THE CONDUCT OF MONETARY POLICY: A CRITICAL REVIEW*

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1. INTRODUCTION

During the past decades there has been a surge in the monetary policy literature. Many issues have been subject to vivid debate, including the empirical extraction of the effects of monetary shocks, the optimal goals of monetary policy, the appropriate monetary policy operating procedures, the option between rules and discretion, the monetary transmission channels or the appropriate way to model a monetary economy.

This note will not survey all the literature related to these issues (an excellent review of these issues is Walsh (1998)). The focus will be on the analysis of what monetary policy can actually do, on the evaluation of the risks arising from discretionary monetary policies and on suggestions of how these policies can be improved upon to best achieve their objective. In undertaking this exercise we will review and build on some recent literature on monetary policy, most notably Woodford (2003). Many of the issues under discussion in this note are subject to criticism and have not yet gained general consensus among the profession. We claim, nonetheless, that they are representative of the current state of the investigation on monetary policy issues.

The note is organized as follows. The next section explores the effects of monetary policy, both in the long and the short run. The subsequent sections will only focus on the short-run conduct of monetary policy. Section 3 describes what affects inflation in the short run. Sections 4 and 5 describe the characteristics, the risks and some possible improvements on the current monetary policy strategy followed by major central banks. Section 6 concludes.

2. WHAT DOES MONETARY POLICY DO?

2.1. In the long-run

The fact that a higher rate of money growth is fully transmitted in the long run into a higher rate of inflation has been recognized for centuries. In the words of Hume (1752) “The absolute quantity of the precious metals is a matter of great indifference”. The root of this result is clear: in the long-run, the economy is not subject to any nominal friction, thus changes in money will be fully incorporated in price changes. In the very long-run, changes in money are similar to changes in the denomination of the numeraire, which obviously have no effect but on prices. Monetary policy is therefore neutral in the long run, i.e., it has no effect on activity or employment in the long run.

This so-called quantity theory of money is one of the most robust relations in macroeconomics. It holds for different periods, different monetary aggregates and different groups of countries (for an overview, see McCandless and Weber (1995) or Monnet and Weber (2001)). Chart 1, taken...
from McCandless and Weber (1995), is a representative example.

In recent decades, major central banks have followed policies aimed at targeting a level for a very short-term interest rate. Does quantity theory continue to apply in this case? The answer is yes. To see this it is important to recall the Fisher equation (Fisher (1896)), which states that the nominal interest rate is equal to real interest rate plus the expected rate of inflation

\[ \text{Nominal Interest Rate} = \text{Real Interest Rate} + \text{Expected Rate of Inflation} \]

In the long run, the real interest rate will be a function of the time preferences of households and the rate of growth of the economy. Assuming that these elements are constant in the long run, a higher nominal interest rate will be associated with a higher expected (and realized) inflation. This positive relation in the long-run between nominal interest rates and inflation should be no surprise. In fact, it is by now a stylized fact: countries with high average nominal interest rates are the ones experiencing high average inflation; countries with low average nominal interest rates are also the ones experiencing low average inflation.

How does this relate to the long run monetary growth in the economy? To support a higher (lower) targeted average interest rate, the central bank has to increase (decrease) the inflation expectations of the economic agents, since it cannot permanently change the real interest rate. To increase (decrease) these expectations the central bank has to permanently increase (decrease) the rate of monetary creation. Therefore, in the long run, there is a positive relation between money growth, nominal interest rates and inflation.

In sum, monetary policy is neutral in the long run. In this time span, monetary policy is able to control the average rate of inflation by targeting a compatible rate of money growth (in case of money-growth targeting) or a corresponding level of interest rates (in case of interest rate targeting). The long run mandate of a central bank is thus clear. Since monetary policy cannot affect real variables, it should focus on promoting a low inflation environment, in particular given that high inflation is costly. All the major central banks share this focus on low inflation.

However, in the short to medium run, there are nominal and real frictions in the economy. In this time span, monetary policy has real effects. Accordingly, the central banks’ mandates typically leave some room for short-run stabilization policy.

2.2. In the short to medium run

To evaluate the performance of a given monetary policy in the short run it is important to understand first what monetary policy can achieve in this time-span. However this is a very difficult question. Since monetary policy actions reflect, in part, policy makers’ responses to non-monetary developments, the effects on the economy combine the effects from the policy shock and the effects from the underlying shocks to which policy is reacting. It is not possible to isolate the policy shock without an a priori idea of the characteristics of those underlying shocks. These so-called identifying assumptions, together with data, allow one to answer the question of “what does monetary policy do in the short to medium run?”. Different identifying assumptions lead to different answers for the same data set. To be able to pursue the analysis it is therefore necessary to rely on some beliefs. Those beliefs, coming mainly from central bank understanding, can be summarized by saying that “monetary policy actions cannot affect current inflation and output, nor inflation and output in the near future.” (Svensson, 2000). Us-
ing these identifying assumptions, the data mainly confirms the remaining beliefs about the effects of monetary policy. As reported in Chart 2, for the US, and Chart 3, for Europe, the identification of monetary shocks using zero contemporaneous effects leads to the “rough benchmark that monetary policy affects output in about one year and inflation in about two years” (Svensson, 2000). This is the sense in which monetary policy acts with lags in aggregate activity and inflation. These figures also confirm that an unanticipated increase in money supply leads to a temporary reduction in interest rates. This is the so-called liquidity effect.

These lagged effects of monetary policy on inflation and output, together with the liquidity effect, correspond to the conventional wisdom on what monetary policy does in the short to medium run.

The impulse response of the identified monetary policy shock also allows us to get some idea on the magnitude of the effects on inflation and output. And the conclusion is that these effects are very small: a 60 basis points decline of the interest rate implies a maximum impact on output of 0.2 per cent (after one year) and on annualized inflation of 10 basis points (after two years)\(^\text{(2)}\). These very small effects of the monetary shock can also be read from the exercises developed by the Bank of England. In its *Inflation Report*, the Bank of England compares the projections of inflation and output based on a constant interest rate scenario with the projections based on an interest rate path taken from market expectations. In its February 2001 issue, for example, market expectations implied an overall decline in interest rates of around 60 basis points for the subsequent 8 quarters. The comparison of the end of period effects of the two interest rate paths allows us to conclude that the 60 basis points decline in the interest rate leads to an increase of inflation of 10-20 basis points and to an increase of output of 0.1-0.2 per cent.

Therefore the conventional wisdom on the short to medium run effects of monetary policy is true.

\(^\text{(2)}\) In the short-run, inflation actually decreases after a decline in interest rates. This response is usually called the “price puzzle” in the literature.
can be summarized by a long lag on output, a very long lag on inflation and very small effects on both variables.

3. WHAT AFFECTS INFLATION IN THE SHORT RUN?

If one of the main focus of monetary policy is the inflation rate it is important to have some idea on the determinants of inflation. In the long-run it should be clear that inflation is “always and everywhere” a monetary phenomenon (Friedman (1992)). Different rates of inflation in the long run are necessarily rooted in different objectives (implicit or explicit) of the monetary authority.

To understand the short run determinants of inflation one has to start by analysing the effects on inflation of the most commonly identified shocks to the economy. Starting with the monetary shock, it was already described above that while money and interest rates move contemporaneously with the shock, inflation only starts to rise more than one and a half years later (see Chart 2). This sluggishness can arise, for example, due to rigidities in the price-setting mechanism of firms, due to frictions in the labour market, or due to a learning process by the agents concerning the policy of the monetary authority.

When a non-monetary shock hits the economy the response of the rate of inflation is always related to the reaction of the monetary authority to the shock. It is not possible to strip down the impulse response of inflation to a shock irrespective of the policy response. Nonetheless, it is still possible to present some ideas that seem broadly consensual in the literature. It is fair to say that the
available evidence suggests that the response of inflation to most shocks commonly identified in the literature (namely technology shocks, preference shocks and real demand shocks) is significant, mostly contemporaneous and relatively short-lived. Representative studies that confirm this result are, for example, Iacovello (2000), Altig et al. (2002) or Canova and Nicoló (2000). It would be difficult to argue that it is the monetary policy action which is producing such contemporaneous effects. We can therefore infer that it is the underlying relation between the shock and inflation that causes these patterns.

4. THE CURRENT STRATEGY OF CENTRAL BANKS

Central banks in the major industrialized countries share a common understanding as to how to conduct monetary policy in an uncertain environment. Despite differences in the goals of monetary policy and in the specific implementation procedures, their monetary policy strategies can be summarized as follows:

a) The recognition that in the long run the monetary authority can only be responsible for nominal aggregates. Since high inflation is known to be costly, an objective of monetary policy is always a low inflation rate in the medium term.

b) The acknowledgement of transmission lags in the policy. Policy is therefore conducted with a forward-looking perspective. Past and current variables are important as predictors of future economic developments.

c) The control of a short term interest rate in the implementation of monetary policy.

d) The absence of a mechanical reaction of the policy instrument to specific developments in the economy.

In short, the instrument is an interest rate. Its level depends on the forecasts of future inflation. There is no rule connecting the instrument and the forecasts. This means that the policy uses the interest rate as an instrument and is discretionary.

4.1. A model to assess monetary policy

An evaluation of monetary policy requires the construction of a structural model where alternatives of conducting monetary policy can be compared. As clearly stated in Lucas (1980) “...we are interested in models because we believe they may help to understand matters about which we are currently ignorant”. The basic model where to conduct this monetary experiment will be chosen taking into account that we should “subject them [models] to shocks for which we are fairly certain how actual economies or parts of economies would react.” (Lucas (1980)) The choice of the model would be more connected with its ability to imitate the economy than with its realism. “A “good” model (...) will not be exactly more “real” than a poor one, but will provide better imitations. Of course, what one means by a “better imitation” will depend on the particular questions to which one wishes answers.” (Lucas (1980)).

Therefore to discuss the conduct of monetary policy we chose a very simple model which replicates quite well what we called in Section 2 the conventional wisdom. Let us look sequentially at the two qualitative effects of monetary policy: first, the existence of transmission lags; second, the reaction of output, leading and peaking first than inflation.

The existence of transmission lags is related to the so-called aggregate-demand block of the economy. This block is summarized in a behavioural equation that relates private expenditures in two consecutive periods, and is therefore forward-looking. Economic agents that give up one unit of real expenditures today save an amount of income equal to the price today, and the application of that amount allows them to increase real expenditures tomorrow by that amount times the interest rate divided by the price level tomorrow. Therefore economic agents are indifferent between consuming today, or delaying expenditures when the change of utility due to the decline of one unit of consumption today is identical to the increase of utility of consumption tomorrow. The assumption that these spending decisions are made in advance, or based on old information, is a

(3) Let us take the period duration as a quarter. Then $t$ and $t+1$ denote respectively the current and next quarter.
The output gap is defined as the deviation of output from the output that would characterize an economy without frictions. This measure is not related to the usual definition of output gap (deviation from a trend).

To close the model we need to build the aggregate-supply block of this economy. In line with the recent literature, this block takes the form of an “expectations-augmented Phillips-curve”. The most popular story behind such a relation comes from the hypothesis that firms set prices in a staggered way. A discrete-time variant of that model was proposed by Calvo (1983). In that model every period only a fraction of firms choose prices optimally. In Calvo’s original work this price would be rigid between adjacent price setting opportunities. These opportunities arise randomly. In this framework, the price setting and inflation are purely forward looking, and therefore react to expected developments of the output gap. This characteristic is at odds with the evidence described above that the effects on output lead the effects on inflation and that the inflation process is therefore characterized by a substantial degree of inertia. In the simple Calvo story the effect on inflation should precede the effect on output, when this effect on output is predictable. The model of staggered pricing can be improved in this respect. One first extension is the hypothesis that prices are set in advance, that is, the firms commit to a price for period $t$ and afterwards in period $t - d$. A second extension is to allow for a backward looking behaviour in the aggregate-supply curve. This is done by assuming that prices are automatically changed in accordance with some mechanical rule between the occasions in which firms can choose prices optimally. Firms that are not allowed to choose prices optimally can re-set previously chosen prices by last period’s inflation.

$$\pi_t = \gamma \pi_{t-1} + \kappa E_{t-d} \pi_{t-1} + \beta E_{t-d} (\pi_{t-1} - \gamma \pi_{t-1}) + \mu_t$$  

This second equation is the so-called Phillips curve where $\gamma$ is the degree of indexation of the firms that do not choose prices and $\mu_t$ is a measure of a cost-push shock. The inflation dynamics implied by this curve replicate the one displayed by the data. In particular, inflation exhibits inertia and a hump-shaped response to a monetary shock.

In summary, the main frictions of the model come from some price stickiness and some real rigidity. The price stickiness is formalized as an extended Calvo price setting mechanism and the real rigidities are summarized by time to build or time to plan constraints.

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4) See Appendix for the derivation.
5) The output gap is defined as the deviation of output from the output that would characterize an economy without frictions. This measure is not related to the usual definition of output gap (deviation from a trend).
It is easy to understand the role of monetary policy in this model. The interest rate affects the intertemporal choices, but cannot affect them contemporaneously since expenditure decisions are taken $d$ periods in advance. The channel through which monetary policy affects the inflation rate is the output gap. The output gap responds to expectations of the interest rate and the inflation rate responds to expectations of the output gap. This model, for $d \geq 1$, is consistent with the identifying assumption that there are no contemporaneous effects of monetary policy. A natural value for $d$ would be the estimated time lag between the monetary shock and the output response. Most empirical studies suggest that this lag is around two quarters.

Notice that the amount of money was not used to determine the equilibrium paths of the output gap and of the inflation rate. The equilibrium of the money market for a certain trajectory of the interest rate and the associated trajectories of prices and output, will determine the trajectory of the monetary aggregate for which it is possible to define a stable demand.

Note also that in the simplified economy represented by the aggregate demand and supply and the money demand, a monetary policy shock leads to a liquidity effect in the impact period. This occurs because, for a given demand for real money, the existence of lags and the associated zero contemporaneous effects of a change of the interest rate on output and prices, leads naturally to a negative relation between the amount of money and the interest rate.

The conduct of monetary policy takes usually into account the identified transmission lags. But the main question is the one that we have been trying to answer, that is: what are the foundations of those lags? The really important result from this model, for the purpose of this note, is that the existence of lags in the transmission of monetary policy does not come from the fact that the policy today directly affects the economy tomorrow, but rather from the fact that the policy today is an indicator of the policy tomorrow. It is this forecastable part of future policy that affects the economy tomorrow. Monetary policy has real effects through the anticipated component and these real effects lead the effects on inflation.

This seems in contrast with the common understanding that the existence of transmission lags comes from the fact that it is today’s change in interest rates that per se affects output and inflation in the future. It is also in contrast with the idea that non-neutrality of monetary policy comes predominantly from surprises, that is from its non-anticipated component.

4.2. Indeterminacy

The conduct of monetary policy by most central banks, that explicitly or implicitly can be characterized as inflation targeters, is purely forward looking. That is “only factors that matter for the central bank’s forecast of the future evolution of its target variables, conditional upon its current and future policy actions, play any role on the decision.” (Svensson and Woodford (2002)). In practice the central bank has to forecast the path of the conditional expectations of inflation that themselves depend upon current policy. This forecast depends only upon information about exogenous disturbances and the policy action. Then the interest rate is set as a function of exogenous disturbances. Although the option for an interest rate is mainly defended to avoid that financial volatility is transmitted to equilibrium prices and quantities, it is well established in the literature, as in the seminal contribution of Sargent and Wallace (1975), that in general this is not the case. When the interest rate is set as a function of exogenous disturbances the policy leads to indeterminacy. Even if policy is imposed as an equilibrium condition where the interest rate today depends on expectations of inflation in a horizon of several quarters, it is well known in the literature that too long a forecast period leads to indeterminacy (Levin et al. (2001)) and that too strong a response to forecasts also leads to indeterminacy\(^6\) (Bernanke and Woodford (1997)). As both things

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\(^{6}\) Suppose monetary policy is being conducted to fight against the impact on inflation of different types of shocks. If these shocks are serially correlated, the more forward looking the policy the larger should be the response of the interest rate (see Gianonni and Woodford (2002)). To be effective, policy is then characterized by large responses to something that is likely to be estimated with considerable error.
occur in the context of inflation-targeting the danger of indeterminacy could be significant.\(^7\)

This indeterminacy is mainly driven by the difficulty of such a procedure to anchor inflation expectations. As the monetary aggregate is not controlled any expectation could in principle be accommodated, that is, it could be self-fulfilled. As can be easily confirmed in equation (1) the same interest rate can be associated with different inflation expectations which lead to different output gaps and, by equation (2), to different realized inflations.

When the interest rate policy does not follow a rule that guarantees local determinacy, as is the case when monetary policy does not react systematically to any endogenous variable, or when local determinacy can be associated with global indeterminacy,\(^8\) the existence of “escape clauses” can help determination of equilibrium. Let us just refer two of those clauses. The first is the one referred as an hybrid rule in Svensson and Woodford (2003). The proposed rule implies that the central bank reacts when private sector expectations deviate systematically from the central bank forecasts. A strong reaction to those deviations implies a determination of equilibrium. A second clause is the one developed in Christiano and Rostagno (2001). The interest rate policy is complemented with a threat to change to a policy that controls the monetary aggregate in case the growth rate of prices (or the growth rate of the monetary aggregate) transposes a certain pre-announced interval. Woodford (2003) develops a similar argument to claim that the threat to change to a policy that controls money supply can avoid deflationary trajectories. The control of the broad aggregate (the one for which a stable relation with prices exists) is a necessary condition for the credibility of such a clause. However, in the current context of financial innovation, it is not clear how an institutional framework that would allow the control of broad monetary aggregates could be designed.

\(^7\) See Batini and Pearlman (2002) where these results are extended for a policy rule that includes a term on the lagged interest rate.

\(^8\) See for example Benhabib et al. (2001).

### 4.3. Implications

In summary, results in the recent literature suggest that the procedures followed by central banks that target the inflation rate have the following implications:

a) Because the interest rate affects inflation through the forecastable interest rate path, any change of the interest rate necessarily implies a period where the surprise is completely ineffective. Since the surprise interest rate is as high as the subsequent path, the initial volatility of the interest rate has costs and no benefits.

b) Because the interest rate is reacting to forecasts with an horizon of several quarters in the future, to be really effective the interest rate should react strongly to these forecasts. However, the longer the forecasting horizon, the more uncertain are the targeted variables and the more conservative should the central bank act. The conservative moves that we observe in the data (and subsequent patterns towards the unchanged policy) can justify the small effects of monetary policy that we initially described.

c) Because the procedure can be described as an interest rate reacting to shocks, the policy leads to indeterminacy and therefore the described responses of the endogenous variables are just one among a multiplicity of possible outcomes.

In conclusion, when the idea that monetary policy affects inflation with long lags leads to a purely forward-looking procedure, monetary policy is costly and has small and uncertain effects.

These characteristics of “inflation targeting” procedures, which are associated with decision-making under discretion are quite robust to different types of rigidities. Their drawbacks come from the fact that, as already referred, these procedures cannot affect inflation expectations due to their pure forward-looking character. Only if policy reacts to past shocks, can it bring about the desired evolution of private sector expectations. The reason is that in this case agents, by observing shocks today, can infer policy tomorrow. If policy does not have this type of “history dependence” opti-
mal outcomes cannot be reached due to the forward-looking behaviour of the private sector. In the model described above money has real effects due to nominal price rigidities and to real frictions in the decision process. However the results are robust to an economy characterized by rigidity in nominal wages. It is well documented in the literature (Woodford (2003)) that allowing for wage stickiness would not add power to the model in replicating the effects of monetary shocks on inflation and output. Stickiness of wages would create a new way through which real shocks can shift the Phillips curve but this could be replicated in the model proposed by a cost push shock.

5. IMPROVEMENTS IN THE CONDUCT OF MONETARY POLICY

According to the recent developments in the literature if central banks committed to a policy rule they would be able to achieve the optimal (given the objective) equilibrium. This is quite an ambitious proposal. The existence of a commitment technology is something exogenous and therefore does not belong to the set of alternatives that can be chosen by the central bank. A modest proposal would be a type of “intermediate commitment” which we believe could anyway improve the way central banks usually conduct monetary policy. We can use the analysis of the risks identified before to propose some improvements that build on the current characteristics of the conduct of monetary policy by the major central banks. First, it is important to recognize that current policy does not affect current inflation, that is, that surprises in the interest rate have no real effects. This implies that even if the central bank, when deciding period t’s policy, has more information than the one that was available to the private sector when it took the decisions for period t, that additional information should not be used. Therefore both the central bank and the private sector should have identical relevant information. This will avoid the costly and ineffective volatility in the interest rate. Second, it is also important to recognize that the policy should aim at influencing the forecastable components of inflation and that therefore the efficacy is through the forecastable component of the interest rate. This implies that efficacy could be improved if the interest rate were perfectly forecastable some periods in advance. Every period t the central bank, instead of fixing an interest rate for that period, would announce an interest rate for period t+d. If we think that d equal to one or two quarters is a reasonable decision lag then the interest rate should be decided for the next, or the following, quarter. In deciding on the value of the interest rate for period t the main forecast would be period t’s inflation.

The model presented in section 4 considered the existence of a sub-set of firms which updated prices with the previous period’s inflation. The existence of this indexation implies that the costs of inflation come not only from the level of inflation but also from changes of the inflation rate. This latter cost arises due to the increased relative price distortion caused by the increase in inflation volatility. Since a sub-set of firms index prices by last period’s inflation while the remaining update prices optimally, sudden changes in inflation have a cost in terms of resource allocation. The central bank should thus try to minimize this additional cost of inflation, by avoiding a high volatility of inflation. This implies that for a given inflation forecast for period t the interest rate should change less the higher the degree of indexation. In particular, the interest rate set at period t−d for period t should react negatively to forecasts of period t + d inflation. For a given inflation forecast, the change in interest rate would be smaller than in the case without indexation.

If the monetary authority committed today to an interest rate d periods from now, the gain would be to avoid costly changes of the interest rate with no effects, and to have a smaller reaction to the inflation forecasts. Although these results are quite robust we cannot forget that “the key stumbling block for policy-formation is limited knowledge of the way the macroeconomy works” (McCallum (1997)). Therefore also from this point of view the characteristic that policy instrument reacts less to obtain the same result seems desirable. Until now, the procedure is described as a reaction of central bank forecasts. Therefore no reaction was imposed when realized inflation, or private sector expectations, deviate in a systematic way from central banks forecasts. As mentioned above, to ensure determinacy the central bank should react to these deviations and therefore
should respond not only to its own forecasts but to the deviations between these forecasts and private sector forecasts (Svensson and Woodford (2003)).

6. IN SUMMARY

We can summarize the last sections as follows:

1. The recognition of lags in the transmission of monetary policy actions to inflation has to be associated with some type of lags of decision of the private sector. Given these lags, the private sector behaviour has a strong forward looking component.

2. The transmission lags should be interpreted as the private sector reacting to the forecastable behaviour of the interest rate. Surprises have no effects. A pure white noise policy would be completely ineffective.

3. The conduct of monetary policy could be improved if the central bank commits to an interest rate in the next period (or two periods from now). The setting of the interest rate for period \( t \) would have as main input the forecast at \( t-d \) of inflation of period \( t \). Note that the existence of lags in the transmission does not imply a forward looking policy. This interest rate policy could be amended by a commitment to change the interest rate when private expectations deviate from central bank forecasts.

REFERENCES


DERIVATION OF THE AGGREGATE DEMAND EQUATION

Here we will present the derivation of the aggregate demand equation

\[ x_t = E_{t-d} x_{t+1} - \sigma E_{t-d} \left( i_t - \pi_{t+1} - r^n_{t+1} \right) + \chi_i \]

Let us begin by assuming \( d = 0 \). To derive this equation we take the first order condition of the household problem in order to present and future consumption given by\(^{\text{(9)}}\)

\[
U_\varepsilon(C_t, \xi_t) = \beta (1 + i_t) E_t \left[ \frac{U_\varepsilon(C_{t+1}, \xi_{t+1})}{\pi_{t+1}} \right] \]

where \( \xi_t \) represents shocks to preferences. When we represent this Euler equation as a function of output instead of consumption we have

\[
U_\varepsilon(Y_t, \xi_t) = \beta (1 + i_t) E_t \left[ \frac{U_\varepsilon(Y_{t+1}, \xi_{t+1})}{\pi_{t+1}} \right] \tag{3}
\]

where \( \xi_t \) contains now not only shocks to preferences but real demand components different from private consumption. Letting government purchases be denoted by \( G_t \), market clearing implies that \( C_t + G_t = Y_t \). Loglinearizing equation (3) around a deterministic steady-state, and representing by \( \hat{G} \) the percentual deviation from the steady state of \( G \), we can write

\[
\hat{Y}_t = \hat{\gamma} G_t + E_t \left[ \hat{Y}_{t+1} - \hat{\gamma} G_{t+1} \right] - \sigma [ \hat{i}_t - \hat{\pi}_{t+1} ]
\]

where \( \sigma \) represents the intertemporal elasticity of substitution.

If we define the output gap as \( x_t = \hat{Y}_t - Y^n_t \) with \( Y^n_t \) being the deviation of the flexible price equilibrium output from the steady state, we can write the above equation as

\[
\hat{x}_t = \hat{\gamma} + E_t [ \hat{x}_{t+1} - \hat{\gamma} G_{t+1} ] - \hat{Y}^n_t + \hat{Y}^n_{t+1} - \sigma [ \hat{i}_t - \hat{\pi}_{t+1} ]
\]

If we define \( \gamma^n_{t+1} \equiv \sigma^{-1} \left[ \hat{\gamma} - \hat{\gamma} G_{t+1} - \hat{Y}_t + \hat{Y}^n_{t+1} \right] \) we obtain equation (1), with \( d = 0 \). The existence of decision delays implies that the decisions for period \( t \) are taken in period \( t-d \). In this case the household's first order condition should be written as

\[
E_{t-d} U_\varepsilon(C_t, \xi_t) = \beta E_{t-d} \left[ (1 + i_t) \frac{U_\varepsilon(C_{t+1}, \xi_{t+1})}{\pi_{t+1}} \right]
\]

Using the same sequence as before we arrive at equation (1) where \( \chi_i = (\hat{\gamma} - \hat{Y}^n_t) - E_{t-d} (\hat{\gamma} - \hat{Y}^n_t) \).

\(^{(9)}\) This condition is a simplification since it assumes that the economy is cashless and that labour is inelastic or that preferences are additively separable between consumption and leisure.