ESTIMATION OF THE OUTPUT GAP: A BIVARIATE APPROACH*

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1. INTRODUCTION

The output gap of a given economy is defined as the difference between the observed output of the economy and its potential output. Although it may have diverse meanings, the term “potential” usually conveys the output level corresponding to the “natural”, normal or average utilisation of primary production factors available in the economy.

Whenever the economy is operating above (below) its potential output level — that is, when the output gap is positive (negative) — budgetary automatic stabilisers will improve (deteriorate) the general government balance compared to the balance one would get in a neutral cyclical situation — where observed output matches potential output. This is why general government balances adjusted for cyclical changes are calculated upon output gap estimates — to abstract from the effects of the automatic stabilisers and to better evaluate the restrictive or expansionary stance of fiscal policy.

In turn, a positive output gap situation will be characterised by inflationary pressures (both in the goods and services markets, but also in the markets for primary factors — specially in the labour market). However, in a very open small economy — as in the case of the Portuguese economy — inflationary pressures in the traded goods markets (i.e., the markets where domestic output is more exposed to international competition) are moderated.

For the reasons presented above, the output gap is a thoroughly used concept in short-term analyses and in justifying economic policy decision making. However, the output gap is not a directly observed variable, and conventional estimation methods bear weaknesses that fragilise conclusions.

This article alerts for the limitations of conventional output gap estimation procedures. We propose an alternative approach to the estimation of the output gap — a bivariate approach —, which tries to solve some of the most important limitations of conventional methods. Obviously, this new methodology also presents drawbacks and is quite discretionary. Nevertheless, we argue that these drawbacks and limitations may prove less serious than those affecting conventional methods.

This article concludes that the Portuguese economy is presently at a fairly neutral cyclical position. We also conclude that no risks of overheating exist in the near future, unless real output grows at more than an annual 4% alongside a slowdown of investment. The current international background of the Portuguese economy — presenting signs of activity slowdown in our leading trade partners — minimises the risk of a positive and significant output gap in the near future.

The remaining of this article is structured as follows: Section 2 discusses the major drawbacks of conventional methodologies of estimation of output gap; the necessary assumptions in building the proposed alternative methodology are presented in Section 3; the fourth Section describes the data and the considered estimation technique; the last sections discuss the findings and evaluate the robustness of conclusions.

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* The opinions of this paper represent the views of the author, they are not necessarily those of the Banco de Portugal.

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2. METHODOLOGICAL CONSIDERATIONS

A simplified typology of conventional methodologies used in estimating the output gap distinguishes two main groups:

— univariate approaches applied to a sample of output observations (using techniques of analysis of time series);
— approaches based on the estimation of an aggregate output function relating output with the primary production factors.

The first class of methodologies encompasses the adjustment of a linear trend to the logarithm of the time series for (real) output — the gap resulting as the residual of the regression — but also algebraically more sophisticated procedures, like trend estimation through the Hodrick-Prescott (HP) filter\(^{(1)}\).

The HP filter became very popular through its extensive application in the context of tests of real business cycles models\(^{(2)}\). This ended up determining its widespread utilisation as a method of estimation of the output gap. The HP filter currently enjoys an “official method” status. It is adopted by most national and international economic institutions whenever the output gap is to be measured. The widespread utilisation of the HP filter — often a-critically — tends to disregard the serious drawbacks of the method. These cover both essential statistical and economic problems.

Among the former, the choice of the “smoothing parameter” (\(\lambda\), as frequently noted) should be highlighted. A consensus existing among users of the HP filter dictates that this parameter should equal 100 for annual data, and 1600 for quarterly data, although justifications behind these values are weak and usually absent. Furthermore, output gap estimates for the last observations of the sample seem to be particularly sensitive to the choice of the parameter\(^{(3)}\). A third criticism of the first nature is that the HP filter, when applied to series generated by integrated processes (which is probably the case of the series of real output, in logarithms), may produce serious cyclical fluctuations (i.e., that do not appear in the original data)\(^{(4)}\).

The major criticism made to the application of the HP filter to the output gap estimation — shared by all univariate methods — is that it is poor from an economic point of view. All methods resorting exclusively to the time series of output for the estimation of the output gap do not take into account that potential output is determined by the productive capacity of the economy. This implies a multivariate approach that relates the economy’s output with the available primary factors, namely the capital stock, through a production function — the second class of approaches referred above. However, the estimation of a macroeconomic production function is far from being free of difficulties. First, there are all the difficulties related with measuring the capital stock — usually measured indirectly through the accumulation of investment flows. But above all, the estimation of potential output through the estimation of a production function requires the previous calculation of the “natural” level of employment. The estimation of natural employment from observed employment using statistical smoothing techniques only transforms the original problem (i.e., the estimation of potential output) into another one, similar to the first (the estimation of the natural amount of employment). The same difficulties are met if the labour force series is used instead of employment, since the former also presents cyclical fluctuations.

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\(^{(1)}\) Hodrick and Prescott (1980) (original reference). See also King and Rebelo (1993). Given a value to smoothing parameter \(\lambda\), the estimated trend \((y')_t\) of series \(y_t\) (in logarithms) obtained through the HP filter corresponds to the solution of the following minimisation problem:

\[
\min \left\{ \sum_{t=1}^{T} (y_t - y')_t^2 + \lambda \sum_{t=1}^{T} [(y_t' - y'_{t-1}) - (y'_{t-1} - y'_{t-2})]^2 \right\}
\]

\(^{(2)}\) See for instance Stadler (1994) for an overview.

\(^{(3)}\) Hodrick and Prescott (1980) suggest that the smoothing parameter should be chosen as to translate approximately the ratio between the variance of the cyclical component and the variance of the growth rate of trend GDP. The sensitivity analysis they promote intends to quiet users of the filter, since the main descriptive statistics (the standard deviation and autocorrelation coefficients) and the impulse response function of the cyclical component remain virtually unchanged in the presence of different smoothing parameters. However, this does not mean that individual estimates for each period in the sample — specially those at its end — do not suffer some changes with changes in parameter \(\lambda\).

\(^{(4)}\) This is the so-called Nelson-Kang critique (on this issue, see for instance Cogley and Nason (1995)).
In the next Section, a mixed methodology for calculating the output gap is presented, as an alternative to the two pure approaches. This alternative builds upon the specification of a Cobb-Douglas production function, but substitutes the prior estimation of natural employment by an assumption about the behaviour of the capital-labour ratio. The proposed formulation leads to output gap estimates that do not differ widely from those obtained through the HP filter, but reveal some interesting particularities.

**3. ASSUMPTIONS OF THE PROPOSED APPROACH**

“Potential” or “natural” output is defined as the output level, which in a given period, corresponds to a normal intensity of utilisation of the primary production factors available in the economy. Assume that potential output is generated according to a Cobb-Douglas production function of two primary production factors, capital and labour, with constant returns to scale:

$$\frac{Y_t}{L_t} = \alpha \left( \frac{K_t}{L_t} \right)^\alpha$$

where $\pi$ and $\alpha$ are unknown constants, $Y_t$ stands for the (real) potential output in period $t$, $K_t$ is the (real) fixed capital stock at the beginning of period $t$ and $L_t$ is the “natural” employment level in period $t$, with $K_t$ and $L_t$ measured in efficiency units. None of these variables is directly observed.

At each moment, the comparison between the observed output level and potential output determines the so-called “output-gap”:

$$Y_t = Y_t^* e^{\pi}$$

where $Y_t$ is the observed level of output in period $t$ and $g_t$ is the output gap in logarithms (to neutralise scale effects and to facilitate algebraic rearranging). For instance, positive values for $g_t$ indicate that the economy is “overheating”, i.e. functioning above the natural output level. The values of the output gap $g_t (t = 1, \ldots, T)$ are assumed to be generated according to a zero-mean stationary stochastic process of type ARMA$(p, q)$ with gaussian innovations, $p$ and $q$ being determined as to maximise the empirical fitness of the model. Usually $g_t$ would be expected to be strongly auto-correlated, to generate the typical cyclical fluctuations associated to the output gap estimates.

To overcome the non-direct observation of the fixed capital stock measured in units of efficiency, we assume the following accumulation equation:

$$K_t' - K_{t-1}' = I_{t-1}' - \delta K_{t-1}'$$

with

$$I_t = I_t e^{\mu}$$

where $\delta$ stands for the rate of depreciation of capital and $I_t$ is the (real) flow of Gross Fixed Capital Formation (GFCF) observed during period $t$. The productive efficiency of investment is assumed to rise in time at a constant rate $\mu$; thus the productive capacity does not depend exclusively on the amount of fixed capital available, but depends also on the average age of capital.

In addition, we admit that the capital-labour ratio follows equation (5)', (with $\mu > 0$)

$$\frac{K_t}{L_t} = \frac{K_{t-1}}{L_{t-1}} e^{\mu \delta g_{t-1}}$$

Bearing in mind that $L_t'$ is the natural employment level measured in terms of efficiency, and that the capital stock is referred to the beginning of period $t$, the expression above assumes a procyclical behaviour of the capital-labour ratio in addition to an eventual “stochastic trend”. Note that if $g_t$ were white noise (which it is not, since it is auto-correlated), equation (5) would imply that the logarithm of the capital-labour ratio would follow a random walk with drift $\mu_t$, since:

$$\ln \left( \frac{K_t}{L_t} \right) - \ln \left( \frac{K_{t-1}}{L_{t-1}} \right) = \mu_t + \mu g_{t-1}$$

where “$\ln$” denotes natural logarithm.

Equation (5) completes the model and functions as an equation of definition of $L_t'$, and is necessary to the model since no reasonable proxy exists for this variable. As stressed in Section 2, the definition of $L_t'$ from the directly observed employment level (for instance, making $L_t' = L_t e^{\pi}$ with $L_t$ being the observed employment) is not a coherent alternative, since observed employment does not reflect the natural employment level in the economy. Moreover, the assumption — made by most
traditional approaches based on the estimation of a production function — that labour efficiency exhibits a deterministic exponential growth in time, independently of the investment effort in the economy (and specially the changes in time of that effort) is too strong.

Despite the improved flexibility when compared with the conventional formulation based on the production function, assumption (5) is clearly the less orthodox of all assumptions made. In fact, it implies a conceptual rupture with the utilisation of information from the labour market. In the Portuguese case this represents an advantage of the proposed approach, from the point of view of labour market data availability and quality, since employment and unemployment time series prior to 1974 are not reliable. Even in the period following to 1974, intertemporal consistency problems would still be present, due to several series breaks existing in 1983, 1992 and 1998.

One could reason that model (1) to (5) is bound to deliver output gap estimates inconsistent with the observed behaviour of labour market indicators, since it does not incorporate directly employment or unemployment statistical data. However, if this were the case, within reasonable limits, results could possibly simply indicate a change in the natural unemployment rate, instead of suggesting the weakness of the model or of the series used. Compared with some traditional approaches through the estimation of a production function, that use previously smoothed employment series assuming a constant NAIRU, the flexibility implicit in assumption (5) seems to be preferable.

4. ESTIMATION

The assumptions described above yield the following reduced form:

\[
\Delta y_{gt} + \theta_0 + \theta_3 g_{t+1} - \ln\left[ e^{-\theta_0} + e^{\theta_3} \left( e^{\theta_0} - e^{\theta_3 - \Delta y_{it}} (\theta_0 - \theta_3) - \theta_3 \right) \right] = g_t
\]

\[
A(L)g_t = B(L)u_t \quad \text{with} \quad u_t \sim n.i.d.(0, \sigma^2)
\]

where \( A(L) \) and \( B(L) \) are polynomials on the lag operator \( L \) (of order \( p \) and \( q \), respectively), \( n.i.d.(0, \sigma^2) \) means “independent and identically distributed following a normal law (and with zero mean and variance \( \sigma^2 \))”, \( \Delta y_t = \ln(Y_t) - \ln(Y_{t-1}) \), \( \Delta i_t = \ln(i_t) - \ln(i_{t-1}) \), \( \theta_0 = (1 - \alpha) \mu_0 \), \( \theta_1 = 1 + (1 - \alpha) \mu_1 \), \( \theta_2 = -\ln(1 - \delta) \) and \( \theta_3 = \lambda (5) \).

Having a sample of observations for Gross Domestic Product (GDP) and GFCF (both in real terms), fixing the values of \( p \) and \( q \) and assuming values of \( g_t \) for the beginning of the sample period, the parameters of reduced form (6A) - (6B) can be estimated through the maximum likelihood method\(^6\). This procedure also gives us the estimated series for the output gap.

Our sample encompasses annual data covering the period 1953-1998, drawn from the Historical Series for the Portuguese Economy (1997, Banco de Portugal), in its revised version, extended to the period 1953-1995\(^7\). These series were then extended to 1998 using estimates released by the Banco de Portugal in its 1997 Annual Report and in this December 1998 Economic Bulletin\(^8\).

The choice of the Historical Series for this exercise is explained by the fact that these series better ensure intertemporal consistency in measuring output and investment. The alternative, based upon the construction of series by “linking” (with change rates) the several segments available for the official national accounts since 1977 in different bases, is not sustainable. Any analysis, regardless of how superficial it may be, concludes that such segments are incompatible — e.g., just by comparing overlaps. Furthermore, there is strong evidence that the official accounts from 1977 up to the early 1990’s underestimate the output growth rates, which affects the estimates for potential output, implying the overestimation of the output gap in recent years.
Likelihood was maximised using the algorithm proposed by Berndt, Hall, Hall and Hausman, as available in econometric package RATS. Several values for \( p \) and \( q \) were tested. We concluded that the most parsimonious model that fits well the data is that with \( p=2 \) and \( q=0 \), i.e., with \( g_t \) generated according to a second order autoregressive process (AR(2)).

As referred above, the dynamic character of the model and the fact that the output gap is not observed directly imply that the values for \( g_t \) in the first years of the sample must be previously provided to the application of the maximum likelihood algorithm. For \( p=2 \) and \( q=0 \), the algorithm was given the first two years of the output gap (1953 and 1954). Estimates proved sensitive to distinct initial values for the output gap. In this context, a careful research must be carried out to ensure the quality of results. The natural criterion of choice of the initial levels for the output gap corresponded to the maximisation of the likelihood function, searching in an array of admissible values. Maximum likelihood is reached when 0.005 and 0.015 are used in 1953 and 1954, respectively.

5. RESULTS FOR THE SAMPLE 1953-1998

Table 1 exhibits the estimated parameters of the reduced form and some statistics associated to these for the estimation period 1955-1998. Parameter \( \theta_4 \) was clearly non-significant in the model estimation. Therefore, equation (3), which represents the dynamics of the stock capital in units of efficiency resumes to:

\[
K_t^* - K_{t-1}^* = I_{t-1}^*
\]  

(3’).

Note that equation (4) implicitly introduces in equation (3’) a depreciation mechanism, by making productive efficiency of capital dependent on the average age of the capital stock.

Chart 1 compares the output gap estimates obtained from applying the HP filter to the 1953-1998 sample (with smoothing parameter 100) with the estimates yielded by the proposed method. Globally, both sets of estimates do not differ substantially from each other, despite some local differences. In the last six years of the sample, for instance, the output gap estimated through the HP filter is more negative than that estimated through the proposed method, which takes into account the limiting effect of the increase in productive capacity resulting from the fall in GFCF in 1993, followed by a stagnation of this variable in 1994. In addition, the HP estimates for output gap in 1997 and 1998 evidence the quick “closing” (of a minimum of -2.1% in 1995 to -0.8 and +0.1%, respectively).

### Table 1

**THE MODEL**

\[
\Delta y_t + \theta_0 + \theta_2 g_{t+1-1}^* - \ln \left[ e^{-\theta_3} + e^{-\theta_4} \left( e^{\theta_1} + e^{\theta_2} g_{t+1-1}^* \right) \right] = g_t,
\]

\[
g_t = \beta_0 g_{t+1-2}^* + \beta_1 u_t,
\]

with \( u_t \sim n.i.d. (0, \sigma^2) \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_0 )</td>
<td>0.049294389</td>
<td>0.00133</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>1.415479418</td>
<td>0.16476</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>( \theta_3 )</td>
<td>0.039156934</td>
<td>0.00051</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>1.141854298</td>
<td>0.1097</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-0.534299733</td>
<td>0.10615</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>0.000362119</td>
<td>0.00012</td>
</tr>
</tbody>
</table>

Sample: 1955-1998

Descriptive statistics of the estimated innovations \( \tilde{u}_t \)

- Average \( m_{(1)} \): 0.00192 (p-value of the hypothesis mean = 0: 0.51)
- Asymmetry \( m_{(2)} \): -0.41854 (p-value of the hypothesis asymm. = 0: 0.27)
- “Kurtosis” \( m_{(3)} \): -0.62894 (p-value of the hypothesis Kurtosis = 0: 0.43)

- 1st order autocorr.: -0.10626 (standard deviation = 0.14586)
- 2nd order autocorr.: 0.12852 (standard deviation = 0.14750)

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In equation (3) with \( \theta_4 = -\ln(1-\delta) \), was clearly non-significant in the model estimation. Therefore, equation (3), which represents the dynamics of the stock capital in units of efficiency resumes to:

\[
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\]  

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Note that equation (4) implicitly introduces in equation (3’) a depreciation mechanism, by making productive efficiency of capital dependent on the average age of the capital stock.

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spectively in 1997 and 1998), while the proposed method suggests the output gap rose from a minimum of -1.1% in 1995 to -0.3% and -0.2% in 1997 and 1998. The recent high growth rates of investment and their expansionary effect on productive capacity — taken into account by the proposed method and ignored by the HP filter — explain this difference between estimates.

6. EXTENSION OF THE SAMPLE TO 1999

Since estimates differ for the end of the sample period, one may question what would be the behaviour of the output gap if the sample were extended to include 1999. This was carried out using the European Commission Autumn 1998 forecasts, which indicate a real output growth for Portugal of 3.4% in 1999. Two alternative scenarios are considered to assess the sensitivity of results to different output growth hypothesis: a less favourable scenery, where output grows 2.9% in 1999, and a more optimistic one, where output grows 3.9% in 1999 (virtually the same as that estimated for 1998, 4%).

Extended the three samples, the output gap estimates given by the proposed bivariate method and by the HP filter with a smoothing parameter of 100 were re-calculated. Estimates are shown in charts 2, 3 and 4. The first conclusion to be drawn, which is specific of output gap estimation methods, is that the addition of a new observation to the sample (1999 in this case) may change significantly estimates for the closest years in the sample.

In the more optimistic scenario, according to the proposed method, the strong dynamism of GFCF since 1996, with growth rates more than twice (or thrice, in 1997) of output, renders a neutral position of the economy from 1997 to 1999, with output gaps of 0.0% + 0.2% and 0.0% in 1997, 1998 and 1999, respectively (chart 4). This happens despite real output growing about 4% in these three years, according to this scenario. Note that the HP filter estimates give a quick “closing” of the output gap in 1997 and 1998, followed by a
positive value in 1999 (-1.0%, -0.2% and +0.6% respectively). Since the HP filter results do not take into account the investment effort of the economy, they indicate a potential output annual growth around only 3.2% per cent in these years, compared with almost 4.0% per cent in the proposed method.

In the central and pessimistic scenarios for 1999, output gap estimates for 1997 and 1998 are substantially revised when the proposed bivariate method is extended to include 1999. Before this extension, as referred in the previous Section, estimates were -0.3% for 1997 and -0.2% for 1998. With 1999 in the sample, the central scenario gives +0.2% and +0.6% and +0.1% respectively for 1997, 1998 and 1999, compared with +0.5%, +0.9% and +0.1 in the pessimistic scenario (charts 2 and 3). These revisions take place because activity slowdown in 1999, implicit in both scenarios, constitutes a “surprise” to the AR (2) process generating the output gap values, implying slight adjustments to the parameters estimated with the sample up to 1998.

Curiously, the 1997 and 1998 estimates of the output gap rendered by the HP filter according to the central and pessimistic scenarios for 1999 remained virtually unchanged from those reported in the previous Section: with the central scenario we obtained -0.8%, 0.0% and +0.2% for 1997, 1998 and 1999 respectively, while the corresponding figures are -0.7%, +0.1% and -0.1% according to the pessimistic one.

Therefore, independently of the calculation method, if the slowdown expected in the central and unfavourable scenarios takes place, no overheating can be expected for the Portuguese economy in 1999.

7. CONCLUSION

This article presents a mixed methodology for the calculation of the output gap, based on the specification of a Cobb-Douglas production function, in alternative to the HP filter or to the conventional estimation of a production function. It substitutes the prior estimation of natural employment by an assumption about the behaviour of the capital-labour ratio. Taking into account the behaviour of both output and GFCF, the proposed formulation presents an important conceptual advantage in relation to the HP filter. The latter does not take into account that potential output is determined by the productive capacity installed in the economy, which basically depends on the past investment effort of the economy.

Despite the conceptual advantages over the HP filter, the proposed method is not exempt of drawbacks — namely some sensitivity of estimates to the output gap values assumed for the first years in the sample. Moreover, it is also somewhat sensitive to changes in data at the end of the sample. Therefore, the available results do not allow for a precise conclusion on the current level and sign of the output gap in the Portuguese economy. Nevertheless, it seems fairly safe to conclude that the output gap was virtually null in 1997 and 1998, to which corresponds a neutral cyclical position of the Portuguese economy. Furthermore, given the deceleration of external demand and the available forecasts pointing to some slowdown of the Portuguese GDP in 1999, the risks of a significantly positive output gap in a near future are much unlikely.
REFERENCES


