A NOTE ON AN “EXPLOSIVE” LAW*

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According to the provisions laid down in paragraph 3 of Article 24 of the Local Finance Law, of 6 August 1998:

“Annual charges with amortization and interest payments of medium and long-term loans, including debenture loans, shall not exceed the highest limit of the value corresponding to three twelfths of the Municipal General Fund and the Municipal Cohesion Fund assigned to the municipality or to 20% of the investment expenditure of the municipality in the previous year.”

Is this law strict enough to discipline the indebtedness of local authorities? Or, on the contrary, unlimited and unsustainable indebtedness may possibly result despite strict compliance with the law? By examining the limits of indiscipline permitted by law, it will be shown that the latter applies. For this purpose we shall analyse the extreme behaviour of a local authority with unlimited propensity for spending, and at the same time, for financing such expenditure exclusively through borrowing, always, of course, within the limits permitted by law.

Thus, let us consider a local authority that finances its investment exclusively through debt, where $K_t$ is the debt stock (and, assuming equivalent amortization, also the capital stock) in $t$. Suppose that the value corresponding to three twelfths of the Municipal General Fund and the Municipal Cohesion Fund assigned to the municipality is low, so that the second limit mentioned in the law is relevant. In this case we have:

$$ (i + \delta)K_{t-1} \leq 0.2 I_{t-1} \quad (1) $$

where $i$ and $\delta$ are the interest rate and the amortization rate applicable to the debt, while $I_t$ represents the gross additional indebtedness, in the period. The left side of (1) thus represents the debt service, while the right side represents the limit set by law. On the other hand, we have the relationship,

$$ K_t = I_t + (1 - \delta) K_{t-1}. \quad (2) $$

Using (2) in (1) we obtain

$$ (i + \delta)K_{t-1} \leq 0.2K_{t-1} - 0.2(1 - \delta)K_{t-2} \quad (3) $$

or, assuming $i + \delta < 0.2$,

$$ K_{t-1} \geq \frac{0.2(1 - \delta)}{0.2 - i - \delta} K_{t-2}. \quad (4) $$

which implies an explosive dynamics for $K$ given that $0.2(1 - \delta) > 0.2 - i - \delta$.

The intuition of this result may improve if (2) is used in (1) to obtain,

$$ (i + \delta)[I_{t-1} + (1 - \delta)K_{t-2}] \leq 0.2I_{t-1}. \quad (5) $$

This expression puts into evidence $I_{t-1}$ as a decision variable in period $t-1$, given the value of $K_{t-2}$.

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* The opinions expressed in this paper are those of the author and not necessarily those of the Banco de Portugal. The author is grateful for the comments and suggestions on a preliminary version of this paper by Paulo Bárcia, Rui Baleiras, Vítor Constâncio, Luís C. Cunha, Mário Páscoa, Maximiano Pinheiro and Vasco Santos. The usual disclaimer applies.

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(1) It should be noted that the dynamic problems of the law discussed in this paper were already present in the Decree-Law no. 98, of 29 March 1984.

(2) Further below in this paper, it will become clear that this assumption is actually not restrictive.
$K_{t-2}$ inherited from the past. The choice of $I_{t-1}$ has an impact on the two sides of the inequality and should therefore be made taking it into account. Lines $E$ and $D$ in the chart below show respectively the left and right sides of the inequality, as functions of $I_{t-1}$, given $K_{t-2}$.

$$K_{t-1} \leq \frac{02(1-\delta)}{02-i-\delta} K_{t-2},$$

(6) a condition impossible to be fulfilled since it implies negative values for $K$.

To sum up, either there is an explosive dynamics of the debt, or a condition impossible to be fulfilled.

We have admitted in inequality (1) that the debt service in period $t$ would apply to the debt stock at the beginning of the period, $K_{t-1}$. However, an alternative could be considered in which in addition to $K_{t-1}$ investment in period, $I_t$ leads to debt service. In this case, (1) should read

$$(i + \delta)(I_t + K_{t-1}) \leq 02I_{t-1}$$

(7) and, again using (2), we obtain, with the proper substitutions,

$$K_t + K_{t-1} \left( \delta - \frac{02}{i + \delta} \right) + \frac{1-\delta}{i + \delta} 02K_{t-2} \leq 0.$$  

(8) This expression, taken in the limit of equality, is a second-order difference equation, which can be expressed in canonical form as

$$\left(1-t_1 L - t_2 L^2 \right) K_t = 0,$$

(9) where $L$ is the lag operator and, in this case,

$$t_1 = \frac{02}{i + \delta} - \delta, \quad t_2 = \frac{\delta - 1}{i + \delta}.$$  

(10)

For plausible values of the parameters $i$ and $\delta$, an explosive dynamics for the debt stock is obtained. Thus, for example, with $i=0.05$ and $\delta=0.07$, which will lead to $t_1=16, t_2=-155$, we obtain an explosive behaviour for $K_t$, with oscillations and negative values.

The question may arise as to whether debt growth may, simultaneously produce faster growth of the local economy and, thereby, of the respective collateral, so that the explosive behaviour of debt is after all not so serious. In fact, if as a result of the investment financed through the debt, the local economy grows faster, the risk of default may be reduced. But can it be eliminated?

Let us consider the variable that seems to be relevant in this context: the debt/local output ratio, $B = K/Y$ where $Y$ is the output of the local authority.(4) Let us admit that this is simply described by the Cobb-Douglas production function

$$Y = K^\gamma N^{1-\gamma}, \quad 0 < \gamma < 1,$$

(11) where it is admitted that the debt is entirely channelled to the accumulation of productive capital stock. In turn, $N$ generically indicates all other productive factors (e.g., labour) used in production, which are supposed to grow annually at the constant rate $n$. Thus, the evolution of ratio $B$ can be described through its growth rate, $b$, as given by

(3) This proves that, should the alternative limit set by law (three twelfths of Municipal Funds) initially prevail, the explosive dynamics of investment will eventually become dominant (obviously within plausible values for the growth of Municipal Funds), causing this restriction to be exceeded by the alternative limit 0.2$I_{t-1}$ which we have been analysing.

(4) No external effects are considered.
where lower case represents the growth rates of the corresponding variables in upper case. The growth rate of local production is in turn, given by

\[ y = \gamma k + (1 - \gamma) n \]  \hspace{1cm} (13)

and, substituting (13) in (12),

\[ b = (1 - \gamma)(k - n). \]  \hspace{1cm} (14)

The debt/output ratio will grow unbounded unless \( k = n \). Taking, for example, expression (4) above,\(^{(5)}\) the minimum value for \( k \) is readily obtained as

\[ k = \frac{2(1 - \delta)}{2 - i - \delta} - 1, \]  \hspace{1cm} (15)

and, with the plausible values \( i = 0.05 \) and \( \delta = 0.07 \) used above, we obtain \( k = 133\% \). That is, the other productive factors would have to grow (at least) at this implausible annual rate so that a collateral commensurable with (explosive) developments in the debt might be generated.

In conclusion, the above analysis is not at all reassuring regarding the stability of the debt over time. The present law contains some severe shortcomings and it should therefore be modified. The key issue here is not to prove that it may be possible to avoid instability through recourse to other healthier financing sources, such as taxes or other effective revenue of the local authority. The critical point is that strict compliance with law may nevertheless give rise to an unsustainable local debt.

\(^{(5)}\) This is the relevant case, since the alternative dynamics described by (6) and (8) involve, as mentioned above, negative values for the capital stock.