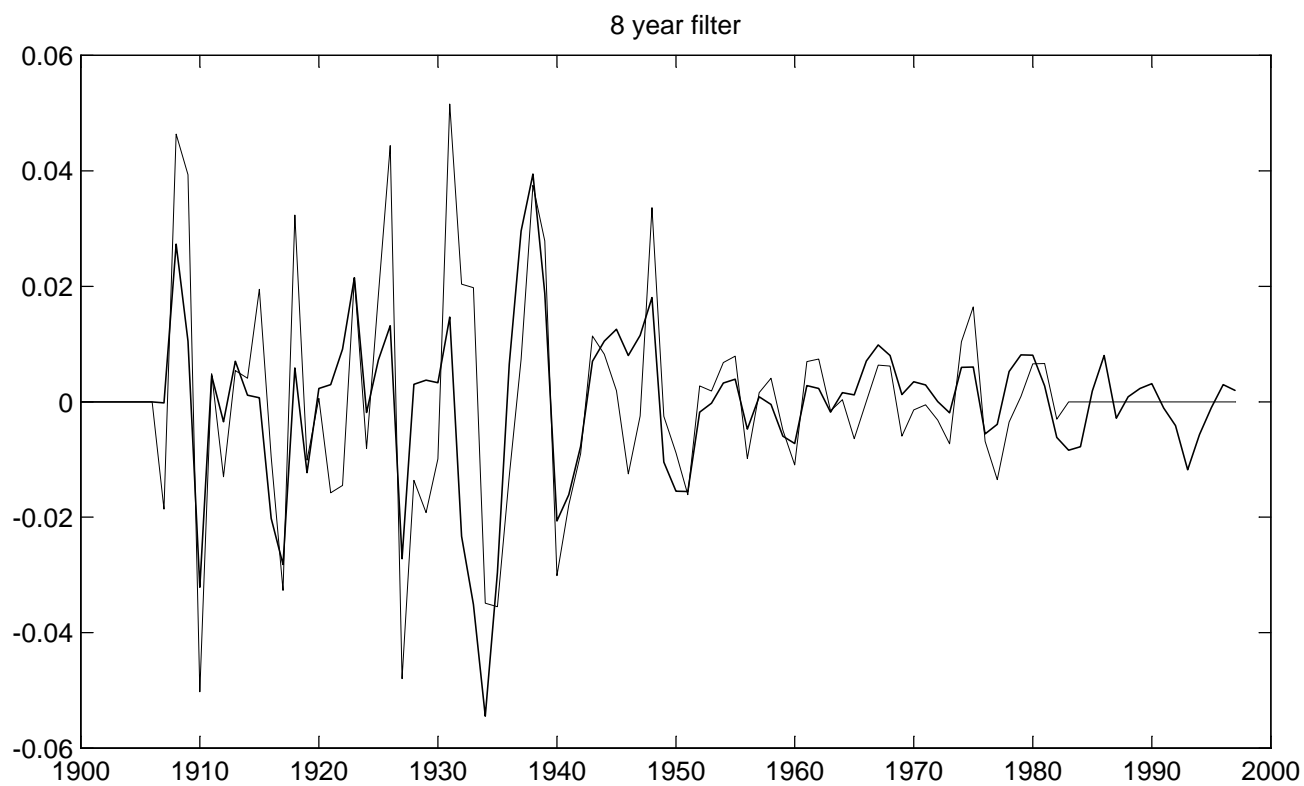Transfer functions for low-pass filters, 8 year cutoff (15 lags)

Figure 3A        Consumption Deviations from Trend

8 year filter



12 year filter



*(The thin line is obtained with a two-sided, 8 year filter)*

Figure 3B     Consumption Deviations from Trend



16 year filter
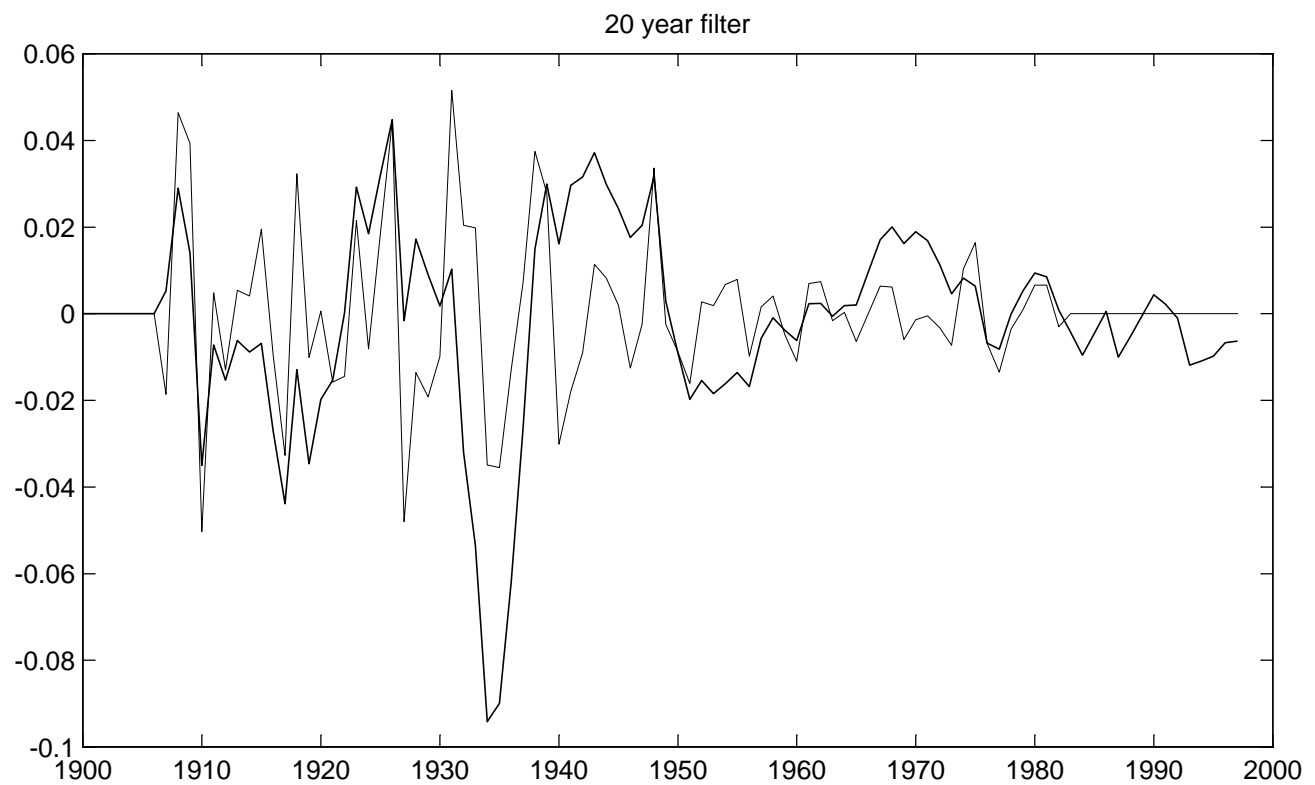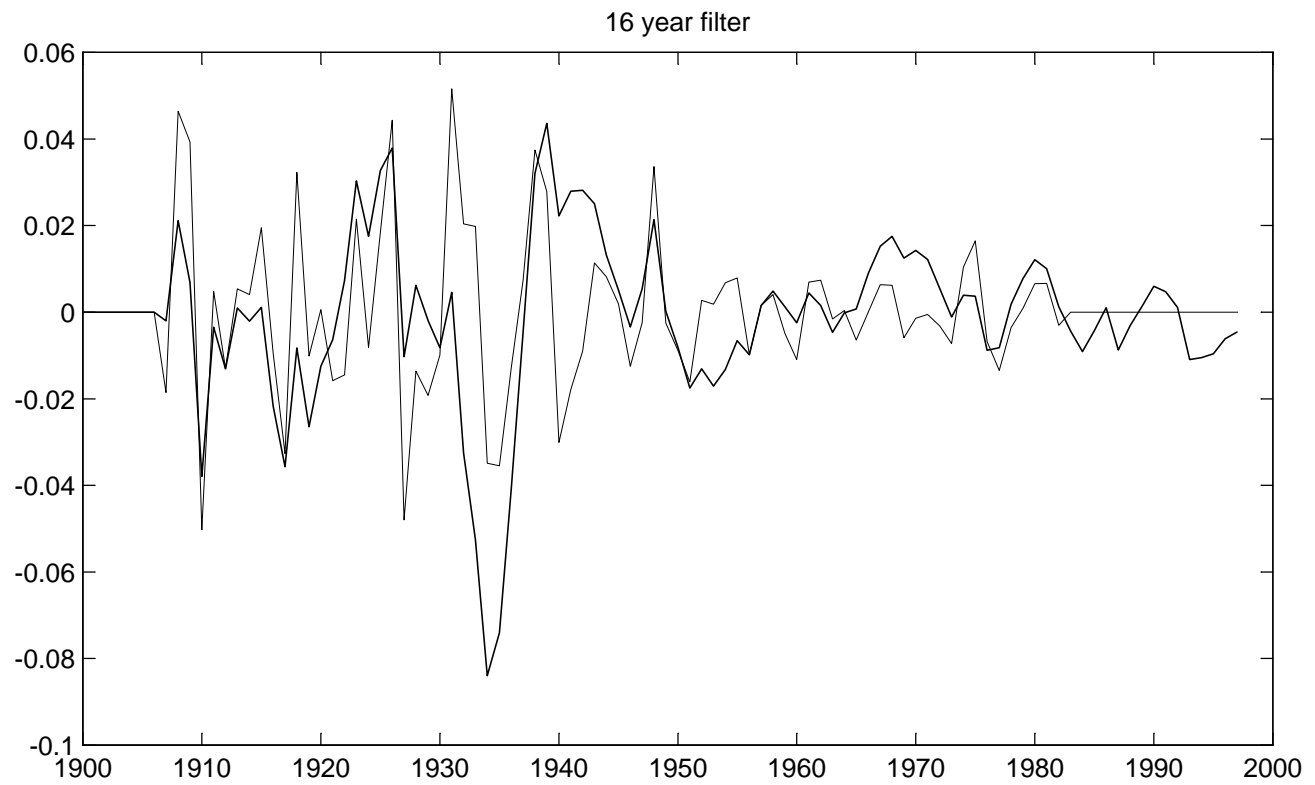


20 year filter

*(The thin line is obtained with a two-sided, 8 year filter)*

Figure 3C    Consumption Deviations from Trend
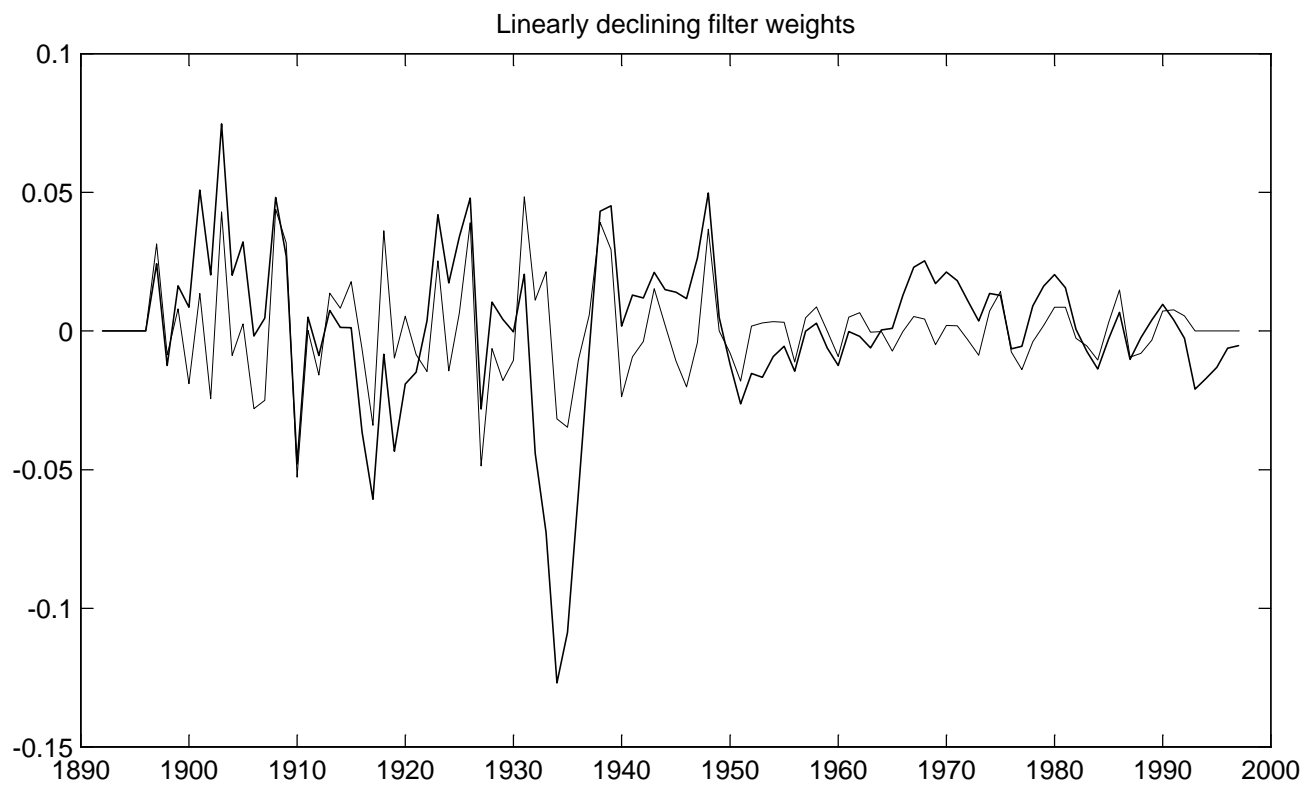
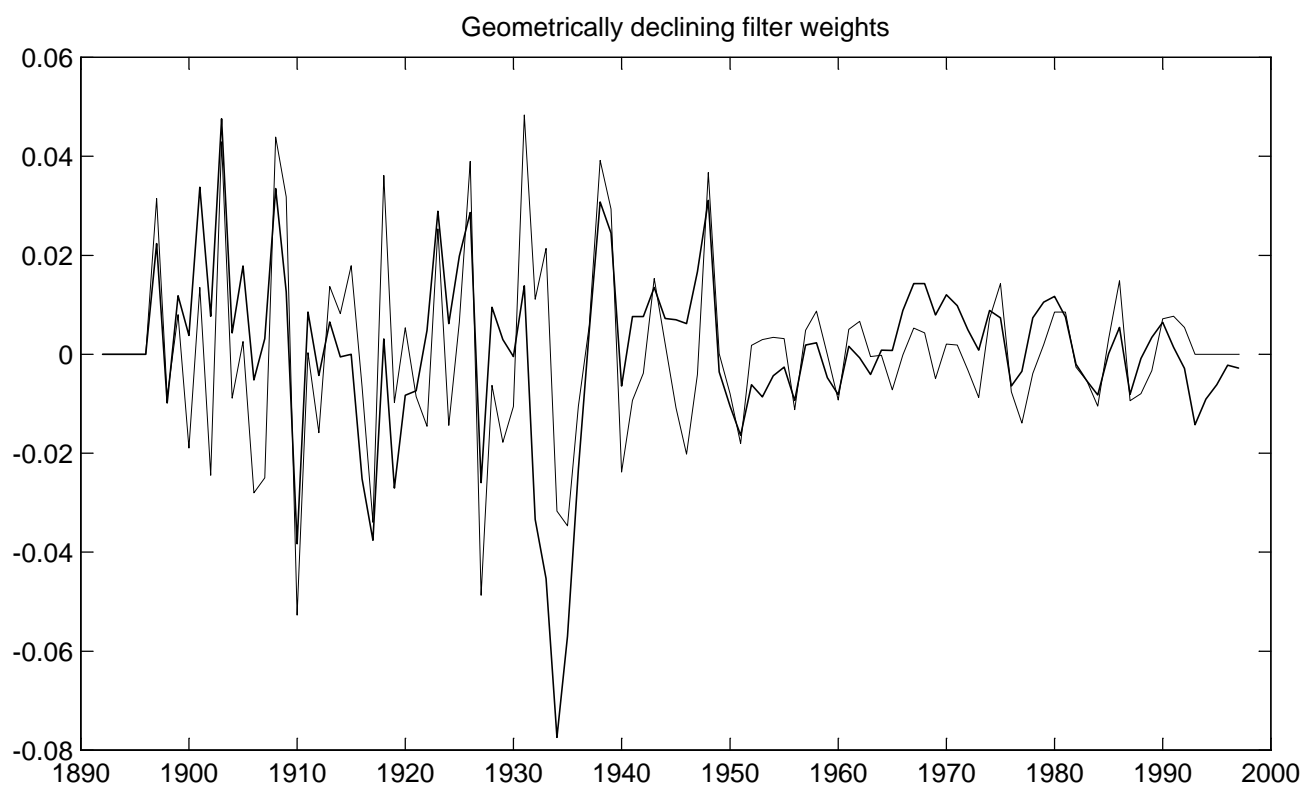*(The thin line is obtained with a two-sided, 8 year filter)*

Figure 4         Transfer function for adjusted one-sided filter (8years, theta = 1.4)