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WITH NATIONAL PRICE ASYMMETRIES

Sandra Gomes

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Please address correspondence to Sandra Gomes, Economic Research Department, Banco de Portugal, Av. Almirante Reis nº. 71, 1150-165 Lisboa, Portugal; Tel: 351 213130719; Fax: 351 213107805; email: sgomes@bportugal.pt
Monetary Policy in a Currency Union With National Price Asymmetries∗

Sandra Gomes†
Banco de Portugal

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Abstract

We investigate the importance of the behaviour of the monetary authority for the dynamics of a currency union where cross-country asymmetries are not necessarily reflected in differences in economic size. We construct a stylised two-country general equilibrium model with sticky-prices to serve as laboratory for studying the operating characteristics of Taylor-type interest rate rules. We consider that the two countries in the union are different in terms of the price-setting practices of firms, and inspect the implications of alternative policy rules for the dynamics of the economy. The experiments carried out show, in general, that the way shocks propagate in the monetary union is linked to the systematic behaviour of the monetary authority. The reaction of the central bank to economic developments is important both at the union level and at the country level, namely to explain cross-country differences in economic behaviour. On the other hand, in our model the policy that stabilizes inflation is not necessarily the same that makes the output in the union less volatile. Also, the policy that reduces aggregate volatility does not necessarily imply the same for each country individually.

Keywords: General equilibrium models, sticky-prices, monetary union, systematic monetary policy, Taylor rules.

JEL classification: E52, E58, E61.

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†Corresponding author: Economic Research Department (DEE), Banco de Portugal, Av. Almirante Reis no. 71, 1150-012 Lisboa Portugal; Tel: +351 213130719; E-mail: sgomes@bportugal.pt.
1 Introduction

According to the Treaty Establishing the European Community, the primary objective of the ECB is to maintain price stability. Eventually, monetary policy is permitted to stimulate growth of the different regions, as long as it does not jeopardize the goal of price stability. Moreover, the Governing Council of the ECB has specified a quantitative definition of the maintenance of price stability, formulated in terms of the euro area HICP, which is a weighted average of the EMU countries’ HICP. The weight of each country corresponds to its share in total consumption in the EMU. So, the ECB has adopted an euro-area wide perspective in the conduct of monetary policy, clarifying that it would not react to regional or national developments.

Taking this framework as the starting point in our analysis, we investigate the importance of the behaviour of the monetary authority in a currency union where cross-country asymmetries are not necessarily reflected in differences in economic size. We consider rules in which the monetary authority sets the interest rate as a function of current inflation and output and the lagged interest rate. There is a wide agreement that the choice on how to conduct monetary policy has important consequences for the dynamics of an economy. Indeed, as pointed out by Dotsey (1999a, b) and Woodford (2003), the way the monetary authority reacts to economic activity has significant implications for the way a model economy responds to shocks. We inspect the implications of alternative policy rules for the economic dynamics of a currency union, therefore extending to a monetary union framework previous work on this subject.

The analysis of monetary policy when the policy instrument is the interest rate has received considerable attention in the literature, namely following the seminal paper of Taylor (1993). A number of prominent examples are included in the volume of Taylor (1999), e.g. Batini and Haldane (1999), McCallum and Nelson (1999) and Rotemberg and Woodford (1999). These papers look at the welfare effects and the variability of output and inflation induced by alternative policy rules. A different approach to the analysis of interest rate rules is taken by Dotsey (1999a, b), McCallum (1999) and Christiano and Gust (1999), who examine the implications of changing the feedback parameters in the monetary policy rule on the responses of a model economy to shocks and the feasibility of parameter choice as to the determinacy, indeterminacy or explosiveness for the economy responses.

Our work is in the spirit of the latter mentioned strand of the literature, specifically we address issues regarding the implications of systematic monetary policy for the dynamics of an economy. While these papers deal
with single economy models, we study the consequences of alternative policy rules in a two-country monetary union, both in terms of aggregate variables and individual country variables. Albeit the mainly positive focus of our analysis, we also look at the impact of alternative policy rules on welfare related indicators, namely the volatility of inflation and output. In a context similar to ours, Benigno (2003) finds that HICP targeting policy as defined by the ECB would only be optimal in the case that the relative asymmetries are well captured by differences in economic dimension. If the degree of price stickiness is different across countries, a (nearly) optimal policy gives higher weight to developments in the country where prices are stickier. This conclusion is in line with previous findings by Aoki (2001), that in an economy with a flexible price sector and a fixed price sector optimal policy involves targeting inflation in the sticky-price sector rather than a broad measure of inflation.

We construct a stylised two-country general equilibrium monetary model, buffeted by monetary and technology shocks, to serve as laboratory for studying the operating characteristics of Taylor-type interest rate rules. Following a considerable part of the literature over the last two decades, we assume the existence of price stickiness in order to get less trivial real effects from monetary policy in the short run, meaning that even though firms choose prices optimally they cannot reset their prices every period in response to shocks that hit the economy. We model price stickiness by considering that prices are subject to changes at random intervals, as in Calvo (1983). Both Gali, Gertler and López-Salido (2001, 2003) using a single equation estimation procedure and Smets and Wouters (2003) based on the estimation of a dynamic general equilibrium model find evidence in favour of a significant degree of price stickiness in the euro area. Micro evidence on the degree of price stickiness for individual euro area countries is somewhat scarce and the extent to which cross-country comparisons are hampered by methodological differences is not clear. Even so, focusing on some studies with similar characteristics (based on information for consumer prices), a somewhat higher degree of price stickiness was found by Aucremanne and Dhyne (2004) for Belgium (roughly one year) compared with the results of Baudry et al. (2004) for France (around 6 months) and Dias, Dias and Neves (2004) for Portugal (8.5 months).

1 This approach to model the dynamics of price adjustment is widely used in the literature. King and Watson (1996) and Yun (1996) were among the first to use the Calvo formulation.

2 These results show a somewhat longer duration than that found by Bils and Klenow (2002) for the US, namely around 4 months. The finding that rigidity in European coun-
A sticky-price model will not replicate all the main features of real world data, as extensively shown by Chari, Kehoe and McGrattan (2000). Actually, King and Watson (1996) and Christiano, Eichenbaum and Evans (1997) conclude that neither a sticky-price model nor a model incorporating only a financial market friction accounts for the main stylised facts in the data. In a recent paper, Papadopoulou (2002) combines sticky prices with a financial market friction into a dynamic general equilibrium model and finds that the unified model performs better than either one of the models including only a single friction. Consequently, several authors have argued that the empirical fitting of a model is improved by combining more than one source of frictions and a number of recent papers tried to ascertain the set of frictions that make a model fit best the empirical evidence. The models of Christiano, Eichenbaum and Evans (2003) for the US economy and Smets and Wouters (2003) for the euro area include a considerable set of nominal and real rigidities because this allows to better replicate the main characteristics of the monetary transmission mechanism.

Given that our analysis is focused on the implications of alternative monetary policy rules in a currency union, we abstract from many features that would approximate the model to the real world, namely we keep a single rigidity in the model. Nevertheless, we do not expect significant changes in major conclusions of the present paper.

The experiments we carry out show, in general, that the way shocks propagate in the monetary union is linked to the systematic behaviour of the monetary authority. Our key findings are as follows. As previously pointed out in the literature, the design of monetary policy plays an important role in the dynamics of an economy. We extend earlier work to a monetary union framework with asymmetric countries. In this context we find that the way the central bank reacts is important both at the union level and at the country level, namely to explain differences in economic behaviour that arise between countries in the union. On the other hand, in our model the policy that stabilizes inflation is not necessarily the same that makes output in the union less volatile. Also, the policy that provides greater...
stabilization in aggregate terms does not necessarily imply the same for each country individually.

Our analysis proceeds as follows. Section 2 outlines the general equilibrium model used. Section 3 briefly discusses the parametrisation. In section 4 we obtain the linearised equilibrium conditions. Section 5 investigates the mechanics of the model, assuming both countries are identical. Section 6 analyses the implications of alternative monetary policy rules in a monetary union comprising two identical countries. Section 7 discusses how alternative policy rules affect the behaviour of member countries in the monetary union in the presence of cross-country asymmetries in the degree of price rigidity. Section 8 concludes.

2 The model

We use a two-country monetary model featuring only one type of friction, namely sticky-prices, and where capital is in fixed supply. The model extends the sticky-price framework in Christiano, Eichenbaum and Evans (1997) to a two-country monetary union model. We change the way price-stickiness is modeled by following Calvo (1983).

The monetary union consists of country A and country B and is populated by a continuum of agents in the interval $[0, 1]$. We assume that the population of segment $[0, n]$ resides in country A while population of the segment $(n, 1]$ resides in country B. A single monetary authority conducts monetary policy in the union. Each country is populated by an infinitely lived representative agent and a continuum of producers of differentiated goods that are competitive monopolists. In addition, we assume complete financial market integration in the monetary union and therefore there exists a single funds market in the union. Each household purchases goods, supplies labour to the firms, and makes deposits to the financial intermediary. The financial intermediary receives deposits from households and lump-sum injections of money from the monetary authority, and makes loans to firms. Firms need loans because they must pay workers before they sell their output, and borrow these funds at the gross interest rate $R_t$. We assume that there is no mobility of labour between countries, therefore each firm hires domestic workers only.

2.1 Households

All households in the monetary union consume goods produced in both countries. Let’s consider the generic agent $j$, where $j \in [0, n]$ if the agent...
resides in country A and \( j \in (n, 1] \) if the agent resides in country B. Given that we will not consider any heterogeneity among consumers we end up having two types of consumers (A and B)\(^4\).

At time \( t \) each (representative) household ranks alternative streams of consumption and hours worked according to the criterion function

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C^j_t, N^j_t)
\]

where \( \beta \) is a discount factor, \( 0 < \beta < 1 \), \( C^j_t \) are units of consumption at time \( t \) and \( N^j_t \) are hours worked at time \( t \), and period utility is given by\(^5\)

\[
U(C^j_t, N^j_t) = \frac{(C^j_t)^{1-\gamma}}{1-\gamma} - \psi_n \frac{(N^j_t)^{1+\psi}}{1+\psi}.
\]  \( (1) \)

\( C^j_t \) is an index of consumption of commodity bundles produced in countries A and B, defined as in Obstfeld and Rogoff (1998):

\[
C^j_t = \left( \frac{C^j_{At}}{n^\sigma} \right)^{1-n} \left( \frac{C^j_{Bt}}{n^\sigma} \right)^{1-n} - \psi_n \frac{(N^j_t)^{1+\psi}}{1+\psi}.
\]  \( (2) \)

where \( C^j_{At} \) is the consumption bundle of the continuum of goods produced in A and \( C^j_{Bt} \) is the consumption bundle of the continuum of goods produced in country B\(^6\). These bundles of the differentiated goods produced in each country (indexed by \( a \) if produced in country A and by \( b \) if produced in country B) are defined as constant elasticity of substitution aggregators (following Dixit and Stiglitz (1977)):

\[
C^j_{At} = \left[ \left( \frac{1}{n^\sigma} \right)^{\frac{1}{\sigma}} \int_0^n c^j_a(a)^{\frac{1}{\sigma}} \, da \right]^{\frac{\sigma}{\sigma-1}}
\]  \( (3) \)

\(^4\)All households within each country are faced with an identical problem. Therefore we can look at the decision problem of the representative household of each country.

\(^5\)We can reconcile our model with balanced growth if we think of this utility function as an indirect utility function for a household whose actual utility function depends on market and non-market consumption and hours (on this issue see for example Greenwood, Rogerson and Wright (1996)).

\(^6\)Thus \( n \) is the size of country A both in terms of population and also in terms of the share on total consumption of the consumption of the differentiated goods produced in A. Similarly, \( 1-n \) is the size of country B both in terms of population and the share in consumption of the differentiated goods produced in B.
where $\sigma > 1$ is the elasticity of substitution across the goods produced within each country, while the elasticity of substitution between bundles $C_{At}$ and $C_{Bt}$ is equal to one.

Given a decision on $C^j_t$, household $j$ will allocate optimally the expenditure on $C_{At}^j$ and $C_{Bt}^j$ by minimising the total expenditure $P_j C^j_t$ under the constraint given by the Cobb-Douglas consumption index (2). The demand functions for the composite goods produced in A and B are:

\begin{align*}
C_{At}^j &= n \left( \frac{P_{At}^j}{P_t^j} \right)^{-\frac{1}{n}} C^j_t \\
C_{Bt}^j &= (1 - n) \left( \frac{P_{Bt}^j}{P_t^j} \right)^{-\frac{1}{n}} C^j_t.
\end{align*}

Then, given the decisions on $C_{At}^j$ and $C_{Bt}^j$, household $j$ allocates the expenditure among the differentiated goods by minimizing $P_{At}^j C_{At}^j$ and $P_{Bt}^j C_{Bt}^j$ under the constraints (3) and (4). The demand for differentiated consumption goods $a$ and $b$ are:

\begin{align*}
c^j_t(a) &= \left( \frac{1}{n} \right) \left( \frac{p^j_t(a)}{P_{At}^j} \right)^{-\sigma} C^j_{At} \\
c^j_t(b) &= \left( \frac{1}{1 - n} \right) \left( \frac{P_{Bt}^j}{p^j_t(b)} \right)^{-\sigma} C^j_{Bt},
\end{align*}

or equivalently, by using (5) and (6)

\begin{align*}
c^j_t(a) &= \left( \frac{p^j_t(a)}{P_{At}^j} \right)^{-\sigma} \left( \frac{P_{At}^j}{P_t^j} \right) C^j_t \\
c^j_t(b) &= \left( \frac{p^j_t(b)}{P_{Bt}^j} \right)^{-\sigma} \left( \frac{P_{Bt}^j}{P_t^j} \right) C^j_t.
\end{align*}

The overall price index $P_t^j$, defined as the minimum expenditure in country $j$ required to purchase goods resulting in consumption index $C^j_t$, is

\[ P_t^j = \left( \frac{P_{At}^j}{P_{Bt}^j} \right)^{n} \left( \frac{P_{Bt}^j}{P_t^j} \right)^{1-n}. \]
with $P^j_A = \left[ \frac{1}{\beta} \int_0^1 p^j_t(a)^{1-\sigma} \, da \right]^{-\frac{1}{\sigma}}$ and $P^j_B = \left[ \frac{1}{\beta} \int_0^1 p^j_t(b)^{1-\sigma} \, db \right]^{-\frac{1}{\sigma}},$

where $p^j_t(a)$ and $p^j_t(b)$ are the prices, at time $t$, of the goods produced in country A and B, respectively, and sold in country $j$. Assuming that there are no transportation costs, prices are set taking the monetary union as a common market. It follows that $P^A_t = P^B_t = P_t$. Given these assumptions and the symmetric structure of preferences the purchasing power parity holds, i.e., $P^A_t = P^B_t = P_t$.

The representative household of country $j$ supplies $N^j_t$ units of labour at the nominal wage rate $W^j_t$. Each household faces a cash-in-advance on its consumption purchases, namely

$$P^j_t C^j_t \leq W^j_t N^j_t + M^j_t - D^j_t$$

(9)

where $W^j_t N^j_t$ are time period $t$ wage earnings, which are paid to the households in time to satisfy its time $t$ cash constraint, and $D^j_t$ are deposits made to the financial intermediaries. The households money holdings evolve according to

$$M^j_{t+1} = \left[ W^j_t N^j_t + M^j_t - D^j_t - P^j_t C^j_t \right] + R^j_{Kt} K^j + R^j_t D^j_t + DivI^j_t + DivI^j_t$$

(10)

where $DivI^j_t$ are the profits received from the financial intermediary and $DivF^j_t$ are the profits received from firms. The representative household is endowed with $K^j_t$ units of capital that it supplies inelastically to a competitive rental market. The rental rate of capital in each country is $R^j_K$. We assume there is no technology for capital accumulation and capital does not depreciate. As a result the aggregate stock of capital is constant, and we set it to unity.

The optimisation problem of the representative household in each country is

$$\max \{ C^j_t, N^j_t, D^j_t \} \quad E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U \left( C^j_t, N^j_t \right) \right\}$$

\[ \text{st} \quad P^j_t C^j_t = W^j_t N^j_t + M^j_t - D^j_t \]

$$M^j_{t+1} = \left[ W^j_t N^j_t + M^j_t - D^j_t - P^j_t C^j_t \right] + R^j_{Kt} K^j + R^j_t D^j_t + DivI^j_t + DivI^j_t.$$
The first order conditions are:

\[-U_{Nj} = W_j^j\]
\[
12
\]

\[E_t \left[ \frac{U_{Cj}^j}{P_t} - \beta R_t \frac{U_{Cj}^{j+1}}{P_{t+1}} \right] = 0.\] (13)

In addition, given our definition of the consumption bundles we conclude that:

\[P_{At} C_{At}^j = n P_t C_j^j\] (14)
\[P_{Bt} C_{Bt}^j = (1 - n) P_t C_j^j\] (15)

### 2.2 Firms

At time \(t\), monopolist competitive firms in each country produce differentiated goods. The representative firm in each country uses the following technology:

\[y_t (i) = \xi_t^j [K_t (i)]^\alpha [N_t (i)]^{1-\alpha}\] (16)

where \(y_t (i)\) denotes the production of good \(i\) (where goods produced in A are indexed by \(a\) and goods produced in B are indexed by \(b\), \(\xi_t^j\) is a time-varying exogenous technology shock in country \(j\), and \(N_t (i)\) is the domestic labour employed by the firm in the production of good \(i\).

Firms rent homogeneous capital and labour in perfectly competitive factor markets. Let \(R_{Ki}^j\) and \(W_t^j\) be the rental rate of capital and the wage rate, respectively. Workers must be paid in advance of production. Consequently firms have to borrow their wage bill from the financial intermediaries at the beginning of period \(t\). Repayment occurs at the end of period \(t\), at the gross interest rate \(R_t\). Economic profits are distributed to the firms owner (the representative household in each country) at the beginning of period \(t + 1\). The problem is completely symmetric between all firms in each country. Each firm’s total costs at time \(t\) are equal to \(R_t W_t^j N_t (i) + R_{Ki}^j K_t (i)\). Since firms take \(W_t^j\) and \(R_{Ki}^j\) as given, one can easily show that the capital to labour ratio is constant across firms and that marginal costs are the same for all firms within a country and equal to

\[MC_t^j = \frac{1}{\xi_t^j} \frac{1}{1 - \alpha} (N_t^j) ^\alpha W_t^j R_t.\]

\[^9\text{Given that no factor is product specific then national factor prices are unique.}\]
We consider that not all firms are able to reoptimize in each period. Specifically, we use a discrete time version of Calvo (1983), by assuming that in each period $t$ a firm faces a constant probability $1 - \theta^j$ ($j = A, B$) of being able to reoptimize nominal prices\(^{10}\) and with probability $\theta^j$ the firm will keep its price constant\(^{11}\). This implies that the fraction of firms reoptimising its price at $t$ is $1 - \theta^j$. The possibility of a firm changing its price is independent of history and we do not need to keep track of firms reoptimising. The expected time over which the price is not reoptimized, i.e., the expected waiting time for the next reoptimisation, is therefore $\frac{1}{1-\theta^j}$ for each country $j$. If the firm reoptimises it will do so after the realization of current period shocks\(^{13}\).

A central issue in Calvo-type price setting models is that when firms reoptimize they understand that they will not be able to do it in every subsequent period. Therefore, taking first the case of country A, firms that reoptimize at time $t$ choose $p^*_t(a)$ to maximize the expected stream of profits:

$$\sum_{i=0}^{\infty} \left\{ V^{A}_{t+i} \left( \beta \theta^A \right)^i \left[ p^*_t(a) y_{t+i}(a) - TC^A_{t+i} \right] \right\}. $$

where $V^{A}_{t+i} = \frac{U^{A}_{t+i+1}}{r_{t+i+1}}$ is the marginal value of a dollar to the representative

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\(^{10}\)The ability to reoptimize is independent across firms and time. The opportunity to reoptimize follows a Bernoulli distribution.

\(^{11}\)Time-dependent models of the type we use, where the number of firms changing prices in any period is specified exogenously, are commonly used in the literature. As argued by Woodford (2003), the exogenous nature of the probability of reoptimization can be rationalized by the existence of costs related to the decision making-process (as documented by Zbaracki et al. (2000)). Even so, some authors argue that a more realistic model should endogenize the timing of price adjustments, as in Caplin and Leahy (1991), Dotsey, King and Wolman (1999), Burnstein (2002), and Golosov and Lucas (2003). These state-dependent models assume firms constantly re-evaluate their price by comparing the benefits and costs (the so-called menu-costs) of changing it. Nevertheless, time-dependent models of the type we use and state-dependent models generate similar results in a moderate inflation framework.

\(^{12}\)The time that elapses between price adjustments follows a geometric distribution.

\(^{13}\)The literature on staggered price-setting has also assumed that new prices are chosen at a date before they first take effect, e.g. Christiano, Eichenbaum and Evans (2003), or that pricing decisions are not always based on current information, as in Mankiw and Reis (2002). Both interpretations imply that optimization is conditional on information available at an earlier date, and therefore a monetary policy shock will affect prices with some lag. Eichenbaum and Fisher (2003) find that, even though a model featuring only sticky-prices does not find much support in US data, allowing for a lag between the time firms reoptimize and the time they implement such decisions (arguably one quarter) improves the fit of the model. Nevertheless, we do not expect that it would mean fundamental changes in our analysis, so we abstract from this fact.
household in country A and is taken as exogenous by the firm, and \( p_t^*(a) \) is the optimal price chosen by a firm that reoptimises at time \( t \). So the optimisation problem of the firms in country A is to choose \( p_t^*(a) \) so that\(^{14}\):

\[
\max_{p_t^*(a)} E_t \left\{ \sum_{i=0}^{\infty} \left[ V_{t+i}^A \left( \beta \theta^A \right)^i \left[ p_t^*(a) y_{t+i}^d(a) - TC_{t+i}^A \right] \right] \right\}
\]

\[
\text{st } TC_{t+i}^A = \frac{1}{\xi_t} \frac{1}{1-\alpha} \left( N_i \right)^{\alpha} W_i^R p_{t+i}(a) \]

\[
y_{t+i}(a) = \frac{1}{n} \left( \frac{p_{t+i}(a)}{P_{At+i}} \right)^{-\sigma} Y_{At+i}
\]

In this case the first order condition is

\[
p_t^*(a) = \mu E_t \left\{ \sum_{i=0}^{\infty} \omega_{t+i}^A MC_{t+i}^A \right\}
\]

where

\[
\omega_{t+i}^A = \frac{V_{t+i}^A \left( \beta \theta^A \right)^i Y_{At+i}^d (P_{At+i})^\sigma}{E_t \left\{ \sum_{i=0}^{\infty} \left[ V_{t+i}^A \left( \beta \theta^A \right)^i Y_{At+i}^d (P_{At+i})^\sigma \right] \right\}}, \quad \mu = \frac{\sigma}{\sigma - 1}.
\]

So, \( p_t^*(a) \) is set as a markup \( \mu \) over a weighted average of marginal costs expected to prevail in future dates at which \( p_t^*(a) \) applies\(^{15}\). The problem for country B firms is similar.

Because firms reoptimising in each country face an identical problem they will choose the same price, that is \( p_t^*(a) \) does not depend on good \( a \) and \( p_t^*(b) \) does not depend on good \( b \). Those firms that do not reoptimise keep their prices fixed. This fraction of prices charged in \( t \) is a subset of the prices charged in \( t - 1 \), with each price showing up with the same relative frequency in the price distribution in \( t \) and \( t - 1 \). Therefore, Calvo price-setting implies the following state equations:

\(^{14}\)The expectation is conditional on information through period \( t \) and taken over states of nature in which the firm cannot reoptimise. Therefore, the supplier of good \( a \) chooses \( p_t^*(a) \) on the basis of information available at date \( t \), as to maximise profits given the expected state-contingent values of the random variables \( V_{t+i}^A, Y_{At}, P_{At} \) and \( \xi_i^A \).

\(^{15}\)Recall that in the case of flexible prices, all firms reoptimise each period and, as usual in a model of monopolistic competition, prices are set as a markup over current marginal costs, i.e., \( p_t(a) = \mu MC_t^A \) and \( p_t(b) = \mu MC_t^B \). Note that when \( \sigma \) gets close to one we approach the perfect competition situation.
\[
P_{At} = \left[ (1 - \theta^A)p^*_t(a)^{1-\sigma} + \theta^A P_{At-1}^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (18a)
\]
\[
P_{Bt} = \left[ (1 - \theta^B)p^*_t(b)^{1-\sigma} + \theta^B P_{Bt-1}^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (18b)
\]

2.3 Financial intermediary and equilibrium in the loan market

Financial markets are completely integrated in the monetary union. Demand in the loan market comes from firms who need to finance their wage bill. Regarding the supply of loans, at the beginning of each period the perfectly competitive financial intermediary receives deposits from households in both countries \( (D_t = nD^A_t + (1 - n)D^B_t) \) plus a monetary injection from the monetary authority \( (X_t) \). At the end of the period the financial intermediary pays \( R_t D^j_t \) to households in country \( j \) in return to their deposits and distributes the profits from the monetary injection. Since we assume that all agents own an equal share of the financial intermediary, they receive an equal share of its profits\(^{16}\). In per capita terms \( X_t = X^A_t = X^B_t \), and therefore the dividends received by each household are \( R_t X_t \). Clearing in the loan market implies

\[
nW^A_t N^A_t + (1-n)W^B_t N^B_t = nD^A_t + (1-n)D^B_t + X_t. \quad (19)
\]

2.4 Goods market equilibrium

Given that in the model the only source of demand for the goods produced in the union is consumption by households\(^{17}\), equilibrium in the goods market implies that overall consumption of each type of good equals its production:

\[
c_t(a) = y_t(a) \quad \text{and} \quad c_t(b) = y_t(b).
\]

Let \( C^A_{At} \) and \( C^B_{At} \) be the consumption of the bundle of goods produced in A by the representative consumer of country A and of country B, respectively, and similarly for the bundle of goods produced in B we have \( C^A_{Bt} \) and \( C^B_{Bt} \). Then total consumption in the monetary union of each bundle of

\(^{16}\)Consequently each country will receive the profits of the financial intermediary in proportion to its relative size in the union.

\(^{17}\)We do not assume the possibility of having consumption by the Government. We only assume the existence of a monetary authority, whose single role is to inject money into the economy, which is channeled to the agents through the financial intermediary.
goods is \( C_{jt} = nC^A_{jt} + (1 - n)C^B_{jt} \) while total production is \( Y_{jt} (j = A, B) \). Consequently, equilibrium in the goods markets in aggregate terms implies

\[
C_{At} = Y_{At} \text{ and } C_{Bt} = Y_{Bt}.
\]

Given the demand functions (5) and (6) this is equivalent to\(^{18}\)

\[
\frac{n}{P_{At}} \left[ nC^A_t + (1 - n)C^B_t \right] = \left( \frac{P_{At}}{P_{At}^{\text{unw}}} \right)^\sigma n \xi^A_t \left[ N^A_t \right]^{1-\alpha} \quad (20)
\]

\[
(1 - n) \frac{P_{Bt}}{P_{Bt}^{\text{unw}}} \left[ nC^A_t + (1 - n)C^B_t \right] = \left( \frac{P_{Bt}}{P_{Bt}^{\text{unw}}} \right)^\sigma (1 - n) \xi^B_t \left[ N^B_t \right]^{1-\alpha} \quad (21)
\]

where \( P_{At}^{\text{unw}} = \left[ \int_0^n \frac{1}{n} (p_t(a))^{-\sigma} \right]^{-\frac{1}{\sigma}} \) and \( P_{Bt}^{\text{unw}} = \left[ \int_{n}^1 \frac{1}{1-n} (p_t(b))^{-\sigma} \right]^{-\frac{1}{\sigma}} \).

### 2.5 Monetary authority

We assume that the monetary authority sets the interest rate according to a specified policy rule. The specification of the rule is known by agents. By assuming that the monetary authority conducts rule-based policy we mean that the central bank commits itself to a systematic way of determining an appropriate response to economic developments, by acting in conformity with a policy rule that it judges adequate at the time the decision is made. We also postulate the existence of a random component in monetary policy.

Our analysis contributes to the debate on the implications of alternative monetary policy rules. It has been common in the literature to describe monetary policy by means of reaction functions that link the nominal short-term interest rate to inflation and output. An early and influential paper was Taylor (1993), and a number of well-known contributions are collected in Taylor (1999). We assume, following Clarida, Galí and Gertler (1998), that the central bank has a target for the (gross) nominal interest rate in period \( t \), denoted by \( R^*_t \), which depends on the gaps between inflation \( (\Pi_t = \frac{P_t}{P_{t-1}}) \) and output \( (Y) \) and their steady-state values\(^{19}\). To capture the tendency of the central banks to smooth changes in interest rates and to allow for some randomness in monetary policy, we assume that the actual interest rate is set according to

\[
R_t = (R_{t-1})^{g_r} (R^*_t)^{1-g_r} \epsilon_{rt}. \quad (22)
\]

\(^{18}\)The proof can be found for example in Christiano, Eichenbaum and Evans (2003).

\(^{19}\)Given a variable \( X_t \) we denote by \( \overline{X} \) its steady-state value.
where $\epsilon_{rt}$ is an exogenous shock to the policy function and the target rate, $R_t^*$, is defined as

$$R_t^* = \Pi_t \left( \frac{\Pi_t}{\Pi} \right)^{g_r} \left( \frac{Y_t}{V_t} \right)^{g_y}.$$  \hspace{1cm} (23)

In equation (22) $g_r$ represents the degree of interest rate smoothing, that is when $g_r \neq 0$ the monetary authority partially adjusts the policy rate to its target, eliminating only a fraction of the gap between the actual and the target interest rate. Gradualism in monetary policy can result from the fact that the central bank seeks to minimize the variability of interest rate changes, in addition to objectives of policy. Traditional explanations given in the literature for gradualism in monetary policy regard the concern of the central bank in avoiding adverse reactions to frequent changes in interest rates in the financial markets or reasons of reputation of the monetary authority and uncertainty about the economy. Also, it has been argued by some authors that interest rate smoothing increases the potency of monetary policy, given the interaction of systematic monetary policy and the forward-looking behaviour of agents. On the other hand, the interest rate smoothing parameter has been estimated to be considerably large in several countries, namely in the euro area (see for example Gerdesmeier and Roffia (2003)).

Even though a policy rule as (22) may seem somewhat restrictive, such simple rules have been found in the literature to be fairly robust to model uncertainty (e.g. Levin, Wieland and Williams (1999) and Levin and Williams (2003)), and in many cases close approximations of fully optimal rules. However, and as argued by Christiano and Gust (1999), the robustness of simple Taylor rules across models may hinge on the source of rigidity considered in the models, namely if one assumes that prices are sticky or if a financial market friction is postulated.

### 3 Parameterisation

We set the parameters values in the model according to existing evidence for euro area countries. We consider two equally sized countries that, apart from the degree of price rigidity, are identical.

Consider first the preferences parameters. The discount rate is set to give an annual steady-state interest rate of 3 per cent (around 1 per cent quarterly). The risk aversion parameter $\gamma$ is set to 1.5 which implies an intertemporal elasticity of substitution of 0.7, following the Real Business Cycle literature. This is broadly consistent with Smets and Wouters (2003).
estimates for the euro area (but one has to bear in mind that their model includes habit in consumption) and is close to the value estimated by Casares (2001). Regarding the inverse elasticity of labour supply $\psi$, a wide range of calibration values has been used in the literature. Micro-labour literature estimates of the elasticity of substitution are relatively low while high labour elasticities are usually used in dynamic general equilibrium models. We take $\psi$ equal to 1.5 and therefore an elasticity of labour supply of 0.7, which falls between the values implied by macro and micro evidence. Finally, we choose $\psi_0$ such that households devote one quarter of their time to market activities\footnote{We arrive at this value by taking the ten-year average of the employment rate in the euro-area (in the period 1990-2000), approximately 59.5 per cent, and considering an average workweek of forty hours.}, given the values chosen for the other parameters and the steady-state of the model.

Regarding the technology parameters, we set the share of capital, $\alpha$, to 0.3. As for the inverse of the elasticity of substitution, $\sigma$, it is set to 6 which implies a steady-state net markup of 20 per cent. The stochastic processes for the technology shocks (which in steady-state are set to 1) are assumed to be autoregressive of order one with the persistence parameters set to 0.8.

Finally, regarding the innovation to the interest rate rule (22) (which is also set to unity in steady-state) we assume that $\log(M_t) = \rho_r \log(M_t) + \epsilon_{rt}$, where $\epsilon_{rt}$ has zero mean and $\rho_r$ is set to 0.5.

The parameter values are summarised in Table 1.

4 Equilibrium dynamics

We will now approximate the equilibrium dynamics of the model, for the case of small enough disturbances, by considering a log-linear approximation of the equilibrium conditions. First, we compute the steady-state and then take a log-linear approximation of the equilibrium conditions around this steady-state. All nominal variables are scaled by $M_t$\footnote{Given a variable $X_t$, we denote by $\tilde{X}_t$ its normalized value and by $\tilde{x}_t$ its log-deviation from steady-state, i.e. $\tilde{x}_t = \log \left( \frac{\tilde{X}_t}{X} \right)$.}. The linearised model was solved using Christiano (2002) undetermined coefficients method.

4.1 Steady-state

We now describe the deterministic steady-state around which the model will be approximated. Shocks are set to their mean value. On the other
hand, nominal variables grow at the same constant rate. We consider a zero inflation and zero money growth steady-state. Note that by assuming inflation is zero in steady-state we are considering the steady-state where there are no relative price distortions\(^{22}\).

Since in steady-state consumption is constant, the steady-state interest rate in the monetary union is tied down by the consumers Euler equations (13), so \(\bar{R} = \frac{1}{\beta}\). On the other hand, equations (18a) and (18b) imply that \(\overline{p}(a) = \overline{P}_A\) and \(\overline{p}(b) = \overline{P}_B\). Using the loan market clearing and goods market clearing conditions we conclude that

\[
\bar{P}_j = \frac{1 + x}{\bar{y}_j} \Leftrightarrow \bar{P}_j \bar{y}_j = 1.
\]

From the demand functions (7) and (8) one can find the steady-state value of each country’s total production, namely \(\bar{Y}_A = n \left(\frac{N^A}{n} \right)^{1-\alpha}\) and \(\bar{Y}_B = (1-n) \left(\frac{N^B}{n} \right)^{1-\alpha}\), so \(\overline{P}_A = \left(\frac{N^A}{n} \right)^{\alpha-1}\) and \(\overline{P}_B = \left(\frac{N^B}{n} \right)^{\alpha-1}\). Given that we assume \(N^A = N^B\) then \(\bar{Y}_A = \bar{Y}_B\) and \(\overline{P}_A = \overline{P}_B = \overline{P}\).

Given that the steady-state the real marginal cost equals the markup then

\[
\overline{W}_j = \frac{\sigma - 1}{\sigma} \beta (1 - \alpha) \frac{1}{\bar{y}_j}
\]

and hence \(\overline{W}_A = \overline{W}_B\). From this and the consumers Euler equations one concludes that \(\overline{C}_A = \overline{C}_B\) which together with the goods market conditions implies

\[
\overline{C}_j = \left(\frac{N_j}{n} \right)^{1-\alpha}.
\]

Finally, taking the budget constraint and the cash-in-advance equation of each household we obtain the steady-state level of deposits

\[
\overline{D}_j = \overline{W}_j \frac{1}{\bar{y}_j}.
\]

### 4.2 Log-linear approximation

Now we approximate the equilibrium conditions around the steady-state described above.

\(^{22}\)Actually, by assuming a zero inflation steady-state means that in steady-state all the firms will update their prices in the same way every period.
Given the utility function (1), the intertemporal conditions from the consumers optimisation problem (13) are approximated by:

\[ E_t \hat{c}_{t+1}^A = \hat{c}_t^A + \frac{1}{\gamma} [\hat{r}_t - E_t (n\hat{\pi}_A + (1-n)\hat{\pi}_B)] \quad (24) \]

\[ E_t \hat{c}_{t+1}^B = \hat{c}_t^B + \frac{1}{\gamma} [\hat{r}_t - E_t (n\hat{\pi}_A + (1-n)\hat{\pi}_B)] . \quad (25) \]

since \(\hat{\pi}_t = n\hat{\pi}_A + (1-n)\hat{\pi}_B\).

Similarly, log-linearising the intratemporal conditions (12), and taking into consideration that \(\hat{b}_p^t = n\hat{b}_p^A + (1-n)\hat{b}_p^B\) we get

\[ \psi n_t^A + \gamma \hat{c}_t^A = \hat{w}_t^A - \left[ n\hat{p}_A + (1-n)\hat{p}_B \right] \quad (26) \]

\[ \psi n_t^B + \gamma \hat{c}_t^B = \hat{w}_t^B - \left[ n\hat{p}_A + (1-n)\hat{p}_B \right] . \quad (27) \]

The log-linearised form of the households cash-in-advance equations is\(^23\):

\[ \bar{D}^A \bar{z}_t^A = \bar{W}^A N^A (\hat{\bar{w}}^A + \hat{n}_t^A) - \left( n\hat{p}_A + (1-n)\hat{p}_B \right) \]

\[ \bar{D}^B \bar{z}_t^B = \bar{W}^B N^B (\hat{\bar{w}}^B + \hat{n}_t^B) - \left( n\hat{p}_A + (1-n)\hat{p}_B \right) . \]

By log-linearising the first order conditions of the firms optimisation problem one derives the aggregate supply equations:

\[ \hat{\pi}_A = \beta E_t \hat{\pi}_{A+1} + (1-\beta \theta^A) \frac{1-\theta^A}{\theta^A} \left( \hat{r}_t + \hat{w}_t^A - \hat{\bar{p}}_A + \alpha \hat{n}_A - \hat{\xi}_t^A \right) \quad (28) \]

\[ \hat{\pi}_B = \beta E_t \hat{\pi}_{B+1} + (1-\beta \theta^B) \frac{1-\theta^B}{\theta^B} \left( \hat{r}_t + \hat{w}_t^B - \hat{\bar{p}}_B + \alpha \hat{n}_B - \hat{\xi}_t^B \right) \quad (29) \]

where \(\hat{\xi}_t^j = \log \left( \frac{\xi_t^j}{\xi_t} \right) = \log (\xi_t^j)\).

The monetary policy rule takes the following log-linear form

\[ \hat{r}_t = g_r \hat{r}_{t-1} + (1-g_r) (g_y \hat{\pi}_t + g_y \hat{y}_t) + \varepsilon_{rt} \quad (30) \]

where\(^24\) \(\varepsilon_{rt} = \log (\xi_{rt}) = \log (\varepsilon_{rt})\).

\(^23\)Given that we have no financial market friction (and since preferences are symmetric) then consumption in country A will always equal consumption in country B and liquidity in the monetary union will always be distributed proportionally to the size of each country in the union.

\(^24\)The exogenous shock \(\varepsilon_{rt}\) equals unity in steady-state.
Regarding the clearing conditions, we have seen that the clearing in the goods market implies

\[ C_{At} = n \cdot \frac{P_t}{P_{At}} \left( nC_t^A + (1 - n)C_t^B \right) = Y_{At}. \]

Log-linearising we get

\[ \hat{p}_t - \hat{p}_{At} + n\hat{c}_t^A + (1 - n)\hat{c}_t^B = \hat{y}_{At}. \]

On the other hand,

\[ Y_{At} = \left( \frac{P_{At}}{\hat{p}_{At}} \right)^{-\sigma} \xi_t^A \left[ N_t^A \right]^{1-\alpha} \]

log-linearising we get

\[ \hat{y}_{At} = -\sigma \left( \hat{p}_{At} - \hat{p}_{At}^{unw} \right) + (1 - \alpha) \hat{n}_t^A + \hat{\xi}_t^A \]

but since \( \hat{p}_{At} - \hat{p}_{At}^{unw} = 0 \)

and \( \hat{p}_t = n\hat{p}_{At} + (1 - n)\hat{p}_{At} \) we get the following clearing in the goods market conditions

\[ n\hat{c}_t^A + (1 - n)\hat{c}_t^B - (1 - n) \left( \hat{p}_{At} - \hat{p}_{Bt} \right) = (1 - \alpha) \hat{n}_t^A + \hat{\xi}_t^A \quad (31) \]

\[ n\hat{c}_t^A + (1 - n)\hat{c}_t^B + n \left( \hat{p}_{At} - \hat{p}_{Bt} \right) = (1 - \alpha) \hat{n}_t^B + \hat{\xi}_t^B. \quad (32) \]

On the other hand, clearing in the loan market implies

\[ nW^A N^A \left( \hat{w}_t^A + \hat{n}_t^A \right) + (1 - n)W^B N^B \left( \hat{w}_t^B + \hat{n}_t^B \right) = Dw_t + x_t \quad (33) \]

where \( D = nD^A + (1 - n)D^B \) and \( Dw_t = nD^A d_t + (1 - n)D^B d_t. \)

5 The mechanics of the model

In this section we describe the main characteristics of the transmission mechanisms at work in the model. Beside describing the responses of the variables in the model, we will also try to rationalise what is happening in the economy. However, one should stress that such reasoning entails a considerable simplification because all the described events happen simultaneously, given the general equilibrium nature of the model.

\[ \text{The proof can be found for example in Christiano, Eichenbaum and Evans (2003).} \]
We assume that the two countries in the union are of the same size and have an equal degree of price rigidity, and examine the responses of the model economy to a common monetary policy shock, simultaneous technology shocks, and country-specific technology shocks. As regards the degree of price stickiness, we consider two cases, namely that prices are unchanged for slightly more than one quarter and for one year.

Since we assume only one type of friction, in the low price rigidity scenario the model works closely to a basic cash-in-advance model where a (persistent) monetary contraction is associated with a rise in output and employment (Figure 1). This happens because in this case the inflation effect, acting as a tax on cash-goods, implies that a persistent monetary contraction (by lowering anticipated inflation) has a positive effect on output and consequently on employment. In the case of a high degree of price stickiness this is no longer true, a monetary contraction exhibits mainly demand side effects and therefore is associated with an output contraction and a decline in prices. As for the interest rate it will follow the assumed rule, namely reacting to inflation so to counter its behaviour according to

\[ \hat{r}_t = 1.5 \hat{\pi}_t + \hat{\epsilon}_{rt}. \] (34)

In the first panel of Figure 1 we present the responses to a shock to the monetary policy rule for the case where prices remain unchanged for roughly one quarter. As regards the interest rate behaviour, even though we consider a positive shock to the interest rate rule, the interest rate actually falls as the effect from the systematic component of monetary policy, namely the contemporaneous response to inflation, dominates the effect of the random component\textsuperscript{26}. On the other hand, the innovation to the interest rate rule means a contraction in the monetary aggregate\textsuperscript{27}. An interest rate reduction implies, other things equal, a decline in firm’s costs (current and future). In this context, a significant downward adjustment of prices occurs and the favourable inflation tax effect explains the observed increase in consumption, as consumers substitute consumption for leisure. Therefore employment and output increase. On the other hand, the labour supply shifts up due to the wealth effect and at the same time firms increase their demand for labour, which brings about an increase in real wages. Regarding the loan market, there is an increase in deposits and a marked contraction of the monetary

\textsuperscript{26}This result has been found elsewhere in the literature, as documented by Woodford (2003).

\textsuperscript{27}Therefore, the model does not predict in this scenario the existence of a liquidity effect.
aggregate. Consequently the supply of loans declines slightly. At the same
time, there is a reduction in the demand for funds by firms, who are faced
with a lower wage bill.

Even though the model does not exhibit a liquidity effect in the low rigid-
ity case just described, this is not a necessary feature of our framework. In
a situation where prices are fairly flexible the inflation tax effect dominates.
On the contrary, if the degree of stickiness is high, prices react less and the
real effects are larger. Consequently the endogenous effect on interest rates
no longer dominates the random component of policy and the interest rate
rises (second row of charts in Figure 1). Due to the monetary contraction,
consumption expenditures fall and given the sluggish response of prices real
consumption also decreases, unlike in the previous case. Lower consumption
entails a decline in output and lower employment. The increased willingness
of consumers to work jointly with the fall in the demand for labour by firms
brings about a reduction in real wages.\footnote{The model implies that after
a monetary contraction we observe an increase in the markup. Christiano,
Eichenbaum and Evans (1997) show that the actual behaviour of
prices is inconsistent with this implication of a sticky-price model of money.
To solve this failure one should dampen the change in marginal
costs (e.g. introducing variable capital utilisation) or adding other type of
frictions, e.g. sticky wages or a financial market friction
as in Christiano, Eichenbaum and Evans (1997) and Papadopoulou (2002).}
Regarding the loan market, the
supply of funds declines, both through the fall in deposits and the monetary
aggregate contraction, which equals a lower demand for funds by firms.

To gain further insight into the model we now analyse the responses to
technology shocks. First, consider the responses to a technology shock that
hits countries A and B at the same time, as displayed in the second panel of
Figure 1. A positive (and persistent) technology shock in both countries in-
duces a reduction of current and expected marginal costs, which leads firms
to adjust prices downward and therefore induces a decline in the interest
rate. Even though all firms face a decline in marginal costs (via both in-
creased productivity and lower interest rate) only a fraction adjusts prices.
As a result, the aggregate price level declines and consumption rises, but
less than proportionally to the increase in productivity. Therefore, firms
may face demand by hiring less labour, since workers are more productive,
and aggregate employment falls. Since the adjustment in demand is weaker
the sticker prices are, then in the case of a low degree of price rigidity we
see employment increasing slightly, while for the case of a higher degree of
price stickiness we observe a significant contraction in employment.\footnote{This result clearly contrasts with the predictions of standard real business cycle mod-
els, but is in line with findings in the literature for sticky prices models, namely Galí
(1999), Rotemberg (1996) and King and Wollman (1996).}
Focusing on the labour market, the productivity shock leads to an upward shift in the labour supply curve due to the increase in consumption and shifts the labour demand curve up because workers are more productive and labour is potentially less costly due to the decline in the interest rate, thus pushing the real wage up. Nevertheless, the effect on labour demand is (partly) offset by the fact that prices are sticky and therefore in equilibrium prices and output will adjust less than they would under perfect price flexibility. Finally, given the increased demand for funds in the period of the shock the monetary authority injects money in the economy.

Consider now an unanticipated positive productivity shock in country A \(^{30}\) (third panel of Figure 1). The increase in productivity pushes down marginal costs of firms operating in country A (other things equal), putting downward pressure on their prices. The observed decline in the aggregate price level generates a reduction in the interest rate, which has a downward effect in marginal costs of firms in both countries (other things the same). On the other hand, consumption of both goods increases. Focusing first on goods produced in A, the decline in marginal costs together with the increase in demand implies that output rises while prices go down. Given that workers in country A are more productive, the response of employment depends on how strong the increase in output is compared with the increase in productivity. An increase in the degree of price stickiness, by dampening the response of demand will make it more likely for employment to experience a large contraction. The labour supply curve shifts up due to the wealth effect and the labour demand curve in A shifts up since workers are more productive and the fall in the interest rate makes labour potentially less costly, pushing the real wage up. However, the effect on labour demand is (partly) offset by the fact that prices are sticky and therefore prices and output will adjust less than they would under perfect price flexibility. As a result while we see the real wage increasing in the low rigidity scenario, in the high rigidity scenario the real wage in A temporarily falls. The movement in relative prices therefore favours consumption of goods produced in A, which consequently experience a stronger increase in demand and a stronger output expansion than in country B. The rise in output in country B necessarily implies that employment increases in this country. Given the increased demand for labour and the lower willingness of consumer to work associated with higher consumption, the real wage also increases in country B. The change in demand (towards good A) together with the behaviour of

\(^{30}\)For the sake of simplicity we assume that productivity shocks are uncorrelated across countries and that there are no spillover effects.

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the costs of firms entails the temporary increase in prices of goods produced in this country (not seen in country A).

6 Monetary policy with symmetric countries

So far we have assumed that the monetary authority only responds to inflation developments. As a first step to assess the importance of the systematic component of monetary policy in the union, we discuss the implications of alternative rules of the form (30) for the dynamics of the model economy, while maintaining the assumption of identical countries. In this case, as countries are completely symmetric, i.e. there are no structural divergences or differences in the transmission mechanisms of shocks, cross-country differentials only arise as a result of asymmetric shocks hitting the two countries. Therefore, changing the design of the policy rule may have an impact on the dynamics of each country but it will have no importance in explaining the magnitude of differentials arising between the two economies. For illustrative purposes we consider an average duration of price contracts of two quarters in each economy and study separately the effects of changing the degree of interest rate smoothing and the responsiveness of the central bank to deviations of inflation and output from their steady-state values.

6.1 Impulse responses

First, consider the case of a stronger response to inflation developments by the monetary authority. To make the comparison clearer we assume a considerable increase in the response coefficient, namely from 1.5, as in rule (34), to 10.0 (first and second rows of charts in Figure 2). Such an increase of the coefficient on inflation results in almost like having an inflation target to be met period by period. Therefore, the central bank offsets the effect in inflation of any shock and consequently the necessary adjustments will be carried out with little response from inflation. If the monetary authority is more responsive to inflation developments then it behaves cautiously by trying to avoid surprising agents so that inflation does not fluctuate much. Consequently, the effects of a shock to the monetary policy rule are considerably reduced.

When simultaneous technology shocks occur (Figure 3) and the monetary authority behaves according to a rule like (34), the interest rate and prices fall and demand increases, resulting in higher output (employment hardly moves). If we increase the response to inflation in the policy rule, we again observe that inflation almost does not respond, which implies that
the interest rate falls by considerably less. However, given that demand expands slightly less than before, the fall in aggregate employment is somewhat higher. In face of country-specific technology shocks (Figure 4) the effects are similar, namely inflation in the union is stabilized, the interest rate response is smaller. To keep inflation close to target in the union, the monetary authority prevents it from falling, which results in a greater increase of prices of goods produced in A while prices of goods produced in B do not fall by as much as they did, though the inflation differential remains unchanged.

Alternatively, as argued in section 2.5, the central bank may want to smooth the behaviour of the policy rate, which means that current decisions explicitly depend on past decisions and therefore interest rate changes are deliberately reduced. When the monetary authority conducts policy by gradually adjusting the interest rate in response to deviations of inflation from its steady-state value (third panel in Figures 2, 3 and 4), the interest rate response on impact to any shock is less pronounced and the change in the interest rate is relatively more persistent (in fact, even though in the no-smoothing case, presented in the first panel, the interest rate reacts more on impact, this response is reverted more quickly than in the case with smoothing, presented in the third panel). In face of a monetary policy shock, the response of inflation and of the interest rate are less pronounced than in the case without smoothing. At the same time, the decline of output and employment is somewhat accentuated by this change in the monetary authority behaviour (given that the shock is more persistent, the favourable inflation tax effect is smaller). Similarly after a technology shock the interest rate falls, but less than in the no-smoothing case and consequently the inflation rate also shows a smaller decline. The increase in demand is lower and the downward effect in marginal costs via the interest rate decline is also weaker, which reduces the demand for labour and consequently we observe a larger contraction of employment.

Finally, one may postulate that the monetary authority besides responding to inflation developments, also adjusts the policy rate when output deviates from its steady-state value. Given that in face of a monetary policy innovation there is no trade-off between stabilizing inflation and output, increasing the response to output works in the same way as increasing the response to inflation, and consequently both output and inflation decline by less (Figure 2). The same is not true in face of technology shocks (Figures 3 and 4). With the first specification for the rule, following technology shocks inflation goes down and output goes up. Therefore adding the output term implies that the central bank tries to counter the output movement,
which implies that output does not increase above its steady-state value by as much as it does when the monetary authority responds only to inflation. At the same, with an output coefficient in the policy rule, inflation shows a stronger decline.

### 6.2 Unconditional second moments

Besides looking at impulse response functions to evaluate the impact of changing the systematic component of monetary policy, one can also look at the unconditional second moments of relevant variables, which combines the information of having all shocks at work at the same time\(^{31}\). Table 2 presents measures of the volatility and correlation of relevant variables under several monetary policy rules.

In accordance with the results presented above, when we increase the response of the monetary authority to inflation developments, the variability of inflation and output decreases and the interest rate is also less volatile. On the other hand, by adding (or increasing) the interest rate smoothing coefficient, most variables present a less volatile behaviour. In contrast, adding a response to output increases the variability of both output and inflation\(^{32}\).

As argued by Dotsey (1999a, b) and Gavin and Kydland (1999), the systematic component of monetary policy may have significant implications for the correlations in a model. In our model economy, output and inflation are negatively correlated, which is associated with the economy’s behaviour in face of technology shocks. On the other hand increasing the response to inflation in the union implies that the cross-country correlation of output is reduced. As for the correlation of goods price inflation across countries, for a low coefficient on inflation the correlation is positive but turns negative for higher values and gets stronger the higher the response to inflation in the case of a rule without smoothing. In the case of a rule with smoothing, this correlation is always negative under the rules considered, and it is also considerably strong for high values of the coefficient on inflation. In fact when the central bank is very responsive to inflation developments in the union, price inflation of goods A and B are almost perfectly negatively correlated, since for stabilizing inflation in the union, inflation of one good offsets in-

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\(^{31}\) The statistics were obtained by running Monte Carlo simulations. The results are therefore conditional on the assumed distribution of shocks.

\(^{32}\) This fact results from the response of the economy to technology shocks. In fact, as seen above, the variability of inflation and output conditional on monetary shocks is reduced when the monetary authority also responds to output developments.
flation developments of the other good. Adding an output response has the opposite effect on output correlation namely it increases the comovement between output in the two countries (while making it more volatile) and increases the comovement of inflation of goods A and B.

7 Monetary policy with asymmetric countries

In a monetary union where the central bank has an area-wide perspective, country-level analyses are justified, among other reasons, by potential differences that may exist in the transmission mechanism of shocks owing to differences in economic and financial structures. Such differences imply that even in the presence of common shocks disparities will emerge among countries in the union. This is illustrated, for example, by Andrés, Ortega and Vallés (2003) for the case of cross-country differences in the degree of competition in the goods market, in the degree of price inertia and in the degree of openness (or preference for foreign goods in consumption). The authors find that small differences in the degree of competition generate considerable inflation differentials (at business cycle frequency) among countries in a monetary union facing a common monetary policy shock. Additionally, they find that cross-country asymmetries in the degree of nominal inertia and openness also contribute to generate substantial inflation differentials in presence of regional shocks. Figure 5 illustrates how the relative degree of price inertia influences the inflation differentials generated by shocks in our model in the impact period of each shock. Our results show that the existence of asymmetries in the degree of price inertia generates sizeable differentials in face of monetary and technology shocks, which are more significant in the case of country specific technology shocks, thus supporting the conclusions of the above mentioned work.

These results involve an assumption regarding the characterization of monetary policy. We assumed that the monetary authority sets the interest rate to prevent deviations of inflation from its steady-state value, ensuring that the interest rate moves smoothly. But the specification of the monetary policy rule also matters for the results obtained, and consequently it is

\[ r_t = 0.7r_{t-1} + 1.5\hat{r}_t + \hat{\epsilon}_r. \]

We assumed that \( \hat{r}_t = 0.7\hat{r}_{t-1} + 1.5\hat{r}_t + \hat{\epsilon}_r \). The authors also assumed a rule with smoothing and an inflation response, though with a somewhat different parameterisation.
important to understand the asymmetries observed across countries in the union. In the previous section we assumed that countries were identical and consequently asymmetries only existed in the presence of country-specific shocks. In contrast, when structural differences exist among countries, common shocks also propagate differently across countries and even though the monetary authority only looks at aggregate variables, the specific choice of how to conduct policy will potentially be important to shape each country response to shocks, either common or country-specific.

In this section we study the importance of systematic monetary policy on the transmission of shocks in a monetary union when countries exhibit different degrees of price inertia, thereby extending the analysis in Andrés, Ortega and Vallés (2003) (even though our model is in some dimensions simpler than their model). In particular, we assume that in country A prices remain unchanged for slightly more than one quarter while in country B prices are fixed for one year. To do that we analyse how the model works under four alternative policy rules, namely

\begin{align*}
\text{Rule 1)} \quad \hat{\pi}_t &= 1.5\hat{\pi}_t + \hat{\epsilon}_{rt} \quad (35) \\
\text{Rule 2)} \quad \hat{\pi}_t &= 0.7\hat{\pi}_{t-1} + 1.5\hat{\pi}_t + \hat{\epsilon}_{rt} \quad (36) \\
\text{Rule 3)} \quad \hat{\pi}_t &= 0.7\hat{\pi}_{t-1} + 1.5\hat{\pi}_t + 0.5\hat{y}_t + \hat{\epsilon}_{rt} \quad (37) \\
\text{Rule 4)} \quad \hat{\pi}_t &= 0.7\hat{\pi}_{t-1} + 1.5\hat{\pi}_t + 1.5\hat{y}_t + \hat{\epsilon}_{rt}. \quad (38)
\end{align*}

The choice of rules is aimed at covering a set of different ways a central bank can react to economic developments, by changing the relative importance assigned by the central bank to deviations of inflation and output from their steady-state values, and are not necessarily based on the results from the estimation of reaction functions for any particular economy in a given period. While rule 1 and 2 postulate that the monetary authority only worries about inflation developments, rules 3 and 4 hypothesize that the central bank responds both to inflation and output, in the spirit of the rule proposed by Taylor (1993). However, we consider the existence of smoothing in interest rates, which did not happen in Taylor’s proposal, since it has been shown to be an important characteristic of central bank behaviour.

We focus on the responses of output and inflation to simultaneous and country-specific shocks and will concentrate on the short-run differences across countries. Additionally, we look at a number of statistics regarding the volatility and the comovement of relevant variables in the model.
7.1 Impulse responses

Figures 6 to 9 show the responses of output and inflation to a monetary policy shock, simultaneous technology shocks and country-specific technology shocks under each of the rules considered. Table 3 summarises information on the magnitude of the responses of output and inflation, in the period of the shock and four quarters after. The dynamics of output and inflation in the union and in each country differ somewhat when the reaction function of the monetary authority is changed. Unlike in the previous section, common shocks also generate cross-country differentials, due to the asymmetry in price inertia. In addition, the behaviour of the central bank plays a role in the cross-country differentials that arise following a shock. Table 4 reports cross-country differentials of output and inflation under the different rules considered. One should stress that the differentials generated on the impact period usually vanish rather quickly because there are no strong propagation mechanisms in the model\textsuperscript{36}. Consequently, our analysis of cross-country differentials will focus on the short-run.

In Figure 6 we present the responses of output and inflation to a shock to the interest rate rule when the central bank follows each of the rules under consideration (equations (35) to (38)). Given the observed contraction in the monetary aggregate, both inflation and output in the union go down. Prices of the goods produced in country A fall by more because the degree of price stickiness is lower in this country, thereby generating on impact a negative inflation differential between good A and good B. Then inflation gradually returns to its steady-state level, but since price adjustment is quicker in the country where prices are more flexible, we then observe a positive differential for some periods. On the other hand, since prices are quite flexible in country A we get little real effects from a monetary policy shock in this country, whereas in country B output falls considerably. The responses of output and inflation to a shock to the interest rate rule are qualitatively similar across the rules under consideration. However, given that the reaction of output in country B is reduced either by adding smoothing or an output response to the first rule, and since the real effects in country A are basically inexistent, then this rule generates higher output differentials than the other three rules. Additionally, the inflation differentials are also higher when the

\textsuperscript{36}We could however increase the persistence of such differentials by moving to a Phillips curve in the spirit of the hybrid New-Keynesian Phillips curve, as in Gali and Gertler (1999) and Gali, Gertler and López-Salido (2001). We could also strengthen the propagation of shocks following Christiano, Eichenbaum and Evans (2003), by introducing a dynamic updating scheme in the Calvo formulation or by improving the real side features of our model, namely with the introduction of variable capital utilisation.
central bank only looks at inflation (rule 1) than when it also responds to output (rules 3 and 4).

The responses to simultaneous technology shocks are presented in Figure 7. When a technology shock hits both countries at the same time, there is a downward pressure on prices of both goods, given lower marginal costs. But the fraction of firms adjusting prices is smaller in country B thus the relative price movement favours consumption of good A. Consequently, even though we see demand for both goods rising, the increase in output of good A is stronger and consequently we observe a positive output differential between A and B. If the central bank aims at stabilizing not only inflation but also output (relative to their steady-state values) we observe greater differences between the output response of the two countries, because output in the country where prices are more flexible partly compensates for the changes in output of the other country which leads to the desired stabilization of output in the union. At the same time, adding a response to output also widens the inflation differentials.

When the technology shock is country-specific the results are considerably different if the shock hits the country where prices are more flexible, country A, or the country where prices are stickier, country B. If the shock hits the country where prices are more flexible (Figure 8), the effect on costs will imply a considerable downward adjustment of the price of good A. Given that the relative price of good A falls, demand for this good increases by more than that for good B and consequently a positive differential between the responses of output in A and B arises. If the central bank responds strongly to output movements in the union, production of good B may actually fall, since the considerable change in relative prices induces consumers to strongly substitute consumption away from good B. This implies a somewhat smaller response of aggregate output and a widening of the output differential. On the other hand, adding a response to output also increases short-run inflation differentials between goods A and B. Moreover, if the central bank prefers interest rates to move smoothly then we observe a narrowing of both output and inflation differentials.

Consider now that the shock hits the sticky price country, i.e., country B (Figure 9). The downward pressure on prices associated with a shock to technology induces a smaller reaction of prices in the country hit by the shock than in the previous case. Given that the fraction of firms that adjust prices in each period in country B is smaller than in A, the pressure on prices associated with the shock induces a smaller reaction of price inflation of good B. On the other hand, output in this country and in the union increases and only slowly returns to the steady-state level, while output
in country A barely moves because in this country most of the necessary adjustment is done through prices. The dynamics of output and inflation are also somewhat altered when we change the monetary authority reaction function, as seen in Figure 9. The rule with stronger response to output induces a more pronounced response of inflation in the union and price inflation of both goods fall considerably more, with the fall in inflation of good B being induced by behaviour of the central bank. On the other hand, and in contrast with the case of a technology shock to country A, increasing the response by the central bank to output generates lower output and inflation differentials across countries. A rule with smoothing again implies lower differentials between countries, both output and goods price inflation.

### 7.2 Unconditional second moments

The results based on impulse response functions can be complemented with statistics that summarise the behaviour of variables in the model under alternative rules, as shown in Table 4. In this section, we concentrate our attention on the implications of alternative rules on the volatility of inflation and output.

According to the results presented in Table 4 and Figure 10, the introduction of smoothing in the reaction function of the central bank, namely going from rule 1 to rule 2, stabilizes inflation and, to a less extent, output, both in the union and in each country. Additionally, it implies, as seen in the previous section, lower short-run differentials between countries. However, the positive correlation between the rates of inflation of goods A and B is considerably reduced\(^{37}\). On the other hand, if the central bank reacts to output developments besides reacting to inflation, as in rules 3 and 4 compared with rule 2, we observe that while aggregate output is less volatile this comes at the cost of increased inflation volatility in the union. Additionally, while the volatility of output in the stickier country is reduced when the policy rule reacts to output, we observe an increase in the volatility of output in country A compared with output in the stickier country. On the contrary, the volatility of good A inflation relative to good B decreases, as the rise in standard-deviation of the inflation rate is more pronounced in the goods produced in the stickier country. Adding an output response also implies a significant change of the correlation of variables across countries (Table 4). Indeed, the stronger the response to output in the union the less output

\(^{37}\) Our results seem to be in line with those of Gavin and Kydland (1999) in that changes in the policy rules have greater implications in the magnitude of the correlation of nominal variables than real variables.
in the two countries will move together. For a sufficiently high response to output developments in the union, production in the two countries actually becomes negatively correlated.

In sum, in aggregate terms the policy that provides the greater stabilization for inflation is not the same that is best for stabilizing output at its steady-state level, a result akin to those in Woodford (2003). On the other hand, a central bank that ensures interest rates move smoothly in general lowers inflation and output differentials between countries. In addition, a rule that improves the results in terms of the aggregate variables volatility does not necessarily imply the same for country-specific variables. Also, the behaviour of the central bank influences the differentials arising between countries, and the rule that implies smaller differentials depends on the shock that hits the union. Finally, the comovement of variables in our model also depends to a considerable extent on the behaviour of the central bank.

The implications of systematic monetary policy in shaping the responses of a model economy to shock are used by Dotsey (1999b) to argue that the results in the literature that attempt to discriminate across models based on correlations or impulse responses are not robust. Using a model similar to that in Galí (1999), the author shows that following technology shocks one may obtain either a positive or a negative correlation between output and inflation or between output and employment depending on the monetary policy rule assumed\textsuperscript{38}. In keeping with his suggestion, we also concluded that in our model the comovement of variables depends on the reaction of the central bank to economic developments. As regards the labour market, Figure 7 shows the responses of labour and the real wage to simultaneous technology shocks. The inspection of the impulse responses makes clear that the adjustment in the labour market also hinges on the specification of policy. Specifically, a common positive technology shock can either entail a quite muted response of employment in both countries or imply a pronounced contraction of employment in country B accompanied by a slight increase in employment in country A, depending on the assumed monetary policy rule.

\textsuperscript{38}In a previous paper, Judd and Trehan (1995) also called attention to the fact that cross-correlations may not be very informative to discriminate among models. The authors show that the signs of correlations between variables, namely prices and output, depend crucially on the model used and therefore provide little information about the shock driving an economy.
8 Concluding remarks

In this paper we investigated the importance of systematic monetary policy for the dynamics of a two-country currency union where agents are asymmetric in the degree of nominal rigidity. We thereby extend previous contributions to a monetary union framework. In particular, we focus on a single rigidity, namely price stickiness, and work with a simple model to keep the analysis straightforward. The main conclusion of our work is that the design of monetary policy matters for the dynamics of the union not only in aggregate terms but also in terms of the behaviour of individual countries and it plays an important role when explaining differences arising between countries in a union. Even though we have assumed that the mandate of the central bank is to focus on developments at the union’s level, a policy implication of our analysis would be that it is important for the central bank to acknowledge how his actions will influence national developments. Specifically, the monetary authority actions may rebalance how the effects of a shock are distributed in the union and may compel a greater adjustment burden on the agents in some countries relative to others.

Our analysis is broadly in line with previous results in the literature. In particular, we conclude that the policy that provides greater inflation stabilization is not the same that also stabilizes output at its steady-state level, as Woodford (2003) pointed out in a slightly different framework. In this paper we take a positive focus and address questions regarding the design of alternative monetary policy rules, which is motivated by the recent attention given to the differences in economic behaviour amid countries in the EMU. Nevertheless, a natural extension of our work would be to study optimal policy in a monetary union framework, a topic that has already been analysed by Benigno (2003) in a similar framework.

Even though we expect that, to some extent, the robustness of our key conclusions do not hinge crucially on our modeling choices, some specific results will depend on the model used. Therefore, an additional step in the analysis will lead us to investigate the robustness of the results, for example, to the introduction of other frictions in the model or of factor mobility (through the inclusion of physical capital with elastic supply) which would improve the reallocation of resources among countries in the union. Most importantly, this would also bring our model closer to the data and would also allow us to take more direct implications for policy makers.
References


Tables and Figures

Table 1: Parameterisation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<td>Discount rate</td>
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<td>Risk aversion</td>
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<td>Elasticity of labour supply</td>
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<td>Share of capital</td>
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Table 2: Symmetric countries - Statistical moments

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37
Table 3: Asymmetric countries - Impulse responses

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Table 4: Asymmetric countries - Differentials and statistical moments

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<th>Differentials</th>
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<th>Simultaneous technology shocks</th>
<th>Technology shock in country A</th>
<th>Technology shock in country B</th>
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<td>Rule 1</td>
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Figure 1: IRF - Symmetric countries - Changing the degree of rigidity

Monetary policy shock

Low rigidity scenario ($\theta = 0.25$)

High rigidity scenario ($\theta = 0.75$)

Simultaneous technology shocks

Low rigidity scenario ($\theta = 0.25$)

High rigidity scenario ($\theta = 0.75$)

Technology shock in country A

Low rigidity scenario ($\theta = 0.25$)

High rigidity scenario ($\theta = 0.75$)
Figure 2: Symmetric countries - Monetary policy shock under alternative rules

$r(t)=1.5\pi(t)$

$r(t)=10.0\pi(t)$

$r(t)=0.7r(t-1)+1.5\pi(t)$

$r(t)=0.7r(t-1)+1.5\pi(t)+0.5y(t)$

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<th>% Dev. from steady-state</th>
<th>Consumption</th>
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<th>Labour</th>
<th>Real Wage</th>
<th>Deposits/M</th>
<th>Inflation</th>
<th>Relative prices</th>
<th>Interest rate and money growth</th>
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<td>wB</td>
<td>dA</td>
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<th>% Dev. from steady-state</th>
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Figure 3: Symmetric countries - Simultaneous technology shocks under alternative rules
Figure 4: Symmetric countries - Technology shock in country A under alternative rules

### Consumption
- One figure showing the percentage deviation from steady-state for consumption over time. The graph indicates a decline in consumption after the shock, with a partial recovery over time.

### Output
- Another figure illustrating the output percentage deviation from steady-state. The output shows a decrease initially, followed by a gradual increase.

### Labour
- A graph depicting the labour force percentage deviation from steady-state. Labour drops initially and then stabilizes at a lower level.

### Real Wage
- A chart showing the real wage percentage deviation from steady-state. Real wages decrease significantly in the short term and then stabilize.

### Deposits/M
- A graph illustrating the percentage deviation of deposits to money supply from steady-state. Deposits decrease sharply initially and then stabilize.

### Inflation
- A figure showing the inflation percentage deviation from steady-state. Inflation increases significantly after the shock and then stabilizes.

### Relative prices
- A graph depicting the relative price percentage deviation from steady-state. Relative prices increase initially and then stabilize.

### Interest rate and money growth
- A chart illustrating the percentage deviation of interest rate and money growth from steady-state. Both interest rates and money growth exhibit a decline initially and then stabilize.

### Deviation from steady-state
- All graphs include a bottom axis indicating the deviation from steady-state, with values ranging from 0 to 5%.

The figures demonstrate the economic impact of a technology shock on various macroeconomic indicators in symmetric countries under different rules.
Note: Theta A is kept unchanged at 0.25.
Figure 6: Asymmetric countries - Monetary policy shock under alternative rules

\[ r(t) = 1.5 \pi(t) \]

\[ r(t) = 0.7r(t-1) + 1.5 \]

\[ r(t) = 0.7r(t-1) + 1.5 + 0.5y(t) \]

\[ r(t) = 0.7r(t-1) + 1.5 \pi(t) + 0.5y(t) \]
Figure 7: Asymmetric countries - Simultaneous technology shocks under alternative rules

$r(t) = 1.5 \pi(t)$

$r(t) = 0.7r(t-1) + 1.5 \pi(t)$

$r(t) = 0.7r(t-1) + 1.5 \pi(t) + 0.5y(t)$

$r(t) = 0.7r(t-1) + 1.5 \pi(t) + 1.5y(t)$
Figure 8: Asymmetric countries - Technology shock in country A under alternative rules

$r(t)=1.5\pi(t)$

$r(t)=0.7r(t-1)+1.5\pi(t)$

$r(t)=0.7r(t-1)+1.5\pi(t)+0.5y(t)$

$r(t)=0.7r(t-1)+1.5\pi(t)+1.5y(t)$
Figure 9: Asymmetric countries - Technology shock in country B under alternative rules
Figure 10: Asymmetric countries - Volatility of output and inflation

**Monetary union**

- Rule 1
- Rule 2
- Rule 3
- Rule 4

**Country A**

- Rule 1
- Rule 2
- Rule 3
- Rule 4

**Country B**

- Rule 1
- Rule 2
- Rule 3
- Rule 4