REAL-TIME NOWCASTING THE US OUTPUT GAP: SINGULAR SPECTRUM ANALYSIS AT WORK

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Real-time nowcasting the US output gap: 
Singular spectrum analysis at work

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Abstract

We explore a new approach for nowcasting the output gap based on singular spectrum analysis. Resorting to real-time vintages, a recursive exercise is conducted so to assess the real-time reliability of our approach for nowcasting the US output gap, in comparison with some well-known benchmark models. For our applied setting of interest, the preferred version of our approach consists of a two-channel singular spectrum analysis, where we use a Fisher $g$ test to infer which components, within the standard business cycle range, should be included in the grouping step. We find that singular spectrum analysis provides a reliable assessment of the cyclical position of the economy in real-time, with the two-channel approach outperforming substantially the univariate counterpart.

Keywords: Band-pass filter; Multivariate singular spectrum analysis; Singular spectrum analysis; US output gap, Real-time data.

JEL classification: C50, E32.

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1 Introduction

The output gap plays a central role in policymaking. Most central banks aim to keep inflation under control, and the output gap is a key source of inflation pressures in the economy. Given that the output gap fluctuates when the economy is overheating or underperforming, the conduct of monetary policy should take it into full consideration. It can also be used to determine and pursue policy measures by governments—as the cyclical position of the economy may influence fiscal policy—, and thus the assessment of the output gap is crucial for many formulations of countercyclical stabilization policy.

Measuring the output gap is however challenging—as it cannot be observed directly—and so cannot be assessed precisely. The revisions to which real-time output gap estimates are subject, present yet another challenge as they can compromise their operational usefulness for policymakers—who need reliable ‘intel’ in real-time. There are by now several studies documenting the large uncertainty of real-time output gap estimates, with this being a common issue for all estimation methods available (see Orphanides & van Norden, 2002, Orphanides, 2003a, Watson, 2007, Marcellino & Musso, 2011, Edge & Rudd, 2012, among others). The policy implications of the effects of output gap uncertainty have been addressed by, for example, Orphanides (2001), Rudebusch (2001), Smets (2002), Orphanides (2003b), and Orphanides and Williams (2007).

In this paper we focus on singular spectrum analysis (SSA), and evaluate its potential contribution for nowcasting output gap in a real-time setup. Despite the potential usefulness of SSA for the analysis of economic phenomena there are only a few applications in the economics and finance literature. In this respect, see the recent work by Hassani, Heravi, and Zhigljavsky (2009), Patterson, Hassani, Heravi, and Zhigljavsky (2011), Hassani, Soofi, and Zhigljavsky (2013a,b), and de Carvalho, Rodrigues, and Rua (2012). In particular, the latter have shown that SSA can deliver output gap estimates that resemble those obtained with band-pass filters while improving the reliability of the corresponding nowcasts. We extend the work by de Carvalho, Rodrigues, and Rua (2012) in several dimensions.

First, to mimic a real-life policymaking scenario, that is, to replicate the problem faced by policymakers at the time policy decisions have to be taken, we consider real-time data. This means considering the vintages of data available at each moment in time. It is by now widely acknowledged that data revisions can affect policy decisions, and although the issue of the importance of data revi-
sions is not recent, there has been a growing interest among practitioners to take on board real-time data into the analysis, since the influential work by Croushore and Stark (2001, 2003)—who compiled and examined real-time data for major US macroeconomic variables. Hence, we focus on the evaluation of output gap nowcasts computed through a recursive exercise using at each period the corresponding available vintage. This allows us to obtain real-time estimates—which are the ones relevant in terms of policymaking—whereas de Carvalho, Rodrigues, and Rua (2012) only considered quasi-real estimates, by considering the latest available vintage.¹

An important issue for which we suggest a novel approach regards the selection of principal components to be used in the reconstruction of the variable of interest. For instance, de Carvalho, Rodrigues, and Rua (2012) use an heuristic approach to select the components to be considered for the reconstruction of the GDP cyclical component. Based on the dominant frequency, they consider the components that reflect periodicities of interest, namely within the business cycle frequency range. In this respect, Hassani, Heravi, and Zhigljavsky (2009) suggest the computation of the periodogram for assessing the dominant periodicity. We propose an alternative inferential procedure to address this issue, by using a spectral-based Fisher *g* test. Although less popular than time domain analysis, Fourier analysis has proven to be quite useful in a wealth of contexts (see, for example, A’Hearn & Woitek, 2001, Rua & Nunes, 2005, Breitung & Candelon, 2006, Lemmens, Croux, & Dekimpe, 2008). Drawing on the periodogram estimator, Fisher (1929) derived an exact test—the so-called Fisher *g* test—which allows for the detection of hidden periodicities of unspecified frequency, by determining whether a peak in the periodogram is significant or not. We use the Fisher *g* test to select the principal components to be aggregated in the reconstruction of the output gap; specifically, we consider all principal components that present a statistically significant peak in the periodogram, within the standard business cycle frequency range. This provides a formal criterion for selecting the principal components relevant for the problem at hand.

Another contribution of our paper rests on the use of information beyond that conveyed by GDP to estimate the output gap. Although, as stressed by Stock and Watson (1999), the cyclical component of real GDP is a useful proxy for the overall business cycle, it is sensible to argue that other

¹More details on the distinction between concepts of real-time and quasi-real estimates of the output gap can be found in Orphanides and van Norden (2002, p. 571).
macroeconomic variables should also reflect business cycle developments (see also the pioneer work of Burns and Mitchell, 1946). In this respect, the industrial production index is one of the macroeconomic indicators more commonly used in the literature for assessing the cyclical position of the economy in the absence of GDP data, and it is actually one of the top indicators used in practice for dating the US business cycle by the National Bureau of Economic Research (NBER) Business Cycle Dating Committee (www.nber.org/cycles/recessions.html). As GDP and industrial production are strongly correlated at business cycle frequencies, the use of industrial production data to complement GDP in the estimation of the output gap seems a natural choice. Typically, the use of macroeconomic data other than GDP does not lead to substantial differences in final output gap estimates, but can potentially improve the real-time assessment (see, for example, Valle e Azevedo, Koopman, & Rua, 2006, Valle e Azevedo, 2011). To take on board information beyond that conveyed by GDP, we extend the output gap estimation from the univariate SSA, considered in de Carvalho, Rodrigues, and Rua (2012), to the multivariate SSA case.

To assess the relative performance of the suggested approach to nowcast the US output gap, we consider alternative econometric techniques, namely the popular Hodrick and Prescott (1997) filter and the band-pass filter of Christiano and Fitzgerald (2003). In line with previous literature, we find that all approaches deliver relatively similar final output gap estimates. In addition, such estimates are in accordance with the US business cycle chronology. Based on a real-time US dataset and resorting to a standard battery of reliability statistics, we evaluate the real-time performance of each approach. The Hodrick–Prescott filter seems to perform the worst, whereas the SSA approach delivers more reliable output gap nowcasts than the alternative filtering techniques. Going beyond the univariate SSA, we conclude that the use of data other than GDP, in particular industrial production, can be very useful for improving output gap nowcasting. Hence, considering a multivariate framework based on SSA can be quite useful for producing reliable real-time estimates of the US output gap.

Our paper is organized as follows. In Section 2 we discuss our SSA-based approach for modeling business cycles. In Section 3 we use our approach for real-time nowcasting the US output gap, and compare it with some popular benchmark methods. We conclude in Section 4.

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2We underscore that although measuring output gap nowcast uncertainty can also be interesting (Garratt, Mitchell, and Vahey, 2014), the focus here is on point estimation.
2 Singular spectrum business cycle analysis

2.1 Modeling concept: One-channel setting

As argued by Morley and Piger (2012) there are two main views for modeling business cycles: An alternating-phases approach (Mitchell, 1927), which considers a rotating sequence of expansions–recessions, and an output gap approach (Beveridge & Nelson, 1981) where the business cycle, \( C_t \), is defined as a transitory deviation from a trend, \( T_t \). Formally, for seasonally adjusted data, the latter approach is based on decomposing GDP, \( Y_t \), as follows

\[
Y_t = T_t + C_t.
\]  

(1)

The target of estimation in an output gap approach is thus naturally \( C_t \). Following the seminal work of Burns and Mitchell (1946), most literature has been concerned with recurring movements ranging from 6 to 32 quarters, so that a more reasonable working assumption is provided by the model

\[
Y_t = T_t + C_t + R_t,
\]  

(2)

where \( R_t \) is a noise term describing recurring movements of frequencies higher than the ones of interest in a business cycle context. The singular spectrum analysis-based approaches to be discussed in the next sections are based on the output gap approach discussed in Eqs. (1) and (2), and the interest is on assessing the performance of the methods in real-time, so that our goal is on nowcasting \( C_t \), using information available until time \( t - 1 \), or in other words, conditionally on \( \mathcal{F}_{t-1} = \sigma(Y_1, \ldots, Y_{t-1}) \), where \( \sigma(\cdot) \) denotes the natural filtration.\(^3\) For a primer on singular spectrum analysis see, for instance, Golyandina, Nekrutkin, and Zhigljavsky (2001) and Hassani, Mahmoudvand, and Patterson (2014).

To make the exposition concrete, below we focus on discussing singular spectrum analysis in the context of our applied econometric problem of interest, so that the expression ‘singular spectrum business cycle analysis’ should be understood as a synonym of an adapted singular spectrum analysis with business cycle applications in mind.

\(^3\)To be precise, in our applied setting of interest, at each period \( t \) we consider a different vintage of data, \( \mathbf{V}_t = (Y_{1,t}, \ldots, Y_{t-1,t}) \), where \( Y_{\tau,t} \) denotes the data at time \( \tau \) as they looked at time \( t \), for \( \tau \in \{1, \ldots, t - 1\} \), and thus our analysis at time \( t \) is actually conditional on \( \mathcal{F}_{t-1} = \sigma(\mathbf{V}_t) \).
2.2 One-channel singular spectrum business cycle analysis

The method entails two phases, namely decomposition and reconstruction, and each of these phases includes two steps; the phase of decomposition includes the steps of embedding and singular value decomposition, which we discuss below.

Embedding. This is the preliminary step of the method. The core concept assigned to this step is given by the GDP trajectory matrix, i.e., a matrix whose columns consist of rolling windows of the GDP time series \( y = (Y_1, \ldots, Y_n) \). The GDP trajectory matrix is defined as

\[
Y = \begin{pmatrix}
Y_1 & \cdots & Y_k \\
Y_2 & \cdots & Y_{k+1} \\
\vdots & \vdots & \vdots \\
Y_l & \cdots & Y_{k+l-1}
\end{pmatrix}
\]

where \( k \) is such that \( Y \) encompasses all the observations in the original time series, i.e., \( k = n - l + 1 \).

We refer to each vector \( y_i = (Y_i, \ldots, Y_{i+l-1})^T \), as a GDP window, where the window length, \( l \), is a parameter to be set by the user. Note that \( Y \) has constant antidiagonals—and thus it is a Hankel matrix—and note further that the GDP series \( y \) relies in the ‘ell’ formed by the first column and the last row; the GDP trajectory matrix can also be thought of as a sequence of \( k \) GDP windows, i.e., \( Y = (y_{1,l} \cdots y_{k,l}) \). A short, yet formal, description of embedding, is that the step resumes to the map

\[
y \mapsto \text{Embed}(y) = Y,
\]

with \( Y \) defined in Eq. (3).

Singular Value Decomposition. In the second step we perform a singular value decomposition of the GDP trajectory matrix. Hence, from an eigenanalysis of \( YY^T \) we obtain the eigenvalues \( \lambda_1 \geq \cdots \geq \lambda_d \), where \( d = \text{rank}(YY^T) \), as well as the corresponding left and right singular vectors, which we respectively denote by \( u_i \) and \( v_i \). This leads us to the following decomposition of the GDP trajectory matrix,

\[
Y = \sum_{i=1}^{d} u_i v_i^T \sqrt{\lambda_i}, \tag{4}
\]
Below we discuss the second phase of the method—reconstruction, which entails the steps of grouping cyclical components and diagonal averaging.

**Grouping Cyclical Components.** Not all summands in Eq. (4) contain relevant information on the business cycle, and hence we confine ourselves to a subset $S$ of $\{1, \ldots, d\}$, so to compute what we define as the cycle matrix,

$$ C = \begin{pmatrix} 
C_{1,1} & \cdots & C_{1,k} \\
\vdots & \ddots & \vdots \\
C_{l,1} & \cdots & C_{l,k} + l - 1 
\end{pmatrix} = \sum_{i \in S} u_i v_i^T \sqrt{\lambda_i}. \quad (5) $$

In practice, we construct $S$ through a Fisher $g$ test on which we provide further details in Section 2.5.

**Diagonal Averaging.** In this step we average over all the elements of the antidiagonals of the cycle matrix in Eq. (13) so to obtain a Hankel matrix. This can be performed through the map

$$ C \mapsto \mathbb{D}(C) = \left( \frac{1}{|A_1|} \sum_{(i,j) \in A_1} c_{i,j}, \ldots, \frac{1}{|A_n|} \sum_{(i,j) \in A_n} c_{i,j} \right), \quad (6) $$

where $|\cdot|$ denotes the cardinal operator, and where the sequence of sets

$$ A_t = \{(i, j) : i + j = t + 1, i \in \{1, \ldots, l\}, j \in \{1, \ldots, k\}\}, \quad t = 1, \ldots, n, \quad (7) $$

defines the elements of the $n$ antidiagonals of the cycle matrix.

The business cycle indicator $\hat{c}$ yielded by the steps above is given by the diagonal averaging of the cycle matrix in (13), i.e.,

$$ \hat{c} = (\hat{C}_1, \ldots, \hat{C}_n) \equiv \mathbb{D}(C) = \left( \frac{1}{|A_1|} \sum_{(i,j) \in A_1} c_{i,j}, \ldots, \frac{1}{|A_n|} \sum_{(i,j) \in A_n} c_{i,j} \right), \quad (8) $$

### 2.3 Modeling concept: Two-channel setting

The method in Section 2.2 is essentially an updated version of the approach in de Carvalho, Rodrigues, and Rua (2012), which we now generalize to the two-channel, or bivariate, setting. The main motivation for this is as follows: From a practical viewpoint, we have reasons to believe that we should be able
to borrow strength from further information available on real time. Particularly, we are interested in constructing a business cycle indicator which combines information of the GDP and the industrial production (IP) index—which is a proxy for measuring economic activity evolution, and it is well known to be strongly correlated with the aggregate activity as measured by GDP (see, for instance, Fagiolo, Napoletano, & Roventini, 2008, de Carvalho & Rua, 2014). With this in mind, we extend the working assumption in (2) to a joint setting, so that the dynamics governing the GDP, $Y_t$, and the IP, $I_t$, are assumed to be of the form

$$Y_t = T_t^Y + C_t^Y + R_t^Y, \quad I_t = T_t^I + C_t^I + R_t^I.$$  \tag{9}

It is important to underscore that the target of estimation is $C_t^Y$, and thus the same as in Section 2.1.

### 2.4 Two-channel singular spectrum business cycle analysis

Two-channel singular spectrum business cycle analysis can be conducted by extending the approach discussed in Section 2.2. As we shall see below, the main modification is that we need to construct a block Hankel trajectory matrix—rather than an ordinary trajectory matrix; the extension to the multivariate setting is analogous.

**Embedding.** In the embedding step we construct a GDP–IP trajectory matrix which consists of a two-block matrix defined as

$$Z = \begin{pmatrix} Y \\ I \end{pmatrix},$$  \tag{10}

where $Y$ is a GDP trajectory matrix, similar to the one defined in Eq. (3),

$$Y = \begin{pmatrix} Y_1 & \cdots & Y_k \\ Y_2 & \cdots & Y_{k+1} \\ \vdots & \vdots & \vdots \\ Y_l & \cdots & Y_{k+l-1} \end{pmatrix} = \begin{pmatrix} T_1^Y + C_1^Y + R_1^Y & \cdots & T_k^Y + C_k^Y + R_k^Y \\ T_2^Y + C_2^Y + R_2^Y & \cdots & T_{k+1}^Y + C_{k+1}^Y + R_{k+1}^Y \\ \vdots & \vdots & \vdots \\ T_l^Y + C_l^Y + R_l^Y & \cdots & T_{k+l-1}^Y + C_{k+l-1}^Y + R_{k+l-1}^Y \end{pmatrix},$$  \tag{11}
and with $I$ being analogously defined, i.e.,

$$I = \begin{pmatrix} I_1 & \cdots & I_k \\ I_2 & \cdots & I_{k+1} \\ \vdots & \vdots & \vdots \\ I_l & \cdots & I_{k+l-1} \end{pmatrix} = \begin{pmatrix} T_i^I + C_i^I + R_i^I \\ T_2^I + C_2^I + R_2^I \\ \vdots \\ T_l^I + C_l^I + R_l^I \end{pmatrix}. $$

**Singular Value Decomposition.** In the second step we perform a singular value decomposition of the GDP–IP trajectory matrix. Hence, from an eigenanalysis of $ZZ^T$ we gather the eigenvalues $\lambda_1 \geq \cdots \geq \lambda_d$, where $d = \text{rank}(ZZ^T)$, as well as the corresponding left and right singular vectors which we respectively denote by $u_i$ and $v_i$. Thus, we decompose the GDP–IP trajectory matrix into

$$Z = \sum_{i=1}^{d} u_i v_i^T \sqrt{\lambda_i}. \quad (12)$$

**Grouping Cyclical Components.** Not all summands in Eq. (12) contain relevant information on the business cycle, and hence we confine ourselves to a subset $S$ of $\{1, \ldots, d\}$, so to produce what we define as the *two-block cycle matrix*,

$$C = \begin{pmatrix} C^Y \\ C^I \end{pmatrix} = \sum_{i \in S} u_i v_i^T \sqrt{\lambda_i}, \quad (13)$$

where

$$C^Y = \begin{pmatrix} c_{1,1}^Y & \cdots & c_{1,k}^Y \\ \vdots & \vdots & \vdots \\ c_{l,1}^Y & \cdots & c_{l,k+l-1}^Y \end{pmatrix}, \quad C^I = \begin{pmatrix} c_{1,1}^I & \cdots & c_{1,k}^I \\ \vdots & \vdots & \vdots \\ c_{l,1}^I & \cdots & c_{l,k+l-1}^I \end{pmatrix}. \quad (14)$$

Similarly to Section 2.2, we construct $S$ through a Fisher $g$ test on which we provide further details in Section 2.5. As we discuss in Section 3 the advantages of our Fisher–$g$ test approach are particularly evident in the two-channel setting, given that we face a larger number of ‘candidate’ components which could potentially be used to construct the cycle.

**Diagonal Averaging.** In this step we average over all the elements of the antidiagonals of the cycle matrix $C^Y$, as defined in Eq. (14), so to obtain a Hankel matrix. This can be performed through the
D mapping defined in (6), so that the business cycle indicator $\hat{c}^Y$ yielded by the steps above is given by the diagonal averaging of the block of the cycle matrix corresponding to GDP, that is to say

$$\hat{c}^Y = (\hat{C}^Y_1, \ldots, \hat{C}^Y_n) \equiv \mathbb{D}(C^Y) = \left( \frac{1}{|A_1|} \sum_{(i,j) \in A_1} C^Y_{i,j}, \ldots, \frac{1}{|A_n|} \sum_{(i,j) \in A_n} C^Y_{i,j} \right).$$  \hspace{1cm} (15)

Here $C^Y$ is defined as in Eq. (13), and $A_t$ is defined as in Eq. (7).

The extension from the two-channel to the $D$-channel setting is trivial, the main difference being that we would need to build a $D$-block Hankel matrix approximation to the trajectory matrix, where the latter would consist of an extension of the GDP–IP trajectory matrix in Eq. (10) to a $D$-block setting; see Hassani, Soofi, and Zhigljavsky (2013b, p. 747).

2.5 Targeted grouping based on the Fisher $g$-statistic

The grouping stage in SSA should take into account the targeted output. In our framework, the aim is to group the components that reflect business cycle developments. In this respect, de Carvalho, Rodrigues, and Rua (2012) have grouped the components that seemed, by visual inspection, to contain information about the standard business cycle frequency range. Here, we suggest a formal inferential approach to address this issue. Underlying the informal approach of de Carvalho, Rodrigues, and Rua (2012) is the idea that one should select the components whose dominant periodicity (or frequency) falls within the range of frequencies of interest. This problem can be more formally addressed using spectral analysis. In particular, one can determine the dominant frequency (or periodicity) by finding the peak in the periodogram, while its statistical significance can be assessed through the so-called Fisher $g$-statistic—to be introduced below. If the frequency at which the peak is observed in the periodogram lies within the business cycle frequency range, and if it is statistically significant according to Fisher $g$-statistic, then that component is selected for the reconstruction of the cyclical component.

As mentioned earlier, the Fisher $g$ test draws on the periodogram; see Priestley (1981, Sec. 6.1.4). The periodogram unveils the power of the signal at various frequencies, so that if the signal is being driven by a certain frequency, the periodogram presents a peak precisely at that periodicity. Basically, the Fisher $g$ test checks for the proportion of power accounted for the frequency associated with the peak in the periodogram, and tests whether such peak is random or not. More formally, if $X_1, \ldots, X_n$...
is an equally-spaced time series, the periodogram consists of the set of points
\[ \{(\omega_j, I(\omega_j)) : j = 1, \ldots, J\}, \quad J = [(n - 1)/2], \]
where \(\lfloor \cdot \rfloor\) denotes the floor function, \(\omega_j = 2\pi j/n\) are the so-called Fourier frequencies, for \(j = 1, \ldots, J\), and
\[ I(\omega) = \frac{1}{n} \left| \sum_{t=1}^{n} X_t e^{-i\omega t} \right|^2 = \frac{1}{n} \left[ \left\{ \sum_{t=1}^{n} X_t \sin(\omega t) \right\}^2 + \left\{ \sum_{t=1}^{n} X_t \cos(\omega t) \right\}^2 \right], \quad \omega \in (0, \pi). \]
If a time series has a significant periodic component with frequency \(\omega^*\), then the periodogram will exhibit a peak at frequency \(\omega^*\). Fisher (1929), in a celebrated paper, derived an exact test for testing the significance of the spectral peak based on the \(g\)-statistic,
\[ g = \frac{\max_j \{I(\omega_1), \ldots, I(\omega_J)\}}{\sum_j I(\omega_j)}, \quad \text{Eq. (16)} \]
In Fisher’s test, the null hypothesis is that the spectral peak is not statistically significant against the alternative hypothesis that there is a periodic component; under the Gaussian assumption, large values of \(g\) lead to the rejection of the null hypothesis. The \(p\)-value of the test under the null hypothesis is given by
\[ p \equiv P(g > g^*) = \sum_{\kappa=1}^{K} \left( \frac{-1}{\kappa!} \frac{J!}{(J - \kappa)!} \right) (1 - \kappa g^*)^{J-1}, \quad \text{Eq. (17)} \]
where \(K\) is the largest integer less than \(1/g^*\) and \(g^*\) is the observed value of the \(g\)-statistic. In practice we proceed as in the following pseudocode implementation. Let \(\Omega \subseteq (0, \pi)\) denote a range of frequencies of interest, and let \(\overline{D}_i = \overline{D}(u_i, v_i^T \sqrt{\lambda_i})\) denote the \(i\)th principal component.

<table>
<thead>
<tr>
<th>Targeted grouping based on the Fisher (g)-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start with (S^{(0)} = \emptyset), and for (i = 1, \ldots, d), do:</td>
</tr>
<tr>
<td>Step 1. Obtain the periodogram of (\overline{D}_i), and compute:</td>
</tr>
<tr>
<td>[ \omega_i^* = \arg \max_{\omega \in \omega_1, \ldots, \omega_J} I_i(\omega). \quad \text{Eq. (18)} ]</td>
</tr>
<tr>
<td>Step 2. if (\omega_i^* \in \Omega) go to Step 3; otherwise increment (i) and go back to Step 1.</td>
</tr>
<tr>
<td>Step 3. use Eq. (16) to compute the (g)-statistic associated with (\overline{D}_i); save the result in (g_i).</td>
</tr>
</tbody>
</table>
Step 4. use Eq. (17) to compute the $p$-value corresponding to the $g_i$ statistic from Step 3; save the result in $p_i$.

Step 5. if $p_i < 0.05$, set $S^{(i)} = S^{(i-1)} \cup \{i\}$; otherwise, set $S^{(i)} = S^{(i-1)}$.

Step 6. if $i = d$, set $S_g = S^{(i)}$ and stop; otherwise, increment $i$ and go back to Step 1.

Following the notation from the pseudocode implementation above, throughout we use the notations $g_i$ denote the Fisher $g$-statistic computed from the periodogram of $\mathbb{E}(u_i' v_i' \sqrt{\lambda_i})$; similarly, $p_i$ is used to denote the $p$-value corresponding to this statistics, while $S_g$ denotes the grouping set selected through our approach. To be able to visualize in a simple way which components have been selected through our method, we propose plotting

$$\{(i, \delta_i(S_g)) : i = 1, \ldots, d\},$$

where $\delta(\cdot)$ denotes the Dirac measure, and $S_g = \{i \in \{1, \ldots, d\} : \omega^*_i \in \Omega, p_i < 0.05\}$; throughout we will call the graph in Eq. (19) as the comb-plot, and the point masses $\delta_i(S_g)$ as Fisher $g$ indicators, for $i = 1, \ldots, d$.

3 Real-time nowcasting the US output gap

3.1 Real-time vintages

As the aim is to assess the real-time performance of several alternative methods to extract the cyclical component of GDP, one requires a real-time dataset for the US. In particular, we use the US data set comprising real-time vintages, based on the work of Croushore and Stark (2001), which is maintained by the Federal Reserve Bank of Philadelphia.\footnote{The data are publicly available at: \url{www.philadephiafed.org/research-and-data/real-time-center/real-time-data/}.} The sample period runs from the first quarter of 1947 up to the fourth quarter of 2013. We consider the real-time vintages since the first quarter of 2000, as this is the earliest date for which GDP is available in all subsequent vintages for the whole sample period. In the case of GDP, we use the first release for a given quarter and for industrial production
Figure 1: Above: Univariate SSA$_{GDP}$ analysis. Below: Bivariate SSA$_{GDP,IP}$ analysis. For each of the analysis: on the left, we present the respective comb-plot, as defined in Eq. (19), with the point masses identifying the indices of the components selected according to the approach discussed in Section 2.5; on the right, we plot the principal components used to construct our business cycle indicators, colored according to the same palette as the one used in the comb-plot on the left.
we consider the available vintage at the time GDP is released. The period under consideration for real-time evaluation is close to the one in de Carvalho, Rodrigues, and Rua (2012), but extended up to the end of 2013, corresponding to 20% of the sample size; this period encompasses the Great Recession which is by all standards challenging in many dimensions.

![GDP and IP over time](image)

**Figure 2:** Latest available vintage for the GDP and IP, released on the first quarter of 2014.

### 3.2 Final output gap estimates

In this section, we compute the so-called final output gap estimates, which are based on the latest available vintage (Orphanides & van Norden, 2002); in Fig. 2 we plot the latest available vintage, which for our case corresponds to the one released on the first quarter of 2014. These estimates will then be used as target variables for the assessment of the real-time nowcasting ability of the alternative methods in the next section. Regarding the well-known and commonly applied filters in business cycle literature, we consider the usual parameter values to extract the GDP cyclical component. In particular, for the Hodrick–Prescott filter we set the smoothing parameter equal to 1600 which is the recommended
value for quarterly data; see Prescott (1986) and also Baxter and King (1999) for a more thorough discussion. In the case of the Christiano–Fitzgerald filter we define the range of periodicities of interest, when extracting the GDP cyclical component, to be between 6 and 32 quarters which corresponds to the standard frequency range considered in the business cycle literature; see, for example, Stock and Watson (1999, 2005).

In the case of the SSA, since we are interested in dynamics of up to 8 years, we set a window length of 32 quarters as in de Carvalho, Rodrigues, and Rua (2012). Regarding the selection of the components in the grouping stage of SSA, we resort to the Fisher $g$-statistic discussed in Section 2.5. Given all the potential components to be considered in the construction of the output gap, we select the components for which the dominant periodicity lies within the standard business cycle frequency range (that is, between 6 and 32 quarters), and which are statistically significant, at the usual 5% significance level, according to the Fisher $g$ test. Once the components are selected, they are aggregated to obtain an output gap measure, by following the steps discussed in Section 2.

The resulting output gap estimates are presented in Fig. 3. All the measures seem to be in accordance with the NBER business cycle chronology and deliver similar qualitative reading concerning the cyclical position of the economy. Note however that, as expected, near the end of the sample there is a higher dispersion of the estimates reflecting the end-of-sample uncertainty. Furthermore, note that the output gap from the Hodrick–Prescott filter is slightly noisier than the remainder reflecting the fact that it acts as a high-pass filter (King & Rebelo, 1993, Baxter & King, 1999). In contrast, the Christiano–Fitzgerald band-pass filter yields a much smoother measure of output gap. In this respect, both the univariate (SSA$_{GDP}$) and bivariate (SSA$_{GDP, IP}$) SSA-based output gap estimates are also smooth over time, reflecting the criterion adopted in the grouping stage which allows us to discard the trending components and components associated with higher frequencies.

Regarding the univariate SSA$_{GDP}$, the Fisher $g$-statistic led to the selection of the 3rd up to the 10th components, for the construction of the GDP cyclical component, i.e., $S_g = \{3, 4, 5, 6, 7, 8, 9, 10\}$. These almost correspond to the components chosen by de Carvalho, Rodrigues, and Rua (2012), through an heuristic approach which led them to obtain $S = \{3, 4, 5, 6, 7, 8, 9\}$. In practice, the two output gap estimates are nearly indistinguishable graphically; this stems from the fact that the 10th component accounts for a negligible part ($\approx 1\%$) of the variance of the output gap.
Figure 3: Comparison of output gap estimators: SSA$_{GDP, IP}$ (— ), SSA$_{GDP}$ (— ), Hodrick–Prescott (– –), and Christiano–Fitzgerald (···).
In the case of the two-channel SSA_{GDP,IP}, from the potential 64 components, 18 have been selected drawing on the Fisher $g$-based approach discussed in Section 2.5; in this case

$$S_g = \{4, 5, 6, 7, 8, 9, 11, 12, 13, 20, 22, 23, 24, 29, 33, 34, 40, 41\};$$

and we summarize this information in the comb-plot in Fig. 1, whose formal definition can be found in Eq. (19).

As can be observed in Fig. 1, in contrast with the univariate SSA_{GDP} case, the selected components are not sequential in terms of the ordering based on the eigenvalues. In fact, the ordering based on the eigenvalues does not necessarily lead to the most relevant components for the problem at hand. This feature highlights the usefulness of the suggested Fisher $g$-based criterion to identify the components of interest. Naturally, increasing the number of variables makes the practical contribution of using this criterion even more striking.

All in all, the resulting output gap estimates are relatively similar across alternative methods. Hence, in the next section, we evaluate the information content of the real-time nowcasts for assessing the output gap.

### 3.3 Real-time nowcasting

In this section, we compute the real-time output gap nowcasts based on a recursive estimation exercise, with an expanding sample window, using the real-time vintages of data. In the cases of SSA_{GDP} and SSA_{GDP,IP}, this also entails the computation of the Fisher $g$ test at each moment in time and corresponding components selection. This truly mimics a real-time scenario in all dimensions. The resulting real-time estimates along with the final output gap estimate for each approach are displayed in Fig. 4.

To evaluate quantitatively the real-time ability of the different methods to nowcast output gap we consider a wide range of performance statistics (see, for example, Orphanides & van Norden, 2002, Marcellino & Musso, 2011). The results are presented in Table 1. In the first column, we report the Mean Absolute Error (MAE), which refers to the average of the absolute difference between the final output gap estimates and the real-time nowcasts. We also present, in the second column, the Root Mean Squared Error (RMSE) which penalizes more larger differences. Both, the MAE and RMSE, are
Figure 4: Comparison of real-time estimates (---) and of final estimates (-----).
Table 1: Real-time performance evaluation.

<table>
<thead>
<tr>
<th>Filter</th>
<th>MAE</th>
<th>RMSE</th>
<th>CORR</th>
<th>NS</th>
<th>NSR</th>
<th>SIGN-LEV</th>
<th>SIGN-CH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hodrick–Prescott</td>
<td>1.08</td>
<td>1.24</td>
<td>0.56</td>
<td>0.90</td>
<td>0.87</td>
<td>64.3</td>
<td>80.4</td>
</tr>
<tr>
<td>Christiano–Fitzgerald</td>
<td>0.71</td>
<td>0.92</td>
<td>0.71</td>
<td>0.71</td>
<td>0.84</td>
<td>73.2</td>
<td>67.9</td>
</tr>
<tr>
<td>SSA_GDP</td>
<td>0.93</td>
<td>1.25</td>
<td>0.92</td>
<td>0.63</td>
<td>0.63</td>
<td>83.9</td>
<td>76.8</td>
</tr>
<tr>
<td>SSA_GDP, IP</td>
<td>0.67</td>
<td>0.92</td>
<td>0.97</td>
<td>0.45</td>
<td>0.48</td>
<td>92.9</td>
<td>80.4</td>
</tr>
</tbody>
</table>

The top real-time performances according to each measure are identified by the cells in gray.

reported in percentage terms. The third column presents the correlation (CORR) between the final and real-time estimates. The next two columns report measures of signal-to-noise of the nowcasts for each method. In particular, NS denotes the ratio of the standard deviation of the revision to that of the final estimate, whereas NSR refers to the ratio of the root mean square of the revision to the standard deviation of the final estimate. In the last two columns we report the sign concordance between the real-time nowcasts and the final estimates. The SIGN-LEV denotes the percentage of times in which the sign of the level of the real-time and final estimates coincide, whereas the SIGN-CH refers to the sign of the changes in output gap estimates.

In terms of the size of the revisions, the Hodrick–Prescott filter seems to perform worse than its competitors, whereas the Christiano–Fitzgerald filter, and the SSA_GDP, IP approach are the top ranked. Among these two, the SSA_GDP, IP method delivers better results than the Christiano–Fitzgerald filter, according to the MAE criterion. In terms of the correlation between the real-time and final estimates, the Hodrick–Prescott filter ranks last. In contrast, both the univariate SSA_GDP and SSA_GDP, IP record the highest correlation coefficients, with the latter presenting a correlation close to one. Concerning the signal-to-noise measures, qualitatively similar findings emerge. The Hodrick–Prescott filter records the highest noise-to-signal whereas there is a striking decrease when one considers the SSA_GDP, IP. Although the univariate SSA_GDP approach already improves on the Hodrick–Prescott and Christiano–Fitzgerald filters, extending the SSA_GDP to the multivariate case results in an even larger decrease of the noise-to-signal. Regarding the sign concordance, in terms of the level, the SSA_GDP approach outperforms the other filters, with the SSA_GDP, IP standing at the top of the ranking. For the sign concordance in terms of the change, the Christiano–Fitzgerald filter ranks last, whereas the Hodrick–Prescott filter,
and SSA_{GDP, IP} present the best performance. Summing up, for all the performance indicators, the SSA_{GDP, IP} always ranks first. The SSA_{GDP} approach outperforms, in overall terms, standard filtering techniques, but further gains can still be achieved with the SSA_{GDP, IP} approach from Section 2.4. Hence, by considering information beyond the one conveyed by GDP, namely the industrial production index, it is possible to improve the real-time performance of the output gap nowcasts in all dimensions. Although it is straightforward to extend the approach in Section 2.4 to a multichannel setting, from an empirical viewpoint it is not clear cut that by enlarging the number of variables considered it will improve the real-time performance. We tried to supplement industrial production with other real-time data, namely non-farm payroll employment and/or real personal income less transfers, which are also among the series closely monitored by the NBER Business Cycle Dating Committee. However, the results, not reported here, do not reveal any improvement in terms of the performance of the real-time nowcasts.

4 Conclusions

This paper explores the performance of SSA-based methods for nowcasting in real-time the US output gap. The assessment in real-time of output gap is of utmost relevance for policymaking, and here we assess the added value of SSA-based nowcasts in a real-life policymaking scenario, by replicating the problem faced by policymakers at the time policy decisions have to be taken. We used real-time vintages, and conducted a recursive study so to evaluate the real-time reliability of our SSA-based approach. For our econometric setting of interest, the preferred specification of our approach consists of a two-channel singular spectrum analysis, where a Fisher $g$ test is used to screen which components—within the standard business cycle range—should be included in the grouping step. Our findings suggest that singular spectrum analysis provides a reliable evaluation of the cyclical position of the US economy in real-time, with the two-channel approach outperforming considerably the univariate counterpart.

Although SSA has been widely applied on many fields of research, there are only a few applications in the economics and finance literature (see, for instance, Hassani, Heravi, & Zhigljavsky, 2009, Patterson, Hassani, Heravi, & Zhigljavsky, 2011, de Carvalho, Rodrigues, & Rua, 2012, Hassani, Soofi, & Zhigljavsky, 2013a,b). We hope that this paper takes another small step in promoting the application
of SSA methods in economics, by stressing the resilience of SSA-based approaches to model macroeconomic data. Applied econometric analysis requires the combination of different methodology, and we hope further applied econometricians may consider taking advantage of SSA-based approaches in a near future.

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**Appendix**

**NBER’s Business Cycle Reference Dates**

This appendix includes the NBER’s business cycle reference dates used in Section 3. This chronology is included here for completeness; the complete chronology can be found at the NBER web site at: [www.nber.org/cycles/cyclesmain.html](http://www.nber.org/cycles/cyclesmain.html)

Table 2: US Business Cycle Reference Dates from Peak to Through, along with duration of corresponding contractions.

<table>
<thead>
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<th>Business Cycle Reference Dates</th>
<th></th>
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<tr>
<td>Peak</td>
<td>Through</td>
</tr>
<tr>
<td>November 1948 (iv)</td>
<td>October 1949 (iv)</td>
</tr>
<tr>
<td>July 1953 (ii)</td>
<td>May 1954 (ii)</td>
</tr>
<tr>
<td>August 1957 (iii)</td>
<td>April 1958 (ii)</td>
</tr>
<tr>
<td>April 1960 (ii)</td>
<td>February 1961 (i)</td>
</tr>
<tr>
<td>December 1969 (iv)</td>
<td>November 1970 (iv)</td>
</tr>
<tr>
<td>November 1973 (iv)</td>
<td>March 1975 (i)</td>
</tr>
<tr>
<td>January 1980 (i)</td>
<td>July 1980 (iii)</td>
</tr>
<tr>
<td>July 1981 (iii)</td>
<td>November 1982 (iv)</td>
</tr>
<tr>
<td>July 1990 (iii)</td>
<td>March 1991 (i)</td>
</tr>
<tr>
<td>March 2001 (i)</td>
<td>November 2001 (iv)</td>
</tr>
<tr>
<td>December 2007 (iv)</td>
<td>June 2009 (ii)</td>
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Here i–iv are used to denote the quarters corresponding to the reference dates; the duration is in months.
References


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