1. INTRODUCTION

We present in this article a methodology that aims to estimate in real-time relevant macroeconomic signals. The methodology is illustrated for the case of two measures of economic activity in the euro area: business cycle fluctuations and smooth component of output growth. In line with Baxter and King (1999), business cycle fluctuations are defined as those oscillations with period between 6 and 32 quarters in output. This definition reflects the knowledge of the typical duration of the phases of expansion and recession in developed economies. The business cycle fluctuations as defined can be interpreted as fluctuations in real Gross Domestic Product (GDP) not attributable either to long-run growth, or to measurement errors or to other short-run fluctuations usually not related to the business cycle phenomena. Thus, are just deviations (not erratic) of GDP on a long-run trend (statistically well defined). This definition has been extensively used in literature (see, e.g., Stock and Watson 1999).

The smooth growth component of output (henceforth smooth growth) is defined as output growth excluding fluctuations with period less than one year. The smooth growth is a measure of GDP growth cleaned of erratic or short-run oscillations that make difficult to assess the aggregate economic situation. This signal was approximated by Altissimo et al. (2007) in the construction of a coincident indicator for the euro area, the EuroCoin (denoted as the medium to long run component of output growth).

The signals just defined can be approximated with arbitrary precision by applying infinite moving averages (or filters) to the series of interest, but this requires the knowledge of all past and future values of that series. Extraction in real-time is therefore restricted by the availability of data and is thus a difficult task. Chart 1 presents approximations to the signals of interest for the euro area, obtained with the filter developed by Baxter and King (1999) (BK filter) in the middle of the sample. Additional (past and future) data would lead to negligible differences between these estimates and the desired signal. Using the BK filter implies, however, loosing relevant estimates at the beginning and end of the sample, whereas our interest is in obtaining estimates of these signals in real-time (e.g., obtain estimates of the signals for the first quarter of 2009 using only the available data up to that moment).

We will show that the methodology here presented has clear advantages in relation to other alternatives, delivering real activity indicators that have several desirable properties:

i) They are timely, since our approach is flexible enough so as to take into account the release delays of all the variables used in the exercise; both that of GDP, which typically arrives three months after the end of the quarter to which it refers, and also of all the other series included in our panel of predictors;
ii) They display little short-run oscillations, thereby giving a clear picture of current cyclical and growth prospects;

iii) Unlike to what usually happens, they are based on a large and comprehensive panel of predictors, whose idiosyncratic and short-run components are eliminated through factor analysis;

iv) They have a remarkable forecasting performance at short horizons (less than one year). To be specific, we view forecasts of the smooth growth indicator as being useful to forecast GDP itself. We highlight and important insight: for forecasting purposes, targeting a smooth version of time series may be more useful than targeting the original series. We offer a possible justification: with conventional models, short-run fluctuations are being approximated despite being possibly unpredictable or idiosyncratic.

Following Stock and Watson (1989), model-based methods assuming a factor structure have also been used to construct growth indicators. There have also been attempts to extract signals similar to those we target in a parametric setting. For instance, Valle e Azevedo, Koopman and Rua (2006) constructed a business cycle indicator which can be seen has a multivariate filter (that eliminates undesirable fluctuations). Although a common factor structure is assumed to describe a small set of time series, the representation is far from general and the method is not aimed at approximating a predefined set of fluctuations. Such is the aim of this paper. The approach closest to ours is that of Altissimo et al. (2007) that resulted in the New EuroCoin indicator. This indicator is obtained by projecting estimated factors on the smooth component of output growth, disregarding the information contained in past observations of GDP. We will later contrast in more detail our approach to theirs.
2. APPROXIMATIONS TO THE SIGNALS OF INTEREST

2.1. How to approximate the signals of interest

Our variable of interest throughout the paper will be (logarithm of) real quarterly GDP, the best available proxy of aggregate economic activity. Define \( x_t \) as the logarithm of real GDP and \( \Delta x_t = x_t - x_{t-1} \) as its growth rate. To be general, suppose that we are interested in isolating the signal \( y_t \) that defines a signal in \( x_t \) (\( \Delta x_t \)). Unlike almost all of the literature (see Section 4.3.1 for an exception and criticisms), we will explore information contained in an arbitrary number of additional series to approximate the signals of interest. Suppose that are available \( c \) series of covariates \( z_1, \ldots, z_c \).

The estimate \( \hat{y}_t \) of the signal \( y_t \) (for example, business cycle fluctuations) will be a weighted sum of elements of \( x \) (or \( \Delta x \)) and elements of \( z_1, \ldots, z_c \):

\[
\hat{y}_t = \sum_{j=-f}^{p} \hat{B}_j x_{t-j} + \sum_{s=1}^{c} \sum_{j=-f}^{p} \hat{R}_{s,j} z_{s,t-j}
\]

or, in the case we are interested in \( y_t \) defined in \( \Delta x_t \) (as smooth growth):

\[
\hat{y}_t = \sum_{j=-f}^{p} \hat{B}_j \Delta x_{t-j} + \sum_{s=1}^{c} \sum_{j=-f}^{p} \hat{R}_{s,j} z_{s,t-j}
\]

where \( p \) denotes the number of observations in the past that are considered and \( f \) the number of observations in the future that are considered. To obtain the estimate of \( y_t \), we choose the weights \( \{\hat{B}_j, \hat{R}_{1,j}, \ldots, \hat{R}_{c,j}\} \) associated with the series of interest and the available covariates that minimize the mean squared error between the estimates \( \hat{y}_t \) and \( y_t \).

Note \( f \) is allowed to be negative, which is of particular interest if at time \( T \) (say, the current quarter) the series of interest \( x_t \) (or \( \Delta x_t \)) is not available. Thus, is straightforward to extract the signal \( y_{T+k} \) for \( k > 0 \). One just needs to set \( f = -k \) in the solution, so that only the available information (that is, up to period \( T \) in this case) is taken into consideration.

In the remainder of the paper, we will set \( p = 50 \) (larger values of \( p \) lead to negligible differences in the approximations). To approximate the signals of interest in real-time we set either \( f = -2 \) (in a first or second month of a given quarter) or \( f = -1 \) (in the third month of a given quarter), taking thus into account the release delays of the final estimate of GDP for the euro area.

The signals that we approximate (smooth growth and business cycle fluctuations) have a quarterly frequency (as GDP), but we will seek in each month an estimate of the current quarter. Thus, within each quarter, new monthly information will be used to update the estimates. It turns out that this additional infra-quarterly information is useful in that it provides more precise estimates of the signals of interest.

As a by-product, we will seek in each month an estimate of current year GDP growth based on the most up-to-date approximations to smooth growth and its relevant forecasts.

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(1) So, e.g., in January we will target the signals regarding the first quarter, in April those of the second quarter and in September those of the third quarter. It would be trivial to target in each month any other quarter.
3. COVARIATES

3.1. Factor Model

In the real-time approximations to the macroeconomic signals presented in this paper, the quarterly covariates, \( z_1, \ldots, z_c \), will be series derived from (estimated) common factors, that summarize the information contained in a large panel of monthly time series.\(^2\) Consider the following panel of monthly variables:

\[
W = \{w_i\} \quad i = 1, \ldots, n; \quad t = 1, \ldots, T^*,
\]

We stress that all the series in the panel are organized in such a way that for month \( t \) they are in fact available. So, if an Industrial Production index refers to a month but it is only and always released one month after, we use the one-period lagged series as an \( w_i^* \). Thus, we effectively take into account the release delays of all the indicators.

Assume that each variable in the panel can be decomposed into a common component, \( \chi_i \), driven by a small number (say \( q \)) of orthogonal shocks (the so-called dynamic factors), and an idiosyncratic component \( \xi_i \), orthogonal to \( \chi_i \), for all \( i = 1, 2, \ldots, n \), at all leads and lags. Specifically:

\[
w_i = \chi_i + \xi_i
\]

where:

\[
\chi_i = \lambda_{1i} F_{1t} + \lambda_{2i} F_{2t} + \ldots + \lambda_{q} F_{rt}
\]

Thus, a potentially (and usually) reduced set of \( F_j \)'s, summarizes an important part of the movement of all the variables (the component \( \chi_i \)), resulting in a possibility of parsimoniously incorporate the information contained in \( W \) in the approximations to the signals of interest.

3.2. Covariates (quarterly)

The common factors \( F_{1t}, F_{2t}, \ldots, F_{rt} \) and their number (\( r \)) can be approximated in a number of ways. For more details on the estimation of the factors, see Valle e Azevedo and Pereira (2008). We will report only the results obtained with the first two estimated static factors (that will be split into 6 quarterly series). Such always delivered the best approximations. In month \( t^* \) (of quarter \( t \)), the set of covariates to be used in the approximations contains 2 estimated static factors as described above (denominated \( \hat{F}_{ij} \)), each split into three quarterly series. We have thus the set of covariates updated with the elements of \( Z_{t^*} \), where:

\[
Z_{t^*} = (\hat{F}_{11}, \hat{F}_{12}, \hat{F}_{13}, \hat{F}_{21}, \hat{F}_{22}, \hat{F}_{23})
\]

and:

\footnote{We thank Giovanni Veronese for providing us with a transformed and realigned version of the dataset used to compute the New EuroCoin Indicator of Altissimo et al. (2007). The dataset encompasses 144 monthly economic variables of national and euro area aggregate economies from May of 1987 through August of 2005 (7* = 220).}
As a reminder, our targets are defined on the GDP of the current quarter, but we compute (or update) the approximations every month.

4. PERFORMANCE OF THE INDICATORS

4.1. Evaluation of the indicators in a pseudo real-time exercise

The proposed indicators of business cycle fluctuations and smooth growth will be assessed by analysing their real-time performance. Specifically, and in line with Orphanides and Van Norden (2002), we will look at the approximation errors observed by using our method in real-time. These approximation errors can be estimated comparing these estimates with those obtained by considering future data of the series of interest. Obviously, once new data is available, the approximations that explore new information vary near the end of the sample. This variation is due to revisions in the data itself, which we do not analyse here, and revisions due to the nature of the one-sided filters used in the end of the sample (and in our case, along with re-estimation of moments and factors). The magnitude of the revisions is often large, even in a multivariate context (Orphanides and Van Norden 2002). The fact that the filters’ performance deteriorates near the end of the sample is not conceptually different from the fact that forecasts generally deteriorate if the forecast horizon is larger. Any approximation to a signal (or yet unobserved variable) will suffer revisions. Our approach is an attempt to mitigate revisions (or approximation errors) in the estimates of signals that we believe are relevant for the policy-maker.

We now make explicit in what dimensions our exercise can be seen as a real-time exercise. We make all the necessary transformations in the data, estimate factors, estimate the second moments necessary to solve the projection problem and compute the filter weights in real-time. Further, we take into account all the data release delays.3 Several statistics will be computed to compare the real-time estimates with the estimates obtained in the "middle" of the sample, that we will denote as "final" estimates. These final estimates are again obtained by approximating the signals using the whole sample and then disregarding a sufficient number of observations, to ensure that only negligible revisions will occur once more data becomes available. The criteria used to evaluate the real-time performance of the indicators were the following:

a) Corr, \[ \left[ y_t, \hat{y}_t \right], \] where \( \hat{y}_t \) is the optimal approximation to the signal \( y_t \) and, as it can be proved, is a good measure of the variance of the approximation error. We compute the sample counterpart of this statistic, using the estimated signal (say \( \hat{y}'_t \)) as \( \hat{y}_t \) and approximating \( y_t \) by the "final" estimates, denoted by \( y^F_t \);

b) Noise to Signal Ratio, computed as \( \frac{\sum (\hat{y}'_t - y^F_t)^2}{\sum (y^F_t - \bar{y}^F)^2} \);

As an example, euro area GDP of a given quarter is only available in the third month of the next quarter. So, in the first two months of that quarter, we only use data until the latest observation of GDP, which refers to two quarters before.
c) For the approximations to business cycle fluctuations, the percentage of times $\hat{y}_t^*$ and $y_t^F$ share the same sign (which gives an indication on whether $\hat{y}_t^*$ indicates correctly if GDP is below or above the long-run trend).

The benchmark will be the univariate filter of Christiano and Fitzgerald (2003). Additionally, we stress that the estimation of second order moments and factors will take into account only information available at each point (real-time approximation) or, alternatively, it uses the whole sample (today onwards approximation), while still setting in the filter $f = -1$ in the third month of each quarter and $f = -2$ in the first and second months. With this exercise, we hope to understand the revisions stemming from second moments and factor space uncertainty. We expect these revisions to be less severe as the sample size grows, whereas the real-time approximations include poorly estimated objects at least in the beginning of the evaluation period.

4.2. Business cycle fluctuations

First, we report the real-time evaluation of our approximations to business cycle fluctuations in the euro area. Table 1 contains the evaluation statistics, just mentioned, for approximations done in a third month of the quarter and the variations considered. Additionally, Table 2 contains the evaluation of the approximations done in the first and second month of the quarter for a selection of the best performing approximations (in real-time and today onwards) of Table 1.

The main conclusions are:

- Both the univariate and multivariate filters perform very well in the (nonetheless short) evaluation period;

- The fit of the today onwards approximations is similar to that of real-time approximations;

- The results obtained with factors estimated by principal components (PC) are very similar to those obtained with generalized principal components (GPC);

- The quality of the approximations is very similar across the months of the quarter, but the multivariate filter performs superiorly in the first two months.

Chart 2 displays the best performing real-time and today onwards approximations (multivariate filter with 2 factors, MBPF PC KERNEL) in a third month of the quarter as well as the final estimates of business cycle fluctuations for the euro area. Additionally, Chart 3 compares the final estimates to the best multivariate approximations in the third month of the quarter when 4, 3, ..., 0 quarterly observations of GDP as well as the corresponding monthly series of the panel are missing, and also when 1, 2, ..., 5 additional quarters of information are available. All the measures improve as more data becomes available and in all cases the multivariate filter has by far the best performance. The differences across methods tend to disappear after 5 additional quarters of data are considered. We notice again that the performance of the univariate filter is very good and similar to that of the multivariate filters, with the latter performing superiorly when additional data becomes available. These approximations with additional data are in practice relevant given the fact that one is interested in detecting a signal that has (persistent) fluctuations with period between 6 and 32 quarters.
### Table 1

**EVALUATION STATISTICS FOR THE APPROXIMATIONS TO BUSINESS CYCLE FLUCTUATIONS IN THE EURO AREA IN THE THIRD MONTH OF THE QUARTER**

Evaluation period: 1999(2) - 2004(3)

<table>
<thead>
<tr>
<th>Performance with respect to Business Cycle fluctuations (3rd month of the quarter)</th>
<th>Correlation</th>
<th>Noise to Signal</th>
<th>Sign Concordance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real</td>
<td>Today</td>
<td>Onwards</td>
</tr>
<tr>
<td>Benchmark Filters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BPF AR</td>
<td>0.87</td>
<td>0.89</td>
<td>0.41</td>
</tr>
<tr>
<td>BPF KERNEL</td>
<td>0.87</td>
<td>0.88</td>
<td>0.43</td>
</tr>
<tr>
<td>with 2 factors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MBPF PC KERNEL</td>
<td>0.86</td>
<td>0.89</td>
<td>0.46</td>
</tr>
<tr>
<td>MBPF GPC KERNEL</td>
<td>0.85</td>
<td>0.88</td>
<td>0.48</td>
</tr>
</tbody>
</table>

**Source:** Authors’ calculations.

**Note:**
- BPF AR - univariate filter with second moments estimated by AR model (BIC criterion for lag length);
- BPF KERNEL - univariate filter with second moments estimated non-parametrically;
- MBPF - multivariate band-pass filter; PC - factor space estimated by principal components; GPC - factor space estimated by generalized principal components;
- KERNEL - non-parametric estimation of second moments.

### Table 2

**EVALUATION STATISTICS IN EVERY MONTH OF THE QUARTER, FOR THE APPROXIMATIONS TO BUSINESS FLUCTUATIONS IN THE EURO AREA**

Evaluation period: 1999(2) - 2004(3)

<table>
<thead>
<tr>
<th>Performance with respect to Business Cycle fluctuations</th>
<th>Correlation</th>
<th>Noise to Signal</th>
<th>Sign Concordance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real</td>
<td>Today</td>
<td>Onwards</td>
</tr>
<tr>
<td>BPF AR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st/2nd month</td>
<td>0.85</td>
<td>0.88</td>
<td>0.46</td>
</tr>
<tr>
<td>3rd month</td>
<td>0.87</td>
<td>0.89</td>
<td>0.41</td>
</tr>
<tr>
<td>MBPF PC KERNEL</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2 factors)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st month</td>
<td>0.88</td>
<td>0.89</td>
<td>0.41</td>
</tr>
<tr>
<td>2nd month</td>
<td>0.87</td>
<td>0.88</td>
<td>0.42</td>
</tr>
<tr>
<td>3rd month</td>
<td>0.86</td>
<td>0.89</td>
<td>0.46</td>
</tr>
</tbody>
</table>

**Source:** Authors’ calculations.

**Note:**
- BPF AR - univariate filter with second moments estimated by AR model (BIC criterion for lag length);
- MBPF PC KERNEL - multivariate band-pass filter; factor space estimated by principal components and non-parametric estimation of second order moments.
4.3. Smooth growth

This subsection focuses on the real-time evaluation of the approximations to smooth growth in the euro area. Table 3 contains the evaluation statistics for the approximations done in a third month of the quarter and considering all the variations under analysis. Additionally, Table 4 contains the evaluation of the approximations done in the first and second month of the quarter for a selection of the best-performing approximations (in real-time and today onwards) of Table 3.
The main conclusions are:

- The multivariate filter clearly outperforms the univariate filters in all the dimensions under consideration. Loosing one observation of GDP, as in a first or second month of the quarter, completely deteriorates the performance of the univariate filter but not so much that of the multivariate filter;

### Table 3

**EVALUATION STATISTICS IN A THIRD MONTH OF THE QUARTER, FOR THE APPROXIMATION TO SMOOTH GROWTH IN THE EURO AREA**

Evaluation period: 1996(4) - 2004(3)

<table>
<thead>
<tr>
<th>Performance with respect to Smooth Growth (3rd month of the quarter) (a)</th>
<th>Correlation</th>
<th>Noise to Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real Time</td>
<td>Today Onwards</td>
</tr>
<tr>
<td>Benchmark Filters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BPF AR</td>
<td>0.77</td>
<td>0.78</td>
</tr>
<tr>
<td>BPF KERNEL</td>
<td>0.77</td>
<td>0.78</td>
</tr>
<tr>
<td>with 2 factors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MBPF PC KERNEL</td>
<td>0.81</td>
<td>0.86</td>
</tr>
<tr>
<td>MBPF GPC KERNEL</td>
<td>0.79</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Source: Authors' calculations.

Note: (a) BPF AR - univariate filter with second moments estimated by AR model (BIC criterion for lag length); MBPF - multivariate band-pass filter; PC - factor space estimated by principal components; GPC - factor space estimated by generalized principal components; KERNEL - non-parametric estimation of second moments.

### Table 4

**EVALUATION STATISTICS IN EVERY MONTH OF THE QUARTER, FOR THE APPROXIMATION TO SMOOTH GROWTH IN THE EURO AREA**

Evaluation period: 1996(4) - 2004(3)

<table>
<thead>
<tr>
<th>Performance with respect to Smooth Growth (a)</th>
<th>Correlation</th>
<th>Noise to Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real Time</td>
<td>Today Onwards</td>
</tr>
<tr>
<td>BPF AR</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st/2nd month</td>
<td>0.38</td>
<td>0.42</td>
</tr>
<tr>
<td>3rd month</td>
<td>0.77</td>
<td>0.78</td>
</tr>
<tr>
<td>MBPF PC KERNEL (2 factors)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st month</td>
<td>0.60</td>
<td>0.75</td>
</tr>
<tr>
<td>2nd month</td>
<td>0.70</td>
<td>0.78</td>
</tr>
<tr>
<td>3rd month</td>
<td>0.81</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations.

Note: (a) BPF AR - univariate filter with second moments estimated by AR model (BIC criterion for lag length); MBPF PC KERNEL - multivariate band-pass filter, factor space estimated by principal components and non-parametric estimation of second moments.

The main conclusions are:
- As referred, the multivariate approximations are more accurate in the third month of the quarter, followed by the second month and then the first. In contrast to the case of business cycle fluctuations, there are now considerable gains if factors are used in the approximations, in particular in the first two months of the quarter;

- The fit of the *today onwards* approximations is very high for the multivariate filters. We found that using exactly 2 monthly factors, split into 6 quarterly series, produced always the best results in *real-time*. Perhaps a larger time dimension would be needed to usefully incorporate a larger number of factors, given the clearly distinct performance of the in-sample (*today onwards*) and out-of-sample (*real-time*) approximations, as documented in Valle e Azevedo and Pereira (2008);

- As before, the results obtained with principal components (PC) are very similar to those obtained with generalized principal components (GPC).

To further inspect the quality of the approximations, Chart 4 displays the final estimates of euro area smooth growth as well as the best performing *real-time* and *today onwards* approximations in a third month of each quarter. As is clear, the multivariate indicator tracks very accurately the signal. Additionally, in Chart 5 is analysed the behaviour of the best indicators in a third month of the quarter when 4, 3, ..., 0 quarterly observations of GDP as well as from the covariates are missing (so we are also analysing the forecasting performance of the approximations to the target), and also when 1, 2, ..., 5 additional quarters of information are available. The main conclusion is that the multivariate approximations still clearly outperform the univariate filter, while giving an informative signal (noise to signal ratio below 1) even when 3/4 quarters of data are missing.

Chart 4

![Chart 4](image-url)

**SMOOTH GROWTH OF EURO AREA GDP FINAL ESTIMATES AND REAL-TIME AND TODAY ONWARDS APPROXIMATIONS (MBPF PC KERNEL, \( k = 2 \) MONTHLY FACTORS)**

**Evaluation period: 1996(4)-2004(3)**

*Source: Authors' calculations.*
4.3.1. Comparison with EuroCoin indicator

The New EuroCoin indicator of Altissimo et al. (2007) targets a monthly measure of quarterly GDP growth free of fluctuations with period less than one year (denoted there as the medium to long run component of output growth). This monthly measure of quarterly GDP is obtained through linear interpolation of quarterly figures, which we view as undesirable for statistical and aesthetic reasons. We target instead quarterly GDP growth short of fluctuations with period less than one year in the three months of each quarter.

The New EuroCoin is obtained by projecting smooth monthly factors on the medium to long run component of output growth. These smooth monthly factors do not contain fluctuations with period less than 12 months. The objective is to have an indicator free of short-run oscillations, just as the target. In our approach this step is not needed and would be undesirable since every vintage of our indicator (that uses all the available information) is by definition smooth, although subject to revisions. The multivariate filter used optimally mitigates those revisions. Furthermore, we can explore the infra-quarter dynamics that arises from the partition of the monthly factors, instrumental to update efficiently the indicator in the three months of each quarter. The series obtained from splitting smooth factors would be almost collinear. But more importantly, we are able to incorporate the available observations of GDP in our solution. It turns out that these are the single most important observations in the approximations (as is easily seen by analysing the performance of the univariate filter).

Chart 5

EVALUATION OF EURO AREA REAL-TIME APPROXIMATIONS TO SMOOTH GROWTH. CORRELATION WITH FINAL ESTIMATES AND NOISE TO SIGNAL RATIO WHEN FUTURE QUARTERS OF DATA ARE CONSIDERED (MBBF PC KERNEL WITH 2 MONTHLY FACTORS)
Evaluation period: 1996(4)-2004(3)

Source: Authors’ calculations.
Note: In the horizontal axis -1 represents the real-time estimate (recall, in the filter \( f = -1 \) in a third month of a quarter since the latest available GDP is from the previous quarter, results are already in Table 4), 1 represents the estimate obtained when one future data point is available and so forth.
5. FORECASTING PERFORMANCE

5.1. Quarterly growth

In this section we report results concerning the capability that real-time approximations to smooth growth have in forecasting quarterly GDP growth. To be clear, we will approximate smooth growth at various horizons (quarters ahead) and compare its estimates with the actual observations of quarterly GDP growth. Although our main objective is to approximate a specific signal, we can justify the use of the method for forecasting if we assume from the onset the impossibility of forecasting the high frequencies of GDP growth (those with period less than one year) given a set of covariates. In theory, if the noisy (or for this matter any other) fluctuations of a time series are unpredictable given a set of covariates, it is not ideal to develop models aimed at approximating them as well. Focusing on approximations to the predictable component of the series may lead to a superior forecasting performance if the assumed restriction (unpredictability at some frequencies) in fact holds. This idea is still far from developed but we believe that important insights are given by our exercise. The exercise focuses on forecasts made in the third month of the quarter, from 1992(1) to 2004(4), considering as competing methods the following:

- Univariate approximation to smooth growth with non-parametric estimation of second moments, denoted BPF KERNEL;

- Univariate approximation to smooth growth with second moments derived from an autoregressive model (BIC criterion for lag length), denoted BPF AR;

- Multivariate approximation to smooth growth with second moments estimated non-parametrically and 2 split monthly factors (estimated by principal components) as covariates, denoted MBPF PC KERNEL;

- The linear projection of 2 split monthly factors (estimated by principal components) on GDP growth, obtained with second moments estimated as in MBPF PC KERNEL, denoted PC KERNEL;

- Regression of GDP on (a maximum of 2) split factors estimated by principal components and past GDP, with lag length determined by the BIC criterion, exactly as in Stock and Watson (2002), denoted DI - AR SW. More factors did not lead to a better performance.

The results are presented in Table 5. The main conclusions are as follows:

- The multivariate approximation to smooth growth ranks very well at one and two steps ahead, dominating overall the no-smoothing comparable approximation, PC KERNEL. That is, using exactly the same estimated second moments while targeting smooth growth instead of GDP growth leads to a superior forecasting performance. This is also generally true for the univariate approximations to smooth growth, but in this case the gains are only relevant at one quarter ahead;

- The regression with factors (DI AR - SW) performs poorly in the euro area;
In the case of 3 quarters ahead (or more, results not reported), all methods perform rather poorly, confirming the well-known difficulty of forecasting quarterly GDP growth at long horizons (for recent overviews, see Runstler et al. 2008, and Angelini et al., 2008). In fact, at 3 quarters ahead the root mean squared error (RMSE) of the autoregressive (AR) forecast is basically the standard deviation of GDP growth, informing us that we are as better off with the mean growth rate as forecast.

Overall, the results suggest that our multivariate approximation to smooth growth, while designed for a different purpose, is useful for short-term forecasting of GDP quarterly growth. Additionally, this approximation can have a much more striking role in the forecasting of yearly GDP growth. First, the results in Chart 5 reveal that our approximation to smooth growth at longer horizons (up to 4 quarters) still reveals some of the signal, whereas the forecasts of quarterly growth in Table 5 reveal the uselessness of all the methods at more than (at most) 3 quarters ahead. Second, the yearly growth rate implied by the quarterly smooth growth rates is indistinguishable from the yearly growth rate implied by the quarterly growth rates (see Chart 6). Thus, by approximating accurately smooth growth at various horizons, we will approximate accurately yearly growth. If we use instead useless forecasts of quarterly growth to forecast yearly GDP growth, the results are not likely to be promising. The next subsection analyses this issue in more detail, presenting the evaluation of forecasts of yearly GDP growth made throughout the months of the year by various methods.

5.2. Yearly growth

In this subsection we evaluate forecasts of yearly GDP growth at various points throughout the year using various methods. We do so by forecasting the missing quarterly GDP growth rates and then deriving the implied yearly growth. When using our approximations to smooth growth, we derive yearly growth from the most up-to-date approximations to smooth growth of the relevant quarters (Chart 6 justifies this approach). For the methods that do not target smooth growth, yearly growth rates are de-
derived from known quarterly rates as well as from the relevant forecasts of quarterly growth. In order to be fair in the comparisons, we substituted the useless (worse than the mean in the evaluation period) quarterly growth forecasts by real-time estimates of the mean growth rate of quarterly GDP. Table 6 displays the results for forecasts made in the end of each quarter. In the end of the first quarter (March), no information on quarterly GDP of the current year is available, so that forecasts for the four quarters are necessary. In the end of the fourth quarter (December) we have information on GDP growth from the first three quarters.

Table 7 analyses the forecasting performance of our approximation in the 12 months of the year and Chart 7 shows the forecasts made in March, June, September and December with the actual observations of yearly GDP growth. The results can be summarized as follows:

Table 6

<table>
<thead>
<tr>
<th>Methods</th>
<th>1st quarter</th>
<th>2nd quarter</th>
<th>3rd quarter</th>
<th>4th quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPF AR</td>
<td>1.00</td>
<td>0.98</td>
<td>1.12</td>
<td>0.72</td>
</tr>
<tr>
<td>MBPF PC KERNEL</td>
<td>1.02</td>
<td>0.83</td>
<td>0.99</td>
<td>0.81</td>
</tr>
<tr>
<td>DI AR - SW</td>
<td>2.08</td>
<td>1.76</td>
<td>2.23</td>
<td>3.56</td>
</tr>
<tr>
<td>RMSE, AR</td>
<td>0.0069</td>
<td>0.0046</td>
<td>0.0018</td>
<td>0.0007</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations.
The results obtained with factor regression (DI AR - SW) are rather poor. There are significant gains if the multivariate filter (MBPF PC KERNEL) is used to forecast yearly growth while the univariate filter (BPF AR) also performs well;

Table 7 reveals that the quality of the forecasts improves mainly due to the release of GDP quarterly figures, as evidenced by the significant decrease in the RMSE of the approximations done in the end of each quarter, exactly when GDP from the previous quarter becomes available. Still, within each quarter new monthly information generally improves the forecasts. We note that only in a multivariate context can we explore this information;

In the period under analysis, only after June can we have a clear and accurate picture of current year growth. Nonetheless, the 1993 recession is clearly well predicted by the end of March.

6. CONCLUSIONS

We have shown how to usefully integrate the recent developments in the analysis of dynamic factor models in the approximation to relevant macroeconomic signals. The resulting multivariate filter, fuelled with factors extracted from a large panel of time series, is reliable and clearly outperforms in various dimensions the optimal univariate approximation. Further, it possesses several advantages over similar (multivariate) attempts to track macroeconomic signals in real-time.

We have analysed in detail the approximations to two relevant macroeconomic signals related to real activity in the euro area: business cycle fluctuations and the smooth growth of real GDP. Our exercise provided real-time approximations that take fully into account the data release delays of all the variables involved. In the analysis of the forecasting performance of smooth growth, we have highlighted...
an important insight: targeting a smooth version of a time series may be more useful than targeting the sometimes erratic (or unpredictable at high frequencies) original series. The conventional forecasting models fit the variables of interest at every frequency, regardless of the predictive content of the available covariates at each frequency. In the future, we plan to further explore this idea.

REFERENCES


