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Please address correspondence to Isabel Correia, Economic Research Department, Banco de Portugal, Av. Almirante Reis nº 71, 1150-165 Lisboa, Portugal, Tel.#351-213128385; Fax#351-213107805; e-mail:mihcarvalho@bportugal.pt; available in www.bportugal.pt.
Optimal Fiscal and Monetary Policy: Equivalence Results∗

Isabel Correia
Banco de Portugal, Universidade Catolica Portuguesa and CEPR

Juan Pablo Nicolini
Universidad Di Tella

Pedro Teles†
Federal Reserve Bank of Chicago, CEPR

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Abstract

In this paper, we analyze the implications of price setting restrictions for the conduct of cyclical fiscal and monetary policy. We consider an environment with monopolistic competitive firms, a shopping time technology, prices set one period in advance, and government expenditures that must be financed with distortionary taxes. We show that the sets of (frontier) implementable allocations are the same independently of the degree of price stickiness. Furthermore, the sets of policies that decentralize each allocation are also the same except in the extreme cases of flexible and sticky prices, where the sets are larger.

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but still include that common set of policies. In this sense we establish an irrelevance or equivalence of environments. We also describe the minimal set of instruments, in the different environments, and thus discuss equivalence and neutrality of fiscal and monetary instruments. In particular we show that state contingent debt is not necessary, provided there are both consumption and labor income taxes.

Key words: Optimal fiscal and monetary policy; sticky prices; state-contingent debt.

JEL classification: E31; E40; E52; E58; E62; E63.

1 Introduction

In this paper, we analyze the implications of price setting restrictions for the conduct of cyclical fiscal and monetary policy. We consider as the benchmark a stochastic dynamic general equilibrium economy with a transactions technology and monopolistic competitive firms that set prices contemporaneously. This economy is compared to the same economy with prices set in advance and to an economy with both flexible and sticky firms. We characterize the sets of fiscal and monetary policies and equilibrium allocations that finance exogenous government expenditures, i.e., the sets of implementable allocations and policies.

We assume that the government can choose state-contingent taxes on consumption and labor income and can issue state-contingent debt. The government can also raise taxes on profits and wealth, and decide on monetary policy in reaction to the contemporaneous shocks. We show that the sets of efficient (frontier) implementable allocations are the same independently of the degree of price rigidity. Furthermore the sets of policies that decentralize each efficient allocation are also the same except in the extreme cases of fully flexible or sticky prices, where the sets are larger but still contain that common set of policies. In this sense, we state, as the major result of the paper, an equivalence, or irrelevance, of environments.

In the economies with both sticky and flexible firms it is feasible to conduct policy so that the resulting allocations are the same as under flexible prices. Fiscal policy would coincide with the policy under flexible prices and monetary policy would undo the price rigidity, so that the price level would not vary with the contemporaneous shocks. The planner can, therefore, un-
der sticky prices, achieve the same level of welfare as under flexible prices.\footnote{This is the case in Adao, Correia and Teles (2000) that analyze the same environment but consider only monetary policy.} The interesting question, however, is whether the price rigidity can be exploited to achieve allocations other than the ones under flexible prices, that may attain a higher level of welfare. It turns out, as we show in this paper, that the policies and allocations with relative price distortions arising from the price stickiness are dominated in welfare terms, for any preferences of the government dependent on aggregate consumption and leisure. The frontier of allocations coincides with the allocations that can be implemented under flexible prices. This result is in line with the one obtained by Diamond and Mirrlees (1971) on optimal taxation of intermediate goods. According to Diamond and Mirrlees if taxes on final consumption goods are available, intermediate goods should not be taxed.

The policies, common across environments, that decentralize each efficient allocation are characterized by the following principles: Monetary and fiscal policy are conducted so that there are no surprises in prices, as under sticky prices, and no surprises in markups, as under flexible prices. Because fiscal policy must be conducted as if prices were flexible, the optimal taxation results under flexible prices of e.g. Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991) or Zhu (1992) hold also when there are sticky prices. Because monetary policy must replicate the equilibria under flexible prices, it is efficient to eliminate gaps, defined as the deviations from the allocation under flexible prices for a given tax policy, also in a second best environment.

Not all the policy instruments described above are necessary to obtain the result of equivalence of environments. We determine the minimal sets of instruments in the different environments. We discuss equivalence and neutrality of fiscal and monetary instruments. In particular, we show that, if the government cannot issue state-contingent debt, it is still possible to implement the common set of allocations with high volatility of consumption and labor income taxes. Consumption taxes play the role that the price level can costlessly play under flexible prices of simulating state-contingent real debt (see Chari, Christiano and Kehoe, 1991).

We also analyze the implications of restricting taxes to depend on the same information set as the sticky prices. This restriction is not binding when the optimal (Ramsey) policy is to set proportionate distortions, or
wedges, that do not depend on the contemporaneous shocks. In line with previous literature, as in Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991), or Zhu (1992), we analyze the conditions for uniform taxation. When the restriction is binding, the optimal policies and allocations will depend on the degree of price rigidity. The equivalence of environments is lost.

This paper extends two literatures. On one hand, it diverges from the literature on Ramsey fiscal and monetary policies, such as Lucas and Stokey (1983), Chari, Christiano and Kehoe (1996), Correia and Teles (1996, 1999), in that it considers nominal rigidities. Fiscal and monetary policy can, thus, be interpreted as, short-run, stabilization policy.

On the other hand, the paper extends the literature on optimal monetary policy under sticky prices by considering the joint decision of fiscal and monetary policy. Most work in that literature either does not consider fiscal variables, such as Ireland (1996), Carlstrom and Fuerst (1998b), Carlstrom and Fuerst (1998a), or else assumes that government expenditures are financed by lump sum taxation as in Goodfriend and King (1997, 2000), King and Wolman (1998), Khan, King and Wolman (2000). Rotemberg and Woodford (1999), Woodford (2001) and Gianonni and Woodford (2002), among others, allow for subsidies, financed by lump sum taxation, that eliminate the distortions and achieve the first best, in the absence of frictions. In Adão, Correia and Teles (2000) government expenditures are financed with lump sum taxes, but it is not possible to subsidize labor to eliminate the markup distortion. In that second best world, because of the non negativity of the nominal interest rate, monetary policy generates optimal deviations from the flexible price allocation.

Siu (2001) and Schmitt-Grohe and Uribe (2001b), even if different in purpose, make our same methodological step of considering both fiscal and monetary policy in a world with nominal rigidities. Those papers are directly related to Chari, Christiano and Kehoe (1991), where it is shown that the Ramsey solution in Lucas and Stokey (1983) can be achieved without state contingent debt. In Chari, Christiano and Kehoe (1991), optimal monetary policy generates movements in the price level in reaction to shocks, therefore affecting the real value of nominal debt, and replicating state contingent real debt. Quantitatively, the necessary volatility of the price level is very high. Siu (2000) and Schmitt-Grohe and Uribe (2001) compute the Ramsey solution in a similar set up but consider in addition that it is costly to change prices. They obtain that the benefit of replicating state contingent debt is
minimal relatively to the costs of changing prices. It turns out that the
presence of this trade-off hinges on the assumption that consumption taxes
are not available, as we show in this paper.

Finally, the paper also builds on Adao, Correia and Teles (1999), where
it is argued that the policies that decentralize the flexible price, or portfo-
lios, allocations are independent of the degree of price, or portfolio, rigidity.
They conclude that if policy aimed at replicating the allocation under full
flexibility, then the strength of the monetary transmission mechanism would
be irrelevant.

The paper proceeds as follows: In Section 2, we describe the model. In
Section 3, we characterize the sets of implementable allocations and policies
and show that the degree of rigidity is irrelevant in determining both alloca-
tions and policies. In Section 4, we determine the minimal sets of instruments
that are necessary to obtain the equivalence results. Section 5 contains the
conclusions

2 The model

The environment is a standard real business cycles model with labor only
to which we add restrictions on transactions and the setting of prices. The
agents are identical households, a continuum of firms indexed by $i \in [0, 1]$, and
a government. Each firm produces a distinct, perishable consumption
good, indexed by $i$. The production uses labor, according to a linear tech-
nology. We impose that transactions must be made according to a shopping
time technology as in Kimbrough (1983) and De Fiore and Teles (1998). A
fraction of firms are restricted to set prices one period in advance. Govern-
ment purchases are exogenous and the tax instruments are consumption taxes
$\tau^c_t \geq -1$, taxes on labor income $\tau^w_t \leq 1$, taxes on profits $\tau^d_t \leq 1$, that may
be state-contingent. We also consider an initial nominal wealth tax $L \leq 1$. Money
supply and nominal interest rates are also state-contingent. State-
contingent nominal debt can be used to smooth proportionate distortions
over time and across states. This is the benchmark for the policy instru-
ments. Further along the paper we analyze whether any of these instruments
is redundant.

The state of the economy is represented by the random variable $\sigma_t \in
\Sigma_t$. There are government purchases shocks, $G_t = G(\sigma_t)$, and productivity
shocks, $s_t = s(\sigma_t)$.
Households  The households have preferences described by

\[ U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t, h_t) \right\} \]  

over leisure \( h_t \) and the composite consumption good

\[ C_t = \left[ \int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1. \]  

They start period \( t \) with wealth \( W_t \), and decide to buy money balances \( M_t \). They also buy \( B^h_t \) units of money in nominal bonds that pay \( R_t B^h_t \) units of money one period later; and \( Z^{h+1}_t \) units of state-contingent nominal securities, that cost \( z^{t+1}_t \) in units of currency today, and each of them pays one unit of money at the beginning of period \( t+1 \) in a particular state. They can also buy \( A(i)_{t+1} \) units of stocks of firm \( i \) that cost \( a(i)_t \) in units of currency.

Real money \( \frac{M_t}{(1+\rho_t)P_t} \) and shopping time \( N^s_t \) must be used to purchase consumption, \( C_t \), according to the transactions technology, as in Kimbrough (1986) or De Fiore and Teles (2002),\(^2\)

\[ N^s_t \geq l(C_t, \frac{M_t}{(1+\rho_t)P_t}) \]  

where \( P_t \) is the aggregate price level,

\[ P_t = \left[ \int P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \]  

and \( P_t(i) \) is the price of each good \( i \) in units of money. The transactions function \( l \) is homogenous of degree \( k \geq 0 \). It can thus be written as \( l(C_t, m_t) = L \left( \frac{m_t}{C_t} \right) C_t^k \), where \( m_t = \frac{M_t}{(1+\rho_t)P_t} \). We restrict \( L' < 0 \), and \( L'' > 0 \). We define the point of full liquidity as \( \frac{m^*}{C^*} \) where \( L' \left( \frac{m^*}{C^*} \right) = 0 \). Furthermore we assume that at that point time used for transactions is zero, \( L \left( \frac{m^*}{C^*} \right) = 0 \). This implies that \( l_0 = 0 \) at that point.

\(^2\)De Fiore and Teles (2002) argue that when there are consumption taxes it is reasonable to assume, contrary to previous literature, that money is unit elastic with respect to the price level gross of consumption taxes.
At the end of the period, the households receive the labor income, $W_tN_t^w$, and the profits from the firms $\int_0^1 A(i)\Pi(i)di$. The two sources of income are taxed, respectively, at the rates $\tau_t^w$ and $\tau_t^d$.

The budget constraints of the households can be written as

$$M_t + B_t^h + E_tZ_{t+1}^h \leq \mathbb{W}_t$$  \hspace{1cm} (5)

$$\mathbb{W}_{t+1} = M_t + R_tB_t^h + Z_{t+1}^h - (1 + \tau_t^c) \int_0^1 P_t(c_t)\xi_t\Pi(c_t)\Pi_d(c_t)\Pi_t + \int_0^1 A(i)\xi_t [a(i)_{t+1} + (1 - \tau_t^d)\Pi(i)_{t+1}]\Pi_t \leq \mathbb{W}_t$$  \hspace{1cm} (6)

Initial nominal wealth, $\mathbb{W}_0 = \mathbb{W}_0 - \int_0^1 a(i)_{0\xi_t}\Pi_t$, is given. We assume that initial nominal wealth is positive $\mathbb{W}_0 \geq 0$ and that it can be taxed at the rate $\mathcal{L} \leq 1$. This will be equivalent to assuming that initial nominal wealth is zero.

Given the total available time, normalized to one, we can write labor, $N_t^w$, as

$$N_t^w = 1 - h_t - l(C_t, \frac{M_t}{(1 + \tau_t^c)\Pi_t})$$  \hspace{1cm} (7)

The first order conditions of the households problem include the following marginal conditions:

$$\frac{c_t(i)}{C_t} = \left( \frac{P_t(i)}{P_t} \right)^{-\theta}$$  \hspace{1cm} (8)

$$\frac{u_C(t) - u_h(t)c_t(t)}{u_h(t)} = \frac{(1 + \tau_t^c)P_t}{(1 - \tau_t^w)W_t}$$  \hspace{1cm} (9)

$$-l_m(t) = \frac{(1 + \tau_t^c)P_t}{(1 - \tau_t^w)W_t} (R_t - 1)$$  \hspace{1cm} (10)

$$\frac{u_h(t)}{(1 - \tau_t^w)W_t} = E_t\left[R_t + \frac{\beta u_h(t + 1)}{(1 - \tau_t^w)W_{t+1}}\right]$$  \hspace{1cm} (11)

$$z_{t+1} = \beta u_h(t + 1) R_{t+1}(1 - \tau_t^w)W_t$$  \hspace{1cm} (12)

$$a(i)_{t+1} = \beta E_t [z_{t+1} [a(i)_{t+1} + (1 - \tau_t^d)\Pi(i)_{t+1}]] = 0$$  \hspace{1cm} (13)

From these conditions we get $E_t z_{t+1} = \frac{1}{R_{t+1}}$.  

7
Firms

In this economy there is a share $0 \leq \alpha \leq 1$ of firms that set prices one period in advance. The remaining firms set prices contemporaneously. Each firm $i$ has the production technology

$$ y_t(i) \leq s_t n_t^w(i) \tag{14} $$

where $y_t(i)$ is the production of good $i$ and $s_t$ is the aggregate technology shock. $y_t(i)$ can be used for private and public consumption $g_t(i)$, so that $y_t(i) = c_t(i) + g_t(i)$.

The problem of the firm is to choose the price in order to maximize profits that can be used for consumption in period $t+1$ taking the demand function

$$ \frac{y_t(i)}{Y_t} = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} \tag{15} $$

as given, where $Y_t = C_t + G_t$, and satisfying the technology constraint (14). The demand is obtained from (8) and from (22).

The firms that can choose prices every period, at each date $t$ maximize the nominal value of profits net of taxes $E_t \left[ z_{t+1}(1 - \tau^d_t)\Pi_t(i) \right] = \frac{1}{R_t} \Pi_t(i)$ where

$$ \Pi_t(i) = P_t(i)y_t(i) - W_t n_t^w(i) $$

Since the consumption tax is restricted to be $\tau^c_t \geq -1$, the maximization of the nominal value of profits also maximizes the value in units of consumption. Since $\tau^d_t \leq 1$, the maximization of $\frac{1}{R_t} \Pi_t(i)$ is equivalent to the maximization of $\Pi_t(i)$. The firms choose a single price according to

$$ P_t(i) = P_t^f = \frac{\theta}{(\theta - 1)} \frac{W_t}{s_t} \tag{16} $$

The price is set equal to a constant mark-up over marginal cost.

We consider now the problem of the firms that set the prices one period in advance. As of time $t$, the firms are constrained in terms of the price at which they can sell, but are not constrained in terms of the quantity. Thus, at time $t$, and given a previously chosen price, they do choose quantities to maximize profits. These firms sell the output on demand in period $t$ as long as the value of profits from so doing is non negative, $E_t \left[ z_{t+1}(1 - \tau^d_t)\Pi_t(i) \right] = \frac{1}{R_t} \Pi_t(i) \geq 0$. If that value was negative the firm would choose to produce zero.\(^3\)

\(^3\)We make assumptions on the markup $\frac{\theta}{(\theta - 1)}$ and the magnitude of shocks to guarantee that profits when the firms sell the output on demand are non negative in every state.
When firm $i$ sets prices one period in advance, it solves the problem of choosing at $t - 1$ the price $P_t(i)$ that maximizes the value of profits

$$E_{t-1} \left[ z_t z_{t+1} (1 - \tau^d_t) (P_t(i)y_t(i) - W_t n^{w_t}_t(i)) \right],$$  \hspace{1cm} (17)

subject to (14) and (15).

The firms choose the price according to the following first order condition

$$E_{t-1} \left[ z_t z_{t+1} (1 - \tau^d_t) y_t(i) \left( 1 - \frac{\theta}{(\theta - 1) s_t P_t(i)} \right) \right] = 0$$  \hspace{1cm} (18)

Using (12), this can be written as

$$E_{t-1} \left[ (1 - \tau^d_t) \frac{u_h(t+1) R_{t+1}}{(1 - \tau^w_{t+1}) W_{t+1}} y_t(i) \left( 1 - \frac{\theta}{(\theta - 1) s_t P_t(i)} \right) \right] = 0$$  \hspace{1cm} (19)

Therefore, the price chosen by the firms that set prices in advance is

$$P_t(i) = P_t^s = \frac{\theta}{(\theta - 1)} E_{t-1} \left[ \frac{W_t}{s_t} \right]$$  \hspace{1cm} (20)

where

$$v_t = \frac{(1 - \tau^d_t) \frac{u_h(t+1) R_{t+1}}{(1 - \tau^w_{t+1}) W_{t+1}} y_t^s}{E_{t-1} \left[ (1 - \tau^d_t) \frac{u_h(t+1) R_{t+1}}{(1 - \tau^w_{t+1}) W_{t+1}} y_t^s \right]}.$$

Firms charge a mark-up over the expected value of a weighted marginal cost, where the weights depend on the taxes, period $t + 1$ marginal utility of leisure, the nominal wages, the nominal interest rates, and period $t$ output of the sticky firms.

**Government**  The government must finance a given path of government purchases $\{G_t\}_{t=0}^{\infty}$, such that

$$G_t = \left[ \int_0^1 g_t(i) \frac{\theta-1}{\theta} \, di \right]^{\frac{1}{\theta}}, \theta > 0$$  \hspace{1cm} (21)

Given the prices on each good, $P_t(i)$, the government minimizes expenditure on government purchases by deciding according to

$$\frac{g_t(i)}{G_t} = \left( \frac{P_t(i)}{P_t} \right)^{-\theta}$$  \hspace{1cm} (22)
A government policy consists of a sequence of government purchases, money supplies, nominal interest rates, taxes and debt supplies indexed by dates and states,
\[ \{G_t, M_t, R_t, \tau^c_t, \tau^w_t, \tau^d_t, B^g_t, Z^g_{t+1}\}_{t=0}^{\infty}, \mathcal{L} \]. The following restrictions apply to the nominal interest rates and tax rates: \( R_t \geq 1, \tau^c_t > -1, \tau^w_t < 1, \tau^d_t \leq 1, \mathcal{L} \leq 1 \). If \( R_t < 1 \), it would be possible to make infinite profits issuing debt and holding money. If \( \tau^c_t \leq -1 \), the consumers would be able to purchase an infinite amount of consumption. If \( \tau^w_t \geq 1 \), labor supply would be zero. If \( \tau^d_t > 1 \), it would be optimal to minimize profits, and if \( \mathcal{L} > 1 \), households would dispose of the initial wealth.

**Market clearing** The market clearing conditions are
\[ c^s_t + g^s_t = y^s_t \]  
(23)
\[ c^f_t + g^f_t = y^f_t \]  
(24)
\[ \alpha y^s_t + (1 - \alpha) y^f_t = s_t (1 - h_t - l(C_t, m_t)) \]
\[ B^h_t = B^g_t \]
\[ Z^h_{t+1} = Z^g_{t+1}, \text{ for all possible states at } t + 1 \]
\[ A(i)_t = 1 \]

### 3 Implementable allocations

In this section we characterize the sets of implementable allocations for the different degrees of price rigidity described by \( \alpha \). We first show that these sets coincide in the two extreme environments, under flexible and sticky prices where \( \alpha = 0, 1 \). Fiscal and monetary policies affect the economy very differently in the two environments. However, even if the nominal rigidity neutralizes the effect of taxes, as we will discuss in the next section, it gives rise to the monetary non-neutrality that can be used to achieve the same set of allocations as under flexible prices.

In the intermediate case where the economy is composed of both firms that set the prices contemporaneously and firms that set prices one period in advance, for \( 0 < \alpha < 1 \), the set of implementable allocations includes
allocations where relative prices are distorted. However, these allocations are not efficient in the sense that they wouldn’t be chosen by a planner that aims at maximizing a function of aggregate consumption and leisure. The efficient set of implementable allocations (frontier) coincides with the set of allocations that can be implemented under flexible or sticky prices. The degree of price rigidity is thus irrelevant for the conduct of policy, in terms of the allocations that can be achieved.

In order to characterize the set of implementable allocations, prices and policies it is useful to manipulate the system equilibrium conditions above and replace some of the equilibrium variables so that the equilibrium conditions can be summarized.

From (16) and (18), and using the law of iterated expectations as well as (11), the price setting conditions for the flexible and sticky firms can be written, respectively, as

$$w^f_t = \frac{\theta - 1}{\theta} s_t, \quad t \geq 0 \tag{25}$$

$$E_{t-1} \left[ u_h(t) \left( \frac{(1-\tau^{d}_{t})}{(1-\tau^{w}_{t})} w^s_t \left( \frac{1}{w^s_t} - \frac{1}{\theta} \frac{1}{s_t} \right) \right) \right] = 0, \quad t \geq 1 \tag{26}$$

where $w^j_t = \frac{w_j}{p^j_t}$, $j = f, s$. Note that condition (26), for the sticky firms, holds from period one on, so that there is no restriction on the value of the real wages in the initial period.

From (4), the real wage, $w_t = \frac{w_t}{p_t}$, is

$$w_t = \left[ \alpha \left( w^s_t \right)^{(\theta-1)} + (1 - \alpha) \left( w^f_t \right)^{(\theta-1)} \right] \frac{1}{\tau^{c}_t} \tag{27}$$

Using the intratemporal condition (9) and the price setting condition (25) we replace $w_t$ and $w^f_t$, respectively, in this equation, to obtain the restriction

$$\frac{(1+\tau^{r}_t)}{(1-\tau^{r}_t)} \left[ \alpha \left( w^s_t \right)^{(\theta-1)} + (1 - \alpha) \left( \frac{\theta - 1}{\theta} s_t \right)^{(\theta-1)} \right] \frac{1}{\tau^{c}_t} \tag{27}$$

The set of implementable allocations must also include the intertemporal budget constraint of the households
\[
E_0 \sum_{t=0}^{\infty} Q_{t+1} \left[ (1 + \tau^c_t) P_tC_t + M_t (\frac{Q_t}{Q_{t+1}} - 1) \right] = E_0 \sum_{t=0}^{\infty} Q_{t+1} \left[ (1 - \tau^w_t) W_t (1 - h_t l(C_t, m_t)) - (1 - \tau^d_t) \Pi_t \right] + W_0^-
\]

where \( Q_t = \prod_{j=0}^t z_j \), \( Q_0 = 1 \) and the profits, \( \Pi_t \), are

\[
\Pi_t = \alpha y_t^s \left( \frac{P_t^s - W_t}{s_t} \right) + (1 - \alpha) y_t^f \left( \frac{P_t^f - W_t}{s_t} \right)
\]

The intertemporal prices \( Q_{t+1} = \frac{\beta^{t+1} R_{t+1} (1 - \tau^w_t) W_t}{R_0 (1 - \tau^w_0) W_{t+1}} u_b(t+1) \), obtained using (12), can be replaced in the budget constraint to obtain

\[
E_0 \sum_{t=0}^{\infty} \beta^t u_h(t) \left\{ \frac{(1 + \tau^c_t)}{(1 - \tau^w_t) w_t} C_t + \frac{(1 + \tau^f_t)}{(1 - \tau^w_t) w_t} (R_t - 1) m_t \right\} = E_0 \sum_{t=0}^{\infty} \beta^t u_h(t) \left\{ (1 - h_t l(C_t, m_t)) + \frac{(1 - \tau^d_t) \Pi_t}{(1 - \tau^w_t) W_t} \right\} + \omega^-\]

where \( \omega^- = \frac{R_0 u_b(0) W_0^-}{R_0 (1 - \tau^w_0) w_0} \). Using the intratemporal conditions (9) and (10) to replace \( w_t \) and \( R_t \) in the budget constraint, as well as (25) and (26), and the fact that the transactions technology is homogeneous of degree \( k \geq 0 \), we obtain the implementability condition

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u_C(t) C_t - u_h(t) (1 - h_t - (1 - k) l(t)) \right\} = E_0 \sum_{t=0}^{\infty} \beta^t u_h(t) \frac{(1 - \tau^d_t) \Pi_t}{(1 - \tau^w_t) W_t} + \omega^-,
\]

where the value of profits is

\[
E_0 \sum_{t=0}^{\infty} \beta^t u_h(t) \frac{(1 - \tau^d_t) \Pi_t}{(1 - \tau^w_t) W_t} = E_0 \sum_{t=1}^{\infty} \beta^t u_h(t) \frac{(1 - \tau^d_t)}{(1 - \tau^w_t)} (1 - h_t - l(t)) \frac{1}{(\theta - 1)} +
\]

\[
\pi_h(0) \frac{(1 - \tau^d_0)}{(1 - \tau^w_0)} \left( \alpha y_0^s \left( \frac{1}{w_0^s} - \frac{1}{s_0} \right) + (1 - \alpha) y_0^f \frac{1}{s_0 (\theta - 1)} \right)
\]

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The value of profits from period one on is independent of the share of sticky firms, since the value of profits of the sticky firms is the same as the one of the flexible firms. For a particular state, the profits of both firms will in general differ, since the optimal pricing rule for sticky-price firms allows for departures between the marginal productivity of labor and real wages. However, the expected value of the real wage, weighted by the state-contingent relative price of consumption must be equal to the expected marginal productivity of labor, using the same weights. Those same state-contingent relative prices of consumption also weigh the real wage in the life-time budget constraint of households, resulting in the same value for profits.

At time zero, the prices of the sticky firms are arbitrarily given, so that the constraint on the real wage (26) only holds from time one on. Because the initial price is given, the government can use monetary policy to affect nominal wages and pin down the value for the real wage. This instrument is not available when prices are flexible, since the real wage must be equal to the marginal productivity of labor. It turns out, however, that this instrument is redundant, as will be shown.

The expression for initial wealth in (28) is

$$w_0^- = \left[ 1 - \frac{l_m(0)u_h(0)}{u_C(0) - u_h(0)} \right] \frac{(u_C(0) - u_h(0))l_C(0)}{(1 + \tau_0^c)P_0},$$

where the price level $P_0$ is

$$P_0 = P_0^s \left[ \alpha + (1 - \alpha) \left( \frac{w_0^s}{\theta^{-1}S_0} \right)^{(\theta - 1)} \right]^{\frac{1}{(\theta - 1)}}.$$

From (2), (8), (21), (22), (23), (24), and (25), we can write the resource constraints as

$$\left[ \alpha \left( \frac{\theta^{-1}S_t}{w_t^p} \right)^{(1 - \theta)} + 1 - \alpha \right]^{\frac{\theta}{1 - \theta}} (C_t + G_t) = y_t^f$$

$$\left[ \alpha + (1 - \alpha) \left( \frac{\theta^{-1}S_t}{w_t^p} \right)^{(\theta - 1)} \right]^{\frac{\theta}{\theta - 1}} (C_t + G_t) = y_t^s$$
\[ \alpha y_t^s + (1 - \alpha) y_t^f = s_t (1 - h_t - l(C_t, m_t)) \]  \hspace{1cm} (34)

Let \( D(\frac{w_f^t}{w_t^f}) \equiv \left\{ \alpha \left[ \alpha + (1 - \alpha) \left( \frac{w_f^t}{w_t^f} \right)^{\theta - 1} \right] + (1 - \alpha) \left[ \alpha \left( \frac{w_f^t}{w_t^f} \right)^{\theta - 1} + (1 - \alpha) \right] \right\}^{-1} \).

Then, from (32), (33) and (34), we have

\[ C_t + G_t = D(\frac{w_f^t}{w_t^f})s_t (1 - h_t - l(C_t, m_t)) \]  \hspace{1cm} (35)

We have used the equilibrium conditions (12), (10), (9), (25) to substitute \( \{Q_{t+1}, R_t, w_t, w_f^t\} \), respectively, in the other equilibrium conditions, and obtain a system of equilibrium conditions that restrict the allocations \( \{C_t, h_t, m_t\} \) and the variables \( \{w_t^s, y_t^s, y_t^f, \tau_t^w, \tau_t^d\} \), \( \tau_c^e \) and \( P_0 \). In the following lemma we specify that system of equations:

**Lemma 1** The restrictions on allocations \( \{C_t, h_t, m_t\} \) and on the allocations, prices and policy variables, \( \{w_t^s, y_t^s, y_t^f, \tau_t^w, \tau_t^d\} \), \( \tau_c^e \) and \( P_0 \), can be described by (26), (27) for \( t = 0 \), as well as (28), (29), (30), (31), and the resource constraints, (32), (33) and (34).

The additional conditions (12), (10), (9), (25), can be used to recover \( \{Q_{t+1}, R_t, w_t, w_f^t\} \), and (27) for \( t \geq 1 \) can be used to recover \( \{\tau_t^c, t \geq 1\} \). The restrictions on taxes and the restriction that profits may not be negative, established above, must also be satisfied.

In the extreme cases with only flexible or sticky firms the sets of allocations defined by the restrictions in Lemma 1 are the same. We state this in the following lemma:

**Lemma 2** The sets of implementable allocations coincide in the cases where \( \alpha = 0 \) and \( \alpha = 1 \).

**Proof:** Under flexible prices, when \( \alpha = 0 \), the set of implementable
allocations \( \{C_t, h_t, m_t\} \), taxes \( \{\tau^d_t, \tau^w_t\} \) and initial price \( P_0 \), is described by

\[
0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u_C(t)C_t - u_h(t)(1 - h_t - (1-k)l(t)) \right\} -
\]

\[
E_0 \sum_{t=0}^{\infty} \beta^t u_h(t) \frac{(1 - \tau^d_t)}{(1 - \tau^w_t)} (1 - h_t - l(t)) \frac{1}{\theta - 1} -
\]

\[
\left[ 1 - \frac{l_m(0)u_h(0)}{u_C(0) - u_h(0)l_C(0)} \right] u_h(0) \frac{\mathbb{W}^-_0}{(1 - \tau^w_t)^{\theta - 1} s_0 P_0},
\]

(39)

together with the resource constraints

\[
C_t + G_t = s_t (1 - h_t - l(C_t, m_t)).
\]

(38)

When \( \alpha = 1 \), the set of implementable allocations \( \{C_t, h_t, m_t\} \), taxes \( \{\tau^d_t, \tau^w_t\} \) and initial real wages \( w_0 \) is described by

\[
0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u_C(t)C_t - u_h(t)(1 - h_t - (1-k)l(t)) \right\} -
\]

\[
E_0 \sum_{t=1}^{\infty} \beta^t u_h(t) \frac{(1 - \tau^d_t)}{(1 - \tau^w_t)} (1 - h_t - l(t)) \frac{1}{\theta - 1} -
\]

\[
\left[ 1 - \frac{l_m(0)u_h(0)}{u_C(0) - u_h(0)l_C(0)} \right] u_h(0) \frac{\mathbb{W}^-_0}{(1 - \tau^w_t)^{\theta - 1} w_0 P_0},
\]

and the resources constraints (38). \( P_0 \) is given.

The two restrictions are different in the value of profits and wealth in period zero. However, it is possible to use the instruments \( \tau^d_t, \tau^w_t, P_0 \) under flexible prices, and \( \tau^d_t, \tau^w_t, w_0 \) under sticky prices to achieve the same values for initial wealth and initial profits.

Under sticky prices, the real wage, \( w_t \), does not have to be equal to its value under flexible prices, \( \left( \frac{\theta-1}{\theta} \right) s_t \). Monetary and fiscal policy can affect the real wage at time 0, thereby affecting profits and the real value of initial wealth. On the other hand, under flexible prices \( P_0 \) can be chosen freely. Using these different instruments as well as the tax rates it is possible to guarantee that the sets of implementable allocations are the same in the two environments.

We now define the frontier of allocations:
Definition 3 The frontier of implementable allocations is the subset of the set of implementable allocations such that aggregate consumption is maximized for any value of leisure.

In the mixed economy with both flexible and sticky firms, where $0 < \alpha < 1$, the frontier of allocations coincides with the implementable set under flexible or sticky prices. The other allocations, that involve relative price distortions, are interior allocations that would not be chosen by a government with preferences on aggregate consumption and leisure.

The proposition follows:

Proposition 4 The set of (frontier) implementable allocations is independent of the degree of price stickiness $0 \leq \alpha \leq 1$.

Proof: We show that it is possible to determine a frontier of allocations for $\{C_t, h_t, m_t\}$ that is independent of $\alpha$.

For any $0 < \alpha < 1$, if policy is conducted so that $w^*_t = w^f_t = \frac{2}{\alpha + 1} s_t$, $t \geq 0$, the implementability conditions are described by (36) for $P_0 = P^s_0$ together with the resources constraints (38). These conditions are the same for any $0 \leq \alpha \leq 1$.

If, for a given $0 < \alpha < 1$, policy is conducted so that $w^*_t \neq w^f_t$, the resource constraints will be more restrictive but the other constraints will not. Indeed, if the policy was such that $w^*_t \neq w^f_t$, then the resources constraint (38) would be satisfied with inequality since $D(\frac{w^f_t}{w^*_t}) < 1$ iff $w^*_t \neq w^f_t$. The restriction on the allocations $\{C_t, h_t, m_t\}$ from implementability conditions (26), (27) for $t = 0$, (28), (29), (30), (31), and (32), (33), (34) is the same as when $w^*_t = w^f_t$, since it is possible to pick $\tau^*_c$ and $\tau^*_w$ to attain the same values for initial profits and real wealth. If the preferences of the government depend only on $\{C_t, h_t\}$, then it is optimal to conduct policy so that the real wages are equated across firms, $w^*_t = w^f_t$. This defines a frontier of allocations. This frontier is the same for any $0 \leq \alpha \leq 1$.

This result is an application of Diamond and Mirrlees (1971) optimal rules according to which it is not optimal to distort production in a second best environment. In the proposition we have stated that it is efficient to set policy so that prices are the same across flexible and sticky firms. This means that the price level should not vary with the contemporaneous shocks, and neither should the ex-post markups $\left(\frac{(u_{C}(t)-u_{h}(t))h_{C}(t)}{u_{h}(t)}\right) / \left(\frac{(1+\tau^*_c)}{1+\tau^*_w}\right)$. We have
stated in the proposition above a result of irrelevance of the price rigidity in terms of the sets of implementable allocations. In the proposition and corollary that follow we extend the irrelevance, or equivalence, result, to the policies that decentralize the frontier set of allocations.

It is clear that the policies that decentralize the set of frontier allocations in the intermediate case where $0 < \alpha < 1$ are the same for any $\alpha$. They are fiscal and monetary policies such that markups do not vary with the shocks as under flexible prices, and prices do not depend on the shocks as under sticky prices. In other words, fiscal policy is conducted as if all prices were flexible; and monetary policy replicates the flexible prices equilibrium. Gaps, defined as the deviations to the allocation under flexible prices for a given tax policy, are eliminated. Clearly these policies also decentralize the set of implementable allocations in the environments with flexible or sticky prices, for $\alpha = 0, 1$. The proposition follows:

**Proposition 5** Let $0 < \alpha < 1$. For each allocation in the frontier set of allocations, if a policy decentralizes that allocation for some $\alpha$, then it decentralizes the same allocation for every $\alpha$. The policy also decentralizes the same allocation for $\alpha = 0, 1$.

**Corollary 6** (Adao, Correia and Teles, 1999)$^4$ The optimal (Ramsey) policies do not depend on the alphas.

The frontier of allocations achieved by following a policy that equates real wages across firms, $w^s_t = w^f_t = \theta^{-1}s_t$, $t \geq 0$, is only a partial characterization of the relevant set of implementable allocations for any government that aims at maximizing a function of aggregate consumption and leisure. Further restrictions have to be imposed on the tax instruments in order to fully characterize the frontier set of implementable allocations. In particular it is also a frontier requirement that profits be fully taxed and the nominal interest rate be set according to the Friedman rule. We proceed to show this.

The choices of $\tau_t^d$ and $m_t$ are independent of government preferences. Because profits cannot be negative, it is optimal to tax profits completely, and set $\tau_t^d = 1$. Since for $\tau_t^d = 1$ the production decisions are indeterminate,

$^4$Adao, Correia and Teles (1999) consider a model with either sticky prices or sticky portfolios. They show that in each environment, the policies that replicate the flexible economy are independent of the rigidity. In this sense, they claim that the monetary transmission mechanism is irrelevant for policy.
we consider the limiting economies as $\tau_t^d$ approaches one. In the limit, the government can decentralize the same allocation as in the perfect competition case.

The optimal choice for real balances $m_t$ that satisfies the frontier implementability conditions (36), with $\tau_t^d = 1$, and (38) is characterized for $t \geq 1$ by

$$-l_m(t) [\varphi(k-1) u_h(t) + \lambda t s_t] = 0, \ t \geq 1$$

where $\varphi$ and $\lambda_t$ are the multipliers, respectively, of (36) and (38). The solution for $t \geq 1$ is, thus,

$$-l_m(t) = 0, \ t \geq 1$$

For $t = 0$, the slope of the lagrangian is positive,

$$-l_m(0) [\varphi(k-1) u_h(0) + \lambda_0 s_0] = l_{mm}(0) u_h(0) \frac{W_0^{-1}}{u_C(0)} \geq 0,$$

since $l_m(0) \leq 0$, $l_{mm}(0) \geq 0$, $[\varphi(k-1) u_h(0) + \lambda_0 s_0] > 0$, and since we assumed that $W_0^{-1} \geq 0$. Thus also in this case the optimal solution is $l_m(0) = 0$.

The efficient solution is decentralized with the Friedman rule, $R_t = 1$. This result was obtained, for the deterministic case, by De Fiore and Teles (1998). They extend the results in Correia and Teles (1996) to the case where the policy instruments include consumption taxes. The result is in contrast with Mulligan and Sala-i-Martin (1996) and Schmitt-Grohé and Uribe (2000). Mulligan and Sala-i-Martin (1996) use the specification of the transactions technology proposed by Kimbrough (1986) that was not restricted to exhibit unitary elasticity of money with respect to the price level gross of consumption taxes. Schmitt-Grohe and Uribe (2000) don’t allow for complete taxation of profits and/or for consumption taxes. In their set up, the inflation tax replaces the consumption tax with an efficiency loss.

Because the choice of real money balances is indeterminate when $R_t = 1$, we consider the limiting case as $R_t$ approaches one.

The solution for real money at time zero would have been simplified if we had imposed to start with the efficiency requirement that initial wealth be fully taxed, $L = 1$. An approximate way to achieve this full tax on the initial nominal wealth is to set the consumption tax $\tau_0^c$ equal to a very large number.

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5See Correia and Teles (1996) for the proof of the sign of this term.
Once we impose the efficiency requirements specified in this section, the set of (frontier) implementable allocations \( \{C_t, h_t\} \) is given by
\[
E_0 \sum_{t=0}^{\infty} \beta^t \{u_C(t)C_t - u_h(t)(1 - h_t)\} = 0
\]  
(40)
and the resource constraints
\[
C_t + G_t = s_t (1 - h_t).
\]  
(41)

4 Minimal sets of instruments

In the last section we have established that, independently of the degree of price rigidity, the planner can use fiscal and monetary policy to implement the same set of allocations. In this sense, the environments with flexible and sticky prices, or with both types of firms, are equivalent; or, in other words, the degree of price rigidity is irrelevant in determining the set of implementable allocations. The result was obtained under the assumption that the government has access to taxes on consumption, labor income, profits, initial wealth, and that it can issue state-contingent nominal debt. It turns out that not all these instruments are necessary. In this section we determine the minimal sets of instruments in the extreme environments with flexible and sticky prices and in the intermediate case with both types of firms. We also extend the result of equivalence or irrelevance of environments to the choice of fiscal and monetary policies.

We first discuss the decentralization of the frontier allocations described by (40) and (41) for the intermediate case with \( 0 < \alpha < 1 \). This frontier of allocation requires that the tax on profits and the initial levy are set at their maximum levels, and that the nominal interest rate is \( R_t = 1 \). For each allocation we can recover the prices, the other taxes, the debt and monetary policies that support the allocation using the following equilibrium conditions, for \( R_t = 1 \),
\[
\frac{u_C(t)}{u_h(t)} s_t = \frac{(1 + \tau^e_t)}{(1 - \tau^{w_t}_t)} \frac{\theta}{\theta - 1}, \quad t \geq 0
\]  
(42)
\[
\frac{u_C(t-1)}{(1 + \tau^{e_{t-1}})} P_{t-1} = \frac{1}{P_t} E_{t-1} \left[ R_t \beta u_C(t) \right], \quad t \geq 1
\]  
(43)
\[ E_t \sum_{s=0}^{\infty} \beta^s \{ u_C(t+s)C_{t+s} - u_h(t+s)(1-h_{t+s}) \} \]

\[ = u_C(t) \frac{W_t^-}{(1+\tau\ell_t^c)P_t}, \quad t \geq 0 \]

where

\[ \frac{W_t^-}{(1+\tau\ell_t^c)P_t} = \frac{M_{t-1} + R_{t-1}B_{t-1}^q + Z_{t-1}^q}{(1+\tau\ell_t^c)P_t} - C_t + \frac{u_h(t)}{u_C(t)}(1-h_t) \]

\[ (1+\tau\ell_t^c)P_t \tau m_t = M_t, \quad t \geq 0 \]

(46)

\( P_t \) cannot depend on contemporaneous shocks and \( P_0 \) is given. Because of the initial levy \( L W_0^- (1+\tau\ell_0^c)P_0^- = 0 \).

The intratemporal condition (42) determines the ratio of taxes, \( \frac{(1+\tau\ell_t^c)}{(1-\tau\ell_t^w)} \), \( t \geq 0 \). If, by assumption, only one of those taxes is available, then it is determined uniquely. The intertemporal condition (43) restricts the average level of inflation for the price level gross of consumption taxes, \( \frac{(1+\tau\ell_t^c)P_t}{(1-\tau\ell_{t-1}^c)P_{t-1}} \). Condition (44) determines the real state-contingent level of public debt, \( \frac{W_t^-}{(1+\tau\ell_t^c)P_t} \), \( t \geq 0 \). From (46), we obtain \( \frac{M_t}{(1+\tau\ell_t^c)P_t}, \quad t \geq 0 \). Not all the policy instruments that we have assumed to be available are necessary to decentralize the set of implementable allocations. In particular, as we will show in this section, nominal state-contingent public debt is not necessary. Real state-contingent debt can be achieved with state-noncontingent nominal debt, through the variability of the price level gross of consumption taxes. One of the taxes, on consumption or labor income, may also be redundant.

If the policy instruments include consumption taxes, labor income taxes and state-contingent public debt, the policies will not be uniquely pinned down. Let \( \Phi_t \) be the number of states at time \( t \geq 0 \), with \( \Phi_0 = 1 \). For \( t = 0 \), since \( L = 1 \), there are only two equations to determine three variables \( \{\tau_0^c, \tau_0^w, M_0\} \). If the levy was not set at its maximum value, the same role could be played by \( \tau_0^c \) that would be determined from the frontier requirement of \( \frac{\mathcal{L}\omega^-}{(1+\tau_0^c)\bar{P}_0} = 0 \). \( \tau_0^w \) is determined using the intratemporal condition (42) and \( M_0 \) is determined using (46). For \( t \geq 1 \) there are \( 3\Phi_t + \Phi_{t-1} \) equations to determine \( 4\Phi_t + \Phi_{t-1} \) variables, \( \{\tau_t^c, \tau_t^w, M_t, Z_t^q\} \) and \( \{P_t\} \).

The indeterminacy indicates that not all the policy instruments are necessary to decentralize the allocations. Since we allow for the initial levy there
is an equivalence between the consumption and labor income taxes for all periods. If only one of those taxes is used the policy variables will be uniquely determined. In period 0 there will be two equations to determine two variables and, for \( t \geq 1 \), \( 3\Phi_t + \Phi_{t-1} \) equations to determine \( 3\Phi_t + \Phi_{t-1} \) variables. For example, if there are no labor income taxes, then (42) determines \( \{\tau_c^t\} \) for any date and state; from (43), we obtain \( \{P_t\} \) for any \( t \geq 1 \) and \( P_0 \) is given; \( \{M_t\} \) are determined using (46) and \( \{Z_g^t\} \) or \( \{W_t^{-}\} \) are determined using (44).

The use of both consumption and labor income taxes can play the role of state-contingent debt in the decentralization of the set of frontier allocations. With state-noncontingent debt and both consumption and labor income taxes there is still one degree of indeterminacy at \( t = 0 \); and for \( t \geq 1 \) there are \( 3\Phi_t + \Phi_{t-1} \) equations to determine \( 3\Phi_t + 2\Phi_{t-1} \) variables, \( \{\tau_c^t, \tau_w^t\} \) and \( \{B_{t-1}^g, P_t\} \), so that \( \Phi_{t-1} \) is the degree of indeterminacy. In this case state-contingent real debt is simulated with the variability of the consumption tax. The average value of consumption taxes, \( \tau_c^t \), is not determined. Thus neither is the average labor income tax, \( \tau_w^t \) and \( M_t \). This indeterminacy also implies the indeterminacy of \( P_t \) and \( B_{t-1}^g \).

To summarize, in order to decentralize the frontier set of allocations, the government will need to use as minimal sets of instruments either both consumption and labor income taxes or state-contingent debt, i. e. \( \{\tau_w^t, M_t, Z_g^t\} \), \( \{\tau_c^t, M_t, Z_g^t\} \) or \( \{\tau_c^t, \tau_w^t, M_t, B_{t-1}^g\} \). In the latter case the policy variables are not determined uniquely. The three minimal sets of instruments are equivalent sets, in the sense that they are alternative policy instruments that decentralize the same set of allocations.

We the following section we consider the extreme cases where either all firms set prices contemporaneously or they all set prices in advance. In those cases it is possible to restrict further the policy instruments and still implement the same set of allocations.

In each extreme environment, the set of policies that decentralize each allocation is larger than in the mixed economy. In particular, under sticky prices, even if there are only consumption taxes, or labor income taxes, they are not uniquely pinned down. In this sense there is short-run neutrality of taxes which is analogous to the neutrality of money under flexible prices. Under flexible prices, there are many money supply policies that decentralize the same allocation. Either this neutrality or the equivalence of the consumption and labor income tax is lost if debt is not state-contingent. This means that either the price level or the variability of the consumption tax
are pinned down when public debt is state-noncontingent.

The neutrality of taxes under sticky prices is also lost when public debt is not state-contingent. In fact real state-contingent debt can be simulated with variability of consumption taxes. As we saw, both the neutrality of money and the neutrality of taxes are not present in the mixed economy. Tax and monetary policy must be conducted in a single way so that there are no surprises in prices or mark ups. When public debt is not state-contingent, the variability of the consumption tax and labor income tax are also pinned down.

4.1 Flexible prices

In the case where all firms set prices contemporaneously, for each allocation in the set of implementable allocations, we can recover the taxes, the debt and monetary policies, as well as the prices, that support that allocation using the equilibrium conditions above, except that \( P_t \) can depend on the contemporaneous shocks. The conditions are, thus, (42), (44), (46) and

\[
\frac{u_C(t-1)}{(1 + \tau^e_{t-1})P_{t-1}} = E_{t-1} \left[ R_t \frac{\beta u_C(t)}{(1 + \tau^C)P_t} \right], \quad t \geq 1
\]

that replaces (43).

The number of equations is the same but there are now \( \Phi_t \) price levels to determine in each period instead of \( \Phi_{t-1} \). The minimal sets of instruments in this case where all prices are flexible are smaller sets. In particular, there is no need to issue state-contingent public debt or use consumption taxes (or, alternatively, labor income taxes). If neither of these two instruments are used, the sequences of policy variables \( \{\tau^e_t, M_t, B^g_{t-1}\} \) and prices \( \{P_t\} \) are uniquely pinned down, except at \( t = 0 \) for \( L = 1 \). From (42), \( \tau^e_t \) is determined in each date and state. At \( t = 0 \), unless \( L \neq 1 \), it is not possible to use (44) to determine \( P_0 \), and, thus, \( M_0 \) is also not pinned down. For \( t \geq 1 \), the \( \Phi_t + \Phi_{t-1} \) variables \( \{P_t, B^g_{t-1}\} \) are jointly determined by (44) and (47). \( M_t, t \geq 1 \), is determined using (46). The intertemporal condition determines average inflation, and the budget constraints determine how inflation is distributed across states.

Alternatively, if, instead of labor income taxes, consumption taxes were used, the policy variables \( \{\tau^e_t, M_t, B^g_{t-1}\} \) and the price levels \( \{P_t\} \) would also be uniquely determined, again except for \( t = 0 \) when \( L = 1 \).
In this case, with flexible prices, the minimal sets of instruments are \( \{ \tau^c_t, M_t, B^q_{t-1} \} \) and \( \{ \tau^w_t, M_t, B^q_{t-1} \} \). In both cases, the price level variability replicates real state-contingent debt.

The role for the price level of replicating state-contingent real debt can be played by consumption taxes, if both labor income and consumption taxes are used. In this case, when debt is state-noncontingent, the price level gross of consumption taxes, \((1 + \tau^c_t)P_t\), \(t \geq 1\), is uniquely pinned down. For \(t \geq 1\), conditions (43) and (44) jointly determine \((1 + \tau^c_t)P_t\) and \(B^q_{t-1}\). Money supply, \(M_t\) for \(t \geq 1\), is also uniquely determined, from (46).

The fact that the sequences of the price level gross of consumption taxes and the money supply are determined means that money is not neutral in this flexible price economy, where the government does not issue state-contingent debt. Money supply is playing a role, replacing the lack of a policy instrument, according to a "fiscal theory of the price level gross of consumption taxes".

When the set of policy instruments includes state-contingent public debt, in addition to the taxes on consumption or labor income, there are multiple paths for the money supply, \(\{M_t\}\), corresponding to multiple paths for the price level gross of consumption taxes, \(\{(1 + \tau^c_t)P_t\}\). Average inflation, for the price level gross of taxes, is determined by the intertemporal condition, (47), but the variability of the price level gross of taxes, and money supply, are not pinned down. This is the case whether only one of the taxes or both are considered. The multiple policies are equivalent policies that exhibit a (fiscal) neutrality of money under flexible prices.

When the set of policy instruments includes state-contingent public debt, or when both consumption and labor income taxes are used, so that the price level is not pinned down, then there is one money supply, tax and debt policy such that the prices are predetermined, as under sticky prices. The variability of the price level gross of consumption taxes required to reproduce real state-contingent debt would result from variability of the consumption tax, and labor income tax, alone. With these policies it is possible to decentralize the same frontier set of allocations for \(0 < \alpha < 1\).

The issue of whether state-contingent public debt is necessary to decentralize the Ramsey allocation under flexible prices was addressed by Chari, Christiano and Kehoe (1991). They show that it is possible to decentralize the second best allocation in an equilibrium where the price level reacts to shocks so as to replicate state-contingent real debt. In their calibrated exercise the volatility of the price level is very high. Questioning the relevance of
this policy recommendation, Siu (2000) and Schmitt-Grohe and Uribe (2001) compute the Ramsey solution when it is costly to change prices. They show that the benefits of replicating state-contingent debt are minimal relatively to the costs of changing prices. As the discussion above makes clear this trade-off is artificial since it hinges on the assumption that consumption taxes are not available. If consumption taxes were available, they could be used to replicate real state-contingent debt in an equilibrium where the price level is constant across states. Chari, Christiano and Kehoe (1991) also assume that there are only labor income taxes. However in their flexible price world, consumption taxes would be redundant. That is not the case under sticky prices.

4.1.1 Sticky prices

Under sticky prices there are only $\Phi_{t-1}$ price levels to determine in each period, but there are also less equilibrium conditions that can be used to recover the policy variables. When all the firms set prices one period in advance, the $\Phi_t$ intratemporal conditions (42) are replaced by $\Phi_{t-1}$ conditions each period $t \geq 1$,

$$E_{t-1} \left[ (1 - \tau^d_t) u_h(t) (1 - h_t - l(t)) \left( \frac{(u_C(t) - u_h(t)h_C(t))}{v_h(t)} \left( \frac{1}{\theta} \frac{1 - l(t)}{1 - \tau^d_t} \right) \right] = 0, \quad t \geq 1$$

This means that the intratemporal conditions only restrict an average of the ratio of tax rates $\frac{(1 + \tau^d_t)}{(1 - \tau^d_t)}$. The other relevant equilibrium conditions are the same as in the intermediate cases, (43), (44), and (46). The initial price level, $P_0$, is given.

If public debt is not state-contingent and labor income taxes are not used, then $\{\tau^r_t, M_t, B^g_t\}$ and $\{P_t\}$ are uniquely determined, except for $t = 0$ when the initial wealth is fully taxed. For $t \geq 1$, there are $2\Phi_t + 2\Phi_{t-1}$ equations, (48), (43), (44) and (46), to determine $2\Phi_t + 2\Phi_{t-1}$ variables, $\{\tau^r_t, M_t\}, \{P_t, B^g_{t-1}\}$.

The intratemporal condition, (48), determines an average value for the consumption tax, $\tau^r_t$. The budget constraints, (44), determine how the tax is distributed across states, as well as the level of public debt, $B^g_t$. Given the consumption tax in the different states, the intertemporal condition, (43), can be used to determine the price level, $P_t$. The money supply, $M_t$, can be
recovered using (46). Since the price level cannot depend on the state, in order to replicate real state-contingent debt, the consumption tax will vary across states. From Chari, Christiano and Kehoe (1991), we would expect the variability of the tax rate that implements the Ramsey allocation to be very high. This variability of the tax rate is not distortionary since only the average tax rate matters.

If instead of consumption taxes, labor income taxes were used, then it would not be possible to decentralize the same set of allocations, unless public debt was state-contingent. This is clear since it would not be possible to decentralize allocations such that the outstanding real wealth, \( W_t - P_t \), depends on the contemporaneous shocks.

If public debt is state-contingent, the consumption taxes are only pinned down on average, according to (48). There are multiple paths for \( \{ \tau_c^t, M_t, Z_g^t \} \) or \( \{ \tau_w^t, M_t, Z_g^t \} \), or yet \( \{ (1 + \frac{\tau_c^t}{1 - \tau_w^t}), M_t, Z_g^t \} \), associated with the same real allocation. One of the tax policies would satisfy (42), as under flexible prices. Whenever the tax policy is different from this, the real wage deviates from the one under flexible prices, according to (9), in order to compensate for the deviations in the tax policy. In this case we can say that there are gaps, defined as the deviations between the sticky price and flexible price allocations for a given tax policy.

In this case where the public debt is state-contingent, there is a short-run neutrality of taxes, under sticky prices, analogous to the neutrality of money under flexible prices. As in that case, the neutrality disappears when there is no state-contingent debt, since consumption taxes will be replacing that missing policy instrument.

### 4.2 Other restrictions on policy instruments

In this section we analyze the implications of restricting the consumption and labor income taxes not to depend on the contemporaneous shocks. Thus, taxes are set in advance as prices are. We also consider restrictions on the taxation of profits and initial wealth.

We have seen above that the restriction on the public debt that it may not be state-contingent is not relevant since it can be replaced by the use of other fiscal instruments. We now inquire of the relevance of the constraint that taxes may not depend on the contemporaneous shocks. In order to discuss this we are going to concentrate on the Ramsey allocations. We state
the conditions under which it is optimal for a Ramsey planner to set proportionate distortions, or wedges, that are independent of the contemporaneous shocks. Under those conditions the constraint is not binding.

When the restriction is imposed that taxes may not depend on the contemporaneous shocks, in the extreme case of sticky prices, $\alpha = 1$, it is still possible to use state dependent monetary policy to achieve an optimal solution characterized by state dependent proportionate distortions. Monetary policy plays the role of state-contingent taxes under flexible prices. The optimal allocation is characterized by gaps. For $\alpha \neq 1$, unless the optimal solution is characterized by proportionate distortions that do not depend on the contemporaneous shocks, it cannot be attained.

In general, the equivalence of environments in terms of the optimal allocations and policies is lost when the restriction is imposed that taxes do not depend on the contemporaneous shocks. Adao, Correia and Teles (2001) is an example of this principle. Since they only consider monetary policy there is a natural restriction on policy, that the nominal interest rate cannot be negative. For this reason the set of allocations under flexible prices is smaller that the set of allocations under sticky prices. In that paper, conditions are provided under which the optimal solution belongs to both sets. In general it doesn’t.

When $\alpha \neq 1$, and taxes must be set in advance, if the optimal solution is characterized by wedges that depend on the contemporaneous shocks, then the second best cannot be achieved. The third best will be characterized by deviations from the flexible price allocation and policies.

In the following subsection we provide the conditions under which it is optimal to set proportionate distortions that do not vary with the contemporaneous shocks.

### 4.2.1 Sticky taxes

In this section we describe the conditions under which it is optimal to smooth proportionate distortions. The objective is to assess the relevance of imposing the same restriction on taxes as the restriction on the setting of prices, that they may not depend on the contemporaneous shocks.

For a given tax policy the frontier allocations in each date and state are described by

\[
\frac{u_C(C_t, h_t)s_t}{u_h(C_t, h_t)} = \frac{\theta (1 + \tau^C_t)}{(\theta - 1) (1 - \tau^w_t)}
\]
and the resources constraint
\[ C_t + G_t = s_t (1 - h_t) \]

We want to determine under what conditions the optimal proportionate distortions, or wedges, \( \frac{u_C(C_t, h_t)}{u_h(C_t, h_t)} \), do not depend on the contemporaneous shocks. From the FOC of Ramsey problem, where utility (1) is maximized subject to (40) and (41), we have
\[
\frac{u_C(t)}{u_h(t)} s_t = \frac{1 + \varphi \left( 1 + \frac{u_{hh}(t)C_t}{u_h(t)} - \frac{u_{hh}(t)(1-h_t)}{u_h(t)} \right)}{1 + \varphi \left( 1 + \frac{u_{CC}(t)C_t}{u_C(t)} - \frac{u_{CC}(t)C_t}{u_C(t)s_t} \frac{1}{C_t} \right)}
\]

where \( \varphi \) is the multiplier of the implementability condition (40).

The optimal wedges \( \frac{u_C(t)}{u_h(t)} s_t \) do not depend on the contemporaneous shocks when the preferences are separable and have constant elasticities of the marginal utilities of consumption and labor,
\[
u = \frac{C_t^{1-\sigma}}{1-\sigma} - \alpha N_t^\psi, \quad \sigma \geq 0, \quad \psi \geq 1.
\]

In this case the optimal wedges are constant across dates and states
\[
\frac{u_C(t)}{u_h(t)} s_t = \frac{1 + \varphi \psi}{1 + \varphi (1 - \sigma)}
\]

For preferences that are consistent with balanced growth
\[
u = \frac{(C_t F(h_t))^{1-\sigma} - 1}{1 - \sigma}, \quad F' > 0, \quad \sigma \geq 0,
\]

which include the isoelastic
\[
u = \frac{(C_t h_t^\psi)^{1-\sigma} - 1}{1 - \sigma}, \quad \sigma \geq 0,
\]

the optimal wedges are such that
\[
\frac{u_C(t)}{u_h(t)} s_t = \frac{1 - \varphi \left( \sigma + \psi (1 - \sigma) \frac{1-h_t}{h_t} \right)}{1 + \varphi (1 - \sigma) \left( 1 - \frac{u_{hh}(t)}{u_C(t)s_t} \frac{1}{s_t(1-h_t)} \right)}
\]
Using the resource constraints \((41)\), the wedges for this utility function are in equilibrium

\[
\frac{u_C(t)}{u_h(t)} s_t = \frac{h_t / (1 - h_t)}{\psi \left( 1 - \frac{G_t}{\mu \gamma (1 - h_t)} \right)}.
\]

If \(\frac{G_t}{\mu \gamma (1 - h_t)}\) does not depend on the contemporaneous shock, then the optimal labor allocations won’t either, and neither will the optimal wedges.

Therefore it is optimal to set taxes that do not depend on the contemporaneous shocks when preferences are separable and constant elasticity in consumption and labor or when preferences are consistent with balanced growth and government expenditures shocks are perfectly correlated with the productivity shock. For the first case it is still possible to establish an equivalence of environments in what concerns the policies that decentralize the optimal allocations provided that state-contingent debt is available. In the second case, the same equivalence is established without that requirement.

4.2.2 Taxes on profits and initial wealth

Suppose that for some arbitrary reason the taxes on profits and initial wealth were bounded away from one. Would the equivalence result still hold for the same minimal sets of instruments? The answer is no if the sets of instruments are either the consumption or the labor income tax and state-contingent debt, \(\{\tau^c_t, M_t, Z^p_t\}\) and \(\{\tau^w_t, M_t, Z^p_t\}\), and yes if the instruments are both taxes and state-noncontingent debt \(\{\tau^c_t, \tau^w_t, M_t, B^g_{t-1}\}\). In the latter case the remaining degrees of indeterminacy could be used for the purpose of fully taxing profits and initial wealth, so that the same frontier set of allocations could be implemented with the same policies independently of the degree of rigidity.

In order to achieve the frontier allocations the consumption tax, and also the labor tax, and nominal debt, would have to be arbitrarily large. The tax on consumption would be replicating the tax on profits that is ruled out by assumption. If the same restrictions imposed on the direct taxes on profits and wealth were to apply to the alternative means of achieving the same goal, then the equivalence result would hold in general.

If we were to impose restrictions not on the tax rates but on the net of
taxes values of both profits and initial wealth

\[
E_0 \sum_{t=0}^{\infty} \beta^t u_h(t) \frac{(1 - \tau_d^t) \Pi_t}{(1 - \tau^w_t) W_t} = E_0 \sum_{t=1}^{\infty} \beta^t u_h(t) \frac{(1 - \tau_d^t)}{(1 - \tau^w_t)} (1 - h_t) \frac{1}{(\theta - 1)} + u_h(0) \frac{(1 - \tau_d^0)}{(1 - \tau^w_0)} \left( \alpha y_0 \left( \frac{1}{w_0} - \frac{1}{s_0} \right) + (1 - \alpha) \frac{y_0}{s_0} \frac{1}{(\theta - 1)} \right) \geq \Pi
\]

and

\[
\varpi_0 = u_h(0) \frac{\bar{W}_0}{(1 - \tau^w_0)w_0P_0} \geq \bar{W}
\]

then the equivalence results would still hold. The implementability conditions would be

\[
E_0 \sum_{t=0}^{\infty} \beta^t \{ u_C(t)C_t - u_h(t) (1 - h_t - (1 - k)l(t)) \} = \Pi + \bar{W},
\]

together with the resource constraints (41) and would not depend on \( \alpha \). The new frontier of allocations would be less efficient.

5 Concluding remarks

In this paper we analyze the implications of nominal rigidities for the conduct of fiscal and monetary policy in response to shocks.

We find that the sets of implementable allocations are the same under flexible and sticky prices. Each allocation can be decentralized with a common policy to both environments. Under flexible prices, money supply policy is conducted so that there are no surprises in prices. Under sticky prices, fiscal policy is conducted so that there are no surprises in mark ups. In each environment there are other policies that decentralize the same allocation. In particular, under sticky prices, tax policy is not uniquely pinned down. This means that there is short-run neutrality of taxes, which is analogous to the neutrality of money under flexible prices. Thus, both under flexible prices and under sticky prices, there are neutral and non neutral fiscal and monetary instruments. The roles are reversed across environments.
In the mixed economy with both flexible and sticky firms, taxation and monetary policies are pinned down. It is possible to determine a frontier where fiscal and monetary policies are conducted so that there are no surprises in prices and mark ups. The frontier is independent of the share of sticky firms and the policies that decentralize those allocations are also independent of the share of sticky firms. Because the frontier allocations and policies are independent of the degree of rigidity, it is possible to use the results on optimal taxation under flexible prices as in Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991), or Zhu (1992).

We characterize further the set of frontier allocations by showing that, independently of the degree of price stickiness, that set is characterized by the Friedman rule, of zero nominal interest rates, and by full taxation of profits and initial wealth.

We address the issue of which are the minimal sets of instruments that allow us to obtain the results of equivalence of environments. In particular, we show that in contrast to Siu (2000) and Schmitt-Grohe and Uribe (2001), the equivalence results still hold when public debt is assumed to be state-noncontingent. In the same way that under flexible prices the volatility of the price level can simulate state-contingent real debt, in this environment where price level volatility is costly, high volatility of consumption and income taxes can serve the same purpose.

In this paper we also analyze the conditions for optimal smoothing of proportionate distortions, or wedges. This is relevant since, under those conditions, we can assume that taxes are set in advance and yet obtain the result that the nominal rigidity is not relevant for the conduct of optimal policy.

The degree of price rigidity is assumed to be exogenous. This is not a natural assumption when computing the efficient, or optimal, policies. However, since we obtain that the sets of efficient policies do not depend on the degree of stickiness, exogeneity of the degree of stickiness is not a drawback.

The results in this paper hold under staggered price setting (or Rotemberg, 1982) if the transactions technology was described by a cash-in-advance constraint instead of the assumed transactions technology. In that case it would be feasible to replicate the flexible price allocation with a nominal interest rate path that follows the path of the real interest rate under flexible prices, and with zero inflation. The distortion from the nominal interest rate being different from zero could be compensated by a lower consumption or income tax, and the nominal interest rate wouldn’t cause additional
The equivalence of environments in terms of the sets of implementable allocations would be lost if we were to consider idiosyncratic shocks. The sets of allocations would, obviously, depend on the degree of price rigidity. However the sets of policies would not. In this stricter sense there is still an irrelevance of environments.

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