

# Gaps and Triangles\*

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## Abstract

We derive principles of optimal short run monetary policy in a real business cycles model, with money, and prices set in advance. The Friedman rule is the optimal policy, but the allocation is indeterminate. We move away from the Friedman rule, take the interest rate path as exogenous, and characterize the optimal money supply policy. In our set up it is feasible to replicate the allocation under flexible prices and we identify the conditions under which it is optimal to do so. In general, sticky prices provide the planner with tools to improve upon a distorted flexible prices allocation.

## 1. Introduction

How should monetary policy be conducted in the short run, if prices are sticky?  
Can monetary policy be used to replicate the fluctuations under flexible prices? Is

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this allocation desirable, when there are other distortions in the economy? In this paper we derive general principles on how to conduct short run monetary policy, by analyzing a simple environment with three main ingredients: an underlying real business cycles model without capital and with monopolistically competitive firms, a cash-in-advance restriction on the households transactions, and a restriction on firms that prices must be set one period in advance. In deriving optimal stabilization policy rules we recall the criticism in Lucas (1980) of a welfare criterion based on "gaps" instead of "triangles":

"...since fluctuations about the system's equilibrium represented disequilibrium behavior, standard welfare propositions could be applied only to the average behavior of the system, and not to fluctuations about the average. This left one free to apply other criteria in evaluating stabilization policies: "gaps" instead of "triangles", as James Tobin [1977] puts it. The general idea was to use policy tools to keep the actual path of the system "close" in one sense or another to its equilibrium path." Lucas (1980, p.)

In our RBC economy with money and flexible prices, as in Cooley and Hansen (1989), monetary policy affects the allocation solely through the path of nominal interest rates. There is a large set of money supply policies consistent with that path of nominal interest rates, and characterized by different paths for the price levels. If prices are set in advance, there is also a large set of money supply policies consistent with the given path for the nominal interest rates. However, those different money supply policies are associated with different real allocations, that in general deviate from the allocation under flexible prices. These deviations from flexible prices, for a particular money supply policy under sticky prices, are our definition of gaps.

By definition, then, a model in which gaps can be addressed must exhibit price stickiness. However, that same friction that creates the gaps in the economy provides the government with an additional policy tool, namely, a short run money supply policy, that can be conducted separately from the nominal interest rate policy. Interestingly enough, the government can always undo the distortions (close the gaps) created by the price stickiness using this additional policy instrument. This is shown in Carlstrom and Fuerst (1998a) and in Adao, Correia and Teles (1999). It is even more interesting though, that under many circumstances, a benevolent government will not want to close those gaps, meaning that it will

achieve a better allocation, from a welfare point of view, than the flexible prices allocation.

To highlight this result, we will use the methodological device of taking as given a nominal interest rate path, and will show how the additional policy instrument ought to be used by a welfare maximizer government. This way we are able to compare the economy under flexible prices that is distorted by an interest rate path, and the same economy under sticky prices, with the same interest rates, but where the money supply policy can be used to replicate the flexible prices allocation or, possibly, to achieve a better allocation. Our results are thus conditioned on an arbitrary nominal interest rate policy. Ireland (1996) shows that it is optimal to follow the Friedman rule, setting the nominal interest rate to zero, in our same set up when the utility function is separable in leisure and logarithmic in consumption. We extend the result to general preferences. However, under the Friedman rule, since the cash in advance constraint is not binding, the money supply cannot be used to determine the allocation. As Carlstrom and Fuerst (1998b) show, the allocation is indeterminate and there are equilibria with sunspot fluctuations. The set of allocations includes the one under flexible prices, which may not be the optimal one. This indeterminacy result, coupled with the irrelevance of the money supply policy, partly justifies our approach of moving away from the Friedman rule and taking as exogenous the interest rate path.

There are three sources of distortions in this economy: monopolistic competition, the nominal interest rate and the price stickiness. In general there could be an optimal "degree" of price stickiness, given the presence of the other distortions. This is not necessarily the case. If we only consider technological shocks, and the nominal interest rates are constant across states, then, for preference structures that are aggregable and consistent with balanced growth, it is optimal to replicate the flexible prices allocation, therefore closing gaps. The arguments are public finance arguments. The ex-post social mark up across all states of the world should be the same if in the optimal taxation problem the solution is to set the same tax across goods in different states of the world. In this second best analysis, where we treat gaps as triangles, we still obtain the policy recommendation that monetary policy should be conducted so that gaps are closed. Based on the apparent robustness of this result, one could be led to conclude that gaps can be treated separately from other distortions. This is not the case. As soon as we relax the assumption that interest rates are not state dependent, allow for velocity shocks, or consider other preference structures, the robustness of the result vanishes.

The literature on optimal monetary policy in an imperfectly competitive and sticky prices world is relatively recent. Papers related to the current analysis include Ireland (1996), Rotemberg and Woodford (1997, 1999), Goodfriend and King (1997), Carlstrom and Fuerst (1998a, 1998b), King and Wolman (1998), Gali and Monacelli (1999) and Erceg et al. (2000).

Our analysis is orthogonal to the analysis in Rotemberg and Woodford (1997, 1999), Gali and Monacelli (1999) and Erceg et al (2000). They analyze an environment with our basic structure but with three main differences. They allow for fiscal instruments that undo the monopolistic competition distortion; one way or another those are cash-less economies, where money is only a unit of account; and prices are set in a staggered fashion. Erceg et al (2000) also consider sticky wages. The nominal rigidities are the only distortions so that the flexible prices allocation, if feasible, is optimal. Since by assumption there is no money demand in these models, money supply does not play a role. The nominal interest rate is the sole instrument of monetary policy. When the flexible prices allocation is feasible, the optimal nominal interest rate policy is to target the real interest rate under flexible prices.

We could extend our analysis to an environment with staggered price setting, as long as the model includes a money demand. For a given path of the nominal interest rates, there will still be multiple money supply paths associated with different real allocations. However it would no longer be the case that one could replicate the allocation under flexible prices for any interest rate path. The only one such path will be the one that coincides with the real interest rate, under flexible prices. In our environment where the agents are restricted from setting the prices contemporaneously the planner can improve upon the allocation if it can use as an instrument the money supply, for a particular path of interest rates. In an environment where some of the agents are restricted from choosing inflation, the planner has to be able to use the interest rate policy in order to reach or to improve upon the flexible prices allocation.

Erceg *et al* (2000) assume that there are monopolistic distortions and staggered nominal contracts both in the labor and the product markets. They show, assuming that the fiscal policy eliminates the monopolistic distortions in the steady state, that there is a trade-off in stabilizing the output gap, price inflation and wage inflation. Moreover, for some specifications strict price inflation targeting produces relatively large welfare losses. When agents have too few degrees of freedom to adjust to the shocks they are subject to, it may not be possible to use monetary policy to replicate the flexible prices environment allocation. That is

precisely what happens in Erceg *et al.* (2000) with sticky contracts in the labor market and in the good market, and in Adao *et al.* (1999) when both the prices in the goods markets and the choice of portfolios are sticky.

King and Wolman (1998) study a model with staggered price setting and keep the monopolistic competition distortion but get rid of the nominal interest rate distortion by allowing interest to be paid on currency. They show that the flexible prices solution is optimal in the deterministic case.

Carlstrom and Fuerst (1998a) assume that prices are set one period in advance. Under an interest rate policy, where the government is restricted from setting the money supply, there is real indeterminacy and sunspots. This is a source of concern for them, while for us it is the means to achieve the optimal allocation, since we do not restrict the planner from using the money supply instrument. They also look at a policy where money supply is chosen so that prices are predetermined, and the allocation is the one under flexible prices. In running this policy they restrict the government from setting the nominal interest rate, so that there are many equilibria consistent with it and characterized by different interest rate paths.

The remainder of the paper is organized as follows. In section 2 we outline the model and characterize the equilibria under flexible and sticky prices. Under flexible prices the allocation is a function of monetary policy only through the nominal interest rate path. Instead, under sticky prices, the allocation depends on the money supply policy for a given nominal interest rate policy. In Section 3 we define the Ramsey problem in the economy with sticky prices and start by determining the optimal interest rate policy (Section 3.2). We then take the process for the nominal interest rates as exogenous and determine the optimal money supply policy consistent with that (Section 3.3). We identify the conditions under which the Ramsey solution coincides with the allocation under flexible prices. We show that for preferences that are aggregable and consistent with balanced growth, the Ramsey solution and the allocation under flexible prices coincide, if interest rates are not state dependent. If interest rates are state dependent it is no longer optimal to replicate the allocation under flexible prices. In Section 4 we determine the Ramsey solution when there are shocks to velocity, and show that, in order for the flexible prices allocation to be optimal, the nominal interest rate will have to fluctuate with the velocity shocks. Section 5 contains the conclusions.

## 2. The economy

Our model economy has a simple structure as in Ireland (1996) or Carlstrom and Fuerst (1998a). The economy consists of a representative household, a continuum of firms indexed by  $i \in [0, 1]$ , and a government or central bank.

Each firm produces a distinct, perishable consumption good, indexed by  $i$ . The production uses labor, according to a linear technology. There are aggregate shocks to this technology,  $s_t$ , that follow a Markov process. The history of these shocks up to period  $t$  is denoted by  $s^t$ .

The government makes a lump-sum monetary transfer  $X_t$  at the assets market to the representative household at each date  $t = 0, 1, 2, \dots$ . The money supply evolves according to  $M_t^s = M_{t-1}^s + X_t$ . If  $M_{t+1}$  denotes the money carried by the household into period  $t + 1$ , market clearing requires that  $M_t^s = M_{t+1}$ , for all  $t = 0, 1, 2, \dots$

### 2.1. The households

The households have preferences over composite consumption  $C_t$ , and leisure  $1 - N_t$ , and described by the expected utility function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t) \right\} \quad (2.1)$$

where  $\beta$  is a discount factor and  $C_t$  is

$$C_t = \left[ \int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1.$$

where  $\theta$  is the elasticity of substitution between any two goods.

The households start period  $t$  with outstanding money balances,  $M_t$ , and decide to buy  $B_{t+1}$  units of money in nominal bonds that pay  $R_t B_{t+1}$  units of money one period later. They also decide to spend  $E_t Q_{t,t+1} A_{t+1}$  in state contingent nominal securities. Each security pays one unit of money at the beginning of period  $t + 1$  in a particular state.  $Q_{t,t+1}$  is the price, normalized by the probability of the occurrence of the state, in the beginning of period  $t$ . The households also receive monetary transfers  $X_t$ . The purchases of consumption,  $\int_0^1 P_t(i) c_t(i) di$ , where  $P_t(i)$  is the price of good  $i$ , in units of money, have to be made with  $M_t - B_{t+1} + R_{t-1} B_t - E_t Q_{t,t+1} A_{t+1} + A_t + X_t$  so that the following cash in advance constraint must be satisfied,

$$\int_0^1 P_t(i)c_t(i)di \leq M_t - B_{t+1} + R_{t-1}B_t - E_t Q_{t,t+1}A_{t+1} + A_t + X_t \quad (2.2)$$

At the end of the period, the households receive the labor income,  $W_t N_t$  where  $W_t$  is the nominal wage rate, that can be used to purchase consumption in the following period. They also receive the dividends from the final goods firms  $\int_0^1 \Pi_t(i)di$ .

The households face the budget constraints

$$M_{t+1} \leq \left[ M_t - B_{t+1} + R_{t-1}B_t - E_t Q_{t,t+1}A_{t+1} + A_t + X_t - \int_0^1 P_t(i)c_t(i)di \right] + W_t N_t + \int_0^1 \Pi_t(i)di \quad (2.3)$$

The Bellman equation that describes the households' problem is

$$V(M_t, B_t, A_t, s_t) = \text{Max} \{u(C_t, 1 - N_t) + \beta E_t V(M_{t+1}, B_{t+1}, A_{t+1}, s_{t+1})\},$$

where the maximization is subject to (2.2) and (2.3).

Let  $P_t = [\int P_t(i)^{1-\theta} di]^{\frac{1}{1-\theta}}$ . For  $R_t > 1$ ,  $t = 0, 1, 2, \dots$ , the following are first order conditions of this problem

$$\frac{c_t(i)}{C_t} = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} \quad (2.4)$$

$$\frac{u_{1-Nt}}{u_{Ct}} = \frac{W_t}{P_t} \frac{1}{R_t}, \quad (2.5)$$

$$\frac{u_{Ct}}{P_t} = R_t E_t \left[ \frac{\beta u_{Ct+1}}{P_{t+1}} \right], \quad (2.6)$$

$$Q_{t,t+1} \frac{P_{t+1}}{P_t} = \frac{\beta u_{Ct+1}}{u_{Ct}}, \quad (2.7)$$

and

$$E_t [Q_{t,t+1}] = \frac{1}{R_t}. \quad (2.8)$$

Condition (2.4) defines the demand for each of the intermediate goods  $i$  and condition (2.5) sets the intratemporal marginal rate of substitution between leisure

and consumption equal to the real wage. Condition (2.6) is a requirement for the optimal savings decision. One additional unit of  $B_{t+1}$  implies the marginal cost of  $\frac{U_{ct}}{P_t}$ , and the benefit of  $R_t E_t \left[ \frac{\beta U_{c,t+1}}{P_{t+1}} \right]$ . Condition (2.7) says that for each state of nature at  $t+1$ , the intertemporal marginal rate of substitution in consumption must be equal to the ratio between the amount of money at  $t$  necessary to buy one unit of the good at  $t+1$  and the amount of money necessary to buy one unit of the good at  $t$ . Condition (2.8) says that at  $t$  the money value of an unit of money at  $t+1$ ,  $\frac{1}{R_t}$ , is equal to the expenditure at  $t$  necessary to get one unit of each contingent nominal security.

## 2.2. Firms

Each firm  $i$  has the production technology

$$y_t(i) \leq s_t n_t(i) \quad (2.9)$$

where  $y_t(i)$  is the production of good  $i$  and  $s_t$  is an aggregate technology shock.

The problem of the firm is to choose the price in order to maximize profits that can be used for consumption in period  $t+1$  taking the demand function

$$\frac{y_t(i)}{C_t} = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} \quad (2.10)$$

obtained from (2.4) as given, and satisfying the technology constraint.

### 2.2.1. Under flexible prices

Under flexible prices, the value of period  $t$  profits in units of utility<sup>1</sup> is  $E_t \frac{\beta u_{C,t+1}}{P_{t+1}} \Pi_t(i)$ . The maximization of this expression is equivalent to the maximization of  $\Pi_t(i)$ , which is given by expression,

$$\Pi_t(i) = P_t(i) y_t(i) - W_t n_t(i)$$

where  $P_t(i)$  satisfies the demand function (2.4). The first order condition of this problem is

$$P_t(i) \left[ 1 + \frac{d \ln P_t(i)}{d \ln y_t(i)} \right] - \frac{W_t}{s_t} = 0$$

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<sup>1</sup>Alternatively, the value of period  $t$  profits in units of money at the beginning of the period  $t$  is  $E_t Q_{t,t+1} \Pi_t(i)$  and in units of consumption in period  $t$  is  $E_t \frac{Q_{t,t+1}}{P_t} \Pi_t(i)$ . All these expressions are equivalent in the sense that they have the same maximizers.



where  $\frac{d \ln P_t(i)}{d \ln y_t(i)} = -\frac{1}{\theta}$ , since  $\theta$  is the demand elasticity. Therefore, it must be that

$$P_t = P_t(i) = \frac{\theta}{\theta - 1} \frac{W_t}{s_t} \quad (2.11)$$

The firms set a common price, which is a constant mark-up over marginal cost. As the elasticity of demand  $\theta$  gets larger, the mark-up converges down to 1.

### 2.2.2. When prices are set in advance

We consider now an environment where firms set the prices one period in advance and sell the output on demand in period  $t$  at the previously chosen price.<sup>2</sup>

When firm  $i$  sets prices one period in advance, it solves the problem of choosing at  $t - 1$ , the price  $P_t(i)$  that maximizes the expected value of profits, i.e.  $E_{t-1} \left[ \beta^2 \frac{u_{C_{t+1}}}{P_{t+1}} \Pi_t(i) \right]$  subject to (2.4) and (2.9).<sup>3</sup>

The solution of the firms' problem must satisfy the first order condition

$$E_{t-1} \left[ \frac{u_{C_{t+1}} C_t (1 - \theta)}{P_{t+1} P_t^{-\theta}} + \frac{u_{C_{t+1}}}{P_{t+1}} C_t \theta \frac{P_t(i)^{-1} W_t}{P_t^{-\theta} s_t} \right] = 0.$$

Thus, the price chosen by the firm is

$$P_t(i) = P_t = \frac{\theta}{(\theta - 1)} E_{t-1} \left[ v_t \frac{W_t}{s_t} \right] \quad (2.12)$$

where  $v_t = \frac{\frac{u_{C_{t+1}}}{P_{t+1}} C_t}{E_{t-1} \left[ \frac{u_{C_{t+1}}}{P_{t+1}} C_t \right]}$ .

### 2.3. Market clearing:

In each period there are markets for loans, goods, labor and state contingent nominal securities. Market clearing conditions are given by the following expressions,

$$c_t(i) = y_t(i),$$

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<sup>2</sup>This only makes sense if the size of the shocks is sufficiently small so that the firms still prefer to satisfy demand instead of shutting down.

<sup>3</sup>Alternatively could have profits in units of money at  $t - 1$ , i.e.  $E_{t-1} [Q_{t-1,t} Q_{t,t+1} \Pi_t(i)]$ . Since  $Q_{t,t+1} = \frac{\beta u_{C_{t+1}} P_t}{u_{C_t} P_{t+1}}$ , this objective function is equivalent to the one in the text as they have the same maximizers.

so that the demand for each of the goods is equal to the supply;

$$N_t = \int_0^1 n_t(i) di$$

so that the total demand of labor is equal to the supply;

$$B_{t+1} = 0$$

and

$$A_{t+1} = 0$$

since nominal bonds and the contingent securities are in zero net supply.

From the cash in advance constraints (2.2) of the households and the market clearing conditions we have, in equilibrium,

$$P_t c_t = M_t + X_t \equiv M_t^s. \quad (2.13)$$

#### 2.4. Imperfectly competitive equilibrium:

An equilibrium are initial  $s^0$ ,  $M_0$ ,  $B_0$ , and  $A_0$ , prices  $\{(P_t, P_t(i), W_t, Q_{t,t+1}, R_t)_{s^t \in S^t}\}_{t=0}^\infty$ , allocations  $\{(C_t, C_t(i), N_t, M_{t+1}, B_{t+1}, A_{t+1}, n_t(i), y_t(i))_{s^t \in S^t}\}_{t=0}^\infty$ , and productivity shocks  $\{s^t \in S^t\}_{t=0}^\infty$  such that:

- (i) given the prices  $\{(P_t, P_t(i), W_t, Q_{t,t+1}, R_t)_{s^t \in S^t}\}_{t=0}^\infty$  the sequences  $\{(C_t, C_t(i), N_t, M_{t+1}, B_{t+1}, A_{t+1})_{s^t \in S^t}\}_{t=0}^\infty$  solve the problem of the representative household;
- (ii) given the restrictions on price setting, the prices  $\{(P_t, W_t)_{s^t \in S^t}\}_{t=0}^\infty$ , and the productivity shocks  $\{s^t \in S^t\}_{t=0}^\infty$ , the sequences  $\{(y_t(i), n_t(i), P_t(i))_{s^t \in S^t}\}_{t=0}^\infty$  solve the problem of the firms;
- (iii) all markets clear;

#### 2.5. Allocations under flexible and under sticky prices

It is useful to notice that the two first order conditions of the households and firms, under flexible prices, (2.5) and (2.11), can be written as

$$\frac{u_{1-N_t}(C_t, 1 - N_t)}{u_{C_t}(C_t, 1 - N_t)} = \frac{W_t}{P_t R_t} = \frac{(\theta - 1)}{\theta R_t} s_t, \quad (2.14)$$

The feasibility constraints are

$$C_t = s_t N_t \quad (2.15)$$

Conditions (2.14) and (2.15) determine the flexible prices allocation for consumption,  $C_t$ , and labor,  $N_t$ , as a function of the nominal interest rate,  $R_t$ , and the shock  $s_t$ .

Using these conditions and the equilibrium cash-in-advance constraint (2.13), we can write an implicit function for a money demand

$$\frac{u_{1-N_t} \left( \frac{M_t^s}{P_t}, 1 - \frac{1}{s_t} \frac{M_t^s}{P_t} \right)}{u_{C_t} \left( \frac{M_t^s}{P_t}, 1 - \frac{1}{s_t} \frac{M_t^s}{P_t} \right)} = \frac{(\theta - 1) s_t}{\theta R_t}, \quad (2.16)$$

If the utility function had the form  $u = \frac{C_t^{1-\sigma}}{1-\sigma} - \alpha N_t$ , then we could write the money demand as

$$\frac{M_t^s}{P_t} = \left[ \frac{(\theta - 1) s_t}{\alpha \theta R_t} \right]^{\frac{1}{\sigma}} \quad (2.17)$$

Given an interest rate, the money supply can be used to determine the price level.

Now, using the first order conditions in the environment with prices set in advance, (2.5) and (2.12), we still have

$$\frac{u_{1-N_t} \left( \frac{M_t^s}{P_t}, 1 - \frac{1}{s_t} \frac{M_t^s}{P_t} \right)}{u_{C_t} \left( \frac{M_t^s}{P_t}, 1 - \frac{1}{s_t} \frac{M_t^s}{P_t} \right)} = \frac{W_t}{P_t R_t}$$

or, in the above example for the utility function

$$\left[ \frac{M_t^s}{P_t} \right]^{\sigma} = \frac{W_t}{\alpha P_t R_t} \quad (2.18)$$

In the environment with sticky prices the money supply has a direct effect on the real wage, since prices are predetermined. The real wage however has to satisfy the following condition, in expected value

$$E_{t-1} \left[ v_t \frac{\frac{W_t}{P_t}}{(\theta-1)s_t} \right] = 1. \quad (2.19)$$

so that, in a loose sense the money demand has to be satisfied only in the average. Given a path for the nominal interest rates, there is a continuum of money supply

policies that are consistent with that path, and are associated with different real allocations. This is the sense in which Carlstrom and Fuerst (1998a) call attention to the real indeterminacy associated with an interest rate rule under sticky prices. Our approach is very different, since we allow the planner to decide on the money supply and, therefore, to use the degrees of freedom implied by (2.19) to achieve the optimal allocation.

### 3. Optimal allocations under sticky prices

In this section we determine allocations under commitment that maximize the representative agent's expected utility subject to the feasibility constraints and the first order conditions of the households' problem and of the firms' problem. Under sticky prices the first order condition of the households' problem (2.5) can be combined with the first order condition of the firms' problem (2.12), to obtain an implementability condition,

$$E_{t-1} \left[ v_t \frac{1}{\theta R_t} \frac{u_{1-Nt}}{s_t} \frac{u_{Ct}}{u_{Ct}} \right] = 1. \quad (3.1)$$

Using the law of iterated expectations in (3.1), we have

$$E_{t-1} \left[ \frac{E_t \left( \frac{u_{Ct+1}}{P_{t+1}} \right) C_t}{E_{t-1} \left[ E_t \left( \frac{u_{Ct+1}}{P_{t+1}} \right) C_t \right]} \frac{1}{\theta R_t} \frac{u_{1-Nt}}{s_t} \frac{u_{Ct}}{u_{Ct}} \right] = 1. \quad (3.2)$$

Using the intertemporal condition (2.6) in (3.2), we get

$$E_{t-1} \left[ \frac{u_{Ct}}{\theta R_t} C_t - u_{1-Nt} N_t \right] = 0. \quad (3.3)$$

This implementability constraint is exactly the expected value of the constraints (2.14) of the flexible prices allocation.

#### 3.1. Ramsey problem with sticky prices

The Ramsey problem with sticky prices is the choice of sequences of consumption,  $C_t$ , labor,  $N_t$ , and interest rates  $R_t$ , that maximize (2.1) subject to the feasibility

constraint (2.15) and the implementability condition (3.3) and the condition that net nominal interest rates are non-negative

$$R_t \geq 1$$

Notice that any allocation under flexible prices satisfies the constraints of the Ramsey problem under sticky prices. Thus, the optimal allocation in an environment with sticky prices is at least as good as the optimal allocation in an environment with flexible prices.

The Lagrangian for the Ramsey problem under sticky prices can be written as

$$\begin{aligned} \mathfrak{L} = & \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \Pr(s^t) u(C_t, 1 - N_t) \\ & + \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \lambda_t(s^t) \Pr(s^t) (s_t N_t - C_t) \\ & + \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \varphi_t \Pr(s^t) \left( \frac{u_{C_t}}{\theta R_t} C_t - u_{1-N_t} N_t \right) \end{aligned}$$

where  $\beta^t \lambda_t(s^t) \Pr(s^t)$ ,  $\beta^t \varphi_t \Pr(s^t)$  are the multipliers of the technology constraint and the implementability condition, respectively, and  $\Pr(s^t)$  are the probabilities of the real shocks.

The formalization of the Ramsey problem in terms of the choice of the allocations and the nominal interest rates allows for a very simple representation of the set of possibilities and allows for the determination of the optimal policy in two steps: In the first step we determine the optimal allocation for a given path of the nominal interest rates. In the second step, the indirect utility function that is a function of the path for the nominal interest rate is maximized.

### 3.2. The choice of the optimal interest rate policy

In this environment it is optimal to follow the Friedman rule, setting the nominal interest rate,  $R_t$ , to one as stated in the following proposition:

**Proposition 3.1.** *The interest rate,  $R_t$ , that allows to achieve the optimal allocation when prices are set one period in advance is always equal to one, i.e. the Friedman Rule is optimal.*

**Proof:**

A marginal increase in the nominal interest rate has a negative impact on utility, which is given by

$$-\beta^t \varphi_t \Pr(s^t) \frac{u_{C_t}}{\frac{\theta R_t^2}{(\theta-1)}} C_t. \quad (3.4)$$

Given the non-negativity constraint on the net nominal interest rate and the fact that in this second best environment  $\varphi_t$  is always positive, it is optimal to set  $R_t^* = 1$ . ■

When the nominal interest rate is zero, the level of real balances is indeterminate as the cash in advance constraint is nonbinding. Under flexible prices there is a unique equilibrium allocation, as the desire of agents to spend cash is matched with fluctuations in the current price level. However, in a sticky prices environment sunspot equilibria have real consequences. The fact that output is demand determined and agents in the economy vary the amount of cash they spend in response to some sunspot event leads to real indeterminacies (see Carlstrom and Fuerst, 1998b).

Thus, if  $R_t = 1$  and prices are sticky, equilibrium allocations are not unique. Real balances are indeterminate implying that the monetary authority cannot select one of these equilibria by the use of money supply. The set of equilibrium allocations includes the one under flexible prices. Some of them are clearly bad in utility terms. In order to avoid the occurrence of these equilibria it can be justified to set the interest rate policy away from the Friedman rule.

**3.3. Optimal policy for a given positive interest rate path**

We will pursue the analysis of the optimal policy assuming that the interest rates  $R_t > 1$  are exogenous. The first order conditions of the Ramsey problem are

$$u_{C_t} - \lambda_t + \varphi_t \frac{(\theta - 1)}{\theta R_t} (u_{C_t} + u_{C_t C_t} C_t) - \varphi_t u_{1-N_t, C_t} N_t = 0 \quad (3.5)$$

$$-u_{1-N_t} + \lambda_t s_t - \varphi_t \frac{(\theta - 1)}{\theta R_t} u_{C_t, 1-N_t} C_t + \varphi_t (u_{1-N_t, 1-N_t} N_t - u_{1-N_t}) = 0 \quad (3.6)$$

Manipulating these first order conditions, we obtain

$$1 + \varphi_t = \frac{u_{C_t}}{u_{1-N_t}} s_t \left( 1 + \varphi_t \frac{(\theta - 1)}{\theta R_t} \right) + \varphi_t \frac{(\theta - 1)}{\theta R_t} \frac{u_{C_t C_t} C_t}{u_{C_t}} \frac{u_{C_t}}{u_{1-N_t}} s_t - \quad (3.7)$$

$$-\varphi_t \frac{u_{1-Nt, Ct} N_t}{u_{1-Nt}} s_t - \varphi_t \frac{(\theta-1) u_{Ct, 1-Nt} C_t}{\theta R_t u_{1-Nt}} + \varphi_t \frac{u_{1-Nt, 1-Nt} N_t}{u_{1-Nt}}$$

For the utility function

$$U = \frac{C^{1-\sigma} - 1}{1-\sigma} - \alpha N$$

$$\frac{u_{Ct}}{u_{1-Nt}} s_t = \frac{1 + \varphi_t}{1 + \varphi_t \frac{(\theta-1)}{\theta R_t} (1-\sigma)} \quad (3.8)$$

When  $\sigma = 1$ , then

$$\frac{u_{Ct}}{u_{1-Nt}} s_t = 1 + \varphi_t \quad (3.9)$$

which means that the optimal proportionate wedge is constant across states.

If  $R_t$  is constant across states, then that wedge is constant across states, so that the allocation under flexible prices will be the optimal allocation.

We can write (3.8) as

$$\frac{u_{1-Nt}}{u_{Ct} s_t} = \frac{1}{1 + \varphi_t} + \frac{\varphi_t (\theta-1)}{1 + \varphi_t \theta R_t} (1-\sigma) \quad (3.10)$$

Under flexible prices, we have  $\frac{u_{1-Nt}}{u_{Ct} s_t} = \frac{(\theta-1)}{\theta R_t}$ . So, if the allocation under flexible prices is optimal, the shadow price of the implementability condition is  $\varphi_t = \frac{1+\theta R_t-\theta}{\sigma(\theta-1)}$ .

If  $R_t$  is not constant across states, then it is never optimal to replicate the flexible prices allocation. In this case, when  $\sigma < 1$ , it is not optimal to completely stabilize  $\frac{u_{1-Nt}}{u_{Ct} s_t}$ , but it moves as in the flexible prices case. When  $\sigma > 1$ , it moves in the opposite direction.

### 3.4. Are flexible prices optimal?

We determine the conditions under which the optimal solution under sticky prices coincides with the allocation under flexible prices. We analyze the case where  $R_t$  is constant across states.

**Proposition 3.2.** *If  $R_t$  is constant across states, the optimal allocation under sticky prices is the flexible prices allocation if and only if the expression*

$$-\frac{u_{Ct} C_t}{u_{Ct}} C_t + \frac{u_{Ct, 1-Nt}}{u_{Ct}} N_t - \frac{u_{1-Nt, 1-Nt}}{u_{1-Nt}} N_t + \frac{u_{Ct, 1-Nt}}{u_{1-Nt}} C_t \quad (3.11)$$

does not depend on  $s_t$ .

**Proof:** Under flexible prices,  $\frac{u_{C_t} s_t}{u_{1-N_t}} = \frac{\theta}{\theta-1} R_t$ , so that (3.7) becomes

$$\varphi_t = \frac{\frac{\theta}{\theta-1} R_t - 1}{-\frac{u_{C_t} C_t}{u_{C_t}} C_t + \frac{u_{C_t, 1-N_t}}{u_{C_t}} N_t + \frac{u_{1-N_t, C_t}}{u_{1-N_t}} C_t - \frac{u_{1-N_t, 1-N_t}}{u_{1-N_t}} N_t} \quad (3.12)$$

Let

$$D \equiv -\frac{u_{C_t} C_t}{u_{C_t}} C_t + \frac{u_{C_t, 1-N_t}}{u_{C_t}} N_t - \frac{u_{1-N_t, 1-N_t}}{u_{1-N_t}} N_t + \frac{u_{C_t, 1-N_t}}{u_{1-N_t}} C_t. \quad (3.13)$$

and suppose  $R_t$  is not state dependent. Then as long as  $D$  is not state dependent, the optimal solution is the flexible prices allocation ■

In this case the optimal solution under sticky prices sets the social mark up constant across states since under flexible prices the allocation has  $\frac{u_{C_t}}{u_{1-N_t}} = \frac{P_t}{W_t} = \frac{\theta}{\theta-1} \frac{R_t}{s_t}$  and  $R_t$  is assumed to be constant across states.

We proceed to analyze the conditions under which  $D$  in (3.13) does not depend on the state, so that flexible prices are optimal. This will be the case when the expression does not depend on the allocation, or the allocation is constant across states. We concentrate on the class of preferences that are Gorman aggregable, i.e. there is a representative household. For time separable preferences<sup>4</sup> we have the following classes, defined by the momentary utility function, that are consistent with balanced growth.:

$$\begin{aligned} 1) \quad u &= \frac{(C^\alpha (1-N)^\psi)^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \quad \sigma \neq 1 \\ 2) \quad u &= \alpha \log C + \psi \log(1-N) \end{aligned}$$

For these classes,  $D = \frac{u_{C_t, 1-N_t}}{u_{C_t}} - \frac{u_{1-N_t, 1-N_t}}{u_{1-N_t}} = \frac{N}{1-N}$ . Since  $N$  is constant across states, in the flexible price equilibrium, then the denominator is constant across states.

Other preferences that are Gorman aggregable but are not consistent with balanced growth, and are commonly used in macroeconomics, also satisfy the condition of a constant  $D$ . Examples are:

$$3) \quad u_t = \frac{C^{1-\sigma} - 1}{1-\sigma} - \alpha(1-N), \quad \sigma > 0$$

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<sup>4</sup>Preferences over consumption and labor are considered too, as the analysis and results of this paper still hold under that type of preferences.



$$4) u_t = \frac{(C - \zeta N^\psi)^{1-\sigma}}{1-\sigma}, \sigma > 0, \sigma \neq 1, \psi > 1$$

$$5) u_t = \log(C - \zeta N^\psi), \psi > 1$$

For the class 3),  $D = \sigma$ . For the function  $C - \gamma_s \zeta N^\psi$ ,  $D = 1 - \psi$ . Since  $D$  is invariant to monotonic transformations of preferences, then  $D$  is a constant for classes 4) and 5).

We have shown that when it is assumed that the nominal interest rates,  $R_t$ , are constant across states, the conditions under which the optimal solution under sticky prices is to replicate the flexible prices allocation are, in a sense, general conditions, for purposes of the description of aggregate behavior. One could be led to conclude on the robustness of a criterion that eliminates gaps, even if treated as triangles. This robustness vanishes when instead the exogenous process for the nominal interest rates is assumed to be state dependent.

In any case, there are still Gorman aggregable preferences that do not satisfy the constancy of  $D$  at the flexible price allocations. One simple example is given by:

$$6) u = \alpha C_t + (1 - N)^{\frac{1}{2}}$$

It is easy to see that here  $D = 1/2 \frac{N}{1-N}$ . As in the flexible price allocation  $N$  depends on the state, in this case  $\varphi_t$  is state dependent and therefore the fixed price optimal allocation dominates the flexible price one. For the parametrization: two equally probable i.i.d. shocks,  $S = \{1, 2\}$ ,  $\alpha = 2$ ,  $\theta = 2$  and  $R_t = 1$ , the flexible price allocation is given by .9375 in the good state and 0.75 in the bad state. However the pair of consumptions, 1.845 in the good state and 0.834 in the bad state, satisfies the restrictions of the Ramsey problem and gives a higher expected utility to the representative agent. In this particular case the social planner prefers a more stable production to the flexible price equilibrium production.

#### 4. Optimal policy with aggregate velocity shocks

In this section we determine the Ramsey policy when there are shocks to velocity,  $v_t$ , that follow a Markov process. Abstracting from the choice of each of the goods  $i$  that must satisfy condition (2.4), the representative household's problem is the following

$$V(M_t, B_t, A_t, s_t, v_t) = \text{Max} \{u(C_t, 1 - N_t) + \beta E_t V(M_{t+1}, B_{t+1}, A_{t+1}, s_{t+1}, v_{t+1})\}$$

subject to

$$P_t C_t \leq (M_t + X_t + B_t R_{t-1} - B_{t+1} + A_t - E_t Q_{t,t+1} A_{t+1}) v_t$$

$$M_{t+1} \leq [M_t + X_t + B_t R_{t-1} - B_{t+1} + A_t - E_t Q_{t,t+1} A_{t+1} - P_t C_t] + W_t N_t + \Pi_t$$

Where  $Q_{t,t+1}$  is the normalized price, at date  $t$ , of one unit of money at  $t + 1$ , in state  $(s_{t+1}, v_{t+1})$ . The following conditions summarize the first order conditions of this problem: The price of the contingent claim is given by:

$$Q_{t,t+1} = \beta \frac{\left( \frac{u_{ct}}{P_{t+1}} v_{t+1} - \frac{u_{1-Nt+1}}{W_{t+1}} (v_{t+1} - 1) \right)}{\left( \frac{u_{ct}}{P_t} v_t - \frac{u_{1-Nt}}{W_t} (v_t - 1) \right)} \quad (4.1)$$

The intratemporal condition is

$$\frac{u_{1-Nt}}{u_{ct}} = \frac{W_t}{P_t R'_t}, \text{ where } R'_t = \frac{R_t - 1 + v_t}{v_t}. \quad (4.2)$$

An intertemporal condition is

$$\frac{u_{ct}}{P_t} = R'_t E_t \left[ \frac{\beta u_{ct+1} R_{t+1}}{P_{t+1} R'_{t+1}} \right] \quad (4.3)$$

The firms choose at  $t - 1$  the price  $P_t(i)$  that maximizes

$$E_{t-1} [Q_{t-1,t} Q_{t,t+1} (P_t(i) y_t(i) - W_t N_t(i))]$$

subject to the production function (2.9) and the demand (2.10). The objective function for firms can be written as

$$E_{t-1} \left[ \left( \frac{u_{ct+1}}{P_{t+1}} v_{t+1} - \frac{u_{1-Nt+1}}{W_{t+1}} (v_{t+1} - 1) \right) (P_t(i) y_t(i) - W_t N_t(i)) \right]$$

or equivalently, using (4.2),

$$E_{t-1} \left[ \left( \frac{u_{ct+1} R_{t+1}}{P_{t+1} R'_{t+1}} \right) (P_t(i) y_t(i) - W_t N_t(i)) \right].$$

The price chosen by the firm is

$$P_t(i) = P_t = \frac{\theta}{(\theta - 1)} E_{t-1} \left[ v_t \frac{W_t}{s_t} \right] \quad (4.4)$$

where  $v_t = \frac{\left( \frac{u_{c(t+1)} R_{t+1}}{P_{t+1} R'_{t+1}} \right) C_t}{E_{t-1} \left[ \left( \frac{u_{c(t+1)} R_{t+1}}{P_{t+1} R'_{t+1}} \right) C_t \right]}$ .

Using (4.2) and (4.4), we have

$$E_{t-1} \left[ \frac{E_t \left( \frac{u_{Ct+1} R_{t+1}}{P_{t+1} R'_{t+1}} \right) C_t}{E_{t-1} \left[ E_t \left( \frac{u_{Ct+1} R_{t+1}}{P_{t+1} R'_{t+1}} \right) C_t \right]} \frac{1}{\frac{(\theta-1)}{\theta R'_t} s_t} \frac{u_{1-Nt}}{u_{Ct}} \right] = 1.$$

Using (4.3), we obtain

$$E_{t-1} \left[ \frac{\frac{u_{Ct}}{R'_t \beta P_t} C_t}{\frac{(\theta-1)}{\theta R'_t} s_t} \frac{u_{1-Nt}}{u_{Ct}} \right] = E_{t-1} \left[ \frac{u_{Ct}}{R'_t \beta P_t} C_t \right]$$

and using (4.2),

$$E_{t-1} \left[ \frac{U_{Ct}}{\frac{\theta R'_t}{(\theta-1)}} C_t - U_{1-Nt} N_t \right] = 0 \quad (4.5)$$

Condition (4.5) is the implementability condition of the Ramsey problem that is the maximization of utility subject to the implementability condition and the feasibility constraint. From the analysis in Section 3, it is straightforward to establish the following propositions, analogous to propositions 3.1 and 3.2:

**Proposition 4.1.** *The interest rate that allows to achieve the optimal allocation when prices are set one period in advance is always unity, i.e. the Friedman Rule is optimal.*

**Proof:** The proof is analogous to the one in Proposition 3.1. Welfare is maximized by choice of  $R'_t$ , when the nominal interest rate is minimized at  $R'_t = R_t = 1$

**Proposition 4.2.** *If  $R'_t$  is constant across states, the optimal allocation under sticky prices is the flexible prices allocation if and only if the expression in proposition 3.1. (3.11) does not depend on the shocks  $s_t$  and  $v_t$ .*

When there are shocks to velocity the nominal interest rate  $R_t$  has to fluctuate with the velocity shocks, in order for the flexible prices allocation to be optimal, for the classes of utility functions described in Section 3.4. This reinforces the previous results that, in general, the policy tools that can be used under sticky prices allow to improve upon the flexible prices allocation.

## 5. Concluding Remarks

We analyze a simple environment with short run non neutrality of money and determine principles for the conduct of monetary policy as stabilization policy. In our environment there are three sources of distortions: monopolistic competition; time and state dependent nominal interest rates; and prices set in advance. The first two distortions are present in the economy with flexible prices, as in RBC models with long run non-neutrality of money, as in Cooley and Hansen (1989).

The optimal monetary policy is to set the nominal interest rate to zero, following the Friedman rule.<sup>5</sup> However, since under that policy it is no longer possible to use the money supply to determine the allocation, this is indeterminate and there can be sunspot fluctuations. We move away from the Friedman rule and determine the optimal money supply policy consistent with a given interest rate path. This way we are able to compare a generic allocation under flexible prices, distorted by an interest rate path, and the allocation under sticky prices, with the same interest rates, but where money supply is determined in order to maximize welfare. The money supply policy that we study here is an instrument that can only be used because of price stickiness. This way we are able to identify a close analogy to the gaps in the traditional stabilization policy debate, treat them as distortions, and obtain policy recommendations on the use of monetary policy as stabilization policy.

We show that, if nominal interest rates are constant across states, then it is optimal to use the money supply policy so that gaps are eliminated and the flexible prices allocation is achieved. However one should not neglect the criticism by Lucas (1980) of a welfare criterion based on gaps rather than triangles. If nominal interest rates are state dependent, or there are shocks to velocity, then it is no longer optimal to use the money supply to close gaps. In that case, sticky prices provide a means to improve upon a distorted flexible prices allocation.

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<sup>5</sup>See Ireland (1996).

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## 6. Appendix A

We want to prove that

$$D = -\frac{u_{Ct}C_t}{u_{Ct}} + \frac{u_{Ct,1-Nt}}{u_{Ct}}N_t - \frac{u_{1-Nt,1-Nt}}{u_{1-Nt}}N_t + \frac{u_{Ct,1-Nt}}{u_{1-Nt}}C_t$$

is invariant to monotonic transformations of  $u$ .

Let us define  $V = F(u)$ .

Then

$$\begin{aligned} V_C &= F' u_C \\ V_{CC} &= F'' u_C^2 + F' u_{CC} \\ V_{C,1-N} &= F'' u_C u_{1-N} + F' u_{C,1-N} \\ V_{1-N} &= F' u_{1-N} \\ V_{1-N,1-N} &= F'' u_{1-N}^2 + F' u_{1-N,1-N} \end{aligned}$$

and

$$\begin{aligned} D^V &= -\frac{(F'' u_C^2 + F' u_{CC})C}{F' u_C} + \frac{(F'' u_C u_{1-N} + F' u_{C,1-N})N}{F' u_C} - \\ &\quad -\frac{(F'' u_{1-N}^2 + F' u_{1-N,1-N})N}{F' u_{1-N}} + \frac{(F'' u_C u_{1-N} + F' u_{C,1-N})C}{F' u_{1-N}} \end{aligned}$$

or

$$D^V = D - \frac{F'' u_C C}{F'} + \frac{F'' u_{1-N} N}{F'} - \frac{F'' u_{1-N} N}{F'} + \frac{F'' u_C C}{F'} = D$$

Time invariant preferences that satisfy simultaneously Gorman aggregation and balanced growth are given by

$$\begin{aligned} 1) \quad u &= \frac{(C^\alpha (1-N)^\psi)^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \quad \sigma \neq 1 \\ 2) \quad u &= \alpha \log C + \psi \log(1-N) \end{aligned}$$

Since these functions are homogeneous, 1) or transformations of homogeneous functions, 2), marginal utility of  $C$  and marginal utility of  $(1-N)$  are homogeneous of the same degree. Then

$$\frac{u_{C_t} C_t}{u_{C_t}} + \frac{u_{C_t, 1-Nt}}{u_{C_t}} (1 - N_t) = \frac{u_{1-Nt, 1-Nt}}{u_{1-Nt}} (1 - N_t) + \frac{u_{C_t, 1-Nt}}{u_{1-Nt}} C_t$$

Using this relation we can write  $D$  as

$$D = \frac{u_{C_t, 1-Nt}}{u_{C_t}} - \frac{u_{1-Nt, 1-Nt}}{u_{1-Nt}}$$

and therefore  $D = \frac{N}{1-N}$ . Because  $N$  is constant across states for these preferences, the denominator is constant.

There is another class of preferences that are aggregable but not compatible with balanced growth:

$$3) u_t = \frac{C^{1-\sigma} - 1}{1 - \sigma} - \alpha(1 - N), \sigma > 0$$

$$4) u_t = \frac{(C - \zeta N^\psi)^{1-\sigma}}{1 - \sigma}, \sigma > 0, \sigma \neq 1, \psi > 1$$

$$5) u_t = \log(C - \zeta N^\psi), \psi > 1$$

For the class represented by 3),  $D = \sigma$ .

For the class given by 4) and 5), the so called GHH preferences, these functions are positive transformations of  $C - \zeta N^\psi$ . As  $D$  for this last form is identical to  $\psi - 1$ ,  $D$  for 4) and 5) is constant and given by  $\psi - 1$ .

Then for 1) to 5) the denominator is constant across states.

## 7. Appendix B

In this appendix, we show that any solution of this problem can be decentralized by an appropriate monetary policy, which we also characterize.

Take an allocation  $(c_t^*(s^t), N_t^*(s^t))$  that satisfies the problem of the social planner. There are price levels  $P_t^*(s^t)$  and money balances  $M_t^{s*}(s^t)$  that satisfy

$$P_t^*(s^t) c_t^*(s^t) = M_t^{s*}(s^t)$$

and

$$\frac{u_c(c_t^*(s^t), N_t^*(s^t))}{P_t^*(s^t)} = k(s^{t-1}), \text{ for } k \geq 0$$

for all  $s^t$ . For these prices and money supplies, the intertemporal first order condition of the households can be written as

$$\frac{u_c(c_t^*(s^t), N_t^*(s^t))}{P_t^*(s^t)} = \beta R_t \frac{u_c(c_{t+1}^*(s^{t+1}), N_{t+1}^*(s^{t+1}))}{P_{t+1}^*(s^{t+1})}.$$

i.e. without the conditional expectations operator.

Observe that this vector of prices and money supplies depend on the current state and therefore cannot be part of the sticky prices economy equilibrium. From this vector of nominal variables we construct another vector with predetermined prices which are part of the equilibrium. Let us call this new pair  $P_t(s^{t-1})$  and  $M_t^s(s^t)$ . This pair is defined as,

$$\frac{P_t^*(s^t)\gamma(s^t)}{P_t(s^{t-1})} = 1, \quad \gamma(s^t) \in \mathcal{R}^{++}, \text{ for all } s^t$$

and

$$P_t(s^{t-1})c_t^*(s^t) = M_t^s(s^t), \text{ for all } s^t \quad (7.1)$$

Observe that this price,  $P_t(s^{t-1})$ , is a price firms could choose at date  $t - 1$  since the allocation is consistent with the firms' first order condition (2.12).

Next, we compute the value for  $\gamma(s^t)$ . The intertemporal first order condition of households can be rewritten as

$$\frac{u_c(c_t^*(s^t), N_t^*(s^t))}{\gamma(s^t)P_t^*(s^t)} = \beta R_t \sum_{s^{t+1}|s^t \in S^{t+1}} \Pr(s^{t+1}|s^t) \frac{u_c(c_{t+1}^*(s^{t+1}), N_{t+1}^*(s^{t+1}))}{\gamma(s^{t+1})P_{t+1}^*(s^{t+1})},$$

or

$$\frac{u_c(c_t^*(s^t), N_t^*(s^t))}{\gamma(s^t)P_t^*(s^t)} = \beta R_t \frac{u_c(c_{t+1}^*(s^{t+1}), N_{t+1}^*(s^{t+1}))}{P_{t+1}^*(s^{t+1})} \sum_{s^{t+1}|s^t \in S^{t+1}} \frac{\Pr(s^{t+1}|s^t)}{\gamma(s^{t+1})}.$$

This implies that,

$$\gamma(s^t) = \left[ \sum_{s^{t+1}|s^t \in S^{t+1}} \frac{\Pr(s^{t+1}|s^t)}{\gamma(s^{t+1})} \right]^{-1}, \text{ for all } s^t$$

and that the price vector

$$P_t(s^{t-1}) = P_t^*(s^t) \left[ \sum_{s^{t+1}|s^t \in S^{t+1}} \frac{\Pr(s^{t+1}|s^t)}{\gamma(s^{t+1})} \right]^{-1}, \text{ for all } s^t$$



or

$$P_{t+1}(s^t) = P_t(s^{t-1})P_t^*(s^t)^{-1} \left[ \sum_{s^{t+1}|s^t \in S^{t+1}} P_{t+1}^*(s^{t+1}) \Pr(s^{t+1}|s^t) \right], \text{ for all } s^t.$$

There is freedom in the choice of the first price level in the economy, and thus we may choose  $P_0 = P_0^*$ , for instance. Given the initial price level, the remaining price levels are obtained according to the equation above.

The monetary policy that implements the allocation  $(c_t^*(s^t), N_t^*(s^t))$  is given by (7.1). Additionally we describe the money growth rate associated with the optimal allocation. From the equation above we get

$$\frac{P_{t+1}(s^t)}{P_t(s^{t-1})} = \frac{\left[ \sum_{s^{t+1}|s^t \in S^{t+1}} P_{t+1}^*(s^{t+1}) \Pr(s^{t+1}|s^t) \right]}{P_t^*(s^t)},$$

using the cash-in-advance conditions we can rewrite it as

$$\frac{M_{t+1}^s(s^{t+1})}{M_t^s(s^t)} = \frac{c_{t+1}^*(s^{t+1})}{c_t^*(s^t)} \frac{\left[ \sum_{s^{t+1}|s^t \in S^{t+1}} P_{t+1}^*(s^{t+1}) \Pr(s^{t+1}|s^t) \right]}{P_t^*(s^t)},$$

or as

$$\frac{M_{t+1}^s(s^{t+1})}{M_t^s(s^t)} = \frac{c_{t+1}^*(s^{t+1})}{c_t^*(s^t)} \left[ \beta R_t \sum_{s^{t+1}|s^t \in S^{t+1}} \Pr(s^{t+1}|s^t) \frac{u_c(c_{t+1}^*(s^{t+1}), N_{t+1}^*(s^{t+1}))}{u_c(c_t^*(s^t), N_t^*(s^t))} \right].$$

The value of  $M_0^s = P_0^* c_0^*$ , and  $M_t^s$  for  $t > 0$  given by the equation above.