1. INTRODUCTION

Understanding the patterns and determinants of inflation persistence is very important for policymakers, because inflation persistence has immediate consequences in the conducting of monetary policy. For instance, the appropriate response to shocks depends on the degree to which their effect on inflation is persistent. Furthermore, the horizon at which monetary policy should aim for price stability depends on the persistence of inflation: with less persistence, inflation can be stabilised in a shorter time following a shock. Accordingly, the degree of inflation persistence is an important factor determining the medium-term orientation of monetary policy.

This paper is a contribution to a recently growing literature which discusses and measures inflation persistence in the context of a simple univariate time-series representation of inflation. The paper discusses the definition of persistence and its implications for the process of persistence evaluation. The need for a proper treatment of the mean of inflation is emphasised, especially the idea that it should be seen as exogenous to the model and allowed to vary over time. The paper also suggests a new measure of persistence which is based on the correspondence between persistence and mean reversion. This new measure has the advantage that it does not require specifying and estimating a model for the inflation process.

This new methodology, including the use of the new measure of persistence, is applied to inflation in the U.S.. It is shown that the evidence on inflation persistence dramatically changes with the assumption on the mean of inflation. In particular, the widespread accepted wisdom that inflation has been more persistent in the sixties and seventies than in the last twenty years is only obtained for the special case of a constant mean, which however, appears to be a counterfactual assumption.

The rest of the paper is organised as follows. Section 2 discusses some issues concerning the definition and measurement of inflation persistence and makes the case for a time varying mean. Section 3 suggests an alternative, simple and intuitive measure of inflation persistence that explores the relationship between mean reversion and persistence. Section 4 re-evaluates the evidence on inflation persistence for the U.S. allowing for a time varying mean and section 5 concludes.

2. DEFINING AND MEASURING INFLATION PERSISTENCE: SOME METHODOLOGICAL ISSUES

For the purpose of this paper we define persistence as the “speed with which inflation converges to equilibrium after a shock”. Such a definition of persistence is similar to alternative definitions available in the literature (see, for instance, Willis...
(2003) or Pivetta and Reis (2003)) but has the advantage of stressing two important ideas: the idea of speed and the idea of equilibrium. If the speed of convergence to the equilibrium after a shock is low we say that inflation is persistent while if the speed is high we say that inflation is not persistent.

One important point when computing inflation persistence regards whether one should assume that the equilibrium level of inflation is exogenous or endogenous to the hypothesised shock to inflation. In the context of a univariate time-series representation of inflation, inflation persistence is computed under the assumption that shocks (to the model) do not affect the mean of the series, so that the long run level of inflation or the central bank inflation target must be seen as exogenous to the shocks (1). Thus, in this framework, evaluating inflation persistence amounts to find an answer to the following question: how slowly does inflation converge to the exogenous central bank inflation target, in response to a shock?

A second important point worth stressing that follows from the definition of persistence is the fact that any estimate of persistence must be seen as conditional on the assumed long-run inflation path. As we shall see below, there is a trade-off between persistence and the degree of flexibility of the assumed long run equilibrium level of inflation: for a given series, we obtain the maximum level of persistence under the assumption of a constant mean, but we can make persistence to converge to zero if we allow enough flexibility to enter into our measure of the long run level of inflation. Thus, it is important to bear in mind that any given estimate of persistence crucially depends on the specific long run level of inflation assumed in its computation and that, as a consequence, the reliability of such estimate ultimately depends on how realistic the assumed long run inflation path is.

The literature has to some extent recognized the liaison between persistence and the way the mean of the series is treated, and has tried to deal with the problem by identifying some structural breaks in the mean of the series using statistical tests (see, for instance, Burdekin and Syklos (1999), Bleaney (2001), Levin and Piger, (2004), O’Reilly and Whelan (2004)). Usually such papers start by investigating persistence assuming a constant mean and then, proceed by testing for a break (or breaks) in that mean level. The general conclusion is that persistence is significantly reduced once breaks in the mean are accounted for. A major limitation of such an approach is that it restricts the mean to be a “constant function” or a “piecewise constant function” so that there would always remain the question of whether the estimated degree of persistence is a real feature of the data or rather a spurious result brought about by these two simple assumptions for the mean. Moreover, it can be argued that we cannot rely exclusively on statistical tests to decide how realistic from an economic point of view is our estimated mean.

As an alternative approach we suggest allowing for the possibility of a time varying mean computed outside the estimated model. At least for some European countries, during the convergence period, that took place in the eighties and the nineties, allowing for the possibility of a time varying mean appears clearly a more realistic alternative (to account for a time varying central bank inflation target) than simply allowing for some discrete breaks in the mean. As we shall see below, this has significant implications for the degree of estimated persistence.

The existence of a time varying mean may be the result of the fact that central banks sometimes change their decision rules or allow inflation to drift. The adoption of new decision rules may be justified in the context of models in which the central bank and private agents are assumed not to know the equilibrium population moments and thus use adaptive methods to learn about the world in which they live (see, for instance, Cogley 2002). As an alternative explanation one may also think of central banks allowing inflation to drift, because of economic or political constraints (2).

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(1) We notice that in the context of the univariate time series representation of inflation, the mean of inflation is the level for which inflation converges after a shock, so that the mean of the series plays the role of the long run equilibrium level of inflation. Moreover, if we assume that in the long run, inflation is determined by monetary policy we can see the long run level of inflation as corresponding to the central bank (implicit or explicit) inflation target. Thus, in what follows the expressions “long run level of inflation”, “central bank inflation target” and “mean of inflation” are used interchangeably.
Bellow, in section 4, we compute persistence conditional on different hypotheses for a time varying mean, which include simple linear trends and the HP filter. Such an exercise allows us to show that, as expected, the estimates of persistence crucially depend on the underlying assumed mean.

3. AN ALTERNATIVE MEASURE OF PERSISTENCE

In this section we suggest a new simple and intuitive measure of persistence, which explores the relationship between persistence and mean reversion. We start by highlighting the relationship between persistence and mean reversion, as it allows a deeper understanding of what persistence implies in terms of the time path for any given stationary time series and helps us to better understand the intuition behind the alternative measure of persistence suggested below.

Let us assume that inflation follows a stationary autoregressive process of order p (AR(p)), which we write as:

$$y_t = \alpha + \sum_{j=1}^{p} \beta_j y_{t-j} + \varepsilon_t$$  (3.1)

and reparameterise as:

$$\Delta y_t = \sum_{j=1}^{p} \delta_j \Delta y_{t-j} + (\rho - 1)[y_{t-1} - \mu] + \varepsilon_t$$  (3.2)

where

$$\rho = \sum_{j=1}^{p} \beta_j$$  (3.3)

$$\delta_j = \sum_{j=1}^{p} \beta_j$$  (3.4)

$$\mu = \frac{\alpha}{1 - \rho}$$  (3.5)

In the context of this model, persistence is defined as the speed with which inflation converges to its mean, after a shock in the disturbance term, $\varepsilon_t$. To compute inflation persistence several scalar measures have been proposed in the literature. These include the “sum of the autoregressive coefficients” $\rho$, as defined in (3.3), but also other measures such as the “spectrum at zero frequency”, the “largest autoregressive root” and the “half-life”(3).

Let us now assume that $y_t$ is a stationary process with $0 < \rho < 1$. One identifying characteristic of any stationary process is that it must exhibit mean-reversion. In equation (3.2) the presence of mean reversion is reflected in the term $(\rho - 1)[y_{t-1} - \mu]$. This implies that if in period (t-1) the series $y$ is above (below) the mean, the deviation $[y_{t-1} - \mu]$ will contribute as a “driving force” to a negative (positive) change of the series in the following period, through the coefficient $(\rho - 1)$, thus bringing it closer to the mean. Of course mean reversion is stronger the larger (in absolute terms) the coefficient $\lambda = (\rho - 1)$. Once we measure persistence by $\rho$ and mean reversion by $\lambda = (\rho - 1)$ we conclude that mean reversion and persistence are inversely related: high persistence implies low mean reversion and vice-versa.

This correspondence between persistence and mean reversion allows us to carry out a simple preliminary evaluation of persistence by visual inspection of two different series: in a graph with two stationary series the one exhibiting the lowest mean reversion, that is the one that crosses the mean less frequently, is the one exhibiting more persistence.

We may now introduce a new measure of persistence, which we denote by $\gamma$, and define as the unconditional probability of a given stationary process not crossing its mean in period $t$, or equivalently as $1$ minus the probability of mean reversion of the process. A natural estimator of $\gamma$ is given by

$$\hat{\gamma} = 1 - \frac{n}{T}$$  (3.6)

where $n$ stands for the number of times the series crosses the mean during a time interval with $T+1$ observations.

(2) During the seventies and the eighties, balance of payments constraints in some countries were sometimes seen as more important than negative consequences emerging from an inflationary environment, so that exchange rate policies were implemented to correct external imbalances even though at the expense of higher future inflation. In the context of our approach similar situations are seen as implying a time varying long run level of inflation consistent with a time varying (implicit) central bank inflation target.

(3) For the definition of these measures of persistence and a detailed discussion of their major limitations, see Andrews and Chen (1994) and Marques (2004).
Intuitively, the use of $\gamma$ as a measure of persistence may be justified as a simple implication following directly from the very definition of persistence. If a persistent series is the one which converges slowly to its equilibrium level (i.e., the mean) after a shock, then such a series, by definition, must exhibit a low level of mean reversion, i.e., must cross its mean only infrequently. Similarly, a non-persistent series must revert to its mean very frequently. And particularly, a non-persistent series must cross its mean in period $t$, and equal to zero otherwise. Now we have $\hat{\gamma} = 1 - \bar{x}$ so that $\hat{\gamma}$ can be computed by regressing $x_t$ on a constant, i.e., by estimating the model $x_t = \alpha + \nu_t$ by OLS, from which we get $\hat{\alpha} = \bar{x} = 1 - \hat{\gamma}$. Now suppose we are investigating persistence for the period $t = 1, 2, \ldots, T$ and we want to test whether there is a change in persistence occurring in period $t = s$, such that persistence for the sub-period $t = 1, 2, \ldots, s - 1$ differs from persistence for the sub-period $t = s, s + 1, \ldots, T$. We may estimate the model

$$x_t = \alpha_1 + \alpha_2 d_t + u_t \quad (3.8)$$

where $d_t$ is a dummy variable which is zero before the date of the break ($t < s$) and equals 1 thereafter ($t \geq s$). In (3.8) we have $\alpha_1 = 1 - \gamma_1$ and $\alpha_2 = \gamma_1 - \gamma_2$ where $\gamma_1$ and $\gamma_2$ are the measures of persistence in the first and second sub-period, respectively. Thus, testing whether persistence has changed from the first to the second sub-period amounts to test whether $\alpha_2$ is significantly different from zero in (3.8). Of course, in general the residuals $u_t$ will be autocorrelated, so that the test of the statistical significance of $\alpha_2$ must be computed based on an autocorrelation consistent estimator for the standard deviation of $\hat{\alpha}_2$.

4. PERSISTENCE AND MEAN REVERSION: RE-EVALUATING INFLATION PERSISTENCE IN THE UNITED STATES

There seems to be a widely accepted view in the literature that inflation has been more persistent during the sixties and seventies than thereafter. For instance, Levin and Piger (2004) write, (5) In a companion paper Dias and Marques (2005) show that in fact $\hat{\gamma}$, for the class of stationary autoregressive processes, is an estimator with better properties than the OLS estimator of $\rho$, namely as regards unbiasedness and robustness against outliers.

(4) See Marques (2004)
“there is widespread agreement that inflation persistence was very high over the period extending from 1965 to the disinflation of the early 1980s. However, there is substantial debate regarding whether inflation persistence continued to be high since the early 1980s, or has declined”(6).

In this section we investigate this claim by re-evaluating inflation persistence for the U.S. We compare the estimates of persistence for the two major sub-periods (the sixties and seventies on the one side, and the eighties and the nineties on the other) that are obtained using first a constant mean for inflation and then some alternative time varying means. As measures of persistence we use $\rho$, the “sum of the autoregressive coefficients”, and $\gamma$, the “unconditional probability of the process not crossing its mean”, introduced in the previous section. Estimates of $\gamma$ are obtained using equation (3.6) while estimates of $\rho$, are obtained by estimating equation (A.2) in the Appendix.

Chart No.1 displays quarterly inflation in the U.S. as from 1960q2 to 2002q4 using GDP deflator. This series has been analysed among others by Taylor (2000), Cogley and Sargent (2001), Pinetta and Reis (2003) and Levin and Piger (2004). Let us start by focussing on the mean of inflation. Simple visual inspection of Chart No.1 suggests that we can basically distinguish three distinct periods. The first period, during which inflation exhibits a clear upward trend, stretches from the beginning of the sample until roughly the end of 1980.

The second period is composed of a very pronounced downward trend that took place during roughly 1981 and 1982. Finally, a third period from 1983 onwards, in which inflation seems not to have exhibited a clear increasing or decreasing trend. Some authors further decompose this latter period into two sub-periods according to the two different average levels of inflation: the first starting in 1983 and ending in mid 1991, (which corresponds to a higher average inflation) and the second covering basically the second half of the eighties to the end of the sample period. For each sub-period, the mean of inflation is assumed constant. The upper panel of Chart No.2 displays inflation as well as the average of inflation for two different sub-periods: 1960q2-1981q4 and 1982q1-2002q4. The lower panel displays the deviations of the series from these two different means. It is the persistence of these series that is analysed in Taylor (2000), with minor differences due to slightly different dates for the cut-off of the series. The conclusion of Taylor (2000) is that persistence has been larger during the first part of the sample(7).

Using $\rho$ as a measure of persistence we get an estimate of 0.92 for the period 1960q2-1981q4 and of 0.73 for the period 1982q1-2002q4 suggesting that persistence may have been higher for the first sub-period (see Table 1). Using $\gamma$ as the measure of persistence we get 0.83 for the first sub-period and 0.80 for the second sub-period.

In formal terms we tested for a change in persistence using both $\gamma$ and $\rho$ as alternative measures of persistence. Tests on $\gamma$ were performed as explained in section 3, by estimating equation (3.8) and computing autocorrelation consistent t-statistics for $\hat{\alpha}_2$. Tests on $\rho$ were performed by estimating models (A.3) to (A.6), which are described in the Appendix.

If we stick to $\gamma$ as the single measure of persistence one would conclude that there is not strong evidence of a significant change in persistence between the two periods (the t-statistic for $\hat{\alpha}_2$ in equation (3.8) is 0.36). The same conclusion is obtained if we use $\rho$ as the measure persistence pro-

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(6) In the same vein see Cogley and Sargent (2001), Willis (2003) and Guerrieri (2002). Against such a view see, however, Pinetta and Reis (2003) and Stock (2001), who argue that there is not enough evidence to conclude for a change in persistence.

(7) Specifically Taylor (2000) considers the periods 1960q2-1979q1 and 1982q1-1999q3, so that the years 1980 and 1981 are excluded from the analysis. The author obtains $\hat{\rho}=0.94$ for the first sub-period and $\hat{\rho}=0.74$ for the second.
vided we stick to models (A.3) and (A.5). However the conclusion, as far as $\rho$ is concerned, is reversed if we rather retain the results of models (A.4) and (A.6). According to these models, which are not likely to suffer from over-parameterisation and thus allow more efficient inference than models (A.3) and (A.5), the null of equal $\rho$s for the two sub-periods can be rejected.

Thus, under the assumption of a constant mean for each sub-period we conclude that inflation in the U.S. appears to have been highly persistent in the sixties and seventies and that there is some evidence (even though highly model dependent) that inflation persistence in the U.S has declined during the last twenty years or so.

The assumption of a constant mean for inflation during each sub-period emerges as the major limitation of the previous approach to persistence evaluation. Most likely, many econometricians would argue that during the first sub-period (1960-1981), rather than exhibiting mean reversion, the GDP inflation series in Chart No.2 is more likely to be a non-mean reverting process. In fact an ADF test for this period reveals that the null of a unit root cannot be rejected thus, casting strong doubts on the usefulness of measuring inflation persistence for the U.S. during this period, assuming a constant mean\(^\text{8}\). Of course, the above tests on the statistical significance for the difference in the estimated $\rho$s and estimated $\gamma$s are not valid if

\(\text{8} \) The ADF statistic is -1.53, so that the null of a unit root in inflation for the sub-period 1960q2-1981q4 cannot be rejected even for a 10% test.
the series of the deviations from the mean is non-stationary.

To see how things can change let us now assume that the mean of inflation during the first two sub-periods (1960q2-1980q4 and 1981q1-1983q1) may be approximated by two linear trends as in Chart No.1. This new possibility is displayed in Chart No.3 (upper panel), which differs from Chart No.1 in that it assumes a constant mean with no break for the whole sub-period 1983q2-2002q4.

Now we have a different picture. If we look at the lower panel of Chart No.3 and think of persistence as the degree of mean reversion, we see that it is no longer so obvious that persistence for the period 1960-1980 has been higher than persistence in the period 1981-2002. In fact, if anything, the results are now the other way round. First, for the whole period we now get an estimate for $\rho$ of 0.58 and for $\gamma$ of 0.70 suggesting the absence of any significant persistence. Second, we get estimates of persistence for the first sub-period, which are lower than the ones for the second sub-period, in contrast with the previous situation. In fact, for the sub-period 1960q2-1980q4 we now have $\rho$ equal to 0.45 and $\gamma$ equal to 0.66 while for the sub-period 1981q1-2002q4 we get $\rho$ equal to 0.79 and $\gamma$ equal to 0.74 (see Table 1). Thus, once we allow for a time varying mean for the period 1960-1983 (proxied by a time trend), we get inflation that, if anything, emerges as less persistent in the sixties and seventies than in eighties and nineties.

It is important to stress this result because it runs against the above-cited widely accepted idea that inflation in the U.S. has been more persistent in the sixties and seventies, than in the last twenty years. Put differently, the so-called widespread evidence claiming that inflation was more persistent in the sixties and seventies crucially depends on the implicit assumption of a time invariant inflation target for inflation in this period, which appears to be a counterfactual assumption. In fact, assuming a constant mean for inflation in the sixties and seventies implies that inflation becomes a unit root process around the central bank inflation target and this, in turn, has the undesirable consequence of implying that monetary policy would be unable to determine inflation in the medium to long run.

Of course this is not the end of the story since by looking at the inflation series we can think of many other reasonable possibilities to measure the mean of inflation. For instance, if we now also assume two different means for the sub-period 1983-2002, as in Levin and Piger (2004) we end up with the situation described in Chart No.1, with

<table>
<thead>
<tr>
<th>Period</th>
<th>Constant</th>
<th>Two linear trends and a constant</th>
<th>Two linear trends and two constants</th>
<th>HP filter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimated $\rho$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1960q2-2002q4</td>
<td>0.91</td>
<td>0.58</td>
<td>0.45</td>
<td>0.42</td>
</tr>
<tr>
<td>1960q2-1980q4</td>
<td>0.92(*)</td>
<td>0.45</td>
<td>0.45</td>
<td>0.41</td>
</tr>
<tr>
<td>1981q1-2002q4</td>
<td>0.73(*)</td>
<td>0.79</td>
<td>0.45</td>
<td>0.30</td>
</tr>
<tr>
<td>1960q2-1983q1</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.43</td>
</tr>
<tr>
<td>1983q2-2002q4</td>
<td>0.8</td>
<td>0.45</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Estimated $\gamma$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1960q2-2002q4</td>
<td>0.81</td>
<td>0.70</td>
<td>0.63</td>
<td>0.60</td>
</tr>
<tr>
<td>1960q2-1980q4</td>
<td>0.83(*)</td>
<td>0.66</td>
<td>0.66</td>
<td>0.61</td>
</tr>
<tr>
<td>1981q1-2002q4</td>
<td>0.80(*)</td>
<td>0.74</td>
<td>0.60</td>
<td>0.59</td>
</tr>
<tr>
<td>1960q2-1983q1</td>
<td>0.64</td>
<td>0.64</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td>1983q2-2002q4</td>
<td>0.77</td>
<td>0.62</td>
<td>0.56</td>
<td></td>
</tr>
</tbody>
</table>

Note:
(a) Case of Chart No.2; (b) Case of Chart No.3; (c) Case of Chart No.1; (d) Case of Chart No.4;
(*) Refers to the period 1960q2-1981q4; (+) Refers to the period 1982q1-2002q4;

Table 1
INFLATION PERSISTENCE IN THE U.S.
the deviations from the mean depicted in the corresponding lower panel. Now again we have a different picture as we get estimates for $\rho$ which display an impressive constancy. From table 1 we that for the all the sub-periods considered we get an estimate of $\rho$ equal to 0.45 and thus the idea we get from the analysis of Chart No.2 is that the persistent process has now evaporated. This conclusion is confirmed by the estimates for $\gamma$ (which vary between 0.60 and 0.66)\(^{(9)}\).

The previous approach may be criticised on the grounds that, from an economic point of view, a linear time trend for the period 1960-1980, does not constitute a sensible proxy for the central bank inflation target. A less subjective solution (in the sense that it is not defined after looking at the data) can be obtained by entertaining the possibility of a pure time varying mean and see what happens to inflation persistence under such circumstances. A reasonable alternative is the well-known HP filter. Using the HP filter to proxy the mean of inflation may be justified as a simple device which ensures that the deviations of inflation from its mean are stationary. And of the deviations of inflation from its mean is a minimum requirement for an inflation persistence evaluation exercise to be worth carrying out. Such a situation is depicted in Chart No. 4.

Now we see that persistence under the assumption of an HP mean for inflation has basically vanished (we get an estimate for $\rho$ of 0.42 and for $\gamma$ of

\(^{(9)}\) We note that $\hat{\gamma} = 0.60$ is close to being not significantly different from 0.50 (zero persistence).
Moreover, once again, according to the tests performed, there seems to be no strong evidence of a difference in persistence for the two periods under analysis, i.e., there seems to be no change in inflation persistence through time.

Summing up, this section shows that the evidence on inflation persistence dramatically changes with the assumption on the mean of inflation. In particular, the evidence on whether inflation persistence was higher in the sixties and seventies than in the two last decades or whether inflation is persistent at all, ultimately hinges on the type of mean assumed when computing persistence. This section considers some statistical measures for the mean of inflation but, of course, other alternatives could have been entertained. However, the real issue is that the reliability of any estimate of inflation persistence ultimately depends on how realistic the assumed long run inflation path is.

5. CONCLUSIONS

This paper is a contribution to the literature on measures of inflation persistence in the context of a simple univariate time-series representation of inflation. The paper discusses the definition of persistence and its implications for the process of persistence evaluation. The need for a proper treatment of the mean of inflation is emphasised, especially the idea that it should be allowed to vary over time to account for monetary policy regime shifts.

The paper suggests a new measure of persistence which is based on the correspondence between persistence and mean reversion. Such a measure is broader in scope than the widely used “sum of the autoregressive coefficients”, and has the advantage of not requiring specifying and estimating a model for inflation. Moreover, an estimator for the new measure is suggested which, by construction, is immune to potential model misspecifications and that, given its non-parametric nature, is expected to be robust against outliers in the data.

We use this methodology to re-evaluate the evidence on inflation persistence in the U.S., allowing for a time varying mean and using the new measure of persistence. We conclude that the evidence on inflation persistence dramatically changes with the assumption on the mean of inflation. In particular, the widespread accepted wisdom that inflation in the U.S. has been more persistent in the sixties and seventies than in the last twenty years only obtains for the special case of a constant mean, which however, appears to be a counterfactual assumption.

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To test for a change in persistence we assume
the general autoregressive model
\[ z_t = \sum_{j=1}^{p} \beta_j z_{t-j} + \varepsilon_t \] (A.1)
reparameterised as
\[ z_t = \sum_{j=1}^{p} \delta_j \Delta z_{t-j} + \rho z_{t-1} + \varepsilon_t \] (A.2)
with
\[ \rho = \sum_{j=1}^{p} \beta_j, \quad \delta_j = -\sum_{i=1+j}^{p} \beta_i \]
where \( z_t \) is the series of deviations from the mean.
The following four models were estimated:
\begin{align*}
  z_t &= \sum_{j=1}^{p} \delta_j \Delta z_{t-j} + \sum_{i=1}^{p-1} \phi_j d_i \cdot \Delta z_{t-j} + \\
  &\quad + \rho_1 z_{t-1} + \rho_2 d_i \cdot z_{t-1} + \varepsilon_t \quad \text{(A.3)} \\
  z_t &= \sum_{i=1}^{p-1} \delta_j \Delta z_{t-j} + \rho_1 z_{t-1} + \rho_2 d_i \cdot z_{t-1} + \varepsilon_t \quad \text{(A.4)} \\
  z_t &= \sum_{j=1}^{p-1} \theta_i \Delta d_{t-j} + \sum_{j=1}^{p-1} \delta_j \Delta z_{t-j} + \\
  &\quad + \sum_{j=1}^{p-1} \phi_j d_i \cdot \Delta z_{t-j} + \rho_1 z_{t-1} + \rho_2 d_i \cdot z_{t-1} + \varepsilon_t \quad \text{(A.5)} \\
  z_t &= \sum_{j=0}^{p-1} \theta_j \Delta d_{t-j} + \sum_{j=1}^{p-1} \delta \Delta z_{t-j} + \\
  &\quad + \rho_1 z_{t-1} + \rho_2 d_i \cdot z_{t-1} + \varepsilon_t \quad \text{(A.6)}
\end{align*}
where \( d_i \) is a dummy variable which is zero before the date of the break \((t < s)\) and equals 1 thereafter \((t \geq s)\) and \( \Delta d_{t-j} \) is a dummy variable which is one for the date of the break \((t = s)\) and zero otherwise.

We note that while models (A.3) and (A.5) allow for the possibility of a break in every autoregressive coefficient, models (A.4) and (A.6) by assuming that \( \phi_1 = \phi_2 = \ldots = \phi_{p-1} = 0 \) basically impose that the change in the persistence parameter (the sum of the autoregressive parameters) stems solely from a change in the first autoregressive parameter, i.e., \( \beta_1 \). Even though this might appear a very restrictive assumption the fact is that models (A.3) and (A.5) turned out also to deliver too many insignificant \( \phi_j \) coefficients suggesting that they might be over-parameterised thus, raising concerns about the power of the test.

Notice also that models (A.3) and (A.4) are misspecified. This misspecification comes from the fact that in models (A.3) and (A.4) data occurring before the break \((t = s)\) are being used to estimate the parameters of the model, which is assumed to be valid only for the data after the break (i.e., \( t > s \)). Introducing the dummy variables \( \Delta d_{t-j} \) allows overcoming this problem because estimating model (A.5) or model (A.6) is basically equivalent to run two independent regressions in which due account is taken of fact that the second model should only be estimated using data generated after the break has taken place.

Whether it is relevant to account for this problem and estimate models (A.5) and (A.6) rather than models (A.3) and (A.4) is basically an empirical issue. In our case it turned out to be important as the conclusions for the test sometimes changed according to the type of model.