THE DISTRIBUTION OF LIQUIDITY
IN A MONETARY UNION
WITH DIFFERENT
PORTFOLIO RIGIDITIES

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The distribution of liquidity in a monetary union with different portfolio rigidities*

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Abstract
This paper analyses the monetary transmission mechanism in a monetary union with a segmented financial market. Differences in the households’ information sets imply that a money supply shock yields permanently heterogeneous allocations across households. The distribution of liquidity is fundamental to this equilibrium. This distribution is also important to understand the response of the macroeconomic variables to a technology shock. In this case, a money supply rule yields heterogeneous allocations between households, while an interest rate peg undoes the portfolio friction, yielding the same allocation across agents.

1 Introduction
During the last decade, there has been a growing interest on the study of monetary policy in general equilibrium models. Lucas (1990) and Fuerst (1992) showed that one way to model the monetary non-neutralities present in the data is to take into account that agents are not all and/or always participating in the financial market. This may arise due to information asymmetries or to the inability of agents to costlessly implement their desired financial flows in all periods. Models with this segmentation in the

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financial markets easily generate a liquidity effect after a monetary shock, which has led to their popularity in the modelling of monetary economies (see for example Christiano (1991), Cooley and Quadrini (1999) and Papadopoulou (2002)). These "limited participation" models were primarily developed to understand the monetary transmission mechanism and the distribution of liquidity in closed economies. The literature on open economy limited participation models is thus rather scarce, with some exceptions being Shlagenhauf and Wrase (1995) and Cooley and Quadrini (2001).

In this paper, we extend the analysis of the monetary transmission mechanism to a two-country monetary union, where countries are differentiated by the degree of portfolio rigidity of the households. In analyzing the impact of monetary shocks, special emphasis is placed on the distribution of liquidity between countries in the union. The emphasis on the wealth redistribution effects of monetary policy was already present in the early contribution of Grossman and Weiss (1983) but was later downplayed due to generalised adoption of the representative household construct of Lucas (1990). In the latter each household consisted of a multi-member family who traded in different markets but which pooled resources and information at the end of the period so that wealth distribution effects did not emerge. More recently, Alvarez, Atkeson and Edmond (2002) presented an inventory theoretic model of money demand. Households are assumed to have the opportunity to transfer funds to their bank accounts only once every $N$ periods. In this context, a monetary injection implies that agents trading in the financial market at that point in time will hold a disproportionate amount of money. Since these agents have a low propensity to consume and hold a disproportionate amount of money, aggregate velocity decreases and prices respond sluggishly to the monetary shock.

In the two-country monetary union model presented below, the simplicity of the representative household construct is preserved by assuming that households are heterogeneous between countries and not within each country. This heterogeneity is modelled by assuming that households have different information sets when making savings decisions, as in Christiano (1991). This allows the analysis of issues related to the distribution of liquidity in the monetary union. We find that the real effects of monetary policy depend on the overall degree of portfolio rigidity. Further, there are heterogeneous real effects in each country due to the existence of heterogeneous portfolio rigidities. In particular, a monetary shock leads to permanent trade effects and capital balance effects.

This paper also studies the impact of technology shocks in each economy, under an interest rate peg and a constant money growth rule. The interest
The study of the impact of a single monetary policy in different countries of a monetary union is not only important for the positive analysis of the monetary transmission mechanism but also to the normative analysis of the optimal monetary policy. In this set-up, an interest rate peg replicates the all-flexible allocation irrespective of the degree of portfolio rigidity in the union. Under a constant interest rate, the allocations after a technology shock are thus independent of the degree of the underlying friction. In a companion paper, it is shown that the optimal policy in this set-up is in general the Friedman rule. It follows that the optimal policy is independent of the degree of portfolio rigidity, as in Adão, Correia and Teles (1999).

This paper is organized as follows. The next section describes the two-country economic environment, the optimal decisions of the agents and the equilibrium of the model. Section 3 describes the parameterization and assesses the quantitative impact of a monetary policy shock while Section 4 focuses on the importance of the liquidity distribution to the various outcomes. In Section 5 the effects of common technology shocks to the union are analyzed under two rules: an interest rate peg and a constant money growth rule. Section 6 summarizes the main conclusions and discusses areas for future research.

2 A two-country monetary union model with a segmented financial market

2.1 Overview

The model follows Christiano, Eichenbaum and Evans (1997) and generalizes it to a monetary union set-up. The monetary union comprises two countries, \( A \) and \( B \). The union is populated by a continuum of agents in the interval \([0, 1]\). We assume that the segment \([0, n]\) corresponds to country \( A \) and the segment \((n, 1]\) to country \( B \). The economy consists of six sectors: the firms in each country; the households in each country; the financial intermediaries, who are completely integrated in the monetary union; and a single monetary authority. The behavior of each agent is described in the subsections below.
To understand the mechanics of this model, it is crucial to describe thoroughly the cash flow in the economy. In the beginning of period $t$, all the money in the economy is held by the households. Each country’s representative household enters the financial market and allocates her money holdings between loans to the financial market - remunerated at a gross interest rate $R_t$ - and money sent to the goods markets. It is assumed that households from country $A$ do not observe the contemporaneous shocks to the economy before entering the financial market, whilst households from country $B$ observe these shocks before making their consumption/savings decisions.

After leaving the financial market, the households participate in the goods markets. They demand goods produced in both countries and face a cash in advance, stating that all nominal consumption must be purchased with cash. The households are also able to use their wage bill to finance consumption.

There is no mobility of labor between countries. Each country’s representative firm $i$ thus hires domestic labor only. In order to pay in advance the wage bill to the households, the firms have to borrow that amount from the financial intermediaries at a gross interest rate $R_t$.

At the end of the period, the households receive the dividends from the firms and the dividends plus the loans (with interest) from the financial intermediaries.

The uncertainty in this economy comes from the monetary authority, who is assumed to inject reserves to the system through a lump-sum transfer to the financial intermediaries. Later, the impact of technology shocks will also be considered.

### 2.2 The representative households

All individuals consume goods produced in both countries. The preferences of household $j$ (where $j \in [0, n]$ corresponds to the representative household in country $A$, while $j \in (n, 1]$ corresponds to the representative household in country $B$) are given by

$$U^j_t = E_0 \sum_{t=0}^{\infty} \beta^t u(C^j_t, 1-N^j_t)$$

where $\beta$ is a discount factor, $1-N^j_t$ is leisure and $C^j_t$ is an index of consumption of commodity bundles in countries $A$ and $B$, defined as in Obstfeld and
Rogoff (1998):

\[ C^j = \left( \frac{C^j_A}{n} \right)^n \left( \frac{C^j_B}{n} \right)^{1-n} \frac{1}{n^{1-n}(1-n)^{1-n}} \]  

(2)

In this expression, \( n \) corresponds to the size of country \( A \), both in terms of population and in terms of the share on total consumption of the consumption good produced in \( A \). The index \( C^j_A \) corresponds to consumption in country \( j \) of the continuum of goods produced in country \( A \) and \( C^j_B \) corresponds to consumption in country \( j \) of the continuum of goods produced in country \( B \). These bundles are generally defined as

\[ C^j_A = \left( \frac{1}{\sigma} \right) \int_0^n c^j(a) \frac{\sigma - 1}{\sigma} da \]  

(3)

and

\[ C^j_B = \left( \frac{1}{1-n} \right) \int_0^1 c^j(b) \frac{\sigma - 1}{\sigma} db \]  

(4)

where \( \sigma > 1 \) is the elasticity of substitution between the goods produced in each country. Note that the elasticity of substitution between the indexes \( C_A \) and \( C_B \) is equal to 1. There is no home-bias in consumption.

By standard procedures the overall price index in each country is given by

\[ P^j = \left( \frac{P^j_A}{P^j_B} \right)^n \left( \frac{P^j_B}{P^j_A} \right)^{1-n} \]  

(5)

with

\[ P^j_A = \left( \frac{1}{\sigma} \right) \int_0^n p^j(a)^{1-\sigma} da \]  

and

\[ P^j_B = \left( \frac{1}{\sigma} \right) \int_0^1 p^j(b)^{1-\sigma} db \]

where \( p^j(a) \) is the price of good \( a \) (which is only produced in \( A \)), sold in country \( j \). An analogous definition applies to \( p^j(b) \).

Assuming there are no transportation costs between countries, prices are set taking into account that the monetary union is a single market. It follows that \( p^A(a) = p^B(a) \) and \( p^B(b) = p^A(b) \). Further, given the symmetric structure of preferences, the purchasing power parity holds, implying \( P^A = P^B \).

Given a decision on \( C^j \), the Cobb-Douglas total consumption index implies that the demands for the composite \( A \) and \( B \) goods are given by

\[ C^j_A = n \left( \frac{P^j_A}{P^j_B} \right)^{-1} C^j \]  

and

\[ C^j_B = (1-n) \left( \frac{P^j_B}{P^j_A} \right)^{-1} C^j \]  

Moreover, we can also

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1 An analogous mnemonic applies to the prices of the goods.
derive the demand of the differentiated good \( a \), produced in country \( A \) and of good \( b \), produced in country \( B \):

\[
c(d) = \left[ \frac{p(d)}{P^d} \right]^{-\sigma} \left( \frac{P^d}{P^j} \right)^{-1} C^j
\]

\[
c(d) = \left[ \frac{p(d)}{P^d} \right]^{-\sigma} \left( \frac{P^d}{P^j} \right)^{-1} C^j
\]

Note that the superscripts in the price indexes are redundant, since prices are the same in both countries.

The households maximize utility subject to a cash in advance constraint and an asset evolution equation. The problem of the representative household in country \( A \) is the following:

\[
\begin{align*}
\max U_A^t &= E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left( C_A^t , 1 - N_A^t \right) \right\} \\
st P_A^t C_A^t &= \left( W_A^t A^t N_A^t + M_A^t - L_A^t \right) \\
M_{t+1}^A &= \left[ W_A^t A^t N_A^t + M_A^t - L_A^t - P_A^t C_A^t \right] + R_t L_A^t + R_t X_A^t + D_A^t
\end{align*}
\]

where \( W_A^t \) is the nominal wage, \( N_A^t \) is aggregate labor, \( M_A^t \) is the beginning-of-period stock of money, \( L_A^t \) are the loans to the financial intermediaries and \( D_A^t \) are the dividends from firms in country \( A \), which are distributed to the respective owners - country \( A \)’s households. The amount \( R_t X_A^t \) corresponds to the profits of the financial intermediaries, which arise due to the monetary injection \( X_t \). Since the financial markets are completely integrated we assume that all agents have an equal share in the financial intermediaries’ capital. Thus, each household in the monetary union receives an equal share of this profit, i.e., in per capita terms we have \( X_A^t = X_B^t = X_t \). In terms of countries, each country will receive the financial intermediaries’ profits in proportion to their relative size in the union.

The first-order conditions can be summarized in the following equations:

\[
P_{A,t}^A C_{A,t}^A = nP_t^A C_t^A
\]

\[
P_{B,t}^A C_{B,t}^A = (1 - n)P_t^A C_t^A
\]

\(^2\)All the derivations of the prices indices and the demand functions are available upon request.
\[
\frac{U_{t-1-N,t}}{U_{C,t}} = \frac{W_t^A}{P_t^A}
\]  
(13)

\[
E_{t-1} \left[ \frac{U_{C,t}}{P_t^A} - \beta R_t \frac{U_{C,t+1}}{P_{t+1}^A} \right] = 0
\]  
(14)

Conditions (11) and (12) state that the nominal share of consumption of the goods produced in each country is proportional to the relative size of the countries. This is due to the Cobb-Douglas specification of the consumption bundles. Condition (13) states that the intratemporal marginal rate of substitution between leisure and consumption is equal to the real wage rate. Condition (14) is the intertemporal condition, equating the marginal utility of a unit of money lent to the financial intermediary at time \( t \) to the expected marginal utility of return from that deposit at time \( t + 1 \). The expectation term in equation (14) is taken at time \( t - 1 \) due to the assumption that country \( A \)’s households participate in the financial market prior to the realization of shocks. In contrast, we assume that country \( B \)’s households observe the shocks before transacting in the financial market. This heterogeneity segments the financial market and is the main subject of study in this paper.

The analogous first-order conditions for country \( B \) are

\[
P_{A,t}^B C_{A,t}^B = n P_t^B C_t^B
\]  
(15)

\[
P_{B,t}^B C_{B,t}^B = (1 - n) P_t^B C_t^B
\]  
(16)

\[
\frac{U_{t-N,t}}{U_{C,t}} = \frac{W_t^B}{P_t^B}
\]  
(17)

\[
E_t \left[ \frac{U_{C,t}}{P_t^B} - \beta R_t \frac{U_{C,t+1}}{P_{t+1}^B} \right] = 0
\]  
(18)

2.3 The representative firms

Firms in each country have access to the following production technology

\[
y_t(i) = s_t^i [n_t(i)]^{1-\alpha} \quad i = a, b
\]

where \( y_t(i) \) is the production of good \( i \), \( s_t^i \) is a technology shock (which can be common or country-specific) and \( n_t(i) \) is labor employed by the firm producing good \( i \). The firms in each economy hire domestic labor at a wage rate \( W_t^j \) and need to borrow their wage bill from the financial intermediaries
at a rate of interest $R_t$. The problem is completely symmetric between all households and firms in each country so $n_t(a) = N_t^A$ and $n_t(b) = N_t^B$.

The firms choose the price to maximize their profits. In the case of country $A$, we have:

$$\text{Max } \pi_t^A = p_t(a) y_t(a) - W_t^A N_t^A - (R_t - 1) W_t^A N_t^A =$$

Taking into account that the demand elasticity facing the firms’ products is $-\sigma$, the first-order condition of this problem implies that firms set their prices according to

$$P_{A,t} = p_t(a) = \frac{\sigma}{\sigma - 1} R_t W_t^A \frac{1}{1 - \alpha s_t^A} (N_t^A)^{1-\alpha}$$

i.e., they set prices as a constant mark-up $\mu = \frac{\sigma}{\sigma - 1}$ over marginal costs. As the elasticity of demand gets larger, the mark-up converges to 1, the case of perfect competition.

The price-setting behaviour of the firms in country $B$ is completely symmetric.

2.4 The financial intermediaries

There is complete integration of the financial markets. The supply of loans corresponds to the sum of the monetary injection $X_t$ with the loans made by the households. The demand for loans comes from the firms and equals the total wage bill. The loan market clearing condition is therefore:

$$n W_t^A N_t^A + (1 - n) W_t^B N_t^B = n L_t^A + (1 - n) L_t^B + X_t$$

2.5 Other equilibrium conditions

Substituting the cash-in-advance constraints into equation (20) yields:

$$nP_t^A C_t^A + (1 - n) P_t^B C_t^B = M_t + X_t$$

where $M_t = n M_t^H + (1 - n) M_t^F$ is the union’s monetary aggregate.

The goods markets equilibria imply that overall consumption of each type of good in both countries equals the respective production:

$$n C_{A,t}^A + (1 - n) C_{A,t}^B = n s_t^A (N_t^A)^{1-\alpha}$$
and

\[ nC_{B,t}^A + (1 - n)C_{B,t}^B = (1 - n)s_t^B (N_t^B)^{-\alpha} \]  

(23)

Replacing equations (11) and (15) in equation (22), equations (12) and (16) in (23), and using equation (21) yields the aggregate demand functions of each type of good:

\[ P_{A,t}^A = P_{A,t}^B = \frac{M_t + X_t}{s_t^A (N_t^A)^{-\alpha}} \]  

(24)

\[ P_{B,t}^A = P_{B,t}^B = \frac{M_t + X_t}{s_t^B (N_t^B)^{-\alpha}} \]  

(25)

3 Response of the union to a monetary policy shock

3.1 Parameterization

As our baseline, we assume the following preferences for the households

\[ U_j^t = E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{1}{1 - \phi} \left( C_j^t - \frac{\left( N_t^j \right)^{1+\chi}}{1 + \chi} \right)^{1-\phi} - 1 \right] \]

These preferences are in the class of GHH preferences, which imply that there are no wealth effects in labor supply. Whenever relevant we will report the impact of this assumption on the results.

Since we will be particularly interested in the distribution of liquidity in the monetary union, we define \( \theta \) as the share of the union’s money held by country A’s households:

\[ \theta_t = n \frac{M_t^A}{M_t} \]

By construction, we also have

\[ 1 - \theta_t = (1 - n) \frac{M_t^B}{M_t} \]

The monetary authority is assumed to follow an autoregressive money supply process, of the form

\[ x_t = \rho x_{t-1} + \varphi_{x,t} \]

where \( \varphi_{x,t} \) has mean \( x \) and standard deviation \( \sigma_{x,t} \).
The parameters were assigned the following values (following closely Christiano et al. (1998)): $\beta = 1.03^{-0.25}$; $\alpha = 0.36$; $\mu = 1.3$; $\chi = 0.6$; $\phi = 2$; $x = 0.01$; $\sigma_x = 0.005$; $\rho = 0.5$. Three features of this parameterization may be highlighted. First, an AR(1) process with a degree of persistence of 0.5 was found by Christiano et al. (1998) to be a good statistical approximation for the response of M2 to monetary policy shocks in the US in the period 1965Q3-1995Q2. Second, the mark-up is within the consensus of the literature (Christiano et al. (1997) use a value of 1.2 and Christiano et al. (1998) choose 1.4 instead). Third, the parameter $\chi = 0.6$ implies, with our GHH preferences, an intratemporal elasticity in labor supply of $\frac{1}{\chi} = 1.7$. This may be considered high when looking at microeconomic studies but low when looking at macro studies (see Domeij and Flodén (2001) for an overview and a description of a downward bias in microeconometric studies ignoring borrowing constraints).

We will study linear approximations to the solution of the model by log-linearizing the equations around the deterministic steady-state, using the undetermined coefficients method presented in Christiano (2002). All the nominal variables were scaled by $M_t$.

3.2 Analysis of the results

Figure (1) reproduces the response of the economy to a 0.5 p.p. shock in the money supply growth path of the monetary union. The baseline parametrization assumes that both countries are of the same size.

After a positive monetary shock we observe the following events. First, the interest rate drops in the period of the shock (the liquidity effect dominates). In the subsequent quarters, the anticipated inflation effect dominates (the model has no endogenous propagation mechanism, so the liquidity effect lasts only 1 period). Second, the monetary union’s overall consumption rises in the impact period. However, consumption rises in country $B$ and falls in country $A$. In subsequent periods, consumption is below steady state in both countries. Third, the monetary union’s overall employment and real wages rise in the impact period. The labor market equilibrium is the same in both countries, due to the assumption of GHH preferences. Fourth, inflation rises persistently after the shock. Fifth, the loans from country $B$’s households to the financial intermediaries fall in the impact period and the money sent to the goods market rises. Country $A$’s households do not change the amount of money sent to the financial or the goods market in the impact period. Finally, the share of money in country $A$ ($\theta_t$) rises in the impact period and stays permanently higher relative to the previous steady state.
Figure 1: Response of the economy to a monetary injection in period 6. Countries are of the same size. Country A’s households have sticky portfolios and Country B’s households have flexible portfolios. The Figure shows the percent deviations from steady state, except for the interest and inflation rates, which are presented as annualised rates.
To understand these events, it is useful to conduct some partial equilibrium analysis, starting with the financial markets, moving to the labor markets and ending in the goods markets (this follows Christiano et al. (1997)). We will first focus on the $n = 0.5$ case and on the impact period. Afterwards we will assess the impact of varying $n$. Finally, we will analyze the distribution of liquidity in the union and its relation with the intertemporal allocations.

### 3.2.1 The financial market

Combining the loan market equilibrium condition (20) with the aggregate cash-in-advance (21), we can write

$$\frac{nW_t^A N_t^A + (1-n)W_t^B N_t^B}{nP_t^A C_t^A + (1-n)P_t^B C_t^B} = \frac{nL_t^A + (1-n)L_t^B + X_t}{M_t + X_t}$$

which can be simplified, using the price setting equation (19) in both countries and the goods market equilibrium, to

$$\frac{(1 - \alpha)}{\mu R_t} = \frac{nL_t^A + (1-n)L_t^B + X_t}{M_t + X_t} \tag{26}$$

This relation shows that the evolution of the interest rate is directly linked to the evolution of the amount of money in the hands of the financial intermediaries. Knowing that $L_t^A$ does not respond contemporaneously to a monetary shock (due to the portfolio rigidities), the existence of a liquidity effect will be determined, on the one hand, by the response of $L_t^B$ and, on the other, by the relative size of the countries. In the absence of persistence in the money supply process (i.e., without an increase in expected inflation), the existence of a fraction of sticky households would be sufficient to generate a dominant liquidity effect. The baseline results suggest that, even with considerable persistence in the money supply process ($\rho = 0.5$), the framework with equally-sized countries is sufficient to generate a dominant liquidity effect.

Compared with traditional limited participation models, the monetary injection in this set-up is not only channelled through the financial intermediaries to the firms but also to the flexible households. Relative to those frameworks, the liquidity effect is thus less pronounced here, since the firms do not have to absorb all the liquidity injected by the monetary authority in the system.
3.2.2 The labor markets

Since labor is immobile between countries, the labor markets are segmented. The demand for labor is given by the first-order conditions of the firms

\[ \frac{W_j^j}{P_j^j} = \frac{1}{R_t} \frac{s_j^j (1 - \alpha) \left( N_j^j \right)^{-\alpha}}{\mu} , \text{ for } j = A, B \]  

(27)

and the supply of labor is given by the household’s first-order conditions

\[ \frac{W_j^j}{P_j^j} = \left( N_j^j \right)^{\chi} , \text{ for } j = A, B \]  

(28)

In the baseline parameterization, the monetary shock implies a reduction in interest rates. Since the decrease in interest rates represents a decrease in the firms’ intermediate costs, the demand for labor rises, *ceteris paribus*. This occurs in both countries. Due to the assumption of GHH preferences, the behavior of the supply schedule in the labor market is unchanged after a monetary shock. Thus, both labor markets share the same equilibrium.

The overall effects are an unambiguous increase of labor and of the real wage in both countries, which occurs as long as the liquidity effect dominates the anticipated inflation effect after a monetary injection\(^3\).

3.2.3 The goods market

The demand function for the good produced in country \(j\) is given by (recall equations (24) and (25))

\[ P_{j,t} = \frac{M_t + X_t}{s_j^j \left( N_j^j \right)^{1-\alpha}} \]  

(29)

After a money injection the demand curve is shifted upwards. This happens in both markets. The same occurs to the supply of goods. The

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\(^3\)When preferences have wealth effects in labor supply, the different behavior of the \(C_j^j\)’s now correspond to different movements of the labor supply curve. In this case, a monetary policy shock implies the following. In country \(A\), both the labor and the supply curve shift rightwards, yielding an unambiguous increase in labor in country \(A\) and an ambiguous change in the real wage. In country \(B\) the supply curve shifts leftwards and the demand curve shifts rightwards yielding a decrease in labor in country \(B\) (for most parameterizations) and an unambiguous increase in the real wage (when the liquidity effect dominates).
overall effects are thus an unambiguous increase in both quantities and prices of each type of goods\(^4\).

An important result of these models is that profits always rise after a monetary policy shock. To see this, we can write the profits of the representative firm of country \(A\) as \(\pi_A^t = p_A^A s_A^A (N_A^A)^{1-\alpha} - R_t w_A^A N_A^A\). Using the price setting condition of the firm, as well as equation (29), we conclude that \(\pi_A^t = (1 + x_t) \left[ 1 - \frac{1-\alpha}{\mu} \right]\). Thus, profits are positively related to money injections. The increase in profits after a monetary injection is in accordance with the empirical evidence (see, in particular, Christiano et al. (1997)).

### 3.3 Impact of changing the size of each country

As expected, the size of the country \(A\) - which in this set-up corresponds to the percentage of sticky households - has a very important impact in the response of the monetary union’s variables to a money injection.

First, for small sizes of country \(A\) \((n < 0.25)\), the interest rate rises following a monetary injection. In this case, a vast majority of households in the monetary union observes the monetary shock, so the anticipated inflation effect dominates the liquidity effect. Conversely, the larger the size of country \(A\), the larger the liquidity effect.

Second, the behavior of \(L^B_t\) is also a function of the relative size of the countries. The larger the proportion of “sticky” households, the smaller the decrease in total loans to the financial intermediaries (Figure (2)). The counterpart of this effect is that the higher the size of \(A\), the smaller the increase in the monetary union’s money sent to the goods markets.

Third, in the labor market, the higher the size of \(A\), the higher the upward shift\(^5\) in the demand curve (because the nominal interest rate is progressively lower) [see Figure (2)]. This implies that the larger the size of the “sticky” country \(A\), the larger the expansion (or the smaller the decrease, for \(n < 0.25\)) in the real wage and the quantity of labor in equilibrium.

In the goods markets, a higher \(n\) implies a larger shift to the right of the supply curve with no impact on the demand curve (Figure (3)). A higher \(n\) is thus associated with a smaller increase in the price of the goods produced in both countries, and consequently with a smaller increase in the overall price

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\(^4\)In case of preferences with wealth effects in labor supply, the behavior of the goods supply curve is different in each market. The demand curve is shifted upwards in both markets. For the baseline calibration, there is a rise in \(P_B\) higher than in \(P_A\), a rise in \(y_A\) and a fall in \(y_B\).

\(^5\)For \(n < 0.25\), the higher \(n\), the smaller the downward shift in the demand curve.
Equilibria in the financial market

<table>
<thead>
<tr>
<th>Loans to the financial intermediaries (divided by M)</th>
<th>Annualised R %</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>0.0%</td>
</tr>
<tr>
<td>9%</td>
<td>0.1%</td>
</tr>
<tr>
<td>8%</td>
<td>0.2%</td>
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<tr>
<td>7%</td>
<td>0.3%</td>
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<tr>
<td>6%</td>
<td>0.4%</td>
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<td>5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>4%</td>
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Equilibria in the labor markets (in countries A and B)

<table>
<thead>
<tr>
<th>W/P</th>
<th>Annualised R %</th>
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<tr>
<td>10%</td>
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<td>9%</td>
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Figure 2: Equilibria in the financial and labor markets in the period of the shock

indexes in both countries. Thus the stickiness in portfolios partly translates in price stickiness after a monetary shock.

4 The distribution of liquidity in the monetary union

The analysis of the distribution of liquidity in the union is crucial to assess the allocations following a monetary policy shock. The distribution of liquidity between countries can be studied by analyzing: (a) the ratio $\frac{M_A^t}{M^t}$ and $\frac{M_B^t}{M^t}$, i.e., the per capita money holdings in country $i$ relative to the monetary union’s per capita holdings and (b) the evolution of the money sent by households to the goods market (“cash”).

The following analysis will be rooted in the equation describing the evolution of the household’s assets. After normalizing the variables in equation (10) by $M^t$ and denoting them with lower cases, the asset evolution equation in country $A$ can be re-written as

$$\frac{M_A^{t+1}}{M_t} = \left[ w^A_t N^A_t + \frac{M^A_t}{M_t} - t^A_t - p^A_t C^A_t \right] + R_t t^A_t + R_t X^A_t \left( M^A_t \right) \quad (30)$$

Above the state variable $\theta_t = \frac{n^A_{M_A^t}}{M_t}$ was already defined. Noting that $\frac{M_A^{t+1}}{M_t} = \frac{M_A^{t+1}}{M^{t+1}} = \frac{\theta^{t+1}}{n} (1 + x_t)$, and recalling that the monetary injection is distributed evenly among households (i.e. $\frac{X^A_t}{M^t} = \frac{X^B_t}{M^t} = \frac{X_t}{M^t}$) equation
Figure 3: Equilibria in the goods markets in the period of the shock

(30) can be rewritten as

\[
\frac{\theta_{t+1}}{n} (1 + x_t) = w_t^A N_t^A (1 - R_t) + \frac{\theta_t}{n} - (1 - R_t) l_t^A - p_t^A C_t^A + R_t x_t + p_t^A s_t^A (N_t^A)^{1-\alpha}
\]

How is liquidity channelled in the monetary union in the period of the shock when countries are of the same size (see Figure (4))? The monetary injection is first absorbed by the financial intermediaries, adding to the supply of funds in the financial market. In this market, the flexible households contemporaneously adjust their decisions to the new liquidity conditions, while the sticky households do not. Thus, the flexible households reduce their supply of funds to the financial market, and channel a larger amount of funds to the goods markets. This occurs because the flexible households perceive that prices will rise with the monetary injection and accordingly send more funds to the goods markets. Meanwhile, since the sticky households made their decisions before observing the monetary shock, they continue to behave as if the shock had not occurred. Thus, there is an increase in liquidity for transactions in country B relative to country A.
Figure 4: Response of the economy to a monetary injection in period 6. Countries are of the same size. Country A’s households have sticky portfolios and Country B’s households have flexible portfolios. The Figure shows the percent deviations from steady state.
A. In parallel, both countries’ firms also absorb the extra liquidity injected by the monetary authority and increase their wage bill. This latter effect occurs since the interest rate drops on impact.

At the end of the period, all money returns to the households. From equation (30) it is straightforward to see that the evolution of $\theta_1$ will depend on the profits of each country’s firms (which are equal due to the symmetry of technology and preferences in both countries) and on the returns from the financial intermediaries. Since country A’s households send relatively more money to the financial markets in the period of the shock, the return of these loans with interest at the end of the period implies that $\frac{M_{A,t+1}}{M_{A,t}} > \frac{M_{B,t+1}}{M_{B,t}}$, where $t$ is the period of the shock.

In the periods after the shock, the intertemporal condition holds equally for both country’s households. Thus the path of $L^A$ is parallel to the path of $L^B$ (they are not equal since they diverge in the period of the shock). This allows households A to finance a higher level of consumption (relative to $B$), through the higher returns on the loans made to the financial intermediaries. Since the households wish to smooth their consumption over time, this effect is permanent. Thus, country A’s households will send a permanently higher level of loans to the financial market and a permanently higher level of money to the goods markets relative to country B’s households.

4.1 The trade balance and the consumption decisions over time

From the cash-in-advance conditions, it is straightforward to see that the contrasting evolution in $C^A_t$ and $C^B_t$ is directly related to differences in the behavior of $L^A_t$ and $L^B_t$. By assumption $L^A_t$ does not react to the monetary injection, while $L^B_t$ does. In an all-flexible world, the households would react to a money injection by decreasing their loans to the financial intermediaries (thus replicating the equilibrium without the central bank’s surprise injection) and increasing the money sent to the goods market. This is the qualitative response of country B’s households. Thus, in the period of the shock the consumption in country B increases more than the consumption

---

6 The cash-in-advance conditions are $P^A_t C^A_t = W^A_t N^A_t + (M^A_t - L^A_t)$ and $P^B_t C^B_t = W^B_t N^B_t + (M^B_t - L^B_t)$. By construction, $P^A_t = P^B_t$. The equations determining $P^A_t$ and $P^B_t$ imply that there is perfect nominal output insurance in the union, since $P^A_t s^A_t (N^A_t)^{1-\alpha} = P^B_t s^B_t (N^B_t)^{1-\alpha}$ always holds. From the price-setting equations of firms from both countries, it is straightforward to conclude that $W^A_t N^A_t = W^B_t N^B_t$. These conditions also imply that firms earn equal profits in both countries. Thus the path of $C^A_t$ and $C^B_t$ must be related to $L^A_t$ and $L^B_t$.  

18
in country A (for \( n = 0.5 \), the consumption in country B increases 0.29%, while consumption in country A actually decreases 0.05%).

How is the partition of \( C^A_t \) in \( C^A_{A,t} \) and \( C^A_{B,t} \) and of \( C^B_t \) in \( C^B_{A,t} \) and \( C^B_{B,t} \) ? Here, two forces are at work: first, the evolution of the overall consumption basket in each country; second, the evolution of the relative price \( p_t^i \). With common shocks and GHH preferences, the relative price between goods is always equal to 1 and the prices of each type of good are equal to the overall price level in both countries. Thus the consumption by each country’s households of each type of good is directly related to the aggregate consumption in each country. In the period of the shock, and for \( n = 0.5 \), this implies that \( C^A_{A,t} \) and \( C^A_{B,t} \) will decrease and that \( C^B_{A,t} \) and \( C^B_{B,t} \) will increase. However, it was already shown that the production (and thus consumption in equilibrium) of both types of goods increases in the period of the shock. It follows necessarily that

\[
\frac{\Delta^{-} C^A_{A,t}}{(1-n) \Delta^{+} C^B_{B,t}} < \frac{\Delta^{-} C^A_{B,t}}{(1-n) \Delta^{+} C^B_{B,t}}.
\]

The trade balance of the sticky country (A), which equals \((1-n)p^B_{A,t}C^B_{A,t} - n p^A_{B,t} C^A_{B,t}\), thus improves in the period of the shock. This reflects the increased demand of (domestic and foreign) goods by country B’s households and the decrease in demand of (domestic and foreign) goods by country A’s households. This result contrasts with the conclusions of standard open-economy models using Cobb-Douglas consumption indexes and zero initial non-monetary wealth (see Obstfeld and Rogoff (1998) and Benigno (2001)). In these models, the trade balance is always zero and consumption in both countries is always equal, reflecting total consumption insurance between countries.

The different result found in this model is rooted in the different timing assumptions of the intertemporal conditions. Without segmentation in the financial markets, the intertemporal conditions in both countries would imply, taking into account that the price index is the same in both countries, that

\[
E_t \left[ \frac{U^A_{C_A,t+1}}{U^C_{C_A,t}} - \frac{U^B_{C_B,t+1}}{U^C_{C_B,t}} \right] = 0.
\]

A positive level of assets at time \( t \) held by country A’s households would imply \( C^A_t < C^B_t \) and \( C^A_{t+1} > C^B_{t+1} \). But this would correspond to \( \frac{U^A_{C_A,t+1}}{U^C_{C_A,t}} < \frac{U^B_{C_B,t+1}}{U^C_{C_B,t}} \), violating the above condition. Without limited participation of some of the households, consumption must always be the same between countries.

In a model with portfolio rigidities, \( \frac{U^A_{C_A,t+1}}{U^C_{C_A,t}} - \frac{U^B_{C_B,t+1}}{U^C_{C_B,t}} \) can be different from zero, since one of the intertemporal conditions only holds in expected
value. Consequently, consumption between agents can differ in the period of the shock and the trade balance can be different from zero.

To understand the evolution of the trade balance after the shock, it is important to note that in the period of the shock the trade balance surplus in country A is associated with an increase in loans to financial intermediaries greater than the observed in country B. As shown in the last subsection, country A’s households are thus able to finance a higher stream of consumption in the following periods. Since they wish to smooth consumption over time, their consumption/savings decisions imply that, relative to country B, they permanently lend more to the financial intermediaries and permanently consume more (by also sending more money to the goods markets). The higher stream of loans to the financial intermediaries allows country A’s households to finance a permanent trade deficit arising from the higher level of consumption. This pattern can be observed in Figure (5).

5 Impact of technology shocks in the monetary union

In the previous section the effects of a monetary policy shock in the context of a monetary union with heterogeneous portfolio rigidities were analysed. It was concluded that monetary policy shocks had an impact not only on the allocations of the union’s agents as a whole, but also the relative allocations between agents. This heterogeneity allocation was rooted in the distribution of liquidity in the monetary union.

However, this analysis of monetary policy shocks is not an appropriate description of how central banks operate around the world. Central banks do not usually randomize the money supply, surprising the agents with liquidity injections/absorptions. On the contrary, monetary policy usually focuses its attention on fundamental shocks hitting the economy. Consequently this section analyses the impact common technology shocks in the monetary union. Two simple monetary policy rules will be compared: an interest rate peg and a money growth peg.

7 In this set up, there is international lending and borrowing, so the current account in both countries also changes after the monetary shock. However, since there is perfect integration in the financial markets, the nationality of the financial intermediaries is not defined. Thus, the evolution of the current account is indeterminate. The trade balance, in contrast, is pinned down.

8 The preferences of the consumer are assumed to be GHH. The case of preferences with wealth effects in labor supply is available upon request.
Figure 5: Response of the economy to a monetary injection in period 6. Countries are of the same size. Country A’s households have sticky portfolios and Country B’s households have flexible portfolios. The Figure shows the percent deviations from steady state.
As a first step in the analysis, it is important to assess the determinacy of equilibrium under each rule. As shown in Appendix A the nominal and real allocations are determinate under a money growth rule. However, the allocations differ between both countries after a technology shock. In contrast, an interest rate peg yields nominal indeterminacy. There is however real determinacy in the model and the allocations are the same between both countries.

5.1 Money-growth rule

Here the focus is on the actual response of the economies to a technology shock. Three benchmark cases are analysed: the case where all households are flexible; the case where only country A’s households are sticky and both countries are of the same size; and the case where all households are sticky. The calibration is the same used in the analysis of the money shock.

The technology shock is assumed to have the following distribution: $s_t = \exp(\tau_t)$, where $\tau_t = 0.5\tau_{t-1} + \epsilon_{\tau,t}$ and $\epsilon_{\tau,t}$ has mean zero and standard deviation 0.01.

A positive technology shock has important effects in all markets. To analyze the various cases, it is important to first depict the dynamics of the credit market in the monetary union (Figure (6)). Recall that the loan supply is given by $\text{Loan}^S = nL^A_t + (1-n)L^B_t + X_t$ and that the loan demand is the wage bill of the firms, equal to $nW^A_tN^A_t + (1-n)W^B_tN^B_t$. By equation (39) the loan demand can also be expressed as $\text{Loan}^D = \frac{(1+z_t)(1-\alpha)}{\mu R_t}$.
With a money growth rule, the change in the supply of loans is a function of the behavior of the private sector. In the case when all households have sticky portfolios, the supply curve is vertical in the period of the shock. In this case the share of funds with the financial intermediaries is fixed and $R_t$ does not change (see equation (39)). The allocation is thus similar to the case of an interest rate rule.

When some (or all) of the households are flexible, the supply curve slopes upwards. In this case, after a technology shock, the flexible agents increase their savings for the next period (i.e., their loans to the financial intermediaries), and this shifts the supply curve rightwards. This implies, for $n < 1$, a decrease in interest rates in the period of the shock, a decrease in total money sent to the goods markets and an increase in loans (all relative to the steady state). A higher fraction of flexible agents in the union corresponds to a flatter supply curve in the credit market. This implies that the higher the number of flexible households, the greater the decrease in the interest rate.

In the labor market, and for $n < 1$, a positive technology shock increases the demand for labor by firms (since the interest rate decreases) indirectly raising the demand for loans (associated with a potentially higher wage bill). Thus, equilibrium labor and real wages increase in the period of the shock. In the goods markets, the demand curve is shifted to the left and the supply curve is shifted to the right (the substitution effect dominates the non-existent wealth effect). This implies an unambiguous decrease in prices and an increase in total output.

When the degree of rigidity varies between countries, their economies behave differently (Figure (7)). Flexible agents adjust their savings decisions in the period of the shock, increasing their loans to the financial intermediaries and decreasing the money sent to the goods market, which is justified since prices fall and households wish to smooth consumption. Sticky agents continue to behave as if the shock had not occurred. This implies that in the period of the shock consumption of the sticky households $A$ is higher relative to country $B$’s households. These events trigger a dynamic process analogous to the one described for a monetary policy shock. This process includes the following elements: a permanent lower share of money held by country $A$’s households; a permanent lower level of loans sent by country $A$’s households to the financial intermediaries; a permanent lower level of money sent by country $A$’s households to the goods markets; and, a trade deficit in country $A$ in the period of the shock followed by a permanent trade surplus thereafter. The heterogeneity in the degree of stickiness thus translates in heterogeneous allocations when a money growth rule is followed.
Figure 7: Response of the economy to a common technology shock in period 6. The monetary authority follows a constant money growth path. Countries are of the same size. Country A’s households have sticky portfolios and Country B’s households have flexible portfolios. The Figure shows the percent deviations from steady state, except for the interest and inflation rates, which are presented as annualised rates.
Two elements with an important impact on the quantitative results are the assumptions of a mark-up equal to 1.3 and the GHH preferences of the households. The smaller the mark-up, the smaller the impact of a monetary injection in the interest rate (i.e., the flatter the demand curve in the loan market). The assumption of GHH preferences impacts on the equilibrium through its effect on the labor market dynamics. With preferences that allow for wealth effects in labor supply, the decrease in interest rates following a monetary injection would imply a shift to the left of the supply curve in the labor market, mitigating the expansionary effects of the fall in interest rates.

5.2 Interest rate peg

With an interest rate peg, the monetary authority stands ready to inject/absorb the amount of reserves needed to support the current interest rate, implying an horizontal supply curve at the targeted interest rate in the financial market. This also implies that the share of the money stock with the financial intermediaries is unchanged. This is straightforward from equation (39)

\[
\frac{(1 - \alpha)}{\mu R_t} = \frac{n l^A_t + (1 - n) l^B_t + x_t}{1 + x_t}
\]  

(32)

By definition, country A’s households do not revise the loans sent to the financial intermediaries since their portfolios are sticky in the period of the shock. In country B, even though portfolios are flexible, agents do not have any incentive to change \( l^B_t \), since they know that ultimately the central bank will always counter their decisions to ensure the interest rate peg. Appendix A shows that the allocation under an interest rate rule is the same in both countries. This implies that country B’s households behave similarly to country A’s households and choose \( l^B_t = l^A_t \). Since in the period of the shock the loans from households do not change from their steady state path, neither do the monetary authority’s injections. This occurs irrespective of the size of each country. Thus the supply of loans is fixed and the wage bill is also unchanged in the period of the shock.

When all agents are flexible, the nominal variables are indeterminate with an interest rate rule. However, when some of the agents have sticky portfolios, these nominal variables are pinned down in the period of the shock, and their path is equal in both countries. In the periods following the shock, there is nominal indeterminacy.

In all cases, there is no heterogeneity in the real allocation between countries. This allocation is identical to the case where portfolios are flexible.
Figure 8: Summary of responses in the impact period to a common technology shock

for all the union’s households.

Figure (8) compares the response of each country to a common technology shock, when countries vary in size and have heterogeneous portfolio rigidities. Two main conclusions may be highlighted: first, the allocation in both countries is the same under an interest rate rule and differs under a money growth rule; second, the allocation is more volatile under a money growth rule.

6 Concluding remarks

With the introduction of the euro the study of the monetary transmission mechanism in a monetary union has been a growing subject of interest. This paper analyzed the transmission of monetary shocks in a monetary union with heterogeneous portfolio rigidities and the response of the union’s economies to technology shocks under some simple policy rules.

The monetary union was modelled as a standard two-country model in
which countries were characterized by distinct degrees of portfolio rigidity. As an extreme, it was assumed that in each country households faced either complete portfolio rigidity (with the consumption/savings decision taken before the shocks hit the economy) or complete portfolio flexibility. It was also assumed that labor markets were separated, whereas the financial markets were completely integrated.

A monetary policy shock in this monetary union yields standard results in terms of the union’s aggregates but implies heterogeneous responses of each country’s macroeconomic variables. Both the union and each country’s allocations are closely related to the distribution of liquidity in the union.

The degree and heterogeneity of portfolio rigidities in a monetary union is thus an important friction to understand the impact of monetary shocks, not only on the overall aggregates, but also on distributive outcomes between agents of the economy.

The distribution of liquidity between countries is also crucial to understand the response of the union to common technology shocks when the monetary authority follows one of two simple policy rules: an interest rate peg or a money growth peg. The main conclusions of this analysis were the following. First, an interest rate peg undoes the portfolio rigidities in the union, replicating the flexible-portfolio allocation. The allocation in each country is the same. In the period of the shock, the nominal aggregates are also pinned down. These results hold irrespective of the functional form of the preferences of the households.

Second, a constant money growth rule yields nominal determinacy, but implies different allocations in each country, since the saving/consumption decision differs with the degree of portfolio rigidity. Further, the union’s flexible-portfolio allocation is not replicated. In terms of the quantitative response to technology shocks, the distribution of liquidity within the union is again crucial in determining the real and nominal variables of the union and of each country.

Since all shocks in the monetary union yield permanent effects in the distribution of liquidity, it is straightforward to conclude that this distribution follows a random walk. It would be interesting to assess the empirical validity of this result. Another interesting line of investigation would be the study of current operating procedures by the ECB, in particular in what concerns the distribution of liquidity throughout the euro area after the refinancing operations conducted by the ECB. Finally, an extension of this work lies in the analysis of the optimal monetary policy in the union. In particular it would be interesting to assess whether the existence of differences in the underlying frictions affects the optimal policy prescription.
References


A Appendix A: Analysis of determinacy of equilibrium under a money growth rule and an interest rate peg

To analyse the determinacy of equilibrium it is useful to start by recalling the equilibrium conditions of the economy. These are the following:

- the two intratemporal conditions,

\[
\frac{U_{1-N,t}^{A}}{U_{C,t}^{A}} = \frac{w_t^{A}}{p_t^{A}} \tag{33}
\]

\[
\frac{U_{1-N,t}^{B}}{U_{C,t}^{B}} = \frac{w_t^{B}}{p_t^{B}} \tag{34}
\]

- the two first-order conditions of the firms

\[
\frac{w_t^{A}}{p_t^{A}} = \frac{p_{A,t}}{p_t^{A}} \frac{1}{\mu R_t} s_t^{A} (1 - \alpha) \left( N_t^{A} \right)^{-\alpha} \tag{35}
\]

\[
\frac{w_t^{B}}{p_t^{B}} = \frac{p_{B,t}}{p_t^{B}} \frac{1}{\mu R_t} s_t^{B} (1 - \alpha) \left( N_t^{B} \right)^{-\alpha} \tag{36}
\]

- the two goods market equilibrium conditions

\[
nC_{A,t}^{A} + (1 - n)C_{A,t}^{B} = ns_t^{A} \left( N_t^{A} \right)^{1-\alpha} \tag{37}
\]

\[
nC_{B,t}^{A} + (1 - n)C_{B,t}^{B} = (1 - n)s_t^{B} \left( N_t^{B} \right)^{1-\alpha} \tag{38}
\]
• the loan market condition

\[ nw^A_t N^A_t + (1 - n)w^B_t N^B_t = n l^A_t + (1 - n)l^B_t + x_t \]

which can be combined with the aggregate cash-in-advance as (26),

\[ \frac{(1 - \alpha)}{\mu R_t} = \frac{n l^A_t + (1 - n)l^B_t + x_t}{1 + x_t} \]  \( (39) \)

• the two cash-in-advance conditions

\[ C^A_t = \frac{w^A_t N^A_t + \theta_t - l^A_t}{p^A_t} \]  \( (40) \)

\[ C^B_t = \frac{w^B_t N^B_t + \frac{1 - \theta_t}{1 - n} - l^B_t}{p^B_t} \]  \( (41) \)

• and the two intertemporal conditions

\[ E_{t-1} \left[ \frac{U_{CA}^A}{p^A_t} - \beta R_t \frac{U_{CA}^{A+1}}{p^A_{t+1}(1 + x_t)} \right] = 0 \]  \( (42) \)

\[ E_t \left[ \frac{U_{CB}^B}{p^B_t} - \beta R_t \frac{U_{CB}^{B+1}}{p^B_{t+1}(1 + x_t)} \right] = 0 \]  \( (43) \)

It is also useful to compute the relative prices between both countries \( \frac{p^A_t}{p^B_t} \). By (24) and (25) we know that

\[ p^A_t = \frac{1 + x_t}{s^A_t (N^A_t)^{1 - \alpha}} \]  \( (44) \)

\[ p^B_t = \frac{1 + x_t}{s^B_t (N^B_t)^{1 - \alpha}} \]  \( (45) \)

\[ p^A_t = p^B_t = \frac{1 + x_t}{(s^A_t)^n (s^B_t)^{1 - n} [(N^A_t)^n (N^B_t)^{1 - n}]^{1 - \alpha}} \]  \( (46) \)

Thus

\[ \frac{p^A_t}{p^B_t} = \frac{s^B_t (N^B_t)^{1 - \alpha}}{s^A_t (N^A_t)^{1 - \alpha}} \]  \( (47) \)

31
This equation implies that there is complete nominal output insurance in the union.

The computation of \( C^A_{t} \) and \( C^B_{t} \) is implemented through the goods market conditions. Combining these with the formulas \( C^A_{A,t} = n_p^A C^A_t \), \( C^A_{B,t} = (1 - n_p^B) C^A_t \), \( C^B_{A,t} = n_p^B C^B_t \), \( C^B_{B,t} = (1 - n_p^B) C^B_t \) yields

\[
\begin{align*}
\left\{ \begin{array}{l}
    n \left( \frac{p_{A,t}}{p_{A,t}} \right)^{1-n} [n C^A_t + (1-n) C^B_t] = n s^A \left( N^A_t \right)^{1-\alpha} \\
    (1 - n) \left( \frac{p_{B,t}}{p_{B,t}} \right) n [n C^A_t + (1-n) C^B_t] = (1 - n) s^B \left( N^B_t \right)^{1-\alpha}
\end{array} \right.
\end{align*}
\]

(48)

With GHH preferences the intratemporal condition for the representative agent in each country \( j = A, B \) is

\[
\frac{U_{1-N^j,t}}{U_{C^j,t}} = \left( N^j_t \right)^\chi
\]

(49)

which does not depend on \( C^j_t \) (there are no wealth effects in labor supply).

Combining equation (47) with the first-order conditions of the firms yields

\[
w^A_t N^A_t = w^B_t N^B_t
\]

(50)

Knowing this, and using the intratemporal first-order conditions yields

\[
p^A_t \left( N^A_t \right)^{1+\chi} = p^B_t \left( N^B_t \right)^{1+\chi}
\]

(51)

Since, by construction \( p^A_t = p^B_t \) (the aggregate price level is the same in both countries, due to the Cobb-Douglas specification of the consumption indexes and the absence of home-bias), labor is the same in both countries:

\[
N^A_t = N^B_t
\]

(52)

A.1 Money growth rule

With a money growth rule, the share of money with the financial intermediary \( m^j_t + (1-n) y^j_t \) is a function of \( l^A_t \) and \( l^B_t \) only. Note that the loans market equilibrium and the aggregate cash-in-advance \( R_t = \frac{m^j_t + (1-n) y^j_t}{1 + \alpha} \) imply that

\[
R_t = f R (l^A_t, l^B_t, x_t)
\]

Equations (47)-(48) and (33)-...-(36) imply
\[
\frac{PA_t}{PB_t} = f^{PA} \left( s^A_t, s^B_t, N^A_t \right) = f^{PB} \left( s^A_t, s^B_t \right) \\
C^A_t = f^{CA} \left( s^A_t, PA_t, N^A_t, C^A_t \right) = f^{CA} \left( s^A_t, s^B_t, N^A_t, C^B_t \right) \tag{53}
\]
\[
C^B_t = f^{CB} \left( s^A_t, PA_t, N^B_t, C^A_t \right) = f^{CB} \left( s^A_t, s^B_t, N^B_t, C^A_t \right) \tag{54}
\]
\[
N^A_t = f^{NA} \left( \frac{w^A_t}{p_t} \right) \tag{55}
\]
\[
N^B_t = f^{NB} \left( \frac{w^B_t}{p_t} \right) \tag{56}
\]
\[
w_t^A = f^{\lambda A} \left( s^A_t, PA_t, N^A_t, l^A_t, l^B_t, x_t \right) = f^{\lambda A} \left( s^A_t, s^B_t, N^A_t, l^A_t, l^B_t, x_t \right) \tag{57}
\]
\[
w_t^B = f^{\lambda B} \left( s^B_t, PB_t, N^B_t, l^A_t, l^B_t, x_t \right) = f^{\lambda B} \left( s^A_t, s^B_t, N^B_t, l^A_t, l^B_t, x_t \right) \tag{58}
\]

and consequently the real allocation depends on the distribution of liquidity in the union (through \(l^A_t\) and \(l^B_t\)).

To determine \(l^A_t\) and \(l^B_t\) we resort to the cash-in-advance conditions (40)-(41) which imply
\[
l^A_t = f^{IA} \left( \frac{w^A_t}{p_t}, N^A_t, C^A_t, \frac{\theta_t}{n}, p_t^A \right) = f^{IA} \left( s^A_t, s^B_t, N^A_t, N^B_t, \frac{w^A_t}{p_t}, C^A_t, \frac{\theta_t}{n}, x_t \right) \tag{59}
\]
\[
l^B_t = f^{IB} \left( \frac{w^B_t}{p_t}, N^B_t, C^B_t, \frac{1 - \theta_t}{1 - n}, p_t^B \right) = f^{IB} \left( s^A_t, s^B_t, N^A_t, N^B_t, \frac{w^B_t}{p_t}, C^B_t, \frac{1 - \theta_t}{1 - n}, x_t \right) \tag{60}
\]

where we used the fact that, as shown in equation (46), \(p_t^A = p_t^B = f^{PA} \left( s^A_t, s^B_t, N^A_t, N^B_t, x_t \right)\).

Note, finally, that \(\theta_t\) is a predetermined variable since it represents the share of money of country A’s households at the beginning of the period.

Let \(\Phi_t\) be the number of states in period \(t\). Under a money growth rule (i.e., for a certain path of \(x_t\)), the \(8\Phi_t\) equations (53) to (60) determine uniquely the \(8\Phi_t\) variables \(C^A_t, C^B_t, N^A_t, N^B_t, \frac{w^A_t}{p_t}, \frac{w^B_t}{p_t}, l^A_t, l^B_t\). Thus there is no real indeterminacy in the allocation. Further, since \(p_t^A = p_t^B = f^{PA} \left( s^A_t, s^B_t, N^A_t, N^B_t, x_t \right)\), there is also no nominal indeterminacy for a certain path of \(x_t\).
However, $l_t^A$ and $l_t^B$ differ in the period of the shock and, consequently, the real allocations in both countries differ. This implies that a money growth rule does not replicate the all-flexible portfolios equilibrium, where the allocations in both countries are the same.

A.2 Interest rate peg

Analogously to the money growth rule case, equations (47)-(48) and (33)-...-(36) imply

$$\frac{PA,t}{PB,t} = \frac{PA,t}{PB,t} \left( s_t^A, s_t^B, \frac{N_t^A}{N_t^B} \right) = \frac{PA,t}{PB,t} \left( s_t^A, s_t^B \right)$$

$$C_t^A = f^{CA} \left( s_t^A, \frac{PA,t}{PB,t}, N_t^A, C_t^B \right) = f^{CA} \left( s_t^A, s_t^B, N_t^A, C_t^B \right) \quad (61)$$

$$C_t^B = f^{CB} \left( s_t^A, \frac{PA,t}{PB,t}, N_t^B, C_t^A \right) = f^{CB} \left( s_t^A, s_t^B, N_t^B, C_t^A \right) \quad (62)$$

$$N_t^A = f^{NA} \left( \frac{w_t^A}{p_t^A} \right) \quad (63)$$

$$N_t^B = f^{NB} \left( \frac{w_t^B}{p_t^B} \right) \quad (64)$$

$$\frac{w_t^A}{p_t^A} = f^{wA} \left( s_t^A, \frac{PA,t}{PB,t}, N_t^A, R_t \right) = f^{wA} \left( s_t^A, s_t^B, N_t^A, R_t \right) \quad (65)$$

$$\frac{w_t^B}{p_t^B} = f^{wB} \left( s_t^B, \frac{PA,t}{PB,t}, N_t^B, R_t \right) = f^{wB} \left( s_t^A, s_t^B, N_t^B, R_t \right) \quad (66)$$

With an interest rate peg, the $6\Phi_t$ equations (61) to (66) uniquely determine the real allocation $C_t^A$, $C_t^B$, $N_t^A$, $N_t^B$, $w_t^A$, $w_t^B$. There is a mapping between the interest rate and the real allocation.

Above it was shown that $w_t^A N_t^A = w_t^B N_t^B$ and $p_t^A = p_t^B$ under both rules. Further, by equation (52) we already know that $N_t^A = N_t^B$ (under both rules). We now formally show that $C_t^A = C_t^B$.

From the cash-in-advance conditions in both countries we have

$$\begin{align*}
C_t^A &= \frac{w_t^A N_t^A + \frac{s_t^A}{p_t^A} - l_t^A}{p_t^A} \\
C_t^B &= \frac{w_t^B N_t^B + \frac{1-s_t^B}{p_t^B} - l_t^B}{p_t^B}
\end{align*} \quad (67)$$
We first focus on country A’s households (we will drop the upperscript in the labor and wage variables since they are equal in both countries). Using the loans market equilibrium and the fact that \( w^A_t N^A_t = w^B_t N^B_t \), (32) can be rewritten as

\[
\frac{nl^A_t + (1 - n)l^B_t + xt}{1 + xt} = \frac{(1 - \alpha)}{\mu R_t}
\]

\[
w^A_t N_t + (1 - n)w^B_t N_t = \frac{(1 + xt)(1 - \alpha)}{\mu R_t}
\]

\[
w^A_t N_t = \frac{(1 + xt)(1 - \alpha)}{\mu R_t} \quad (68)
\]

Using (46) and (68), and assuming a common technology shock \( s_t \), the cash-in-advance in country A can be rewritten as.

\[
C^A_t = \frac{w^A_t N_t + \frac{\theta_t}{n} - l^A_t}{\theta_t + 1}
\]

\[
C^A_t (1 + xt) = s_t N^A_t^{1-\alpha} \left[ w^A_t N_t + \frac{\theta_t}{n} - l^A_t \right] \quad (69)
\]

Since by (35)

\[
1 + xt = w^A_t N_t \frac{\mu R_t}{1 - \alpha}
\]

the above can be rewritten as

\[
C^A_t w^A_t N_t \frac{\mu R_t}{1 - \alpha} = s_t N^A_t^{1-\alpha} \left[ w^A_t N_t + \frac{\theta_t}{n} - l^A_t \right]
\]

\[
C^A_t = \frac{1 - \alpha}{\mu R_t} \frac{1}{w^A_t N_t} s_t N^A_t^{1-\alpha} \left[ w^A_t N_t + \frac{\theta_t}{n} - l^A_t \right] \quad (70)
\]

This expression can be further simplified by noting that the first order conditions of the firms and the intratemporal conditions imply (since \( p_{A,t} = p_{B,t} \) by (47)) :

\[
(N_t)^{\alpha+\chi} = s_t \frac{1 - \alpha}{\mu R_t} \quad (71)
\]

Substituting (71) in (70) yields

\[
C^A_t = \frac{1 - \alpha}{\mu R_t} \frac{1}{w^A_t N_t} s_t \left( s_t \frac{1 - \alpha}{\mu R_t} \right)^{\frac{1-\alpha}{\alpha+\chi}} \left[ w^A_t N_t + \frac{\theta_t}{n} - l^A_t \right]
\]

\[
C^A_t = \left( s_t \right)^{\frac{1+\chi}{\alpha+\chi}} \left( \frac{1 - \alpha}{\mu R_t} \right)^{\frac{1+\chi}{\alpha+\chi}} \left[ 1 + \frac{\theta_t}{n} - l^A_t \right] \quad (72)
\]
The analogous equation for country $B$ is

$$C_t^B = (s_t)^{\frac{\frac{1+\chi}{\alpha+\chi}}{\frac{\frac{1+\chi}{\alpha+\chi}}}} \left[ 1 + \frac{1}{w_t N_t} \right]$$ (73)

By the equilibrium conditions in the goods markets (see (48)), we know that

$$nC^A_t + (1 - n)C^B_t = s_t (N_t)^{1-\alpha}$$ (74)

We also know by (71) that the response of labor is the same in both countries and independent of whether the respective households have flexible or sticky portfolios. Thus the right hand side is independent of the fraction of sticky households $n$. Equations (72) and (73) show that $C^A_t$ and $C^B_t$ may only differ through different $l^A_t$ and $l^B_t$. But since $l^A_t$ does not react to the shock, this implies, by (74), that the response of $C^A_t$ and $C^B_t$ is the same. It also implies that the flexible agents do not have any incentive to change their loans to the financial intermediaries. The real allocation is thus the same in both countries and independent of $n$. This allocation is the all-flexible portfolios allocation.

The monetary injections that support the interest rate peg are

$$1 + x_t = \frac{\mu R_t (1 - nl^A_t - (1 - n)l^B_t)}{\mu R_t - (1 - \alpha)}$$

Since the flexible households behave as if they were sticky, $l^A_t$ and $l^B_t$ do not move in the period of the shock. With an interest rate peg $x_t$ is independent of the technology shocks in the period of the shock.

Concerning the nominal determinacy of the interest rate peg, the two cash-in-advance conditions, the two intertemporal conditions and the loan market equilibrium form $3\Phi_t + 2$ equations to solve for $5\Phi_t$ variables - $p^A_t$, $p^B_t$, $l^A_t$, $l^B_t$ and $x_t$. Thus there is nominal indeterminacy with an interest rate rule. A lower $l^B_t$ corresponds to a higher $x_t$ and a higher $p^A_t$, all supporting the same real allocation. Note, however, that this nominal indeterminacy does not arise in the period of the shock.
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