Expectation Traps and Monetary Policy

Preliminary Draft

Stefania Albanesi[*], V.V. Chari[†], Lawrence J. Christiano[‡]

May 20, 2000

[*]Northwestern University

[†]University of Minnesota and Federal Reserve Bank of Minneapolis.

[‡]Northwestern University and Federal Reserve Bank of Chicago.

I Introduction

Many countries have gone through prolonged periods of costly, high inflation, as well as prolonged periods of low inflation. The United States and other industrialized countries went through a high inflation episode during the Great Inflation of the 1970s and are now in a low inflation episode. A central question in monetary economics is why high inflation episodes occurred and what can be done to prevent them from occurring again.

In this paper we examine whether standard monetary general equilibrium models with benevolent monetary authorities acting under discretion can generate persistent episodes of high and low inflation. Specifically, we ask whether private agents' expectations of high or low inflation can lead these agents to take actions which then make it optimal for monetary authorities to validate these expectations. Following Chari, Christiano and Eichenbaum (1998), we call such an outcome an expectation trap. Chari, Christiano and Eichenbaum (1998) showed that expectation traps could occur in conventional general equilibrium monetary models. They relied, however, on trigger strategies on the part of the monetary authority to support such outcomes. One criticism of trigger strategies is that because of folk theorem-like reasons, virtually any inflation outcome can be rationalized as an equilibrium. In this paper we restrict attention to Markov equilibria and ask whether conventional general equilibrium models can generate expectation traps.

We analyze versions of Lucas and Stokey (1983)'s cash-credit good model. In the models, the benefit of unexpected growth in the money supply is a rise in output and the cost is the misallocation of resources arising from a distortion in relative prices. The monetary authority optimally balances the benefit and costs. In our benchmark model, we obtain the following three results:

- for a surprisingly large range of parameter values, there is an equilibrium in which the outcome is the same as under commitment.
- if the interest elasticity of money demand exceeds unity, there are at least two equilibria.
- the monetary authority smooths the response of interest rates to technology and fiscal shocks and allows interest rates to respond to money demand shocks.

We now briefly explain the economic mechanisms in our benchmark model and the intuition underlying our results. In the model, goods are produced in monopolistically competitive markets. The monopoly power of firms causes the level of economic activity to be inefficiently low. A subset of monopolists set their prices before the monetary authority selects the money growth rate, while the rest of the monopolists set prices afterward. Because of the preset prices, a monetary expansion greater than expected can raise output. Such a monetary expansion tends to raise welfare because output is inefficiently low. A monetary expansion also has costs. In our model, some goods must be purchased with previously accumulated cash. A monetary expansion, by raising prices, reduces the consumption of cash goods and welfare. These aspects of the model formalize old ideas with an extensive literature.[1] An equilibrium requires that the expansion chosen by the monetary authority coincides with expectations.

To understand our first result, suppose that the public expects a low inflation, the one associated with the absence of an inflation bias. Then, for a large range of parameter values, it turns out that the price distortions induced by deviating to higher money growth outweigh the associated gains.

The basic finding underlying our second result is that the marginal cost of unanticipated inflation is non-monotone in the expected inflation rate. In our model it turns out that the marginal cost of unexpected inflation is roughly proportional to rM/P, where r is the net nominal interest rate, and M/P denotes real balances. In our model, real balances are bounded and since the nominal interest rate is increasing in the expected inflation rate, it follows that the marginal cost of unanticipated inflation is low at low expected inflation. A key feature of our model is the behavior of money demand at high nominal interest rate. specifically, rM/P goes to zero as r goes to infinity. This feature implies that the marginal cost of unexpected inflation is low at high levels of expected inflation. We conjecture that the relationship between the marginal cost of inflation and rM/P lies in the fact that a monetary expansion acts as a distorting tax on real balances. Because the marginal cost of inflation has an inverted 'U' shape, as in a Laffer curve, while the marginal benefit is roughly constant, there is more than one value of expected inflation in which the marginal benefit of unanticipated inflation equals the marginal cost.

As the reasoning in the previous paragraph suggests, we find that the properties of money demand at high levels of inflation are crucial to the question of the multiplicity of equilibria. We confirm this reasoning by developing a model in which rM/P does not go to zero as r goes to infinity. In this model we find that there is a unique equilibrium.

The multiplicity of equilibria in our benchmark model raises the possibility of expectation traps. If private agents expect the monetary authority to pursue an expansionary monetary policy, they set prices sufficiently high so that it is optimal for the monetary authority to validate their expectations. Conversely, if private agents expect low inflation, then the monetary authority optimally validates those expectations. The possibility of expectation traps in our model is promising because it may help account for the observed prolonged periods of high inflation as well as prolonged periods of low inflation. This possibility depends in a crucial way on the properties of money demand. At an abstract level, it should not be surprising that the behavior of the monetary authority depends in an essential way on the determinants of demand for the object they supply, namely money. But, to our knowledge, this connection has not been made as yet in the literature. We go on, in this draft of the paper, to develop a model with a financial intermediation technology. This version of the model we believe can account for a number of key features of the data on money demand. These include the observed trend in velocity, the differences between short and long-run interest elasticities of money demand and persistence of estimated money demand shocks. This version also holds out promise for other types of expectation traps.

The plan of the paper is as follows. The following section summarizes results in the existing literature. Section 3 presents results based on our benchmark general equilibrium model. Second 4, describes a different version of our model, in which the interest elasticity of money demand can be lower. In this model, when the elasticity of money demand is below unity, the Markov equilibrium is often unique. Section 5 analyzes a model in which the cash-credit good designation is endogenous. We argue that the implications for money demand are better than those of the model in section 3. Still, that model is like the one in section 3 in that it predicts expectation traps and the possibility of a high inflation bias. The final section concludes.

II The Kydland-Prescott/Barro-Gordon Model

Most of the literature on the inflation bias hypothesis has been conducted in reduced form models. We briefly describe one such model, the KP-BG model to motivate our analysis. The model is static, and the payoff for the monetary authority is given by $a\left[y-x\right]^2 - b\pi^2$, $a, b, x > 0$, where y is a measure of output, $x > 0$ is a measure of 'full employment output' and π is the realized rate of inflation. Output is given by $y = \alpha\left(\pi - \pi^e\right)$, $\alpha > 0$,where π^e is expected inflation. The timing is that private agents choose π^e first and the monetary authority then chooses π. The policymaker's best response, $\pi(\pi^e, x)$ maximizes the monetary authority's payoff given π^e. The model is completed by imposing rational expectations on private agents, namely, $\pi^e = \pi(\pi^e, x)$. The equilibrium inflation rate is given by the solution to this equation, and is denoted by $\pi^*(x)$. Under commitment, the monetary authority chooses π first, and then private agents set $\pi^e = \pi$. The inflation rate implied by this Ramsey-like programming problem is $\pi = 0$. The *inflation bias* is $\pi^*(x)$.

Figure 1 plots the best response function $\pi(\pi^e, x)$ for a fixed x. The equilibrium inflation rate, π^* is the intersection of the best response function with the 45 degree line. The figure shows the two basic results in this literature. First, since the best response function crosses the 45 degree line only once, the equilibrium value of inflation is determined solely by the variable, x, that shift the best response function. If the best response function crossed the 45 degree line more than once, then private sector expectations play an independent role in determining the equilibrium. Chari, Christiano and Eichenbaum (1998) refer to equilibria of this type as 'expectation traps'. The KP-BG model excludes expectation traps. Second, since $a, b, x > 0$ there is always an inflation bias, and it may be substantial.

These two findings hold up in repeated versions of the model as long as attention is restricted to equilibria without reputational mechanisms or trigger strategies. These reputational mechanisms have the unattractive feature that they depend sensitively on the horizon being infinite. In addition, the folk theorem from repeated games implies that the set of equilibria is extremely large and it is difficult to know which of these equilibria are plausible. For these reasons, in our research we focus on stationary Markov equilibria, which rule out such reputational mechanisms.

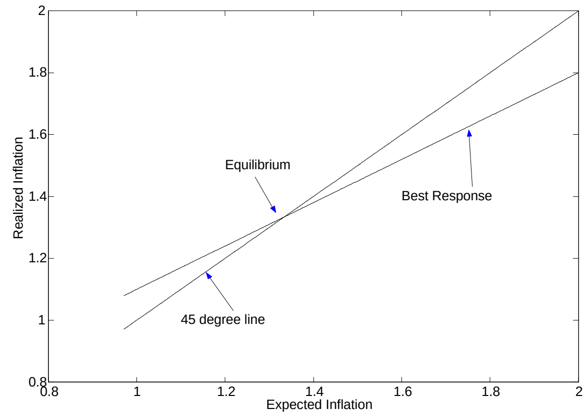

Figure 1: Best Response Function, KP-BG Model

III The Benchmark Cash-Credit Goods Model

Our economy is composed of firms, a representative household and a monetary authority. There is a continuum of firms, each of which is a monopolist in the production of some good, $j \in (0, 1)$. The sequence of events within a period is as follows. First, a shock, θ, to technology, a shock, g, to government consumption, and a shock, z, to money demand, are realized. Then, a fraction μ of the firms (the 'sticky price firms') set their prices. We let $P^e(\theta, g, z)$ denote the average price set by sticky price firms. After that, the monetary authority makes its policy decision. Finally, all private decisions are made.

We denote the state of the economy at the time the monetary authority makes its decision as follows:

$$S = (\theta, g, z, P^e). \tag{1}$$

We define this state for arbitrary P^e rather than just its equilibrium value, $P^e(\theta, g, z)$. This is not necessary for purposes of defining an equilibrium.[2] We do it because it is useful for purposes of computing and characterizing equilibrium. Notice also that we do not include the aggregate stock of money in the state. In our economy, all equilibria are neutral in the usual sense that if the initial money stock is doubled, there is an equilibrium in which real allocations and the interest rate are unaffected and all nominal variables are doubled. This consideration leads us to focus on equilibria which are invariant with respect to the initial money stock. We are certainly mindful of the possibility that there can be equilibria which depend on the money stock. For example, if there are multiple equilibria in our sense, it is possible to construct 'trigger strategy-type' equilibria which are functions of the initial money stock. In our analysis we exclude such equilibria and we normalize the aggregate stock of money at the beginning of each period to unity.

The monetary authority makes its money growth decision conditional on S. We denote the gross money growth rate by x and its policy rule by $X(S)$. The state of the economy after the monetary authority makes its decision is (S, x). With these definitions of the economy's state variables, we proceed now to discuss the decisions of firms, households and the monetary authority.

A Firms

There is a continuum of goods denoted by $\omega \in (0, 1)$. Each good is produced by a monopolist using the following production technology:

$$y(\omega) = \theta n(\omega),$$

where $y(\omega)$ denotes output, $n(\omega)$ denotes employment, and θ is the technology shock. The price set by the $1-\mu$ firms which set their prices after the monetary authority makes is decision ('flexible price firms') is denoted by $\hat{P}(S, x)$. The prices $P^e(\theta, g, z)$ and $\hat{P}(S, x)$ maximize profits subject to a demand curve discussed below. Because of the constant elasticity form of this demand curve, firms set prices as a fixed markup above marginal cost, the wage rate. For the μ sticky price firms and the $1 - \mu$ flexible price firms, this means:

$$\begin{aligned} P^e(\theta, g, z) &= \frac{W(\theta, g, z, P^e(\theta, g, z), X(\theta, g, z, P^e(\theta, g, z)))}{\theta\rho}, \\ \hat{P}(S, x) &= \frac{W(S, x)}{\theta\rho}, \; 0 < \rho < 1, \end{aligned} \tag{2}$$

where $W(S, x)$ denotes the nominal wage rate.

B Household

The preferences of the representative household are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{3}$$

where

$$c_t = \left[\int_0^1 c_t(\omega)^\rho d\omega \right]^{\frac{1}{\rho}},$$

and $c_t(\omega)$ denotes consumption of type ω good and l_t denotes labor time. Goods with $\omega > z$, (where z is a parameter between 0 and 1) are credit goods and goods with $\omega \leq z$ are cash goods. We adopt the following utility function specification:

$$u(c, l) = \frac{\left[c(1-l)^\psi \right]^{1-\sigma}}{1-\sigma} \tag{4}$$

The sequence of events during the period is as follows. The household begins the period with nominal assets, which it divides into money and bonds. Let A, M, B denote the household's nominal assets, money and bonds, respectively, scaled by the aggregate stock of money. The household's constraint is:

$$M + B \leq A. \tag{5}$$

It also faces a cash in advance constraint:

$$M - \left[P^e \mu z c_{11} + \hat{P}(S, x)(1-\mu) z c_{12} \right] \geq 0, \tag{6}$$

where c_{11} and c_{12} denote quantities of cash goods purchased from sticky and flexible price firms, respectively.[3] Nominal assets evolve over time as follows:

$$\begin{aligned} (7)\ 0 \leq\ & W(S,x)n + (1 - R(S,x))M - z\left[P^e \mu c_{11} + \hat{P}(S,x)(1-\mu)c_{12} \right] \\ & -(1-z)\left[P^e \mu c_{21} + \hat{P}(S,x)(1-\mu)c_{22} \right] + R(S,x)A + (x-1) + D(S,x) - xA', \end{aligned}$$

where c_{21} and c_{22} denote the quantities of credit goods purchased from sticky and flexible price goods, respectively. In (7), M denotes the individual household's beginning of period stock of money, $R(S,x)$ denotes the gross nominal rate of return on bonds, and $D(S,x)$

denotes profits after lump sum taxes. Finally, B has been substituted out in the asset equation using (5). Notice that A' is multiplied by x. This reflects the way we have scaled the stock of nominal assets.

Consider the household's asset, goods and labor market decisions. Given that the household expects the monetary authority to choose policy according to X in the future, the household solves the following problem:

$$v(A, S, x) = \max_{n,M,A',c_{ij};i,j=1,2} u(c, n) + \beta E_{\theta',g',z'}[v(A', S', X(S'))|\theta, g, z] \tag{8}$$

subject to (6), (7), $0 \leq M \leq A$,[4] non-negativity on consumption and labor, and

$$c = [z\mu c_{11}^{\rho} + z(1-\mu)c_{12}^{\rho} + (1-z)\mu c_{21}^{\rho} + (1-z)(1-\mu)c_{22}^{\rho}]^{\frac{1}{\rho}}. \tag{9}$$

Also,

$$S' = (\theta', g', z', P^e(\theta', g', z')).$$

In (8), v is the household's value function. The solution to (8) yields decision rules of the form $n(A, S, x)$, $M(A, S, x)$, $A'(A, S, x)$, and $c_{ij}(A, S, x)$, $i, j = 1, 2$. To keep our notation simple, we suppress the dependence of these functions on the wage and price functions, and the monetary policy rule.

C Private Sector Equilibrium

We now define an equilibrium for the private sector, taking as given the current monetary action, x, and the rule governing future monetary policy, $X(S)$. This equilibrium requires that households and firms optimize and markets clear.

Definition Given a monetary policy rule, $X(S)$, and a current money growth rate, x, a *Private Sector Equilibrium* is a collection of functions $P^e(\theta, g, z)$, $\hat{P}(S_1)$, $W(S_1)$, $v(A, S_1)$, $c_{ij}(A, S_1)$, $n(A, S_1)$, $M(A, S_1)$, $A'(A, S_1)$, $R(S_1)$, where $S_1 = (\theta, g, z, P^e(\theta, g, z), x)$, such that:

1. The functions v, c_{ij}, n, M, A' solve (8),
2. Firms optimize, i.e., (2) is satisfied,
3. The asset markets clear, i.e., $A'(1, S_1) = 1$ and $M(1, S_1) = 1$,
4. The resource constraint is satisfied, i.e., $\theta n(1, S_1) = g + z[\mu c_{11} + (1-\mu)c_{12}] + (1 - z)[\mu c_{21} + (1-\mu)c_{22}]$,

D Monetary Authority

The monetary authority chooses the money growth rate to maximize the utility of the representative household. Taking future monetary policy and private sector allocation rules as given, the monetary authority chooses the current money growth rate, x, to solve the problem:

$$\begin{aligned}(10) \qquad & \max_x u(c(1,S,x), n(1,S,x)) \\ & +\beta E_{\theta',g',z'}[v(1,S',X(S'))|\theta,g,z],\end{aligned}$$

where $S' = (\theta', g', z', P^e(\theta', g', z'))$ is the state at the beginning of the next period. We restrict x to be to be at least as large as $\underline{x}$, where $0 < \underline{x} < \beta$.

Definition A *Markov equilibrium* is a private sector equilibrium and a monetary policy rule such that $X(S)$ solves (10).

E Characterization

In this section we exploit the fact that a Markov equilibrium is the fixed point of a particular mapping. We study the properties of that mapping to determine the uniqueness properties of the equilibrium.

Following is our algorithm for computing a Markov equilibrium. It is best described by considering a particular fixed point problem. For each fixed (θ, g, z), the fixed point is of the mapping from an expectation of government policy to actual government policy. It can be verified that in any Markov equilibrium, $\hat{P}(\theta, g, z, P^e(\theta, g, z), x)$ is a strictly monotonic function of x.[5] This feature of the equilibrium implies that it is equivalent to characterize policy as a choice of the price, $\hat{P}$, rather than the money growth rate, x. As a practical matter, it is convenient to characterize policy in terms of the relative price

$$q \equiv \frac{\hat{P}}{P^e}.$$

The government's decision problem maps P^e into q. The government's decision problem is simplified in our setting because its choice of q has no impact on future allocations. As a result, the government faces a static problem. An equilibrium is a P^e such that $q = 1$.

Obviously, this price is the equilibrium pricing function, $P^e(\theta, g, z)$. The mapping from this function to the remaining allocations and prices and to money growth is straightforward.

The mapping can be defined more precisely as follows. For each fixed (θ, g, z), start with an arbitrary value of P^e and map to q in three steps. First, for arbitrary q, calculate the mapping from (θ, g, z, P^e, q) to the private allocations and prices that are determined after the monetary authority makes its decision. This allows us to define the policy maker's objective function, $U(\theta, g, z, P^e, q)$. Second, optimize this objective with respect to q, and denote the solution, $q = q(\theta, g, z, P^e)$. Finally, adjust P^e until $q = 1$. This value of P^e corresponds to the function, $P^e(\theta, g, z)$.

E.1 Private Allocations and Prices

We now solve for the private allocations given $(\theta,\, g,\, P^e,\, q)$. Let ν and λ denote the multipliers on (6) and (7), respectively.[6] The first order necessary and sufficient conditions for household optimization are:

$$
\begin{aligned}
\frac{u_{11}}{u_{12}} &= \frac{u_{21}}{u_{22}} = \frac{\mu}{1-\mu}\frac{1}{q}, \\
\frac{u_{11}}{u_{21}} &= \frac{u_{12}}{u_{22}} = \frac{z}{1-z}R, \\
-u_n &= \frac{\theta\rho u_{22}}{(1-\mu)(1-z)}, \\
\frac{x u_{21}}{P^e \mu (1-z)} &= \beta E_{\theta', g', z'}[v_1(1, S', X(S'))|\theta, g, z].
\end{aligned}
\tag{11}
$$

Here, u_{ij} denotes the partial derivative of u with respect to c_{ij}, and v_1 denotes the partial derivative of v with respect to its first argument. In the labor Euler equation, we have used the fact, $W = \theta\rho\hat{P}$.

In addition, the cash in advance constraint can be written as

$$P^e \mu z c_{11} + q P^e (1-\mu) z c_{12} \leq 1,$$

so that the complementary slackness condition is

$$\{1 - [P^e \mu z c_{11} + q P^e (1-\mu) z c_{12}]\}\, [R - 1] = 0.$$

The resource constraint is:

$$g + z\,[\mu c_{11} + (1-\mu) c_{12}] + (1-z)\,[\mu c_{21} + (1-\mu) c_{22}] = \theta n. \tag{12}$$

There are 7 unknowns, c_{ij}, $i, j = 1, 2$, n, x, R, and 7 independent equations. (Of the first four first order conditions, only three are independent.) Notice that x only appears in the last equation in (11), so that the allocations and the interest rate can be computed as functions of P^e, q, θ, g and x can be solved for at the last stage. Denote the allocations defined in this way by:

$$c_{ij}(\theta, g, z, P^e, q),\ i, j = 1, 2,\ R(\theta, g, z, P^e, q),\ n(\theta, g, z, P^e, q). \tag{13}$$

We denote the set, (P^e, q) for which there exists a solution to the seven equations by D.

For later reference, it is useful to know that when utility is given by (4), then (11) reduce to:

$$\begin{aligned} c_{12} &= c_{11} q^{\frac{-1}{1-\rho}},\ c_{22} = c_{21} q^{\frac{-1}{1-\rho}}, \\ R &= \left(\frac{c_{22}}{c_{12}}\right)^{1-\rho},\ \theta(1-n) = \frac{\psi}{\rho} c^{\rho} c_{22}^{1-\rho}. \end{aligned} \tag{14}$$

Incorporating this into the cash in advance constraint:

$$c_{11}\left\{\mu + (1-\mu) q^{\frac{-\rho}{1-\rho}}\right\} \leq \frac{1}{zP^e}. \tag{15}$$

E.2 Government Problem

We can summarize the previous discussion as providing functions:

$$c = c(\theta, g, z, P^e, q),\ n = n(\theta, g, z, P^e, q),$$

where c is obtained by substituting (13) into (9). These functions can be substituted into the utility function,

$$U(\theta, g, z, P^e, q) = u\left[c(\theta, g, z, P^e, q), n(\theta, g, z, P^e, q)\right].$$

Define

$$q(\theta, g, z, P^e) = \arg\max_{q \in D} U(\theta, g, z, P^e, q).$$

The function, $q(\theta, g, z, P^e)$, is the monetary authority's best response, given θ, g, z, P^e. Equilibrium requires that $q(\theta, g, z, P^e) = 1$. This equilibrium requirement allows us to construct the equilibrium function, $P^e(\theta, g, z)$.

E.3 Qualitative Characteristics of Equilibrium

We are interesting in determining how many equilibrium functions there are. Notice from (14) that $c_{11} = c_{12} = c_1$, say, in an equilibrium. Define c_2 similarly. We proceed in two steps. We first look for equilibria with the property that $c_1/c_2 < 1$, or, equivalently, that $R > 1$. We then consider the case, $c_1/c_2 = 1$. In our analysis, we assume that U is concave in q. The findings in our computational experiments are consistent with this assumption.

When $R > 1$, the cash in advance constraint is binding and the allocations are determined by (14), (15) with equality, and (12). It is easy to see that the interest rate and allocation functions, (13) are differentiable in q.

Equilibrium requires

$$U_q(P^e, q, \theta, g, z) = u_c c_q\left(P^e, q, \theta, g, z\right) + u_n n_q\left(P^e, q, \theta, g, z\right) = 0, \tag{16}$$

for $q = 1$, where U_q is the derivative of U with respect to q. From here on we suppress the arguments of functions, where there is no possibility of confusion. It is useful to rewrite the expression for U_q by adding and subtracting $u_c\theta(c/c_2)^{(1-\rho)}n_q$:

$$U_q = u_c c_q - \theta u_c\left(\frac{c}{c_2}\right)^{1-\rho} n_q + \left[u_n + \theta u_c\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q = 0. \tag{17}$$

We now develop formulas for c_q and n_q. Consider first c_q. Differentiating (9), and evaluating the derivatives at $q = 1$, we obtain:

$$c_q = \left(\frac{c}{c_1}\right)^{1-\rho} z\left[\mu c_{11,q} + (1-\mu)c_{12,q}\right] + \left(\frac{c}{c_2}\right)^{1-\rho}(1-z)\left[\mu c_{21,q} + (1-\mu)c_{22,q}\right], \tag{18}$$

where $c_{ij,q}$ is the derivative of c_{ij} with respect to q. From the resource constraint, we obtain:

$$n_q = \frac{z\left[\mu c_{11,q} + (1-\mu)c_{12,q}\right] + (1-z)\left[\mu c_{21,q} + (1-\mu)c_{22,q}\right]}{\theta}. \tag{19}$$

Substituting for c_q and n_q in (17) we obtain

$$U_q = u_c\left[\left(\frac{c}{c_1}\right)^{1-\rho} - \left(\frac{c}{c_2}\right)^{1-\rho}\right] z\left[\mu c_{11,q} + (1-\mu)c_{12,q}\right] + \left[u_n + \theta u_c\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q. \tag{20}$$

Using (14), we obtain

$$U_q = u_c\left(\frac{c}{c_2}\right)^{1-\rho}(R-1)\, z\left[\mu c_{11,q} + (1-\mu)c_{12,q}\right] + \left[u_n + \theta u_c\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q. \tag{21}$$

From (15) and (14),

$$c_{11,q} = c_1 \frac{(1-\mu)\rho}{1-\rho}, \; c_{12,q} = -c_1 \frac{1-(1-\mu)\rho}{1-\rho}, \tag{22}$$

so that

$$\mu c_{11,q} + (1-\mu) c_{12,q} = -c_1(1-\mu). \tag{23}$$

Substituting this into (21):

$$U_q = -u_c \left(\frac{c}{c_2}\right)^{1-\rho} (R-1)\, z c_1 (1-\mu) + \left[u_n + \theta u_c (\frac{c}{c_2})^{1-\rho} \right] n_q. \tag{24}$$

Notice that if the government could commit itself to monetary policy, it would follow the Friedman rule and set $R = 1$, so that the first term would be zero. In light of this, we call this term the inflation distortion. We call the second term the monopoly distortion because, if firms behaved competitively, the marginal rate of substitution between credit good consumption and leisure would be set equal to the corresponding marginal rate of transformation, so the second term would be zero.

Specifically, the inflation distortion is $(R-1)\, z\, (c_1/c_2)\, (1-\mu)$. Since $R = (c_1/c_2)^{\rho-1}$, we can write the inflation distortion as

$$\psi_{ID}\left(\frac{c_1}{c_2}\right) = \left[\left(\frac{c_1}{c_2}\right)^{\rho-1} - 1 \right] z \frac{c_1}{c_2} (1-\mu). \tag{25}$$

Then, it is easy to see that the first term in (24) is

$$-u_c \left(\frac{c}{c_2}\right)^{1-\rho} c_2 \psi_{ID}\left(\frac{c_1}{c_2}\right).$$

In the appendix we show that n_q is of the form $\psi_{MD}(c_1/c_2)c_2/(\theta(1-\rho))$, where $\psi_{MD}(c_1/c_2)$ is the monopoly distortion. Using the labor first order condition in (14), (24) can be rewritten as

$$U_q = u_c \left(\frac{c}{c_2}\right)^{1-\rho} c_2 \left[-\psi_{ID}\left(\frac{c_1}{c_2}\right) + \psi_{MD}\left(\frac{c_1}{c_2}\right) \right]. \tag{26}$$

Consider the inflation distortion function, ψ_{ID}. From inspection of (25), it is immediate that

$$\psi_{ID}(0) = 0.$$

That is, there is no inflation distortion when expected inflation rates are high. This feature of the model plays a central role in generating multiplicity of Markov equilibria. In the next section, we consider a model in which there is an inflation distortion, even when expected inflation rates are high. In that model, it turns out that there is often a unique equilibrium.

In the appendix, we show that $\psi_{MD}(0) > 0$. Thus, $U_q > 0$ for $c_1/c_2 = 0$. From (15), we have that $c_1 \rightarrow 0$ as $P^e \rightarrow \infty$. Using concavity of U, we conclude that for P^e sufficiently large, $q(\theta, g, z, P^e) > 1$. High levels of P^e correspond to low levels of real balances and high levels of expected inflation. So, we conclude that at high levels of expected inflation, the monopoly distortion dominates in the monetary authority's objective, creating an incentive to make the inflation rate even higher than expected.

Next we examine the behavior of U_q when $c_1/c_2 = 1$. In this case, the allocation functions are not differentiable functions of q. The problem is that for $q > 1$ the cash in advance constraint is binding, while it is not binding for $q < 1$. The allocation functions are different in the two cases because the equations characterizing a private sector equilibrium are different. Previously, we discussed these equations when the cash in advance constraint is binding. When this constraint is not binding, we replace the cash in advance constraint by $R = 1$ in (14).

In the appendix, we establish that the right derivative of U with respect to q, denoted by $U_{q\downarrow 1}$, is identical to (24). We also establish that the left derivative of U, denoted by $U_{q\uparrow 1}$, is strictly positive.

These observations imply that $R = 1$ is a Markov equilibrium outcome if, and only if, the right derivative of U is non-positive. To see this, notice that, from concavity of U, when the right derivative of U is non-positive, the monetary authority has no incentive to raise q. Since the left derivative is strictly positive, the authority also has no incentive to reduce q.

The observation that the right derivative of U when $c_1/c_2 = 1$ is given by (24) can be used to establish that there are at least two Markov equilibria. We have that

$$\psi_{ID}(0) = \psi_{ID}(1) = 0, \ \psi_{MD}(0) > 0.$$

Let

$$\psi(\frac{c_1}{c_2}) = -\psi_{ID}(\frac{c_1}{c_2}) + \psi_{MD}(\frac{c_1}{c_2}).$$

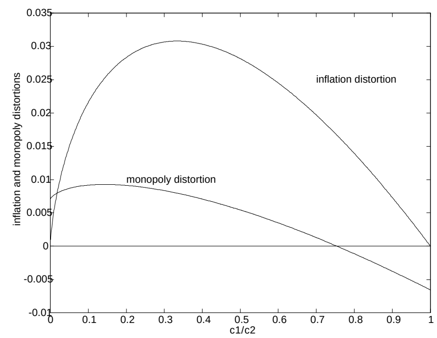

Figure 2: ψ_{ID} and ψ_{MD} for $c_1/c_2 \in (0,1)$

Suppose first that $\psi_{MD}(1) < 0$. Then, $\psi(0) > 0$ and $\psi(1) < 0$. By continuity of ψ there is at least one value of $c_1/c_2 < 1$ such that $\psi(c_1/c_2) = 0 = U_q$. This value corresponds to an equilibrium. Furthermore, $R = 1$ is also an equilibrium, since the right derivative of U is negative.

Suppose next that $\psi_{MD}(1) > 0$. Then, $\psi(0) > 0$ and $\psi(1) > 0$. With one exception, by continuity of ψ there are at least two values of c_1/c_2 such that $\psi(c_1/c_2) = 0 = U_q$. The exceptional case occurs when the graph of ψ is tangent to the horizontal axis. This case is clearly non-generic. A final non-generic case occurs when $\psi_{MD}(1) = 0$. We have established:

Proposition 1: Generically, there are at least two Markov equilibria.

To illustrate this multiplicity, we constructed a numerical example in which there are no shocks. Figure 2 displays the monopoly distortion, ψ_{MD}, and the inflation distortion, ψ_{ID}, for $c_1/c_2 \in (0,1)$. An equilibrium value of c_1/c_2 is one for which the two are equal or, $c_1/c_2 = 1$ and $\psi_{MD} \leq 0$. The figure illustrates an example in which $R = 1$ in the low inflation equilibrium and $R = 1.97$ in the high inflation equilibrium.[7]

We find it useful to develop the money demand relationship in our model. Denote private purchases of consumption goods by

$$c^p = zc_1 + (1 - z)c_2.$$

Notice this is the value of consumption goods purchased in markets by the households in the model since both cash and goods sell for the same price. Using (14) and that $zc_1 = M/P$ from the cash in advance constraint, we have that, in a Markov equilibrium, the following relationship must hold:

$$\frac{c^p}{M/P} = 1 + \left[\frac{1-z}{z} R\right]^{\frac{1}{1-\rho}}.$$

This relationship can be interpreted as a money demand equation. Notice that z appears in this equation and that movements in z can therefore be interpreted as money demand shocks. Next, we show that only shocks to money demand affect the equilibrium interest rate.

Proposition 2: In a stationary Markov Equilibrium, the interest rate, R, does not depend on the realization of technology, θ, and of government consumption, g, but it does depend on the money demand shock, z.

Proof: Notice from (25) that ψ_{ID} does not depend on θ or g, but does depend on z. This is true of ψ_{MD} as well (see (46) and (48)). Thus, c_1/c_2 does not depend on θ or g,while it does depend on z. The result follows because $R = (c_2/c_1)^{1-\rho}$ (see (14)).

IV Cash Credit Goods Models with Inelastic Money Demand

In the previous analysis, we found that the equilibria of our model depend upon the elasticity of substitution in utility between cash and credit goods. In that model, this is the same as the elasticity of demand faced by suppliers. To ensure that their profit function is bounded above, it is necessary that that elasticity be no less than unity. To understand the robustness of our results to situations in which the elasticity of substitution between cash and credit goods is low, we break the link between the elasticity of demand faced by suppliers and the elasticity of substitution between cash and credit goods. We do this by modifying the household's utility function. The market structure of the firm sector, and firm technology remain the same. In addition, the sequence of events in the period is also unchanged. That is, at the beginning of the period a fraction of firms set their prices. Then, the monetary authority selects its action. Finally, the remaining prices and quantities for the period are determined in a private sector equilibrium. We abstract from uncertainty in this section.

A Firms

The firm sector is essentially identical to what it was before. There is a continuum of goods. Each good is produced by a monopolist who faces a demand curve with elasticity denoted here by $1/(1-\lambda)$, where $0 < \lambda < 1$. Some firms ('sticky price firms') set prices before the monetary authority takes it current period action, and other firms ('flexible price firms') set their price afterward. All firms operate competitively in homogeneous factor markets. As before, sticky and flexible price firms set prices as follows:

(27)
$$P^e = \frac{W(P^e)}{\lambda}, \ \hat{P}(S,x) = \frac{W(S,x)}{\lambda}.$$

where the state S now consists only of the price set by the sticky price firms, P^e. With one exception, all the notation is the same as before. The exception is that ρ has been replaced by λ.

B Households

Preferences are as in (3), with

$$c = \left[zc_1^\rho + (1-z)\,c_2^\rho\right]^{\frac{1}{\rho}}, \quad -\infty < \rho \leq 1$$

and

(28)
$$c_1 = \left[\int_0^1 c_1(x)^\lambda dx\right]^{\frac{1}{\lambda}}, \ c_2 = \left[\int_0^1 c_2(x)^\lambda dx\right]^{\frac{1}{\lambda}}.$$

The individual goods, $c_i(x)$, $x \in (0,1)$, $i = 1,2$, are produced by the firms discussed in the previous section.

To purchase cash goods, $c_1(x)$, households must use cash accumulated in advance:

(29)
$$M - \left[P^e \mu_1 c_{11} + \hat{P}(S,x)\,(1-\mu_1)\,c_{12}\right] \geq 0,$$

where c_{1i} denotes consumption of the sticky price cash goods when $i=1$ and of the flexible price cash goods when $i=2$ (c_{2i} is the corresponding notation for credit goods.)[8] Also, μ_1 denotes the fraction of cash goods whose prices are sticky (μ_2 is the corresponding fraction sticky price credit goods).

As before, the household begins the period with nominal assets, A. It then goes to the asset market where it faces constraint, (5). The household's nominal assets evolve as follows:

$$
\begin{aligned}
(30) \qquad xA' \leq\; & W(S,x)n + R(S,x)A + (x-1) + D(S,x) - (R(S,x)-1)M \\
& - \left[P^e \mu_1 c_{11} + \hat{P}(S,x)(1-\mu_1)c_{12}\right] - \left[P^e \mu_2 c_{21} + \hat{P}(S,x)(1-\mu_2)c_{22}\right],
\end{aligned}
$$

where W, R, D and x are as defined before.

The household problem is formally identical to (8), with (7) replaced by (30), (6) replaced by (29), and (9) replaced by:

$$
(31) \qquad c = \left[z\left[\mu_1 c_{11}^{\lambda} + (1-\mu_1)c_{12}^{\lambda}\right]^{\frac{\rho}{\lambda}} + (1-z)\left[\mu_2 c_{21}^{\lambda} + (1-\mu_2)c_{22}^{\lambda}\right]^{\frac{\rho}{\lambda}}\right]^{\frac{1}{\rho}}.
$$

As before, the solution to the household's problem yields decision rules of the form, $n(A,S,x)$, $M(A,S,x)$, $A'(A,S,x)$, and $c_{ij}(A,S,x)$, $i,j=1,2$.

Note how our specification of the household problem disentangles the elasticity of substitution between cash and credit goods, c_1 and c_2, from the elasticity of demand for the individual goods, $c_i(x)$, $i=1,2$, $x \in (0,1)$.

C Markov Equilibrium

Our definition of a Markov equilibrium coincides with the definition given in the previous section, with the obvious modifications. For example the labor market clearing condition is:

$$
n(1,S,x) = \mu_1 c_{11} + (1-\mu_1)c_{12} + \mu_2 c_{21} + (1-\mu_2)c_{22},
$$

where c_{ij} is as previously defined.

D Characterization

This section displays the qualitative properties of the Markov equilibrium of our economy. We proceed as in section on the benchmark model. In particular, we first derive the equations which characterize a Markov equilibrium. For this, we need to first construct the private sector allocation rule, the mapping from government policies to the prices and quantities that define a private sector equilibrium. We then need to express the first order conditions for the monetary authority, who optimizes subject to the private sector allocation rule. In the second section we use our equations to characterize the set of Markov equilibria for the model.

D.1 Private Allocations and Prices

The monetary authority's action, x, is taken at a time when P^e is known. The private sector prices and quantities to be determined are c_{ij}, $i = 1, 2$, q, n, w, R, where $q = \hat{P}/P^e$. As before, we find it convenient to think of the government's policy variable as q instead of x. So, we compute c_{ij}, $i = 1, 2$, n, w, R as a function of q and P^e.

We proceed now to pin down the seven unknowns, c_{ij}, $i = 1, 2$, n, w, R, conditional on q and P^e. For this, we use 7 equations that characterize the equilibrium. As before, the 7 equations depend upon whether or not the cash in advance constraint is binding.

The resource constraint is:

$$g + \mu_1 c_{11} + (1 - \mu_1) c_{12} + \mu_2 c_{21} + (1 - \mu_2) c_{22} = n. \tag{32}$$

Given our utility function, the first order conditions can be written as follows:

$$c_{12} = c_{11} q^{\frac{-1}{1-\lambda}}, \tag{33}$$

$$c_{22} = c_{21} q^{\frac{-1}{1-\lambda}}, \tag{34}$$

$$R = \frac{z}{1-z} \left(\frac{c_1}{c_2} \right)^{(\rho-\lambda)} \left(\frac{c_{21}}{c_{11}} \right)^{1-\lambda}, \tag{35}$$

$$\lambda = \frac{\psi c^{\rho} c_2^{\lambda-\rho} c_{22}^{1-\lambda}}{(1-n)(1-z)}. \tag{36}$$

The cash in advance constraint is given by:

$$P^e \mu_1 c_{11} + q P^e (1 - \mu_1) c_{12} \leq 1. \tag{37}$$

When the cash in advance constraint is not binding, then we impose $R = 1$, i.e.,

$$\frac{z}{1-z} \left(\frac{c_1}{c_2} \right)^{(\rho-\lambda)} \left(\frac{c_{21}}{c_{11}} \right)^{1-\lambda} = 1. \tag{38}$$

As before, we use these equations to define the private sector allocation rules, $c_{ij}(P^e, q)$, $i = 1, 2$, $R(P^e, q)$, $n(P^e, q)$.

D.2 Government Problem

We can summarize the previous discussion as providing functions:

$$c = c(P^e, q), \ \ n = n(P^e, q),$$

where c is obtained by substituting (31) into (9). These functions can be substituted into the utility function,

$$U(P^e, q) = u\left[c(P^e, q), n(P^e, q)\right].$$

Define

$$q(P^e) = \arg\max_{q \in D} U(P^e, q).$$

The function, $q(P^e)$, is the monetary authority's best response, given P^e. Equilibrium requires that $q(P^e) = 1$. This equilibrium requirement allows us to construct the equilibrium price, P^e.

D.3 Qualitative Characteristics of Markov Equilibria

As in section E.3, we begin by considering equilibria which satisfy $R > 1$. In this case, the allocation rules are differentiable and U_q is given by

$$U_q = u_c c_q + u_n n_q$$

Adding and subtracting $u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho} n_q$, we obtain

$$U_q = u_c\left[c_q - (1-z)\left(\frac{c}{c_2}\right)^{1-\rho} n_q\right] + \left[u_n + u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q. \tag{39}$$

Differentiating (31) and evaluating the result at $q = 1$,in which case $c_{ij} = c_i$, $i, j = 1, 2$, we obtain:

$$c_q = \left(\frac{c}{c_1}\right)^{1-\rho} z\left[\mu_1 c_{11,q} + (1-\mu_1)c_{12,q}\right] + \left(\frac{c}{c_2}\right)^{1-\rho}(1-z)\left[\mu_2 c_{21,q} + (1-\mu_2)c_{22,q}\right],$$

which is just (18). We now derive n_q, by differentiating (32):

(40) $$n_q = \mu_1 c_{11,q} + (1-\mu_1)c_{12,q} + \mu_2 c_{21,q} + (1-\mu_2)c_{22,q}.$$

Substituting out for c_q and n_q in the first set of square brackets in (39), we obtain:

$$\begin{aligned} U_q &= u_c\left[\mu_1 c_{11,q} + (1-\mu_1)c_{12,q}\right]\left[\left(\frac{c}{c_1}\right)^{1-\rho} z - \left(\frac{c}{c_2}\right)^{1-\rho}(1-z)\right] \\ &\quad + \left[u_n + u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q. \end{aligned}$$

Using the expression for the interest rate, (35), we obtain:

$$\begin{aligned} U_q &= u_c\left[\mu_1 c_{11,q} + (1-\mu_1)c_{12,q}\right](1-z)\left(\frac{c}{c_2}\right)^{1-\rho}(R-1) \\ &\quad + \left[u_n + u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q. \end{aligned}$$

Differentiating (37) with respect to q, and evaluating the result at $q = 1$:

(41) $$\mu_1 c_{11,q} + (1-\mu_1)c_{12,q} = -(1-\mu_1)c_1.$$

Substituting this into the preceding expression, we obtain:

(42) $$\begin{aligned} U_q &= -u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}(R-1)\, c_1(1-\mu_1) \\ &\quad + \left[u_n + u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q. \end{aligned}$$

Notice that this expression is essentially the same as the corresponding expression, (24), in the benchmark model. In this case, however, it is no longer true that the first term in (42) is zero when $c_1/c_2 = 0$. To see this, it is convenient to write (42) as

(43) $$\begin{aligned} U_q &= -\left[u_c c_2(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}\right]\psi_{ID}\left(\frac{c_1}{c_2}\right) \\ &\quad + \left[u_n + u_c(1-z)\left(\frac{c}{c_2}\right)^{1-\rho}\right] n_q, \end{aligned}$$

where

$$\psi_{ID}\left(\frac{c_1}{c_2}\right) = \left[\frac{z}{1-z}\left(\frac{c_1}{c_2}\right)^{(\rho-1)} - 1\right]\frac{c_1}{c_2}(1-\mu_1).$$

When $\rho > 0$, it is possible to use exactly the same kind of argument used in the previous section to demonstrate that there are at least two Markov equilibria. When $\rho < 0$, $\psi_{ID}(c_1/c_2) \to \infty$ as $c_1/c_2 \to 0$. When $\rho = 0$, ψ_{ID} converges to a constant as $c_1/c_2 \to 0$. Therefore, in these cases, it is not possible to use the same argument as earlier to demonstrate multiplicity of equilibria.

Indeed, in the log case ($\rho = 0$), for certain values of the model parameters, it is possible an analytical expression for the best response. This expression shows that there is a unique equilibrium, which also yields the outcome, $R = 1$. We have also constructed robust numerical examples in which it appears that the Markov equilibrium is unique.

To summarize, a key result of the two previous sections is that the issue of multiplicity of equilibria turns on the elasticity of money demand at very high rates of inflation. Specifically, we found that the multiplicity issue depends on the behavior of

$$(R-1)\frac{c_1}{c_2}.$$

In both economies, c_1 is proportional to M/P, and when inflation rates are high, c_2 is approximately proportional to aggregate consumption. Thus, the multiplicity of equilibria depends on the behavior of $(R-1)M/(Pc)$ at high inflation rates. This expression is equivalent in a sense to the magnitude of the inflation tax, where the net nominal interest rate is interpreted as the tax rate and the base, of course, is the stock of real balances. One interpretation of our results is that if these inflation tax revenues go to zero, as inflation goes to infinity, there are necessarily multiple equilibria, while if the inflation tax revenues do not go to zero, there are often unique equilibria.

V Model with Financial Intermediation

Section III displayed a cash-credit good model in which expectation traps and a high inflation bias are possible. This model has an important deficiency, however, in that it does not reproduce the dynamics of observed money demand. We propose a framework which offers some hope of resolving these difficulties. Because the framework gives households greater flexibility in their cash decisions, it is perhaps not surprising that it also implies the existence of expectation trap equilibria and a potentially high inflation bias.

We extend the model with differentiated goods studied in the previous section. The novelty here is that households can choose whether to purchase each good with cash or with credit. Cash purchases are costly because households forego interest, while credit purchases require payment of a fixed cost which differs depending on the type of good. For the sake of tractability, we assume that this fixed cost is zero for a subset of the goods and is the same and positive for the rest. We refer to this model as the endogenous cash-credit good model and the previous model as the exogenous cash-credit good model. In spirit, our endogenous cash credit good model is similar to the financial intermediation models of Aiyagari, Braun and Eckstein (1998), Cole and Stockman (1992), Freeman and Kydland (1994), Ireland (1994), Lacker and Schreft (1996), Schreft (1992) and others. All of these emphasize the tension between the use of costly transactions technologies and the interest cost of using money to finance transactions. These models and ours have the feature that at low levels of expected inflation, households use cash in a relatively large number of transactions, while at high levels little cash is used. In our preliminary research we assume that in each period the household chooses whether to use cash or credit for a given good before that period's monetary policy decision is made.

A Firms

As in section III, we assume that intermediate good firms sell output directly to households, who then aggregate individual consumption goods using a CES aggregator function. A fraction, μ, of these firms, called sticky price firms, set prices before the monetary authority's policy is chosen, and the rest, the flexible price firms, set their prices afterward. Utility maximization by households leads to the usual constant elasticity demand functions. The firms have the same production functions as the intermediate good firms in section III. Profit maximization leads to the usual condition that they set price to a constant markup over marginal cost, which is the wage rate. Let $p^e(\bar{M})$ denote the pricing rule of the sticky price firms.

B Households

The preferences of the representative household are given by: $\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$, $c_t = \left[\int_0^1 c_t(j)^\rho dj\right]^{\frac{1}{\rho}}$, where $c_t(j)$ denotes consumption of type j good and l_t denotes labor time. Households choose whether a given consumption good, $c_t(j)$, $j \in (0,1)$, is a cash good or a credit good. For goods with $j > \bar{z}$, (where $\bar{z}$ is a parameter between 0 and 1), the fixed cost of purchasing with credit is zero, and we assume these goods are always purchased with credit. To purchase each good with $j \leq \bar{z}$ using credit, a fixed cost in terms of time, η, must be paid. Without loss of generality, we suppose that the household selects a parameter, $z \in (0, \bar{z})$, such that goods with an index greater than z are purchased on credit and goods with an index less than z are purchased with cash. For a given value of z, the household's labor time, including time spent working in the market, n, is $l = n + (\bar{z} - z)\,\eta$.

The sequence of events during the period is as follows. The household begins the period with nominal assets, A. It then selects a value for z. It then goes to the asset market where it divides A into money holdings, M, and bonds, B subject to (5). Its cash in advance constraint is (6). As before, S is the state of the economy after the monetary authority has made its choice, i.e., $S = (P^e, Z, x)$, where Z denotes the economy-wide average value of z. The asset accumulation equation is the analog of (7).

Consider the household's consumption and employment decisions for a given value of z. These decisions solve the following problem:

$$w(A, z, S) = \max_{c,n,A'} u(c, n + (\bar{z} - z)\,\eta) + \beta v(A'),$$

subject to (6) and the analog of (5). Here, v is the household's value function at the beginning of the next period, before it chooses next period's z. Solving this problem yields decision rules of the form $c(A, z, S)$, $n(A, z, S)$ and $A'(A, z, S)$. The choice of z solves the following dynamic programming problem:

$$v(A) = \max_z w(A, z, S^e), \tag{44}$$

where S^e is the state of the economy if the monetary authority follows its money growth rule, i.e., $S^e = (P^e, Z, X(P^e, Z))$. The solution to (44) yields a decision rule of the form $z(A)$. These objects can be used to define a private sector equilibrium.

The monetary authority's policy function, $X(P^e, Z)$, maximizes the household's utility function. $w(1, Z, (P^e, Z, x))$ by choice of x. With one additional condition, $z = Z$, a stationary Markov equilibrium is defined analogously to that in the benchmark model.

C Results

For a range of parameter values it turns out that there are two Markov equilibria in the endogenous cash-credit good model: one with a low inflation bias and the other with a high inflation bias. To develop intuition for this result, in Figure 3, we plot the best response of the monetary authority in terms of its optimal growth rate of the money supply, x, against the corresponding expected growth rate, X.[9] (Note that x is labelled G and X is labelled g in Figure 3.) The points labelled A and C correspond to the two Markov equilibria. Over the range A to B the model behaves in exactly the same way as the exogenous cash-credit goods model. When X rises above the point corresponding to B, households find it optimal to pay the fixed costs and purchase an increasing fraction of goods using credit. As a result, the marginal cost of unexpected inflation - the implicit tax on cash goods - is actually falling as expected money growth rises, since households react to higher expected money growth by reducing the fraction of goods purchased with cash. This explains why the best response function has a steeper slope after the point B, and why it intersects the 45 degree line at point C. This multiplicity of equilibria implies that it is possible for the economy to get caught in an expectations trap. In such a trap, if private agents, for whatever reason, expect the high money growth associated with point C, they choose to purchase a small fraction of goods with cash and set prices at a high level. The monetary authority optimally chooses to validate these expectations for two reasons. First, the small number goods purchased with cash implies that the cost - in terms of the implicit tax on cash goods - of high money growth is small. Second, the fact that agents have posted high prices means that if the monetary authority does not validate expectations, output will be inefficiently low. The existence of multiple, stationary Markov equilibria also raises the possibility that the economy could be subject to excessive volatility. We conjecture that, in this economy, there are 'sunspot' equilibria in which inflation rates and output fluctuate erratically over time. For example,

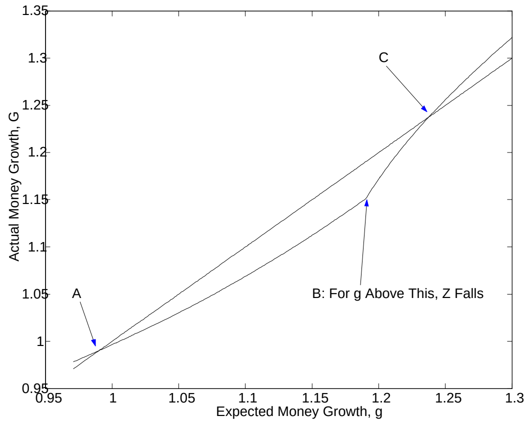

Figure 3: Monetary Authority's Best Response

suppose that at the beginning of each period, agents observe a random variable which can take on two values, h, l, and is very persistent over time. In this case, we conjecture that inflation rates will fluctuate around A for a long period of time, occasionally change by a large amount and then fluctuate around C for a long period of time.

D Money Demand and Extensions

The money demand relationship in this model is:

$$\frac{c}{M/P} = 1 + \left[\frac{1-z}{z}R\right]^{\frac{1}{1-\rho}},$$

where c is total consumption, $c = zc_1 + (1-z)c_2$, and c_1 and c_2 denote consumption of each cash and credit good, respectively. We believe that this money demand function represents an improvement over the one in the exogenous cash credit good model, and studied empirically in Lucas and Stock and Watson. For example, it is likely to be consistent with the well known observation that short run money demand elasticities are smaller than long run money demand elasticities. That is, the immediate impact on real balances of a change in the interest rate is smaller than the longer run impact. To see why, consider a stochastic version of our model in which money shocks cause interest rates to be positively serially correlated. Since the current interest rate is not known at the time that z chosen, then the immediate response of velocity is exactly the same as in the cash credit good model. Dynamically, however, the response to velocity is likely to be stronger, as z falls in response to the rise in R.

Another feature of empirical money demand equations is that shocks tend to be persistent and heteroscedastic. This is likely to be a feature of our model, since money demand shocks are functions of the product of z and R. Finally, an interesting capital-theoretic extension is one in which z in the current period is determined by its past value and by incurring current costs. In such an extension velocity may exhibit a trend, and the data suggests that some empirical measures do too.

We plan to consider stochastic versions of this model. Since the nonstochastic version has two equilibria, it seems likely that in the stochastic version monetary policy will exhibit regime shifts. Clarida, Gali and Gertler (1999) and Taylor (1999) argue that the data seems to show regime shifts. We plan to investigate whether these resemble the regime shifts in the model.

VI Conclusion

The results in this paper show that absence of commitment in monetary policy is in principle capable of rationalizing a high inflation bias, as well as prolonged periods of low and high inflation. The analysis suggests that whether it can do so depends on how high is the elasticity of money demand with respect to the interest rate.

An analysis of an extended version of our benchmark model, reported in section V convinces us that there is a good chance that it is indeed high enough. In the extended version of the model we endogenized the cash-credit good designation of a given good. Basically, any good can be treated as a cash good, but the payment of a fixed cost makes it possible to treat it as a credit good instead. This model gives people even more opportunities to substitute out of money (cash goods) than the model emphasized in the text, and so - in view of the intuition developed here - it is not surprising that that model also has multiple equilibria.

The particular empirical virtue of this model is that it holds the potential to account quantitatively for a variety of puzzles noted in the empirical literature on money demand. Specifically, the data suggest that short run interest elasticities of money demand are lower than longer run elasticities, that shocks to money demand are persistent, and that there have been secular shifts in velocity. Standard monetary models like the one emphasized in this draft of the paper cannot reproduce these features of the data, even qualitatively. The endogenous cash credit good model can do so qualitatively and seems promising quantitatively. We plan to explore this further.

A convincing test of the inflation bias hypothesis requires showing that our results are robust to various extension of our models. As we noted in the introduction, the credibility of the model's implications for inflation depend on how well it does in accounting for the dynamics of a broader set of macroeconomic variables. We intend to extend the type of model described here to allow for capital accumulation, shocks to technology, government consumption, financial intermediation and the like. Furthermore, we plan to study other types of sticky price models, such as those of Calvo (1983), Chari, Kehoe and McGrattan (1998), Taylor (1999), Gust (1998), Caplin and Spulber (1987), Erceg, Henderson and Levin (1999), King and Wolman (1996), Rotemberg (1996), Rotemberg and Woodford (1999) and others.

In addition, in this paper we have abstracted from government debt and taxes. We plan to incorporate these into our analysis because they change the incentives to generate unexpected inflation and so are likely to have an impact on any assessment of the inflation bias hypothesis.

Finally, a feature of the models considered here is that the ability of the monetary authority to affect output is not substantially different at high and low inflations. If firms could do something to make their prices more flexible at high inflation, then our multiple equilibrium result may possibly go away. This is something that needs to be investigated.

A Appendix

In this appendix we prove four results used in the text. First, we show that n_q is of the form:

$$n_q = \frac{c_2 \psi_{MD}\left(\frac{c_1}{c_2}\right)}{(1-\rho)\theta}.$$

Second, we show that $\psi_{MD}(0) > 0$. Third, we show that $U_{q\downarrow 1}$ is identical to (24). Fourth, we show that $U_{q\uparrow 1}$ is strictly positive.

A First result

To establish the first result, we begin by differentiating (14) to obtain,

$$c_{22,q} = c_{21,q} - \frac{c_2}{1-\rho}. \tag{45}$$

Combining this and (23) with (19), we obtain:

$$\frac{(1-\rho)\theta}{c_2} n_q = (1-\rho)\left\{-\frac{c_1}{c_2} z(1-\mu) + (1-z)\left[\frac{c_{21,q}}{c_2} - \frac{1-\mu}{1-\rho}\right]\right\} \tag{46}$$

To get $c_{21,q}$ we work with the labor first order condition in (14), after substituting out for θn from (12) and for c using (9) and for c_{12} and c_{22} from (14) to obtain:

$$\begin{aligned} \theta &= g + [zc_{11} + (1-z)c_{21}]\left[\mu + (1-\mu) q^{\frac{-1}{1-\rho}}\right] \\ &\quad + \frac{\psi}{\rho}\left[zc_{11}^{\rho} + (1-z)c_{21}^{\rho}\right]\left[\mu + (1-\mu) q^{\frac{-\rho}{1-\rho}}\right] c_{21}^{1-\rho} \frac{1}{q}. \end{aligned}$$

Totally differentiating this expression, we obtain:

$$c_{21,q} = \frac{-c_{11,q} z \left[1 + \psi\left(\frac{c_2}{c_1}\right)^{1-\rho}\right] + [zc_1 + (1-z)c_2]\frac{1-\mu}{1-\rho} + \frac{\psi}{\rho} c^{\rho} c_2^{1-\rho} \frac{1-\rho\mu}{1-\rho}}{(1-z)(1+\psi) + \frac{\psi}{\rho}(1-\rho)\left(\frac{c}{c_2}\right)^{\rho}} \tag{47}$$

Using (22), this reduces to:

$$\frac{c_{21,q}}{c_2} = \frac{-\frac{c_1}{c_2}\frac{(1-\mu)\rho}{1-\rho} z \left[1 + \psi\left(\frac{c_1}{c_2}\right)^{\rho-1}\right] + \left[z\frac{c_1}{c_2} + 1 - z\right]\frac{1-\mu}{1-\rho} + \frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\frac{1-\rho\mu}{1-\rho}}{(1-z)(1+\psi) + \frac{\psi}{\rho}(1-\rho)\left(\frac{c}{c_2}\right)^{\rho}}. \tag{48}$$

Substituting (48) into (46) it is easily verified that n_q is of the desired form.

B Second Result

To verify $\psi_{MD}(0) > 0$, it is sufficient to establish that the expression in square brackets in (46) is positive when $c_1/c_2 = 0$. Evaluating this, taking into account (48):

$$\begin{aligned}
\frac{c_{21,q}}{c_2} - \frac{1-\mu}{1-\rho} &= \frac{(1-z)\frac{1-\mu}{1-\rho} + \frac{\psi}{\rho}(1-z)\frac{1-\rho\mu}{1-\rho}}{(1-z)(1+\psi) + \frac{\psi}{\rho}(1-\rho)(1-z)} - \frac{1-\mu}{1-\rho} \\
&= \frac{1}{1-\rho}\frac{1-\mu+\frac{\psi}{\rho}(1-\rho\mu)}{1+\psi+\frac{\psi}{\rho}(1-\rho)} - \frac{1-\mu}{1-\rho} \\
&= \frac{1}{1-\rho}\left\{\frac{1-\mu+\frac{\psi}{\rho}(1-\rho\mu) - (1-\mu)(1+\psi) - (1-\mu)\frac{\psi}{\rho}(1-\rho)}{1+\psi+\frac{\psi}{\rho}(1-\rho)}\right\} \\
&= \frac{1}{1-\rho}\frac{\psi}{\rho}\frac{(1-\rho)\mu}{1+\psi+\frac{\psi}{\rho}(1-\rho)} > 0.
\end{aligned}$$

This establishes the desired result.

C Third Result

To establish the third result, it suffices to establish that the interest rate is increasing in q at the point $c_1/c_2 = 1$. That is, since $R = (c_{22}/c_{12})^{1-\rho}$, we need $c_{21,q} \geq c_{11,q}$ at the point $c_{21} = c_{11} = c_{22} = c_{12} = c$. Substituting for $c_{21,q}$ from (47), we need to show that

$$\frac{-c_{11,q}z\left[1+\psi\right] + c\frac{1-\mu}{1-\rho} + \frac{\psi}{\rho}c\frac{1-\rho\mu}{1-\rho}}{(1-z)(1+\psi) + \frac{\psi}{\rho}(1-\rho)} \geq c_{11,q},$$

or

$$\left\{\frac{1-\mu}{1-\rho} + \frac{\psi}{\rho}\frac{1-\rho\mu}{1-\rho}\right\} \geq \frac{c_{11,q}}{c}\left\{z(1+\psi) + (1-z)(1+\psi) + \frac{\psi}{\rho}(1-\rho)\right\},$$

or, substituting for $c_{11,q}$ and simplifying,

$$\left\{\frac{1-\mu}{1-\rho} + \frac{\psi}{\rho}\frac{1-\rho\mu}{1-\rho}\right\} \geq \frac{(1-\mu)\,\rho}{1-\rho}\left\{(1+\psi) + \frac{\psi}{\rho}(1-\rho)\right\}$$

or,

$$1-\mu+\frac{\psi}{\rho}(1-\rho\mu) \geq (1-\mu)\,\rho(1+\psi) + (1-\mu)\,\psi(1-\rho).$$

Dividing through by $1-\mu$, we need to show that

$$1 + \frac{\psi}{\rho}\frac{1-\rho\mu}{1-\mu} \geq \rho(1+\psi) + \psi(1-\rho)$$

or

$$1 + \frac{\psi}{\rho}\frac{1 - \rho\mu}{1 - \mu} \geq \rho + \psi.$$

Since $\rho \leq 1$ and $(1 - \rho\mu) / [\rho(1 - \mu)] = (1/\rho - \mu) / (1 - \mu) \geq 1$, we have the desired result.

D Fourth Result

To obtain the fourth result, we can see from (21) that, since the first term is zero and the term in square brackets is positive, the result follows if $n_q > 0$. We establish this result here.

Inspecting (18) and (19), and evaluating the derivatives at $c_1 = c_2 = c$, it follows that

$$c_q = \theta n_q.$$

From (14), we have

$$c_{12,q} = c_{11,q} - \frac{c}{1 - \rho}, \quad c_{22,q} = c_{21,q} - \frac{c}{1 - \rho}$$

Totally differentiating the equation, $R = 1$, i.e., $c_{22} = c_{12}$, we obtain $c_{22,q} = c_{12,q}$. Using this result in the previous equation, we obtain

$$c_{11,q} = c_{21,q}.$$

Using these results in (18), we obtain

$$c_q = \mu c_{11,q} + (1 - \mu) c_{12,q},$$

or

(49) $$c_q = c_{12,q} + \frac{\mu c}{1 - \rho}.$$

Next, totally differentiating the labor first order condition in (14) and using $c_q = \theta n_q$ and $c_{22,q} = c_{12,q}$, we obtain

(50) $$c_q = -\frac{\psi(1 - \rho)}{\rho(1 + \psi)} c_{12,q}.$$

Substituting for $c_{12,q}$ from (50) into (49), we obtain

$$c_q = \frac{\mu c}{1 - \rho} \frac{1}{1 + \frac{\rho(1+\psi)}{\psi(1-\rho)}} > 0.$$

Since $n_q = c_q / \theta$, the desired result follows.

References

[1] Aiyagari, S. Rao, R. Anton Braun and Zvi Eckstein, 1998, 'Transactions Services, Inflation and Welfare,' *Journal of Political Economy*, vol. 106, no. 6, pp. 1274-1301.

[2] Barro, Robert J., and David B. Gordon, 1983, 'A Positive Theory of Monetary Policy in a Natural Rate Model,' *Journal of Political Economy* 91 (August): 589-610.

[3] Bental, Benjamin, and Zvi Eckstein, 1995, 'A Neoclassical Interpretation of Inflation and Stabilization in Israel,' Foerder Institute for Economic Research Working Paper no. 28.

[4] Blanchard, O.J. and N. Kiyotaki, 1987, 'Monopolistic Competition and the Effects of Aggregate Demand,' *American Economic Review*, Vol. 77, No. 4, 647-666.

[5] Cagan, Phillip, 1991, 'The Monetary Dynamics of Hyperinflation,' in Edmund S. Phelps, ed., *Recent Developments in Macroeconomics*, pp. 181-203, Elgar.

[6] Calvo, Guillermo, 1978, 'On the Time Consistency of Optimal Policy in a Monetary Economy,' *Econometrica,* Vol. 46, No. 6, 1411-1426.

[7] Chari, V. V., Lawrence J. Christiano and Martin Eichenbaum, 1998, 'Expectation Traps and Discretion,' *Journal of Economic Theory* , Vol. 81 No. *2,* pp. *462-492.*

[8] Cochrane, John, 1998, 'A Frictionless View of US Inflation,' *NBER Macroeconomics Annual 1998*, MIT Press, pp. 323-384.

[9] Cole, Harold, and Alan Stockman, 1992, 'Specialization, Transactions Technologies and Money Growth,' *International Economic Review*, vol. 33, no. 2, pp. 283-298.

[10] Clarida, Richard, Jordi Gali and Mark Gertler, 1999, 'The Science of Monetary Policy: A New Keynesian Perspective,' National Bureau of Economic Research Working Paper 7147, May.

[11] Erceg, Christopher, Dale Henderson and Andrew Levin, 1999, 'Optimal Monetary Policy with Staggered Wage and Price Contracts,' manuscript, Federal Reserve Board of Governors.

[12] Freeman, Scott and Finn Kydland, 1998, 'Monetary Aggregates and Output,' Federal Reserve Bank of Cleveland Working Paper no. 9813.

[13] Gust, Christopher, 1998, 'Staggered Price Contracts and Factor Immobilities: The Persistence Problem Revisited,' manuscript, Federal Reserve Board of Governors.

[14] Ireland, Peter, 1994, 'Money and Growth: An Alternative Approach,' *American Economic Review*,' Vol. 84, No. 1, 47-65.

[15] Ireland, Peter N., 1998, 'Does the Time-Consistency Problem Explain the Behavior of Inflation in the United States?,' manuscript, Department of Economics, Boston College.

[16] King, Robert G. and Alexander Wolman, 1996, 'Inflation Targeting in a St. Louis Model of the 21st Century,' National Bureau of Economic Research Working Paper number 5507.

[17] Kydland, Finn E., and Edward C. Prescott, 1977, 'Rules Rather Than Discretion: The Inconsistency of Optimal Plans,' *Journal of Political Economy*, vol. 85, no. 3, June, pages 473-91.

[18] Lacker, Jeffrey, and Stacey Schreft, 1996, 'Money and Credit as Means of Payment,' *Journal of Monetary Economics*, vol. 38, no. 1, pp. 3-23.

[19] Lucas, Robert E., Jr., and Nancy L. Stokey, 1983, 'Optimal Fiscal and Monetary Policy in an Economy Without Capital,' *Journal of Monetary Economics*, vol. 12, pp. 55-93

[20] McCallum, Bennett T., 1997, 'Crucial Issues Concerning Central Bank Independence,' Journal of Monetary Economics 39, June, pp. 99-112.

[21] Persson, Torsten and Guido Tabellini, 1993, 'Designing Institutions for Monetary Stability,' Carnegie-Rochester Series on Public Policy, vol. 39, pp. 53-84.

[22] Rotemberg, Julio, 1996, 'Prices, Output and Hours: An Empirical Analysis Based on a Sticky Price Model,' *Journal of Monetary Economics*, vol. 37, no. 3, pp. 505-533.

[23] Rotemberg, Julio, and Michael Woodford, 1999, 'Interest-Rate Rules in an Estimated Sticky Price Model,' in Taylor (1999a).

[24] Sargent, Thomas J., 1999, *The Conquest of American Inflation*, Princeton University Press.

[25] Schreft, Stacey, 1992, 'Welfare-Improving Credit Controls,' *Journal of Monetary Economics*, vol. 30, no. 1, pp. 57-72.

[26] Svensson, Lars E. O., 1995, 'Optimal Inflation Targets, "Conservative Central Bankers", and Linear Inflation Targets,' National Bureau of Economic Research Working Paper 5251.

[27] Taylor, John B., 1999, 'An Historical Analysis of Monetary Policy Rules,' in John B. Taylor (1999a).

[28] Vining, D., and T. Elwertowski, 1976, 'The Relationship Between Relative Prices and the General Price Level,' *American Economic Review*, 66, pp. 699-708.

[29] Woodford, Michael, 1998, 'Comment on Cochrane,' *NBER Macroeconomics Annual 1998*, MIT Press, pp. 390-418.

[30] Zarazaga, Carlos, 1992, 'Inflation Processes in Latin America,' University of Minnesota Ph.D. dissertation.

Notes

[1]Our model formalizes the idea in Kydland and Prescott (1977) and in Barro and Gordon (1983) that an unanticipated monetary expansion raises output and can raise welfare. For models and evidence on the effects of inflation on relative allocations, see Cukierman 1983, Parks 1978, and Vining and Elwertowski 1976.

[2]A more compact definition of the state would include only θ, g, and $P^e(\theta, g)$, where $P^e(\theta, g)$ is the equilibrium pricing function. Effectively, this means that S need only include θ and g. To see why this more compact definition of a state is adequate in our environment, see Chari and Kehoe (1991).

[3]We have adopted a slight change in the notation. Concavity of the function that aggregates $c(\omega)$ into c in the utility function implies that $c(\omega)$ is optimally chosen to be constant for ω associated with sticky price cash goods, flexible price cash goods, etc. Thus, we let c_{12} denote $c(\omega)$ for those ω such that $\omega \leq z$ and which are produced by flexible price firms.

[4]What sort of constraint should we place on household debt?

[5]The result will be presented in a subsequent draft of this paper.

[6]Actually, we scale the multipliers by $\bar{M}$. That is, if $\tilde{\lambda}$ is the multiplier on (7), then $\lambda = \tilde{\lambda}\bar{M}$. This scaling is adopted to assure that λ is not a function of the stock of money.

[7]The parameter values used in the example are $\mu = 0.1$, $z = 0.5$, $\rho = 0.83$, $\psi = 4$, $\theta = 1$, $g = 0.05$.

[8]As in the previous section, concavity of the utility function guarantees that households optimally choose to consume all sticky price cash goods at the same rate, and similarly for the flexible price cash goods and the sticky and flexible price credit goods.

[9]The underlying parameter values are $\beta = 1/1.03$, $\eta = .065$, $\psi = 1.64$, $\rho = .83$, $\mu = 0.1$, $\bar{z} = 0.3$, $\sigma = 1.01$.